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Strategic Default Jump as Impulse Control in Continuous Time

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This paper presents a new approach for modeling an optimal debt contract in continuous time. It examines a competing contract design in a continuous-time environment with Markov income shocks and costly verifiable information. It shows that an optimal contract has the form of a long-term debt contract that permits a debtor's strategic default and debt restructuring. The default is characterized by a recurrent, optimal impulse control beyond default. Numerical examples show that the equilibrium probability of the default is decreasing in the monitoring technology level when the default causes a big wealth loss.

Key Words: Default, Costly verification, Continuous time, Competing contract design, Impulse control.
JEL Codes: C73, D82, G33.

1. INTRODUCTION

Since debt defaults in Latin America, Asia, and Russia repeatedly caused serious financial turmoil in the world economy in the 1990s, financial markets have advanced the growth of credit derivatives (e.g., developments in credit default swap markets and trading of equity market volatility) to improve a technology level of risk management dramatically. They seemed to have been successful in slicing up credit risk in some sophisticated ways and dispersing it in their deep pocket. However, most recently, defaults on subprime loans in a weak U.S. mortgage market triggered another serious crisis in the global financial markets, although they were originally just a specific domestic problem. Based on the experiences, a question is now raised: How is the value of defaultable bonds affected by the function of information disclosure in financial intermediation when firms' information is private to them?

Despite such importance of the information disclosure, surprisingly, economists do not know much about the dynamic role of information disclosure in credit risk valuation. Specifically, asset pricing literature has explored default risk in a "reduced form," in which a default time arrives based on an exogenously given intensity probability distribution – called an intensity-based credit-risk approach.[1] Due to mathematical tractability, this literature is often successful in capturing the effect of default in debt pricing empirically

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[1]Theoretically, see, for example, Duffie and Singleton [16], Jarrow and Turnbull [22]. As for empirical applications, see Duffee [13], Duffie, Pedersen, and Singleton [15].

from actual financial data. However, due to a lack of a game-theoretic treatment of default, it fails to detect strategic default incentives of debtors and the role of information disclosure on the default.[2]

On the other hand, there is a large financial contracting literature on strategic default that uses contingent-claim models of Black and Cox [5] and Merton [27] in continuous time – called a structural credit-risk approach.[3] This literature typically studies strategic default under exogenously given (mostly, complete) security structures of equity and debt. In this approach, equity owners trigger a default decision to maximize the equity value when the firm's asset value becomes below a sufficiently low level (Leland and Toft [24]). This approach is successful in explaining this specific type of endogenous default behavior theoretically.[4] However, this approach is unrealistic in the sense that the lenders are assumed to decide the default through observing directly the firm value under the complete security structures. In fact, due to the assumption of such symmetric information, this literature understates short-term credit spreads empirically (Capponi and Cvitanić [7]). To summarize, there is a big gap between the financial contracting theory on the one hand and the asset pricing literature and actual financial data on the other.

The purpose of this paper is to bridge this gap by presenting a new approach for modeling an optimal debt contract in continuous time. In particular, this paper looks at an optimal competitive design of a continuous-time communication game in environments with Markov income shocks and costly verifiable information. It shows that an optimal contract takes the form of a debt contract that permits a debtor's ex post strategic default. The equilibrium default is characterized by a discontinuous, downward jump (i.e, "impulse") of equilibrium payment path. This equilibrium payment reduction (i.e., forgiveness) is interpreted as debt restructuring.[5] To avoid the debtor's unnecessary, intentional default, the re-contracting requires his voluntary costly verification of bad shape in equilibrium. The debtor then tries to re-contract when the advantage of the re-contracting exceeds the disadvantage of the costly disclosure. Otherwise, he chooses to repudiate his debt without disclosure and lives in financial autarky from then onwards. Thus, from a financial-contracting perspective, this model can examine strategic decisions between debt repudiation and restructuring by synthesizing the costly monitoring models and the costly diversion models of Bolton and Scharfstein [6]. Also, in this model, difference of the information sets between the debtor and the creditor is optimally designed and reconciles the contract theory and the asset pricing theory.[6]

[2]For example, Duffee [13] shows in United States corporate data that, when debtors' credit qualities or conditions change, there appears to be some instability of estimated default intensity parameters. This may be, at least partly, because their strategic default actions change.

[3]Also, there is enormous financial contracting literature in finite-period discrete-time models. See below.

[4]Furthermore, some papers study debt renegotiation in a structural form (for example, Anderson and Sundaresan [1], Mella-Barral and Perraudin [26], Fan and Sundaresan [18].

[5]In practice, debt restructuring can take the form either of a cut in principal, a lengthening of maturity, or a reduction in interest payments. This paper focuses only on the form of a reduction in interest payments.

[6]In this respect, structural credit-risk models with noisy, delayed, or distorted accounting information has the same spirit as mine. They reconcile the structural credit-risk model and the intensity-based one by incorporating difference of information sets between lenders (or markets) and borrowers (e.g., Duffie and Lando [14], Capponi and Cvitanić [7], Cetin et al. [8]). However, in those models, the difference of the information sets is exogenous. In my model, by contrast, the difference of measurability of the state processes between the lenders and the borrower is optimally and endogenously formed under an optimal security design.

Also, this paper is mathematical tractable for numerical analyses. It shows that the equilibrium probability of the default is decreasing in a level of the disclosure technology when the default causes a big wealth loss. Many previous models imposes default-boundary conditions exogeneously. Hence, the effect of default costs (including disclosure costs) on the default probability depends on the exogenous boundary value. This paper, by contrast, makes clear the effect structurally by endogenizing the default boundary values.

This paper presents a model that is an infinite-horizon, continuous-time extension of a classical finite-period (typically, two-period) costly state verification (CSV, hereafter) model, which is presented seminally by Townsend [36]. In particular, my dynamic model uses the finite-period CSV model as one component game. Wang [37] also studies a dynamic CSV model in infinite horizon.[7] A borrower's project produces single non-storable goods by using one unit of capital that a lender invests. The income process from the project is uncertain and its realization is privately observable to the borrower. A deterministic state verification (or disclosure) technology is available and costly.[8] Income is allocated according to the terms in a contract.

Previous CSV models have been successful in capturing the role of default as a threat of finitely costly inspection to make promised repayments and induce truth revelation. These models have been often used for examining empirically default costs.[9] However, most of them examine short-term debts that expire at one period. This is unrealistic. This paper, by contrast, looks at optimal strategic default behavior under long-term debt contracts when the default value is determined endogenously based on rational expectations on optimal future re-contracting.

This model extends Wang [37]'s model mainly in three points. First, with regard to the technological environment, the income process is Markov in infinite horizon, whereas Wang presumes individually and independently distributed (i.i.d., hereafter) income shocks. In his model, due to a lack of intertemporal links across stages, the equilibrium disclosure strategy is static, in that only a current shock triggers a disclosure in a history-independent way. Nakamura [29] extends Wang's model to have two-state Markov chain shocks, which are also restrictive. In contrast, this paper generalizes the Markov shock process to have a continuum of states. The Markovian income shocks are more realistic than the technological assumptions in the previous models and result in more relevant equilibrium default behavior under long-term debt contracts.

Second, this model has a common agency structure like the one presented by Epstein and Peters [17] and Peters [31]. There are four infinitely-lived risk-averse players: two borrowers and two lenders. Each of the borrowers maximizes his ex ante lifetime utility by designing a contract non-cooperatively (or competitively) to acquire the participation of one lender. This competitive structure enables us to draw competitive implications of the optimally designed debt. In particular, in contrast to previous default literature, the bor-

[7] For key differences of my model from his, see shortly below.

[8] Deterministic disclosure means that, if a player demands disclosure, then the disclosure is undertaken with probability one. Note that, contrary to Wang [37]'s model, the borrower has the right to demand disclosure by incurring the costs. This twist simplifies the outcome function form in the contract, in that the less informed lender designs a contract *ex ante* whereas the fully informed borrower undertakes all the *ex post* actions.

[9] For example, Bernanke, Gertler, and Gilchrist [2], Levin, Natalucci, and Zakrajšek [25].

rower's strategic default decision may not depend on exogenous autarky utility, but rather on competitively determined re-contracted continuation utility. Accordingly, this paper can overcome the above-mentioned underestimation of actual default probability.

Finally, this model has a continuous-time game structure.[10] The best feature of the continuous-time framework is mathematical tractability.[11] In particular, this continuous-time contracting model has two big advantages over discrete-time ones. First, from a theoretical standpoint, this continuous-time framework makes complex, dynamic Bayesian games tractable to achieve complete characterizations of the equilibrium by solving stochastic differential equations based on highly-established mathematical techniques.[12] In particular, it makes it possible to use a convenient Markov operator method, which provides observable implications of potentially rich families of Markov processes.[13] Second, from a practical viewpoint, by using those well-established continuous-time stochastic process techniques, this framework is useful in numerical applications to actual financial data. It can incorporate many essential, practical features of asset pricing and corporate finance such as (1) asset pricing implications, jump processes, and term structures of interest rates, (2) hidden entrepreneurial efforts, (3) human capital accumulation of disclosure technology, and (4) an optimal mix of debt and equity. Still, to show the dynamic role of default intuitively, this paper focuses on a relatively simple dynamic CSV situation. Such simplification enables us to obtain closed-from solutions regarding the positive role of default in credit-risk evaluation.

