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Abstract

This paper investigates unique implementation in large economies with incomplete information and interdependent values; we degenerate the common knowledge assumptions and assume that a central planner is unaware of the specifications of an environment. With a minor restriction on the class of environments, we demonstrate that there exists a detail-free mechanism that virtually implements competitive allocations with complete information in twice iterative dominance, irrespective of how the environment is specified.

Keywords: Large Economies, Interdependent Values, Unique Implementation, Detail-Free Mechanisms, Twice Iterative Dominance.

Journal of Economic Literature Classification Numbers: C70, D60, D78, D82

1. Introduction

This paper investigates the *unique* implementation problem in economic environments with incomplete information and *interdependent values*, wherein a sufficiently large number of players exist with single-unit supplies and demands. Each player receives her respective private signals that are correlated with each other through some *unobservable* macro shock. Each player's value for the commodity depends not only on her private signal but also on this macro shock, which implies interdependent values. The central planner

attempts to design a well-behaved mechanism in that, irrespective of the macro shock that occurs, every strategy profile that satisfies a defined equilibrium concept can induce a competitive allocation with complete information at least approximately and almost certainly.

The standard approach in the implementation literature with incomplete information has assumed that the central planner has complete knowledge of the specifications of the environments, such as the payoff and signal structures, and tailors a mechanism for the finer details of these specifications. See Jackson (1991), Palfrey (1992), Matsushima (1990, 1993), Abreu and Matsushima (1992, 1994), and Maskin and Sjöström (2002). Equilibrium concepts such as the Nash equilibrium and iterative dominance have generally been used; these concepts rely on the strong rationality assumption that it is not only mutual knowledge but also *common knowledge* for the players to avoid playing dominated strategies. These assumptions are regarded as the drawbacks of the standard implementation problem and have been criticized from the practical and experimental viewpoints. In fact, many authors studying auction design from the practical standpoint have confined their attention to auction protocols that are *detail-free*, i.e., independent of the finer details of the specifications. See Krishna (2002), Milgrom (2004), and Klemperer (2004). Several experimental studies have reported that subjects in laboratory experiments like guessing games made a maximum of two or three iterative removals of dominated strategies. For instance, see Camerer (2003, Chapter 5).

Based on the above viewpoints, this paper considers the situation in which the central planner is unaware of the specifications of the environment. From the practical viewpoint,

we consider the central planner who, instead of tailoring a mechanism based on the specifications, attempts to design a detail-free mechanism that implements the desired allocations irrespective of how the environment is specified.[1] Further, based on the experimental viewpoint, we use the equilibrium concept of *twice* iterative dominance, which requires players to eliminate dominated strategies only two times iteratively. Twice iterative dominance is based solely on the weak rationality assumption that it is *not* common but mutual knowledge among the players that they do not play dominated strategies. The main theorem of this paper is permissive, which reveals that with a minor restriction on the class of possible environments, there exists a detail-free mechanism that can virtually implement competitive allocations with complete information in twice iterative dominance, irrespective of how the environment is specified.

The constructed mechanism describes the following three-stage procedure. The central planner divides the players into a sufficient number of distinct groups, each of which includes an adequate number of sellers and buyers. The central planner then requires each player to announce *three price bids* as follows. In stage 1, each player announces the first bid. After the first bid announcement, she observes the first bid announcements in the preceding group. In stage 2, she announces the second bid. After the second bid announcement, she observes the first bid announcements in all groups other than her own.

[1] The detail-free concept in this paper implies that the mechanisms are independent of the fine details of value functions, signal spaces, and probability functions. The mechanisms, however, depend on several model settings, such as the finiteness of the signal spaces, single unit demands and supplies, and the numbers of sellers and buyers.

Finally, in stage 3, she announces the third bid.

In order to determine the allocation, the central planner randomly uses two rules: the *random fixed price rule* and the *price-taking rule*. With a positive but very low probability, the central planner uses the random fixed price rule, according to which she selects a trading price randomly and independent of the players' announcements. The random fixed price rule incentivizes the players to announce their expected values honestly. In particular, their first bids completely reveal their private signals. Further, on account of the law of large numbers, their second and third bids can approximate their true values associated with the private signals and the unobservable macro shock.

With a very high probability, the central planner uses the price-taking rule instead of the random fixed price rule. The central planner balances the transfers within each group. For each group, she calculates the market-clearing price that equalizes the supplies and demands associated with the second bids within that group. In order to conduct the transfers within each group, the central planner sets the trading price for this group such that it is equal to the market-clearing price for its *preceding* group and accords the priority to players whose third bids are greater than this price. Since the trading price for each group is determined independent of the announcements in this group, and the signals possessed by the other players in this group are relatively uninformative with respect to all the signals possessed by the players in the other groups, it follows that all players in this group are willing to adopt price-taking behaviors in the virtual sense for their third bid announcements. Hence, they have the incentive to almost honestly announce their conditional expected values that approximate their true values.

The standard analysis of competitive markets with interdependent values has assumed the form of a non-strategic price taker and investigated a rational expectation equilibrium; for instance, see Mas-Colell, Whinston, and Green (1995, Chapter 19). Each trader updates her belief based on the market price and the forecast function that maps private signal profiles to trading prices, and then maximizes the updated expected utility as a non-strategic price taker. In order to demonstrate a strategic foundation for a rational expectation equilibrium, Reny and Perry (2006) investigated the standard model of a large double auction and established the existence of a Bayesian Nash equilibrium that resembles the rational expectation equilibrium. Also, see McLean, Peck, and Postlewaite (2005). These studies have not considered the issue of uniqueness, which is the central theme of the implementation theory. Moreover, since the trading price for each player is determined independent of her price bids, we do not require any price grid device to restrict the players' price manipulations as employed by Reny and Perry (2006).

In the implementation literature, Matsushima (2006a) investigated the possibility that a detail-free mechanism implements social choice functions in general environments. Matsushima (2006a) assumed that each player has an intrinsic preference for honest reporting. This paper focuses on players who are motivated only by their material interests.

Another paper by Matsushima (2006b) investigated implementation possibilities of efficient allocations on the assumption that players can conduct a maximum of three iterative removals of dominated strategies. Matsushima (2006b) used mechanisms that are not detail-free and therefore depend on the finer details of the specifications.

Bergemann and Morris (2005a, 2005b) studied the robustness of mechanism design on

the assumption that the environment is not common knowledge among the players and the central planner. In contrast, the present paper permits the common knowledge assumption among the players with respect to the environment and focuses on the case in which the central planner is unaware of the environment specifications.

The remainder of this paper is organized as follows. Section 2 presents definitions and notations. Section 3 demonstrates the main theorem. Section 4 presents a brief sketch of the proof of the main theorem. Section 5 presents the full proof.

2. The Model

Let n and r denote two positive integers. Let $N = \{1,...,nr\} \bigcup \{nr+1,...,2nr\}$ denote the set of players, where the first nr players are regarded as the *sellers* and the last nr players are regarded as the *buyers*. We assume that both n and r are sufficiently large. Each seller possesses one unit of the commodity, and therefore, the total amount of the commodity equals nr. Let $A \subset \{0,1\}^{2nr}$ denote the set of alternatives, where $a \equiv (a_i)_{i \in N} \in \{0,1\}^{2nr}$ is included in A if and only if $\sum_{i \in N} a_i = nr$, and $a_i = 1$ ($a_i = 0$) implies that player i consumes one (zero) unit of the commodity. Let $\Delta(A)$ denote the set of lotteries over alternatives. A *mechanism* is defined by $G = (n, r, M, g, x)$, where $M = \prod_{i \in N} M_i$, M_i is the set of messages for player $i \in N$, $m_i \in M_i$, $m = (m_i)_{i \in N} \in M$, $g : M \to \Delta(A)$, $x = (x_i)_{i \in N}$, $x_i : M \to R$, and $x_i(m)$ implies the monetary transfer to

player i. We focus on mechanisms that are *budget-balancing* in $\sum_{i \in N} x_i(m) = 0$ for all $m \in M$. The central planner chooses any alternative $a \in A$ with probability $g(m)(a) \in [0,1]$.

