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Habit Formation and the Present-Value Model of the Current Account: Yet Another Suspect

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Abstract

A recent paper claims that habit formation in consumption plays an important role in current account fluctuations in selected developed countries, extending the present-value model of the current account (PVM) with consumption habits. In this paper, however, I show that the habit-forming PVM is observationally equivalent to the PVM augmented with persistent transitory consumption, which is induced by world real interest rate shocks. Based on a small open-economy real business cycle (SOE-RBC) model endowed with consumption habits as well as world real interest rate shocks, this paper seeks effects of habit formation on current account fluctuations in a typical small open economy, Canada, by a Bayesian calibration approach. Results reveal no clear evidence that habit formation plays a crucial role in current account fluctuations.

Key Words : Current account; Habit formation; World real interest rate; Present-value model; Small open economy model; Bayesian analysis.

JEL Classification Number : $F32$, $F41$, $E32$.

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1. Introduction

A small open-economy model endowed with rational, forward-looking economic agents serves as a benchmark for studying current account dynamics in the recent literature of open-economy macroeconomics. This model, as known as the intertemporal approach to the current account or, more recently, a small open-economy real business cycle (SOE-RBC) model, stresses the consumption-smoothing behavior of economic agents in the determination of the current account in a small open economy.[1] When they expect changes in future income, forward-looking agents smooth their consumption by borrowing or lending in international financial markets and hence by generating current account movements. This role of consumption-smoothing behavior in current account determination is clearly expressed by the present-value model (PVM) of the current account, which is a closed-form solution of the canonical SOE-RBC model. For example, the PVM predicts that the current account of a small open economy moves into deficit when the economy's income is expected to decline temporarily, while no change in the current account occurs if the decline in income is expected to be permanent.[2]

Many empirical studies including Sheffrin and Woo (1990), Otto (1992), Ghosh (1995) and Bergin and Sheffrin (2000), however, fail to find empirical support for the standard PVM of the current account in postwar data of many of the G-7 economies. The cross-equation restrictions the standard PVM imposes on unrestricted vector autoregressions (VARs) are statistically rejected for all of the G-7 economies except the United States. Moreover, the forecasts of the standard PVM are too smooth to track actual current account movements. The empirical failures of the standard PVM have led some researchers to explore the role of consumption-tilting motives in current account movements: the current account might be adjusted to factors that deviate consumption away from the random-walk, permanent income level.

One way to introduce consumption-tilting motives into the standard SOE-RBC model is habit formation in consumption. Habit formation makes optimal consumption decisions of households depend not only on permanent income but also on past consumption. Habit-forming households tend to maintain their past consumption levels against unexpected shocks to permanent income; therefore, habit formation makes consumption smoother and more sluggish than in the basic permanent income hypothesis (PIH). The sluggishness of consumption in turn implies more volatile current account movements than the standard PVM predicts. Gruber (2004) augments the otherwise standard PVM with consumption habits.[3] Estimating a parameter capturing the degree of habit formation by GMM, he finds that consumption habits help improve the ability of the PVM to track actual current account movements in postwar quarterly G-7 data. He concludes that habit

[1] Obstfeld and Rogoff (1995) provide a most detailed survey of the intertemporal approach to the current account.

[2] A crucial prediction of the PVM is that only country-specific shocks matter for the current account of a small open economy. A global shock does not give a small open economy an opportunity to borrow or lend in international financial markets because all economies have identical preferences, technologies and endowments and hence react to a global shock symmetrically. For empirical tests of this prediction, see Glick and Rogoff (1995), İşcan (2000), Nason and Rogers (2002), and Kano (2005).

[3] A similar modification of the standard PVM with consumption habits is also found in Bussière et al. (2004).

formation plays an important role in current account dynamics.

Habit formation, however, is not the only source of the consumption-tilting behavior. For example, the stochastic world real interest rate introduces a consumption-tilting motive into the PVM of the current account. Expected changes in the world real interest rate tilt the optimal consumption path away from the random-walk, permanent income level and, as a result, create consumption-tilting in the PIH.[4] Blankenau, Kose and Yi (2001) and Nason and Rogers (2006, hereafter NR) provide evidence that world real interest rate shocks play a crucial role in explaining net trade balance/current account movements in small open economies. In particular, NR examine several economic factors in a canonical SOE-RBC model as "usual suspects" that might lead to the empirical failure of the standard PVM in postwar Canadian data. Among the suspects, which do not incorporate habit formation, their Bayesian Monte Carlo exercise shows that world real interest rate shocks, when combined with an internalized risk premium in international financial markets, can explain the rejection of the standard PVM in Canadian data best.

In this paper, I show that the PVM augmented with habit formation (hereafter, the habit-forming PVM) is observationally equivalent to a canonical PVM modified with an arbitrary transitory consumption component that follows an AR(1) process. The two PVMs, thus, imply the same time series of the current account. Perhaps more importantly, observational equivalence also holds between the habit-forming PVM and a PVM predicted by an SOE-RBC model with an AR(1) world real interest rate.[5] Since the two PVMs, which are derived as closed-form solutions of different small open-economy models, yield identical sample statistics, tests of the habit-forming PVM are not informative for detecting the role of habit formation in current account movements; rather, statistics of the habit-forming PVM might capture effects of world real interest rate shocks on current account fluctuations.[6]

The identification problem comes from the fact that the habit-forming PVM, as a partial equilibrium model, imposes no restrictions on the stochastic dynamics of net output growth. SOE-RBC models, on the other hand, impose restrictions on stochastic processes of both net output growth and the current account. This paper, hence, exploits restrictions of SOE-RBC models imposed on joint dynamics of net output growth and the current account to identify the role of habit formation in current account fluctuations. To do so, I add habit formation to NR's list of "usual suspects" by extending their model with consumption habits. I then investigate the extended model by a Bayesian calibration approach, which is developed by DeJong et al.(1996) and Geweke

[4]See Campbell and Mankiw (1989) for tests of the PIH, and Bergin and Sheffrin (2000) and Kano (2005) for tests of the current account PVM. In particular, Bergin and Sheffrin (2000) extend the standard PVM with stochastic variations in real interest rates as well as real exchange rates, which yield a serially-correlated transitory consumption component independent of permanent income. They observe that the extension improves the PVM's forecasts particularly in Australia and Canada.

[5]The list of other potential sources of transitory consumption shocks includes transitory government expenditure shocks affecting the utility function, real exchange rate shocks, and terms of trade shocks.

[6]In other words, tests of the habit-forming PVM potentially lead an econometrician to commit a Type II error by accepting the null hypothesis of habit formation when habit formation is a false specification of important consumption-tilting motives.

(2006). Allowing for uncertainty in structural parameters, this Monte Carlo exercise simulates SOE-RBC models endowed with habits (hereafter, the Habit model) and without habits (hereafter, the Non-Habit model) to generate theoretical probability distributions of population moments of Gruber's GMM statistics for the habit-forming PVM. Comparing theoretical distributions with the corresponding empirical posterior distributions estimated by a VAR with post-Bretton Woods Canadian data, I choose one of the two SOE-RBC models that yields theoretical distributions overlapping with the empirical counterparts to a better degree as an underlying data generating process (DGP) of the current account in Canada. Results from the Bayesian calibration exercise in this paper reveal no evidence for an important role of habit formation in current account movements: the Non-Habit model explains the post-Bretton Woods Canadian current account data better than does the Habit model.

The rest of this paper is organized as follows. Section 2 introduces the SOE-RBC model of NR augmented with habit formation, derives the habit-forming PVM as a closed-form solution, and explains its observational equivalence property. Section 3 draws Bayesian posterior inferences on the habit-forming PVM with Canadian data. Section 4 reports the results of the Bayesian Monte Carlo exercise of this paper. Section 6 concludes.

2. An SOE-RBC model with habit formation and the habit-forming PVM

2.1 An SOE-RBC model with habit formation

In this section, I introduce the SOE-RBC model this paper uses for identifying habit formation in consumption. Under particular assumptions, the habit-forming PVM is shown as a closed-form solution of the SOE-RBC model. Consider a small open-economy endowed with the representative household. The problem of the representative household, who buys or sells state non-contingent bonds in incomplete international financial markets, is to maximize expected discounted lifetime utility. Let C_t and N_t denote consumption and hours worked of the household at period t, respectively. The lifetime utility of the household then is

$$U_t = E_t \sum_{i=0}^{\infty} \beta^i \left[\phi \ln(C_{t+i} - hC_{t+i-1}) + (1-\phi)\ln(1 - N_{t+i})\right], \; 0 \le h < 1, 0 < \phi < 1, \tag{1}$$

where $\beta \in (0,1)$ and E_t are the subjective discount factor and the mathematical expectation operator conditional on the information set at period t, respectively. Equation (1) implies that the lifetime utility is separable between consumption and leisure in each period. NR observe that non-separability of utility between consumption and leisure is not crucial for explaining the standard PVM's forecasts of the Canadian current account. Following their observation, this paper assumes the logarithmic instantaneous utility function separable over consumption and leisure.[7]

[7] While Gruber exploits a quadratic instantaneous utility function, this paper specifies the instantaneous utility

More importantly, the lifetime utility function (1) implies that past consumption decreases current utility, and only current consumption over and above the habit level hC_{t-1} effectively increases utility. Current and past levels of consumption thus are complements. This fact makes the habit-forming households averse to large swings in their consumption: given $h \in (0,1)$, the optimal consumption path becomes smoother than that predicted by a model with time-separable utility. As in Constantinides (1990), habit formation is specified as being *internal*, where habits depend on the household's own consumption and the household takes habits into account when choosing the optimal amount of consumption.[8]

Let Y_t, I_t, G_t, B_t, and r_t denote output, investment, government consumption expenditure, net foreign assets (i.e., net foreign bond holdings), and the real interest rate the representative household faces at period t, respectively. The household's budget constraint is

$$B_{t+1} = (1 + r_t)B_t + Y_t - I_t - G_t - C_t. \tag{2}$$

Output Y_t is produced by a Cobb-Douglas production function

$$Y_t = K_t^{\psi}[A_t N_t]^{1-\psi} \quad 0 < \psi < 1, \tag{3}$$

where K_t and A_t are the stock of capital and the level of county-specific, labor-augmenting technology at period t. The law of motion of capital is represented by

$$K_{t+1} = (1-\delta)K_t + \left(\frac{K_t}{I_t}\right)^{\varphi} I_t, \quad 0 < \varphi, \quad 0 < \delta < 1, \tag{4}$$

where δ is the depreciation rate. Equation (4) includes capital adjustment costs with parameter φ: φ is the inverse of the price elasticity of the investment-capital ratio.