Technically, a solution method that this paper uses is new. Precisely, I solve for the optimal contract via an impulse control method of Øksendal and Sulem [30]. Contrary to continuous control problems, the state of the system is subject to jumps (i.e., "impulses") in an impulse control problem. The timing, number, jump size, and intensity of impulses are decision variables in the control problem. The borrower's default decision in this dynamic CSV model is a typical impulse control. Based on this method, I show that, under several mild regularities, there is a stationary equilibrium in which default occurs recurrently beyond re-contracting over time. In a mathematical context, this paper has a contribution of accomplishing particular closed-form solutions of the optimal impulse control problem under relevant economic environments, although such accomplishment is not easy in general. In particular, as a part of the solution, I characterize the borrower's optimization program as a stopping-time problem under information asymmetry via a stochastic maximization principle of Bismut [4] given some boundary conditions on a finite horizon $[0, \tau]$ where τ denotes the first default time.[14] The boundary conditions are optimally determined as a function of the

[10] Nakamura [28] formalizes this dynamic CSV game as a continuous limit of discrete-time games with fine grids.

[11] For example, DeMarzo and Sannikov [12], Biais et al. [3], Williams [38]. For more details, see below.

[12] There is one caveat: in general, we must be careful when constructing a continuous-time game. A discrete-time dynamic model can form an extensive-form game in a straightforward way by defining some relevant time interval of each stage (e.g., day, week, month, year, etc.) and the timing of events during a component game. In contrast, a continuous-time game has no natural notion of a "previous" stage before a point of time (Fudenberg and Tirole [19]). Thus, it often faces some difficulty with extensive-form interpretations. Against this problem, this paper shows that $\{t_-\} \cup [t, t + \triangle)$ is a relevant grid of an infinitesimal component game at a point of time t.

[13] For example, see Hansen and Scheinkman [21].

[14] The method of Bismut [4] is also used in Williams [38]. His paper assumes that the reported state variables are removed in

fully revealed state variables in the process of the optimal re-contacting. Two Hamiltonian adjoint processes (i.e., differentials of the Hamiltonian) associated with the income and the payment stand for the borrower's endogenous reservation utility and the shadow price of a hidden state. These adjoint processes encode history dependence for the borrower's default decision and incentive compatibility. Due to mathematical tractability, this paper is successful in showing reasonable sufficient conditions for the existence of solutions of the complex dynamic model and in proving the positive, dynamic role of default.

In relationships to previous literature, this paper is in line with a large literature on dynamic optimal contracting using recursive methods under asymmetric information environments, which started with a seminal paper of Green [20]. Recently, DeMarzo and Fishman [11] study a dynamic optimal capital structure in finitely horizontal discrete time when a borrower privately observes independent cash flows from his investment project and is able to enjoy costly diversion from them. Tchistyi [35] extends DeMarzo and Fishman [11]'s model into a two-state Markov chain model. Furthermore, DeMarzo and Sannikov [12] extend Tchistyi [35]'s model into an infinitely horizontal continuous-time framework. The paper of DeMarzo and Sannikov [12] is close to my paper in the sense of studying an optimal long-term defaultable contract under serially correlated technological environments in continuous time. Biais et al. [3] and Williams [38] also explore continuous-time contract models in a different context. Those papers study standard principal-agent communication games. In contrast, this paper explores a continuous-time CSV model in which costly disclosure causes strategic default jumps on an equilibrium payment path recurrently. This is a methodological contribution of my paper.

This paper is organized as follows. The next two sections define the physical and institutional environments. Section 4 defines the contracts and the strategies. Section 5 defines the equilibrium, solves for the optimal contract, and characterizes it numerically. The final section concludes.

2. ENVIRONMENT

Consider an economy with single non-storable consumption goods under uncertainty in infinite-horizon continuous time $\mathbb{T} = \{t | t \in [0, \infty)\}$.[15] The stochastic basis is a filtered space $(\Omega, \mathcal{F}, P)$, which satisfies the usual conditions for the filtration, characterized completely by a two-dimensional standard Brownian motion $\begin{bmatrix} W_t^s & W_t^0 \end{bmatrix}^\top$, the elements of which are independent of each other.[16]

mechanisms, in advance, in a communication game under information asymmetry. The assumption is restrictive. By contrast, this paper looks at a more general mechanism space that includes the reported and disclosed state variables in the CSV environment. Rather, as in standard communication games, the optimal revelation is the center of my attention in a default context.

[15] A technical appendix in the preceding working paper of this paper (namely Nakamura [28]) includes longer, technical descriptions of this section.

[16] As I will describe later, W^0 drives a sequence of income shocks while W^s drives a sequence of payment randomization shocks. Also, the superscript $\top$ represents a transpose of the matrix.

2.1. Players

The economy is populated with four infinitely-lived players: two identical borrowers and two identical lenders, indexed by $i = 1, 1', 2, 2'$. Each player i ranks a consumption profile $\{\gamma_i(t) \in \mathbb{R}_+\}_{t\in\mathbb{T}}$ by a time-separable utility of consumption characterized by an instantaneous utility function $f_i : \mathbb{R}_+ \to \mathbb{R}$ and a common instantaneous discount rate δ. In particular, for simplicity, f_i is of a CARA type with the absolute risk aversion parameter α for player $2, 2'$ and of a log type for player $2, 2'$: Given $\{\gamma_i(t)\}_{t\in\mathbb{T}}$, player i's ex ante lifetime utility level is $E_0^i \left[\int_{\mathbb{T}} e^{-\delta t} \{-\exp(-\alpha\gamma_i(t))\} dt\right]$ for $i = 1, 1'$ and $E_0^i \left[\int_{\mathbb{T}} e^{-\delta t} \ln(\gamma_i(t))dt\right]$ for $i = 2, 2'$ where E_0^i denotes an expectation operator conditional player i's information set at a point of time 0. Also, player $1, 1'$'s autarky (i.e., reservation) utility level is $U_0 \in \mathbb{R}$; player $2, 2'$'s is $V_0 \in \mathbb{R}$. For notational convenience, I will use female pronouns for the lenders (player $2, 2'$), and male for the borrowers (player $1, 1'$).

2.2. Technology

Each of player $2, 2'$ has one unit of indivisible physical input (or capital), but has no investment project. Each of player $1, 1'$ has an investment project that, if either player 2 or $2'$ (not both) transfers one unit of capital, could produce a predictable income process of the goods, denoted by $\{X(t)\}_{\mathbb{T}}$ for each $\omega \in \Omega$ (otherwise, zero production), but has no capital. The two projects are the same. The capital transfer takes place at (re-)contracting that can follow after default in which player 1 misses a promised payment. For simplicity, the capital does not depreciate over time. The investment project then starts its production with the capital. The income evolves as follows: with the initial value $X(0) \in \mathbb{R}_+$ given,

$$dX(t) = \mu_0 dt + \sigma_0 dW_t^0$$

where $\mu_0, \sigma_0 \in \mathbb{R}_+$ are constant. Whereas (μ_0, σ_0) are public information, the realization of the income is private information of player $1, 1'$ except for the initial level $X(0)$.

Also, a state verification (or disclosure) technology reveals the current income level to both player $2, 2'$ with perfect accuracy. The technology is available to player $1, 1'$. A disclosure process, denoted by d, is predictable. Specifically, player 1 can undertake disclosure at t_- for any $t > 0$. If $d(t_-) = 1$, disclosure is undertaken (otherwise, no disclosure). A point of time t is said to be a disclosure time if $d(t_-) = 1$. When player 1 undertakes disclosure, this technology requires a constant amount of the resources $C_X \in \mathbb{R}_+$ – call it *disclosure costs* – from his current income at the disclosure time and causes the time path of his income to decrease permanently relative to what it would otherwise be:

$$X(\tau) = X(\tau_-) - C_X.$$

The resource loss is deadweight loss. Higher costs mean a lower level of the disclosure technology (and vice versa). The value C_X is public information. By using this costly disclosure technology, player $1, 1'$ can control his own income process downward in such a discontinuous way (i.e., "impulsively").

3. INSTITUTIONAL STRUCTURE

This section describes an institutional structure of the (re-)contracting and the income allocation between the players in continuous time. Each player i's information set is denoted by $\mathcal{F}_{i,t_-}$ for t_- $(t \geq 0)$ – call it player i's private filtration – which is generated by the processes distinguishable to player i prior to t (i.e., at or prior to the left-limit time t_-). Since player $1, 1'$ try to take informational advantage in this CSV environment, $\mathcal{F}_{2,t_-}$ is coarser than $\mathcal{F}_{1,t_-}$ for all t. Let $E\left[\cdot|\mathcal{F}_{i,t_-}\right] = E_t^i\left[\cdot\right]$ denote player i's conditional expectation operator given $\mathcal{F}_{i,t_-}$. There is an arbitrarily small time duration dt for t such that during a time interval $\{t_-\} \cup [t, t + dt)$, an instant-t component game is played – call this interval grid t as well.[17]

A contract prescribes (1) a recommended participation probability $p \in [0, 1]$ and (2)a payment rule to a lender, in which $\{S(t), t \in \mathbb{T}\}$ denotes the payment trajectory as a continuous function of player 1's messages, the observed actions and outcomes, and the calendar time conditional on the participation. In particular, the institutional structure consists of two parts: (1) (re-)contracting and (2) dynamic games under a contracted mechanism.