Each player observes a *private signal,* ω_i. Let Ω_i denote the finite set of player i's private signals, where we assume that $\Omega_i = \Omega_1$ for all $i \in N$. Let $\omega = (\omega_i)_{i \in N}$ denote a private signal profile, and let $\Omega = \underset{i \in N}{\times} \Omega_i$ denote a set of private signal profiles. Let $\omega_{-i} = (\omega_j)_{j \in N \setminus \{i\}}$ and $\Omega_{-i} = \underset{j \in N \setminus \{i\}}{\times} \Omega_j$. We assume that there exists a *macro shock* ω_0 that is *unobservable* to the players. Let Ω_0 denote the finite set of macro shocks. A combination of the private signal profile and the macro shock (ω_0, ω) is randomly determined according to the probability function $f : \Omega_0 \times \Omega \to (0,1]$. We assume *conditional independence* in that there exist $f_0 : \Omega_0 \to (0,1]$ and $f_1(\cdot \mid \omega_0) : \Omega_1 \to (0,1]$ for each $\omega_0 \in \Omega_0$ such that

$$f(\omega_0, \omega) = f_0(\omega_0) \prod_{i \in N} f_1(\omega_i \mid \omega_0).$$ [2]

[2] Reny and Perry (2006) assumed the conditional independence in the same manner as in this paper, where each player's private signal is determined according to the identical conditional distribution function. However, we can relax this assumption by permitting the sellers' private signals and the buyers' private signals to be determined according to the different conditional distribution functions. The paper by Fudenberg, Mobius, and Szeidl (2007) investigated large double auctions on this

The *value function* is defined by $v:\Omega_0 \times \Omega_1 \to (0,1]$.[3] Each player $i's$ value for one unit of the commodity depends on the macro shock $\omega_0 \in \Omega_0$ as well as her private signal $\omega_i \in \Omega_i$, and is given by $v(\omega_0, \omega_i) \in (0,1]$. We assume quasi-linearity and risk neutrality in that the player $i's$ utility is given by $a_i v(\omega_0, \omega_i) + t_i$, where $t_i \in R$ is the monetary transfer to player i.

A combination $e \equiv (\Omega_0, \Omega_1, f_0, f_1, v)$ defines an *environment*. We use E to denote a set of environments that satisfies the following three assumptions, which are generically very weak restrictions.

Assumption 1: For every $\omega_0 \in \Omega_0$ and every $\omega_0' \in \Omega_0 \setminus \{\omega_0\}$,

$$f_1(\cdot \mid \omega_0) \neq f_1(\cdot \mid \omega_0').$$

Assumption 1 implies that the different macro shocks correspond to the different probability distributions of the private signal.

Assumption 2: For every $\omega_0 \in \Omega_0$ and every $v \in (0,1]$,

$$\sum_{\omega_1 \in \Omega_1 : v(\omega_0, \omega_1) \geq v} f_1(\omega_1 \mid \omega_0) \neq \frac{1}{2}.$$

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relaxed conditional independence assumption; however, this paper focused only on the private value cases.

[3] The mapping into the interval $[0,1]$ simply implies that the payoffs are uniformly bounded.

Since Ω_0 and Ω_1 are finite sets, it holds generically in the class of conditional distribution functions f_1 that for every $\omega_0 \in \Omega_0$, there exists no subset $\Gamma \subset \Omega_1$ such that $\sum_{\omega_1 \in \Gamma} f_1(\omega_1 \mid \omega_0) = \frac{1}{2}$, which automatically implies Assumption 2.

Let

$$v(\omega_i) \equiv \sum_{\omega_0 \in \Omega_0} v(\omega_0, \omega_i) \tilde{f}(\omega_0 \mid \omega_i)$$

denote the expected value conditional on the private signal $\omega_i \in \Omega_i$, where

$$\tilde{f}(\omega_0 \mid \omega_i) \equiv \frac{f_1(\omega_i \mid \omega_0) f_0(\omega_0)}{\sum_{\omega_0' \in \Omega_0} f_1(\omega_i \mid \omega_0') f_0(\omega_0')}$$

is the probability of the macro shock ω_0 occurring conditional on the private signal $\omega_i \in \Omega_i$.

Assumption 3: For every $\omega_1 \in \Omega_1$ and every $\omega_1' \in \Omega_1 \setminus \{\omega_1\}$,

$$v(\omega_1) \neq v(\omega_1').$$

Since Ω_1 is a finite set, it holds generically in the class of environments that the different private signals correspond to the different expected values, which is exactly the same as what Assumption 3 implies.

A *strategy* for player $i \in N$ is denoted by $s_i : \Omega_i \to M_i$. Let S_i denote the set of

strategies for player i. Let $S \equiv \underset{i \in N}{\times} S_i$, $s = (s_i)_{i \in N} \in S$, and $s(\omega) = (s_i(\omega_i))_{i \in N} \in M$. The expected utility for player i when the players play a strategy profile $s \in S$ is denoted by

$$u_i(s) \equiv \sum_{(\omega_0, \omega) \in \Omega_0 \times \Omega} \{\sum_{a \in A} a_i v(\omega_0, \omega_i) g(s(\omega))(a) + x_i(s(\omega))\} f(\omega_0, \omega).$$

A strategy $s_i \in S_i$ for player i is said to be *undominated* in (G, e) if there exists no $s_i' \in S_i$ such that

$$u_i(s_i', s_{-i}) > u_i(s) \text{ for all } s_{-i} \in S_{-i}.$$

Let $S_i^1 \subset S_i$ denote the set of undominated strategies for player i in (G, e). A strategy $s_i \in S_i$ for player i is said to be *twice iteratively undominated* in (G, e) if $s_i \in S_i^1$ and there exists no $s_i' \in S_i^1$ such that

$$u_i(s_i', s_{-i}) > u_i(s) \text{ for all } s_{-i} \in S_{-i}^1.$$

For every environment $e \in E$ and every macro shock $\omega_0 \in \Omega_0$, we define $p^*(\omega_0, e) \in [0,1]$ by

$$\sum_{\omega_1 \in \Omega_1 : v(\omega_0, \omega_1) < p^*(\omega_0, e)} f_1(\omega_1 \mid \omega_0) < \frac{1}{2} \text{ and } \sum_{\omega_1 \in \Omega_1 : v(\omega_0, \omega_1) > p^*(\omega_0, e)} f_1(\omega_1 \mid \omega_0) < \frac{1}{2},$$

where we used Assumption 2 to guarantee its existence.[4] Since the total amount of the

[4] Note from Assumption 2 that for each $\omega_0 \in \Omega_0$, there exists $\omega_1 \in \Omega_1$ such that $v(\omega_0, \omega_1) = p^*(\omega_0, e)$. This ensures that with sufficient players, it is almost certain that the trading price is set equal to $p^*(\omega_0, e)$ in the mechanisms constructed in this paper. With the exception of this point, Assumption 2 is not considerably relevant to the main theorem of this paper.

commodity nr is merely half the number of the players $2nr$, we can regard $p^*(\omega_0, e)$ as the *competitive price at the macro shock* $\omega_0 \in \Omega_0$ in the environment $e \in E$ when the number of players is sufficiently large. Note that the proportion of the players who observe the private signal ω_1 is approximated by $f_1(\omega_1 \mid \omega_0)$. Hence, the first inequality implies that less than a half of the players are willing to sell the commodity for the price $p^*(\omega_0, e)$. The second inequality implies that less than a half of the players are willing to buy the commodity for $p^*(\omega_0, e)$. The rest of the players are indifferent to buying the commodity for $p^*(\omega_0, e)$. This implies that $p^*(\omega_0, e)$ equalizes the demand and supply.

