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Timing of Convertible Debt Financing and Investment

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Abstract

In this paper, we examine the optimal investment policy of the firm which is financed by issuing equity, straight debt and convertible debt. We extend the model in Mauer and Sarkar [7] over financing with convertible debt. We examine two different investment policies that maximize the equity value and the firm value and show the agency cost as the difference between each policy value. Furthermore, we investigate how the issuance of convertible debt affects investment.

Keywords: Real options, convertible debt, investment, agency cost

1 Introduction

Real options theory, pioneered by Brennan and Schwartz [2], and McDonald and Siegel [9], and summarized in Dixit and Pindyck [3], has attracted growing attention because it enables us to account for the value of flexibility under uncertainty. In standard real options models, all-equity financing is assumed, and the interactions between investment and financing decisions have been not analyzed.

Recently many researchers have studied the interaction among firm's investment and financing decisions under uncertainty by means of real option framework. In some literatures, the investment problems for the firm with growth options, which is financed with equity and debt are investigated (e.g. Lyandres and Zhdanov [4], Mauer and Ott [6], Mauer and Sarkar [7], and

Sundaresan and Wang [10]). Although the debt used in these studies is straight debt, there also exists a previous work on the effect of convertible debt financing on the investment decisions. Lyandres and Zhdanov [5] suggests the model for analyzing the investment problem of the firm with outstanding convertible debt and discusses the accelerated investment effect arising from the issuance of convertible debt by the optimal investment policy to maximize the equity value. In their model, the value of the firm, which is the sum of the equity and debt values, and the leverage ratio are not analyzed.

In this paper, we examine the optimal investment policy of the firm which is financed by issuing equity, straight debt and convertible debt. We extend the model in Mauer and Sarkar [7] over financing with convertible debt in the following section. As in Mauer and Sarkar [7], we examines two different investment policies that maximize the equity value and the firm value. In Sec. 3 we discuss a difference of the optimal investment policies maximizing between the value of equity and the firm value by issuing convertible debt, and then show the agency cost as the difference between each policy value by using numerical results. Furthermore, we investigate how the issuance of convertible debt affects investment. Finally, in Sec. 4 we summarize this paper with some concluding remarks.

2 The Model

Consider a firm with an option to invest at any time by paying a fixed investment cost I. The firm partially finances the cost of investment with straight debt and convertible debt. Denote K_s as the total issue value of straight debt with the instantaneous contractual coupon payment of s and infinite maturity, and K_c as that of convertible debt with coupon payment of c and infinite maturity. These coupon payments are tax-deductible at a constant corporate tax rate τ. Once the investment option is exercised, we assume that the firm can receive the instantaneous profit

$$\pi(x_t) = (1-\tau)(x_t - s - c), \tag{1}$$

where x_t is the firm's instantaneous EBIT. Suppose that x_t is given by a geometric Brownian motion

$$dx_t = \mu x_t dt + \sigma x_t dW_t, \tag{2}$$

where μ and σ are the risk-adjusted expected growth rate and the volatility of x_t, respectively, and W_t is a standard Brownian motion defined on a probability space $(\Omega, F, \mathbb{P})$. We assume that the holders of convertible debt can convert the debt into a fraction η of the original equity, where $\eta = \alpha c$ and α is a constant. In this paper we deal with only non-callable convertible debt.

In order to examine two different investment policies which maximize the equity value and the firm value and investigate how the issue of convertible debt influences investment, we consider several settings. First, we present a benchmark model in which the investment is financed with all-equity. Second, we examine the case in that the investment is financed with equity and straight debt, based on the analysis in Mauer and Sarkar [7]. Third, we model the investment financed with equity and convertible debt. Finally, we consider the firm with an option of investment that is financed with equity and both debt.

2.1 All-Equity Financing

In this section we assume that the investment is financed entirely with equity ($s = 0$ and $c = 0$). This case has been studied in the literature on real options (e.g. Dixit and Pindyck [3] and McDonald and Siegel [9]).

The optimal investment rule is to exercise the investment option at the first passage time of the stochastic shock to an upper threshold x^*. Assuming that the pre-investment profit of the firm is zero, the value of an investment option $F(x_0)$ can be formulated as

$$F(x_0) = \sup_{T^*>0} \mathbb{E}^{x_0} \left[e^{-rT^*} \left\{ \int_{T^*}^{\infty} e^{-r(u-T^*)}(1-\tau)x_u du - I \right\} \right], \tag{3}$$

where T^* is a stopping time (investment time) when x_t reaches the investment threshold x^*, $\mathbb{E}^{x_0}$ is the conditional expectation operator given that the EBIT at time 0 is equal to x_0, and r is the risk-free interest rate. For convergence, we assume $r > \mu$.