As studied by NR and Schmitt-Grohé and Uribe (2003), the stochastic real interest rate r_t is decomposed into two components. The first component q_t is the unique, exogenous stochastic rate of return common across economies around the world. In this paper, q_t follows a covariance stationary process. The second component is a risk premium specific to this small open economy. The risk premium is given as a linear function of the economy's bond-output ratio. This paper, hence, specifies the stochastic real interest rate r_t as

$$r_t = q_t - \eta \frac{B_t}{Y_t}, \quad 0 < \eta. \tag{5}$$

function by the logarithmic function. In addition to NR's observation, it is worth mentioning two reasons why this paper adopts the logarithmic utility function. First, it will be explicitly shown below that under a power utility function, the habit-forming PVM is observationally equivalent to the PVM augmented with the stochastic world real interest rate. Second, a power utility function leads to a PVM with respect to the current account-net output ratio, while a quadratic utility function results in a PVM with respect to the level of the current account. This difference is important for empirical investigation of PVMs that generally imply stationary current account series. It, however, is well known that the unit root null in the level of the current account is hard to be rejected with the standard unit root test in data of OECD economies. To the contrary, the null of a unit root in the current account-net output ratio is rejected more frequently.

[8] On the other hand, as in Abel (1990) and Campbell and Cochrane (1999), habit formation is *external* or called *catching up with the Joneses* if habits depend on aggregate consumption unaffected by any representative household decision.

Equation (5) implies that if the small open economy is a debtor (i.e. $B_t < 0$), the economy must pay a premium above q_t.[9] Furthermore, as in NR, this paper assumes that the representative household internalizes the effect of a change in the bond-output ratio on the country-specific risk premium. The result of NR strongly supports the internalized risk premium as an important mechanism for explaining the standard PVM's rejection in the Canadian current account. This is because the internalized risk premium shuts off the consumption-smoothing role of employment adjustments, which is not captured by the standard PVM.

The processes of the three exogenous impulses, G_t, A_t, and q_t, are specified as follows. Government consumption expenditure G_t is proportional to output Y_t with a stochastic, time-varying ratio $g_t = G_t/Y_t$.[10] The stochastic transitory component of government expenditure, g_t, follows an AR(1) process in the logarithmic term:

$$\ln g_t = (1-\rho_g)\ln g^* + \rho_g \ln g_{t-1} + \epsilon_t^g \quad |\rho_g| < 1, \quad \epsilon_t^g \sim i.i.d.\mathbf{N}(0, \sigma_g^2). \tag{6}$$

The log of the country-specific, labor-augmenting technology A_t is a random-walk with a drift

$$A_t = A_{t-1}\exp(\alpha + \epsilon_t^a), \quad \alpha > 0, \quad \epsilon_t^a \sim i.i.d.\mathbf{N}(0, \sigma_a^2). \tag{7}$$

Finally, the gross world real interest rate $1 + q_t$ follows an AR(1) process in the logarithmic term

$$\ln(1+q_t) = (1-\rho_q)\ln(1+q^*) + \rho_q \ln(1+q_{t-1}) + \epsilon_t^q, \quad |\rho_q| < 1, \quad \epsilon_t^q \sim i.i.d.\mathbf{N}(0, \sigma_q^2), \tag{8}$$

where q^* is the deterministic steady state value of q_t. In the following analysis, i.i.d. shocks ϵ_t^g, ϵ_t^a, and ϵ_t^q are assumed to be uncorrelated at all leads and lags.

The first-order necessary conditions for the household's problem of maximizing the life-time utility function (1) subject to the constraints (2)-(8) must be satisfied at any equilibrium of this small open economy. The transversality conditions $\lim_{i\to\infty}\beta^i E_t\{\lambda_{K,t+i}K_{t+1+i}\} = 0$ and $\lim_{i\to\infty}\beta^i E_t\{\lambda_{B,t+i}B_{t+1+i}\} = 0$ must be satisfied at any equilibrium for sufficiency, where $\lambda_{K,t}$ and $\lambda_{B,t}$ are shadow prices corresponding to the law of motion of capital (4) and the budget constraint (2), respectively. The first-order necessary conditions and transversality conditions then establish the unique equilibrium path of the small open economy.

2.2 The habit-forming PVM as a closed-form solution

The habit-forming PVM is derived as a closed-form solution of the SOE-RBC model under three assumptions. The first assumption is that the real interest rate r_t is constant to a fixed value r and there is no country-specific risk premium (i.e., $\eta = 0$ and $r = q$). The second assumption requires that labor supply is perfectly inelastic (i.e., $\phi = 1$). The third assumption is that net output NY_t, which is defined by output Y_t minus domestic investment I_t minus government expenditure

[9] The endogenous risk premium in equation (5) excludes an explosive/unit root path of international bonds in the linearized solution of the equilibrium. Moreover, it solves the well-known problem in SOE-RBC models that the deterministic steady state depends on the initial condition.

[10] For example, consider the government budget that G_t is financed by lump-sum tax T_t satisfying $T_t = g_t Y_t$.

G_t, is exogenous. Notice that given the country-specific, random-walk technology A_t as a driving force, net output NY_t follows a non-stationary process.

Under the above assumptions, the optimality conditions for the maximizing problem include the Euler equation

$$(C_t - hC_{t-1})^{-1} - \beta h E_t (C_{t+1} - hC_t)^{-1} = \beta E_t \left[(C_{t+1} - hC_t)^{-1} - \beta h E_{t+1} (C_{t+2} - hC_{t+1})^{-1} \right], \quad (9)$$

and the budget constraint $B_{t+1} = (1+r)B_t + NY_t - C_t$. As shown in the appendix of Dynan (2000), if the real interest rate is constant, Euler equation (9) can be simplified as follows:

$$(C_t - hC_{t-1})^{-1} - \beta(1+r)E_t (C_{t+1} - hC_t)^{-1} = 0. \quad (10)$$

This is because equation (9) implies that the LHS of the simplified Euler equation (10) is forward explosive.

To derive a closed-form solution of a current account measure, this paper follows the linear approximation exercise conducted by Kano (2005) and Bouakez and Kano (2006). The forward iteration of the budget constraint yields the intertemporal budget constraint

$$\sum_{i=0}^{\infty} \left(\frac{1}{1+r} \right)^i E_t C_{t+i} = (1+r)B_t + \sum_{i=0}^{\infty} \left(\frac{1}{1+r} \right)^i E_t NY_{t+i}.$$

The linear approximation exercise starts by dividing the intertemporal budget constraint by NY_t to obtain

$$\tau_t \left\{ 1 + \sum_{i=1}^{\infty} E_t \left[\sum_{j=t+1}^{t+i} \exp \left(\sum_{j=t+1}^{i} (\Delta c_j - \mu) \right) \right] \right\}$$
$$= \exp(\mu - \Delta ny_t)\, b_t + \left\{ 1 + \sum_{i=1}^{\infty} E_t \left[\sum_{j=t+1}^{t+i} \exp \left(\sum_{j=t+1}^{i} (\Delta ny_j - \mu) \right) \right] \right\},$$

where $c_t \equiv \ln C_t$, $ny_t \equiv \ln NY_t$, $\tau_t \equiv C_t/NY_t$, $b_t \equiv B_t/NY_{t-1}$, and $\mu \equiv \ln(1+r)$ are the log of consumption, the log of net output, the consumption-net output ratio, the bonds-net output ratio, and the log of the gross world real interest rate, respectively. The intertemporal budget constraint then is linearly approximated around the balanced growth path with a constant growth rate $\alpha = \Delta c_t = \Delta ny_t$. Let τ and b, respectively, denote the consumption-net output ratio and the international bonds-net output ratio on the balanced growth path. Furthermore, for any variable x_t, let x_t^* denote the deviation of x_t from its value on the balance growth path, x (i.e., $x_t^* = x_t - x$). The linearly approximated intertemporal budget constraint then is given by

$$\tau_t^* = \frac{1-\kappa}{\kappa} (b_t^* - b\Delta ny_t^*) + \sum_{i=1}^{\infty} \kappa^i E_t \Delta ny_{t+i}^* - \sum_{i=1}^{\infty} \kappa^i E_t \Delta c_{t+i}^*, \quad (11)$$

where we assume $\kappa = \exp(\alpha - \mu) < 1$.[11]

[11] The condition $\kappa < 1$ is required to satisfy boundedness of expected discounted value terms of the linearly approximated intertemporal budget constraint (11). In the following analysis, this paper imposes this condition. Intuitively, this condition implies that on the balanced growth path the economy is dynamically efficient.

In this paper, I also take a linear approximation of Euler equation (10) as follows. Notice that Euler equation (10) can be rewritten as

$$[\exp(\Delta c_t) - h]^{-1} \exp(\Delta c_t) = \beta(1+r)E_t \left[\exp(\Delta c_{t+1}) - h\right]^{-1}.$$

Assuming that $\beta(1+r) = 1$ and taking the first-order Taylor expansion of this Euler equation around the balanced growth path together yield

$$E_t \Delta c^*_{t+1} = h \Delta c^*_t. \tag{12}$$

With the linear approximation of the Euler equation (12), the intertemporal budget constraint (11) becomes

$$\tau^*_t = \frac{1-\kappa}{\kappa}\left(b^*_t - b\Delta ny^*_t\right) + \sum_{i=1}^{\infty} \kappa^i E_t \Delta ny^*_{t+i} - \frac{\tau\kappa h}{1-\kappa h}\Delta c^*_t. \tag{13}$$

This paper defines the current account CA_t conventionally by net trade plus net international interest payment: $CA_t \equiv rB_t + NY_t - C_t$. Dividing the current account identity by NY_t provides $ca_t = 1 + \frac{\exp(\mu)-1}{\exp(\Delta ny_t)} b_t - \tau_t$, where $ca_t \equiv CA_t/NY_t$ is the current account-net output ratio. Taking a linear approximation of the above equation around the balance growth path yields $ca^*_t = \left[\kappa^{-1} - \exp(-\alpha)\right](b^*_t - b\Delta ny^*_t) - \tau^*_t$. Substituting equation (13) into τ^*_t in the linearly-approximated current account identity and noting that $\exp(-\alpha)$ takes a value close to one lead to the following behavior function of the current account-net output ratio:

$$ca^*_t = -\sum_{i=1}^{\infty} \kappa^i E_t \Delta ny^*_{t+i} + \frac{\tau\kappa h}{1-\kappa h}\Delta c^*_t. \tag{14}$$

Equation (14) implies that the current account-net output ratio is determined by two factors. The first factor, which is captured by the first term of the RHS of equation (14), represents the consumption-smoothing behavior of the representative household. As in the standard PVM, this factor reflects the fact that the representative household smoothes income shocks by borrowing or lending in international financial markets. The second factor, which corresponds to the second term of the RHS in equation (14), is the consumption-tilting behavior of the representative household that is caused by habit formation. Habit formation makes the optimal consumption deviate from its smoothed level. Suppose that the growth rate of consumption rises at period t. Euler equation (12) implies that other things being equal, the household desires to keep the period $t+1$ consumption growth rate being positive. This requires that given the expected future path of net output, the household lends out in international financial markets today in order to finance the desired increase in consumption tomorrow. Therefore, the current account at period t moves into surplus.