3. The Theorem

Fix an infinite sequence $(n^{(l)}, r^{(l)})_{l=1}^{\infty}$ arbitrarily, where $n^{(l)}$ and $r^{(l)}$ are increasing with respect to l, and $\lim_{l\to\infty}(n^{(l)}, r^{(l)}) = (\infty, \infty)$. An infinite sequence of mechanisms is denoted by $(G^{(l)})_{l=1}^{\infty}$, where $G^{(l)} = (n, r, M, g, x)$ and $(n, r) = (n^{(l)}, r^{(l)})$ for all $l \geq 1$.

The Theorem: *There exists an infinite sequence of mechanisms* $(G^{(l)})_{l=1}^{\infty}$ *such that for every* $e \in E$, *every* $\varepsilon > 0$, *and every sufficiently large* $l \geq 1$, *any twice iteratively undominated strategy profile* $s \in S$ *in* $(G^{(l)}, e)$ *satisfies the following properties. Fix* $i \in N$ *and* $(\omega_0, \omega_i) \in \Omega_0 \times \Omega_i$ *arbitrarily.*

(i) *Suppose* $v(\omega_0, \omega_i) > p^*(\omega_0, e)$ *; then,*

(1) $$\sum_{\omega_{-i} \in \Omega_{-i}} \sum_{a \in A} a_i g(s(\omega))(a) f_{-i}(\omega_{-i} \mid \omega_0) \geq 1 - \varepsilon ,$$

(2) $$\sum_{\omega_{-i} \in \Omega_{-i} : |x_i(s(\omega))| \leq \varepsilon} f_{-i}(\omega_{-i} \mid \omega_0) \geq 1 - \varepsilon \qquad \text{if } i \in \{1, ..., nr\},$$

and

(3) $$\sum_{\omega_{-i} \in \Omega_{-i} : |x_i(s(\omega)) + p^*(\omega_0, e)| \leq \varepsilon} f_{-i}(\omega_{-i} \mid \omega_0) \geq 1 - \varepsilon \qquad \text{if } i \in \{nr+1, ..., 2nr\}.$$

(ii) *Suppose* $v(\omega_0, \omega_i) < p^*(\omega_0, e)$ *; then,*

(4) $$\sum_{\omega_{-i} \in \Omega_{-i}} \sum_{a \in A} a_i g(s(\omega))(a) f_{-i}(\omega_{-i} \mid \omega_0) \leq \varepsilon ,$$

(5) $$\sum_{\omega_{-i} \in \Omega_{-i} : |x_i(s(\omega)) - p^*(\omega_0, e)| \leq \varepsilon} f_{-i}(\omega_{-i} \mid \omega_0) \geq 1 - \varepsilon \qquad \text{if } i \in \{1, ..., nr\},$$

and

(6) $$\sum_{\omega_{-i} \in \Omega_{-i} : |x_i(s(\omega))| \leq \varepsilon} f_{-i}(\omega_{-i} \mid \omega_0) \geq 1 - \varepsilon \qquad \text{if } i \in \{nr+1, ..., 2nr\},$$

where $f_{-i}(\omega_{-i} \mid \omega_0) \equiv \prod_{j \in N \setminus \{i\}} f_1(\omega_j \mid \omega_0)$ *denotes the probability of* $\omega_{-i} \in \Omega_{-i}$ *occurring conditional on* $\omega_0 \in \Omega_0$.

The left-hand side of inequalities (1) and (4) represents the probability of player i consuming one unit of the commodity at the macro shock ω_0. Inequality (1) implies that any player $i \in N$ almost certainly consumes the commodity if $v(\omega_0, \omega_i) > p^*(\omega_0, e)$. Inequality (4) implies that any player $i \in N$ rarely consumes the commodity if

$v(\omega_0, \omega_i) < p^*(\omega_0, e)$.

The left-hand side of inequalities (2) and (6) represents the probability that the monetary transfer to player i is close to zero. Inequality (2) implies that it is almost certain that the monetary transfer to any seller $i \in \{1,...,nr\}$ is close to zero if $v(\omega_0, \omega_i) > p^*(\omega_0, e)$. Inequality (6) implies that it is almost certain that the monetary transfer to any buyer $i \in \{nr+1,...,2nr\}$ is close to zero if $v(\omega_0, \omega_i) < p^*(\omega_0, e)$.

The left-hand side of inequality (3) represents the probability that the monetary transfer to player i is close to $-p^*(\omega_0, e)$. Inequality (3) implies that it is almost certain that the monetary transfer to any buyer $i \in \{nr+1,...,2nr\}$ is close to $-p^*(\omega_0, e)$ if $v(\omega_0, \omega_i) > p^*(\omega_0, e)$. The left-hand side of inequality (5) represents the probability that the monetary transfer to player i is close to $p^*(\omega_0, e)$. Inequality (5) implies that it is almost certain that the monetary transfer to any seller $i \in \{1,...,nr\}$ is close to $p^*(\omega_0, e)$ if $v(\omega_0, \omega_i) < p^*(\omega_0, e)$.

Based on the above observations, the theorem states that when the number of players is sufficiently large, competitive allocations with complete information are virtually implementable in twice iterative dominance. Moreover, the theorem requires the used mechanisms to be detail-free, i.e., to be designed independent of the specifications of the environments.

The proof of this theorem is constructive; an outline of the proof is provided in the following section.

4. Outline of the Proof

The central planner divides the players into r distinct groups. Each group $\beta \in \{1,...,r\}$ is defined as

$$N(\beta) \equiv \{(\beta-1)n+1,...,\beta n\} \bigcup \{(r+\beta-1)n+1,...,(r+\beta)n\} \subset N ,$$

which includes n sellers and n buyers.[5] Let $\beta(i) \in \{1,...,r\}$ denote the index of the group that includes player i, i.e., $i \in \beta(i)$. We consider the following three-stage procedure, according to which the central planner requires each player to announce price bids *thrice*.

Stage 1: Each player i announces the *first* bid $m_{i,1} \in [0,1]$ and observes the first bid announcements $m_{N(\beta(i)-1),1} \equiv (m_{j,1})_{j \in N(\beta(i)-1)} \in [0,1]^n$ in the *preceding* group $\beta(i)-1$.

Stage 2: Each player i announces the *second* bid $m_{i,2}(m_{N(\beta(i)-1),1}) \in [0,1]$. After this

[5] This paper assumes that the numbers of buyers and sellers are the same. The theorem, however, does not depend on this assumption. Suppose that the number of buyers is $n_b r$ and that of sellers is $n_s r$ and $n_b \neq n_s$. In this case, we can show the results of the theorem without any substantial change by defining r distinct groups, each of which consists of n_b buyers and n_s sellers and replicates the whole market.

announcement, each player i observes the first bid announcements $m_{N\setminus N(\beta(i)),1} \equiv (m_{j,1})_{j\in N\setminus N(\beta(i))} \in [0,1]^{n(r-1)}$ in all groups other than group $\beta(i)$.

Stage 3: Each player i announces the *third* bid $m_{i,3}(m_{N\setminus N(\beta(i)),1}) \in [0,1]$.

During this procedure, each player $i \in N$ cannot observe the first bid announcements of the other players in the same group $\beta(i)$ and the second bid announcements of all the other players. The central planner determines an allocation and monetary transfers according to the *random fixed price rule* and the *price-taking rule* as follows. The central planner generally employs the price-taking rule. The random fixed price rule is used only with a positive but very low probability. As shown later in this paper, the first bid announcements reveal the private signals truthfully; the second bid announcements are utilized to calculate the competitive price with complete information; and the third bid announcements are generally the basis for determining the allocation.