Since the ordinary differential equation, which is satisfied by the value of investment option in Eq. (3), is derived from Bellman equation[1]:

$$\frac{1}{2}\sigma^2 x^2 \frac{d^2F}{dx^2} + \mu x \frac{dF}{dx} - rF = 0 \tag{4}$$

for $x < x^*$, the general solution of Eq. (4) is given by

$$F(x) = a_1 x^{\beta_1} + a_2 x^{\beta_2}, \quad x < x^*, \tag{5}$$

where $\beta_1 = \frac{1}{2} - \frac{\mu}{\sigma^2} + \sqrt{\left(\frac{1}{2} - \frac{\mu}{\sigma^2}\right)^2 + \frac{2r}{\sigma^2}} > 1$ and $\beta_2 = \frac{1}{2} - \frac{\mu}{\sigma^2} - \sqrt{\left(\frac{1}{2} - \frac{\mu}{\sigma^2}\right)^2 + \frac{2r}{\sigma^2}} < 0$. Using standard arguments, $a_2 = 0$ and the investment threshold x^* is given by

$$x^* = \frac{\beta_1}{\beta_1 - 1} \frac{r-\mu}{1-\tau} I. \tag{6}$$

Then, the value of the investment option $F(x_0)$ is given by

$$F(x_0) = \left(\frac{x_0}{x^*}\right)^{\beta_1} \left(\epsilon(x^*) - I\right), \quad x_0 < x^*, \tag{7}$$

where $\epsilon(x)$ is the total post-investment profit in which the investment is financed entirely with equity,

$$\epsilon(x) = \frac{(1-\tau)x}{r-\mu}. \tag{8}$$

From $\beta_1 > 1$ and $r > \mu$, the investment threshold $x^* > I$. This means that the investment is made when the EBIT is higher than the investment cost.

[1] See, e.g., Dixit and Pindyck [3].

2.2 Equity and Straight Debt Financing

Next we consider a firm which has an option of the investment that is financed with equity and straight debt ($s > 0$ and $c = 0$), introduced in Mauer and Sarkar [7], and Sundaresan and Wang [10].

2.2.1 Optimal Policies after Investment

In this section we model the values of equity and straight debt after the exercise of investment option. Once the investment option has been exercised, the optimal default policy is established from the issue of debt. The optimal default strategy of the equity holders maximizes the equity value, selecting the default threshold x_d. Letting the earnings x_t at investment time t equal x, the optimization problem of the equity holders can be given by

$$E(x) = \sup_{T_d > 0} \mathbb{E}^x \left[\int_t^{T_d} e^{-r(u-t)}(1-\tau)(x_u - s)du \right], \tag{9}$$

where T_d is the stopping time on reaching the default threshold x_d. Using standard arguments as in Sec. 2.1, the equity value $E(x)$ is given by

$$E(x) = \epsilon(x) - \frac{(1-\tau)s}{r} - \left(\frac{x}{x_d}\right)^{\beta_2} \left(\epsilon(x_d) - \frac{(1-\tau)s}{r}\right) \tag{10}$$

for $x > x_d$ and the default threshold x_d is

$$x_d = \frac{\beta_2}{\beta_2 - 1} \frac{s(r-\mu)}{r}. \tag{11}$$

Let $D_s(x)$ be the total value of straight debt issued at investment time t. Since the holders of straight debt can receive the continuous coupon payment of s, the value of straight debt is given by

$$D_s(x) = \mathbb{E}^x \left[\int_t^{T_d} e^{-r(u-t)} s du + e^{-r(T_d - t)}(1-\theta)\epsilon(x_{T_d}) \right], \tag{12}$$

where θ is the proportional bankruptcy cost, $0 \leq \theta \leq 1$. The holders of straight debt are entitled to the unlevered value of the firm net of proportional bankruptcy cost, $(1-\theta)\epsilon(x)$ at bankruptcy. Then, the value of straight debt is given by

$$D_s(x) = \frac{s}{r} - \left(\frac{x}{x_d}\right)^{\beta_2} \left(\frac{s}{r} - (1-\theta)\epsilon(x_d)\right), \quad x > x_d. \tag{13}$$

The sum of $E(x)$ in Eq. (10) and $D_s(x)$ in Eq. (13) gives the firm value as

$$V_s(x) = E(x) + D_s(x) = \epsilon(x) + \frac{\tau s}{r} \left\{ 1 - \left(\frac{x}{x_d}\right)^{\beta_2} \right\} - \theta\epsilon(x_d). \tag{14}$$

The firm value $V_s(x)$ is decomposed into the value of unlevered firm $\epsilon(x)$, the expected present value of straight debt tax shields and the expected present value of bankruptcy cost.

