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# BUSINESS CYCLE IMPLICATIONS OF INTERNAL CONSUMPTION HABIT FOR NEW KEYNESIAN MODELS

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Abstract

We study the implications of internal consumption habit for new Keynesian dynamic stochastic general equilibrium (NKDSGE) models. Bayesian Monte Carlo methods are employed to evaluate NKDSGE model fit. Simulation experiments show that internal consumption habit often improves the ability of NKDSGE models to match the spectra of output and consumption growth. Nonetheless, the fit of NKDSGE models with internal consumption habit is susceptible to the sources of nominal rigidity, to spectra identified by permanent productivity shocks, to the choice of monetary policy rule, and to the frequencies used for evaluation. These vulnerabilities indicate that the specification of NKDSGE models is fragile.

*Key Words*: Consumption Habit; New Keynesian; Propagation; Monetary Transmission; Posterior Predictive Analysis; Bayesian Monte Carlo.

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of  $k_t$ ,  $W_t(\ell)$  is the nominal wage of household  $\ell$ ,  $R_t$  is the nominal return on  $B_t$ ,  $D_t$  is dividends received from firms,  $u_t \in (0, 1)$  is the capital utilization rate, and  $a(u_t)$  is its cost function. A change in  $u_t$  forces household  $\ell$  to forgo  $a(\cdot)$  units of consumption per unit of capital. The investment adjustment costs specification, which is adapted from CEE, is placed in the law of motion of household capital

$$k_{t+1} = (1 - \delta)k_t + \left[1 - S\left(\frac{1}{\alpha}\frac{x_t}{x_{t-1}}\right)\right]x_t, \quad \delta \in (0, 1), \quad 0 < \alpha,$$
 (4)

where  $\delta$  is the capital depreciation rate and  $\alpha$  (= ln  $\alpha$ \*) is deterministic TFP growth. The cost function  $S(\cdot)$  is strictly convex, where S(1) = S'(1) = 0 and  $S''(1) \equiv \varpi > 0$ .

Given  $k_0$ ,  $B_0$ , and  $c_{-1}$ , the expected discounted lifetime utility function of household  $\ell$ 

$$\mathbf{E}_{t} \left\{ \sum_{i=0}^{\infty} \beta^{i} \mathcal{U}\left(c_{t+i}, c_{t+i-1}, n_{t+i}(\ell), \frac{H_{t}}{P_{t}}\right) \right\}$$

$$(5)$$

is maximized by choosing  $c_t$ ,  $k_{t+1}$ ,  $H_{t+1}$ ,  $B_{t+1}$ , and  $W_t(\ell)$  subject to period utility (1), budget constraint (3), the law of motion of capital (4), and downward sloping labor demand.

Households charge firms  $W_t(\ell)$  per unit of differentiated labor services in a monopolistic market in which a Calvo-staggered nominal wage mechanism operates. Given  $\theta$  is the wage elasticity, the labor supply aggregator is  $N_t = \left[\int_0^1 n_t(\ell)^{(\theta-1)/\theta} d\ell\right]^{\theta/(\theta-1)}$ . Labor market monopoly imposes downward sloping labor demand schedules,  $n_t(\ell) = \left[W_t/W_t(\ell)\right]^{\theta} N_t$ , on firms, where the nominal wage index is  $W_t = \left[\int_0^1 W_t(\ell)^{1-\theta} d\ell\right]^{1/(1-\theta)}$  and its aggregator is  $W_t = \left[(1-\mu_W)W_{c,t}^{1-\theta} + \mu_W\left(\alpha^*\varsigma_{t-1}W_{t-1}\right)^{1-\theta}\right]^{1/(1-\theta)}$ . This Calvo-staggered nominal wage mechanism has households updating their nominal wage to  $W_{c,t}$  at probability  $1-\mu_W$ . At probability  $\mu_W$ , households receive the date t-1 nominal wage indexed by steady-state TFP growth,  $\alpha^*$ , and  $\varsigma_{t-1}$ . In this case, the optimal nominal wage condition is

