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# Technology Shocks and Asset Price Dynamics: The Role of Housing in General Equilibrium

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**Abstract**

A general equilibrium model, that incorporates endogenous production and local housing markets, is developed in order to explain the price relationship among human capital, housing, and stocks, and to uncover the role of housing in asset pricing. Housing serves as an asset as well as a durable consumption good. It is shown that housing market conditions critically affect asset price correlations and risk premia. The first result is that the covariation of housing prices and stock prices can be negative if land supply is elastic. Data from OECD countries roughly support the model's predictions on the relationship among land supply elasticity, asset price correlations, and households' equity holdings. The second result is that housing rent growth serves as a risk factor in the pricing kernel. The risk premium becomes higher as land supply becomes inelastic and as housing services become more complementary with other goods. Finally, the housing component in the pricing kernel is shown to mitigate the equity premium puzzle and the risk-free rate puzzle.

JEL Classification: G12, E32, R20, R30

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# 1 Introduction

Household wealth typically consists of human capital, housing, and financial assets. The covariance of prices among these broad asset classes is critical to portfolio choice, asset pricing and consumption behavior. For example, a high covariance of stock prices with other asset prices suggests that a low weight be given to stocks, given that holdings of human capital and housing are constrained at some positive levels. A low or negative covariance among the assets, in turn, stabilizes household wealth and consumption.[1]

The actual covariance structure varies across countries as well as over time. In particular, in the U.S. housing and stock prices are negatively correlated, while in Japan they are positively correlated.[2] However, our theoretical understanding of the covariance structure among these broad asset classes is limited. General theories of asset pricing such as the Arrow-Debreu equilibrium and the no-arbitrage pricing condition are too general to yield concrete insights into the covariance structure, while more detailed models have been either purely empirical (with a focus on a particular financial asset) or else built on simplistic assumptions regarding the production process.[3]

In this paper, I develop a general equilibrium model in order to address two questions. First, what is the covariance structure among asset prices when we incorporate endogenous responses of production sectors to technology shocks? Second, what is the role of housing in the determination of equilibrium asset prices? By relying only on straightforward economic mechanisms, I derive the direct links between primitive technology shocks and the asset price responses.

The first of three main results is the finding of an equilibrium relationship among asset prices for different types of technology shocks. In particular, I show that the covariation of housing prices and stock prices can be negative if the supply of local inputs for housing production (e.g., land) is elastic and vice versa. This finding is supported by data from seventeen OECD countries. The key to understanding the result is dynamics of housing rents driven by housing supply. The result is suggestive of

[1]For example, it is widely believed that U.S. consumption since 2000 has been sustained in spite of depressed values of human capital and financial assets by the appreciation of housing prices.

[2]Cocco (2000) and Flavin and Yamashita (2002) find a negative correlation in the U.S. between stock and real estate prices using PSID. Chicago Mercantile Exchange also reports that housing displayed a negative correlation with the other asset classes over a ten-year period from February 1995 to February 2005. In contrast, Quan and Titman (1999) and Mera (2000) find a high correlation in Japan.

[3]Empirical models such as the Fama-French three factor model for equity returns are not based on complete theories. Theoretical models often reduce production processes to simply endowments (e.g., Lucas (1978)), render them implicit to the consumption process (e.g., Breeden (1979)), or posit an exogenous return/production process (e.g., Cox et al. (1985)).

the housing price appreciation observed under economic contraction in the U.S. between 2001 and 2003. This result also implies that an economy with inelastic land supply should exhibit either more limited stock-market participation or less homeownership because of positive covariation among asset prices. Data from seven OECD countries support the prediction by showing a positive relationship between land supply elasticity and households' equity holdings.

The second result is that housing market conditions influence the volatility of the pricing kernel, and thus the risk premium on any risky asset. Specifically, the risk premium becomes higher as land supply becomes inelastic, when housing services are relatively complementary to other goods. The risk premium further increases as two goods become more complementary. I show that growth of housing rent is a component of the asset pricing kernel if utility function is non-separable in housing and other goods. The pricing kernel is the ratio of marginal utility of consumption in different states of nature. The housing component enters into the pricing kernel because housing consumption affects marginal utility of consumption, depending on substitutability between housing services and other goods. The land supply determines supply elasticity and complementality determines demand elasticity of housing services. When either supply or demand of housing services is inelastic, housing rent is volatile, and so is the pricing kernel. Since the volatility of the pricing kernel determines the price of risk, risk premia are high under such conditions.

Finally, I present the possibilities that the rent growth factor in the pricing kernel mitigates the equity premium puzzle and the risk-free rate puzzle by either magnifying consumption variation or imposing a downward bias on the estimate of the elasticity of inter-temporal substitution (EIS). The model opens an empirical opportunity to apply a new data set to the Euler equation.

To derive these results, I introduce two key components: endogenous production and housing. The first component, endogenous production, characterizes asset prices and the pricing kernel in relation to different types of technology shocks. The pricing kernel is usually characterized by the consumption process without a model of endogenous production. Although real business cycle models are built on primitive technology shocks, they do not focus on asset prices but predominantly on quantity dynamics.[4] In this paper, I analyze shocks along three dimensions: time, space, and sector. On the time dimension, there are three types of shocks: 1) current, temporary shocks, 2)

[4] A few exceptions include Rouwenhorst (1995), Jermann (1998), and Boldrin et al. (2001) who study asset price implications of technology shocks. The current model differs from theirs in several ways, including the presence of local goods. Empirically, Cochrane (1991) and Cochrane (1996) relate marginal product of capital to the discount factor.

anticipated, temporary shocks, and 3) current, permanent shocks. Along the space dimension, shocks can occur in the "home" city or in the "foreign" city. In the sector dimension, shocks may have an effect on either consumption-goods production or housing production.

The second component of the model is housing. Housing is the major component of the household asset holdings, but it also has at least three unique characteristics.[5] First, housing plays a dual role: as a consumption good and as an investment asset. The portfolio choice is constrained by the consumption choice and vice versa. In particular, when the utility function is not separable in housing and other consumption goods, the housing choice affects consumption and asset pricing through the pricing kernel. Second, housing is a durable good, which introduces an inter-temporal dependence of utility within the expected utility framework. Inter-temporal dependence, which is also introduced via habit formation and through Epstein-Zin recursive utility, improves the performance of the asset pricing model. Third, housing is a local good, or a good that is not traded across different locations. Housing is supplied by combining a structure, which is capital traded nationally, and land, which is a local good. The demand for housing is also local since regionally distinct industrial structures generate regional variations in income. Localized housing generates important effects on the asset prices.

To give a clearer idea about the economics of the model, I illustrate the mechanisms that transmit a technology shock throughout the economy. A country is composed of two cities, each of which is formed around a firm. The capital and goods markets are national, while the labor, housing, and land markets are local. Technology shocks may have direct effects only on one city. For instance, suppose that a positive technology shock to goods-producing firms in a city raises the marginal products of capital and of labor, and hence changes interest rates and wages. The housing demand is affected by a higher lifetime income as well as a price change. The housing supply is also affected by the altered capital supply through the shifted portfolio choice. The other city, without the shock, is influenced through the national capital market. The capital supply to the other city is reduced due to the shifting portfolio choice across cities, and thus production and wages are reduced. Therefore, the responses of housing prices and the firms' use of capital become geographically heterogeneous. The shock also affects the next period through the inter-temporal consumption choice. The saving, or the capital supply to the next period, changes depending on the elasticity of the inter-temporal

[5] Real estate accounts for 30% of measurable consumer wealth, while equity holdings, including pension and mutual funds, are only 3/5 of real estate holdings based on 2002-4 Flow of Funds Accounts of the United States. Cocco (2004) reports, using PSID, that the portfolio is composed of 60-85% human capital, 12-22% real estate, and less than 3% stocks.

substitution. In sum, a shock has effects on the whole economy through consumption substitution between goods and between periods, and through capital substitution or portfolio selection between sectors and between cities. Different effects on the economy are analyzed for different types of technology shocks, whether temporary or permanent and whether in goods production or housing production.

The paper is organized as follows. Section 2 is a review of the related literature. In section 3 the model and the equilibrium are specified. In section 4 the equilibrium results under perfect foresight are presented. Section 5 presents the results when risks on technologies are introduced. Section 6 concludes and details my plan for extensions.

## 2 Related Literature

Most models of production economies are built on the assumption of a single homogeneous good; they focus on quantities rather than asset prices. Still, a small number of recent papers introduce home production, non-tradable goods or sector-specific factors, which are all relevant in the case of housing.

In a closed economy, home production of consumption goods helps explain a high level of home investment and a high volatility of output.[6] In these models, labor substitution between home production and market production plays an important role, while in the present model, capital substitution between sectors and between cities plays an important role. The housing service sector is introduced by Davis and Heathcote (2005) and two empirical regularities are explained: 1) the higher volatility of residential investment and 2) the comovement of consumption, nonresidential investment, residential investment, and GDP. They emphasize the importance of land in housing production and the effects of productivity shocks on the intermediate good sectors. However, the authors do not examine asset prices, which are the main concern here.

In an open economy, non-traded goods are introduced in the multi-sector, two-country, dynamic, stochastic, general-equilibrium (DSGE) model.[7] Non-traded goods in an open economy are comparable to local housing services and land in the current model. The important findings in this literature are that non-traded goods may help explain 1) the high correlation between savings and investment, 2) the low cross-country correlation of consumption growth, and 3) home bias in the investment port-

[6]See Greenwood and Hercowitz (1991) and Benhabib et al. (1991) among others. Boldrin et al. (2001) use a different division of production into the consumption-good sector and investment-good sectors.

[7]See Tesar (1993), Stockman and Tesar (1995), and Lewis (1996), among others.

folio. Again, price dynamics are not considered in this literature.

The asset pricing literature typically relies on a single good by implicitly assuming the separability of the utility function.[8] Accordingly, most empirical works put little emphasis on housing as a good, relying on a single category of good defined in terms of non-durable goods and services.[9] Housing is often taken into account in the portfolio choice problem in partial equilibrium.[10] Incorporating the high adjustment cost of housing leads to interesting results such as high risk aversion and limited stock-market participation. However, the implications of the analyses are limited in scope since covariance structures of returns are exogenously given. Others examine the lifecycle profiles of the optimal portfolio and consumption when housing is introduced.[11] These works are complementary to the research reported in this paper since they address non-asset pricing issues in general equilibrium.

Only a few papers examine the effects of housing on asset prices. Piazzesi et al. (2007) start from the Euler equation and examine the pricing kernel when the intra-period utility function has a constant elasticity of substitution (CES) form, which is non-separable in consumption goods and housing services. They show that the ratio of housing expenditure to other consumption, which they call composition risk, appears in the SDF. They then proceed to conduct an empirical study taking the observed consumption process as the outcome of a general equilibrium. Two key differences from the present model are 1) they do not include the link with technologies and 2) their housing is not distinct from other durable goods. Lustig and van Nieuwerburgh (2004) focus on the collateralizability of housing in an endowment economy. They use the ratio of housing wealth to human capital as indicating the tightness of solvency constraints and explaining the conditional and cross-sectional variation in risk premia. Their result is complementary to those reported below, as they show that another unique feature of housing, collateralizability, is important in asset pricing. Kan et al. (2004), using a DSGE model, show that the volatility of commercial property prices is higher than residential property prices and that commercial property prices are positively

[8] See for example Lucas (1978), Breeden (1979), Cox et al. (1985), Rouwenhorst (1995), and Jermann (1998).

[9] Exceptions include Dunn and Singleton (1986), Pakos (2003), and Yogo (2006), who take account of durable consumption. However, their durable consumption ignores housing in favor of motor vehicles, furniture, appliances, jewelry, and watches.

[10] The demand for housing or mortgages are considered by Henderson and Ioannides (1983), Cocco (2000), Sinai and Souleles (2004), Cocco and Campbell (2004), and Shore and Sinai (2004). The effects of housing on the portfolio of financial assets are considered by Brueckner (1997), Flavin and Yamashita (2002), Cocco (2004), and Chetty and Szeidl (2004), among others.

[11] See for example, Ortalo-Magne and Rady (2005), Platania and Schlagenauf (2000), Cocco et al. (2005), Fernandez-Villaverde and Krueger (2003), Li and Yao (2005), and Yao and Zhang (2005).

correlated with the price of residential property. Although housing is distinguished from commercial properties, its locality is not considered. In addition, their focus is also not on asset pricing in general but is limited to property prices.

# 3 The Model

## 3.1 Technologies

There are two goods: a composite good ($Y_t$) and housing services ($H_t$). The latter is a quality-adjusted service flow; larger service flows are derived either from a larger house or from a higher quality house.

Composite goods are produced by combining business capital ($K_t$) and labor ($L_t$), while housing services are produced by combining housing structures ($S_t$) and land ($T_t$).[12] The production functions are both Cobb-Douglas:

$$Y_t = Y(A_t, K_t, L_t) = A_t K_t^{\alpha} L_t^{1-\alpha}, \tag{1a}$$

$$H_t = H(B_t, S_t, T_t) = B_t S_t^{\gamma} T_t^{1-\gamma}, \tag{1b}$$

where $A_t$ and $B_t$ are total factor productivities of goods and housing production, respectively.[13] Parameters $\alpha$ and $\gamma$ are the share of capital cost in the outputs of composite goods and housing services, respectively.[14]

The production functions exhibit a diminishing marginal product of capital (MPK) so that the return depends on production scale, unlike in the linear technology case. This property, together with changing productivities, allows the return to vary over time and across states. Note also that a technology shock to housing production can be interpreted as a preference shock in the current model. This is because produced housing services directly enter into the utility function. A higher $B_t$ could be interpreted as implying that a greater utility is derived from the same level of structures and land and that the households are less willing to pay for housing due to their reduced marginal utility.

[12] The land should be interpreted as the combination of non-structural local inputs. In particular, it includes all local amenities raising the quality of housing service, such as parks. The land supply function is explained as a part of the households' problem.

[13] With the Cobb-Douglas production function, a total factor productivity shock can be described in terms of a shock to the capital-augmenting technology or as one to the labor-augmenting technology. For example, we can rewrite the production function as $Y = AK^{\alpha}L^{1-\alpha} = \left(A^{1/\alpha}K\right)^{\alpha} L^{1-\alpha} = K^{\alpha}\left(A^{1/(1-\alpha)}L\right)^{1-\alpha}$.

[14] These parameters also represent the elasticity of output with respect to capital in the Cobb-Douglas production function.

## 3.2 Resource Constraint

Composite goods are used either for consumption or investment. The resource constraint is

$$Y_t = C_t + I_t + J_t, \tag{2}$$

where $C_t$ is consumption, $I_t$ and $J_t$ the investment in business capital and housing structures, respectively. The equations defining the accumulation of business capital and housing structures are

$$K_{t+1} = (1-\delta_K) K_t + I_t, \text{ and} \tag{3a}$$

$$S_{t+1} = (1-\delta_S) S_t + J_t, \tag{3b}$$

where $\delta_K$ and $\delta_S$ are the constant depreciation rate of business capital and housing structures, respectively. I assume $\delta_K = \delta_S = \delta$ for simplicity.

Note that the inclusion of the housing structures makes housing services a durable good. Consumption of housing services is directly linked with the accumulated structures while the amount of the composite goods consumption is chosen under the constraint (2). This makes housing services different from other goods.

## 3.3 Preferences

Consumers' preferences are expressed by the following expected utility function:

$$U = E_0 \left[ \sum_{t=1}^{\infty} \beta^t u(C_t,\ H_t) \right] \tag{4}$$

where $E_0$ is the conditional expectation operator given the information available at time 0, $\beta$ is the subjective discount factor per period, $u(\bullet)$ is the intra-period utility function over composite goods ($C_t$) and housing services ($H_t$). In a two-period model with perfect foresight, the lifetime utility becomes

$$U = u(C_1, H_1) + \beta u(C_2, H_2).$$

The CES-CRRA (constant relative risk aversion) intra-period utility function is adopted:

$$u(C_t, H_t) = \frac{1}{1-\frac{1}{\theta}} \left( C_t^{1-\frac{1}{\rho}} + H_t^{1-\frac{1}{\rho}} \right)^{\left(1-\frac{1}{\theta}\right)/\left(1-\frac{1}{\rho}\right)}, \tag{5}$$

where $\rho > 0$ is the elasticity of intra-temporal substitution between composite goods and housing services, and $\theta > 0$ is the parameter for the elasticity of inter-temporal substitution. The simplest special case is that of separable log utility, $u(C_t, H_t) = \ln C_t + \ln H_t$, which corresponds to $\rho = \theta = 1$.

The non-separability between composite goods and durable housing in the CES specification delinks the tight relationship between the relative risk aversion and the elasticity of inter-temporal substitution. Even though the lifetime utility function has a time-additive expected utility form, the durability of housing makes the utility function intertemporally dependent.[15] With the non-separability of the CES function, the relative risk aversion is not simply $1/\theta$; it is defined as the curvature of the value function, which depends on durable housing. CRRA utility over a single good is a special case in which the curvature of the value function coincides with the curvature of the utility function.[16]

Other specifications that also break the link between relative risk aversion and EIS include habit formation and Epstein-Zin recursive utility. Habit formation is similar to durable consumption, but past consumption in the habit-formation model makes the agent less satisfied, while past expenditure on durables makes the agent more satisfied. Both habit formation and Epstein-Zin recursive utility are known to resolve partially the equity premium puzzle.

## 3.4 Cities

There are two cities of the same initial size, in each of which households, goods-producing firms, and real estate firms operate competitively. The variables and parameters of the city with technology shocks ("home" city) are denoted by plain characters ($C_t$, etc.) and those of the other ("foreign") city are denoted by starred characters ($C_t^*$, etc.).

Each "city" should not be interpreted literally. Instead, a "city" is understood to be a set of cities or regions that share common characteristics in their industrial structure and land supply conditions. For example, a technology shock to the IT industry mainly affects the cities whose main industry is the IT industry. A "city"

[15] It might seem that the utility is not specified over housing as a durable but as contemporaneous housing services produced by real estate firms. However, housing services depend on the real estate firms' past investments in the housing structure, which are analogous to the households' expenditure on durable housing. Indeed, "real estate firms" can be characterized as the internal accounts of households. These "real estate firms" are set up just to derive explicitly the housing rent.