Notice that with forecast error $v_{t+1} = \Delta c^*_{t+1} - E_t \Delta c^*_{t+1}$, Euler equation (12) can be rewritten as

$$\Delta c^*_{t+1} = h\Delta c^*_t + v_{t+1}, \tag{15}$$

where $E_t v_{t+1} = 0$. Substituting equation (15) into equation (14) provides

$$ca^*_t = -\sum_{i=1}^{\infty} \kappa^i E_t \Delta ny^*_{t+i} + \frac{\tau\kappa h}{1-\kappa h}(\Delta c^*_{t-1} + v_t).$$

Taking one-period lag of equation (14) and substituting the result back into the above equation leads to the habit-forming PVM

$$ca_t^* = hca_{t-1}^* + h\kappa\Delta ny_t^* - (1-h\kappa)\sum_{i=1}^{\infty}\kappa^i E_t\Delta ny_{t+i}^* + \epsilon_t, \quad (16)$$

where ϵ_t equals $\frac{\tau h\kappa}{1-h\kappa}\upsilon_t - h\kappa\sum_{i=0}^{\infty}\kappa^i(E_t - E_{t-1})\Delta ny_{t+i}^*$, which is serially uncorrelated and orthogonal to the date $t-1$ information set.

The habit-forming PVM of the current account-net output ratio, equation (16), corresponds to Gruber's habit-forming PVM with respect to the level of the current account (cf., equation 6 in Gruber 2004). In this case, the current account-net output ratio depends on its own past value. This makes the process of the current account more persistent than in the standard PVMs. Furthermore, the current account-net output ratio is sensitive to the current change in net output. This makes the current account-net output ratio more volatile compared to the standard PVM.

2.3 Observational Equivalence

In this paper, I claim that the habit-forming PVM (16) is observationally equivalent to a PVM augmented with an arbitrary transitory consumption component that follows an AR(1) process. To show the observational equivalence property of the habit-forming PVM (16), assume that $h = 0$ in the utility function of the representative household. Notice that in this case, the optimal consumption-net output ratio consists only of the perfectly-smoothed, permanent-income level. The linearly approximated intertemporal budget constraint (13) then turns out to be

$$\tau_{t,p}^* = \frac{1-\kappa}{\kappa}(b_t^* - b\Delta ny_t^*) + \sum_{i=1}^{\infty}\kappa^i E_t\Delta ny_{t+i}^*, \quad (17)$$

where $\tau_{t,p}^*$ is the perfectly-smoothed, permanent income level of the optimal consumption-net output ratio. Suppose that the observed consumption-net output ratio τ_t^* is decomposed into the perfectly-smoothed, permanent-income level $\tau_{t,p}^*$ and a transitory consumption component $\tau_{t,s}^*$, i.e., $\tau_t^* = \tau_{t,p}^* + \tau_{t,s}^*$.[12] Substituting the intertemporal budget constraint (17) into this decomposition and using the linearly-approximated current account identity provides

$$ca_t^* = -\tau_{t,s}^* - \sum_{i=1}^{\infty}\kappa^i E_t\Delta ny_{t+i}^*.$$

Suppose that the transitory component of the consumption-net output ratio follows an AR(1) process $\tau_{t,s}^* = \rho\tau_{t-1,s}^* + a_t$ where $0 < \rho < 1$ and a_t is serially uncorrelated and orthogonal to the information set at period $t-1$. Substituting the AR(1) process of the transitory consumption-net output ratio into the above current account-net output equation results in the following PVM:

$$ca_t^* = \rho ca_{t-1}^* + \rho\kappa\Delta ny_t^* - (1-\rho\kappa)\sum_{i=1}^{\infty}\kappa^i E_t\Delta ny_{t+i}^* + z_t, \quad (18)$$

[12] Because the underlying SOE-RBC model has the unique stochastic trend, i.e. the country-specific, permanent, technology shock, it is possible to decompose consumption into a random-walk, permanent-income component and a transitory component: see King, Plosser and Rebelo (1988).

where z_t is a serially uncorrelated disturbance such that $z_t = -a_t - \rho\kappa \sum_{i=0}^{\infty} \kappa^i (E_t - E_{t-1})\Delta ny^*_{t+i}$. Note that z_t is orthogonal to the date $t-1$ information set.

Given expectations of future changes in net output, when $h = \rho$, the habit-forming PVM (16) and the PVM with a transitory consumption component following an AR(1) process, equation (18), imply identical time-series properties of the current account-net output ratio. This fact means that the two PVMs impose the same cross-equation restrictions on an unrestricted VAR — the two PVMs, therefore, are observationally equivalent. To see this more precisely, assume that the joint dynamics of net output growth and the current account-net output ratio are well approximated by a pth-order VAR (hereafter, I also express a VAR by model $\mathcal{M}_0$): $Y_t = A_1 Y_{t-1} + A_2 Y_{t-2} + \cdots + A_p Y_{t-p} + v_t$, where Y_t is the information set that includes the first difference of the log of net output, Δny^*_t, and the current account-net output ratio ca^*_t as a part of the elements, and v_t is an i.i.d. normally distributed disturbance vector with mean zero and a symmetric positive definite variance-covariance matrix Σ. Let $\mathcal{A}$ denote a companion matrix of a first-order representation of the pth-order VAR: $\mathcal{Y}_t = \mathcal{A}\mathcal{Y}_{t-1} + u_t$ where $\mathcal{Y}_t = [Y'_t \quad Y'_{t-1} \quad \cdots \quad Y'_{t-p+1}]'$ and $u_t = [v'_t \quad 0 \quad \cdots \quad 0]'$. Suppose that Δny^*_t and ca^*_t are the ith and jth elements of vector Y_t, respectively. As shown in Appendix 1, the habit-forming PVM (16) then implies the following forecast of the current account-net output ratio, ca^f_t,

$$ca^f_t \equiv \mathcal{H}(h, \mathcal{A}; \kappa) Y_t = \{(1-h)\mathbf{e}_j + [\mathbf{e}_j - h\kappa\mathbf{e}_i + (1-h\kappa)\mathbf{e}_i \kappa\mathcal{A}(I - \kappa\mathcal{A})^{-1}]\mathcal{A}\} Y_t, \tag{19}$$

where $\mathbf{e}_i$ is the $1 \times n(p-1)$ row vector such that the ith element is one and all the other elements are zero. If the habit-forming PVM (16) holds in data, the model's forecast of the current account-net output ratio must equal the actual one, i.e., $ca^f_t = ca^*_t$. This means that under the null, row vector $\mathcal{H}(h, \mathcal{A}; \kappa)$ should contain one as the jth element and zeros as all the other elements: $\mathcal{H}(h, \mathcal{A}; \kappa) = \mathbf{e}_j$. Appendix 1 also discusses that the PVM with a transitory consumption component, equation (18), imposes cross-equation restrictions $\mathcal{H}(\rho, \mathcal{A}; \kappa) = \mathbf{e}_j$ on the unrestricted VAR. When $h = \rho$, the two PVMs, therefore, imply the same likelihood value of the VAR restricted by the identical cross-equation restrictions. In this sense, the two PVMs are observationally equivalent.

2.4 An example: the PVM with the stochastic world real interest rate

One of the most important candidates for the transitory consumption component in the literature of the current account is the stochastic world real interest rate, as emphasized by NR with their Bayesian Monte Carlo exercise based on an SOE-RBC model. In this subsection, I show that the habit-forming PVM (16) indeed is observationally equivalent to the PVM augmented with the stochastic world real interest rate, when the world real interest rate r_t follows an AR(1) process, $r_t = (1-\rho_r)r + \rho_r r_{t-1} + w_t$, where $0 < \rho < 1$ and w_t is serially uncorrelated.

As observed by Kano (2005) and Bouakez and Kano (2006), if the instantaneous utility is time separable (i.e., h=0) and the world real interest rate is allowed to vary stochastically, the current account-net output ratio has a linearly approximated closed-form solution such that

$$ca^*_t = br^*_t + \sum_{i=1}^{\infty} \kappa^i E_t r^*_{t+i} - \sum_{i=1}^{\infty} E_t \Delta ny^*_{t+i}.$$

Notice that the AR(1) process of the world real interest rate rewrites this behavior function of the current account-net output ratio as $ca^*_t = r^*_t - \sum_{i=1}^{\infty}\kappa^i E_t \Delta ny^*_{t+i}$ where $\gamma = b + \kappa\rho_r(1-\kappa\rho_r)^{-1}$. It then is straight-forward to see that applying the AR(1) process of the world real interest to this behavior function of the current account-net output ratio produces

$$ca^*_t = \rho_r ca^*_{t-1} + \rho_r\kappa\Delta ny^*_t - (1-\rho_r\kappa)\sum_{i=1}^{\infty}\kappa^i E_t\Delta ny^*_{t+i} + z^r_t, \tag{20}$$

where z^r_t is a serially uncorrelated disturbance such that $z^r_t = \gamma w_t - \rho_r\kappa\sum_{i=0}^{\infty}\kappa^i(E_t - E_{t-1})\Delta ny^*_{t+i}$. Observe the equivalence between the habit-forming PVM (16) and the PVM augmented with the stochastic world real interest rate (20).

3. Posterior inferences on the habit-forming PVM

The generalized method of moments (GMM) estimator Gruber proposes provides a consistent estimate of habit parameter h conditional on the habit-forming PVM (16). To construct the GMM estimator, define a new variable d_t by $d_t \equiv ca^*_t - \Delta ny^*_t - \kappa^{-1}ca^*_{t-1}$, where $\kappa = 0.9936$ is calculated from calibration $\alpha = 0.0033$ and $\mu = 0.0071$.[13] As shown in Appendix 2, the habit-forming PVM (16) implies that variable d_t follows a stochastic process $d_t = hd_{t-1} + \epsilon_t - \kappa^{-1}\epsilon_{t-1} + e_t$, where e_t is orthogonal to the information set at period $t-1$. This fact in turn means unconditional moment conditions $EW_{t-2}\otimes(d_t - hd_{t-1}) = 0$, where E is the mathematical unconditional expectation operator, W_{t-2} is a $p\times 1$ vector containing instrument variables in the information set at period $t-2$, and $\otimes$ is the mathematical operator of the Kronecker product. Following Gruber, I include Δny_{t-3}, ca_{t-3}, Δny_{t-4}, and ca_{t-4} in instrument vector W_{t-2}. These unconditional moment conditions make it possible to estimate habit parameter h by GMM. In particular, I use the two-step, two-stage least square (2S-2SLS) estimator applying the Newey-West (1987) covariance matrix estimator to the optimal weighting matrix in the second stage.[14] The 2S-2SLS procedure provides the point estimate of the habit parameter, $\hat{h}$, and the corresponding Hansen's (1982) J statistic $\hat{\mathcal{J}}$ for testing overidentifying restrictions. Under the null of the orthogonality of $d_t - hd_{t-1}$ to W_{t-2}, statistic $\hat{\mathcal{J}}$ is asymptotically distributed with the χ^2 function with degree of freedom $p-1$.

Given an $n\times 1$ column vector of data, Y_t, I simulate the posterior distributions of the population means of habit parameter h, J statistic $\mathcal{J}$, cross-equation restrictions $\mathcal{H}$, and forecast of the current account-net output ratio ca^f_t by using a Bayesian Monte Carlo simulation method.[15] To derive the posterior distributions of the population means, I exploit the pth-order VAR of Y_t.