2.2.2 Optimal Investment Policy

Here we consider the optimal investment policy. First, we examine the optimal policy maximizing the value of equity, not total firm value. Denote x_2^* as the second-best investment threshold. By the optimal policy selecting x_2^* to maximize the equity value (9), the value of the investment option $F_2(x_0)$ is given by

$$F_2(x_0) = \sup_{T_2^*>0} \mathbb{E}^{x_0} \left[e^{-rT_2^*} \left\{ E(x_{T_2^*}) - (I - K_s) \right\} \right], \tag{15}$$

where T_2^* is the stopping time on reaching the investment threshold x_2^*, and K_s is the total values of straight debt issued at investment. Using the value matching and smooth-pasting conditions at the investment threshold, the value of the investment option is given by

$$F_2(x_0) = \left(\frac{x_0}{x_2^*} \right)^{\beta_1} \left\{ E(x_2^*) - (I - K_s) \right\} \tag{16}$$

for $x_0 < x_2^*$ and the investment threshold x_2^* is given by the numerical solution of

$$E(x_2^*) - \frac{x_2^*}{\beta_1} \frac{dE}{dx}(x_2^*) - (I - K_s) = 0, \tag{17}$$

where K_s is given by the value of straight debt at the investment threshold x_2^*,

$$K_s = D_s(x_2^*) = \frac{s}{r} - \left(\frac{x_2^*}{x_d} \right)^{\beta_2} \left(\frac{s}{r} - (1-\theta)\epsilon(x_d) \right). \tag{18}$$

Noticing that $K_s = D_s(x_2^*)$ in Eq. (18) and $V_s(x_2^*) = E(x_2^*) + D_s(x_2^*)$, Eq. (16) can be rewritten as

$$F_2(x_0) = \left(\frac{x_0}{x_2^*} \right)^{\beta_1} \left\{ V_s(x_2^*) - I \right\}. \tag{19}$$

Next, we analyze the optimal investment policy maximizing the firm value. By choosing the first-best investment threshold x_1^* and maximizing the firm value $V_s(x)$ in Eq. (14), the value of the investment option $F_1(x_0)$ is given by

$$F_1(x_0) = \sup_{T_1^*>0} \mathbb{E}^{x_0} \left[e^{-rT_1^*} \left\{ V_s(x_{T_1^*}) - I \right\} \right], \tag{20}$$

where T_1^* is the stopping time on reaching the threshold x_1^*. For $x_0 < x_1^*$ the value of the investment option is

$$F_1(x_0) = \left(\frac{x_0}{x_1^*} \right)^{\beta_1} \left\{ V_s(x_1^*) - I \right\} \tag{21}$$

and the investment threshold x_1^* is numerically solvable from

$$V_s(x_1^*) - \frac{x_1^*}{\beta_1} \frac{dV_s}{dx}(x_1^*) - I = 0. \tag{22}$$

2.3 Equity and Convertible Debt Financing

In this section we consider a firm which has an option of the investment that is financed with equity and convertible debt ($s = 0$ and $c > 0$). In this case the optimal problems of the equity holders and the convertible debt holders have to be solved simultaneously. The optimal policy of the equity holders maximizes the equity value selecting the default threshold. On the other hand, the optimal policy of the convertible debt holders maximizes the value of convertible debt selecting the conversion threshold. We follow Brennan and Schwartz [1] and assume block conversion. This means that all convertible debt holders exercise the conversion option at the same time.

2.3.1 Optimal Policies after Investment

We examine the values of equity and convertible debt issued at investment time. The equity holders optimally select the default threshold x_d, maximizing the equity value. The equity value at investment time is given by

$$E(x) = \sup_{T_d>0} \mathbb{E}^x \left[\int_t^{T_c \wedge T_d} e^{-r(u-t)}(1-\tau)(x_u - c)du + 1_{\{T_c<T_d\}} \frac{1}{1+\eta} \int_{T_c}^{\infty} e^{-r(u-t)}(1-\tau)x_u du \right], \tag{23}$$

where T_d is the stopping time on reaching the default threshold x_d, T_c is the stopping time on reaching the conversion threshold x_c selected by the convertible debt holders, and $1_{\{T_c<T_d\}}$ is an indicator function that is equal to one if $T_c < T_d$ and is equal to zero otherwise. By converting the equity value is diluted, that is, $\frac{1}{1+\eta}$ is the dilution factor.

Let $D_c(x)$ be the total value of convertible debt issued at investment time t. The holders of convertible debt receive the continuous coupon payment of c and choose the optimal conversion threshold x_c, maximizing the value of convertible debt. Then, the total value of convertible debt issued at investment time is given by

$$\begin{aligned} D_c(x) = \sup_{T_c>0} \mathbb{E}^x \Bigg[& \int_t^{T_c \wedge T_d} e^{-r(u-t)} c du + 1_{\{T_d<T_c\}} e^{-r(T_d-t)}(1-\theta)\epsilon(x_{T_d}) \\ & + 1_{\{T_c<T_d\}} \frac{\eta}{1+\eta} \int_{T_c}^{\infty} e^{-r(u-t)}(1-\tau)x_u du \Bigg]. \end{aligned} \tag{24}$$

The holders of convertible debt are entitled to $(1-\theta)\epsilon(x)$ at bankruptcy.