$$\left[\frac{W_{c,t}}{P_{t-1}}\right]^{1+\theta/\gamma} = \left(\frac{\theta}{\theta-1}\right) \frac{\mathbf{E}_{t} \sum_{i=0}^{\infty} \left[\beta \mu_{W} \alpha^{*-\theta(1+1/\gamma)}\right]^{i} \left[\left[\frac{W_{t+i}}{P_{t+i-1}}\right]^{\theta} N_{t+i}\right]^{1+1/\gamma}}{\mathbf{E}_{t} \sum_{i=0}^{\infty} \left[\beta \mu_{W} \alpha^{*(1-\theta)}\right]^{i} \lambda_{t+i} \left[\frac{W_{t+i}}{P_{t+i-1}}\right]^{\theta} \left[\frac{P_{t+i}}{P_{t+i-1}}\right]^{-1} N_{t+i}}.$$
(6)

Equation (6) smooths nominal wage growth, which forces labor supply to absorb TFP and monetary policy shocks. Since shifts in labor supply alter production and intra- and intertemporal margins, shock volatility and persistence are realized, for example, as output and consumption fluctuations.

Monopolistically competitive firms produce final goods that households consume. The consumption aggregator is  $c_t = \left[\int_0^1 y_{D,t}(j)^{(\xi-1)/\xi} dj\right]^{\xi/(\xi-1)}$ , where  $y_{D,t}(j)$  is household final good demand for

the output of a firm with address j on the unit interval. The jth final good firm aims to meet this demand with its output,  $y_t(j)$ , by mixing capital,  $K_t(j)$ , rented and labor,  $N_t(j)$ , hired from households net of fixed cost  $N_0$  given labor-augmenting TFP,  $A_t$ , in the constant returns to scale technology,  $\left[u_tK_t(j)\right]^{\psi}\left[\left[N_t(j)-N_0\right]A_t\right]^{1-\psi}$ ,  $\psi\in(0,1)$ . Fixed labor cost  $N_0$  satisfies the needs of monopolistic competition in the final goods market. For the NKDSGE model to have a permanent shock, TFP is a random walk with drift,  $A_t=A_{t-1}\exp\{\alpha+\varepsilon_t\}$ , with its innovation,  $\varepsilon_t\sim\mathcal{N}\left(0,\,\sigma_\varepsilon^2\right)$ .

Firm j maximizes profits by choosing its price  $P_t(j)$ , subject to  $y_{D,t}(j) = \left[P_t/P_t(j)\right]^\xi Y_{D,t}$ , where  $\xi$  is the price elasticity,  $Y_{D,t}$  is aggregate demand, and the price index is  $P_t = \left[\int_0^1 P_t(j)^{1-\xi}\right]^{1/(1-\xi)}$ . Calvo-staggered price setting restricts a firm to update its optimal price  $P_{c,t}$  at probability  $1-\mu_P$ . Or with probability  $\mu_P$ , firms are stuck with  $P_{t-1}$  scaled by  $\zeta_{t-1}$ , which defines the price aggregator  $P_t = \left[(1-\mu_P)P_{c,t}^{1-\xi} + \mu_P\left(\zeta_{t-1}P_{t-1}\right)^{1-\xi}\right]^{1/(1-\xi)}$ . The firm's problem yields the optimal forward-looking price

$$\frac{P_{c,t}}{P_{t-1}} = \left(\frac{\xi}{\xi - 1}\right) \frac{E_t \sum_{i=0}^{\infty} (\beta \mu_P)^i \lambda_{t+i} \phi_{t+i} Y_{D,t+i} \varsigma_{t+i}^{\xi}}{E_t \sum_{i=0}^{\infty} (\beta \mu_P)^i \lambda_{t+i} Y_{D,t+i} \varsigma_{t+i}^{\xi - 1}}$$
(7)

of a firm able to update its price. Inflation is smoothed by equation (7). The same responses are induced in output and consumption in response to TFP and monetary policy shocks because of the reaction of monopolistically competitive firms to variation in the aggregate price level.