[13] The balanced growth rate α is calibrated to match to the sample mean of the quarterly Canadian net output growth rate. The calibrated value of the constant world real interest rate, μ, comes from the prior mean of the constant real interest rate used in NR.

[14] The detailed description of the 2S-2SLS estimator is found in Davidson and MacKinnon (1993, p599).

[15] By focusing on the population means of statistics h, $\mathcal{J}$, ca^f_t, and $\mathcal{H}$, instead of the statistics themselves, I follow the minimal econometric interpretation of DSGE models formalized by Geweke (2006). This interpretation requires an athoretical statistical model that yields the posterior distributions of unobservable population moments of statistics of interest. I exploit a VAR as such a statistical model.

The population means of h and $\mathcal{J}$, which are respectively denoted by $\mathbf{m}_h$ and $\mathbf{m}_J$, are obtained as the mathematical means of the 2S-2SLS estimates conditional on the VAR: $\mathbf{m}_h = E(\hat{h}|\mathcal{M}_0)$ and $\mathbf{m}_J = E(\hat{\mathcal{J}}|\mathcal{M}_0)$. The population mean of vector $\mathcal{H}$, which is denoted by $\mathbf{m}_{\mathcal{H}}$, is given as the mathematical conditional mean $\mathbf{m}_{\mathcal{H}} = E(\hat{\mathcal{H}}|\mathcal{M}_0)$. Similarly, the model's forecast of the current account-net output ratio, which is denoted by $\mathbf{m}_{ca,t}$, is aquired as the mathematical mean of ca_t^f conditional on the VAR and the time t information set: $\mathbf{m}_{ca,t} = E(\hat{\mathcal{H}}|\mathcal{M}_0)Y_t$.

Let $p(\mathbf{m}_h|\mathbf{Y}, \mathcal{M}_0)$, $p(\mathbf{m}_J|\mathbf{Y}, \mathcal{M}_0)$, $p(\mathbf{m}_{\mathcal{H}}|\mathbf{Y}, \mathcal{M}_0)$, and $p(\mathbf{m}_{ca,t}|\mathbf{Y}, \mathcal{M}_0)$ denote the posterior probability densities of $\mathbf{m}_h$, $\mathbf{m}_J$, $\mathbf{m}_{\mathcal{H}}$, and $\mathbf{m}_{ca,t}$ conditional on the whole data $\mathbf{Y} = \{Y_t\}_{t=1}^T$ and the VAR $\mathcal{M}_0$. The posterior joint density of the VAR parameters is given by $p(\mathcal{A}, \Sigma|\mathbf{Y}) \propto p(\mathcal{A}, \Sigma)p(\mathbf{Y}|\mathcal{A}, \Sigma)$, where $p(\mathcal{A}, \Sigma)$ and $p(\mathbf{Y}|\mathcal{A}, \Sigma)$ are the prior joint density of the VAR parameters and the probability density of $\mathbf{Y}$ conditional on the VAR parameters (i.e., the likelihood of $\mathbf{Y}$), respectively. Notice that given the VAR, variables d_t and W_{t-2} can be drawn from the joint density $p(d_t, W_{t-2}|\mathcal{A}, \Sigma)$ by a Monte Carlo simulation. Furthermore, recall that statistics $\hat{h}$ and $\hat{\mathcal{J}}$ are functions of $\mathbf{d} = \{d_t\}_{t=1}^T$ and $\mathbf{W} = \{W_{t-2}\}_{t=3}^T$: $\hat{h} = \hat{h}(\mathbf{X})$ and $\hat{\mathcal{J}} = \hat{\mathcal{J}}(\mathbf{X})$ where $\mathbf{X} = [\mathbf{d},\ \mathbf{W}]$. This consideration then leads to the following Bayesian Monte Carlo integration to approximate the posterior densities of $\mathbf{m}_h$, $\mathbf{m}_J$, $\mathbf{m}_{\mathcal{H}}$, and $\mathbf{m}_{ca,t}$ conditional on the VAR:

$$p(\mathbf{m}_h|\mathbf{Y}, \mathcal{M}_0) = \int_\Sigma \int_\mathcal{A} \int_\mathbf{X} \hat{h}(\mathbf{X}) p(\mathbf{X}|\mathcal{A}, \Sigma) p(\mathcal{A}, \Sigma|\mathbf{Y}) \partial\mathbf{X} \partial\mathcal{A} \partial\Sigma,$$

$$p(\mathbf{m}_J|\mathbf{Y}, \mathcal{M}_0) = \int_\Sigma \int_\mathcal{A} \int_\mathbf{X} \hat{\mathcal{J}}(\mathbf{X}) p(\mathbf{X}|\mathcal{A}, \Sigma) p(\mathcal{A}, \Sigma|Y^T) \partial\mathbf{X} \partial\mathcal{A} \partial\Sigma,$$

$$p(\mathbf{m}_{\mathcal{H}}|\mathbf{Y}, \mathcal{M}_0) = \int_\Sigma \int_\mathcal{A} \int_\mathbf{X} \mathcal{H}(\hat{h}(\mathbf{X}), \mathcal{A}; \kappa) p(\mathbf{X}|\mathcal{A}, \Sigma) p(\mathcal{A}, \Sigma|Y^T) \partial\mathbf{X} \partial\mathcal{A} \partial\Sigma,$$

and

$$p(\mathbf{m}_{ca,t}|\mathbf{Y}, \mathcal{M}_0) = \left\{ \int_\Sigma \int_\mathcal{A} \int_\mathbf{X} \mathcal{H}(\hat{h}(\mathbf{X}), \mathcal{A}; \kappa) p(\mathbf{X}|\mathcal{A}, \Sigma) p(\mathcal{A}, \Sigma|Y^T) \partial\mathbf{X} \partial\mathcal{A} \partial\Sigma \right\} Y_t.$$

In this paper, I scrutinize post-Bretton Woods Canadian data spanning the period Q1:1973-Q4:2005. All data are real and seasonally adjusted at annual rates.[16] Data vector Y_t includes the first difference of the log of net output, Δny_t, and the current account-net output ratio ca_t as well as the log of the consumption-output ratio, $c_t - y_t$: $Y_t = [\Delta ny_t,\ c_t - y_t,\ ca_t]'$. This specific information set is chosen because consumption data might provide better identification of habit formation: habit formation should have strong implications on consumption dynamics. Given a prior distribution constructed by the OLS estimates, the posterior joint density of VAR(2) parameters, $p(A, \Sigma|\mathbf{Y})$, are simulated with 5,000 Markov chain Monte Carlo draws generated by a Metropolis-Hastings algorithm.[17] For each posterior draw of the VAR parameters, I simulate 100 series of vector $\mathbf{X}$ and calculate 100 sets of statistics $\hat{h}$, $\hat{\mathcal{J}}$, $\hat{\mathcal{H}}$, and $\{ca_t^f\}_{t=1}^T$. A posterior draw of the set of population means $\mathbf{m}_h$, $\mathbf{m}_J$, $\mathbf{m}_{\mathcal{H}}$, and $\{\mathbf{m}_{ca,t}\}_{t=1}^T$ is approximated by taking the average of the 100 sets of $\hat{h}$,

[16] Appendix 3 provides the detailed description of the source and construction of the data used in this paper.

[17] This paper uses John Geweke's BACC software to generate the posterior joint distribution of the VAR parameters A and Σ. The software is available at *http : //www2.cirano.qc.ca/ bacc/bacc2003/*. The second order is selected as the optimal lag length because the VAR(2) yields the highest marginal likelihood among VARs with different orders.

$\hat{\mathcal{J}}$, $\hat{\mathcal{H}}$, and $\{ca_t^f\}_{t=1}^T$. I repeat this process for the 5,000 posterior draws of the VAR parameters to construct the posterior distributions of population means, $\mathbf{m}_h$, $\mathbf{m}_J$, $\mathbf{m}_{\mathcal{H}}$ and $\{\mathbf{m}_{ca,t}\}_{t=1}^T$.

Figure 1 plots non-parametric probability density estimates of the posterior densities of the population means of the habit parameter and the J statistics, $p(\mathbf{m}_h|\mathbf{Y}, \mathcal{M}_0)$ and $p(\mathbf{m}_J|\mathbf{Y}, \mathcal{M}_0)$.[18] The posterior means of $\mathbf{m}_h$ and $\mathbf{m}_J$ are 0.708 and 17.658, which are accompanied by 90 percent Bayesian credible intervals $[0.384, 0.9576]$ and $[8.941, 30.254]$, respectively.[19] Therefore, one might conclude that the habit parameter is strictly positive and less than 1, although the overidentifying restrictions are likely to be rejected as implied by the 90 percent credible interval of $\mathbf{m}_J$, which is greater than the critical value of 7.82 for the 5 percent-size test based on the χ^2 statistic with the third degree of freedom. Nevertheless, the observation that the posterior draws of the population mean of the habit parameter are concentrated strictly within the unit interval between 0 and 1 might lead a researcher to conclude a significant role of habit formation in current account fluctuations.

Figure 2 summarizes the posterior inference on the population mean of the forecast of the current account-net output ratio, $\mathbf{m}_{ca,t}$. The left window of the figure depicts the posterior mean of $\mathbf{m}_{ca,t}$ (the green line) as well as the actual current account-net output ratio (the blue line) for the sample period. The striking observation is that the posterior mean can track the actual current account-net output ratio very precisely. The right window of Figure 2 plots the 5 and 95 percentiles of the pointwise posterior distributions of $\mathbf{m}_{ca,t}$. The reported 90 percent Bayesian credible intervals are very narrow and include the actual data points almost always. The result of the excellent forecast ability of the habit-forming PVM, however, does not necessarily implies that the posterior mean of $\mathbf{m}_{\mathcal{H}}$ in equation (19) is equal to the one under the null of the habit-forming PVM (16), i.e., $\mathbf{e}_3 = [0,\ 0,\ 1,\ 0,\ 0,\ 0]$. In fact, the posterior mean of $\mathbf{m}_{\mathcal{H}}$ is $[0.161,\ 0.450,\ 0.954,\ 0.159,\ -0.403,\ 0.139]$ with the standard deviation of $[0.073,\ 0.186,\ 0.103,\ 0.040,\ 0.090,\ 0.059]$. Thus, the cross-equation restrictions of the habit-forming PVM, particularly with respect to the second, fourth, and fifth elements of vector $\mathcal{H}$, are far from their hypothesized counterparts.

Overall, as claimed by Gruber, the above results imply that the habit-forming PVM does a fairly good job for explaining the Canadian current account fluctuations, although more stringent piecewise tests of the theoretical restrictions are against the model. It should be recalled, however, that because of the zero-power property of the statistics based on the habit-forming PVM, the good fit to the data is not necessarily attributed to habit formation — persistent world real interest rate shocks imply the identical dynamics of the current account-net output ratio, given expectations of future changes in the log of net output.

4. Identifying habit formation with SOE-RBC models

[18] Following NR, I use the normal kernel and the automatic bandwidth proposed by Silverman (1986).