Once the convertible debt has been converted, the firm becomes an all-equity entity. It follows from the optimal problems of the equity holders and convertible debt holders in (23) and (24), respectively, that the values of equity and convertible debt prior to default and conversion are given by

$$E(x) = a_3 x^{\beta_1} + a_4 x^{\beta_2} + (1-\tau)\left(\frac{x}{r-\mu} - \frac{c}{r}\right), \tag{25}$$

$$D_c(x) = a_5 x^{\beta_1} + a_6 x^{\beta_2} + \frac{c}{r}. \tag{26}$$

Constants a_i, $i = 3, \cdots, 6$, the default threshold x_d and the conversion threshold x_c must be determined using the following boundary conditions:

$$a_3 x_d^{\beta_1} + a_4 x_d^{\beta_2} + (1-\tau)\left(\frac{x_d}{r-\mu} - \frac{c}{r}\right) = 0, \tag{27}$$

$$\beta_1 a_3 x_d^{\beta_1 - 1} + \beta_2 a_4 x_d^{\beta_2 - 1} + \frac{1-\tau}{r-\mu} = 0, \tag{28}$$

$$a_3 x_c^{\beta_1} + a_4 x_c^{\beta_2} + (1-\tau)\left(\frac{x_c}{r-\mu} - \frac{c}{r}\right) = \frac{1}{1+\eta}\frac{(1-\tau)x_c}{r-\mu}, \tag{29}$$

$$a_5 x_d^{\beta_1} + a_6 x_d^{\beta_2} + \frac{c}{r} = (1-\theta)\frac{(1-\tau)x_d}{r-\mu}, \tag{30}$$

$$a_5 x_c^{\beta_1} + a_6 x_c^{\beta_2} + \frac{c}{r} = \frac{\eta}{1+\eta}\frac{(1-\tau)x_c}{r-\mu}, \tag{31}$$

$$\beta_1 a_5 x_c^{\beta_1 - 1} + \beta_2 a_6 x_c^{\beta_2 - 1} = \frac{\eta}{1+\eta}\frac{1-\tau}{r-\mu}. \tag{32}$$

Eqs. (27), (28) and (30) represent conditions in default. Eq. (27) is the value matching condition which ensures that the value of equity at the default threshold is equal to zero. Eq. (28) is the smooth-pasting condition that ensures the optimality of the the default threshold x_d. Eq. (30) is the value-matching condition which ensures that the value of convertible debt at default threshold equals the unlevered value of the firm net of proportional bankruptcy cost. Eqs. (29), (31) and (32) represent conditions in conversion. Eq. (29) is the value matching condition requiring that the value of equity at the conversion threshold is equal to a proportion of the unlevered value of the firm possessed by the original equity holders after conversion. Eq. (31) is the value matching condition which ensures that the value of convertible debt at the conversion threshold is equal to the value of new equity issued in conversion. Eq. (32) is the smooth-pasting condition that ensures the optimality of the the conversion threshold x_c. Six equations (27)–(32) have six unknown variables ($a_i, i = 3, \cdots, 6, x_d, x_c$). We can solve these equations numerically.

Then, the firm value $V_c(x)$ is given by

$$V_c(x) = E(x) + D_c(x) = \epsilon(x) + \frac{\tau c}{r} + (a_3 + a_5)x^{\beta_1} + (a_4 + a_6)x^{\beta_2}. \tag{33}$$

2.3.2 Optimal Investment Policy

We examine two optimal investment policies to maximize the equity value (23) and the firm value (33). From Sec. 2.2.2 both the values of the investment option are given by

$$F_i(x_0) = \left(\frac{x_0}{x_i^*}\right)^{\beta_1} \{V_c(x_i^*) - I\} \tag{34}$$

for $x_0 < x_i^*$ and $i = 1, 2$. The second-best investment threshold x_2^* is given by the numerical solution of

$$E(x_2^*) - \frac{x_2^*}{\beta_1}\frac{dE}{dx}(x_2^*) - (I - D_c(x_2^*)) = 0, \tag{35}$$

and the first-best investment threshold x_1^* is given by the solution of

$$V_c(x_1^*) - \frac{x_1^*}{\beta_1}\frac{dV_c}{dx}(x_1^*) - I = 0. \tag{36}$$

2.4 Equity, Straight Debt and Convertible Debt Financing

In this section we consider a firm with an option of the investment that is financed with equity, straight debt and convertible debt ($s > 0$ and $c > 0$). For simplicity reasons, we assume that straight debt and convertible debt have the same priority. Hence, the holders of straight debt are entitled to $\frac{s}{s+c}(1-\theta)\epsilon(x)$ at pre-conversion bankruptcy and $(1-\theta)\epsilon(x)$ at post-conversion bankruptcy. Similarly, the convertible debt holders are entitled to $\frac{c}{s+c}(1-\theta)\epsilon(x)$ at pre-conversion bankruptcy.