3.1. Contracting

This economy starts with a contracting stage. At the initial point of time 0, each of player $1, 1'$ announces a contract. During the announcement, neither player 1 nor $1'$ can observe the contract announced by the other, while player $2, 2'$ can observe the two announced contracts. Next, each of player $2, 2'$ independently communicates with player $1, 1'$ by reporting player 1 (or $1'$)'s contract, denoted by $M(0) \in \mathbb{M}_\gamma$ ($\mathbb{M}_\gamma$ denotes a well-defined direct message space regarding contracts), privately to player $1'$ (or 1). Player $2, 2'$ does not necessarily tell the truth. In turn, each of player $1, 1'$ reports a recommended participation probability into his own contract. Let $P = \{0, 1\}$ denote the set of player 1's participation decision (1 denotes participation; 0 no participation). Let $\triangle(P) = [0, 1]$ (its element p_γ given a contract γ) denote the space of the recommended participation probability that player 2 announcing γ gives to player 1 at time 0 after receiving player 2's message $M(0) \in \mathbb{M}_\gamma$. Player 2 then chooses one contract from the two announced ones and enters into a bilateral contract with a chosen lender. The capital transfer then follows. If a contracting fails, then

[17] As Simon and Stinchcombe [34] discuss, there is no natural notion of the previous stage before a point of time t in a continuous-time game. In fact, generally, there may not exist a sequence of the discrete-time games that would converge to the continuous-time game (with some relevant topology) as the discrete-time grid goes to zero. This model is not an exception. That is because, by construction, player 1's report and strategic actions at a stage may cause some information flows across the following stages in an infinitesimal component game. Nakamura [28] shows this continuous-time game as a continuous limit by defining an appropriate topology. That is, for each t, I can define the very fine time grid $\{t_-\} \cup [t, t + dt)$ during which a component game is played.

the players must live in autarky. The process until the agreement is called a contracting stage (Figure 1). The contracts are exclusive so long as the contract is committed to after the agreement. Still, there exists

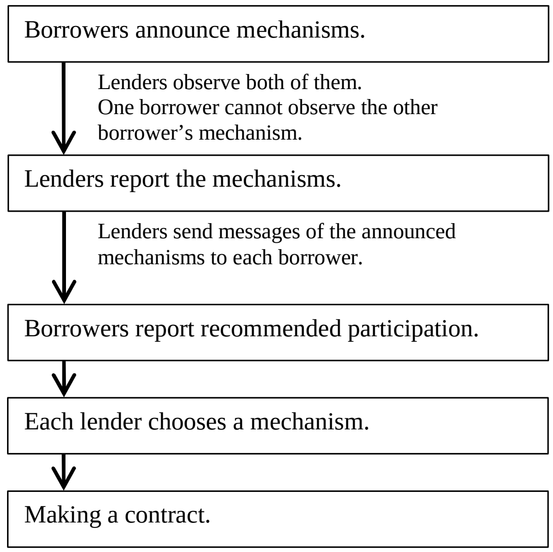

FIG. 1 Timing of events in a contracting stage

contractual externality through the participation probability ex ante. As I will describe below, this paper focuses attention on competition at this contracting stage and draws competitive implications of an optimal contract.

3.2. Dynamic game under a contracted mechanism

Next, a dynamic game starts between two contracting players (say, player $1, 2$) according to terms in the contract. Player 1's production starts with the invested capital, and the output is allocated between them. For $t > 0$, a component game evolves for a very short fine duration $\{t_-\} \cup [t, t + dt)$ (or grid t) (for the details of the "very fine time", see Nakamura [28]). The component game consists of three stages: commitment, production, and payment stages (Figure 2).[18] First, at the commitment stage t_- for $t > 0$, player 1 decides whether or not to terminate the contract. If player 1 does not terminate the contract, then the stage game moves on to the next stage. Or, if player 1 does, then player 2 repossesses the capital from player 1. The contract then ends; both the players are separated. This event is a specific meaning of *default* in this paper. At this point, player 1 has an additional option to disclose his true state or not. If player 1 does not disclose, he must live in autarky, without producing anything forever, as in Bolton and Scharfstein [6]. On the other hand, if disclosure is undertaken, then defaulting player 1 is given a chance to enter into a new contract with a lender only if disclosure occurs. That is, the component game then immediately moves back to a contracting

[18] I interpret that, at the initial point $t = 0$, the game starts with the production stage. This interpretation is relevant, because due to the disclosure costs, disclosure does not occur in grid 0 in equilibrium.

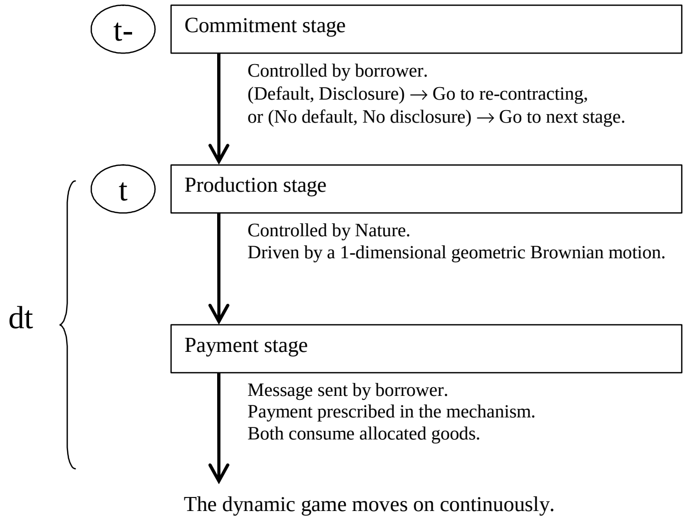

FIG. 2 Timing of events in a component game

stage at τ – call it *re-contracting.* Precisely, after both player $2, 2'$ observe player 1's current state $X(t_-)$, the income path is lowered discontinuously by the amount of the disclosure costs: $X(\tau) = X(\tau_-) - C_X$ at a disclosure time τ. In addition, he loses reputation, denoted by $R(X(\tau_-), S(\tau_-), S(\tau)) \in \mathbb{R}$, at the default time:

$$R(X(\tau_-), S(\tau_-), S(\tau)) \triangleq C_R \left[K + \{S(\tau_-) - S(\tau)\} - \{X(\tau_-) - S(\tau_-)\}\right].$$

where $C_R, K \in \mathbb{R}_+$ are constant. In other words, higher payment allowance beyond default (i.e., $S(\tau_-) - S(\tau)$) relative to his consumption level (i.e., $X(\tau_-) - S(\tau_-)$) results in higher reputation loss, multiplied by C_R, in addition to the constant reputation loss (i.e., $C_R K$). The game then goes back to a (re-)contracting stage.

The borrowers have an incentive to continue to keep a contractual relationship beyond default, so long as the contract is expected to promise them no smaller than the reservation utility. At the same time, they have no incentive to undertake disclosure while committing to the contract to save the disclosure costs. Also, not only the borrowers but also the lenders have an incentive to minimize the disclosure costs over time in order to maintain as much income as possible. Accordingly, player $1, 1'$ draw up a contract that seeks to balance two goals conditional on player $2, 2'$'s participation: (1) to make player $1, 1'$ reveal his true state as frequently as possible in order to prevent player $1, 1'$ from excessive exploitation of the informational rents and (2) to make player $1, 1'$ reveal his true state as infrequently as possible in order to reduce the disclosure costs. Thus, in equilibrium, disclosure does not occur when he does not default. Therefore, player 1's actions at this stage are characterized by two compound actions: {default, disclosure} and {no default, no disclosure}, unless the contract breaks the reservation utility.

Next, when default does not occur at the commitment stage, this component game moves on to a production stage. At this stage, the grid-t output is produced, as defined above, and reveals the true output amount (i.e., grid-t true state) only to player 1: $dX(t) = \mu_0 dt + \sigma_0 dW_t^0$.

Third, the component game proceeds to a payment stage. At this stage, player 1 sends a message of his current state, $M(t) \in \mathbb{M}_X$ ($\mathbb{M}_X$ denotes player 1's dated message space regarding his own current income levels), to player 2. The message is not necessarily true. Player 1 then makes a payment to player 2 as prescribed by the contract. At the end of the grid, they consume the allocated goods. The dynamic game moves on continuously.

4. CONTRACT FORMS AND STRATEGIES

With the above specifications, characterize the contract (say, between player $1, 2$) at date τ ($\tau \geq 0$[19]) as: $\gamma : \mathbb{R}_+ \times \mathbb{M}_\gamma \to \left\{\Phi^{\mathbb{M}_X}\right\}^P \times \triangle(P)$. That is, given the initial income level $X(\tau) \in: \mathbb{R}_+$, Φ denotes the set of payment processes S from player 1 to player 2, $P = \{0, 1\}$ (1 denotes participation; 0 no participation), and $\triangle(P) = [0, 1]$ (its element p_γ given a contract γ) denotes the space of the recommended participation probability that player 1 announcing γ gives to player 2 at τ after receiving player 2's message $M(\tau) \in \mathbb{M}_\gamma$. $\Phi^{\mathbb{M}_X}$ denotes the set of the payment process as a map from $\mathbb{M}_X$ to Φ, and $\left\{\Phi^{\mathbb{M}_X}\right\}^P$ denotes the set of the payment processes conditional on player 1's participation. I also call the outcome function γ as a contract. To make clear the contracting time τ_-, a subscript τ may be added to γ. Furthermore, define $\gamma := (\gamma_s, \gamma_p)$ where $\gamma_s \in \Phi^{\mathbb{M}_X}$ denotes a payment rule after the participation given the latest disclosed income level $X(\tau)$ and γ_p denotes a recommended participation probability. Accordingly, Date-t consumption levels of player 1 and player 2 can be written as $\gamma_1(t) = X(t) - \gamma_s(t) = X(t) - S(t)$ and $\gamma_2(t) = \gamma_s(t) = S(t)$.