Random Fixed Price Rule: The central planner randomly selects a trading price $p \in [0,1]$. Consider the following six cases:

$$m_{i,1} < p\,,\ \ m_{nr+i,1} > p\,,$$

$$m_{i,2}(m_{N(\beta(i)-1),1}) < p\,,\ \ m_{nr+i,2}(m_{N(\beta(i)-1),1}) > p\,,$$

$$m_{i,3}(m_{N\setminus N(\beta(i)),1}) < p\,, \text{ and } \ m_{nr+i,3}(m_{N\setminus N(\beta(i)),1}) > p\,.$$

For each $k \in \{0,...,6\}$, each seller i sells the commodity to buyer $i+nr$ for p with a

probability of $k/6$ if and only if any k cases among these six cases occur at once.

Price-Taking Rule: The central planner calculates the *market-clearing* price $p(\beta) \in [0,1]$ for *each* group $\beta \in \{1,...,r\}$, which is defined as the *nth* highest bid among the *second* bids announced in group β. Within each group β, the central planner sets the trading price for the group equal to the market-clearing price $p(\beta-1)$ of its *preceding* group $\beta-1$. Every seller in each group β is compelled to sell the commodity to the central planner for $p(\beta-1)$. The central planner then sells these n units back to the players in the same group β whose *third* bids are greater than $p(\beta-1)$. If less than n players in group β make their third bids greater than $p(\beta-1)$, the central planner randomly selects and compels players to buy the unsold commodities. If more than n players make their third bids greater than $p(\beta-1)$, the central planner randomly selects and sells the commodities to n players among them.

Note that the above procedure is detail-free, i.e., it does not depend on the finer details of the specifications. Fix an environment $e \in E$ arbitrarily. We show that each player's first bid reveals her private signal truthfully. Note that each player's first bid influences her payoff only through the random fixed price rule. Since the trading price p is randomly determined, each player i is willing to make the first bid $m_{i,1}$ equal to the expected value $v_i(\omega_i)$ conditional on her private signal ω_i as being undominated. This, along with

Assumption 3, implies that each player's first bid reveals her private signal truthfully.

Next, we show that, almost certainly, each player $i's$ second bid approximates the true value $v(\omega_i, \omega_0)$, and the market-clearing price $p(\beta)$ for each group β approximates the competitive price $p^*(\omega_0, e)$. Note that each player's second bid influences her payoff only through the random fixed price rule. Before announcing the second bid, each player i has complete knowledge of the private signals $\omega_{N(\beta(i)-1)} \equiv (\omega_j)_{j \in N(\beta(i)-1)} \in \Omega_1^n$ for the players in the preceding group $\beta(i)-1$. Since the trading price p is randomly determined, it follows that each player i is willing to make the second bid $m_{i,2}(m_{N(\beta(i)-1),1})$ equal to the expected value conditional on the private signals $\omega_{N(\beta(i)-1)}$ for the members of the preceding group $\beta(i)-1$ and her private signal ω_i, i.e.,

$$v(\omega_i, \omega_{N(\beta(i)-1)}) \equiv \sum_{\omega_0 \in \Omega_0} v(\omega_0, \omega_i) \tilde{f}(\omega_0 \mid \omega_i, \omega_{N(\beta(i)-1)})$$

as being twice iteratively undominated, where

$$\tilde{f}(\omega_0 \mid \omega_i, \omega_{N(\beta(i)-1)}) \equiv \frac{\prod_{j \in \{i\} \cup N(\beta(i)-1)} f_1(\omega_j \mid \omega_0) f_0(\omega_0)}{\sum_{\omega_0' \in \Omega_0} \prod_{j \in \{i\} \cup N(\beta(i)-1)} f_1(\omega_j \mid \omega_0') f_0(\omega_0')}$$

denotes the probability of ω_0 occurring conditional on $(\omega_i, \omega_{N(\beta(i)-1)})$. Since the size of each group $2n$ is sufficiently large, it is almost certain from Assumption 1 that each player can almost correctly infer the true macro shock ω_0 from $\omega_{N(\beta(i)-1)}$, and therefore, $v(\omega_i, \omega_{N(\beta(i)-1)})$ can approximate $v(\omega_i, \omega_0)$. Since the size $2n$ of each group is

sufficiently large, it is almost certain that each group β approximately replicates the whole, which, along with Assumption 2, implies that $p(\beta)$ approximates $p^*(\omega_0, e)$.

Finally, we show that each player $i's$ third bid almost certainly approximates the true value $v(\omega_i, \omega_0)$, and the competitive allocations are approximately achieved. Before announcing the third bid, each player i has full knowledge of the private signals $\omega_{N\setminus N(\beta(i))} \in \Omega_1^{n(r-1)}$ for all players out of group $\beta(i)$. Similarly, the random fixed price rule incentivizes each player i to make the third bid $m_{i,3}(m_{N\setminus N(\beta(i)),1})$ equal to the expected value conditional on $\omega_{N\setminus N(\beta(i))}$, i.e.,

$$v(\omega_i, \omega_{N\setminus N(\beta(i))}) \equiv \sum_{\omega_0 \in \Omega_0} v(\omega_0, \omega_i) \tilde{f}(\omega_0 \mid \omega_i, \omega_{N\setminus N(\beta(i))})$$

as being twice iteratively undominated, where

$$\tilde{f}(\omega_0 \mid \omega_i, \omega_{N\setminus N(\beta(i))}) \equiv \frac{\prod_{j \in \{i\} \cup N\setminus N(\beta(i))} f_1(\omega_j \mid \omega_0) f_0(\omega_0)}{\sum_{\omega_0' \in \Omega_0} \prod_{j \in \{i\} \cup N\setminus N(\beta(i))} f_1(\omega_j \mid \omega_0') f_0(\omega_0')}$$

denotes the probability of ω_0 occurring conditional on $(\omega_i, \omega_{N\setminus N(\beta(i))})$. Since the size of each group $2n$ is sufficiently large, it follows from Assumption 1 that $v(\omega_i, \omega_{N\setminus N(\beta(i))})$ almost certainly approximates $v(\omega_i, \omega_0)$.

Since the price-taking rule as well as the random fixed price rule is relevant to the third bid announcement, the above argument is not sufficient for the incentive to reveal any approximate value of $v(\omega_i, \omega_0)$ as the third bid announcement. However, we can demonstrate that the price-taking rule does not provide each player with the incentive to

deviate from revealing any approximate value of the true value $v(\omega_i, \omega_0)$ as follows. Let

$$v(\omega_i, \omega_{N\setminus\{i\}}) \equiv \sum_{\omega_0 \in \Omega_0} v(\omega_0, \omega_i)\, \tilde{f}(\omega_0 \mid \omega)$$