[16] See Deaton (2002) and Flavin and Nakagawa (2004) for detailed discussions on the delinking of EIS and risk aversion. Yogo (2006) shows the importance of non-separability between durables and non-durables in explaining the equity premium. Limitations caused by homotheticity induced by the CES form are discussed in Pakos (2003).

in this paper represents the collection of such cities that are affected by the same technology shock.

## 3.5 Market Institutions and Equilibrium in a Two-Period Model with Perfect Foresight

I first derive the decentralized market equilibrium in a two-period model with perfect foresight. In section 5, I will introduce technological risks to the model. Figure 1 presents the time-line of economic activities.

[Figure 1: Time-line]

**(Goods-producing firm)** Goods-producing firms competitively produce composite goods by combining capital and labor. Each goods-producing firm in the home city solves the following problem in each period, taking as given interest rates $(i_1, i_2)$, wages $(w_{1,} w_2)$, and total factor productivities $(A_1, A_2)$. The firms in the foreign city solve the identical problem with possibly different variables and parameters.

$$\max_{K_t, L_t} Y\left(A_t, K_t, L_t\right) - \left(i_t - 1 + \delta\right) K_t - w_t L_t, \quad t = 1, 2.$$

This objective function is a reduced form in which the firm's capital investment decision does not explicitly show up and in which the firm only recognizes the periodic capital cost. (This simplification is possible because there is no stock adjustment cost.)

The first-order conditions define the factor demands of the goods-producing firm:

$$K_t \;:\; i_t - 1 + \delta = \frac{\partial Y_t}{\partial K_t} = \alpha A_t \left(\frac{L_t}{K_t}\right)^{1-\alpha}, \tag{6a}$$

$$L_t \;:\; w_t = \frac{\partial Y_t}{\partial L_t} = \left(1 - \alpha\right) A_t \left(\frac{K_t}{L_t}\right)^{\alpha}. \tag{6b}$$

As usual, the interest rate is equal to $1-\delta$ plus the marginal product of capital (MPK), and the wage is equal to the marginal product of labor. In equilibrium with perfect foresight, the national market for capital implies that capital allocations are adjusted until the interest rates are equated across sectors and cities. Wages are unique to the city since the labor market is local.

**(Real estate firm)** Real estate firms produce housing services by combining land and structures. Each real estate firm solves the following problem in each period, taking as given the housing rent $(p_1, p_2)$, the interest rate $(i_1, i_2)$, the land rent $(r_{1,}r_2)$, and the total factor productivity $(B_1, B_2)$. The firms in the foreign city solve identical problems with starred variables.

$$\max_{S_t,T_t} p_t H(B_t, S_t, T_t) - (i_t - 1 + \delta) S_t - r_t L_t, \quad t = 1, 2.$$

As noted, these "real estate firms" can be also interpreted as the internal accounts of households since homeowners are not distinguished from renters. Nevertheless, I prefer describing the real estate industry in order to obtain explicitly the housing rent.

The first-order conditions define the factor demands of housing production:

$$S_t \quad : \quad i_t - 1 + \delta = p_t \frac{\partial H_t}{\partial S_t} = \gamma B_t p_t \left(\frac{T_t}{S_t}\right)^{1-\gamma}, \tag{7a}$$

$$T_t \quad : \quad r_t = p_t \frac{\partial H_t}{\partial T_t} = (1-\gamma) B_t p_t \left(\frac{S_t}{T_t}\right)^{\gamma}. \tag{7b}$$

The interest rate and the land rent are equal to the marginal housing product of structure (MHPS) and of land (MHPL), respectively, in units of the numeraire. Again, the interest rate will be equated across sectors and cities in equilibrium, while the land rent is locally determined.

**(Households)** Households are endowed with initial wealth $(W_0)$ and land. They provide capital, land, and labor in each period to earn financial, land, and labor income, respectively, and spend income on consumption of composite goods, housing services, and savings $(W_1)$. The savings can be freely allocated among sectors and cities.

Labor is inelastically supplied and normalized at one. Households are assumed to be immobile across cities. This assumption is reasonable since most of the population does not migrate across regions. The immobility of labor will result in wage differentials across cities. The free mobility of households would make labor more like capital and render the production function linear in inputs. The costs of capital and labor would be equated across cities and the price responses would become more moderate. While the mobility would generate more moderate results on the asset price, it would not greatly change the overall results as long as homothetic CES preferences are maintained.[17]

Land supply is assumed to be iso-elastic:

$$T_t = r_t^{\mu}, \quad t = 1, 2,$$

[17] With CES preferences, the income elasticity of housing demand is one. Therefore, even if the housing demand per household is altered by the wage income, the offsetting change in the population will limit the effects on total housing demand.

where $\mu$ is the price elasticity of supply. $\mu = 0$ represents a perfectly inelastic land supply at one and $\mu = \infty$ represents perfectly elastic land supply. By this simple form, land supply elasticity and asset prices are linked in a straightforward way. The land supply function reflects the marginal cost of making land in good condition for residential use, which is implicit in the model. While the land supply is obviously constrained by the topographic conditions of the city, other conditions such as zoning regulations and current population densities are also critical. For example, the infill development and the conversion from agricultural to residential use make the land supply elastic. The elasticity can also be understood as reflecting short-run and long-run elasticities. For example, if eminent domain is politically hard to use in providing a local amenity or if the current landlords rarely agree on redevelopments, the housing supply process may take longer than a business cycle, in which case the land supply is more inelastic.[18]

Each household solves the following problem, taking as given the housing rents, land rents, interest rates, and wages.

$$\begin{gathered}\max_{\{C_t,H_t\}} u\left(C_1,H_1\right)+\beta u\left(C_2,H_2\right)\\ s.t.\ C_1+p_1H_1+W_1=i_1W_0+r_1T_1+w_1\\ C_2+p_2H_2=i_2W_1+r_2T_2+w_2.\end{gathered}$$

The above dynamic budget constraints can be rewritten as the lifetime budget constraint:

$$\begin{aligned}C_1+p_1H_1+\frac{1}{i_2}\left(C_2+p_2H_2\right) &= i_1W_0+r_1T_1+w_1+\frac{1}{i_2}\left(r_2T_2+w_2\right)\\ &\equiv Inc.\end{aligned}$$

The RHS of the lifetime budget constraint is defined as the lifetime income, $Inc$.

[18] Many development projects in Japan take more than twenty years to complete. This is an example of an inelastic supply due to the slow development process.

The first-order conditions for the CES-CRRA utility are[19]

$$
\begin{aligned}
p_t^{\rho} H_t &= C_t, \text{ and} && \text{(8a)}\\
i_2 &= \left(\beta \frac{\partial u/\partial C_2}{\partial u/\partial C_1}\right)^{-1} \\
&= \frac{1}{\beta}\left(\frac{C_2}{C_1}\right)^{\frac{1}{\theta}} \left[\frac{1+(H_2/C_2)^{1-1/\rho}}{1+(H_1/C_1)^{1-1/\rho}}\right]^{\frac{\theta-\rho}{\theta(1-\rho)}}. && \text{(8b)}
\end{aligned}
$$

The interest rate is the reciprocal of the inter-temporal marginal rate of substitution (IMRS). That is, the IMRS is the pricing kernel in this economy. In the log utility case, the interest rate is proportional to consumption growth because of the unit elasticity of inter-temporal substitution. The inter-temporal consumption substitution expressed by this Euler equation, together with the intra-temporal substitution between two goods, is a key driver of the economy. The IMRS is discussed, in greater detail, in Section 5.2 since it is a key to understanding the economy.

With the lifetime budget constraint, I obtain the consumption demands:[20]

$$
\begin{aligned}
C_1 &= \left(1+p_1^{1-\rho}\right)^{-1}\left[1+\beta^{\theta} i_2^{-(1-\theta)}\left(\frac{1+p_2^{1-\rho}}{1+p_1^{1-\rho}}\right)^{\frac{1-\theta}{1-\rho}}\right]^{-1} Inc, && \text{(9a)}\\
C_2 &= \beta^{\theta} i_2^{\theta}\left(\frac{1+p_2^{1-\rho}}{1+p_1^{1-\rho}}\right)^{\frac{\rho-\theta}{1-\rho}} C_1, && \text{(9b)}\\
H_t^{dem} &= \frac{C_t}{p_t^{\rho}}. && \text{(9c)}
\end{aligned}
$$

Note that the housing rents have an effect on the consumption demand in general, while they have no effect in the log utility case. It is also clear that the expenditure ratio of housing, $p_t H_t / C_t$, is $p_t^{1-\rho}$ in general, while it is always 1 in the log case.

## 3.6 Definition of the Equilibrium

Markets are for composite goods, housing services, land, labor, and capital. Walras' law guarantees market clearing in the goods market, and the market-clearing conditions are imposed for the other markets. The multi-sector structure necessitates a numerical solution. Detailed derivation of the equilibrium is shown in the appendix.

[19] In the log-utility case, they reduce to $p_t H_t = C_t$ and $i_2 = \left(\beta \, \frac{\partial u}{\partial C_2} \Big/ \frac{\partial u}{\partial C_1}\right)^{-1} = (1/\beta)\,(C_2/C_1)$.

[20] In the log-utility case, they reduce to $C_1 = Inc/\left[2\,(1+\beta)\right], C_2 = \beta i_2 C_1$, and $H_t^{dem} = C_t/p_t$.

**Definition 1** ***A competitive equilibrium*** *in this 2-period, 2-city economy with perfect foresight is the allocation* $\{C_t, C_t^*,\ H_t, H_t^*,\ W_1, W_1^*,\ Y_t, Y_t^*,\ K_t, K_t^*,\ L_t, L_t^*,\ S_t, S_t^*,\ T_t, T_t^*\}_{t=1,2}$ *and the prices* $\{p_t, p_t^*,\ w_t, w_t^*,\ i_t,\ r_t, r_t^*\}_{t=1,2}$ *such that*
*1. optimality is achieved for households, goods-producing firms, and real estate firms and*
*2. all market-clearing conditions and resource constraints are met.*

# 4 Results with Perfect Foresight

The goals are to understand 1) the observed dynamic relationship among various asset classes, 2) the relationship between asset prices and business cycles, and 3) the role of housing in the economy. Different types of technology shocks are introduced as follows.

| | Goods Production | Housing Production |
|---|---|---|
| Temporary, current | $\Delta A_1 \gtrless 0$ | $\Delta B_1 \gtrless 0$ |
| Temporary, anticipated | $\Delta A_2 \gtrless 0$ | $\Delta B_2 \gtrless 0$ |
| Permanent, current | $\Delta A_1 = \Delta A_2 \gtrless 0$ | $\Delta B_1 = \Delta B_2 \gtrless 0$ |

Technology shocks are given to the home city. Different parameter values are allowed for

$\mu$ : Elasticity of land supply,
$\rho$ : Elasticity of intra-temporal substitution between $C$ and $H$,
$\theta$ : Parameter for inter-temporal substitution.

## 4.1 Effects on the Pricing Kernel

Let $\phi_{t,t+1}$ denote the pricing kernel for time $t+1$ as of time $t$. The price of any asset is expressed as the expected return in units of the numeraire multiplied by the pricing kernel. For example, the ex-dividend equity price of a firm, $e_t$, is expressed in terms of the dividend stream $D_t$ and the pricing kernel as

$$e_t = E_t\left[\sum_{j=1}^{\infty} \phi_{t,t+j} D_{t+j}\right].$$

The one-period risk free rate of return, $i_t$, is obtained by considering a bond that pays off 1 unit of numeraire good in the next period:

$$\frac{1}{i_t} = E_t\left[\phi_{t,t+1}\right].$$

Without uncertainty, the relationship in expectation becomes the exact relationship:

$$\begin{aligned} e_t &= \sum_{j=1}^{\infty} \phi_{t,t+j} D_{t+j}, \\ \frac{1}{i_t} &= \phi_{t,t+1}. \end{aligned}$$

The pricing kernel in the current model is expressed in three different ways by manipulating (6a), (7a), and (8b):[21]

$$\begin{aligned} \phi_{1,2} &= \left[1 + \frac{\partial Y_2}{\partial K_2} - \delta\right]^{-1} && \text{(Reciprocal of MPK)} \qquad (10a) \\ &= \left[1 + p_2 \frac{\partial H_2}{\partial S_2} - \delta\right]^{-1} && \text{(Reciprocal of MHPS)} \qquad (10b) \\ &= \beta \left(\frac{C_2}{C_1}\right)^{-\frac{1}{\theta}} \left[\frac{1 + p_2 H_2/C_2}{1 + p_1 H_1/C_1}\right]^{\frac{\rho-\theta}{\theta(1-\rho)}} && \text{(IMRS).} \qquad (10c) \end{aligned}$$

Analogous relationships hold for the foreign city as well. Indeed, the pricing kernel is the center piece that is common to all agents in the economy. The first equation (10a), which is empirically exploited by Cochrane (1991), is used to understand the effect of goods-sector shocks. The second equation (10b) is useful when considering housing shocks. The third equation (10c) includes the expenditure share of housing consumption, which Piazzesi et al. (2007) call the composition risk and empirically exploit.

The consumption growth, however, is not independent of housing expenditure. The consumption of composite goods, housing consumption, and housing rents are determined in general equilibrium and their changes cannot be identified merely with reference to the first-order conditions. Indeed, I show that the relationship between the consumption growth and the pricing kernel changes signs depending on parameter values and the type of shock involved.

The analyses on equilibrium responses to a technology shock provide a fresh look at several related results: Tesar (1993), who considers an endowment shock to the

[21] For the log utility, IMRS reduces to $\phi_{1,2} = \beta\left(C_2/C_1\right)^{-1}$.

non-tradables; and Piazzesi et al. (2007), who consider the relationship between the pricing kernel and the expenditure share of housing. In particular, it is shown that the housing component may mitigate the equity premium puzzle and the risk-free rate puzzle. The characterization of the pricing kernel using the housing component provides an opportunity to use different data sets in empirical analyses.[22]

Figure 2 presents selected comparative statics of the interest rate and savings. They serve as the basis for understanding the asset price relationship. With a positive shock to goods production ($\Delta A_t > 0$), the marginal product of capital becomes higher at any level of capital. The equilibrium interest rate ($i_t$) rises, or equivalently, the pricing kernel ($\phi_{t-1,t}$) falls although more capital ($K_t$) is allocated from the foreign city. These effects hold regardless of parameters (Figure 2-a). The interest rate in the other period is also affected via savings, as an increase in the lifetime income motivates households to smooth consumption by adjusting their savings ($W_1$). With $\Delta A_1 > 0$, the savings at $t = 1$ (capital supply for $t = 2$) are raised and $i_2$ falls ($\phi_{1,2}$ rises) (Figure 2-b). With $\Delta A_2 > 0$, the reduced savings at $t = 1$ allow a greater demand for goods at $t = 1$ and generally raise $i_1$ (lowers $\phi_{0,1}$) although the effects are much smaller due to the fixed capital supply.

[Figure 2: Effects on the interest rate and savings]

If a positive shock is given to housing production ($\Delta B_t > 0$), the effects are much smaller. Although housing production ($H_t$) increases, expenditures ($p_t H_t$) are less affected since the rent ($p_t$) decreases. The marginal housing product (i.e., the interest rate) may even fall if the housing rent falls enough. The effects on the contemporaneous pricing kernel depend on the rate of substitution between the goods. If the intra-temporal substitution ($\rho$) is low (i.e., the two goods are complements), the contemporaneous pricing kernel ($\phi_{t-1,t}$) rises.[23] The reason is as follows. A low intra-temporal substitution means a low price elasticity of housing demand. The increased housing consumption necessitates a much greater reduction in housing rent ($p_t$) so that the housing expenditure ($p_t H_t$) decreases. The marginal housing product of structure also falls, which means that the pricing kernel rises. If the substitution is high, the opposite is true and the pricing kernel falls. With the log utility, $\Delta B_t$ has no effect on the pricing kernel (Figure 2-c).

[22] Housing rent data have several advantages over housing consumption data in terms of their availability and accuracy.

[23] To be precise, $\theta$ also has a secondary effect on $\phi_{0,1}$ since the inter-temporal substitution affects capital demand. The effect of $\theta$ is more apparent when the shock is temporary.

The other period is again affected through inter-temporal substitution. Since the effects on lifetime income are quite small, the inter-temporal substitution rather than the consumption smoothing may come into play if $\theta$ is large. Consider $\Delta B_1 > 0$ (Figure 2-d). As $\theta$ becomes large, future resources are shifted toward the current period as savings are reduced. This raises $i_2$. If $\theta$ is small, the savings are increased (for consumption smoothing) and $i_2$ falls.[24] With $\Delta B_2 > 0$, the same mechanism affects savings although the effects on $\phi_{0,1}$ are small due to the fixed capital supply. As $\theta$ becomes large, the current capital demand is reduced by the increased savings, and $i_1$ falls. The general equilibrium effects on the pricing kernel are summarized in Table 1.

[Table 1: Effects on the pricing kernel]

## 4.2 Effects on Asset Prices

Three asset classes are considered: financial assets, housing, and human capital. The prices of housing and human capital are defined as the present discounted values of housing rent and wages, respectively, for a unit amount of the asset:

$$\text{(Housing Price)}_0 = \phi_{0,1}p_1 + \phi_{0,1}\phi_{1,2}p_2, \tag{11a}$$

$$\text{(Human Capital Price)}_0 = \phi_{0,1}w_1 + \phi_{0,1}\phi_{1,2}w_2. \tag{11b}$$

The change in the asset price is determined by possibly competing factors on the RHS of (11a) and (11b).

The financial asset price is equivalent to the price of the installed business capital because firms are fully equity-financed. However, without capital adjustment costs as in the current model, the price of business capital is always one. If adjustment costs are introduced, the financial asset price will change in the same direction as the equilibrium quantity of capital employed in goods production ($K_t$), as disscussed by Geanakoplos et al. (2002) and Abel (2003). It is because the price of capital deviates from one during the capital adjustment process toward a new equilibrium. The price gradually approaches one as capital reaches the equilibrium. Since I am interested in the sign of price correlations, I take the change in equilibrium capital as a proxy for the change in financial asset price.

[24] To be precise, $\rho$ has a secondary effect on $\phi_{1,2}$ since intra-temporal substitution affects capital demand.