A contract is said to be *feasible* if the payment is not larger than the whole income in any state at each time, i.e., $0 \leq \gamma_s(M(t); X(\tau)) \leq X(t)$ for any $M(t) \in \mathbb{M}_X$, and $t \geq \tau$ given $X(\tau)$ almost everywhere (a.e., hereafter), almost surely (a.s., hereafter) conditional on the participation. As I will show later, in an equilibrium, the borrower's strategic default necessarily ensures his strictly possible consumption. Therefore, the assumption is not restrictive. Let Γ_0 denote the set of the feasible contracts, endowed with some topology.

Moreover, this paper focuses on a particular form of contracts as follows. The payment $S(t)$ is predictable and is stationary Markovian in the sense that it is dependent only on the current actions and outcomes that player 2 can distinguish. In particular, I focus on continuous payment profiles except for discontinuities on the sample paths caused by default. Precisely,

[19] For notational convenience, the initial point 0 is interpreted as a default time in order to achieve the recursion of the model. As small abuse of mathematical language, 0_- is set to be its imaginary disclosure time.

Assumption 4.1. *For some predictable processes μ, σ and for $\tau_m \leq t < \tau_{m+1}$ ($m = 1, 2, ...$),*

$$dS(t) = \mu dt + \sigma dW_t^s$$

where τ_m denotes the mth state verification time.

Accordingly, μ represents the drift of the payment; dW_t^s is a randomization to conceal a pure choice of the payment level at each t. σ is the amplitude of the randomization. In particular, $\sigma = 0$ would mean no randomization in the contract.

This existence assumption of these predictable processes μ, σ is not quite restrictive for the continuous payment time path during a non-default phase. In other words, basically, the continuity of the payment process is the main restriction here.

Furthermore, let $\Sigma := \mathbb{R}_+ \cup \{0\}$. Let $\mathbb{A}$ denote an equicontinuous family of real-valued functions on $\mathbb{M}_X$ that is uniformly bounded on any closed interval on $\mathbb{M}_X$. By the Ascoli-Arzelá Theorem, $\mathbb{A}$ is relatively compact in a set of all the continuous mappings (Royden [32], theorems 40,41, p. 169).

Assumption 4.2. *$\sigma \in \Sigma$ and $\mu(M(t)) \in \mathbb{A}$.*

In other words, μ depends on player 1's messages in a stationary Markovian structure, whereas σ is constant, independent of player 1's messages. As to the constant σ, Assumption 4.2 looks restrictive. However, since the payment path is observable to both the players, based on the quadratic valuation of the process, the player 2 can infer the observed payment path at each instant. Therefore, Assumption 4.2 is not quite restrictive (also, see Cvitanić, Wan, and Zhang [10]). Accordingly, the continuous payment profile is characterized by $S(\tau_m) \in \mathbb{R}_+$ and its subsequent evolution by a geometric Brownian motion unless state verification occurs: for $\tau_m \leq t < \tau_{m+1}$ ($m = 1, 2, ...$),

$$dS(t) = \mu(M(t))dt + \sigma dW_t^s.$$

Also, the re-contracted payment is characterized by a deterministic function:

$$S(\tau) = \hat{S}(S(\tau_-), X(\tau_-)) \text{ for a disclosure time } \tau$$

where $S(\tau_-)$ denotes the payment level prescribed at the "previous" stage and $X(\tau_-)$ denotes the disclosed income level.

In summary, at time 0, γ_s is characterized by $(S(0), \mu, \sigma)$ in $\mathbb{R}_+ \times \mathbb{A} \times \Sigma$, rather than designs S complexly. Write $\gamma_s = (S(0), \mu, \sigma)$ as well. A contract is said to be *continuous* if it satisfies those particular specifications. Let Γ denote the set of continuous contracts in Γ_0. Assume that the set of continuous contracts Γ is nonempty. Define Γ_s as the set of γ_s corresponding to each $\gamma \in \Gamma$. Assume that there is no randomization across the elements of Γ.

5. OPTIMAL CONTRACT DESIGN

5.1. Equilibrium

This section solves for an optimal contract. I simplify the model by using several results that are obtained in standard CSV models (e.g., Townsend [36]) and in standard competitive mechanism design problems (e.g., Epstein and Peters [17], Peters [31]) and confine attention to a specific form of equilibria in the following three points.[20]

The first simplification is about a revelation principle in the common agency setting. The principals' contracts can depend on one another in complex ways: player 2's contract may depend whether player $2'$'s contract depends on player 2 contract depends...and so on. Thus the set of the agent's true states that matter in the contract designs must come from an infinite dimensional space even when the set of states are finite in a conventional sense. In particular, the announced contracts reveal some information beyond the reports of his own type. In other words, the surjection of the strategy mapping of the reporting is lost. Therefore, a standard direct-revelation principle does not hold for the communication games Γ. Thus, instead of resorting to a standard revelation principle, this paper confines attention only to the set of "menus" of payoff-relevant actions, like Peters [31] does.[21] That is, for any set of indirect mechanisms feasible for the lenders, and for any equilibrium relative to the set, there is an equilibrium in menus that preserves the corresponding equilibrium allocation, although *some* equilibria relative to such optimally designed menu may not be in the equilibrium allocations in an indirect mechanism. This paper achieves such a modified revelation principle by assuming that there always exists an equilibrium relative to the set of feasible indirect mechanisms such that equilibrium allocations relative to the set of the menus are preserved. In this sense, this paper focuses on a smaller set of indirect mechanisms than in standard revelation principles.

Second, with regard to the optimal payment rule, from the results of standard CSV models (e.g., Townsend [36]), μ should be written in a form that is independent of the borrower's messages. In CSV environments, player 1's messages are not able to deliver credible information because of the disclosure costs. That is, in CSV environments, any payment rule that could depend on ex post control is not in equilibrium, because such a rule would cause the borrower to report the lowest value that could avoid default when the outcome is in non-default region; hence, it is not incentive compatible. At the same time, player 2 has an incentive to minimize the disclosure costs from a dynamic perspective because he does not

[20] Note that Nakamura [28] studies a generalized version of this model and shows that such simplification does not lose generality.

[21] Epstein and Peters [17] proves the existence of a revelation principle by using a recursive method. However, as Peters [31] says, their method is not quite tractable practically.

like excessive shrinkage of the whole "pie" to be shared between the players. Thus player 2 permits player 1 to take some informational advantage without revealing the true state, but rather tries to secure a certain critical level of payment. Player 1's welfare must be indifferent with respect to whether or not to disclose at the critical payment level. Accordingly, the threshold is ex-post observable to player 2. Also, if μ depends on player 1's report, then he would always report the lowest income level among the permissible set. That is not incentive compatible. According to such standard CSV discussions, μ is constant between immediate disclosures. Player 2 prescribes the set of a payment process based on the payoff-relevant variables. The menu is independent of the reports. Player 1 is said to keep (break) his payment promise if he makes a payment that is consistent with the prescribed menu (otherwise). Player 1 keeps the payment promise unless the continuation utility goes below liquidation value that he receives when he defaults. In this model, the defaulting borrower decides whether or not to disclose and keep access to the contractual relationship. Hence, the liquidation value can take on two values: the financially autarkic value U_0 and the expected re-contracted continuation utility that is endogenously determined under a competitive contract design between player $2, 2'$ after the costly disclosure. According to standard CSV discussions, we can guess that, in equilibrium, the borrower chooses disclosure only in his bad shape to achieve a lower payment liability beyond the default. The verification plays a role of a credible excuse for the payment allowance. Furthermore, I can focus on a deterministic payment rule in equilibrium. Since the payment path is observable to both the players, any randomization in payment rule (i.e., $\sigma > 0$) would increase uncertainty under their contractual relationship, without increasing expected income returns. In equilibrium, it is unfavorable not only to the borrower but also to the lender. Therefore, $\sigma = 0$. In summary, $\{S(\tau), \mu\} \in \mathbb{R}^2$ characterizes the re-contracted payment rule.

Third, this section restricts attention only to a rational expectations equilibrium. In this framework, the re-contracted payment rule $\{S(\tau), \mu\}$ (and so the re-contracted continuation utility) is determined as a result of the competition among the players. At a re-contracting stage, each of the borrowers offers a contract that promises himself his willing-to-pay level so as not to lose in competition with the other borrower. On the other hand, each of the lenders accepts the offer to avoid loss in competition with the other lender. Based on the symmetry assumption of the strategies, this paper focuses on a rational expectations equilibrium in which each borrower enters into a symmetric contract with a lender, so long as it promises larger than the reservation utility.[22] The biggest advantage of this competitive treatment is that the contract is not necessarily threatened by the exogenous autarky threat, but by the borrower's willing-to-pay continuation utility. In standard principal-agent models, exogenous autarky utility level often plays a role of a threat. Since the threat is often unrealistically strong, most of those models underestimate actual default probability. In contrast, in my competitive contract design model, the re-contracted continuation utility is determined

[22] Strictly speaking, the negotiation may not reach at the equilibrium. See the detailed discussions in Epstein and Peters [17].

in an endogenous, competitive way. Reservation utility U_0, V_0 may not bind in the equilibrium. This setting seems more realistic in practice.