denote the expected value conditional on the private signals for all players, where $\tilde{f}(\omega_0 \mid \omega_i, \omega_{N\setminus\{i\}}) \equiv \dfrac{\prod_{j\in N} f_1(\omega_j \mid \omega_0)\, f_0(\omega_0)}{\sum_{\omega_0' \in \Omega_0} \prod_{j\in N} f_1(\omega_j \mid \omega_0')\, f_0(\omega_0')}$ is the probability of ω_0 occurring conditional on $(\omega_i, \omega_{N\setminus\{i\}})$. Note that the trading price for each group β is set such that it is equal to the market-clearing price $p(\beta-1)$ for its preceding group $\beta-1$, which does not depend on the price bid announcements in group β. Hence, each player i has the incentive to announce any value between $\min_{\omega_{N(\beta(i))\setminus\{i\}}} v_i(\omega_i, \omega_{N\setminus\{i\}})$ and $\max_{\omega_{N(\beta(i))\setminus\{i\}}} v_i(\omega_i, \omega_{N\setminus\{i\}})$ as the third bid. Since the number of distinct groups r is sufficiently large, it follows that both $\min_{\omega_{N(\beta(i))\setminus\{i\}}} v_i(\omega_i, \omega_{N\setminus\{i\}})$ and $\max_{\omega_{N(\beta(i))\setminus\{i\}}} v_i(\omega_i, \omega_{N\setminus\{i\}})$ are close to $v(\omega_i, \omega_{N\setminus\{i\}})$. In other words, $v(\omega_i, \omega_{N\setminus\{i\}})$ does not depend to a large extent on $\omega_{N(\beta(i))\setminus\{i\}}$, and therefore, $v(\omega_i, \omega_{N\setminus N(\beta(i))})$ certainly approximates $v(\omega_i, \omega_{N\setminus\{i\}})$. This, along with the fact that the trading price for each group is independent of the price bid announcements in this group, implies that the price-taking rule does not provide player i with the incentive to deviate from announcing any approximate value of $v(\omega_0, \omega_i)$ as her third bid. Note that *the third bid is not necessarily equal to* $v(\omega_i, \omega_{N\setminus N(\beta(i))})$*; however, it can be approximated by* $v(\omega_i, \omega_{N\setminus N(\beta(i))})$. Since the trading prices almost certainly approximate the competitive price $p^*(\omega_0, e)$, we have shown that the competitive allocations with complete information are

almost certainly implemented in twice iterative dominance. We now present the full proof of the theorem in the following section.

5. Full Proof of the Theorem

We model the three-stage procedure in Section 4 as a mechanism $G = G^{(n,r,\varepsilon)} = (n, r, M, g, x)$ as follows. Let $\varepsilon \in (0,1)$,

$M_i = M_{i,1} \times M_{i,2} \times M_{i,3}$, $M_{i,1} = [0,1]$,

$M_{i,2}$ denote the set of continuous functions $m_{i,2} : [0,1]^n \to [0,1]$,

$M_{i,3}$ denote the set of continuous functions $m_{i,3} : [0,1]^{(r-1)n} \to [0,1]$,

$g = \frac{\varepsilon}{6}(g^{S,1} + g^{B,1} + g^{S,2} + g^{B,2} + g^{S,3} + g^{B,3}) + (1-\varepsilon)g^3$, and

$x_i = \frac{\varepsilon}{6}(x_i^{S,1} + x_i^{B,1} + x_i^{S,2} + x_i^{B,2} + x_i^{S,3} + x_i^{B,3}) + (1-\varepsilon)x_i^3$.

Here,

$g^{D,l} : M \to \Delta(A)$ for all $(D,l) \in \{S,B\} \times \{1,2,3\}$, and

$g^3 : M \to \Delta(A)$.

$x_i^{D,l} : M \to R$ for all $(D,l) \in \{S,B\} \times \{1,2,3\}$, and

$x_i^3 : M \to R$.

Each player i simultaneously chooses a message $m_i = (m_{i,1}, m_{i,2}, m_{i,3})$, where $m_{i,1} \in [0,1]$ corresponds to the first bid, the value $m_{i,2}(m_{N(\beta(i)-1),1}) \in [0,1]$ of the function $m_{i,2}$

corresponds to the second bid, and the value $m_{i,3}(m_{N\setminus N(\beta(i)),1}) \in [0,1]$ of the function $m_{i,3}$ corresponds to the third bid. In the following two subsections, we specify $(g^{D,l},(x_i^{D,l})_{i\in N})_{(D,l)\in\{S,B\}\times\{1,2,3\}}$ and $(g^3,(x_i^3)_{i\in N})$, which define the random fixed price rule and the price-taking rule, respectively. The central planner uses the random fixed price rule with probability ε and the price-taking rule with probability $1-\varepsilon$. Hence, the central planner chooses any alternative $a \in A$ with the following probability:

$$g(m)(a) = \frac{\varepsilon}{6} \sum_{(D,l)\in\{S,B\}\times\{1,2,3\}} g^{D,l}(m)(a) + (1-\varepsilon) g^3(m)(a).$$

The monetary transfers conducted by the central planner are set such that they are equal to the *expected* values of the monetary transfers induced by the three-stage procedure.[6]

5.1. Random Fixed Price Rule

For every $(m,a) \in M \times A$, let

$$g^{S,1}(m)(a) = \prod_{i=1}^{nr} g_i^{S,1}(m)(a_i, a_{nr+i}),$$

where

[6] Here, we assume quasi-linearity and risk neutrality. Without these assumptions, we need to modify the construction of mechanisms by using lotteries over combinations of an alternative and transfers. It might be important to verify the robustness of our results in terms of a non-expected utility, which is beyond the purpose of the present paper.

$$g_i^{S,1}(m)(1,0) = m_{i,1}, \quad g_i^{S,1}(m)(0,1) = 1 - m_{i,1},$$

$$g_i^{S,1}(m)(0,0) = 0, \text{ and } \quad g_i^{S,1}(m)(1,1) = 0.$$

With probability $1 - m_{i,1}$, seller $i's$ commodity is transferred to buyer $nr + i$. Let

$$x_i^{S,1}(m) \equiv \int_{m_{i,1}}^{1} pdp = \frac{1 - (m_{i,1})^2}{2} \quad \text{and} \quad x_{nr+i}^{S,1}(m) \equiv -x_i^{S,1}(m).$$

The central planner randomly selects a price p from the interval $[0,1]$, for which the commodity is transferred from seller i to buyer $nr + i$ if and only if $m_{i,1} < p$. Note that $x_i^{S,1}(m)$ corresponds to the expected monetary transfer from buyer $nr + i$ to seller i. For every $(m, a) \in M \times A$, let

$$g^{B,1}(m)(a) = \prod_{i=1}^{nr} g_i^{B,1}(m)(a_i, a_{nr+i}),$$

where

$$g_i^{B,1}(m)(1,0) = 1 - m_{nr+i,1}, \quad g_i^{B,1}(m)(0,1) = m_{nr+i,1},$$

$$g_i^{B,1}(m)(0,0) = 0, \text{ and } \quad g_i^{B,1}(m)(1,1) = 0.$$

With probability $m_{nr+i,1}$, seller $i's$ commodity is transferred to buyer $nr + i$. Let

$$x_i^{B,1}(m) \equiv \int_{0}^{m_{nr+i,1}} pdp = \frac{(m_{nr+i,1})^2}{2} \quad \text{and} \quad x_{nr+i}^{B,1}(m) \equiv -x_i^{B,1}(m).$$

The central planner randomly selects p from $[0,1]$, for which the commodity is transferred from seller i to buyer $nr + i$ if and only if $m_{nr+i,1} > p$. Note that $x_i^{B,1}(m)$ corresponds to the expected monetary transfer from buyer $nr + i$ to seller i.