2.4.1 Optimal Policies after Investment

We now model the values of equity, straight debt and convertible debt after the investment option has been exercised. The equity holders optimally select two default thresholds; the pre-conversion default threshold x_d and the post-conversion default threshold $x_{d,c}$, maximizing the equity value. The optimal post-conversion default threshold $x_{d,c}$ is not equal to the pre-conversion default one x_d, because debt decreases when convertible debt is converted into equity. The total value of equity at investment time is given by

$$
\begin{aligned}
E(x) = \sup_{T_d, T_{d,c} > 0} \mathbb{E}^x \Bigg[& \int_t^{T_c \wedge T_d} e^{-r(u-t)}(1-\tau)(x_u - s - c)du \\
& + 1_{\{T_c < T_d\}} \frac{1}{1+\eta} \int_{T_c}^{T_{d,c}} e^{-r(u-t)}(1-\tau)(x_u - s)du \Bigg], \qquad (37)
\end{aligned}
$$

where T_d and $T_{d,c}$ are the stopping times when x_t reach the default thresholds x_d and $x_{d,c}$, respectively, and T_c is the stopping time on reaching the conversion threshold x_c.

The value of the straight debt issued at investment time is given by

$$
\begin{aligned}
D_s(x) = \mathbb{E}^x \Bigg[& \int_t^{T_c \wedge T_d} e^{-r(u-t)} s du + 1_{\{T_d < T_c\}} e^{-r(T_d - t)} \frac{s}{s+c}(1-\theta)\epsilon(x_{T_d}) \\
& + 1_{\{T_c < T_d\}} \left(\int_{T_c}^{T_{d,c}} e^{-r(u-t)} s du + e^{-r(T_{d,c} - t)}(1-\theta)\epsilon(x_{T_{d,c}}) \right) \Bigg]. \qquad (38)
\end{aligned}
$$

Similarly, the value of convertible debt at investment time is given by

$$
\begin{aligned}
D_c(x) = \sup_{T_c > 0} \mathbb{E}^x \Bigg[& \int_t^{T_c \wedge T_d} e^{-r(u-t)} c du + 1_{\{T_d < T_c\}} e^{-r(T_d - t)} \frac{c}{s+c}(1-\theta)\epsilon(x_{T_d}) \\
& + 1_{\{T_c < T_d\}} \frac{\eta}{1+\eta} \int_{T_c}^{T_{d,c}} e^{-r(u-t)}(1-\tau)(x_u - s)du \Bigg]. \qquad (39)
\end{aligned}
$$

Once the convertible debt has been converted, the firm becomes an entity that issues equity and straight debt and the optimal default policy is established as in Sec. 2.2.1. Let $E_a(x)$ and $D_{s,a}(x)$ be the post-conversion total values of equity and straight debt. Letting x_t at conversion time t equal x, the post-conversion default threshold $x_{d,c}$, the equity value $E_a(x)$ and the value of straight debt $D_{s,a}(x)$ are given by

$$x_{d,c} = \frac{\beta_2}{\beta_2 - 1} \frac{s(r-\mu)}{r}, \tag{40}$$

$$E_a(x) = \epsilon(x) - \frac{(1-\tau)s}{r} - \left(\frac{x}{x_{d,c}}\right)^{\beta_2} \left(\epsilon(x_{d,c}) - \frac{(1-\tau)s}{r}\right), \tag{41}$$

$$D_{s,a}(x) = \frac{s}{r} - \left(\frac{x}{x_{d,c}}\right)^{\beta_2} \left(\frac{s}{r} - (1-\theta)\epsilon(x_{d,c})\right). \tag{42}$$

Next, we consider the values prior to conversion. It follows from the optimal problems of the equity holders, straight debt holders and convertible debt holders in (37), (38) and (39), respectively, that the values of equity, straight debt and convertible debt prior to default and conversion are given by

$$E(x) = a_7 x^{\beta_1} + a_8 x^{\beta_2} + (1-\tau)\left(\frac{x}{r-\mu} - \frac{s+c}{r}\right), \tag{43}$$

$$D_s(x) = a_9 x^{\beta_1} + a_{10} x^{\beta_2} + \frac{s}{r}, \tag{44}$$

$$D_c(x) = a_{11} x^{\beta_1} + a_{12} x^{\beta_2} + \frac{c}{r}. \tag{45}$$

Constants a_i, $i = 7, \cdots, 12$, the pre-conversion default threshold x_d and the conversion threshold x_c are determined by the boundary conditions:

$$a_7 x_d^{\beta_1} + a_8 x_d^{\beta_2} + (1-\tau)\left(\frac{x_d}{r-\mu} - \frac{s+c}{r}\right) = 0, \tag{46}$$

$$\beta_1 a_7 x_d^{\beta_1 - 1} + \beta_2 a_8 x_d^{\beta_2 - 1} + \frac{1-\tau}{r-\mu} = 0, \tag{47}$$

$$a_7 x_c^{\beta_1} + a_8 x_c^{\beta_2} + (1-\tau)\left(\frac{x_c}{r-\mu} - \frac{s+c}{r}\right) = \frac{1}{1+\eta} E_a(x_c), \tag{48}$$

$$a_9 x_d^{\beta_1} + a_{10} x_d^{\beta_2} + \frac{s}{r} = \frac{s}{s+c}(1-\theta)\frac{(1-\tau)x_d}{r-\mu}, \tag{49}$$