We close the NKDSGE model with one of two monetary policy rules. CEE identify monetary policy with a  $MA(\infty)$  money growth process. It is equivalent to the AR(1) money growth rule (MGR)

$$m_{t+1} = (1 - \rho_m)m^* + \rho_m m_t + \mu_t, \quad |\rho_m| < 1, \quad \mu_t \sim \mathcal{N}\left(0, \sigma_\mu^2\right),$$
 (8)

where  $m_{t+1} = \ln(M_{t+1}/M_t)$  and  $m^*$  is its mean. A model using the MGR (8) is labeled NKDSGE-MGR. The mnemonic NKDSGE-TR refers to models closed by the Taylor rule (TR)

$$(1 - \rho_R \mathbf{L}) R_t = (1 - \rho_R) \left( R^* + a_{\varsigma} \mathbf{E}_t \varsigma_{t+1} + a_{\widetilde{\Upsilon}} \widetilde{Y}_t \right) + \upsilon_t, \quad \left| \rho_R \right| < 1, \quad \upsilon_t \sim \mathcal{N} \left( 0, \sigma_v^2 \right), \tag{9}$$

where  $R^* = \exp(m^* - \alpha)/\beta$ . The TR (9) assumes the Taylor principle,  $1 < a_{\varsigma}$  is obeyed,  $0 < a_{\widetilde{Y}}$ , and expected inflation,  $\mathbf{E}_t \varsigma_{t+1}$ , and transitory output,  $\widetilde{Y}_t$ , are computed without measurement errors.

The government finances  $B_t$ , interest on  $B_t$ , and a lump-sum transfer  $\tau_t$  with new bond issuance  $B_{t+1} - B_t$ , lump-sum taxes  $\tau_t$ , and money creation,  $M_{t+1} - M_t$ . These sources and uses of funds give the

government the budget constraint  $P_t \tau_t = [M_{t+1} - M_t] + [B_{t+1} - (1 + R_t)B_t]$ . We assume government debt is in zero net supply,  $B_{t+1} = 0$  and  $P_t \tau_t = M_{t+1} - M_t$ , along the equilibrium path at all dates t.

The decentralized economy requires goods, labor, and money markets to clear in equilibrium. Equilibrium has  $K_t = k_t$  given  $0 < r_t$ ,  $N_t = n_t$  given  $0 < W_t$ , and  $M_t = H_t$  given  $0 < P_t$ ,  $R_t$ . The aggregate resource constraint,  $Y_t = C_t + I_t + a(u_t)K_t$ , follows, where  $C_t = c_t$  and  $I_t = x_t$ .

#### 3. BAYESIAN MONTE CARLO STRATEGY

Population moments are used to judge the fit of 12 variants of the NKDSGE model. Bayesian Monte Carlo simulations are run that apply sample data, a structural vector moving average (SVMA), its priors, and a Markov chain Monte Carlo (MCMC) simulator to create posterior distributions of SDs of output growth,  $\Delta Y$ , and consumption growth,  $\Delta C$ . Prior distributions of SDs of  $\Delta Y$  and  $\Delta C$  are approximated using a SVMA estimated on synthetic data that are simulated from NKDSGE models with parameters drawn from independent priors. The econometric link between the posterior and prior SDs and the sample data is the SVMAs. The multidimensional posterior and prior SDs are collapsed into scalar Kolmogorov-Smirnov (KS) goodness of fit statistics to compute the confidence interval criterion (CIC) of DeJong, Ingram, and Whiteman (1996). A CIC measures the overlap of posterior and prior KS statistic distributions. We use the CIC to quantify the fit of the NKDSGE models.

#### 3.1 Output and consumption moments

The NKDSGE models are evaluated on permanent and transitory SDs of  $\Delta Y$  and  $\Delta C$ . The SDs are grounded on SVMAs just-identified by the LRMN restriction embedded in the NKDSGE model and orthogonality of permanent and transitory shock innovations. Given these restrictions, the SVMAs are computed by applying the Blanchard and Quah (1989) decomposition to second-order VARs, VAR(2)s, of  $[\Delta \ln Y_t \ \Delta \ln P_t]'$  and  $[\Delta \ln C_t \ \Delta \ln P_t]'$ . The Blanchard and Quah (BQ) decomposition identifies the SVMAs because the TFP innovation  $\varepsilon_t$  is the permanent shock and the transitory shock is either the MGR innovation  $\mu_t$  or TR innovation  $\upsilon_t$ . The identified  $\Delta \ln Y_t - \Delta \ln P_t$  ( $\Delta \ln C_t - \Delta \ln P_t$ ) system recovers a vertical long-run aggregate supply (PIH) curve and a serially correlated "output (consumption) gap."