For every $(m,a) \in M \times A$, let

$$g^{S,2}(m)(a) = \prod_{i=1}^{nr} g_i^{S,2}(m)(a_i, a_{nr+i}),$$

where

$$g_i^{S,2}(m)(1,0) = m_{i,2}(m_{N(\beta(i)-1),1}), \quad g_i^{S,2}(m)(0,1) = 1 - m_{i,2}(m_{N(\beta(i)-1),1}),$$

$$g_i^{S,2}(m)(0,0) = 0, \text{ and } \quad g_i^{S,2}(m)(1,1) = 0.$$

With probability $1 - m_{i,2}(m_{N(\beta(i)-1),1})$, seller $i's$ commodity is transferred to buyer $nr+i$. Let

$$x_i^{S,2}(m) \equiv \int_{m_{i,2}(m_{N(\beta(i)-1),1})}^{1} pdp = \frac{1 - m_{i,2}(m_{N(\beta(i)-1),1})^2}{2}, \text{ and}$$

$$x_{nr+i}^{S,2}(m) \equiv -x_i^{S,2}(m).$$

The central planner randomly selects p from $[0,1]$, for which the commodity is transferred from seller i to buyer $nr+i$ if and only if $m_{i,2}(m_{N(\beta(i)-1),1}) < p$. Note that $x_i^{S,2}(m)$ corresponds to the expected monetary transfer from buyer $nr+i$ to seller i. For every $(m,a) \in M \times A$, let

$$g^{B,2}(m)(a) = \prod_{i=1}^{nr} g_i^{B,2}(m)(a_i, a_{nr+i}),$$

where

$$g_i^{B,2}(m)(1,0) = 1 - m_{nr+i,2}(m_{N(\beta(i)-1),1}), \quad g_i^{B,2}(m)(0,1) = m_{nr+i,2}(m_{N(\beta(i)-1),1}),$$

$$g_i^{B,2}(m)(0,0) = 0, \text{ and } \quad g_i^{B,2}(m)(1,1) = 0.$$

With probability $m_{nr+i,2}(m_{N(\beta(i)-1),1})$, seller $i's$ commodity is transferred to buyer $nr+i$.

Let

$$x_i^{B,2}(m) \equiv \int_0^{m_{nr+i,2}(m_{N(\beta(i)-1),1})} pdp = \frac{m_{nr+1,2}(m_{N(\beta(i)-1),1})^2}{2}, \text{ and}$$

$$x_{nr+i}^{B,2}(m) \equiv -x_i^{B,2}(m).$$

The central planner randomly selects p from $[0,1]$, for which the commodity is transferred from seller i to buyer $nr+i$ if and only if $m_{nr+i,2}(m_{N(\beta(i)-1),1}) > p$. Note that $x_i^{B,2}(m)$ corresponds to the expected monetary transfer from buyer $nr+i$ to seller i.

For every $(m,a) \in M \times A$, let

$$g^{S,3}(m)(a) = \prod_{i=1}^{nr} g_i^{S,3}(m)(a_i, a_{nr+i}),$$

where

$$g_i^{S,3}(m)(1,0) = m_{i,3}(m_{N\setminus N(\beta(i)),1}), \quad g_i^{S,3}(m)(0,1) = 1 - m_{i,3}(m_{N\setminus N(\beta(i)),1}),$$

$$g_i^{S,3}(m)(0,0) = 0, \text{ and } \quad g_i^{S,3}(m)(1,1) = 0.$$

With probability $1 - m_{i,3}(m_{N\setminus N(\beta(i)),1})$, seller $i's$ commodity is transferred to buyer $nr+i$.

Let

$$x_i^{S,3}(m) \equiv \int_{m_{i,3}(m_{N\setminus N(\beta(i)),1})}^{1} pdp = \frac{1 - m_{i,3}(m_{N\setminus N(\beta(i)),1})^2}{2}, \text{ and}$$

$$x_{nr+i}^{S,3}(m) \equiv -x_i^{S,3}(m).$$

The central planner randomly selects p from $[0,1]$, for which the commodity is transferred from seller i to buyer $nr+i$ if and only if $m_{i,3}(m_{N\setminus N(\beta(i)),1}) < p$. Note that

$x_i^{S,3}(m)$ is the expected payment from buyer $nr+i$ to seller i. For every $(m,a) \in M \times A$, let

$$g^{B,3}(m)(a) = \prod_{i=1}^{nr} g_i^{B,3}(m)(a_i, a_{nr+i}),$$

where

$$g_i^{B,3}(m)(1,0) = 1 - m_{nr+i,3}(m_{N\setminus N(\beta(i)),1}), \quad g_i^{B,3}(m)(0,1) = m_{nr+i,3}(m_{N\setminus N(\beta(i)),1}),$$

$$g_i^{B,3}(m)(0,0) = 0, \text{ and } \quad g_i^{B,3}(m)(1,1) = 0.$$

With probability $m_{nr+i,3}(m_{N\setminus N(\beta(i)),1})$, seller $i's$ commodity is transferred to buyer $nr+i$. Let

$$x_i^{B,3}(m) \equiv \int_0^{m_{nr+i,3}(m_{N\setminus N(\beta(i)),1})} pdp = \frac{m_{nr+1,3}(m_{N\setminus N(\beta(i)),1})^2}{2}, \text{ and}$$

$$x_{nr+i}^{B,3}(m) \equiv -x_i^{B,3}(m).$$

The central planner randomly selects p from $[0,1]$, for which the commodity is transferred from seller i to buyer $nr+i$ if and only if $m_{nr+i,3}(m_{N\setminus N(\beta(i)),1}) > p$. Note that $x_i^{B,3}(m)$ corresponds to the expected monetary transfer from buyer $nr+i$ to seller i.

5.2. Price-Taking Rule

Let $A_{N(\beta)} \equiv \underset{i \in N(\beta)}{\times} A_i$ and $a_{N(\beta)} = (a_i)_{i \in N(\beta)} \in A_{N(\beta)}$. For every $(m,a) \in M \times A$, let

$$g^3(m)(a) = \prod_{\beta=1}^{r} g_\beta^3(m)(a_{N(\beta)}),$$

where $g_\beta^3 : M \to \Delta(A_{N(\beta)})$ will be specified later. For every $(\beta, m) \in \{1,...,r\} \times M$, let $\mu(\beta, m) : \{1,...,2n\} \to N(\beta)$ denote the one-to-one mapping defined by $m_{\mu(\beta,m)(1),2}(m_{N(\beta-1),1}) \geq m_{\mu(\beta,m)(2),2}(m_{N(\beta-1),1}) \cdots \geq m_{\mu(\beta,m)(2n),2}(m_{N(\beta-1),1})$, where for every $l \in \{1,...,2n-1\}$, if $m_{\mu(\beta,m)(l),2}(m_{N(\beta-1),1}) = m_{\mu(\beta,m)(l+1),2}(m_{N(\beta-1),1})$, then $\mu(\beta, m)(l) < \mu(\beta, m)(l+1)$. For every $(\beta, m) \in \{1,...,r\} \times M$, let

$$p(\beta) = p(\beta, m) \equiv m_{\mu(\beta,m)(n),2}(m_{N(\beta-1),1})$$

denote the market-clearing price for group β that is calculated on the basis of the second bids $(m_{i,2}(m_{N(\beta(i)-1),1}))_{i \in N(\beta)}$. Let $n(\beta, p) \in \{1,...,2n\}$ denote the integer such that

$$m_{\mu(\beta,m)(n(\beta,p)),3}(m_{N \setminus N(\beta),1}) > p \geq m_{\mu(\beta,m)(n(\beta,p)+1),3}(m_{N \setminus N(\beta),1}),$$

which represents the number of players in group β whose third bids are greater than p. Let $\tilde{n}(\beta, p) \in \{1,...,2n\}$ denote the integer such that

$$m_{\mu(\beta,m)(n(\beta,p)),3}(m_{N \setminus N(\beta),1}) \geq p > m_{\mu(\beta,m)(n(\beta,p)+1),3}(m_{N \setminus N(\beta),1}),$$

which is the number of players in group β whose third bids are greater than or equal to p. Clearly, $n(\beta, p) \geq \tilde{n}(\beta, p)$. For each $(\beta, m) \in \{1,...,r\} \times M$, we specify $g_\beta^3(m)$ as follows. Let $p = p(\beta - 1, m)$.

Case 1: Suppose $n(\beta, p) \geq n$. In this case, for every $a_{N(\beta)} \in A_{N(\beta)}$,

$$g_\beta^3(m)(a_{N(\beta)}) = \frac{n!\{n(\beta, p) - n\}!}{n(\beta, p)!}$$

$$\text{if } [a_i = 1] \Rightarrow [m_{i,3}(m_{N \setminus N(\beta),1}) > p] \text{ for all } i \in N(\beta),$$

whereas $g^3_\beta(m)(a_{N(\beta)}) = 0$ otherwise.