$$a_9 x_c^{\beta_1} + a_{10} x_c^{\beta_2} + \frac{s}{r} = D_{s,a}(x_c), \tag{50}$$

$$a_{11} x_d^{\beta_1} + a_{12} x_d^{\beta_2} + \frac{c}{r} = \frac{c}{s+c}(1-\theta)\frac{(1-\tau)x_d}{r-\mu}, \tag{51}$$

$$a_{11} x_c^{\beta_1} + a_{12} x_c^{\beta_2} + \frac{c}{r} = \frac{\eta}{1+\eta} E_a(x_c), \tag{52}$$

$$\beta_1 a_{11} x_c^{\beta_1 - 1} + \beta_2 a_{12} x_c^{\beta_2 - 1} = \frac{\eta}{1+\eta} \frac{dE_a}{dx}(x_c), \tag{53}$$

where $E_a(x)$ and $D_{s,a}(x)$ are given in Eqs. (41) and (42). Eqs. (46), (47), (49) and (51) are conditions in default. On the other hand, Eqs. (48), (50), (52) and (53) are conditions in conversion. Eq. (46) is the value matching condition which ensures that the equity value at the default threshold equals zero. Eq. (47) is the smooth-pasting condition that ensures the optimality of the default threshold x_d. Eqs. (49) and (51) are the value matching conditions which ensure that the values of debt are equal to respective fractions of the unlevered value of the firm net of proportional bankruptcy cost. Eq. (48) is the value matching condition requiring that the value of equity at the conversion threshold equals a proportion of the post-conversion value of equity given in Eq. (41). Eq. (50) is the value matching condition which ensures that the pre-conversion value of straight debt equals the post-conversion value of straight debt. Equation (52) is the value matching condition which ensures that the value of the convertible debt at the conversion threshold is equal to a proportion of the post-conversion value of equity. Eq. (53) is the smooth-pasting condition that ensures the optimality of the conversion threshold x_c. Eight equations (46)-(53) have eight unknown variables $(a_i, i = 7, \cdots, 12, x_d, x_c)$. These equations can be solved numerically.

Then, the firm value $V_{s+c}(x)$ is given by

$$V_{s+c}(x) = E(x)+D_s(x)+D_c(x) = \epsilon(x)+\frac{\tau(s+c)}{r}+(a_7+a_9+a_{11})x^{\beta_1}+(a_8+a_{10}+a_{12})x^{\beta_2}. \quad (54)$$

2.4.2 Optimal Investment Policy

By the optimal investment policies to maximize the value of equity and the firm value, both the values of the investment option are

$$F_i(x_0) = \left(\frac{x_0}{x_i^*}\right)^{\beta_1} \{V_{s+c}(x_i^*) - I\} \quad (55)$$

for $x_0 < x_i^*$ and $i = 1, 2$. The second-best investment threshold x_2^* is numerically solvable from

$$E(x_2^*) - \frac{x_2^*}{\beta_1}\frac{dE}{dx}(x_2^*) - (I - D_s(x_2^*) - D_c(x_2^*)) = 0, \quad (56)$$

and the first-best investment threshold x_1^* is given by the solution of

$$V_{s+c}(x_1^*) - \frac{x_1^*}{\beta_1}\frac{dV_{s+c}}{dx}(x_1^*) - I = 0. \quad (57)$$

3 Numerical Analysis

3.1 Investment Option and Agency Cost

In this section, the calculation results of the value of equity, each debt, and the investment option are presented in order to quantify the agency cost. We use the following base case parameters: $\mu = 0.01,\ \sigma = 0.2,\ r = 0.05,\ I = 5,\ s = 0.15,\ c = 0.15,\ \alpha = 1.5,\ \theta = 0.3,\ \tau = 0.3.$

Fig. 1 shows the values of equity, straight debt and convertible debt as functions of the earning x at issue time and the post-conversion values of equity and straight debt as functions of the earning x at conversion time in the case of Sec. 2.4, that is, the investment is financed with equity, straight debt and convertible debt. As can be seen in this figure, the threshold values for the pre-conversion default, the post-conversion default and the conversion are 0.140, 0.069 and 2.229, respectively. These values can provide the investment value of the firm financed by issuing equity, straight debt and convertible debt.

In Fig. 2, the investment values and the threshold values of the investment for the firm-value-maximizing and the equity-value-maximizing policies are shown. It turns out that the threshold value of the equity-value-maximizing policy is smaller than that of the firm-value-maximizing policy. Thus, like the model in Mauer and Sarkar [7], the equity-value-maximizing policy overinvests compared to the firm-value-maximizing policy. Furthermore, at the current value x_0 of 0.3, the investment option values for the firm-value-maximizing and equity-value-maximizing policies are 1.959 and 1.882, respectively. The difference between these values for each policy, that is, the agency cost of overinvestment is 0.077. A proportion of the agency cost to the equity-value-maximizing policy, which is the loss in firm value, is 4.1%. It seems that the agency cost in this case is relatively large compared with that in the case of the firm which has only straight debt as in Mauer and Sarkar [7]. In this section, we consider the case that the investment is financed with equity, straight debt and convertible debt. In order to compare the property of convertible debt with that of straight debt, we explore the investment financed with equity and convertible debt (but no straight debt), and equity and straight debt (but no convertible debt) in the next section.