[19] Gruber reports that the point estimate of the habit parameter is 0.775 with standard error 0.335, and the corresponding J statistic is 7.731 for his Canadian sample spanning the period Q2:1958-Q3:2002.

In this paper, I tackle the identification problem of the habit-forming PVM (16), using the SOE-RBC model introduced in Section 2. Notice that the habit-forming PVM says nothing on the dynamics of net output. The SOE-RBC model, on the other hand, characterizes the equilibrium joint dynamics of net output as well as the current account. Different calibrations of the SOE-RBC model lead to different stochastic processes of net output and, as a result, different expectations of future net output growth. In other words, within the SOE-RBC model, habit formation and other persistent transitory-consumption components imply distinct VARs of vector Y_t and different values of statistics $\mathbf{m}_h$, $\mathbf{m}_J$, $\mathbf{m}_{\mathcal{H}}$, and $\{\mathbf{m}_{ca,t}\}_{t=1}^{T}$, even if they share the identical cross-equation restrictions of the habit-forming PVM. This fact induces identification of the role of habit formation in current account fluctuations.

4.1 A Bayesian approach to calibration

To implement the above identification strategy, I employ a Bayesian approach to calibration, which is developed by DeJong et al.(1996), formalized by Geweke (2006), and applied to the literature of current account fluctuations by NR.[20] Given the prior distributions of the structural parameters of the SOE-RBC model, I simulate the probability distributions of population means $\mathbf{m}_h$, $\mathbf{m}_J$, $\mathbf{m}_{\mathcal{H}}$, and $\{\mathbf{m}_{ca,t}\}_{t=1}^{T}$ conditional on two models — the Habit and Non-Habit models. In the Habit model, setting the corresponding prior distribution to the Beta distribution with mean 0.710 and standard deviation 0.176, I allow habit parameter h to take a positive value always through Bayesian Monte Carlo simulations.[21] In this model, to eliminate the role of stochastic world real interest rates, I also select a very small number of $1e-10$ for both the mean and standard deviation of the inverse Gamma prior distribution of world real interest rate shocks, σ_q, as well as the mean and standard deviation of the Beta prior distribution of the AR(1) parameter of world real interest rates, ρ_q. Table 1 summarizes the prior distributions of the structural parameters of the Habit model. The Non-Habit model, on the other hand, is constructed by abstracting habit formation from the SOE-RBC model introduced in Section 2. This model is identical to that of NR. Table 2 reports the prior distributions of the Non-Habit model. Note that the two models share the identical prior distributions of the structural parameters except the habit parameter and the world real interest rate process. With respect to the structural parameters common between the two models, I choose Beta or inverse Gamma distributions, following the prior means adopted by NR and setting the prior standard deviations to make the distributions tight around the corresponding prior means.

The role of habit formation in Canadian current account movements then is identified by comparing the theoretical distributions of the population means implied by the two SOE-RBC models (denoted by $\mathcal{T}_H$s for the Habit model and $\mathcal{T}_{NH}$s for the Non-Habit model, henceforth) with the empirical counterparts estimated by the VAR (i.e., the posterior distributions reported in Section 3, which are denoted by $\mathcal{E}$s henceforth). Only if theoretical distributions implied by

[20] Exploiting a Bayesian approach to calibration, Kano and Nason (2006) investigate business cycle implications of habit formation in several closed-economy DSGE models.

[21] The specific values for the prior mean and standard deviation of the habit parameter h are selected so as to match to the empirical posterior mean and standard deviation of the population moment $\mathbf{m}_h$ estimated by the VAR.

the Habit model, $\mathcal{T}_H$s, overlap with empirical distributions $\mathcal{E}$s significantly better than theoretical distributions implied by the Non-Habit model, $\mathcal{T}_{NH}$s, do, this exercise concludes that there is a significant role of habit formation in the Canadian current account fluctuations.[22] The degree of overlapping is measured in two ways in general. Firstly, I plot non-parametric density estimates of $\mathcal{T}_H$s and $\mathcal{T}_{NH}$s and visually compare each of them with the corresponding empirical distributions $\mathcal{E}$s. Secondly, I calculate the confidence interval criterion (CIC) statistics proposed by DeJong et al. (1996). The CIC with $1-\omega$ percent confidence level counts the fraction of a theoretical distribution $\mathcal{T}_i$ for $i \in \{H, NH\}$ that exists within an interval from the lower 0.5ω quantile to the upper $1-0.5\omega$ quantile of the corresponding empirical distribution $\mathcal{E}$. The larger CIC, the better the fit of the corresponding SOE-RBC model to the actual data. Following NR, I also report the standardized difference in means (SDMs) statistics when evaluating the fits of the two models with respect to $\mathbf{m}_{\mathcal{H}}$. The SDM is a t-ratio-like statistic that is constructed by taking the difference between the means of theoretical and empirical distributions and dividing the result by the empirical standard deviation. The closer the SDM to zero, the better the fit of the model with respect to the statistical dimension of interest.[23]

Theoretical distributions implied by the two SOE-RBC models, $\mathcal{T}_i$s for $i \in \{H, NH\}$, are derived by Monte Carlo simulations as follows. Given a model, I draw a set of the model's structural parameters from the corresponding prior distributions. Conditional on the draw of the parameters, I simulate the model to generate 100 synthetic series of vector $\mathbf{X}$, which then yield 100 repetitions of statistics $\hat{h}$, $\hat{\mathcal{J}}$, $\hat{\mathcal{H}}$, and $\{ca_t^f\}_{t=1}^T$. Taking the averages over the 100 repetitions of $\hat{h}$, $\hat{\mathcal{J}}$, $\hat{\mathcal{H}}$, and $\{ca_t^f\}_{t=1}^T$ constructs a synthetic draw of population means $\mathbf{m}_h$, $\mathbf{m}_J$, $\mathbf{m}_{\mathcal{H}}$, and $\{\mathbf{m}_{ca,t}\}_{t=1}^T$. I repeat this process 5,000 times to yield theoretical distributions of the population means implied by the underlying model. Simulation of artificial series $\mathbf{X}$ is based on the state-space representation of the model, which is obtained by taking a log-linear approximation of the stochastically detrended first-order necessary conditions around a deterministic steady state and solving the resulting linear-rational expectations system by Sims's (2002) algorithm.

4.2 Results

Figure 3(a) plots the theoretical distribution of population mean $\mathbf{m}_h$ implies by the Habit model as the green line and the empirical counterpart as the blue dot-dashed line. The mean of the theoretical distribution is 0.719; therefore, the 2SLS estimator of the habit parameter is almost unbiased under the Habit model as the data generating process. The theoretical distribution overlaps the empirical counterpart significantly with the CIC of 1.10. The successful result of the Habit model with respect to $\mathbf{m}_h$ is also repeated by the Non-Habit model. Figure 3(b) depicts the empirical and theoretical distributions of population mean $\mathbf{m}_h$ for the Non-Habit model. This specification of the SOE-RBC model is also quite successful in replicating the empirical posterior

[22] Geweke (2006) formally shows that comparing two DSGE models with respect to the degree of overlapping of theoretical distributions of population moments with the empirical counterparts can be interpreted as calculating a Bayes factor between the two DSGE models where the posterior model probabilities are conditional on data $\mathbf{Y}$ as well as an empirical reference model (i.e., the VAR in this paper).

[23] NR describe the constructions of the CIC and SDM statistics in detail.

distribution. The theoretical distribution is almost symmetric with mean 0.622 and short two-side tails, most parts of which are inside the admissible range of the habit parameter between zero and one. In fact, the CIC of 0.90 formally implies that the Non-Habit model does a good job in explaining the empirical distribution of the population mean of the habit parameter.

Figures 4(a) and (b) show that the two SOE-RBC models, by and large, perform poorly in replicating the empirical distribution of the population mean of the J statistic, $\mathbf{m}_J$. Figure 4(a) plots the theoretical distribution of $\mathbf{m}_J$ implied by the Habit model and its empirical counterpart. Observe that the theoretical distribution is sharply skewed toward zero with the CIC of 0.38. The Non-Habit model, on the other hand, implies a more defused theoretical distribution of $\mathbf{m}_J$ with the CIC of 0.25, as shown in Figure 4(b). The Habit model does a better job in replicating the empirical distribution of $\mathbf{m}_J$ than does the Non-Habit model. However, the advantage of the Habit model over the Non-Habit model with respect to this statistical dimension is marginal.

Figure 5(a) reports the result with respect to the habit-forming PVM's forecast of the current account-net output ratio, $\{\mathbf{m}_{ca,t}\}_{t=1}^{T}$, for the Habit model. In this figure, the actual current account-net output ratio is represented by the blue solid line, the lower and upper bounds of the 90 percent theoretical pointwise credible intervals the green dashed lines, and the lower and upper bounds of the 90 percent empirical pointwise credible intervals the blue dotted lines, respectively. Figure 5(a) reveals that the Habit model implies the habit-forming PVM's current account forecast tracking the actual current account-net output ratio fairly well. In almost all of the periods in the sample, the 90 percent theoretical credible intervals include the actual current account-net output and overlap with the 90 percent empirical credible intervals. The similar inference is drawn from the Non-Habit model. Figure 5(b) uncovers that the Non-Habit SOE-RBC model produces the 90 percent credible intervals of the habit-forming PVM's forecast that contain the actual current account-net output ratio in all the sample periods and overlap with the 90 percent empirical credible intervals quite precisely.

An important difference between the two models, however, is observed in the volatility of $\{\mathbf{m}_{ca,t}\}_{t=1}^{T}$. Comparing Figure 5(a) with Figure 5(b) reveals that the Habit model implies more volatile current account forecasts of the habit-forming PVM than does the Non-Habit model. In fact, the standard deviation of the period-by-period means of the theoretical distributions of $\mathbf{m}_{ca,t}$ is 0.044 for the Habit model and 0.039 for the Non-Habit model. This observation is confirmed more clearly with Table 3, which summarizes the empirical and theoretical distributions of cross-equation restrictions $\mathbf{m}_{\mathcal{H}}$. The rows of the table correspond to the elements of vector $\mathcal{H}$. The second column reports the empirical posterior means of the elements of $\mathbf{m}_{\mathcal{H}}$, while the third and fourth columns of the table describe the theoretical means of the elements of $\mathbf{m}_{\mathcal{H}}$ implied by the Habit and Non-Habit models, respectively, which are accompanied by the corresponding SDM statistics. Observe that for five out of the six elements of $\mathbf{m}_{\mathcal{H}}$, the Habit model yields larger SDMs in absolute value than does the Non-Habit model. In particular, the former implies the SDMs of the first, third, and sixth elements of $\mathbf{m}_{\mathcal{H}}$ almost three times as large as those implied by the Non-Habit model in absolute value. This means that the Habit model tends to result in excess sensitivity of the current account-net output ratio to variations in the current net output growth rate and the past current account-net output ratio. The volatile current account forecast implied by the Habit model, which is exhibited in Figure 5(a), comes from this excess sensitivity. As a result, the Habit model does a worse job in replicating the habit-forming PVM's forecast of the current account-net output ratio than does the Non-Habit model.