Now, this section studies player 1's optimal contract design that permits his strategic default, which is formulated as an impulse control in an incentive-comparative way when he can observe the true states privately, subject to reservation utility U_0, V_0, with $X(0), S(0)$ given.

Specifically, an impulse control for this system is a triple (possibly finite) sequence:

$$\upsilon \triangleq \{\tau_1, \tau_2, ..., \tau_j ...; \kappa_1, \kappa_2, ..., \kappa_j, ...; \mu_1, \mu_2, ..., \mu_j, ...\}$$

where τ_j denotes jth default time; $\kappa_j = \kappa(\tau_j) \triangleq S(\tau_{j-}) - S(\tau_j) = -dS(\tau_j)$ denotes a downward jump of the payment path (i.e., the payment allowance beyond default) at τ_j; additionally, in this model, player 1 controls $\{\mu_1, \mu_2, ..., \mu_j, ...\}$. Due to the disclosure costs, $\tau_1 < \tau_2 < \cdots < \tau_j < \cdots$. Let $Y(t) \triangleq \begin{bmatrix} X(t) & S(t) \end{bmatrix}^\top$ for $t \in \mathbb{T}$ define the *uncontrolled* state process. With an impulse control υ given, the corresponding state process $Y^{(\upsilon)}(t) \triangleq \begin{bmatrix} X^{(\upsilon)}(t) & S^{(\upsilon)}(t) \end{bmatrix}^\top$ is defined inductively by:

$$Y^{(\upsilon)}(t) = Y(t) \text{ for } 0 \leq t < \tau_1; \tag{5.1}$$

$$Y^{(\upsilon)}(\tau_1) = \begin{bmatrix} X^{(\upsilon)}(\tau_1) \\ S^{(\upsilon)}(\tau_1) \end{bmatrix} = \begin{bmatrix} X(\tau_{1-}) - C_X \\ S(\tau_{1-}) - \kappa_1 \end{bmatrix} \text{ for } t = \tau_1;$$

$$dY^{(\upsilon)}(t) = \begin{bmatrix} dX^{(\upsilon)}(t) \\ dS^{(\upsilon)}(t) \end{bmatrix} = \begin{bmatrix} \mu_0 \\ \mu_1 \end{bmatrix} dt + \begin{bmatrix} \sigma_0 & 0 \end{bmatrix} \begin{bmatrix} dW_t^0 \\ dW_t^s \end{bmatrix} \text{ for } \tau_1 \leq t < \tau_2;$$

$$Y^{(\upsilon)}(\tau_j) = \begin{bmatrix} X^{(\upsilon)}(\tau_j) \\ S^{(\upsilon)}(\tau_j) \end{bmatrix} = \begin{bmatrix} X(\tau_{j-}) - C_X \\ S(\tau_{j-}) - \kappa_j \end{bmatrix} \text{ for } t = \tau_j \ (j = 2, 3, ...);$$

$$dY^{(\upsilon)}(t) = \begin{bmatrix} dX^{(\upsilon)}(t) \\ dS^{(\upsilon)}(t) \end{bmatrix}$$
$$= \begin{bmatrix} \mu_0 \\ \mu_j \end{bmatrix} dt + \begin{bmatrix} \sigma_0 & 0 \end{bmatrix} \begin{bmatrix} dW_t^0 \\ dW_t^s \end{bmatrix} \text{ for } \tau_j \leq t < \tau_{j+1} \ (j = 2, 3, ...);$$

Therefore, the controlled process is lowered discontinuously and permanently at each default time than it would be otherwise. Note that, for physical consistency, this model imposes the following physical constraint:

$$S^{(\upsilon)}(t) \geq 0, X^{(\upsilon)}(t) \geq 0, X^{(\upsilon)}(t) - S^{(\upsilon)}(t) \geq 0 \text{ a.s. for all t.} \tag{5.2}$$

Define the explosion time of $Y^{(\upsilon)}(t)$ as:

$$\tau^* \triangleq \tau^*(\omega) = \lim_{R\to\infty}\left(\inf\left\{t>0; \left|Y^{(\upsilon)}(t)\right| \geq R\right\}\right)$$

Assume, then, that we are given a set $\mathcal{V}$ of *admissible* impulse controls that is included in the set of υ such that a unique solution $Y^{(\upsilon)}$ of the state evolution equations 5.1 exists and

$$\tau^* = \infty \text{ a.s.}$$

Also, before such an explosion, in this model, the contract may end when it cannot promise the reservation utility to either player. Let $\mathcal{S} \subset \mathbb{R}$, called a solvency region, define the set of the values of the state variables that promise no smaller than the reservation utilities to both the contracting players. Implicitly, this model presumes that the continuation utility (i.e., the remaining utility from now onwards) of the players is represented by a certain function of the current state variables levels in the Markovian environment. Assume that there exists such an open set $\mathcal{S}$ that $Y(0) \in \mathcal{S}$. Define another $\mathcal{F}_t$-stopping time $\tau_{\mathcal{S}}$

$$\tau_{\mathcal{S}} \triangleq \inf\left\{t \in (0,\tau^*), Y^{(\upsilon)}(t) \notin \mathcal{S}\right\}$$

That is, if either player is given smaller continuation utility than his reservation utility at the first time $\tau_{\mathcal{S}}$, then the contract is terminated and both players live in autarky from now onwards with U_0, V_0 given.

Now, define player 1's expected utility with an impulse control υ and $Y(0)$ given:

$$J^{(\upsilon)}(Y(0)) \triangleq E_0^1 \left[\begin{array}{l} \int_0^{\tau_{\mathcal{S}}} e^{-\delta t}\left[-\exp\left\{-(X(t)-S(t))\right\}\right] dt \\ -\sum_{\tau_j \leq \tau_{\mathcal{S}}} e^{-\delta\tau_j} C_R\left[K+\kappa_j - \left\{X\left(\tau_{j-}\right) - S\left(\tau_{j-}\right)\right\}\right] \\ +e^{-\delta\tau_{\mathcal{S}}} U_0 \chi_{\{\tau_{\mathcal{S}}<\infty\}} \end{array}\right] \tag{5.3}$$

subject to the state evolution equations (5.1) where $\chi_{\{\tau_{\mathcal{S}}<\infty\}}$ is an index indicator that takes 1 if $\tau_{\mathcal{S}} < \infty$ (otherwise 0). The expected value of each term on the right hand side of Equation (5.3), operated by E_0^1, is assumed to be finite for $Y(0) \in \mathcal{S}, \upsilon \in \mathcal{V}$. The impulse control problem is written as: Find player 1's value function, optimized expected discounted utility at 0 given $Y(0) \in \mathcal{S}$, denoted by $U(Y(0)) = U(X(0), S(0))$, and $\upsilon^* \in \mathcal{V}$ such that

$$U(Y(0)) = \sup_{\upsilon^* \in \mathcal{V}}\left\{J^{(\upsilon)}(Y(0))\right\} = J^{(\upsilon^*)}(Y(0)). \tag{5.4}$$

In the remaining, I suppress superscript (υ) on those controlled state variables, unless it causes any confusion.

5.2. Characterization of the optimal contract

Following Øksendal and Sulem [30] (in particular, Theorem 6.2, p.83)), this subsection shows below that player 1's optimization problem (5.4) is rewritten in the form of Hamilton-Jacobi-Bellman equations as follows: Find player 1's value function $U(Y(0)) = U(X(0), S(0)) = u(Y(0)) = u(X(0), S(0))$ and an optimal impulse control $v^* \in \mathcal{V}$ characterized inductively by $\{\tau, k(\tau), \mu\}$, in which $k(\tau) = S(\tau_-) - S(\tau)$ (time-τ payment allowance) and $u : \bar{\mathcal{S}} \to \mathbb{R} \in C^1(\mathcal{S}) \cap C(\bar{\mathcal{S}})$[23], such that

$$\mathcal{T}u\left(X\left(0\right), S\left(0\right)\right)$$

$$= \max_{\tau, k(\tau), \mu} E_0^1 \left[\begin{array}{l} \int_0^\tau e^{-\delta t} \left\{-\exp\left[-\alpha\left\{X\left(t\right) - S\left(t\right)\right\}\right]\right\} dt \\ +e^{-\delta\tau} \left\{ \begin{array}{l} u\left(X\left(\tau\right), S\left(\tau\right)\right) \\ -C_R\left[K + \kappa(\tau) - \left\{X\left(\tau_-\right) - S\left(\tau_-\right)\right\}\right] \end{array} \right\} \end{array} \right]$$

s.t. for $0 \le t < \tau$,

$$dX\left(t\right) = \mu_0 dt + \sigma_0 dW_t^0,$$

$$dS\left(t\right) = \mu dt;$$

for $t = \tau$

$$X\left(\tau\right) = X\left(\tau_-\right) - C_X;$$

$$u(X\left(0\right), S\left(0\right)) \ge U_0$$

$$V\left(X\left(0\right), S\left(0\right)\right) \triangleq E_0^2\left[\int_0^{\tau_{\mathcal{S}}} e^{-\delta t} \ln\left[S\left(t\right)\right] dt + e^{-\delta\tau_{\mathcal{S}}} V_0 \chi_{\{\tau_{\mathcal{S}} < \infty\}}\right] \ge V_0$$

where, for $\mathcal{H}$: the space of all measurable functions $h : \mathcal{S} \to \mathbb{R}$, $\mathcal{T} : \mathcal{H} \to \mathcal{H}$ denotes a one-step default operator. As in standard impulse control problems, put some technical assumptions:

(1) there is a region in $\mathcal{S}$, called the continuation region D, such that:

$$D \triangleq \left\{Y \in \mathcal{S}; u(Y) > \mathcal{T}u(Y)\right\}.$$

Intuitively, player 1 does not undertake default and keeps the promised payment so long as the state path is in this region.