### 4.2.1 Effects on Housing Prices

The equilibrium housing price goes up in the following cases.

$$
\begin{array}{ll}
\text{Case 1}\left\{\begin{array}{l}\text{A positive shock to goods production } (\Delta A_t > 0)\text{, and} \\ \text{inelastic land supply (small } \mu).\end{array}\right. \\
\text{Case 2}\left\{\begin{array}{l}\text{A negative shock to goods production } (\Delta A_t < 0)\text{, and} \\ \text{elastic land supply (large } \mu). \\ \text{For } \Delta A_2 < 0\text{; additionally, small } \rho \text{ and small } \theta.\end{array}\right. \\
\text{Case 3}\left\{\begin{array}{l}\text{A negative shock to goods production in the foreign city.} \\ \text{For } \Delta A_2^* < 0\text{; additionally, elastic land supply (large } \mu), \\ \qquad\qquad\qquad\qquad\qquad \text{small } \rho\text{, and small } \theta.\end{array}\right. \\
\text{Case 4}\left\{\ \text{A negative shock to housing production } (\Delta B_t < 0).\right.
\end{array}
$$

In case 1, the housing rent ($p_t$) rises at the time of a shock since the numeraire good becomes cheaper. The rent increase is greater if the land supply is more constrained (small $\mu$), since the shift in housing demand results in a greater price change.[25] Although the pricing kernel ($\phi_{t-1,t}$) and rent may be lower in the other period, the overall effect on housing prices is positive because of a large positive response of rent. With the elasticity of land supply around 0.8 or less, a positive shock leads to the appreciation of housing prices (Figure 3-a). If land supply is more elastic, housing prices exhibit the opposite response, which constitutes Case 2 (Figures 3-b and 3-c). A negative shock to housing production also results in the appreciation of housing prices by increasing rent (Figure 3-d).

[Figure 3: Effects on housing prices]

Cases 2 and 3, in which a negative shock to goods production leads to housing price appreciation, provide an interesting insight into the appreciation of housing prices in the United States after 2000. This appreciation occurred in a stagnant economy and with stock prices at a low. A key driver in the model is high future rents induced by reduced housing supply in the future.

Consider a current negative shock to goods production of the home city ($\Delta A_1 < 0$) in a land-elastic economy (Case 2). There are competing forces in the housing-price

[25] The intra-temporal substitution ($\rho$) also has a secondary effect. If the intra-temporal substitution is low, the price elasticity of housing demand is also low and the rent is more responsive to a shift in supply.

equation (11a):

$$\underset{}{(\text{Housing Price})_0} = \underset{(+)}{\phi_{0,1}}\ \underset{(-)}{p_1} + \underset{(+)}{\phi_{0,1}}\ \underset{(-)}{\phi_{1,2}}\ \underset{(+)}{p_2}.$$

The shock lowers the MPK and raises the pricing kernel (high $\phi_{0,1}$), which helps raise the housing price. The negative shock makes the numeraire good more precious and reduces the current housing rent (low $p_1$), but the rent reduction is relatively moderate in a supply-elastic city (large $\mu$). The households cash out part of their savings ($W_1$) in order to support their period 1 consumption (consumption smoothing motive) so that the capital supply at $t = 2$ is reduced. The reduced capital supply results in a higher interest rate or a lower pricing kernel at $t = 2$ (low $\phi_{1,2}$). When land supply is elastic, the housing rent is more affected by the negative supply shift than by the demand shift, which leads to a rise in rent (high $p_2$). When a higher $\phi_{0,1}$ and $p_2$ surpass the other competing forces, the housing price appreciates.

In Case 2, we should observe 1) a bull-steepening of the term structure of interest rates (a lower rate at the short end of the yield curve), 2) higher expected rent growth, 3) a lower current capitalization rate, or "cap rate", for housing, and 4) reduced savings (attributable to a cashing out of the investment portfolio).[26] Case 2 is also consistent with the negative covariation of the housing price and the interest rate noted by Cocco (2000) and positive covariation of business investment and housing investment noted by Davis and Heathcote (2005). While standard two-sector models generate a negative covariation of investments due to the sectoral substitution of capital, the model generates a positive relationship by dint of the capital allocation across cities. A positive covariation between investments, however, means stagnation in near-term construction activity after 2000, which is slightly counterfactual.

Improved results are obtained by combining an anticipated negative shock to housing production ($\Delta B_2 < 0$, Case 4) with Case 2. The negative effect of $\Delta A_1 < 0$ on housing structures is mitigated or may even be reversed. All other effects are enhanced: higher housing prices, lower financial asset prices, a steeper slope of yield curve, a higher rent growth, a lower cap rate, and lower savings. This combined case is also appealing because of a better match to a cross-regional observation that housing price appreciation is pronounced in areas with rich housing amenities such as San Diego and Miami. Housing price appreciation seems to be partly driven by a local shock to preference for

[26] All of these responses were actually observed during the process of housing price appreciation after 2000.

housing, which is equivalent to a shock to housing production in the model.

Table 2 summarizes the model predictions for all four cases. Either Case 2 with $\Delta A_1 < 0$ (elastic land supply) or Case 3 with $\Delta A_1^* < 0$ (a negative shock in the foreign city) provides the predictions that fit best the situation after 2000. Case 3 is driven by the capital flow from the home city under recession. Case 4 is mainly driven by a higher rent due to less efficient housing production. In this case the covariation of investments is negative due to capital substitution between sectors.

[Table 2: Predictions in four cases of housing price appreciation]

### 4.2.2 Effects on Human Capital and Financial Assets

Table 3 presents the effects of various technology shocks on asset prices. The value of human capital rises with a positive shock to goods production ($\Delta A_t > 0$) mainly because of a large increase in wages ($w_t$) with an inelastic labor supply (note the second column of Table 3). A positive shock to housing production ($\Delta B_t > 0$) generates parameter-dependent effects. When the two goods are complementary (small $\rho$), the value of human capital rises because greater demand for composite goods ($Y_t$) increases wages. Inter-temporal substitution ($\theta$) also affects the value via variations in the pricing kernel that are discussed in Section 5.2.

The price of the financial asset exhibits very similar responses as the value of human capital. The price rises with a positive shock to goods production (the third column of Table 3). A positive shock at $t = 1$ ($\Delta A_1 > 0$ or $\Delta A_1 = \Delta A_2 > 0$), for example, will raise the price of the financial asset since the equilibrium levels of $K_1$ and $K_2$ are higher. A higher productivity leads to more capital, either due to the substitution for housing production in the same city or the substitution for foreign production. A positive shock to housing production also generates the parameter-dependent effects that are very similar to the case of human capital.

[Table 3: Effects of technology shocks on asset prices]

## 4.3 Covariation of Asset Prices

Now we examine the covariation of different asset prices. The covariation in response to a shock is measured in terms of the product of the percentage changes in the two prices.

### 4.3.1 Financial Assets and Human Capital

As seen in Table 3, most of the time the price of financial assets and the value of human capital move in the same direction. This is because a change in productivity affects both capital demand and labor demand in the same way when a shock is given at $t = 1$ ($\Delta A_1$ and $\Delta A_1 = \Delta A_2$). When a shock is anticipated in the future ($\Delta A_2$ and $\Delta B_2$), they may move in opposite directions. For example, given a positive shock to goods production in period 2 ($\Delta A_2 > 0$), the household also wants to consume more at $t = 1$ if the inter-temporal substitution is low (small $\theta$). However, housing services must be produced locally while composite goods can be imported from the foreign city. Therefore, capital at $t = 1$ is allocated more to housing production and the amount of capital dedicated to goods production ($K_1$) is reduced. Therefore, prices of financial assets and human capital may move in opposite directions when inter-temporal substitution is low.

### 4.3.2 Housing and Other Assets

The covariation of housing price and the value of human capital depends on the supply elasticity of land ($\mu$) and the elasticities in the utility function ($\rho$ and $\theta$). The effect of a shock to goods production ($\Delta A_t$) on this covariation is determined by the sign of the change in housing prices since the response of human capital is uniform. For example, in response to a positive shock, the human capital always appreciates due to wage increases. As seen in Figure 4-a, housing prices and human capital vary together when an inelastic land supply (small $\mu$) makes the housing rent more responsive to a positive demand shock. Conversely, the covariation is negative when relatively elastic land supply (large $\mu$) makes the rent more stable (Figure 4-b). The critical value of $\mu$ is different for different types of shocks but is not so large for $\Delta A_1$ (Figure 4-c) and $\Delta A_1 = \Delta A_2$. ($\mu \cong 0.8$ for $\Delta A_1 > 0$ and $\mu \cong 2$ for $\Delta A_1 = \Delta A_2 > 0$)

[Figure 4: Covariation of asset prices]

With a shock to housing production ($\Delta B_t$), the link between housing prices and human capital is determined by the effect on human capital. Housing prices always depreciate with a positive shock and appreciate with a negative shock, regardless of parameters. The covariation of housing prices and human capital is generally negative when the two goods are more complementary (small $\rho$) and when the inter-temporal substitution is low (small $\theta$) (Figure 4-d). With a positive shock, for example, human capital appreciates if the two goods are complementary. This is because reduced housing expenditures lead to a lower interest rate, which stimulates production of composite goods.

The covariation between the prices of housing and the financial asset is similar to that between the housing price and the human capital. This is because of the general comovement of human capital and financial assets.

**Proposition 2** *Housing assets are a hedge against human capital risk and the financial risk if*

$$1)\begin{cases}\textit{the land supply is sufficiently elastic (large } \mu\textit{)}\\ \textit{when the source of risk is a current shock to goods production, or}\end{cases}$$
$$2)\begin{cases}\textit{the two goods are more complementary (small } \rho\textit{)}\\ \textit{when the source of risk is a shock to housing production.}\end{cases}$$

A positive production shock causes declines in both the housing price and the value of human capital in the foreign city due to a lower pricing kernel and diminished production of both goods. A housing production shock has a very small impact on the foreign city, so that the covariation is close to zero.

#### 4.3.3 Cross-Country Differences in Asset Price Covariation

A stylized fact, in the US, is that the correlation between the housing prices and stock prices is negative, or at least close to zero. These empirical findings suggest that housing assets provide at least a good diversification benefit and may even be a hedge against the financial risk.[27] An illustrative sample period is after 2000, during which stock prices were depressed and housing prices appreciated. In contrast, the correlation is much higher in Japan.[28] Illustrative periods are the 1980's and the 90's. In the 80's both stock prices and housing prices appreciated, but in the 90's both were depressed. The relationships between housing and human capital, and between human capital and stock are probably positive in both countries although the results are mixed.[29]

[27] Cocco (2000) and Flavin and Yamashita (2002), among others, note the negative correlation. Goetzmann and Spiegel (2000) find a negative Sharpe ratio for housing, which is consistent with the opportunity for hedging.

[28] Quan and Titman (1999) report a high correlation in Japan between stock and commercial real estate, which is positively correlated with housing prices. Casual observation after 1970 also confirms this.

[29] Cocco (2000) reports a positive correlation between housing and labor income. Davidoff (2006) also obtains a positive point estimate but it is not significantly different from zero. The correlation between return to capital and return to labor is positive and very high (Baxter and Jermann (1997)),

Such variations in asset price correlations are typically explained by different macroeconomic policies, and sometimes by "cultural" differences. For example, a standard explanation for a positive covariation of housing prices and stock prices in Japan relies on monetary policy. It treats both stocks and real estate the same, focusing on the nominal values of these assets. However, it does not explain why we observe negative covariation in the U.S. Another explanation is more "behavioral." Japanese households and investors are somehow more prone to irrational exuberance and an investment boom spreads across assets.

This paper provides a rational foundation to explain this difference between countries in the covariation structure among the three assets. The explanation is based on differences in land supply elasticity, and it is more natural and matches a key difference across countries.

Figure 5 presents correlation coefficients between housing prices and 4-quarter lagged stock price for seventeen OECD countries, plotted against the natural log of per capita habitable area. The per capita habitable area is a measure of land supply elasticity, albeit a crude one.[30] The habitable area is "Land Area" minus "Inland Water" and "Forest and Woodland" in FAOSTAT 2003-2005. Each country's population is taken from OECD statistics in 2005. The asset price data are BIS calculations based on quarterly national data from 1970 to 2006.[31] In calculating correlation coefficients between stock price and housing prices, I account for systematic lags in real estate price indices, which have been pointed out in a number of researches, by taking 4-quarter lags of stock prices.

[Figure 5: Land Supply Elasticity and Asset Price Correlation]

Figure 5 exhibits a negative relationship between asset price correlation and land supply elasticity, as predicted by the model. The correlation coefficient is -0.44. The line represents fitted values from a bivariate regression of the price correlation on the log habitable area. The slope is -0.0344 (standard errors are 0.0183 and the t-statistic is 1.88), with adjusted R-squared of 0.137. The coefficient is statistically significant at

especially for proprietary business income (Heaton and Lucas (2000)) and in the long run (Benzoni et al. (2007)).

[30] Quigley and Raphael (2005) and Green et al. (2005) find that population density and housing-market regulation are key determinants of housing supply elasticity in the U.S. Edelstein and Paul (2000) discuss factors that severely limit land supply in Japan.

[31] For detailed descriptions and analysis of the data, see Borio and McGuire (2004) and Tsatsaronis and Zhu (2004).

the 10 percent level. Ireland has a large, negative disturbance, while Japan and New Zealand have large, positive disturbances. If different lags in stock prices are used, the relationship becomes significantly weaker. For example, the correlation coefficients become -0.14 and -0.20 if contemporary and 2-quarter lagged stock prices are used, respectively. The weak relationship is not surprising, given that the measure of land supply elasticity is crude. Overall, a weak support for the model is obtained by using simple per capita habitable area.

It is important to understand properly the land supply in the current model. The land supply is obviously most restricted by the topographic conditions and population densities. The ability to supply housing, whether by land development or via infill, is much more limited if population density is high. That is why I use per capita habitable area as a proxy for the land supply elasticity. However, other important supply constraints are imposed by the regulatory system and the adjustment speed of housing stock. Some countries such as Germany generally impose stricter environmental and historical restrictions on new developments. Such restrictions make the land supply more inelastic than the level implied by population densities. The adjustment speed of housing stock is also affected by negotiation practices. For example, many Japanese redevelopment projects take more than ten years to complete due to the prolonged negotiation process. Such slow adjustment functions as a short-run inelasticity of supply.

## 4.4 Implications for Households' Equity Holdings and Homeownership

Positive covariations among three broad asset classes have important implications for the optimal equity holdings and homeownership. With positive covariations, the optimal portfolio choice results in a small position (or even a short position) in the asset that can be adjusted more freely.[32] In general, there are few constraints on financial asset holdings, while human capital and homeownership are constrained at some positive levels.

Under these constraints, positive covariations in prices lead to less holdings of financial assets, or limited stock-market participation, as derived by Benzoni et al. (2007). They note an empirical fact that human capital and stock prices are more highly correlated in the long run, and they show that, assuming co-integrated prices of these two assets, the optimal portfolio strategy may be even to short-sell stocks, especially for

[32] The partial equilibrium portfolio choice literature leads to the conclusion that less holding of stock is optimal if the exogenously given covariance is positive and vice versa. See Flavin and Yamashita (2002), Cocco (2004), and Cauley et al. (2005).

younger investors.

Similarly, if the rental housing market is well functioning and households are relatively free to choose their level of housing asset holdings, positive covariations lead to less homeownership. This is examined by Davidoff (2006), who shows that households with a higher correlation between labor income and housing prices own less housing.

The current model derives a positive covariation between human capital and financial assets, rather than just assuming one, for most cases, and between housing and financial assets depending on the parameters. Thus, the model identifies fundamental factors that underlie low equity holdings and low homeownership. Interestingly, the model predicts that households in a land-inelastic economy put smaller weights on stocks, since all three asset classes (human capital, housing, and stock) are positively related in such an economy. In contrast, in a land-elastic economy, housing assets serve as a hedge against the other assets and households should be more willing to hold stocks in their portfolio.

Figure 6 presents the share of equity holdings in households' total assets for seven OECD countries at the end of 2001, plotted against per capita habitable area in log scale (Figure 6-a) and against correlation coefficient between housing prices and 4-quarter lagged stock price (Figure 6-b). Seven countries (Canada, France, Germany, Italy, Japan, the UK, and the US) are selected by the Bank of Japan to compare flow of funds accounts.(of Japan (2003)) The BOJ data on flow of funds are used to calculate each country's share of equity holdings in households' total assets.[33] Seven countries have wide variations in the share of equity holdings: the USA (0.34), France (0.29), Canada (0.28), Italy (0.22), Germany (0.14), the UK (0.13), and Japan (0.07). The per capita habitable area and asset price correlations are calculated in the same way as for Figure 5. Table 4 summarizes the data used in Figure 6.

[Figure 6: Households' Stock Holdings]

[Table 4: Households' stock holdings, asset price correlations, and land supply elasticity]

Figure 6 shows a positive relationship between equity holdings and land supply elasticity and a negative relationship between equity holdings and asset price correlations, as suggested by the current model. The correlation coefficients are 0.76 and

[33] of Japan (2003) makes various adjustments on raw national data so that different countries become comparable. An example is whether to include the equity share of private businesses in households' assets.

-0.85, respectively. The lines in Figures 6-a and 7-b represent fitted values from bivariate regressions of equity holdings on log habitable area and on asset price correlations, respectively. In Figure 6-a, the slope is 0.0459 (standard errors are 0.0175 and the t-statistic is 2.62) with adjusted R-squared of 0.495. In Figure 6-b, the slope is -0.566 (standard errors are 0.159 and the t-statistic is -3.57) with adjusted R-squared of 0.661. Both coefficients are significant at 5 percent levels.

Although seven countries are not enough to make a decisive conclusion, the available data seem to support the model's predictions. In particular, the link between households' portfolio choice and land supply elasticity has not been explored before. For example, the US households have twenty-four times larger per capita habitable area and 27 percentage points higher share of equity holdings than Japanese households have. The current model connects these two seemingly unrelated observations through equilibrium asset price correlations. Even if we use population densities (i.e., inverse of per capita habitable area) for explanatory variable, the relation remains strong and statistically significant; the correlation coefficient is -0.80. If mutual funds are added to equity holdings, these relations become slightly weaker, but the results do not change; the correlation coefficients are 0.49 and -0.81 with regard to land supply elasticity and asset price correlations, respectively. However, including mutual funds is not necessarily desirable since mutual funds contain fixed income and global investments.

The current model provides a plausible explanation for the fact that Japanese households put smaller portfolio weights on stocks than other OECD countries' households do. A low elasticity of land supply in Japan leads to positive correlations between housing prices and other assets. Nevertheless, economic institutions encourage households to hold large housing assets. For example, the Japanese rental housing markets have not functioned well due to the tenancy law that heavily protects tenants' rights. The government also favors homeownership through subsidized financing and tax treatments of housing.[34] As a consequence, the optimal portfolio includes less stock.