4.2 Are small habits needed?

Finally, in this subsection, I check the robustness of the inferences drawn in this section toward changing the prior distributions of the structural parameters of the SOE-RBC model. Recall that the results of the Bayesian Monte Carlo exercises reported in the last subsection depends on the prior distributions of the structural parameters. In particular, given the better performance of the Non-Habit model in explaining the habit-forming PVM's forecast of the current account-net output ratio, it is an interesting exercise to see what will occur if a small degree of habits is introduced into the Non-Habit model. Doing this additional exercise, I can check whether or not small habits improve the fit of the SOR-RBC model of NR to the statistics of the habit-forming PVM. Specifically, in this Bayesian calibration exercise (denoted as the Small-Habit model, hereafter), I allow the habit parameter h to take a small positive value in the Non-Habit model assuming a Beta prior distribution of h with mean 0.310 and standard deviation 0.176.

The results of this exercise clearly uncover that even a small degree of habit formation deteriorates the fit of the Non-Habit model with respect to all the statistical dimensions of the habit-forming PVM. Figures 6(a), (b), and (c) plot the theoretical distributions of $\mathbf{m}_h$, $\mathbf{m}_J$, and $\mathbf{m}_{ca,t}$ implied by the Small-Habit model. As shown in Figure 6(a), the Small-Habit model yields a symmetric distribution of $\mathbf{m}_h$ with a great part of the two side tails being outside the admissible range of the habit parameter between zero and one. The CIC of the Non-Habit model falls from 0.90 to 0.71 due to the inclusion of habits. Figure 6(b) shows that habits sharply worsen the fit of the Non-Habit model with respect to $\mathbf{m}_J$: the corresponding theoretical distribution is very flat with the CIC of 0.15. Comparing Figure 6(c) with Figure 5 exhibits that the Small-Habit model has the almost same implication on the current account forecast of the habit-forming PVM as does the Habit model. The fifth column of Table 3 indeed demonstrates that the Small-Habit model produces larger SDMs of all of the cross-equation restrictions $\mathbf{m}_{\mathcal{H}}$ in absolute value than does the Non-Habit model. Therefore, consumption habits in general lead to excess sensitivity of the habit-forming PVM's forecasts of the current account.

5. Conclusion

In a recent paper, Gruber extends the standard PVM of the current account with habit formation in consumption, and claims that this feature improves the ability of the PVM to track actual current movements of selected developed economies.

In this paper, however, I argue that the habit-forming PVM is observationally equivalent to the canonical PVM augmented with persistent transitory consumption shocks, which are well represented by persistent world real interest rate shocks. This finding implies that the test statistics Gruber proposes to estimate and evaluate the habit-forming PVM are not informative for identifying the role of habit formation in current account dynamics: given the process of the net output growth rate, the alternative PVM predicts the identical time-series of the current account measure as does the habit-forming PVM. Therefore, the good forecasting ability of the habit-forming PVM could be interpreted as the results of persistent world real interest rates shocks, instead of habit formation.

This identification problem is attributed to the partial equilibrium approach of the habit-forming PVM. The habit-forming PVM imposes no restrictions on the stochastic dynamics of net output growth. In this paper, therefore, I identify the role of habit formation in current account dynamics by exploiting the restrictions SOE-RBC models impose on net output growth as well as the current account. Conducting Bayesian calibration exercises with the SOE-RBC model with and without habit formation, I find no conclusive support for a significant role habit formation plays in Canadian current account fluctuations in post-Bretton Woods periods. In fact, adding habit formation to the canonical SOE-RBC model of NR makes the model's fit to actual data much worse. Therefore, the results of this paper cast a doubt that habit formation could be a prime suspect for generating current account fluctuations in Canada.

Appendices

A.1. Deriving vector $\mathcal{H}$

Taking one-period lead of the habit-forming PVM (16) and the conditional expectation of the result yields

$$E_t ca^*_{t+1} = hca^*_t + h\kappa E_t \Delta ny^*_{t+1} - (1-h\kappa)\sum_{i=1}^{\infty}\kappa^i E_t \Delta ny^*_{t+i+1}. \tag{A.1}$$

Exploiting the first-order representation of the VAR(p) and unit vectors $\mathbf{e}_i$ and $\mathbf{e}_j$ rewrite equation (A.1) as

$$\mathbf{e}_j \mathcal{A} Y_t = hca^*_t + h\kappa \mathbf{e}_i \mathcal{A} Y_t - (1-h\kappa)\mathbf{e}_i \mathcal{A}^2 (I-\kappa\mathcal{A})^{-1} Y_t. \tag{A.2}$$

Equation (A.2) implies

$$\begin{aligned}
ca^*_t &= (1-h)ca^*_t + \mathbf{e}_j \mathcal{A} Y_t - h\kappa \mathbf{e}_i \mathcal{A} Y_t + (1-h\kappa)\mathbf{e}_i \kappa \mathcal{A}^2 (I-\kappa\mathcal{A})^{-1} Y_t \\
&= (1-h)\mathbf{e}_j Y_t + \{[\mathbf{e}_j - h\kappa \mathbf{e}_i + (1-h\kappa)\mathbf{e}_i \kappa \mathcal{A}(I-\kappa\mathcal{A})^{-1}]\mathcal{A}\} Y_t \\
&= \mathcal{H}(h, \mathcal{A}; \kappa) Y_t. \\
&\equiv ca^f_t
\end{aligned} \tag{A.3}$$

Notice that under the null of the habit-forming PVM, $ca^f_t = ca^*_t$ and $\mathcal{H}(h, \mathcal{A}; \kappa) = \mathbf{e}_j$. Moreover, observe that if the PVM with a transitory consumption component, equation (18), is the case, $\mathcal{H}(\rho, \mathcal{A}; \kappa) = \mathbf{e}_j$. Therefore, the two PVMs impose the same cross-equation restrictions on the unrestricted VAR.

A.2. Deriving the stochastic process $d_t = hd_{t-1} + \epsilon_t - \kappa^{-1}\epsilon_{t-1} + e_t$

Substituting the habit-forming PVM (16) into the definition of d_t yields

$$\begin{aligned}
d_t &\equiv ca^*_t - \Delta ny^*_t - \kappa^{-1} ca^*_{t-1} \\
&= hca^*_{t-1} + h\kappa \Delta ny^*_t - (1-h\kappa)\sum_{i=1}^{\infty}\kappa^i E_t \Delta ny^*_{t+i} + \epsilon_t - \Delta ny^*_t - \kappa^{-1} ca^*_{t-1} \\
&= -(\kappa^{-1} - h)ca^*_{t-1} - (1-h\kappa)\sum_{i=0}^{\infty}\kappa^i E_t \Delta ny^*_{t+i} + \epsilon_t \\
&= hd_{t-1} - hd_{t-1} - (\kappa^{-1} - h)ca^*_{t-1} - (1-h\kappa)\sum_{i=0}^{\infty}\kappa^i E_t \Delta ny^*_{t+i} + \epsilon_t.
\end{aligned} \tag{A.4}$$

Substituting the definition of d_{t-1} into the second term of the RHS of equation.(A.4) and using the habit-forming PVM (16) to eliminate the resulting term ca^*_{t-1} further rewrites equation.(A.4) as

$$\begin{aligned}
d_t &= hd_{t-1} - \kappa^{-1} ca^*_{t-1} + h\Delta ny^*_{t-1} + h\kappa^{-1} ca^*_{t-2} - (1-h\kappa)\sum_{i=0}^{\infty}\kappa^i E_t \Delta ny^*_{t+i} + \epsilon_t \\
&= hd_{t-1} + \epsilon_t - \kappa^{-1}\epsilon_{t-1} - (1-h\kappa)\sum_{i=0}^{\infty}\kappa^i (E_t - E_{t-1})\Delta ny^*_{t+i} \\
&= hd_{t-1} + \epsilon_t - \kappa^{-1}\epsilon_{t-1} + e_t
\end{aligned}$$

Notice that the last term of the above equation $e_t = -(1-h\kappa)\sum_{i=0}^{\infty} \kappa^i (E_t - E_{t-1})\Delta ny_{t+i}^*$ is the revision of the expectation with respect to the current and future net output growth rates between periods $t-1$ and t. Therefore, e_t should be orthogonal to the information set at period $t-1$.

A.3. Data description and construction

All the data are distributed by Statistics Canada CANSIM II (*http://www.statcan.ca/*). The current account series CA_t are constructed by net foreign interest payment plus net export. As net foreign interest payment, this paper uses *Net Investment Income from Non Residents*(v499687). The net export series are obtained by *Exports of Goods and Services* (v1992249) minus *Imports of Goods and Services* (v1992253). The net output series NY_t are given by *GDP*(v1992259) minus *Business Gross Fixed Capital Formation*(v1992238) minus *Business Investment in Inventories*(v1992245) minus *Government Current Expenditure on Goods and Services*(v1992235) minus *Government Gross Fixed Capital Formation*(v1992236) minus *Government Investment in Inventories*(v1992237) minus *Personal Expenditure on Durable Goods*(v1992230) minus *Personal Expenditure on Semi-Durable Goods*(v1992231). The series of the log of the consumption-output ratio $c_t - y_t$ are constructed by dividing the sum of *Personal Expenditure on Non-Durable*(v1992232) and *Personal Expenditure on Services* (v1992233) by *GDP* and taking the log of the result. All the series are seasonally adjusted at annual rates, divided by *Estimates of Population*(v1), and at 1997 constant prices except for *Net Investment Income from Non Residents*, which is converted to real series with the GDP deflator.

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Table 1: Prior distributions of the Habit model

Parameters	Distribution	Mean	S.D.	95 % interval
h	Beta	0.710	0.176	[0.375, 0.952]
β	Beta	0.990	0.001	[0.988, 0.992]
ϕ	Beta	0.372	0.020	[0.339, 0.405]
ψ	Beta	0.350	0.020	[0.317, 0.383]
η	Beta	0.007	0.002	[0.004, 0.011]
φ	Beta	0.050	0.010	[0.034, 0.067]
δ	Beta	0.020	0.005	[0.013, 0.029]
g^*	Beta	0.232	0.020	[0.199, 0.265]
q^*	Beta	0.007	0.001	[0.005, 0.008]
ρ_q	Beta	1e-10	1e-10	—
ρ_g	Beta	0.952	0.010	[0.934, 0.967]
σ_a	Inverse Gamma	0.012	0.010	[0.004, 0.028]
σ_q	Inverse Gamma	1e-10	—	—
σ_g	Inverse Gamma	0.012	0.010	[0.004, 0.028]

Note 1. The prior of the balanced growth rate α is degenerated at the point $\alpha = 0.0033$ which is calibrated to the sample mean of the net output growth rate.

Note 2. The inverse gamma priors are of the form $p(\sigma^2|s,v) \propto \sigma^{-(2+v)}e^{-\frac{s}{2\sigma^2}}$ where $E\sigma^2 = s/(v-2)$ and $Var(\sigma^2) = 2(E\sigma^2)^2/(v-4)$.