(2) $u \in C^2(\mathcal{S} \backslash \partial D)$ with locally bounded derivatives near ∂D where ∂D denote the boundary of the set D. These assumptions are satisfied in numerical examples discussed below.

[23] For any set U, $C(U)$ means the continuous functions from U to $\mathcal{S}$. $C^k(U)$ denotes the functions in $C(U)$ with continuous derivatives up to order k. $\bar{U}$ denotes the closure of the set U.

Now, look at the characteristics of the equilibria in more details. Fix a triple $\{\tau_-, X(\tau_-), S(\tau_-)\}$ as given. Focus on the borrower's optimization at the left-limit time τ_-:

$$\max_{\kappa(\tau)} u\left(X\left(\tau\right), S\left(\tau\right)\right) - C_R\left[K + \kappa(\tau) - \left\{X\left(\tau_-\right) - S\left(\tau_-\right)\right\}\right]$$

The first-order condition with respect to $\kappa(\tau)$ (or $S(\tau)$) is:

$$u_s\left(X\left(\tau\right), S\left(\tau\right)\right) + C_R = 0. \tag{5.5}$$

where $u_s \triangleq \frac{\partial u}{\partial S}$. Assume that, for $\{\tau_-, X(\tau_-), S(\tau_-)\}$, there exists such a maximand $S(\tau)$, which is denoted by $S^*(\tau)$. Also, let $\kappa^*(\tau) \triangleq S(\tau_-) - S^*(\tau)$ denote the optimal downward jump of the payment path (i.e., $-dS(\tau_-)$) given $X(\tau_-)$ and $S(\tau_-)$, that is, date-τ payment allowance that is given to the borrower optimized conditional on $X(\tau_-)$ and $S(\tau_-)$. Assume also that the second-order condition is satisfied:

$$u_{ss}\left(X\left(\tau\right), S\left(\tau\right)\right) < 0$$

where $u_{ss} \triangleq \frac{\partial^2 u}{\partial S^2}$.

Define $Z(t) = X(t) - S(t)$. Correspondingly, with small abuse of language, change the notations of the value functions $U(Z(t)) = U(X(t), S(t))$, $u(Z(t)) = u(X(t), S(t))$, and $V(Z(t)) = V(X(t), S(t))$ for all t. Now, we can guess that the borrower's program is rewritten as follows: with $Z(0) = X(0) - S(0)$ given, find value function $U(Z(0)) = u(Z(0))$ and an optimal impulse control $\upsilon^* \in \mathcal{V}$ characterized inductively by $\{\tau\}$

$$\mathcal{T}u\left(Z\left(0\right)\right) = \max_{\tau} E_0^1 \left[\begin{array}{l} \int_0^\tau e^{-\delta t}\left\{-\exp\left[-\alpha Z\left(t\right)\right]\right\} dt \\ +e^{-\delta\tau}\left\{u\left(Z^*\left(\tau\right)\right) - C_R\left[K + \kappa^*(\tau) - Z\left(\tau_-\right)\right]\right\} \end{array}\right] \tag{5.6}$$

s.t. for $0 \leq t < \tau$,

$$dZ\left(t\right) = \left(\mu_0 - \mu\right) dt + \sigma_0 dW_t^0$$

for $t = \tau$,

$$Z^*\left(\tau\right) = Z\left(\tau_-\right) + \kappa^*(\tau) - C_X$$

$$u(Z(0)) \geq U_0, V(Z(0)) \geq V_0.$$

The problem is reduced into a stopping-time problem. Correspondingly, assume that $u(Z(t))$ is continuous and twice differentiable with respect to $Z(t)$. In particular, assume that for an arbitrarily large finite time

T, there exists the first stopping time τ in $[0, T]$. If this is not true, player 1 could exploit informational rents over time. That would not be in equilibrium. Hence, without loss of generality, I can focus the discussions on how player 1 undertakes the first default τ under a given contract and how player 2 designs the contract to induce player 1's default in his desirable way from his long-run (i.e., stationary) perspective. The above first-order and second-order condition are rewritten as:

$$u_z\left(Z^*\left(\tau\right)\right) = C_R \text{ and } u_{zz}\left(Z^*\left(\tau\right)\right) < 0. \tag{5.7}$$

where $u_z \triangleq \frac{du}{dZ}$ and $u_{zz} \triangleq \frac{d^2u}{dZ^2}$.

Next, this section characterizes the implementability of contract. Specifically, given such $S(0), \mu, \kappa^*(\tau)$, I change the measure to make the incentive-compatibility problem tractable. Define a predictable process:

$$\Pi(t) = \exp\left(\int_0^t \sigma_0^{-1}\left(\mu_0 - \mu\right) d\bar{W}_0(u) - \frac{1}{2}\int_0^t \left|\sigma_0^{-1}\left(\mu_0 - \mu\right)\right|^2 dt\right)$$

where $W_t^0 = \bar{W}_t^0 - \int_0^t \sigma_0^{-1}\left(\mu_0 - \mu\right) du$. By the above hypotheses, Novikov's condition is satisfied. Therefore, $\Pi(t)$ is a martingale with $E\left[\Pi(\tau_-)\right] = \Pi(0) = 1$. By the Girsanov theorem, I have a new measure $\bar{P}^0$:

$$\frac{dP^0}{d\bar{P}^0} = \Pi(\tau_-).$$

Call the distribution process Π the relative density process. Replace the original state variable Z with a new state variable pair Π. The evolution equations of the new state variables are rewritten as: for $0 \leq t < \tau$, given $\Pi(0) = 1$,

$$d\Pi(t) = \Pi(t)\sigma_0^{-1}\left(\mu_0 - \mu\right) d\bar{W}_t^0$$

By the measure change, Problem (5.6) is rewritten into: given $\Pi(0) = 1$,

$$u(Z(0)) = \max_{\tau} \bar{E}_0^1 \left[\begin{array}{l} \int_0^\tau e^{-\delta t}\Pi(t)\left\{-\exp\left[-\alpha Z(t)\right]\right\} dt \\ +e^{-\delta\tau}\Pi(\tau)\left\{u\left(Z^*\left(\tau\right)\right) - C_R\left[K + \kappa^*(\tau) - Z\left(\tau_-\right)\right]\right\} \end{array}\right] \tag{5.8}$$

subject to

$$d\Pi(t) = \Pi(t)\sigma_0^{-1}\left(\mu_0 - \mu\right) d\bar{W}_t^0 \text{ for } 0 \leq t < \tau;$$

$$Z^*\left(\tau\right) = Z\left(\tau_-\right) + \kappa^*(\tau) - C_X \text{ for } t = \tau;$$

$$u(Z(0)) \geq U_0, v(Z(0)) \geq V_0$$

where $\bar{E}_0^1$ denotes the expectation operator conditional on the information set $\mathcal{F}_{1,0}$ under the changed measure.

Following Bismut [4], I use a stochastic maximization principle in continuous time for $0 \leq t \leq \tau_-$. Let Ψ, Λ, respectively, denote the adjoint processes associated with Π and the target volatility of the adjoint process. Given τ, for $0 \leq t < \tau$, the Hamiltonian for this problem with the adjoint equations is:

$$\mathcal{H}_\Pi(\tau) = \Lambda \Pi \sigma_0^{-1} (\mu_0 - \mu) + \Pi \{-\exp(-\alpha Z)\} \tag{5.9}$$

subject to for

$$\begin{aligned} d\Pi(t) &= \Pi(t)\sigma_0^{-1}(\mu_0 - \mu)\, d(t)\bar{W}_t^0 \\ d\Psi(t) &= -\left[\frac{\partial \mathcal{H}_\Pi}{\partial Z}(t)\right] dt + \Lambda(t) d\bar{W}_t^0 \end{aligned} \tag{5.10}$$

given $\Pi(0) = 1$ and $\Psi(\tau_-) = \dfrac{\partial \{\Pi(\tau_-)u(Z(\tau_-))\}}{\partial Z(\tau_-)}$.

By a stochastic maximum principle,

Lemma 5.1. *There exist $\mathcal{F}_{1,t}$-predictable adjoint processes $\{\Psi(t), \Lambda(t)\}$, which satisfy the evolution equation (5.10). In addition, given $\kappa^*(\tau)$ induced by Equations (5.7), τ^* satisfies for almost every $t \in [0, \tau_-]$ a.s., $\mathcal{H}_\Pi(\tau^*) = \max_{\{\tau\}} \mathcal{H}_\Pi(\tau)$ in the Hamiltonian (5.9).*

Let the superscript $*$ of a variable denote its optimal value. From the boundary conditions with respect to the backward variables,

$$\Psi^*(\tau_-) = u^*(\tau_-). \tag{5.11}$$

For $0 \leq t \leq \tau_-$, $\Psi^*(t)$ represents player 1's reservation continuation utility level, which player 1 would accept without requesting a default at t. Hence, they constitute additional state variables in this optimization program. More specifically, the condition implies incentive compatibility, which only binds at τ_-, in terms of contract theory; at the same time, it implies a value matching condition in terms of control theory.