Previous explanations tend to rely on the "irrationality," differences in "culture" or preferences, or differences in investment skills.[35] In fact, based on such arguments, the Japanese government has adopted policies to encourage equity investments, measures supported by the financial industry. The result of this research provides a counter argument: namely, that a smaller weight on stock is a perfectly rational choice for households in the land-inelastic Japanese economy.

[34] Kanemoto (1997) discusses in detail homeownership and limited rental markets in Japan.

[35] For example, of Japan (2003) attributes low stock holdings in Japan to a greater risk aversion of Japanese households.

# 5 Results with Risks

## 5.1 Introducing Technological Risks to the Model

A simplest form of risk is introduced to the model by considering stochastic technologies at $t = 2$; $A_2, B_2, A_2^*$, and $B_2^*$ can be random variables. At $t = 1$, households make their decisions not only on the total amount of savings, but also on the allocation of their funds. Households determine their portfolio weights on business capital in two cities and their housing structure, based on rational expectations on equilibrium asset returns. When a particular state is realized at $t = 2$, asset returns are determined so that capital demand in each production sector is equilibrated with invested funds. At the same time, markets for composite goods, housing, land, and labor also clear and prices of these goods and factors are determined.

Precisely, the problem of households in the home city is modified as follows by using expectation. Households in the foreign city solve a symmetric problem.

$$\max_{\{C_t, H_t, q_f, q_h\}} u\left(C_1, H_1\right) + \beta E u\left(C_2, H_2\right)$$
$$s.t.\ C_1 + p_1 H_1 + W_1 = i_1 W_0 + r_1 T_1 + w_1$$
$$C_2 + p_2 H_2 = i_{p2} W_1 + r_2 T_2 + w_2,$$

where $q_f$ and $q_h$ are portfolio weights on equities of goods-producing firms and real estate firms, respectively. The remainder, $1 - q_f - q_h$, is invested in goods-producing firms in the foreign city. There is no short-sale constraint (i.e., portfolio weights can be negative). These portfolio weights are chosen before time 2 uncertainty is resolved. $i_{p2}$, the portfolio return at $t = 2$, is defined as $i_{p2} \equiv q_f\ i_{f2} + q_h\ i_{h2} + \left(1 - q_f - q_h\right)\ i_{f2}^*$, where $i_{f2}$ and $i_{h2}$ are equity returns to goods-producing firms and real estate firms, respectively. Variables of the foreign city are denoted by starred characters.

The first-order conditions of the households' problem are

$$1 = E\left[\phi_{1,2}\ i_{f2}\right] = E\left[\phi_{1,2}\ i_{h2}\right] = E\left[\phi_{1,2}\ i_{f2}^*\right], \text{ and} \quad (12a)$$
$$p_2(s) = \frac{\partial u / \partial H_2(s)}{\partial u / \partial C_2(s)}, \quad (12b)$$

where $s \in \mathbb{S}$ denotes states of nature in the second period. Quantities and prices at time 2 are now random variables. A variable $X$ in a particular state $s$ in the second period is denoted by $X_2(s)$. The first line exhibits Euler equations with respect to different asset returns. The second line states that price of housing services is equal to the intra-temporal marginal rate of substitution in each state of nature.

Firms determine their factor inputs and production levels after observing technology shocks. Therefore, the first-order conditions on the production side are the same as those with perfect foresight. In particular, equity return of a firm is equal to its marginal product of capital in each state of nature:

$$i_{f2}(s) = 1 + \frac{\partial Y_2(s)}{\partial K_2(s)} - \delta$$

and

$$i_{h2}(s) = 1 + p_2(s) \frac{\partial H_2(s)}{\partial S_2(s)} - \delta.$$

Equilibrium is numerically solved. I consider three states at $t = 2; h, m,$ and $l$. Fundamental risk is present in goods-producing technology; $A_2(h) = 1.2$, $A_2(m) = 1$, and $A_2(l) = 0.8$. Physical probabilities of $h$, $m$, and $l$ are 0.25, 0.5, and 0.25, respectively. Other parameter values are the same as in the perfect foresight case. In equilibrium, optimality conditions of each agent and market-clearing conditions in each state are satisfied.

## 5.2 The Pricing Kernel and the Role of Rent Growth

Before examining equilibrium results when risks are present, I analyze the role of housing in asset pricing by taking a close look at the pricing kernel. Consider discrete states of nature in the second period. Starting from the IMRS (10c), manipulations by means of (12b) lead to a different expression of the pricing kernel that includes only consumption growth and housing rents:

$$\begin{aligned} \phi_{1,2}(s) &= \beta \left\{ \left( \frac{C_2(s)}{C_1} \right) \left[ \frac{\left(1 + p_2(s)^{1-\rho}\right)^{\frac{1}{1-\rho}}}{\left(1 + p_1{}^{1-\rho}\right)^{\frac{1}{1-\rho}}} \right]^{\theta-\rho} \right\}^{-\frac{1}{\theta}} \\ &\equiv \beta \left\{ g_{c,2}(s) \cdot g_{p,2}(s)^{\theta-\rho} \right\}^{-\frac{1}{\theta}}, \end{aligned} \tag{13}$$

where $g_{c,2}(s) \equiv C_2(s) / C_1$ is the consumption growth, and $g_{p,2}(s) \equiv \left(1 + p_2(s)^{1-\rho}\right)^{\frac{1}{1-\rho}} \Big/ \left(1 + p_1{}^{1-\rho}\right)^{\frac{1}{1-\rho}}$ is the growth of the CES-aggregated price index. Note that $g_{p,2}(s)$ is a monotonically increasing function of the rent growth. The IMRS basically has the same form as in the single good CRRA case: a modified consumption growth appearing in the braces is raised to the power of $-1/\theta$ and multiplied by the subjective discount factor.

This equation gives a new insight into the meaning of rent growth in the context of the asset pricing. The IMRS, $\phi_{1,2}(s)$, measures how "under-satisfied" the household is in state $s$ in the second period, relative to the current state. In a state of high consumption growth, the household is more satisfied and the marginal utility is lower. However, in the current model of nonseparable housing services, the level of satisfaction is not simply measured by consumption growth but by the consumption growth augmented by the growth of the aggregate price $g_{p,2}(s)$ raised to the power of $\theta - \rho$.

A high growth of price index $g_{p,2}(s)$ means that the numeraire good in state $s$ is relatively abundant and cheap, or equivalently that housing is relatively precious and expensive. When the two goods are relatively substitutable ($\rho > \theta$), a high growth of price index reduces the satisfaction gained from a given level of composite goods because of their abundance. Put differently, households have additional willingness to consume cheap composite goods in place of housing services when housing rent is high. A low growth of price index, in contrast, raises satisfaction from consuming composite goods because they are more precious. Households substitute cheap housing services for expensive composite goods.

When the two goods are relatively complementary ($\rho < \theta$), the opposite is the case: a high growth of price index (i.e., precious housing) reduces willingness to consume composite goods. The satisfaction from consuming composite goods is adjusted upward. Conversely, a low growth of price index (i.e., abundant housing) creates more willingness to consume composite goods, so that households' satisfaction level is adjusted downward.

It is important to note that equilibrium housing rent is equal to the user cost of housing in a competitive and frictionless market for housing services. If actual housing rents do not exhibit much volatility due to some rent rigidity, the user cost is a more relevant measure of housing rent in the present model. Historically, the user cost tends to be more volatile than housing rent indices.

In sum, the housing rent measures the relative abundance of composite goods. This abundance affects the ma rginal utility of composite goods differently depending on the relative substitutability between the goods. Based on partial derivatives of the pricing kernel, the following proposition is obtained.

**Proposition 3** *Housing rent growth, measured by the growth of the CES-aggregated price index ($g_{p,2}$), is a component of the pricing kernel if the utility function is nonseparable in housing and the numeraire good. The sign of the relationship between rent growth and the pricing kernel, holding consumption growth fixed, is determined by* *relative substitutability between the two goods:*

$$\frac{\partial \phi_{1,2}}{\partial g_{p,2}} > 0 \ (< 0) \quad \textit{for } \rho > \theta \ (\rho < \theta). \tag{14}$$

This intuition is also confirmed by examining the cross-partial derivative of the intra-period utility function: $sgn\left(\partial^2 u\left(C_t, H_t\right)/\partial H_t \partial C_t\right) = sgn\left(\theta - \rho\right)$.[36] Abundant housing raises the marginal utility of consumption when $\theta > \rho$ (i.e., when two goods are complementary). Although this partial equilibrium analysis is interesting in its own respect, a general equilibrium must be solved in order to examine equilibrium risk premia of asset returns.

## 5.3 Risk Premia, Return Volatility, and Volatility of the Pricing Kernel

In this section, I present the equilibrium outcome of the model when risks are present. Figure 7 shows comparative statics of risk premia, return volatility and volatility of the pricing kernel when goods-producing technology is stochastic. The figure focuses on cases in which two goods are relatively complementary: intra-temporal substitution ($\rho$) is lower than inter-temporal substitution ($\theta$). In Figure 7-a, the results are plotted against different levels of land supply elasticity, while intra- and inter-temporal substitutions are fixed at $\rho = 0.2, \theta = 1.8$. In Figure 7-b, the results are plotted against different levels of intra-temporal substitution, while land supply elasticity and inter-temporal substitution are fixed at $\mu = 0, \theta = 1.8$.

[Figure 7: Risk premia, return volatility, and volatility of the pricing kernel ($\rho < \theta$)]

On the upper panel of Figure 7, a solid line and a dotted line represent the risk premium on business capital of goods producing firms and the risk premium on the housing structure of real estate firms, respectively. The most notable result is that risk premia for both types of assets increase as land supply becomes inelastic and as the two goods become complementary. The pattern is monotonic but the effect is stronger at the lower end of each parameter. The risk premium on the housing structure is more

[36] Specifically, the cross-partial derivative is

$$\frac{\partial^2 u\left(C_t, H_t\right)}{\partial H_t \partial C_t} = \left(\frac{\theta - \rho}{\rho\theta}\right)\left(C_t H_t\right)^{-\frac{1}{\rho}}\left[C_t^{1-\frac{1}{\rho}} + H_t^{1-\frac{1}{\rho}}\right]^{\left(1-\frac{1}{\theta}\right)/\left(1-\frac{1}{\rho}\right)-2}.$$

responsive to the values of land supply elasticity and intra-temporal substitution than that on business capital.

The middle panel and the lower panel of each figure show key components of the risk premium: volatility of the pricing kernel and return volatility, respectively. The risk premium is composed of the risk-free rate, correlation coefficient between the asset return and the pricing kernel, return volatility, and volatility of the pricing kernel. To see this, (12a) can be rearranged to express the risk premium on any asset return $i_{i2}$ as

$$E\left[i_{i2}\right] - i_2 = -i_2 \; Corr\left[\phi_{1,2}, i_{i2}\right] \; \sigma_i \; \sigma_\phi,$$

where $i_2$ is the risk-free rate, $Corr\left[\phi_{1,2}, i_{i2}\right]$ is the correlation coefficient between asset return and the pricing kernel, $\sigma_i$ is return volatility, and $\sigma_\phi$ is volatility of the pricing kernel. When goods-producing technology is the source of risks, the correlation coefficients are uniformly positive for both business capital and housing structure, and so are the risk premia.

The volatility of the pricing kernel exhibits the same pattern as risk premia: the pricing kernel becomes more volatile as land supply becomes inelastic and as two goods become complementary. The volatility of the pricing kernel is shown to be the main driver of risk premia in this economy. To see this, note that the risk premium on business capital rises, as land supply becomes inelastic, in tandem with volatility of the pricing kernel, while return volatility is rather reduced. It is consistent with past empirical findings that variation in the risk premium cannot be explained by return volatility.

The volatility of the pricing kernel is influenced by land supply elasticity and intra-temporal substitution through the housing market. Land supply elasticity determines housing supply elasticity while intra-temporal substitution determines housing demand elasticity. When either supply or demand of housing is inelastic, housing rent is more responsive to technology shocks. Since the rent growth factor is a component of the pricing kernel, the pricing kernel also becomes responsive to technology shocks.

Figure 8 shows comparative statics of two components of the pricing kernel: volatility of consumption growth ($g_{c,2}$) and volatility of rent growth factor (${g_{p,2}}^{\theta-\rho}$). It is confirmed that the rent growth factor is driving the volatility of the pricing kernel. The consumption growth is extremely stable with respect to land supply elasticity and intra-temporal substitution. The stable consumption growth is consistent with past empirical findings, and it is the source of the equity premium puzzle, which I discuss in detail in the next section.

It might be surprising at first that inelastic housing markets make the pricing kernel more volatile, especially if equation (8b) is in mind. In the equation, the additional component includes quantity of housing services rather than housing rent. More stable housing services seem to stabilize the pricing kernel. However, what matters is the ratio of housing services to consumption. If housing supply is inelastic, a fixed amount of housing services creates a sort of leverage in the consumption ratio, making the consumption ratio more volatile. If housing demand is inelastic due to a low level of intra-temporal substitution, the consumption ratio is raised to the power of $1 - 1/\rho$, which is a large negative number. In both cases, the second component of the pricing kernel becomes volatile when housing demand or supply is inelastic.

This issue can also be understood by referring to equation (10c), in which the second component is the expenditure ratio of housing. When housing services are elastic, a change in rent is mitigated by a change in housing services in the opposite direction so that the expenditure ratio becomes more stable. When quantity of housing services is fixed, variation in housing rent directly drives the expenditure ratio.

Figure 9 shows the same comparative statics as in Figure 7, except that intra-temporal substitution is higher than inter-temporal substitution. In this case, risk premia, volatility of the pricing kernel, and return volatility are more stable across the range of examined parameter values. Furthermore, volatility of the pricing kernel decreases as land supply becomes inelastic and as the two goods become complementary, contrary to the cases in Figure 7. This is because the rent growth factor is negatively correlated with consumption growth, and it stabilizes the pricing kernel more strongly at the lower end of parameter values. The negative correlation is created by a negative value of $\theta - \rho$ that raises $g_{p,2}$. Although the relative level of intra-temporal substitution is an empirical question, a low intra-temporal substitution makes more interesting cases with respect to the equity premium, as seen in the next section.

[Figure 9: Risk premia, return volatility, and volatility of the pricing kernel ($\rho > \theta$)]

## 5.4 Mitigating the Equity Premium Puzzle and the Risk-Free Rate Puzzle

The equity premium puzzle is the observation that the historical risk premium associated with equity is too high to be explained by the covariance between the consumption-based pricing kernel and the return under plausible levels of risk aversion. Since the puzzle arises from too little variation in the consumption growth, any factor that magnifies the variation of the consumption growth in the Euler equation helps to resolve the puzzle. A closely related issue is a low estimate of EIS, since the coefficient of relative risk aversion is the reciprocal of EIS with a single good power utility specification. Although a high risk aversion must be associated with a low EIS with power utility, a low EIS implies a much higher interest rate than the historical level, in order to account for historical consumption growth. This is called the risk-free rate puzzle. Previous estimates of EIS are typically quite low and even negative in some researches.

In this section, I present two kinds of exercise on how housing may mitigate the equity premium puzzle and the risk-free rate puzzle. The first exercise is to examine equilibrium relationship among consumption growth, rent growth and the pricing kernel in response to a technology shock within a perfect foresight framework. It is shown that the consumption growth augmented by rent growth can be much more volatile than simple consumption growth. The second exercise is to compare, in the risky environment, the volatility of the pricing kernel with the housing component and that without the housing component. It is shown that a higher level of risk aversion than the true level is needed when the housing component is ignored. In each exercise, the housing component scales up the volatility of consumption growth in the pricing kernel.

In the first exercise, responses of consumption growth and rent growth to a specific type of technology shock are examined. There are two cases in which the equity premium puzzle and the risk-free rate puzzle are mitigated.

**Proposition 4** *The equity premium puzzle and the risk-free rate puzzle are mitigated in the following two cases.*

| | |
|---|---|
| *Case 1:* | *Variation of consumption growth in response to anticipated shocks to goods production is magnified when $\theta > \rho$.* |
| *Case 2:* | *Estimates of EIS are biased downward by shocks to housing production.* |

Case 1 is based on a positive covariation of consumption growth ($g_{c,2}$) with the rent growth factor (${g_{p,2}}^{\theta-\rho}$) in (13). Figure 10-a presents the variation of augmented consumption growth ($g_{c,2} \cdot {g_{p,2}}^{\theta-\rho}$) and its components in response to anticipated shocks to goods production ($\Delta A_2$) when $\rho = 0.2$ and $\theta = 1.8$. Augmented consumption growth exhibits much greater variation than plain consumption growth since the rent growth factor changes in the same direction. The covariation of $g_{c,2}$ and ${g_{p,2}}^{\theta-\rho}$ has a positive sign when the two goods are relatively complementary ($\theta > \rho$) (Figure 10-b). Suppose that a positive future shock to goods production is anticipated ($\Delta A_2 > 0$). Both consumption ($C_2$) and rent ($p_2$) increase in the future, which drives both consumption growth ($g_{c,2}$) and rent growth higher. The rent growth factor (${g_{p,2}}^{\theta-\rho}$) also increases if $\theta - \rho$ is positive and vice versa. With other types of shocks, the covariation is mainly negative and variation of consumption growth is dampened. Therefore, this case applies if asset prices are mainly driven by news about future productivity shocks, and if the two goods are relatively complementary. This exercise under perfect foresight confirms the results obtained in the previous section under uncertainty (i.e., the rent growth factor magnifies the variation of the pricing kernel if $\rho < \theta$, but dampens it if $\rho > \theta$).