3.2 Comparison of Convertible Debt and Straight Debt

Here we analyze the investment decision in which the firm issues convertible debt. In this section, the following set of parameters is used: $x_0 = 0.3,\ \mu = 0.01,\ \sigma = 0.2,\ r = 0.05,\ I = 5,\ \alpha = 1.5,\ \theta = 0.3,\ \tau = 0.3$.

Fig. 3 shows the first-best and second-best investment thresholds as functions of coupon payment, c or s, in the case that the firm is financed with equity and either straight debt or convertible debt in Sec.2.2 and 2.3. For the first-best investment policy, convertible debt leads to underinvestment relative to straight debt. Once the investment is financed with debt, the firm can enjoy interest tax shields, and so can have a tax incentive to accelerate the investment. Since convertible debt includes the option to convert debt into equity, the presence of the option reduces the magnitude of the tax shield effect. This leads to an incentive to speed down investment. On the other hand, for the second-best investment policy, convertible debt leads to overinvestment relative to straight debt. The equity holders are not affected by the benefit of the debt holders, and are therefore indifferent to increased risk of default resulting from the earlier investment. As can be seen in Fig. 4, since the convertible debt includes the conversion option, the default probability of convertible debt is higher than that of straight debt. This means that the equity holders optimally hope to exercise the option prior to the convertible debt holders. Similarly, the convertible debt holders also wish to exercise the option, optimally, prior to the equity holders. Hence, by investing earlier, the equity holders can shift the default risk to the debt holders, can raise the probability of conversion for the convertible debt holders, and can mitigate the disposition of the welfare to debt holders.

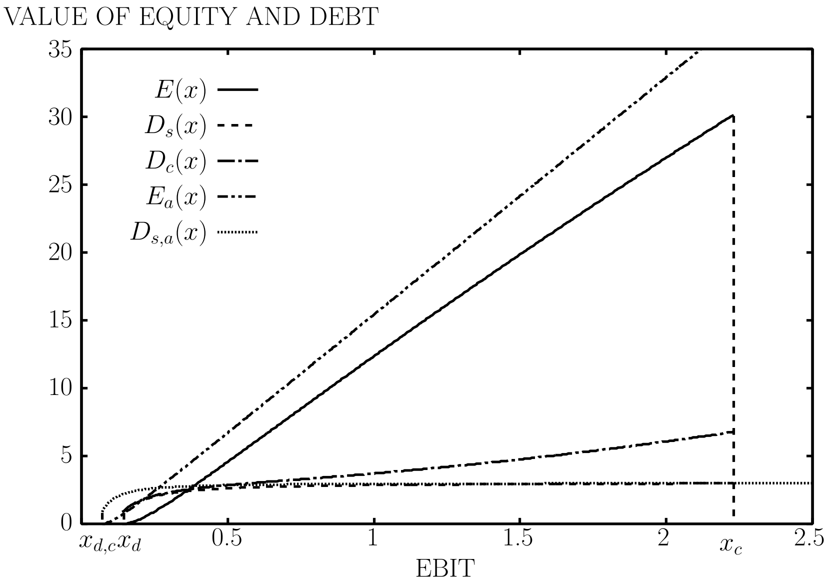

Figure 1: Equity, straight debt and convertible debt

$\mu = 0.01,\ \sigma = 0.2,\ r = 0.05,\ s = 0.15,\ c = 0.15,\ \alpha = 1.5,\ \theta = 0.3,\ \tau = 0.3$

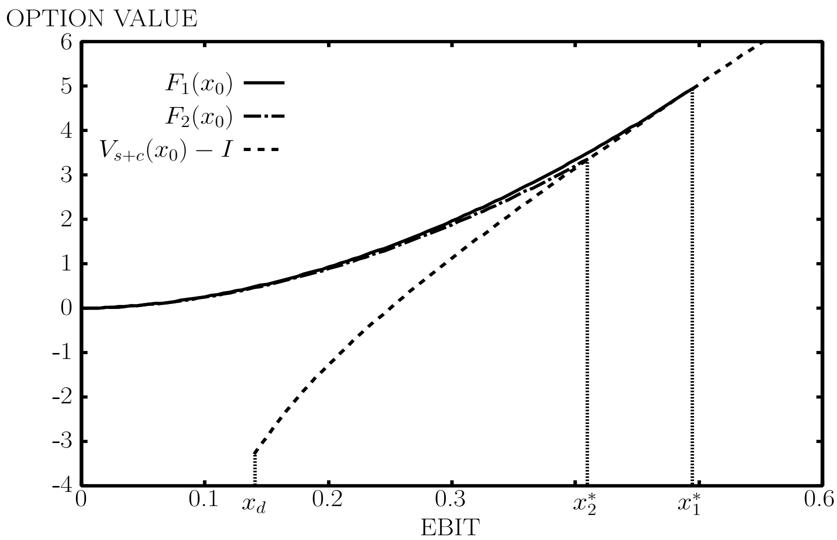

Figure 2: Investment option

$\mu = 0.01,\ \sigma = 0.2,\ r = 0.05,\ I = 5,\ s = 0.15,\ c = 0.15,\ \alpha = 1.5,\ \theta = 0.3,\ \tau = 0.3$