<sup>9.</sup> A rational expectations equilibrium equates, on average, firm and household subjective forecasts of  $r_t$ , and  $A_t$  to the objective outcomes produced by the decentralized economy. The list includes  $\mu_t$  and  $R_t$  under the MGR (8) or  $v_t$  for the TR (9). A flexible price regime (spot labor market) adds  $P_t$  ( $W_t$ ).

<sup>10.</sup> The appendix of Blanchard and Quah (1989) includes a theorem that establishes necessary and sufficient conditions under which bivariate ARs identify the correct responses to a permanent shock and a transitory shock when truth is that there are several of these shocks. The BQ decomposition is satisfied, according to the theorem, when responses, say, of  $\Delta Y$  and  $\Delta P$  to permanent or transitory shocks are equivalent up to a scalar lag operator. Since the shocks found in NKDSGE models are often AR(1)s, the theorem predicts that adding these shocks to a NKDSGE model do not create spurious identification.

As an example consider the SVMA

$$\begin{bmatrix} \Delta \ln Y_t \\ \Delta \ln P_t \end{bmatrix} = \sum_{j=0}^{\infty} \mathbb{B}_j \begin{bmatrix} \varepsilon_{t-j} \\ v_{t-j} \end{bmatrix}, \text{ where } \mathbb{B}_j = \begin{bmatrix} \mathbb{B}_{\Delta Y, \varepsilon, j} & \mathbb{B}_{\Delta Y, v, j} \\ \mathbb{B}_{\Delta P, \varepsilon, j} & \mathbb{B}_{\Delta P, v, j} \end{bmatrix}, \tag{10}$$

that equates the monetary policy shock with the innovation  $v_t$  of the Taylor rule (9). Elements of  $\mathbb{B}_j$  are just-identified by imposing (i) orthogonality on  $\varepsilon_t$  and  $v_t$  and (ii) the LRMN restriction  $\mathbb{B}_{\Delta Y,v}(\mathbf{1}) = 0$  (i.e.,  $\ln Y_{t+j}$  is independent of  $v_t$  at the infinite horizon,  $j \to \infty$ ). These assumptions decompose the SVMA (10) into univariate SMA( $\infty$ )s,  $\mathbb{B}_{\Delta Y,\varepsilon}(\mathbf{L})\varepsilon_t$  and  $\mathbb{B}_{\Delta Y,v}(\mathbf{L})v_t$ , for  $\Delta Y$ . The former (latter) SMA( $\infty$ ) is the IRF of  $\Delta Y$  with respect to the permanent shock  $\varepsilon_t$  (transitory shock  $v_t$ ).

We grab the SMA( $\infty$ ) of  $\mathbb{B}_{\Delta Y, \varepsilon}(\mathbf{L})\varepsilon_t$  and  $\mathbb{B}_{\Delta Y, \upsilon}(\mathbf{L})\upsilon_t$  from the SVMA (10) to compute permanent and transitory SDs of  $\Delta Y$ . Since the SVMA (10) is also a Wold representation of  $[\Delta \ln Y_t \ \Delta \ln P_t]'$ , its SD (at frequency  $\omega$ ) is  $SD_{[\Delta Y \ \Delta P]}(\omega) = (2\pi)^{-1}\Gamma_{[\Delta Y \ \Delta P]}\exp(-i\omega)$ , where  $\Gamma_{[\Delta Y \ \Delta P]}(l) = \sum_{j=0}^{\infty} \mathbb{B}_j \mathbb{B}'_{j-l}$ . The convolution  $\Gamma_{[\Delta Y \ \Delta P]}(l)$  is expanded at horizon j to obtain