The condition $\theta > \rho$ is not unrealistic, although previous estimates of the elasticities of substitution are mixed. Regarding intra-temporal substitution ($\rho$), most studies define durables as motor vehicles, furniture, jewelry, and so on. The estimates of $\rho$ for these goods range from 0.4 to 1.2. A smaller number of studies estimate $\rho$ for housing, whose estimates range from 0.2 to 2.2.[37] However, a large literature on the price elasticity of housing demand indicates that intra-temporal substitution is well below one. The estimates of EIS ($\theta$) are also mixed: although a quite low EIS (close to zero or even negative) is usually estimated, much higher estimates (from 1 to 3) are also presented.[38]

[Figure 10: Mitigating the equity premium puzzle and the risk-free rate puzzle]

Case 2 explains a bias arising from a misspecification in estimating EIS. In equilibrium, housing shocks lead to positive covariation of the pricing kernel and consumption growth (Figure 10-c). The pricing kernel ($\phi_{1,2}$) and consumption growth ($g_{c,2}$) move in the same direction since both of them move inversely with the rent growth factor, which sharply responds to housing shocks (Figure 10-d). The inverse relationship between consumption growth and the rent growth factor is generated as follows. Suppose a positive housing shock occurs at $t = 2$ ($\Delta B_2 > 0$). Rent growth declines and the rent growth factor (${g_{p,2}}^{\theta-\rho}$) also declines when $\theta > \rho$ and vice versa. On the other hand, the consumption at $t = 2$, and thus consumption growth, increases when $\theta > \rho$ because of the complementality of the two goods. An analogous mechanism works with $\Delta B_1$.

If a model of a single good power utility, $\phi_{1,2} = \beta\, {g_{c,2}}^{-1/\theta}$, is applied to this situation, the positive covariation results in a negative estimate of EIS, $\theta$, since the rent growth

[37] See Tesar (1993) and Yogo (2006) for non-housing durables, and Lustig and van Nieuwerburgh (2004), Piazzesi et al. (2007), and Davis and Martin (2005) for housing.

[38] Among the large body of literature on the EIS estimation, Hall (1988) finds it to be negative and Yogo (2006) estimates it at 0.02, while Vissing-Jorgensen and Attanasio (2003) find it between 1 and 2 and Bansal and Yaron (2004) estimate the EIS between 1.9 and 2.7.

factor ($g_{p,2}{}^{\theta-\rho}$) is ignored in (13). Therefore, if housing shocks are mixed with shocks to goods production, the estimate of $\theta$ is biased downward. This implies an ambiguous relationship between consumption growth and the pricing kernel, which in turn cautions us not to make an immediate inference about the pricing kernel by looking only at the consumption growth.

In the second exercise, I compare the true pricing kernel with the housing component (13) and a misspecified pricing kernel under a single-good assumption ($\phi_{1,2} = \beta\ g_{c,2}{}^{-1/\theta}$), using the equilibrium outcome of the model with risks. Figure 11 compares volatilities of two types of pricing kernel. Panel A focuses on cases in which two goods are complementary ($\rho < \theta$), while Panel B focuses on cases in which two goods are substitutable ($\rho > \theta$). In each panel, the misspecified pricing kernel is uniformly less volatile and the volatility is stable over different parameter values. In particular, when $\rho = 1.8$, $\theta = 0.2$, and $\mu = 5$, volatility of the true pricing kernel is about twenty-two times higher than that of the misspecified one. When two goods are complementary and land supply is inelastic ($\rho = 0.2, \theta = 1.8, \mu = 0$), the volatility ratio is 4.6. The lowest ratio within the examined range of parameter values is 1.2 when $\rho = 1.4, \theta = 1.8$, and $\mu = 0$. Since the volatility of the pricing kernel directly affects the equity premium, ignorance of the housing component is a strong candidate for a cause of the equity premium puzzle.

[Figure 11: Volatilities of two types of the pricing kernel]

# 6 Conclusion

In this paper, I build a model of a production economy to study the general equilibrium relationship between the business cycles and asset prices, with an emphasis on implications to the risk premium and portfolio choice.

The first of my main results is that the supply elasticity of land plays a significant role in determining the covariations of asset prices. In particular, a negative productivity shock to goods production may lead to a housing price appreciation if land supply is elastic. A key driver is a higher housing rent expected in the future due to a reduced housing supply. The model predicts that an economy with an inelastic housing supply is more likely to exhibit a positive price correlation between housing and other assets and thus, either less stock-market participation or less homeownership. Some of these predictions are supported by the data from OECD countries.

The second result is that the pricing kernel becomes more volatile as land supply becomes inelastic. The effect is enhanced as two goods become more complementary. It is because growth of housing rent alters the marginal utility of consumption when the utility function is non-separable in housing and other goods. For example, the marginal utility will be adjusted upward (implying that consumers are less satisfied) if rent growth is lower, provided that the two goods are complementary. Housing market conditions critically affect the risk premium on any risky asset. The rent growth factor may mitigate the well-known puzzles on the equity premium and the risk-free rate, by either magnifying consumption variation or imposing a downward bias on the estimate of the EIS.

This paper suggests a rich array of opportunities for empirical analysis. For instance, the new characterization of the pricing kernel allows us to use housing rent data to estimate elasticities of substitution. Housing rent data have several advantages over housing consumption data: 1) rent data may be more accurately collected, 2) rents may respond more sharply to changes in economic conditions, and 3) more detailed regional data are available for rents.

The next task of this research will be to work on a fully dynamic stochastic setting. The system will be solved by either second-order approximation or numerical methods. Although the method often used in the literature is linear approximation, the certainty equivalence property resulting is not suitable for the study of asset prices. By calibration, the levels of variables can be discussed rather than just the direction and the relative magnitudes as done in the present paper.

Data analysis can also be improved. The measure of land supply elasticity used in the current paper is admittedly crude. The measure can be refined by incorporating regulatory environment and other factors. An expanded international dataset that includes elasticities of substitution and volatility of the pricing kernel will allow other testing of the model.

Other directions of future extension include examining time-varying volatility of the pricing kernel. Understanding time-varying risk premia is one of the most important tasks for financial economists since it also provides understansing on the long-run predictability of stock returns. In this paper, supply elasticity of housing is found to drive the volatility of the pricing kernel. Asymmetric adjustment costs in housing, for example, can make housing supply inelastic under economic contraction. Inelastic housing supply is associated with a higher volatility of the pricing kernel, and thus a higher risk premium of assets, provided that housing and other goods are relatively complementary. Therefore, the housing component in the pricing kernel may also help explain the time-variation of risk premia, which are associated with business cycles.

# 7 Appendix: Derivation of the Equilibrium with Perfect Foresight

In this appendix, I describe how to solve for the equilibrium that is defined in the paper.

**(Labor markets)** Labor supply is $L_t^{\sup} = 1$. Labor demand is derived from the first-order condition of a goods-producing firm (6b): $w_t = (1-\alpha) A_t (K_t/L_t)^{\alpha}$. Using the capital demand from another first-order condition (6a), the equilibrium wage is derived as a function of $A_t$ and $i_t$ :

$$w_t^{eq}(A_t, i_t) = (1-\alpha)\,\alpha^{\frac{\alpha}{1-\alpha}} A_t^{\frac{1}{1-\alpha}} (i_t - 1 + \delta)^{-\frac{\alpha}{1-\alpha}}.$$

**(Land markets)** Land supply is $T_t = r_t^{\mu}$. Land demand is derived from the first-order condition of a real estate firm (7b): $T_t^{dem} = \{(1-\gamma) B_t p_t / r_t\}^{\frac{1}{\gamma}} S_t$. Using demand for housing structures from another first-order condition (7a), the equilibrium land rent and the quantity of land is derived as a function of $B_t, p_t, i_t$ :

$$\begin{aligned} r_t^{eq}(B_t, p_t, i_t) &= \gamma^{\frac{\gamma}{1-\gamma}} (1-\gamma) B_t^{\frac{1}{1-\gamma}} p_t^{\frac{1}{1-\gamma}} (i_t - 1 + \delta)^{-\frac{\gamma}{1-\gamma}}, \\ T_t^{eq}(B_t, p_t, i_t) &= \gamma^{\frac{\gamma\mu}{1-\gamma}} (1-\gamma)^{\mu} B_t^{\frac{\mu}{1-\gamma}} p_t^{\frac{\mu}{1-\gamma}} (i_t - 1 + \delta)^{-\frac{\gamma\mu}{1-\gamma}}. \end{aligned}$$

Although both $r_t$ and $T_t$ depend on the housing rent ($p_t$), $r_t$ and $T_t$ can be written as functions of $B_t, A_1, A_2, i_1, i_2$ after deriving the equilibria of the other markets.

**(Housing markets)** Housing supply is $H_t^{\sup}(p_t; B_t, i_t) = B_t S_t^{eq}(B_t, p_t, i_t)^{\gamma} \times T_t^{eq}(B_t, p_t, i_t)^{1-\gamma}$. Housing demand is derived as (9c) from the first-order conditions of the households. Analytical solution to the housing market equilibrium is available for the log case:

$$\begin{aligned} p_1^{eq}(i_1, Inc) &= \{2(1+\beta)\}^{-\frac{1-\gamma}{1+\mu}} \gamma^{-\gamma} (1-\gamma)^{-\frac{\mu(1-\gamma)}{1+\mu}} B_1^{-1} (i_1 - 1 + \delta)^{\gamma} Inc^{\frac{1-\gamma}{1+\mu}}, \\ H_1^{eq}(i_1, Inc) &= \{2(1+\beta)\}^{-\frac{\gamma+\mu}{1+\mu}} \gamma^{\gamma} (1-\gamma)^{\frac{\mu(1-\gamma)}{1+\mu}} B_1 (i_1 - 1 + \delta)^{-\gamma} Inc^{\frac{\gamma+\mu}{1+\mu}}. \end{aligned}$$

For the CES-CRRA case, a numerical solution must be used to derive $p_1, p_2, p_1^*, p_2^*$ jointly with $i_1$ and $i_2$.

**(Capital markets)** After obtaining $p_1^{eq}(i_1, Inc)$ and $H_1^{eq}(i_1, Inc)$ for the log case, I can rewrite $r_t^{eq}(i_t, Inc)$ and $T_t^{eq}(i_t, Inc)$ and further derive $Inc$ as

$$\begin{aligned}
Inc(A_1, A_2, i_1, i_2) &= i_1 W_0 + r_1 T_1(i_1, Inc) + w_1(A_1, i_1) \\
&\quad + \frac{1}{i_2}\{r_2 T_2(i_2, Inc) + w_2(A_2, i_2)\} \\
&= 2(1+\gamma)^{-1} \alpha^{\frac{\alpha}{1-\alpha}} (1-\alpha) \\
&\quad \times \left\{ A_1^{\frac{1}{1-\alpha}} (i_1 - 1 + \delta)^{-\frac{\alpha}{1-\alpha}} + A_2^{\frac{1}{1-\alpha}} i_2^{-1} (i_2 - 1 + \delta)^{-\frac{\alpha}{1-\alpha}} \right\}.
\end{aligned}$$

Note that $B_t$ does not appear in land rents or land quantity in the log-utility case, while it does appear in the CES-CRRA case.

Now the capital supply for period 2, $W_1$, is derived. Given $Inc(A_1, A_2, i_1, i_2)$, the consumption becomes $C_t(A_1, A_2, i_1, i_2)$ and the households' saving after period 1 is

$$\begin{aligned}
W_1(A_1, A_2, i_1, i_2) &= i_1 W_0 + r_1 T_1(A_1, A_2, i_1, i_2) + w_1(A_1, i_1) \\
&\quad - C_1(A_1, A_2, i_1, i_2) - p_1 H_1(A_1, A_2, i_1, i_2).
\end{aligned}$$

The market-clearing conditions in capital markets are

$$\begin{aligned}
W_0 + W_0^* &= \begin{bmatrix} K_1(A_1, i_1) + K_1^*(A_1^*, i_1) \\ +S_1(A_1, A_2, i_1, i_2) + S_1^*(A_1^*, A_2^*, i_1, i_2) \end{bmatrix} \text{ (for t=1)}, \\
\begin{bmatrix} W_1(A_1, A_2, i_1, i_2, W_0) \\ +W_1^*(A_1^*, A_2^*, i_1, i_2, W_0) \end{bmatrix} &= \begin{bmatrix} K_2(A_2, i_2) + K_2^*(A_2^*, i_2) \\ +S_2(A_1, A_2, i_1, i_2) + S_2^*(A_1^*, A_2^*, i_1, i_2) \end{bmatrix} \text{ (for t=2)}.
\end{aligned}$$

With these two equations, in principle two unknowns $(i_1, i_2)$ can be solved for in terms of the exogenous variables $(A_1, A_2, A_1^*, A_2^*, W_0, W_0^*)$. Numerical solutions must be used to obtain the actual solutions.

In the case of CES-CRRA, capital markets' equilibria will depend additionally on the housing rents. Therefore, the housing-market equilibrium and the capital-market equilibrium are solved simultaneously.

In this paper, basic parameters are set as follows: $\alpha = 1/3$, $\beta = 0.9$, $\gamma = 0.7$, and $\delta = 0.5$.

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Table 1: Effects on the discount factor

| | $\Delta\phi_{0,1}$ | $\Delta\phi_{1,2}$ |
|---|---|---|
| $\Delta A_1 > 0$ | $-$ | $+$ |
| $\Delta A_2 > 0$ | $\approx 0$ | $-$ |
| $\Delta A_1 = \Delta A_2 > 0$ | $-$ | $-$ |
| $\Delta B_1 > 0$ | $+$ if $\rho$ is small<br>$-$ if $\rho$ is large | $+$ if $\theta$ is small<br>$-$ if $\theta$ is large |
| $\Delta B_2 > 0$ | $+$ if $\theta$ is large<br>$-$ if $\theta$ is small | $+$ if $\rho$ is small<br>$-$ if $\rho$ is large |
| $\Delta B_1 = \Delta B_2 > 0$ | $+$ if $\rho$ is small<br>$-$ if $\rho$ is large | $+$ if $\rho$ is small<br>$-$ if $\rho$ is large |

Table 1 presents general-equilibrium effects of different types of technology shock on the discount factor. Each row corresponds to different types of shock. $\Delta A_t > 0$ and $\Delta B_t > 0$ refer to a positive shock at $t$ to the production of goods and housing, respectively. $\Delta\phi_{0,1}$ and $\Delta\phi_{1,2}$ refer to the response of the discount factor for the first and the second period, respectively. $\rho$ and $\theta$ are the parameters for intra- and inter-temporal substitution, respectively.

Table 2: Predictions in four cases of housing price appreciation

Case 1: A positive shock to goods production with inelastic land supply

| | $\Delta A_1 > 0$ | $\Delta A_1 = \Delta A_2 > 0$ | $\Delta A_2 > 0$ |
|---|---|---|---|
| Term structure | bear-flattening | bear-parallel shift | bear-steepening |
| Future rent growth | $-$ | $-$ | $+$ |
| Current cap rate | $+$ | $+$ | $+$ if $\theta$ small<br>$-$ if $\theta$ large |
| Savings | $+$ | $+$ | $-$ |
| $\mathrm{Cov}(K_1, S_1)$ | $+$ | $+$ | $\approx 0$ |

Case 2: A negative shock to goods production with elastic land supply

| | $\Delta A_1 < 0$ | $\Delta A_1 = \Delta A_2 < 0$ | $\Delta A_2 < 0$ |
|---|---|---|---|
| Term structure | bull-steepening | bull-parallel shift | bull-flattening |
| Future rent growth | $+$ | $+$ | $-$ |
| Current cap rate | $-$ | $-$ | $+$ if $\theta$ large<br>$-$ if $\theta$ small |
| Savings | $-$ | $-$ | $+$ |
| $\mathrm{Cov}(K_1, S_1)$ | $+$ | $+$ | $\approx 0$ |

Case 3: A negative shock to foreign city

| | $\Delta A_1^* < 0$ | $\Delta A_1^* = \Delta A_2^* < 0$ | $\Delta A_2^* < 0$ |
|---|---|---|---|
| Term structure | bull-steepening | bull-parallel shift | bull-flattening |
| Future rent growth | $+$ | $+$ | $-$ |
| Current cap rate | $-$ | $-$ | $+$ if $\theta$ large<br>$-$ if $\theta$ small |
| Savings | $-$ | $-$ | $-$ if $\theta$&$\rho$ small<br>$+$ otherwise |
| $\mathrm{Cov}(K_1, S_1)$ | $+$ | $+$ | $\approx 0$ |

Case 4: A negative shock to housing production

| | $\Delta B_1 < 0$ | $\Delta B_1 = \Delta B_2 < 0$ | $\Delta B_2 < 0$ |
|---|---|---|---|
| Term structure | mixed | mixed | mixed |
| Future rent growth | $-$ | $+$ if $\rho > 1$<br>$-$ if $\rho < 1$ | $+$ |
| Current cap rate | $+$ | $+$ if $\rho > 1$<br>$-$ if $\rho < 1$ | $-$ |
| Savings | $+$ if $\theta$ small<br>$-$ if $\theta$ large | $+$ if $\rho > 1$<br>$-$ if $\rho < 1$ | $+$ if $\theta > 1$<br>$-$ if $\theta < 1$ |
| $\mathrm{Cov}(K_1, S_1)$ | $-$ | $-$ | $-$ |

Table 2 presents model predictions in the four cases of housing price appreciation. Predictions are about 1) term structure of interest rates, 2) future rent growth, 3) current cap rate, 4) savings, and 5) covariation of investments in business capital and in housing structure. $\Delta A_t > 0$ and $\Delta B_t > 0$ refer to a positive shock at $t$ to the production of goods and housing, respectively. "Mixed" response refers to more complex comparative statics.