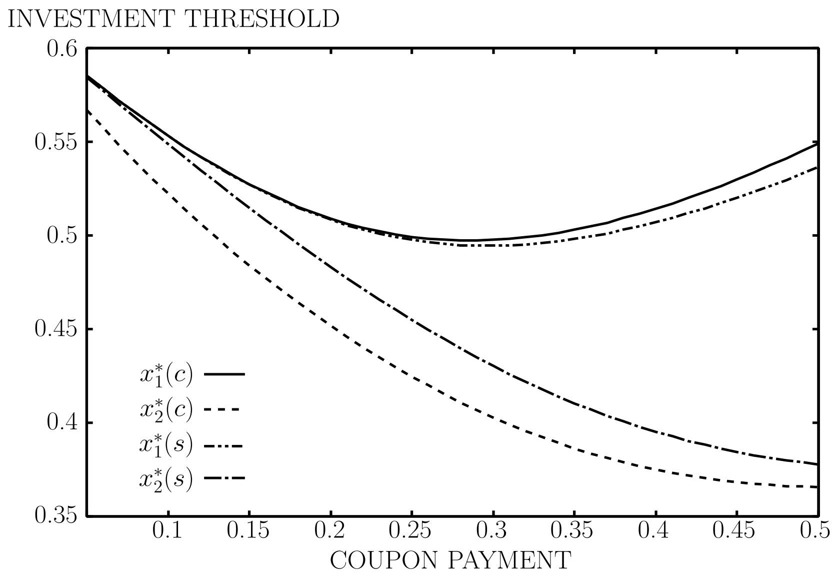

Figure 3: Investment threshold

$\mu = 0.01,\ \sigma = 0.2,\ r = 0.05,\ I = 5,\ \alpha = 1.5,\ \theta = 0.3,\ \tau = 0.3$

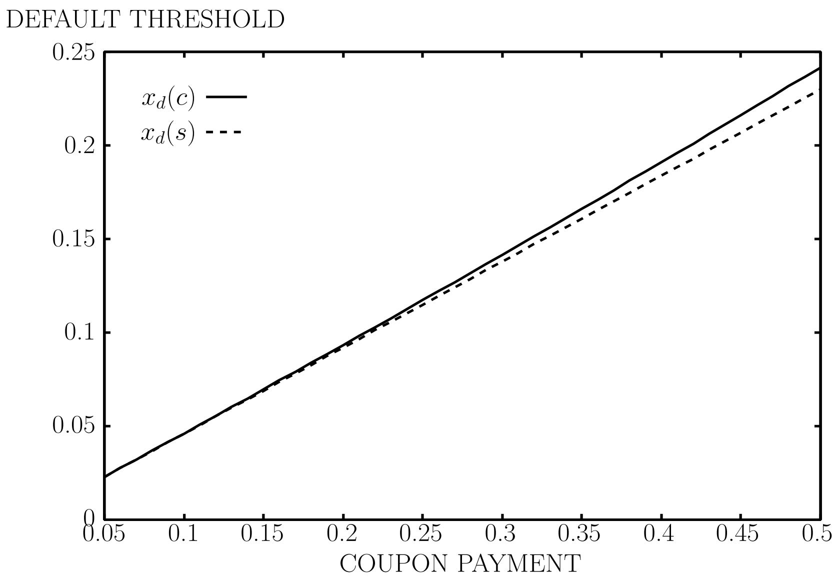

Figure 4: Default threshold

$\mu = 0.01,\ \sigma = 0.2,\ r = 0.05,\ I = 5,\ \alpha = 1.5,\ \theta = 0.3,\ \tau = 0.3$

4 Summary

In this paper, we have investigated the optimal investment policy of the firm financed by issuing equity, straight debt and convertible debt. The values of equity, straight debt and convertible debt after exercising the investment option were shown. We also showed the investment option value and the threshold value for the firm-value-maximizing and equity-value-maximizing policies. In particular, we found that the issue of convertble debt for the firm-value-maximizing policy leads to underinvestment relative to that of straight debt. On the other hand, the issue of convertble debt for the equity-value-maximizing policy leads to overinvestmeCCCCnt relative to that of straight debt.

Many convertible debt contracts include call provisions which entitle the firm to repurchase its debt. For future works, therefore, we will examine the effect of callable convertible debt on investment decision as in Lyandres and Zhdanov [5]. In addition, as discussed in Mayers [8], convertible debt provides a firm with sequential investments with an advantage. Possible extension of this study also includes an analysis of multi-stage investment project.

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