Note 3. The 95 percent intervals of σ_a, σ_q, and σ_g are constructed based on 10,000 Monte Carlo repetitions.

Table 2: Prior distributions of the Non-Habit model

Parameters	Distribution	Mean	S.D.	95 % interval
h	—	—	—	—
β	Beta	0.990	0.001	[0.988, 0.992]
ϕ	Beta	0.372	0.020	[0.339, 0.405]
ψ	Beta	0.350	0.020	[0.317, 0.383]
η	Beta	0.007	0.002	[0.004, 0.011]
φ	Beta	0.050	0.010	[0.034, 0.067]
δ	Beta	0.020	0.005	[0.013, 0.029]
g^*	Beta	0.232	0.020	[0.199, 0.265]
q^*	Beta	0.007	0.001	[0.005, 0.008]
ρ_q	Beta	0.907	0.050	[0.813, 0.973]
ρ_g	Beta	0.952	0.010	[0.934, 0.967]
σ_a	Inverse Gamma	0.012	0.010	[0.004, 0.028]
σ_q	Inverse Gamma	0.004	0.010	[0.000, 0.011]
σ_g	Inverse Gamma	0.012	0.010	[0.004, 0.028]

Note 1. The Non-Habit model is constructed by abstracting the habit parameter from the SOE-RBC model introduced in Section 2.

Table 3: Empirical and theoretical distributions of $\mathcal{H}$

	Empirical	Habit	Non-Habit	Small Habit
$\mathcal{H}_1$	0.161	-0.343 (-6.941)	-0.017 (-2.454)	-0.112 (-3.757)
$\mathcal{H}_2$	0.450	-0.178 (-3.370)	-0.021 (-2.527)	-0.029 (-2.571)
$\mathcal{H}_3$	0.954	1.334 (3.694)	1.005 (0.496)	1.101 (1.434)
$\mathcal{H}_4$	0.159	0.005 (-3.827)	0.001 (-3.929)	0.000 (-3.951)
$\mathcal{H}_5$	-0.403	-0.036 (4.090)	-0.041 (4.039)	-0.001 (4.483)
$\mathcal{H}_6$	0.139	-0.303 (-7.512)	-0.027 (-2.827)	-0.116 (-4.345)

Note 1. The numbers in parentheses are the standardized difference of means (SDM) measures proposed by DeJong et al.(1996) and Nason and Rogers (2006). The closer the SDM to zero, the better the fit of the model.