Case 2: Suppose $\tilde{n}(\beta, p) \geq n > n(\beta, p)$. Then, for every $a_{N(\beta)} \in A_{N(\beta)}$,

$$g^3_\beta(m)(a_{N(\beta)}) = \frac{(\tilde{n}(\beta, p) - n)!\{n - n(\beta, p)\}!}{\{\tilde{n}(\beta, p) - n(\beta, p)\}!}$$

if $[a_i = 1] \Rightarrow [m_{i,3}(m_{N\setminus N(\beta),1}) \geq p(\beta, m)]$ and

$[m_{i,3}(m_{N\setminus N(\beta),1}) > p] \Rightarrow [a_i = 1]$ for all $i \in N(\beta)$,

whereas $g^3_\beta(m)(a_{N(\beta)}) = 0$ otherwise.

Case 3: Suppose $n \geq \tilde{n}(\beta, p)$. Then, for every $a_{N(\beta)} \in A_{N(\beta)}$,

$$g^3_\beta(m)(a_{N(\beta)}) = \frac{n!\{n - \tilde{n}(\beta, p)\}!}{\{2n - \tilde{n}(\beta, p)\}!}$$

if $[m_{i,3}(m_{N\setminus N(\beta),1}) \geq p] \Rightarrow [a_i = 1]$ for all $i \in N(\beta)$,

whereas $g^3_\beta(m)(a_{N(\beta)}) = 0$ otherwise.

The central planner sets the trading price for each group β such that it is equal to the market-clearing price $p(\beta - 1)$ for its preceding group $\beta - 1$. Those players in each group β whose bids are greater than $p(\beta - 1)$ have a higher priority to consume the commodity than do the others. Players whose bids are equal to $p(\beta - 1, m)$ have a higher priority than those whose bids are less than $p(\beta - 1)$.

One interpretation is as follows. Every seller in each group β is compelled to sell the commodity to the central planner for $p(\beta-1)$. The central planner then sells these n units back to the players in the same group β whose *third* bids are greater than $p(\beta-1)$. If the third bids of more than n players are bids greater than $p(\beta-1)$, the central planner randomly selects from among these players and sells the commodities to them. This corresponds to Case 1. If less than n players in group β make their third bids greater than or equal to $p(\beta-1)$, the central planner randomly selects from among the rest of the players and compels the selected players to buy the unsold commodities. This corresponds to Case 3. If less than n players make their third bids greater than $p(\beta-1)$ and more than n players in group β make their third bids greater than or equal to $p(\beta-1)$, the central planner sells the commodities to the players who make their third bids greater than $p(\beta-1)$, and then randomly selects from among the players who equal their third bids to $p(\beta-1)$ and compels them to buy the unsold commodities. This corresponds to Case 2.

For every $i \in N(\beta) \bigcap \{1,...,nr\}$, let

$$x_i^3(m) = \{1 - \sum_{a_{N(\beta)} \in A_{N(\beta)}} a_i g_\beta^3(m)(a_{N(\beta)})\} p(\beta-1,m).$$

For every $i \in N(\beta) \bigcap \{nr+1,...,2nr\}$, let

$$x_i^3(m) = -\sum_{a_{N(\beta)} \in A_{N(\beta)}} a_i g_\beta^3(m)(a_{N(\beta)}) p(\beta-1,m).$$

Any buyer $i \in N(\beta)$ who consumes the commodity must pay $p(\beta-1)$. Any seller $i \in N(\beta)$ who does not consume the commodity can receive $p(\beta-1)$. Hence, we have

completed the construction of the mechanism $G = G^{(n,r,\varepsilon)} = (n, r, M, g, x)$.

Let $(\varepsilon^{(l)})_{l=1}^{\infty}$ denote an infinite sequence in which $\varepsilon^{(l)} \in (0, 1/6)$, $\varepsilon^{(l)}$ decreases with respect to l, and $\lim_{l\to\infty} \varepsilon^{(l)} = 0$. For each $l = 1, 2, ...,$ let $G^{(l)} \equiv G^{(n^{(l)}, r^{(l)}, \varepsilon^{(l)})}$. In the following subsections, we will demonstrate that $(G^{(l)})_{l=1}^{\infty}$ satisfies the properties of the theorem.

5.3. Twice Iterative Dominance

All the functions relevant to player $i's$ utility are $g_i^{D,l}$ and $x_i^{D,l}$ for all $(D, l) \in \{S, B\} \times \{1, 2\}$, $g_{\beta(i)}^3$, and $x_{\beta(i)}^3$. Suppose that player i is a seller, i.e., $i \in \{1, ..., nr\}$. Then, among these functions, only $g_i^{S,1}$ and $x_i^{S,1}$ depend on player $i's$ first bid $m_{i,1}$ and only on this bid. According to the specifications of $g_i^{S,1}$ and $x_i^{S,1}$, player i will choose $m_{i,1}$ to maximize $m_{i,1} v(\omega_i) + \frac{1 - m_{i,1}^2}{2}$, i.e., $m_{i,1} = v(\omega_i)$. Suppose that player i is a buyer, i.e., $i \in \{nr + 1, ..., 2nr\}$. Then, among these functions, only $g_i^{B,1}$ and $x_i^{B,1}$ depend on player $i's$ first bid $m_{i,1}$ and only on this bid. According to the specifications of $g_i^{B,1}$ and $x_i^{B,1}$, player i will choose $m_{i,1}$ to maximize $m_{i,1} v(\omega_i) - \frac{m_{i,1}^2}{2}$, i.e., $m_{i,1} = v(\omega_i)$. Hence, we have proved that for every $i \in N$, if s_i is undominated, then it must hold that $s_{i,1}(\omega_i) = v(\omega_i)$ for all $\omega_i \in \Omega_i$. From Assumption 3, if all players play undominated strategies, then each player's first bid truthfully reveals her private signal. Assuming that all

players play undominated strategies, their first bids truthfully reveal their private signals.

Suppose that player i is a seller. Then, among the functions $g_i^{D,l}$ and $x_i^{D,l}$ for all $(D,l) \in \{S,B\} \times \{1,2\}$, $g_{\beta(i)}^3$, and $x_{\beta(i)}^3$, only $g_i^{S,2}$ and $x_i^{S,2}$ depend on player $i's$ second bid $m_{i,2}(m_{N(\beta(i)-1),1})$ and only on this bid. The specifications of $g_i^{S,2}$ and $x_i^{S,2}$, along with the truthful revelation via $m_{N(\beta(i)-1),1}$, imply that player i will choose $m_{i,2}(m_{N(\beta(i)-1),1})$ to maximize $m_{i,2}(m_{N(\beta(i)-1),1})v(\omega_i, \omega_{N(\beta(i)-1)}) + \frac{1-m_{N(\beta(i)-1),1}^2}{2}$, i.e., $m_{i,2}(m_{N(\beta(i)-1),1}) = v(\omega_i, \omega_{N(\beta(i)-1)})$. Suppose that player i is a buyer. Then, among these functions, only $g_i^{B,2}$ and $x_i^{B,2}$ depend on player $i's$ second bid $m_{i,2}(m_{N(\beta(i)-1),1})$ and only on this bid. The specifications of $g_i^{B,2}$ and $x_i^{B,2}$, along with the truthful revelation via $m_{N(\beta(i)-1),1}$, imply that player i will choose $m_{i,2}(m_{N(\beta(i)-1),1})$ to maximize $m_{i,2}(m_{N(\beta(i)-1),1})v(\omega_i, \omega_{N(\beta(i)-1)}) - \frac{m_{N(\beta(i)-1),1}^2}{2}$, i.e., $m_{i,2}(m_{N(\beta(i)-1),1}) = v(\omega_i, \omega_{N(\beta(i)-1)})$. Hence, for every $i \in N$, if s_i is twice iteratively undominated, then

$$s_{i,2}(\omega_i)(m_{N(\beta(i)-1),1}) = v(\omega_i, \omega_{N(\beta(i)-1)}) \quad \text{for all} \quad \omega_i \in \Omega_i$$

$$\text{if} \quad m_{j,1} = v(\omega_j) \quad \text{for all} \quad j \in N(\beta(i)-1).$$