Now, I characterize a class of the implementable contracts. Define player 1's target controls as $\{\hat{\tau}, \kappa^*(\hat{\tau})\}$ under a given contract $S(0), \mu$. Let the association between a contract γ_s and player 1's target controls $\{\hat{\tau}, \kappa^*(\hat{\tau})\}$ be denoted by a contract correspondence $\Gamma_{\{\hat{\tau}, \kappa^*(\hat{\tau})\}}$, which is induced as a result of the optimal contract designs for each target $\{\hat{\tau}, \kappa^*(\hat{\tau})\}$. A contract is said to be *implementable* if $\{\hat{\tau}, \kappa^*(\hat{\tau})\}$ is an optimal control when player 1 faces the contract correspondence $\Gamma_{\{\hat{\tau}, \kappa^*(\hat{\tau})\}}$. By Lemma 5.1 above and the stochastic maximum principle of Bismut [4], Proposition 5.1. *A contract is implementable if and only if (1) the contract satisfies* $u(Z(0)) \geq U_0$ *and* $V(Z(0)) \geq V_0$*, (2) the contract and its optimal control* $\{\hat{\tau}, \kappa^*(\hat{\tau})\}$ *satisfies the solutions of the Hamiltonian (5.9) for* τ*, and (3) for almost every* $t \in [0, \tau_-]$*, a.s.* $\mathcal{H}_\Pi(\tau^*) = \max_{\{\tau\}} \mathcal{H}_\Pi(\tau)$.

The basic logic of the proof is the same as the one in Williams [38], because, from standard CSV discussions, the optimal mechanisms are independent of the borrower's reports except for default time in this CSV model. Let Γ^* denote the set of implementable contract γ. Correspondingly, define Γ_s^* with elements γ_s^*.

Due to the log utility of player 2, if $\mu < 0$, the payment becomes negative with some probability. That is not in Γ^*. Since player 1 competitively write the contract in a favorable way to himself,

Lemma 5.2. *An optimal contract in* Γ^* *has* $\mu = 0$.

This means that, if an optimal contract exists, it would pay a fixed coupon at every instant.

Finally, following Øksendal and Sulem [30] (in particular, Theorem 6.2, p.83)), I impose several technical assumptions (see the details in Nakamura [28]). Then, this paper obtains my main result:

Theorem 5.1. *There exist player* 1*'s value function* $U(Z(0)) = u(Z(0))$ *and an optimal impulse control* $\upsilon^* \in \mathcal{V}$ *characterized inductively by* $\{\tau^*, k^*(\tau^*), \mu = 0\}$ *via Equations (5.7),(5.11) and Lemma 5.2.*

This optimal contract takes the form of a *debt* contract in the sense that, (1) the contract promises the lender a fixed repayment, (2) the borrower has the control, (3) the borrower has the right to default strategically, and (4) debt reorganization is possible. More concrete characteristics are discussed in the next subsection.

5.3. Quantitative Example

The previous subsection characterized the optimal contract and the equilibrium default behavior under it. However, since I posed several high-level assumptions there, it is uncertain whether such equilibria exist in relevant environments. This subsection specifies the contract and the default behavior in a quantitative example.

First, for characterizing the allocations between the players completely, I confine attention to the following specific forms of the allocations:

Assumption 5.1. $u_z(Z(0)) = C_R$.

I.e., $Y(0) \in \partial D$. Especially, define $D_Z \triangleq \{Z \in \mathbb{Z}; Y \in D\}$ where $\mathbb{Z}$ denotes the set of $Z \in \mathbb{R}$ that is consistent with $Y \in \mathbb{R}^2$. This means that Equation (5.7) holds at initial point of time 0. Intuitively, since this paper focuses on stationary (i.e., long-run) characteristics of the equilibria, this model can presume that the contract is determined in equilibrium even at the initial point. In other words, we can imagine that this model picks up one default point as an initial point when the contractual relationship converges already to a stationary equilibrium. Also,

Assumption 5.2. *The shadow price of the payment is continuous at each default time for player* 1 *when the reservation utility is not binding.*

This condition is called the smooth-pasting condition in terms of control theory. More precisely, for a boundary $b \in \partial D$,

$$\lim_{z \in D \to b} u_z(Z) = u_z(b) = u_z(Z^*) + C_R = 2C_R \tag{5.12}$$

where $Z^* \triangleq b + \kappa^*(\tau) - C_X$ represents the borrower's renewed share after the costly default. The last equality uses the result of Equation (5.7).

Also, the value-matching condition, characterized by Equation (5.11), holds for implementability of the contract: at the same $b \in \partial D$,

$$\lim_{z \in D \to b} u(Z) = u(b) = u(Z^*) - C_R(K + \kappa^*(\tau) - b) \tag{5.13}$$

Furthermore, I confine attention of the impulse control problem with a one-sided barrier that triggers player 1's strategic default and debt restructuring. In standard impulse control models, mathematically, two barriers are possible for D_Z: $D_Z = (b, B) \subset \mathbb{R}$ $(b < B; b, B \in \mathbb{R})$ – call b lower barrier (or, floor) and B upper barrier (or, ceiling). In my framework, by contrast, due to sufficiently large disclosure costs, when player 1 has informational advantage over player 2 in his good shape, re-contracting would not improve player 1's welfare. In addition, liquidation would lead to financial autarky U_0. Hence, in this model, the barrier that triggers strategic default in equilibrium: $D_Z = (b, +\infty)$. In other words, the lower barrier provides player 1 a "put" opportunity: there is no upper barrier, whereas the floor b means that, if player 1's production becomes so low that his after-repayment income Z is below b, he undertakes default. More precisely, there exists an interval $(b, +\infty)$ and a point $Z^* \in (b, +\infty)$ such that, if z is in $(0, b]$, then the path immediately jumps to z^*, whereas if for some t, z is inside $(b, +\infty)$ and subsequently the path hits b from above, then it also jumps to z^*.

Inside the continuation region D (i.e., $Z \in (b, +\infty)$), the evolution of player 1's value function $u(Z)$ follows the Hamiltonian-Jacobi-Bellman (or HJB) equation (Figure 3):

$$\delta u(z) = \{-\exp(-\alpha Z)\} + \mu_0 u_z(Z) + \frac{\sigma_0^2}{2} u_{zz}(Z). \tag{5.14}$$

Now, under the physical constraint (5.2) and Assumptions 5.1,5.2, this subsection finds positive $\tau^*, S^*(\tau), Z^*(\tau)$

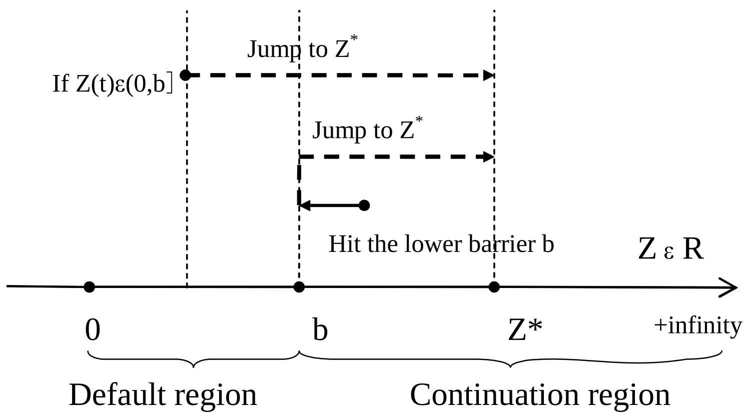

FIG. 3 Impulse control

characterized by Theorem 5.1 (in particular, Equations (5.7),(5.12) and (5.13)) in the following example:

$$X(0) = 4, \quad \mu_0 = -0.05, \quad \sigma_0 = 1, \quad C_X = 0.2, \quad C_R = 0.1, \quad K = 0.5,$$

$$\alpha = 1, \quad \delta = 0.03.$$

In consequence, this example reaches at a stationary equilibrium:

$$u(Z) = -0.1904 \cdot \exp(-Z) - 0.9521 \cdot \exp(\nu_1 Z) - 0.3014 \cdot \exp(\nu_2 Z)$$

where $\{\nu_1, \nu_2\} = \{-0.2, 0.3\}$ are the roots of the equation $\frac{\sigma_0}{2}\nu^2 + \mu_0\nu - \delta = 0$, and

$$u(Z(0)) = -1.2534, \quad Z^* = Z(0) = 1.0285, \quad b = 0.4076, \quad \kappa^*(\tau) = 0.8209.$$

Also, $u_{zz}(Z^*) = -0.136 < 0$ satisfies the second order condition at Z^*. Hence, by Assumption 5.1, $Z(\tau_j)$ and b_j are constant for each default number j. Correspondingly, the amount of the payment allowance is constant at each default time. Therefore, the optimal contract takes the form of a debt contract that pays a fixed coupon $2.9715 \cdot dt$ $(= \{X(0) - Z(0)\} \cdot dt = (4 - 1.0285) \cdot dt)$ for a fine time interval dt. The payment allowance beyond default is 0.8209 (Figure 4).[24] In particular, regardless of player 2's nonlinear utility, the optimality of the debt structure is preserved.