Table 3: Effects of technology shocks on asset prices

<table>
<tr><th></th><th>Housing Asset</th><th>Human Capital</th><th>Financial Assets</th></tr>
<tr><td>$\Delta A_1 > 0$</td><td>+ if $\mu$ small<br>− if $\mu$ large</td><td>+</td><td>+</td></tr>
<tr><td>$\Delta A_2 > 0$</td><td>$- \text{ if } \begin{cases} \mu \text{ large} \\ \rho \text{ small} \\ \theta \text{ small} \end{cases}$<br>+ otherwise</td><td>+</td><td>+ if $\theta$ large<br>− if $\theta$ small</td></tr>
<tr><td>$\Delta A_1 =$<br>$\Delta A_2 > 0$</td><td>+ if $\mu$ small<br>− if $\mu$ large</td><td>+</td><td>+</td></tr>
<tr><td>$\Delta B_1 > 0$</td><td>−</td><td>$- \text{ if } \begin{cases} \rho \text{ large} \\ \theta \text{ large} \end{cases}$<br>+ otherwise</td><td>$- \text{ if } \begin{cases} \rho \text{ large} \\ \theta \text{ large} \end{cases}$<br>+ otherwise</td></tr>
<tr><td>$\Delta B_2 > 0$</td><td>−</td><td>$- \text{ if } \begin{cases} \rho \text{ large} \\ \theta \text{ small} \end{cases}$<br>+ otherwise</td><td>+ if $\theta$ large<br>− if $\theta$ small</td></tr>
<tr><td>$\Delta B_1 =$<br>$\Delta B_2 > 0$</td><td>−</td><td>+ if $\rho < 1$<br>− if $\rho > 1$</td><td>+ if $\rho < 1$<br>− if $\rho > 1$</td></tr>
</table>

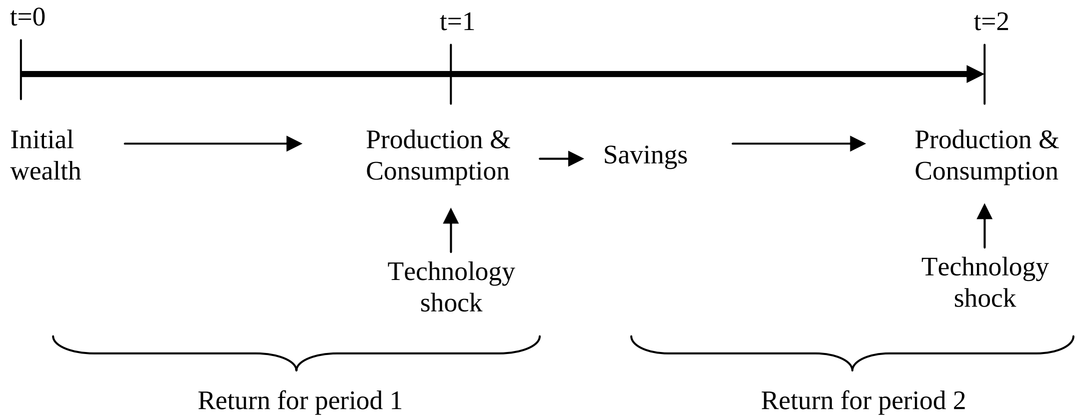
Table 3 presents effects of different types of technology shock on asset prices. Each row corresponds to different types of shock. $\Delta A_t > 0$ and $\Delta B_t > 0$ refer to a positive shock at $t$ to the production of goods and housing, respectively. $\mu$ is elasticity of land supply. $\rho$ and $\theta$ are parameters for intra- and inter-temporal substitution, respectively.

Table 4: Households' stock holdings, asset price correlations, and land supply elasticity

| | Stock holdings (As of end of 2001) | Price correlations (Quarterly, 1970-2006) | Land supply elasticity (per capita habitable area, 2003-2005) |
|---|---|---|---|
| Japan | 0.07 | 0.36 | 0.08 |
| Germany | 0.14 | 0.05 | 0.28 |
| UK | 0.13 | 0.09 | 0.36 |
| Italy | 0.22 | 0.06 | 0.38 |
| France | 0.29 | -0.03 | 0.65 |
| USA | 0.34 | -0.04 | 1.93 |
| Canada | 0.28 | -0.08 | 14.13 |

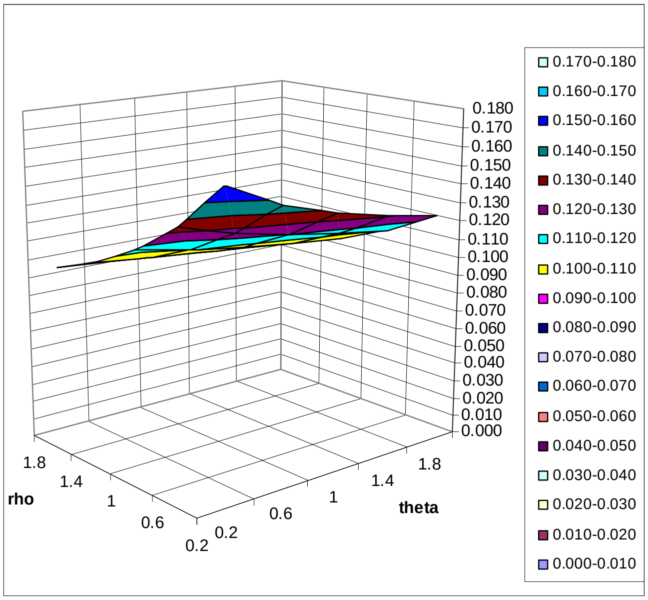
Table 4 compares households' stock holdings, asset price correlations, and land supply elasticity for seven OECD countries. Stock holdings are the ratio of shares and other equities to total assets in the flow of funds account of each country as of the end of 2001 (Bank of Japan (2003)). The price correlations are correlation coefficients between housing prices and four-quarter lagged stock price. Asset prices are BIS calculations based on quarterly national data from 1970 to 2006. The land supply elasticity is proxied by per capita habitable area, calculated from FAOSTAT and OECD data during 2003 and 2005. The habitable area is "Land Area" minus "Inland Water" and "Forest and Woodland."

Figure 1: Time line

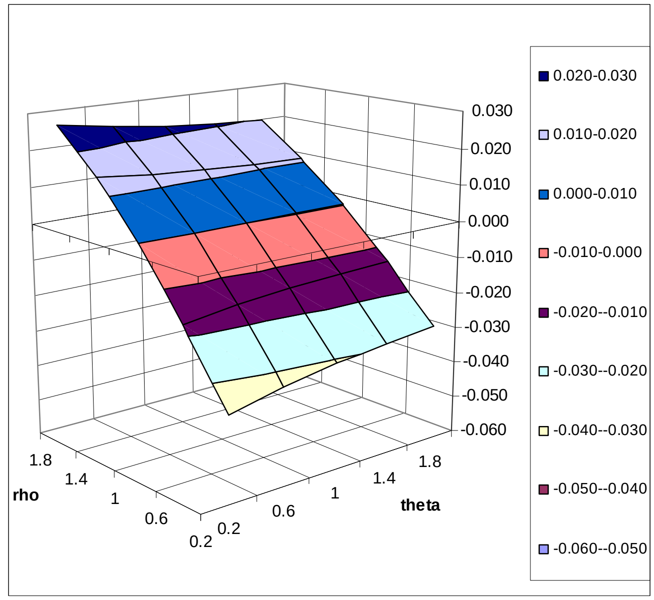


## Figure 2: Effects on the interest rate and savings

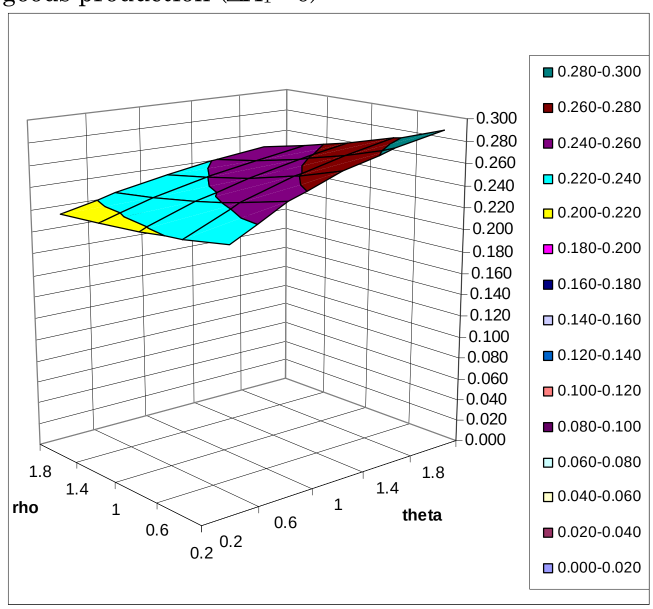
2-a) Response of interest rate ($\Delta i_1 = 1 / \Delta\varphi_{0,1}$) to a positive shock to goods production ($\Delta A_1 > 0$)




2-c) Response of interest rate ($\Delta i_1 = 1 / \Delta\varphi_{0,1}$) to a positive shock to housing production ($\Delta B_1 = \Delta B_2 > 0$)




2-b) Response of savings ($\Delta W_1$) to a positive shock to goods production ($\Delta A_1 > 0$)




2-d) Response of savings ($\Delta W_1$) to a positive shock to housing production ($\Delta B_1 > 0$)

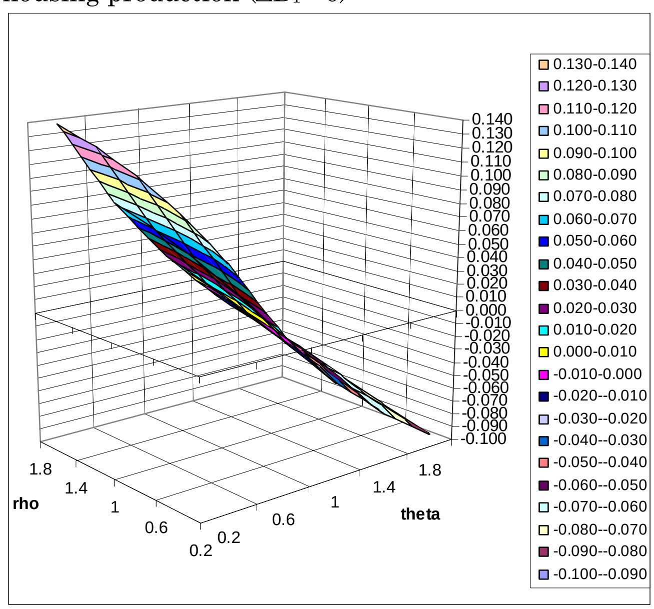
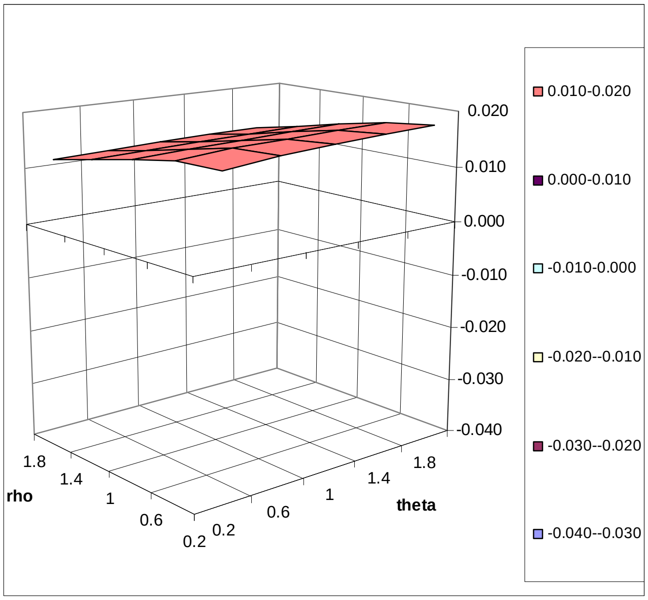


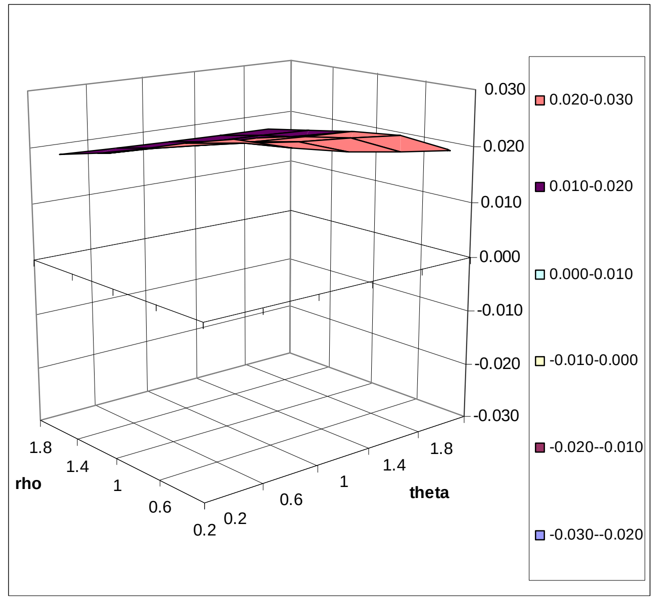

Figure 2 presents selected comparative statics of the interest rate and savings. Figures 2-a and 2-b show the response of the interest rate and savings, respectively, to a positive shock to the goods production for different values of ρ and θ. Figures 2-c and 2-d show the response of the interest rate and savings, respectively, to a positive shock to housing production for different values of ρ and θ.

## Figure 3: Effects on housing prices

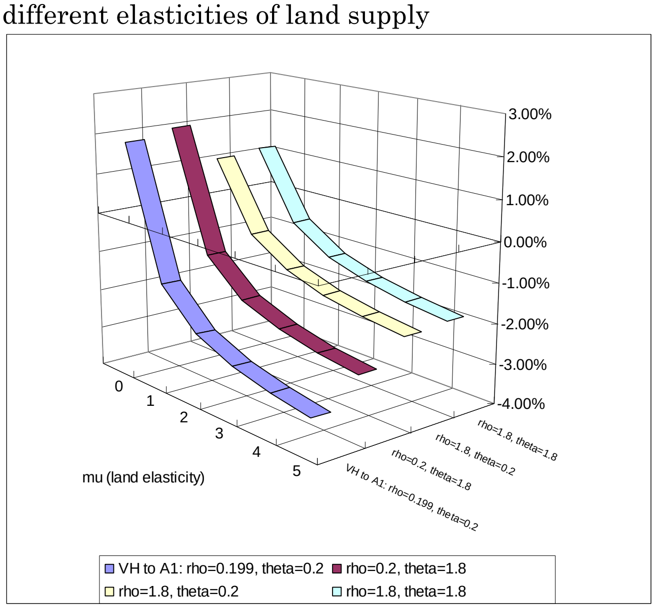
3-a) Case 1: Response of housing prices to a positive shock to goods production ($\Delta A_1 > 0$) if land supply is inelastic




3-c) Case 2: Response of housing prices to a negative shock to goods production ($\Delta A_1 < 0$) if land supply is elastic




3-b) Case 1 and Case 2: Response of housing prices to a positive shock to goods production ($\Delta A1 > 0$) for different elasticities of land supply




3-d) Case 4: Response of housing prices to a negative shock to housing production ($\Delta B_1 < 0$) if land supply is inelastic

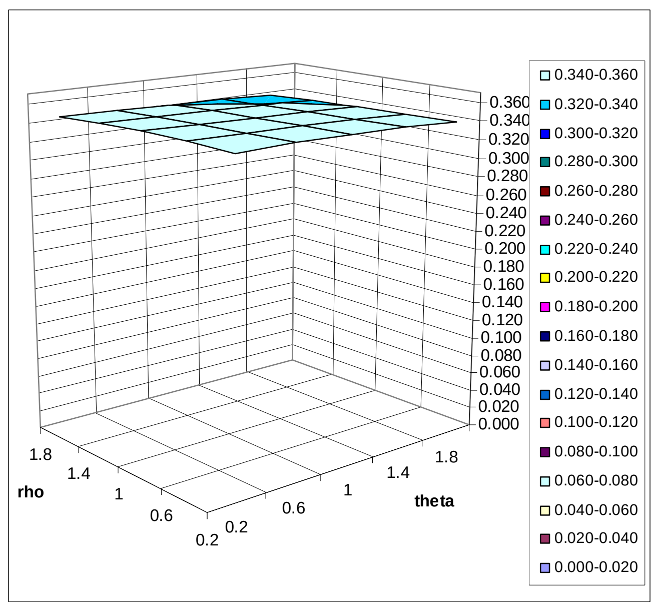
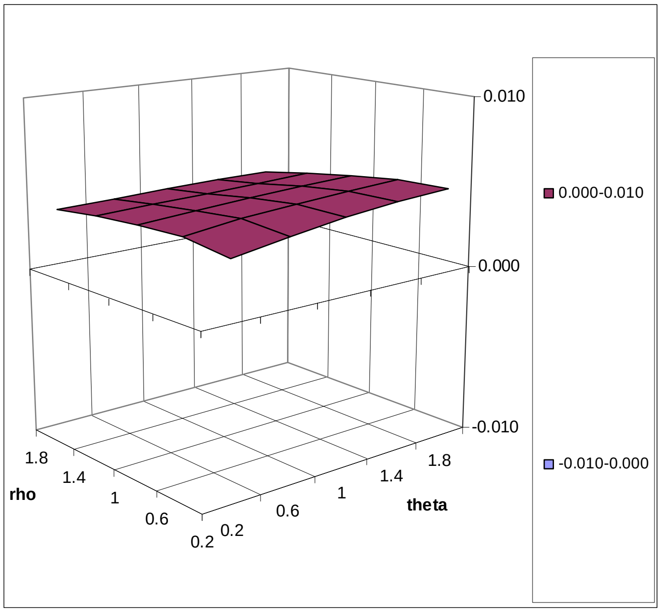


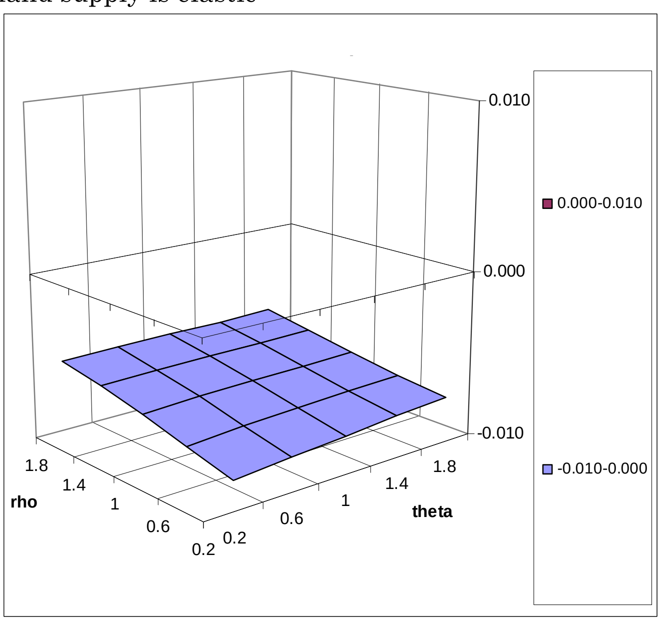

Figure 3 presents selected comparative statics of the response of housing prices. Figure 3-a shows the response of home prices to a positive shock to goods production for different values of $\rho$ and $\theta$ in an economy with $\mu = 0$. Figure 3-b shows the response of home prices to a positive shock to goods production for different values of $\mu$, $\rho$, and $\theta$. Figure 3-c shows the response of home prices to a negative shock to goods production for different values of $\rho$ and $\theta$ in an economy with $\mu = 5$. Figure 3-d shows the response of home prices to a negative shock to housing production for different values of $\rho$ and $\theta$ in an economy with $\mu = 0$.