Figure 1: Posterior Distributions of $\mathbf{m}_h$ and $\mathbf{m}_J$

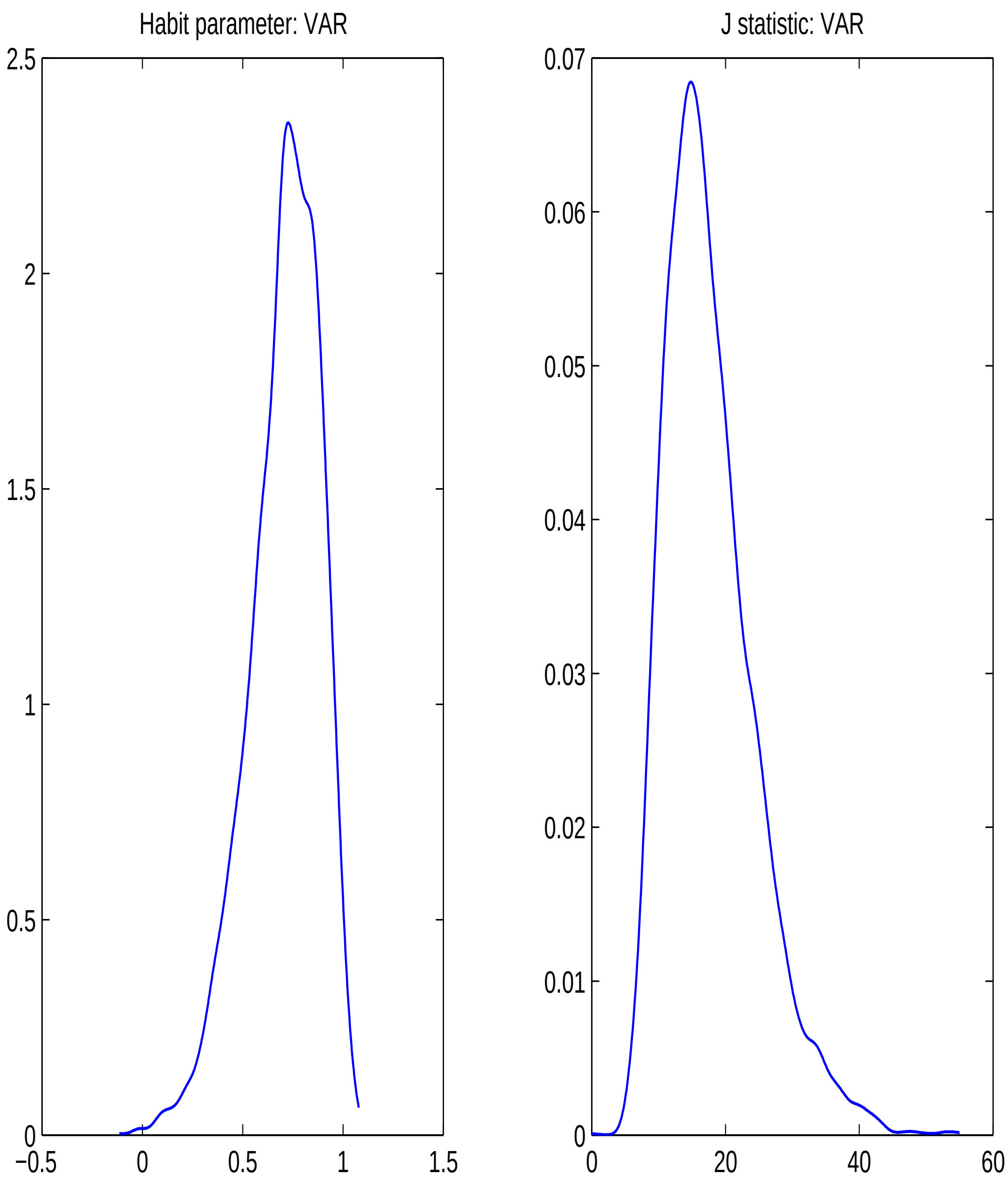

Figure 2: Posterior Distributions of $\mathbf{m}_{ca,t}$

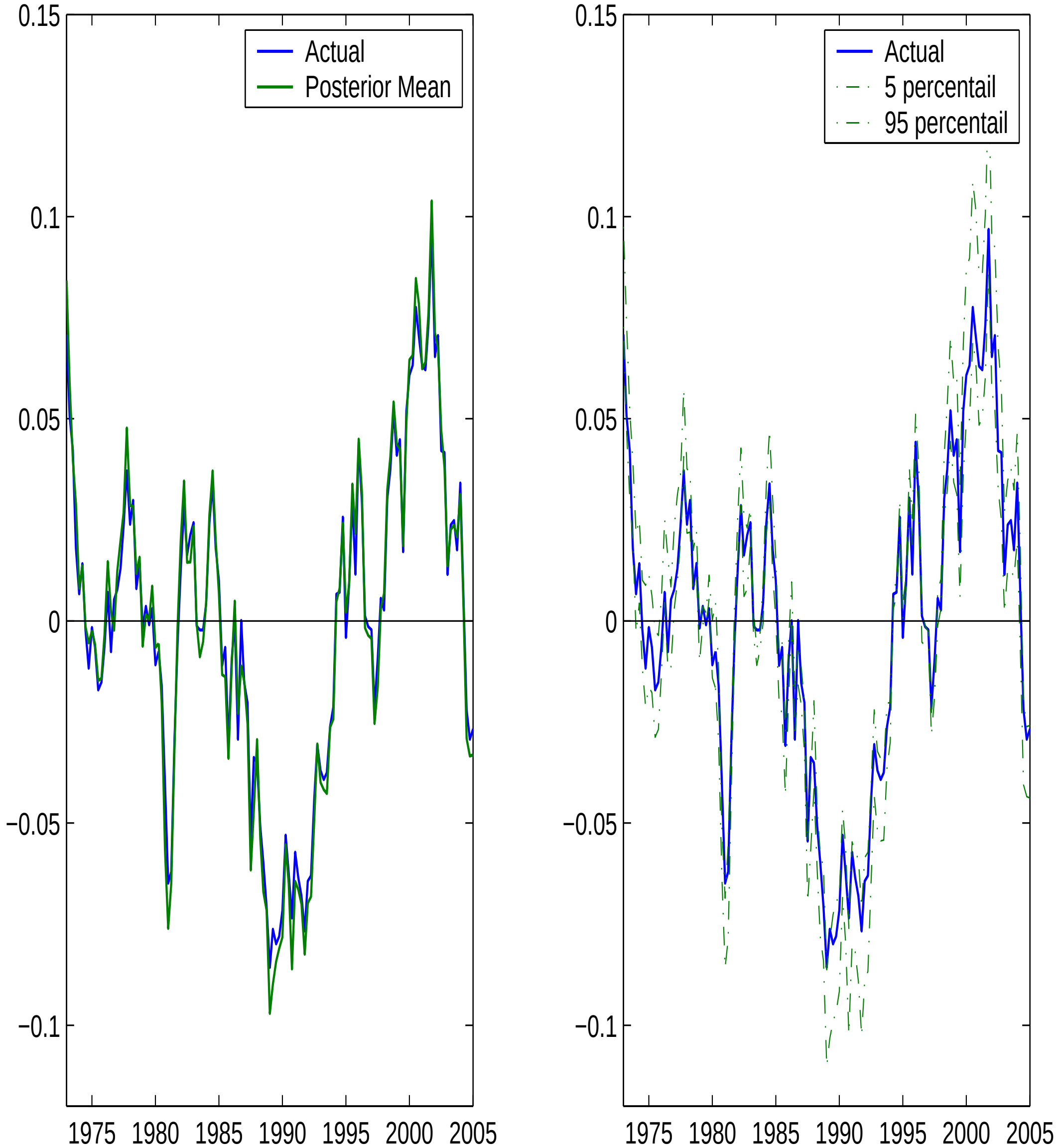

Figure 3: Empirical and Theoretical Distributions of $\mathbf{m}_h$

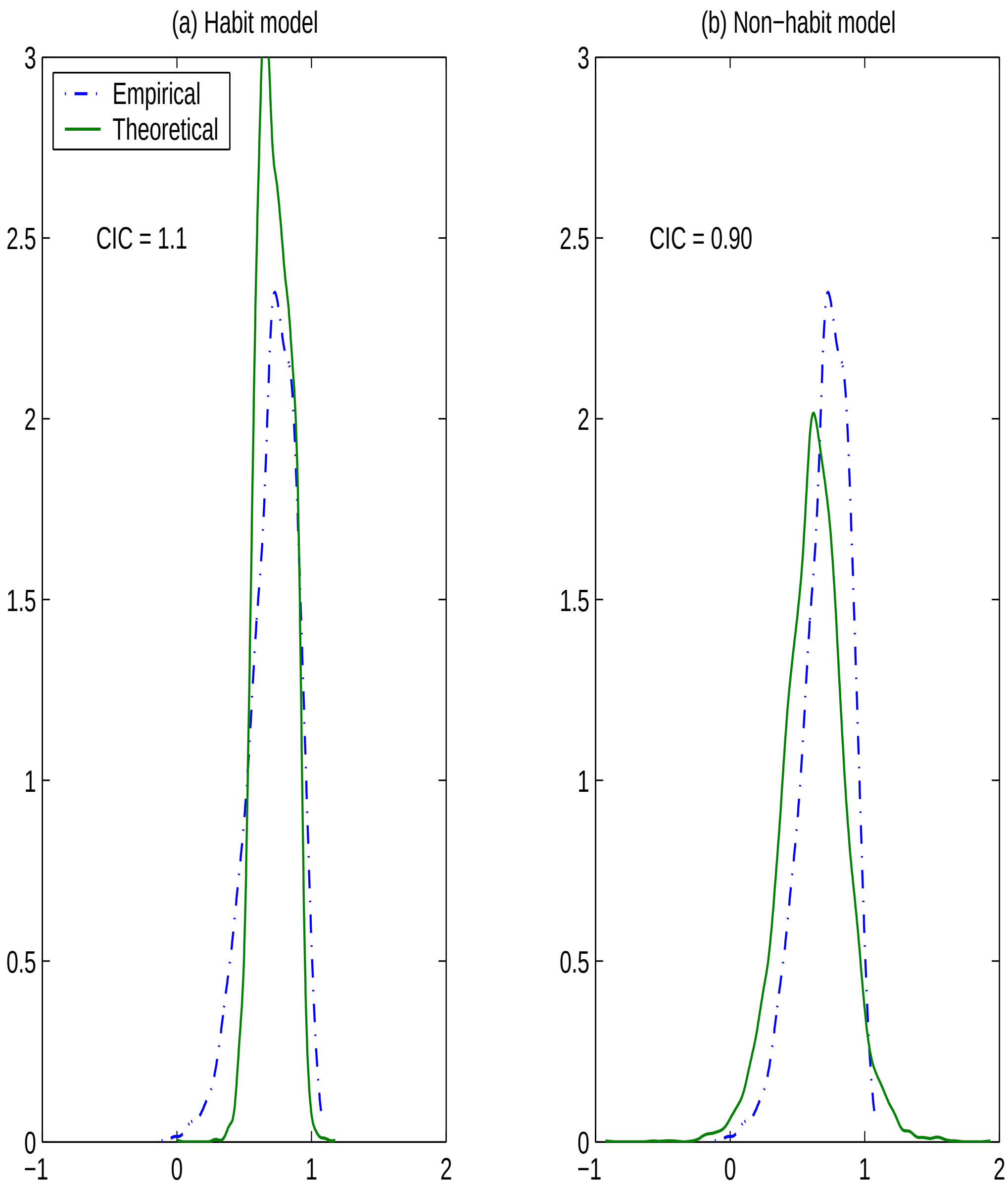

Figure 4: Empirical and Theoretical Distributions of $\mathbf{m}_J$

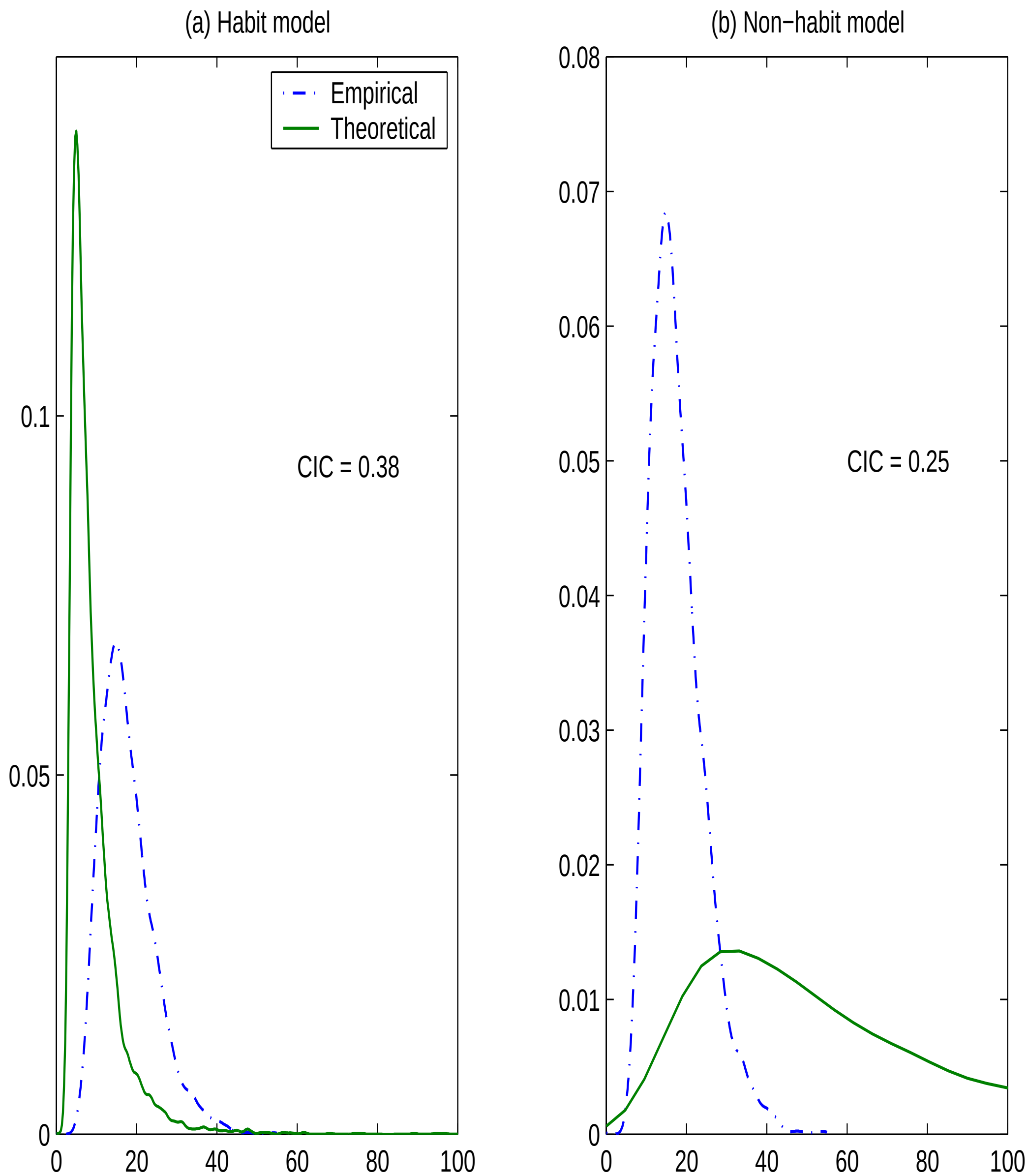

Figure 5: Empirical and Theoretical Distributions of $\mathbf{m}_{ca,t}$

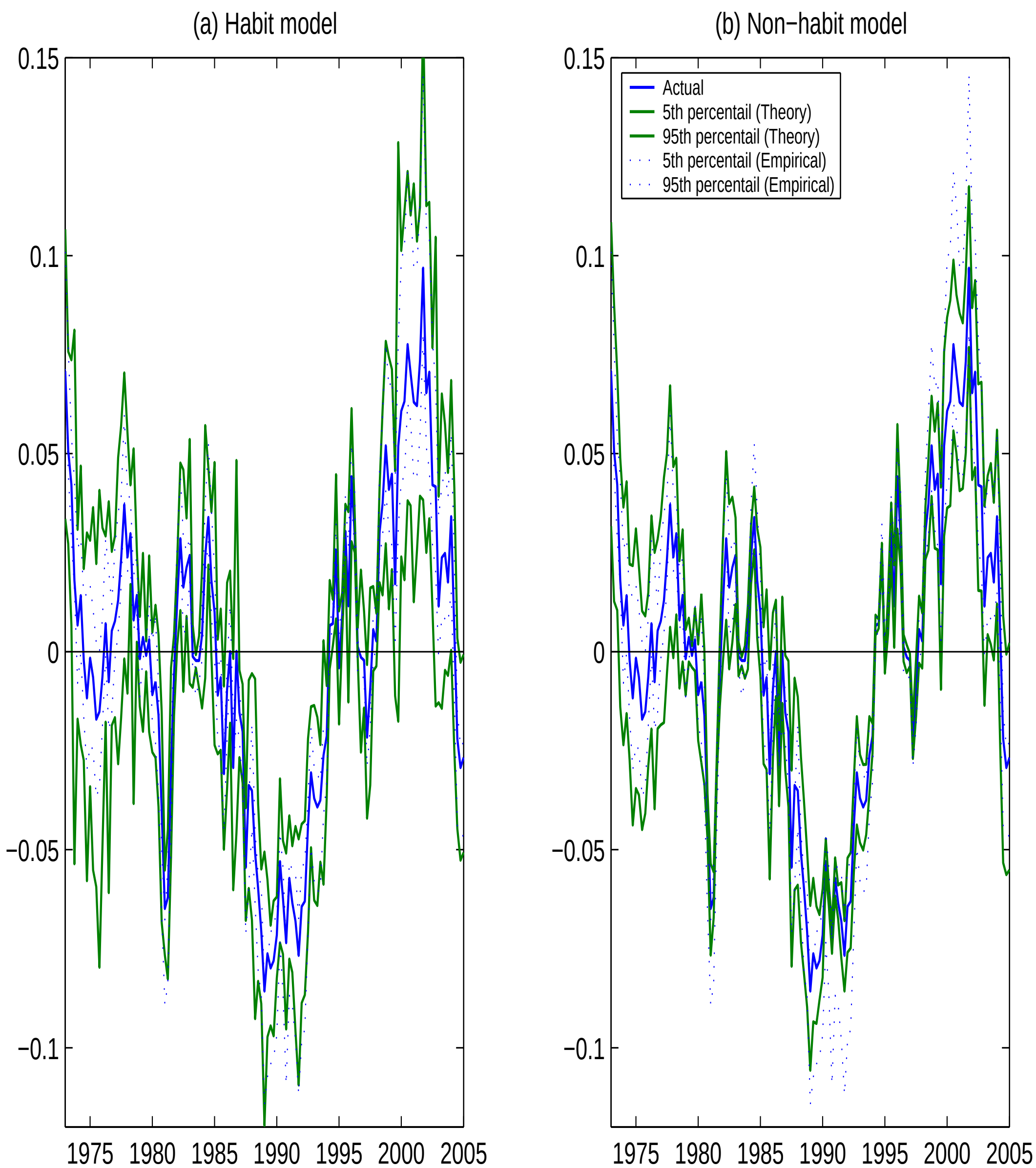

Figure 6: Empirical and Theoretical Distributions of $\mathbf{m}_h$, $\mathbf{m}_J$ and $\mathbf{m}_{ca,t}$: Small Habit model