$$\mathbb{B}_{j}\mathbb{B}_{j-l}' = \begin{bmatrix} \mathbb{B}_{\Delta Y,\varepsilon,j}\mathbb{B}_{\Delta Y,\varepsilon,j-l} + \mathbb{B}_{\Delta Y,\upsilon,j}\mathbb{B}_{\Delta Y,\upsilon,j-l} & \mathbb{B}_{\Delta Y,\varepsilon,j}\mathbb{B}_{\Delta P,\varepsilon,j-l} + \mathbb{B}_{\Delta Y,\upsilon,j}\mathbb{B}_{\Delta P,\upsilon,j-l} \\ \mathbb{B}_{\Delta P,\varepsilon,j}\mathbb{B}_{\Delta Y,\varepsilon,j-l} + \mathbb{B}_{\Delta P,\upsilon,j}\mathbb{B}_{\Delta Y,\upsilon,j-l} & \mathbb{B}_{\Delta P,\varepsilon,j}\mathbb{B}_{\Delta P,\varepsilon,j-1} + \mathbb{B}_{\Delta P,\upsilon,j}\mathbb{B}_{\Delta P,\upsilon,j-l} \end{bmatrix}.$$

The matrix's off-diagonal entries are elements of the cross-covariance function of  $\Delta Y$  and  $\Delta P$  and, therefore, map to co- and quad-spectra. The autocovariance function of  $\Delta Y$  with respect to  $\varepsilon_t$  ( $\upsilon_t$ ) is the upper left diagonal,  $\mathbb{B}_{\Delta Y,\varepsilon,j}\mathbb{B}_{\Delta Y,\varepsilon,j-l}$  ( $\mathbb{B}_{\Delta Y,\upsilon,j}\mathbb{B}_{\Delta Y,\upsilon,j-l}$ ). We exploit the SMAs  $\mathbb{B}_{\Delta Y,\varepsilon}(\mathbf{L})\varepsilon_t$  and  $\mathbb{B}_{\Delta Y,\upsilon}(\mathbf{L})\upsilon_t$ , that are along the diagonal, to parameterize permanent and transitory SDs of  $\Delta Y$  at frequency  $\omega$ 

$$SD_{\Delta Y,\iota}(\omega) \,=\, \frac{1}{2\pi} \left|\, \mathbb{B}_{\Delta Y,\iota,0} + \mathbb{B}_{\Delta Y,\iota,1} e^{-i\omega} + \mathbb{B}_{\Delta Y,\iota,2} e^{-i2\omega} + \ldots + \mathbb{B}_{\Delta Y,\iota,j} e^{-ij\omega} + \ldots \,\right|^2, \quad \iota \,=\, \varepsilon,\, \upsilon,$$

given the BQ decomposition assumption that the structural shock innovations have unit variances. Before computing  $SD_{\Delta Y, \iota}(\omega)$ , we truncate its polynomial at j=40, or a 10-year horizon.<sup>11</sup>

## *3.2 The moments to match: Mean posterior distributions*

We engage MCMC software of Geweke (1999) and McCausland (2004) to create posterior distributions of SVMAs. These posterior distributions consist of  $\mathcal{J}=5000$  SVMA parameter vectors that are grounded on unrestricted VAR(2)s, LRMN, the BQ decomposition, priors, and a 1954Q1-2002Q4 sample (T=196) of  $\Delta Y$ ,  $\Delta C$ , and  $\pi_t$ . These  $\mathcal{J}$  vectors are used to calculate distributions of posterior permanent and transitory SDs of  $\Delta Y$  and  $\Delta C$ . We label these posterior moments  $SD_{\mathcal{P},\Delta Y}$  and  $SD_{\mathcal{P},\Delta C}$ .

<sup>11.</sup> This approach to estimating SDs extends the ideas of Akaike (1969) and Parzen (1974).

# 3.3 Bayesian simulation methods II: Prior distributions

It takes multiple steps to solve and simulate the NKDSGE models. Since these models have a permanent TFP shock, stochastic detrending of optimality and equilibrium conditions is needed before log-linearizing around deterministic steady states. We engage an algorithm of Sims (2002) to solve for log linear approximate equilibrium laws. Synthetic samples are created by feeding sequences of the shock innovations  $\varepsilon_t$  and  $\mu_t$  or  $v_t$  into the equilibrium laws of motion given initial conditions and draws from the priors of NKDSGE model parameters. The appendix describes these procedures in detail.