## Figure 4: Covariation of asset prices

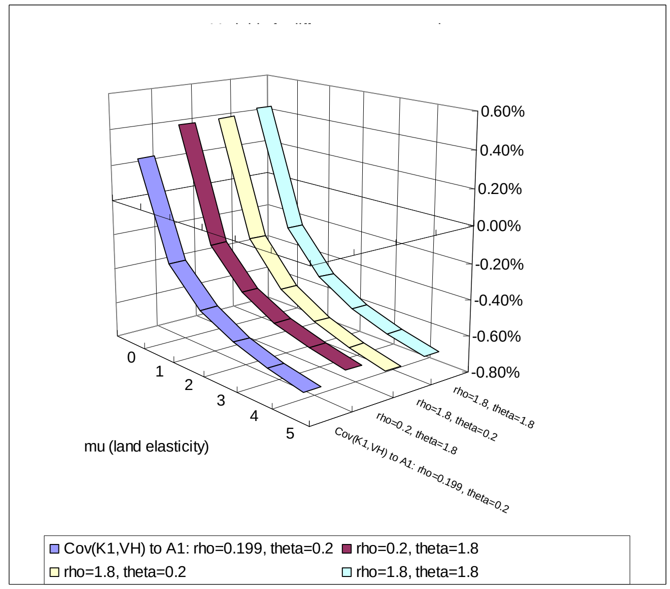
4-a) Covariation of housing prices and human capital in response to a shock to goods production ($\Delta A_1$) if land supply is inelastic




4-b) Covariation of housing prices and human capital in response to a shock to goods production ($\Delta A_1$) if land supply is elastic




4-c) Covariation of housing prices and human capital in response to a shock to goods production ($\Delta A_1$) for different elasticities of land supply




4-d) Covariation of housing prices and human capital in response to a shock to housing production ($\Delta B_1 = \Delta B_2$) if land supply is inelastic

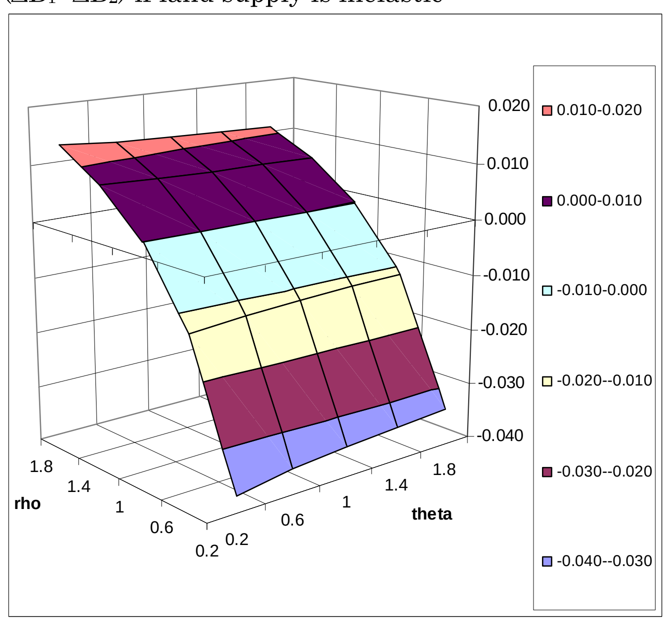



Figure 4 presents covariation of housing prices and human capital. Covariation is measured in terms of the product of percent changes in housing prices and human capital. Figure 4-a shows the covariation in response to a shock to goods production for different values of $\rho$ and $\theta$ in an economy with $\mu = 0$. Figure 4-b shows the same case as 4-a except that $\mu = 5$. Figure 4-c shows the covariation in response to a shock to goods production for different values of $\mu$. Figure 4-d shows covariation in response to a shock to housing production for different values of $\rho$ and $\theta$ in an economy with $\mu = 0$.

Figure 5: Land supply elasticity and asset price correlation, 1970-2006

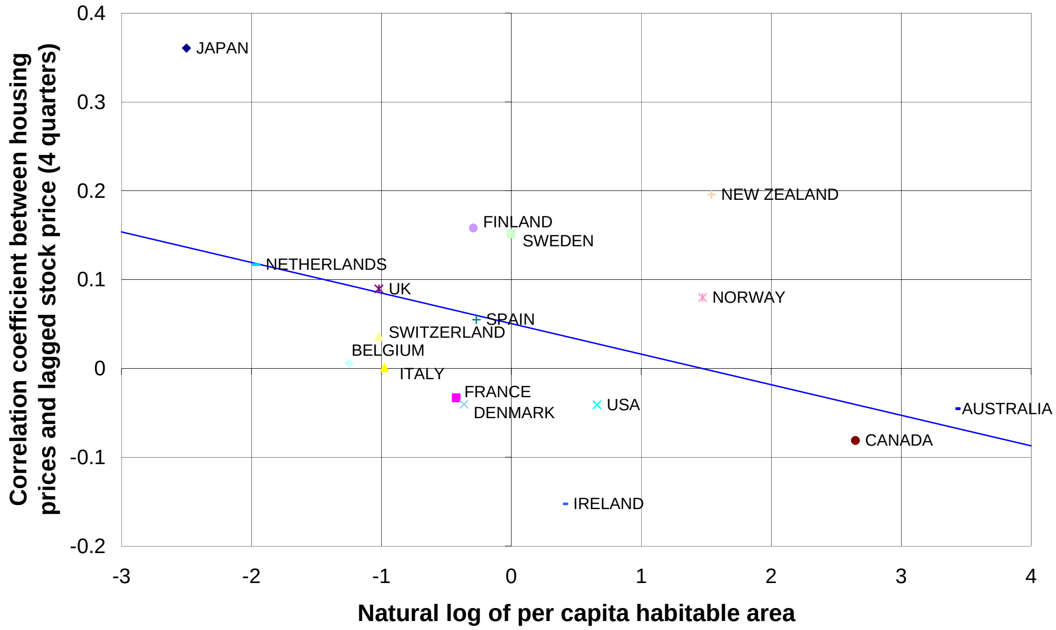
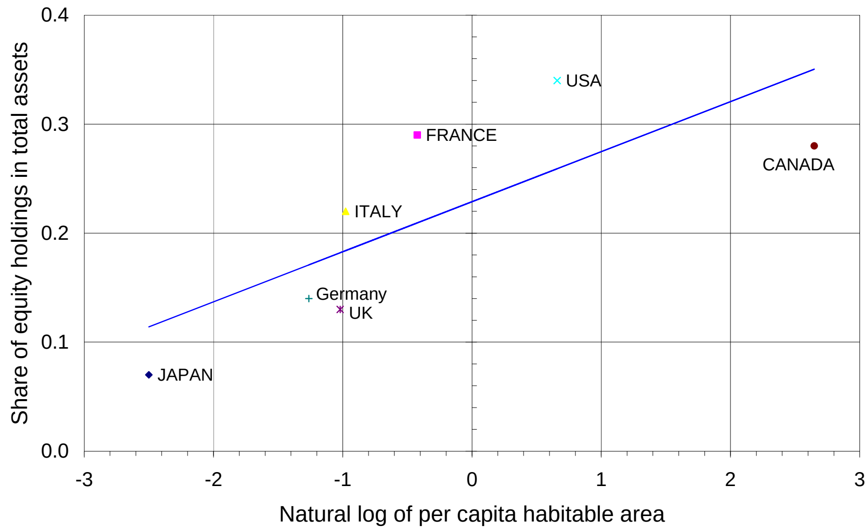


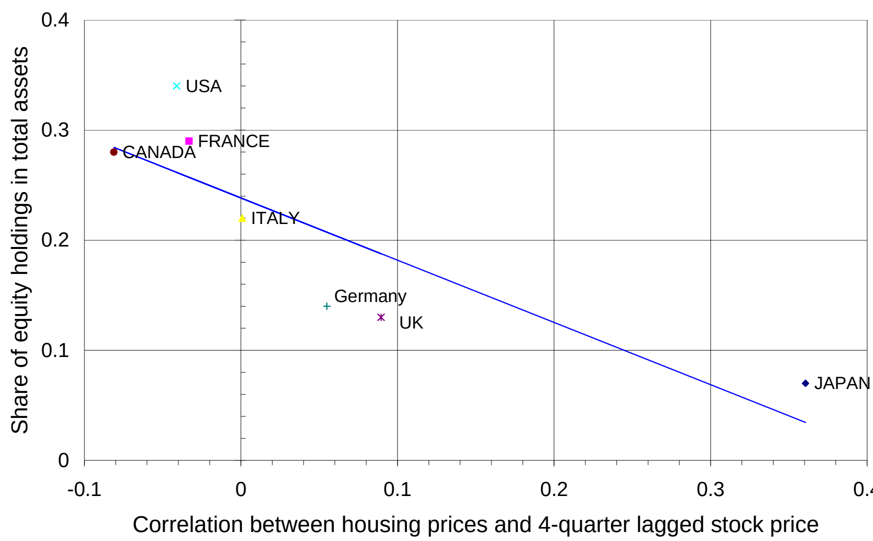

Figure 5 presents correlation coefficients between housing prices and four-quarter lagged stock price for seventeen OECD countries, plotted against the natural log of per capita habitable area, which is a measure of land supply elasticity. Asset prices are BIS calculations based on quarterly national data from 1970 to 2006. The per capita habitable area, a measure of land supply elasticity, is calculated from FAOSTAT and OECD data. The habitable area is "Land Area" minus "Inland Water" and "Forest and Woodland." The correlation coefficient between price correlations and land supply elasticity is -0.44. The line represents fitted values from a bivariate regression of the price correlation on the log habitable area. The slope is −0.0344 (standard errors are 0.0183 and the t-statistic is 1.88), and adjusted R-squared is 0.137.

Figure 6: Households' stock holdings

6-a) Land supply elasticity and households' stock holdings




6-b) Asset price correlations and households' stock holdings

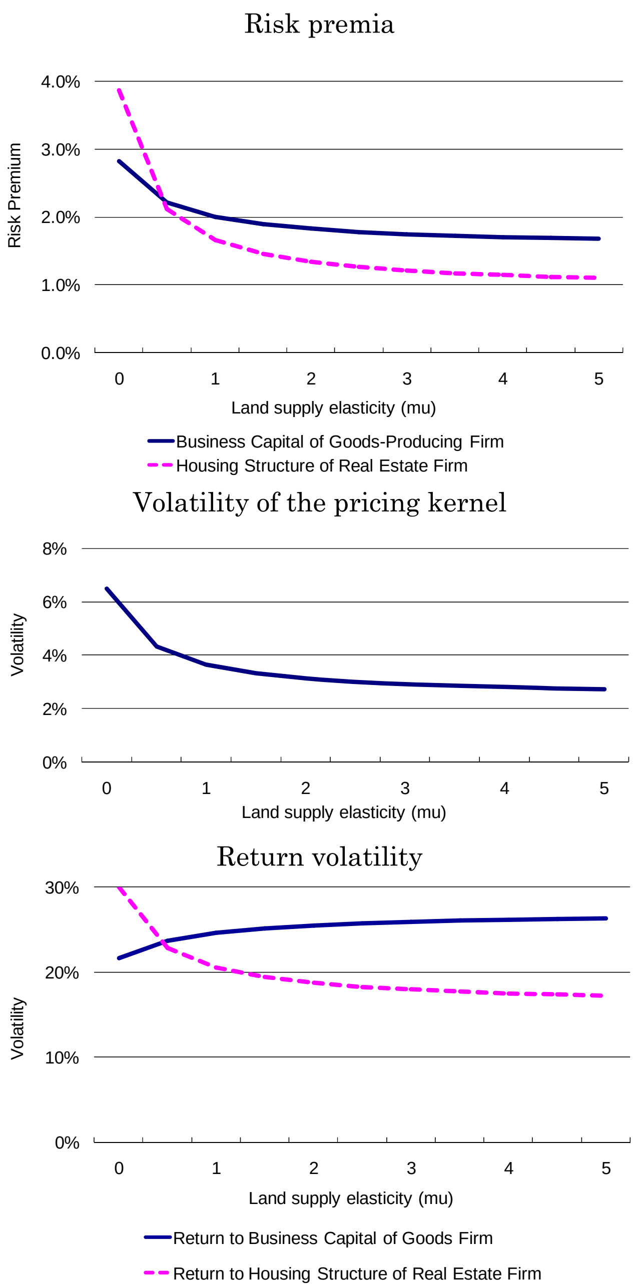



Figures 6-a and 6-b present share of equity holdings in households' total assets for seven OECD countries, plotted against the natural log of per capita habitable area (6-a) and against the correlation between housing prices and four-quarter lagged stock prices (6-b). The data on equity holdings are BOJ calculations based on the flow of funds account of each country (Bank of Japan (2003)). The per capita habitable area, a measure of land supply elasticity, is calculated from FAOSTAT and OECD data. The habitable area is "Land Area" minus "Inland Water" and "Forest and Woodland." Asset prices are BIS calculations based on quarterly national data from 1970 to 2006.

The correlation coefficients are 0.76 and -0.85 for Figures 6-a and 6-b, respectively. The lines represent fitted values from bivariate regressions. In Figure 6-a, the slope is 0.0459 (standard errors are 0.0175 and the t-statistic is 2.62), with adjusted R-squared of 0.495. In Figure 6-b, the slope is -0.566 (standard errors are 0.159 and the t-statistic is -3.57), with adjusted R-squared of 0.661.

Figure 7:
Risk premia, return volatility, and volatility of the pricing kernel (ρ < θ)

7-a) Risk premia, volatility of the pricing kernel, and return volatility for different values of land supply elasticity when goods productivity ($\Delta A_2$) is stochastic (ρ = 0.2, θ= 1.8)




7-b) Risk premia, volatility of the pricing kernel, and return volatility f for different values of intra-temporal substitution when goods productivity ($\Delta A_2$) is stochastic (μ = 0, θ = 1.8)

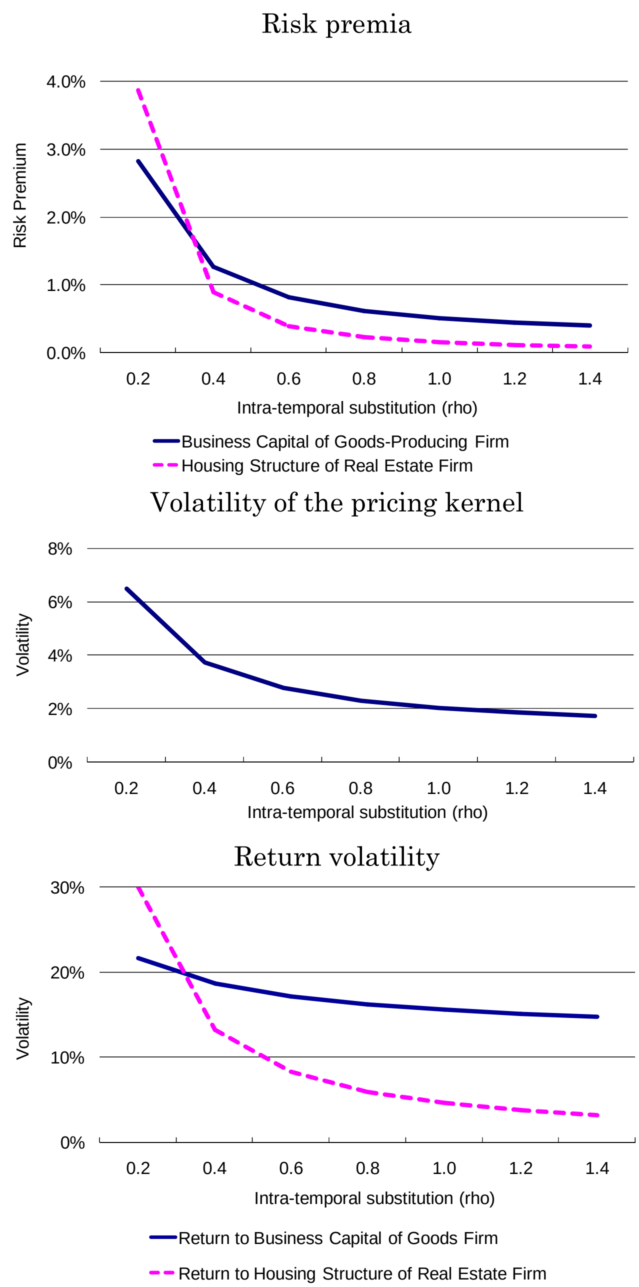



Figure 7 shows comparative statics of risk premia, return volatility, and volatility of the pricing kernel when goods-producing technology is stochastic. The figure focuses on cases in which two goods are relatively complementary: intra-temporal substitution (ρ) is lower than inter-temporal substitution (θ). In Figure 7-a, the results are plotted against different levels of land supply elasticity while intra- and inter-temporal substitutions are fixed at ρ=0.2 and θ=1.8. In Figure 7-b, the results are plotted against different levels of intra-temporal substitution while land supply elasticity and inter-temporal substitution are fixed at μ=0 and θ=1.8.