This model presumes that $Y(0) \in \mathcal{S}$ in this model. Specifically, this model sets U_0, V_0 such that U_0 is smaller than $u(Z(0)) = -1.2534$ and V_0 is smaller than the continuation utility promised by the stream of the fixed coupon $2.9715 \cdot dt$ with the expected default probability given. Hence, players enter into the

[24]This simulation is based on a simple discretization method. Hence, when the after-payment income path hits the lower barrier, the path goes below the b-line on the figure. In a continuous-limit, such crossover does not occur.

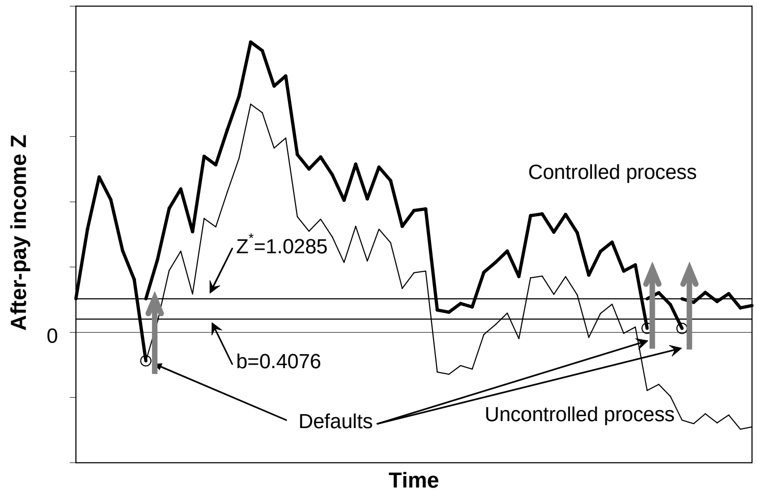

FIG. 4 Simulation

contract necessarily at the initial point 0. However, when μ_0 is not so high, recurrent disclosure results in shrinkage of the economy due to the disclosure costs. Because the payment allowance is constant, the share of each player decreases at every disclosure time. In the end, either of $U(Y(t)), V(Y(t))$ hits the reservation utility; this exit time is $\tau_{\mathcal{S}}$. The contract is then liquidated forever. On the other hand, when μ_0 is very high, default may not occur at all. That means that player 1 exploits excessive informational advantage throughout time.

With respect to the equilibrium default behavior, a default time is expected to arrive based on an endogenously formed probability distribution, while from player 1's viewpoint, a default occurs strategically. When the contract keeps to promise the reservation utility, the expected default probability at the initial point is:

$$\Xi(0) \triangleq \text{Prob}_{\{Z(0)=Z^*\}}\left[Z(\tau)=b\right] = \exp\left\{\nu_1(Z^*-b)\right\} = 0.8832.$$

In addition, with regard to *ex post* default probability expected by the less informed lender, in the continuation region D after default, player 2 observes the payment sample-path but does not observe the true income sample-path. Therefore, she expects the default based on the expected income. That is, at a point of time t with $\tau \le t < \tau^+$ (τ^+ is a default time that follows after τ), her expected default probability $\Xi(t)$ is:

$$\Xi(t) = \exp\left\{\nu_1\left[z^* + \left\{\mu_0 + \frac{\sigma_0^2}{2}\right\}(t-\tau) - b\right]\right\}$$

This is the Laplace transform of the default intensity, which is often used in previous credit-risk literature. These results give strategic insights into an exogenous default intesity that has been often used in the reduced-form credit-risk model (Duffie and Singleton [16]). In particular, such synthesis between the contract theory and the asset pricing theory is due to endogenously formed difference of the information sets (i.e., measurability) between the lender and the borrower.

Finally, look at comparative statics with respect to the disclosure technology. In general, the effect of the disclosure technology on the contract terms and the default probability is uncertain, because a higher level of the disclosure technology (i.e., lower C_X) causes two opposite effects: substitution effect and wealth effect. Specifically, on the one hand, lower C_X means less default costs and results in more frequent default (and debt restructuring) (substitution effect). On the other hand, a discontinuous downward jump of income path causes bigger wealth loss and makes the economy more risk averse to the default shock (wealth effect). Therefore, the total effect of the two is uncertain, and so depends on parametric assumptions. A comparative static with respect to the disclosure cost $C_X \in (0, 3.5]$ in the above example shows that, when the optimal contract promises the reservation utility, $\Xi(0)$ (the distance from player 1's renewed consumption level Z^* to the floor b, resp.) is increasing (decreasing) in the disclosure cost parameter C_X, while the payment allowance is correspondingly increasing (Figure 5). In this numeric analysis, the wealth effect overwhelms

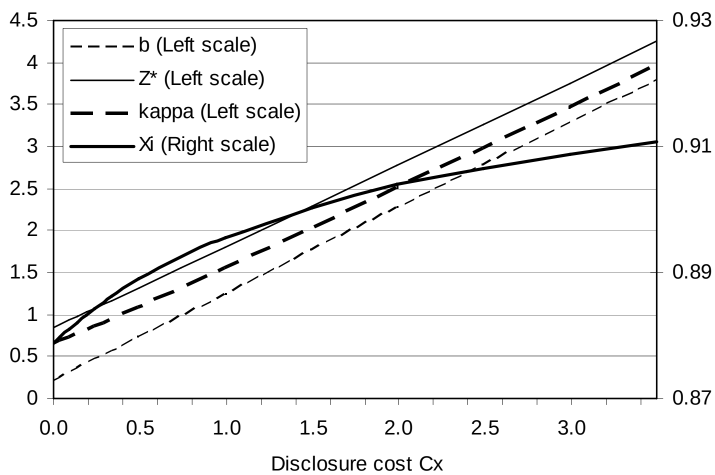

FIG. 5 Comparative static with respect to disclosure cost

the substitution effect. A discontinuous downward jump of income path in default tends to cause some big wealth effect.

6. CONCLUDING REMARKS

This paper studied dynamic CSV in continuous time in competitive environments and established a continuous-time, competitive model of the Markov communication game in costly verifiable information environments. This paper shows, first, that an optimal contract takes the form of a debt, in the sense that the payment profile is deterministic almost everywhere except for a countable, discrete set of the downward discontinuities at the default times and that the contract permits the borrower to default at any instant.

The optimal contract is ex ante describable, although the costly default itself is incontractible. Second, with respect to the equilibrium default behavior, state verification occurs when, and only when, a default occurs. Less informed lender expects equilibrium default time to arrive based on a endogenously formed default probability, while the fully informed borrower defaults strategically.

Based on mathematical tractability, this paper provides an analytical framework to examine the contract terms and the default probability under the optimally designed debt contract in relationships with structural parameters of technology (income growth and uncertainty, disclosure technology levels, and reputation costs) and of utility (risk aversion, intertemporal elasticity of substitution, and time preference). Accordingly, this model has many possibilities of future extensions and provides a better framework than before to analyze actual financial data, although they are out of the scope of this paper. For example, first, due to a general equilibrium framework with non-linear utility, this model is applicable to asset pricing models by incorporating security trading. It may tract the effect of ratings on credit-risk evaluation. Also, since lenders have standard CRRA-type power utility, this model can be extended directly into a long-run risk model with recursive utility in which intertemporal elasticity of substitution is unity and deal with term structures of interest rates.

Second, the income process can be modified. Since this model presumes a continuous income process. a continuous decrease in income triggers default predictably. In particular, financial autarky may occur only after recurrent debt restructuring. On the other hand, if income process involves jump terms, then a big negative jump in income may result in abrupt default. Financial autarky may then occur abruptly without trying to restructure debt.

Third, due to mathematical tractability, this framework can tract hidden entrepreneurial efforts of the borrower, together with the above-specified asymmetric information. In practice, a borrower's *ex post* lazy performance often seems to cause unnecessary default when lenders cannot verify his efforts. This is a serious moral hazard problem under a actual debt contract. Since this model can deal with hidden action and hidden information simultaneously, it can examine interactions between the two effects in corporate finance.

Fourth, human capital accumulation of the disclosure technology can be studied. In practice, rating agencies, auditors, and financial intermediation undertake a disclosure technology. The technology tends to deteriorate when financial innovations are advancing dramatically. Especially, in economic recessions, the disclosure technology is intensively demanded to measure economic uncertainty and evaluate bad-shaped firms. In fact, the productivity of the disclosure technology is time-varying. Several empirical results show that the disclosure costs are negatively correlated with business cycles in the US: disclosure procedures tend to be more costly in a recession period than in a booming one. Such pro-cyclicality of disclosure productivity implies that, in an economic recession, deteriorated disclosure ability might delay an economic recovery. Although this paper assumes a fixed level of disclosure technology, this framework can incorporate endogenous human capital accumulation of the disclosure technology. The application enables us to examine monitoring ability of financial intermediation and the regulation problems of rating agencies and auditors more effectively.

Fifth, this framework can deal with the optimality of a mix of debt and equity due to mathematical tractability. Due to such simplified structure, the above model is unable to explain optimal ownership of equity claims by outside lenders, as in standard CSV models. This result is unrealistic. In fact, this mitigates the direct applicability of this model to actual financial data. By contrast, by assuming that a part of income is observable, this model can be modified to explore an optimal mix of debt and equity in corporate finance.

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