Table 1 lists the priors of the NKDSGE models. These priors reflect our uncertainty about NKDSGE model parameters. For example, the first item listed in table 1 is the consumption habit parameter h. We give h an uninformative prior that is drawn from a uniform distribution with end points 0.05 and 0.95 in table 1. The uninformative prior reflects a belief that any  $h \in [0.05, 0.95]$  is as likely as another. Non-habit NKDSGE models are defined by the degenerate prior h = 0.

Priors are also taken from earlier DSGE model studies.<sup>12</sup> We set the means of the priors of  $\left[\beta \ \delta \ \alpha \ \psi\right]' = \left[0.9930\ 0.0200\ 0.0040\ 0.3500\right]'$  that are consistent with the Cogley and Nason (1995b) calibration. Uncertainty about  $\left[\beta \ \gamma \ \delta \ \alpha \ \psi\right]'$  is captured by 95 percent coverage intervals, which contain values in Nason and Cogley (1994) and Hall (1996). We equate the prior of the investment cost of adjustment parameter  $\varpi$  to estimates reported by Bouakez, Cardia, and Ruge–Murcia (2005).<sup>13</sup> The standard deviation of TFP shock innovations,  $\sigma_{\varepsilon}$ , is endowed with a uniform prior because the DSGE literature suggests that any draw from [0.0070, 0.0140] is equally likely. The source of the prior mean of the Frisch labor supply elasticity,  $\gamma = 1.55$ , and its 95 percent coverage interval of [0.5, 2.6] is Kimmel and Kniesner (1998). They estimate labor supply functions on U.S. panel data.

There are 4 sticky price and wage parameters to calibrate. The prior means are  $[\xi \ \mu_P \ \theta \ \mu_w]' = [12.0\ 0.55\ 15.0\ 0.7]'$ . The mean of  $\xi$  implies a steady-state price markup,  $\xi/(\xi-1)$ , of 9 percent with a 95 percent coverage interval that runs from 5 to 33 percent. This coverage interval blankets estimates found in Basu and Fernald (1997) and CEE. More uncertainty surrounds the priors of  $\mu_P$ ,  $\theta$ , and  $\mu_w$ . Sbordone (2002), Nason and Slotsve (2004), Lindé (2005), and CEE suggest a 95 percent coverage interval for  $\mu_P$  of [0.45, 0.65]. Likewise, a 95 percent coverage interval of [0.04, 0.25] suggests substantial uncertainty around the 7 percent prior mean household wage markup,  $\theta/(\theta-1)$ . The degenerate

<sup>12.</sup> Several priors are centered on sample means of the consumption-output ratio, labor input, federal funds rate, and inflation using the 1954Q1-2002Q4 sample. We also fix  $N_0=0.1678$  and  $r^*=1.0050$ .

<sup>13.</sup> The capacity utilization rate,  $u_t$ , and its cost function,  $a(\cdot)$ , are fixed at the steady state,  $u^* = 1$  and a(1) = 0. Determinate solutions are achieved by setting a''(1)/a'(1) = 1.174.

mean of  $\mu_w$  and its 95 percent coverage interval reveals stickier nominal wages than prices, as found by CEE and Rabanal and Rubio-Ramírez (2005), but we imbue it with greater uncertainty.

The money growth rule (8) is calibrated to estimates from a 1954Q1-2002Q4 sample of the monetary base. The point estimates are degenerate priors for  $\left[m^* \ \rho_m \ \sigma_\mu\right]' = \left[0.011\ 0.628\ 0.006\right]'$ . We give these prior means less precision than found for the sample estimates. For example, the lower end of the 95 percent coverage interval of  $\rho_m$  is near 0.46. CEE note that  $\rho_m \approx 0.5$  implies the AR(1) money growth rule (8) mimics the persistence of their MA( $\infty$ ) money growth shock policy process.