Suppose that player i is a seller. Then, among the functions $g_i^{D,l}$ and $x_i^{D,l}$ for all $(D,l) \in \{S,B\} \times \{1,2\}$, $g_{\beta(i)}^3$, and $x_{\beta(i)}^3$, only $g_i^{S,3}$, g^3, $x_i^{S,3}$, and x_i^3 depend on player $i's$ third bid $m_{i,3}(m_{N\setminus N(\beta(i)),1})$. Moreover, $g_i^{S,3}$ and $x_i^{S,3}$ depend only on this bid. The specifications of $g_i^{S,3}$ and $x_i^{S,3}$, along with the truthful revelation via $m_{N\setminus N(\beta(i)),1}$, imply

that the expected utility for player i induced by $g_i^{S,3}$ and $x_i^{S,3}$ is

$$m_{i,3}(m_{N\setminus N(\beta(i)),1})v(\omega_i,\omega_{N\setminus N(\beta(i))})+\frac{1-m_{N\setminus N(\beta(i)),1}^2}{2},$$

which is concave with respect to $m_{i,3}(m_{N\setminus N(\beta(i)),1})$ and is maximized by

$$m_{i,3}(m_{N\setminus N(\beta(i)),1})=v(\omega_i,\omega_{N\setminus N(\beta(i))}).$$

Suppose that player i is a buyer. Then, among these functions, only $g_i^{B,3}$, g^3, $x_i^{B,3}$, and x_i^3 depend on $m_{i,3}(m_{N\setminus N(\beta(i)),1})$. Moreover, $g_i^{B,3}$ and $x_i^{B,3}$ depend only on $m_{i,3}(m_{N\setminus N(\beta(i)),1})$. According to the specifications of $g_i^{B,3}$ and $x_i^{B,3}$, along with the truthful revelation via the first bids $m_{N\setminus N(\beta(i)),1}$ by all the other groups, it follows that the expected utility for player i induced by $g_i^{B,3}$ and $x_i^{B,3}$ is

$$m_{i,3}(m_{N\setminus N(\beta(i)),1})v(\omega_i,\omega_{N\setminus N(\beta(i))})-\frac{m_{N\setminus N(\beta(i)),1}^2}{2},$$

which is concave with respect to $m_{i,3}(m_{N\setminus N(\beta(i)),1})$ and is maximized by

$$m_{i,3}(m_{N\setminus N(\beta(i)),1})=v(\omega_i,\omega_{N\setminus N(\beta(i))}).$$

Fix $i\in N$ arbitrarily, and consider the expected utility for player i induced by g^3 and x_i^3. Let $p=p(\beta(i)-1,m)$. From the specifications of g^3 and x_i^3, for each $\omega\in\Omega$, there exist $\overline{v}$ and $\tilde{v}$ such that

$$g^3(m)v_i(\omega)-x_i^3(m)=\overline{v} \quad \text{for all} \quad m_{i,3}(m_{N\setminus N(\beta(i)-1),1})>p,$$

$$g^3(m)v_i(\omega)-x_i^3(m)=\tilde{v} \quad \text{for all} \quad m_{i,3}(m_{N\setminus N(\beta(i)-1),1})<p,$$

$$\min[\overline{v},\tilde{v}]\le g^3(m)v_i(\omega)-x_i^3(m)\le\max[\overline{v},\tilde{v}] \quad \text{for} \quad m_{i,3}(m_{N\setminus N(\beta(i)-1),1})=p,$$

$$\overline{v} > \tilde{v} \qquad \text{if} \quad v_i(\omega) > p\,,$$

and

$$\overline{v} < \tilde{v} \qquad \text{if} \quad v_i(\omega) < p\,.$$

From the above observations, it is clear that if all players play undominated strategies, then

$$\min_{\omega_{N(\beta(i))\setminus\{i\}}} v_i(\omega_i, \omega_{N\setminus\{i\}}) \le m_{i,3}(m_{N\setminus N(\beta(i)),1}) \le \max_{\omega_{N(\beta(i))\setminus\{i\}}} v_i(\omega_i, \omega_{N\setminus\{i\}})$$

$$\text{whenever} \quad m_{j,1} = v(\omega_j) \quad \text{for all} \quad j \in N \setminus N(\beta(i))\,.$$

Hence, we have proved that if all players play any twice iteratively undominated strategy profile $s \in S$, then, for every $i \in N$,

$$s_{i,1}(\omega_i) = v(\omega_i) \quad \text{for all} \quad \omega_i \in \Omega_i\,,$$

$$s_{i,2}(\omega_i)(m_{N(\beta(i)-1),1}) = v(\omega_i, \omega_{N(\beta(i)-1)}) \quad \text{for all} \quad \omega_i \in \Omega_i$$

$$\text{if} \quad m_{j,1} = v(\omega_j) \quad \text{for all} \quad j \in N(\beta(i)-1)\,,$$

and

$$\min_{\omega_{N(\beta(i))\setminus\{i\}}} v_i(\omega_i, \omega_{N\setminus\{i\}}) \le m_{i,3}(m_{N\setminus N(\beta(i)),1}) \le \max_{\omega_{N(\beta(i))\setminus\{i\}}} v_i(\omega_i, \omega_{N\setminus\{i\}})$$

$$\text{if} \quad m_{j,1} = v(\omega_j) \quad \text{for all} \quad j \in N \setminus N(\beta(i))\,.$$

5.4. Competitive Allocations

Since the size $2n$ of each group is sufficiently large, it is almost certain from the law of large numbers and Assumptions 1 and 3 that $m_{i,2}(m_{N(\beta(i)-1),1}) = v(\omega_i, \omega_{N(\beta(i)-1)})$

approximates $v_i(\omega_0, \omega_i)$, and from Assumption 2 that $p(\beta, m)$ approximates $p(\omega_0, e)$. Since the number r of distinct groups is sufficiently large, it is almost certain that both $\min_{\omega_{N(\beta(i))\setminus\{i\}}} v_i(\omega_i, \omega_{N\setminus\{i\}})$ and $\max_{\omega_{N(\beta(i))\setminus\{i\}}} v_i(\omega_i, \omega_{N\setminus\{i\}})$ approximate $v_i(\omega_0, \omega_i)$, and therefore, $m_{i,3}(m_{N\setminus N(\beta(i)),1})$ approximates $v_i(\omega_0, \omega_i)$. From the specifications of $(G^{(l)})_{l=1}^{\infty}$, for sufficiently large l, the price-taking rule (g^3, x^3) almost certainly determines the allocation, which implies that the probability of each player i receiving one unit is close to 1 (close to zero) if $v_i(\omega_0, \omega_i) > p^*(\omega_0, e)$ (if $v_i(\omega_0, \omega_i) < p^*(\omega_0, e)$). The monetary transfer to seller i approximates $p^*(\omega_0.e)$ (zero) if $v_i(\omega_0, \omega_i) < p^*(\omega_0, e)$ ($v_i(\omega_0, \omega_i) > p^*(\omega_0, e)$). Moreover, the monetary transfer to buyer i approximates $-p^*(\omega_0, e)$ (zero) if $v_i(\omega_0, \omega_i) > p^*(\omega_0, e)$ ($v_i(\omega_0, \omega_i) < p^*(\omega_0, e)$). Thus, we have presented the full proof.

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