Figure 8:
Volatility of consumption growth and rent growth factor

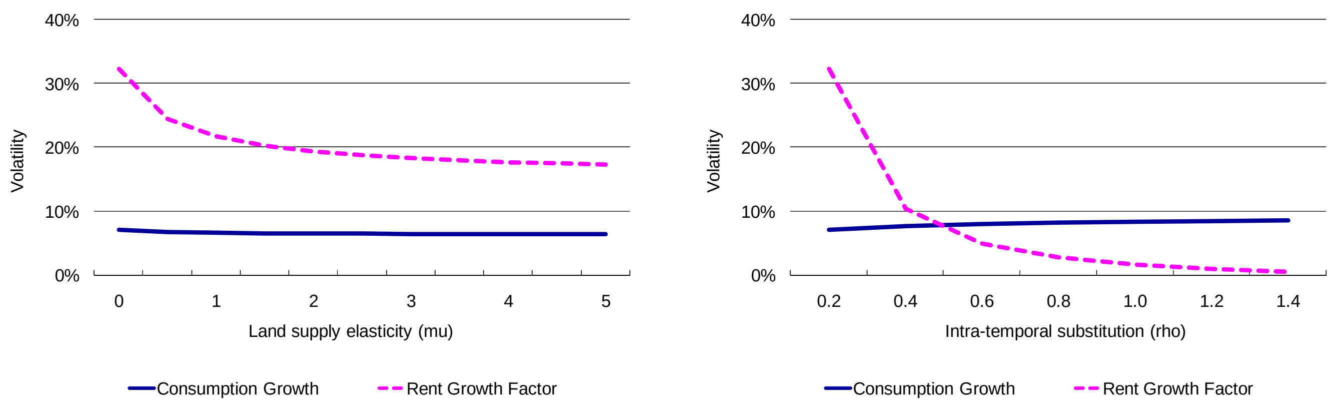
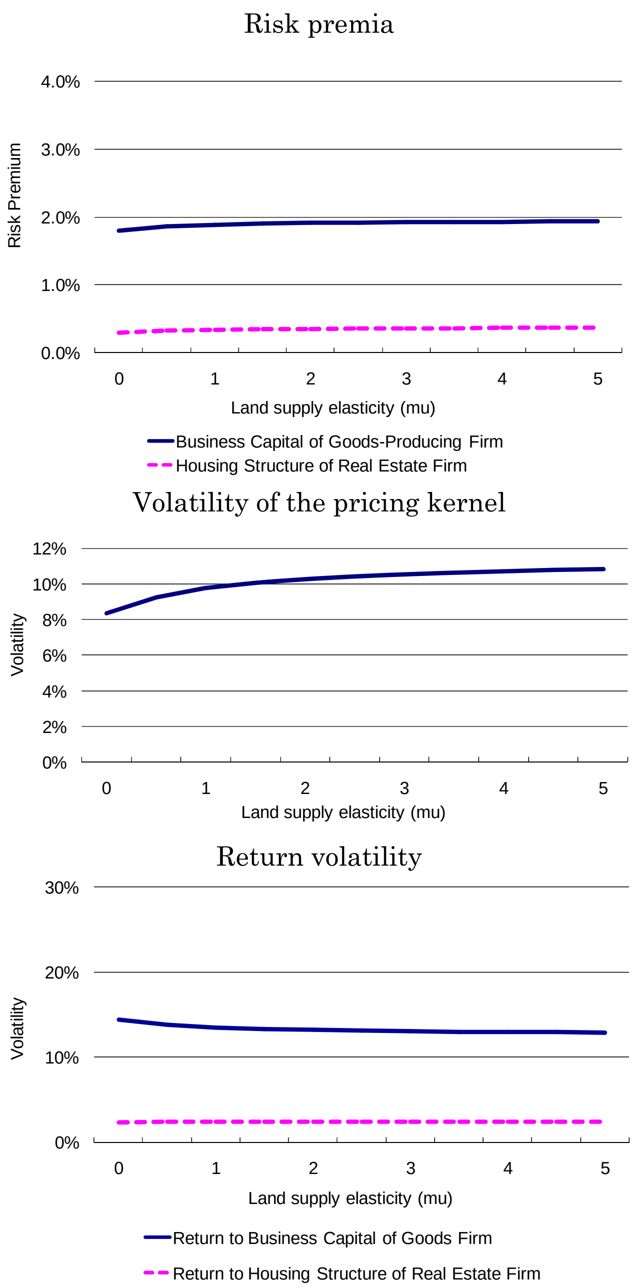



Figure 8 shows comparative statics of two components of the pricing kernel: volatility of consumption growth ($g_{c,2}$) and volatility of rent growth factor ($g_{p,2}{}^{\theta-\rho}$) when goods-producing technology is stochastic. The figure focuses on cases in which two goods are relatively complementary: intra-temporal substitution ($\rho$) is lower than inter-temporal substitution ($\theta$). On the left panel, the results are plotted against different levels of land supply elasticity while intra- and inter-temporal substitutions are fixed at $\rho$=0.2 and $\theta$=1.8. On the right panel, the results are plotted against different levels of intra-temporal substitution while land supply elasticity and inter-temporal substitution are fixed at $\mu$=0 and $\theta$=1.8.

## Figure 9:
## Risk premia, return volatility, and volatility of the pricing kernel ($\rho > \theta$)

9-a) Risk premia, volatility of the pricing kernel, and return volatility for different values of land supply elasticity when goods productivity ($\Delta A_2$) is stochastic ($\rho = 1.8$, $\theta = 0.2$)




9-b) Risk premia, volatility of the pricing kernel, and return volatility f for different values of intra-temporal substitution when goods productivity ($\Delta A_2$) is stochastic ($\mu = 0$, $\theta = 0.2$)

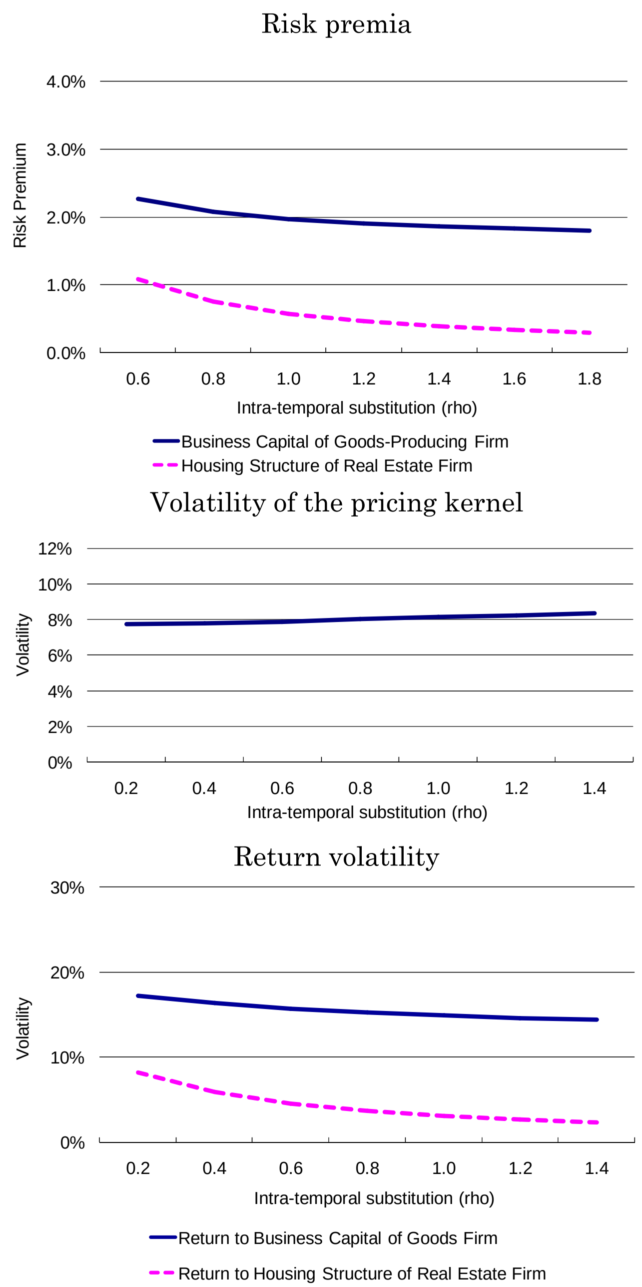
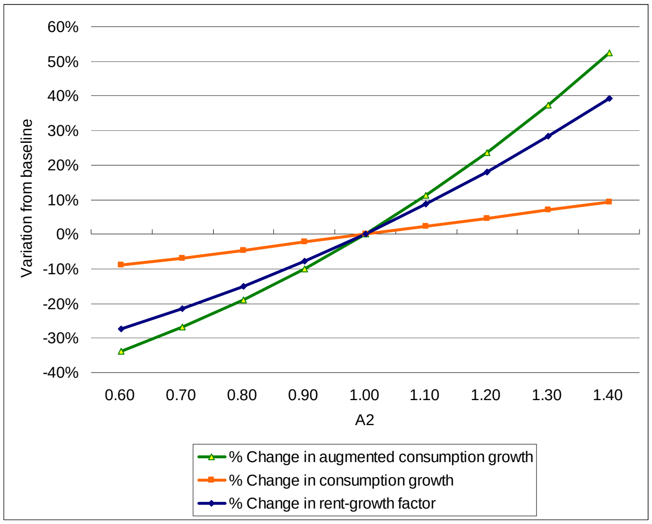


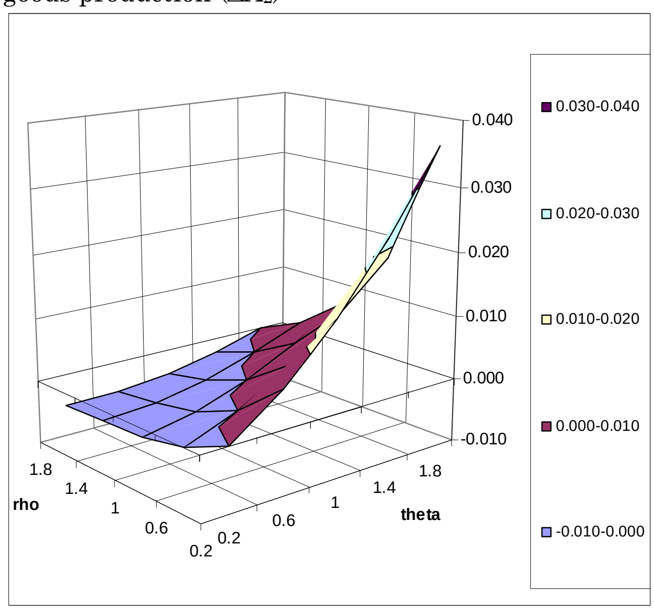

Figure 9 shows comparative statics of risk premia, return volatility, and volatility of the pricing kernel when goods-producing technology is stochastic. The figure focuses on cases in which two goods are relatively substitutable: intra-temporal substitution ($\rho$) is higher than inter-temporal substitution ($\theta$). In Figure 9-a, the results are plotted against different levels of land supply elasticity while intra- and inter-temporal substitutions are fixed at $\rho$=1.8 and $\theta$=0.2. In Figure 9-b, the results are plotted against different levels of intra-temporal substitution while land supply elasticity and inter-temporal substitution are fixed at $\mu$=0 and $\theta$=0.2.

## Figure 10:
## Mitigating the equity premium puzzle and the risk-free rate puzzle

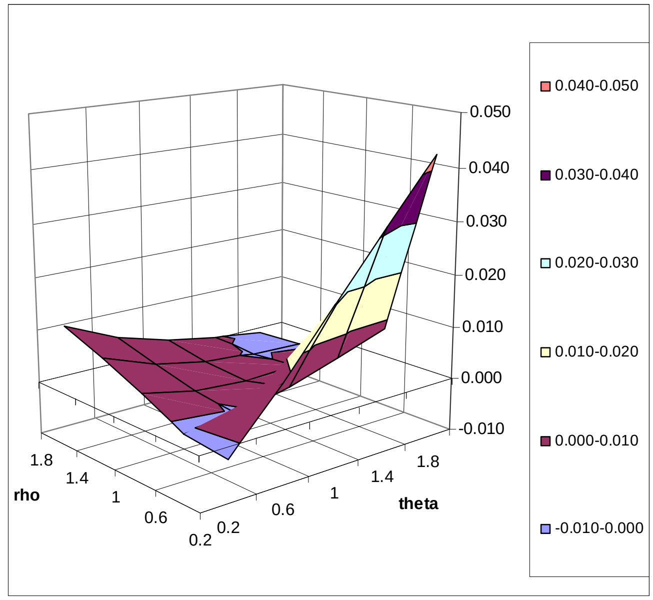
10-a) Variation of consumption growth to an anticipated shock to goods production ($\Delta A_2$) ($\rho = 0.2$, $\theta = 1.8$)




10-b) Covariation of consumption growth ($g_{c,2}$) and rent growth factor ($g_{p,2}{}^{\theta-\rho}$) to an anticipated shock to goods production ($\Delta A_2$)




10-c) Covariation of consumption growth ($g_{c,2}$) and the pricing kernel ($\varphi_{1,2}$) to an anticipated shock to housing production ($\Delta B_2$)




10-d) Variation of the pricing kernel ($\varphi_{1,2}$), consumption growth ($g_{c,2}$), and rent growth factor ($g_{p,2}{}^{\theta-\rho}$) to an anticipated shock to housing production ($\Delta B_2$) ($\rho = 0.2$, $\theta = 1.8$)

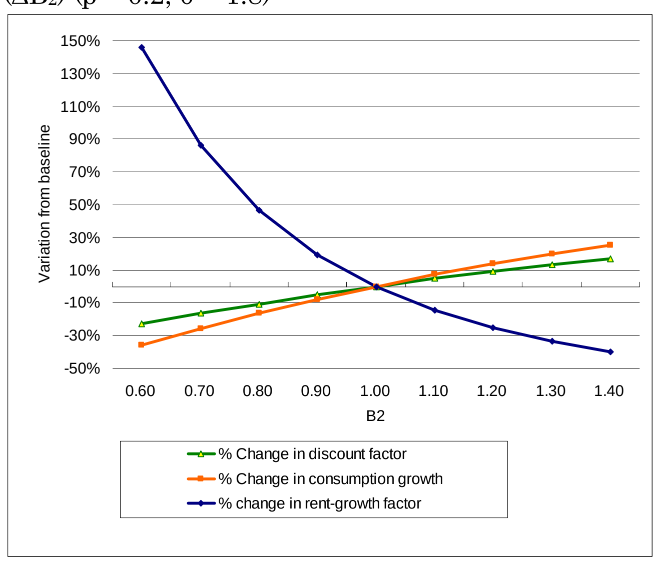



Figure 10 presents two cases in which the equity premium puzzle and the risk-free rate puzzle are mitigated. Figure 10-a presents percentage changes in $g_{c,2}$, $g_{p,2}{}^{\theta-\rho}$, and $g_{c,2}\, g_{p,2}{}^{\theta-\rho}$ from their baselines against different levels of $A_2$. Consumption growth is magnified by the rent growth factor. Figures 10-b and 10-c show covariation of $g_{c,2}$ with $g_{p,2}{}^{\theta-\rho}$ and $g_{c,2}$ with $\varphi_{1,2}$, respectively, against different values of $\rho$ and $\theta$. Figure 10-d shows percentage changes in $g_{c,2}$, $g_{p,2}{}^{\theta-\rho}$, and $\varphi_{1,2}$ from their baselines against different levels of $A_2$.

Figure 11:
Volatilities of two types of the pricing kernel

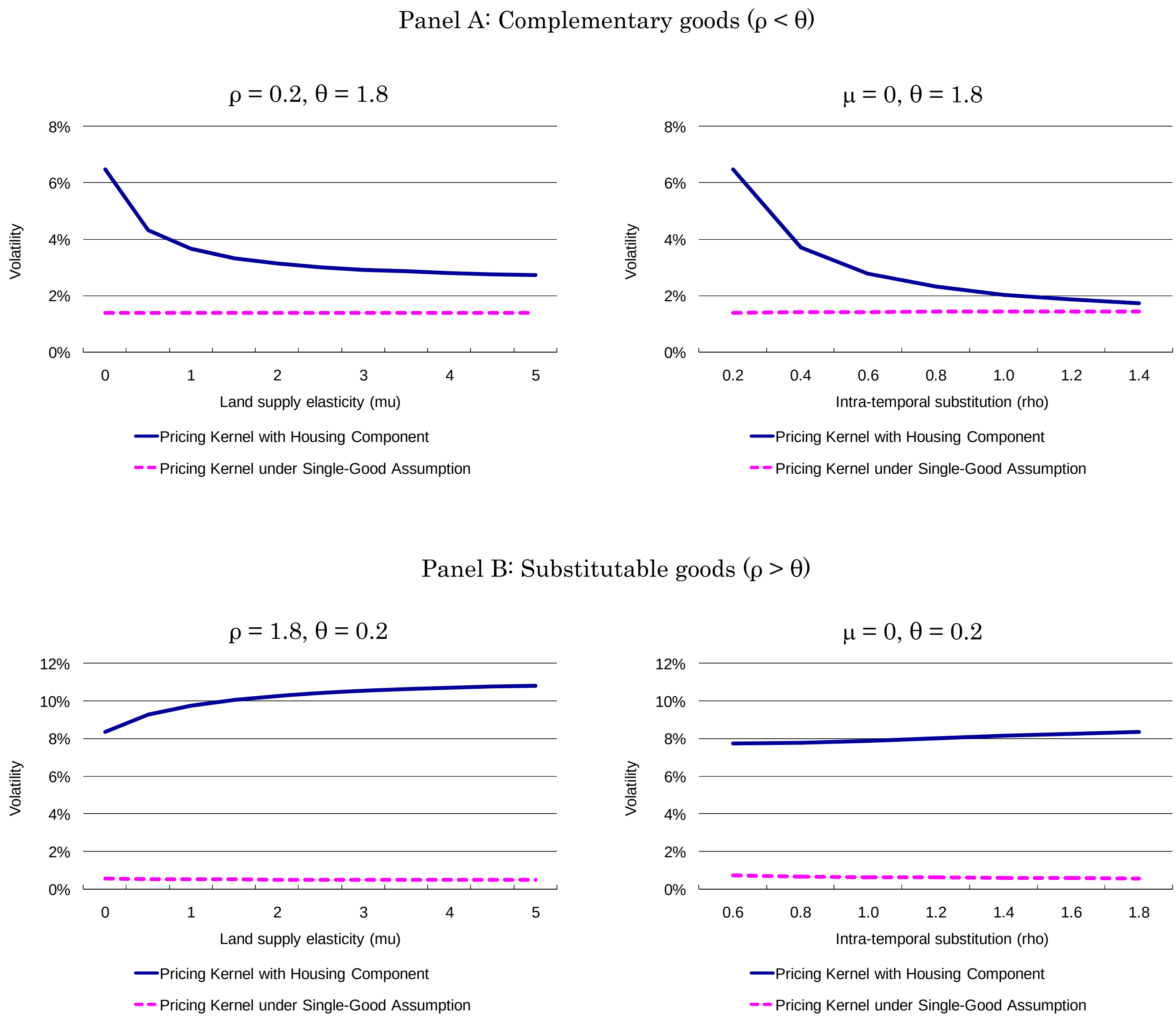



Figure 11 compares volatilities of two different types of the pricing kernel when goods-producing technology is stochastic. Panel A depicts cases in which two goods are relatively complementary: intra-temporal substitution ($\rho$) is lower than inter-temporal substitution ($\theta$). Panel B depicts substitutable cases ($\rho > \theta$). On the left panel, the results are plotted against different levels of land supply elasticity while intra- and inter-temporal substitutions are fixed. On the right panel, the results are plotted against different levels of intra-temporal substitution while land supply elasticity and inter-temporal substitution are fixed.