The calibration of the interest rate rule (9) obeys the Taylor principle and  $a_y \in (0,1)$ . The degenerate prior of  $a_\pi$  is 1.80. We assign a small role to movements in transitory output,  $\widetilde{Y}$ , with a prior mean of 0.05 for  $a_y$ . The 95 percent coverage intervals of  $a_\pi$  and  $a_y$  rely on estimates reported by Smets and Wouters (2007). The interest rate rule (9) is also calibrated to smooth  $R_t$  given a prior mean of 0.65 and a 95 percent coverage interval of [0.55, 0.74] that incorporates estimates found in Guerron-Quintana (2010). Ireland (2001) is the source of the prior mean of the standard deviation of the monetary policy shock,  $\sigma_v = 0.0051$ , and its 95 percent coverage interval, [0.0031, 0.0072]. We assume all shock innovations are uncorrelated at leads and lags (*i.e.*,  $\mathbf{E}\{\varepsilon_{t+i}, v_{t+q}\} = 0$ , for all i, q).

Prior draws of NKDSGE model parameter are applied to its log linear approximation to generate a synthetic sample of length  $\mathcal{M} = \mathcal{W} \times T$ . We set  $\mathcal{W} = 5$  to approximate prior population distributions. After estimating VAR(2)s on these samples, LRMN and the BQ decomposition are applied to construct SVMAs.<sup>14</sup> The SVMAs are employed to compute prior population permanent and transitory SDs of  $\Delta Y$  and  $\Delta C$ . Since our uncertainty about the theory – the parameters of the NKDSGE models – is embedded in these prior population moments, the subscript  $\mathcal{T}$  is used to denote  $SD_{\mathcal{T},\Delta Y}$  and  $SD_{\mathcal{T},\Delta C}$ .

#### 3.4 Measures of fit

An intermediate step in measuring NKDSGE model fit is to construct a KS goodness of fit statistic similar to one used by Cogley and Nason (1995a). They evaluate the fit of a RBC model fit to the  $SD_{\Delta Y}$  with the KS. Since multidimensional SDs are mapped into a scalar KS statistic, it is an efficient metric for summarizing the information in SDs when conducting a model evaluation exercise.

Bayesian Monte Carlo experiments produce distributions of permanent and transitory  $SD_{\mathcal{P},\Delta Y}$ ,  $SD_{\mathcal{P},\Delta C}$ ,  $SD_{\mathcal{T},\Delta Y}$ , and  $SD_{\mathcal{T},\Delta C}$ . We convert the multidimensional SDs into scalar posterior and prior KS statistics,  $KS_{\mathcal{P}}$  and  $KS_{\mathcal{T}}$ . The first step normalizes permanent and transitory  $SD_{\mathcal{P},\Delta Y}$ ,  $SD_{\mathcal{P},\Delta C}$ ,  $SD_{\mathcal{T},\Delta Y}$ 

<sup>14.</sup> The appendix constructs the map from SVAR to SVMA. The SVARs satisfy the invertibility condition of Fernández-Villaverde, Rubio-Ramírez, Sargent, and Watson (2007) across all Bayesian Monte Carlo simulations of the 12 NKDSGE models.

and  $SD_{\mathcal{T},\Delta C}$  on sample permanent and transitory SDs of  $\Delta Y$  or  $\Delta C$ ,  $\widehat{SD}_{\Delta Y}$  and  $\widehat{SD}_{\Delta C}$ . The sample SDs are calculated using a SVMA estimated on the actual data with length T. Define the normalization  $\mathcal{R}_{\mathcal{D},j}(\omega) = \widehat{SD}(\omega)/SD_{\mathcal{D},j}(\omega)$  at replication j and frequency  $\omega$ , where  $\mathcal{D} = \mathcal{P}$ ,  $\mathcal{T}$ . Next, compute the partial sum  $\mathcal{V}_{\mathcal{D},j}\left(2\pi q/\mathcal{H}\right) = 2\pi\mathcal{H}^{-1}\sum_{\ell=1}^{q}\mathcal{R}_{\mathcal{D},j}\left(2\pi\ell/\mathcal{H}\right)$ , where  $\mathcal{H} = T$  when  $\mathcal{D} = \mathcal{P}$  and  $\mathcal{H} = \mathcal{M}$  otherwise. The partial sums are used to construct the partial difference  $\mathcal{B}_{\mathcal{D},j}(\kappa) = 0.5\pi^{-1}\sqrt{2\mathcal{H}}\left[\mathcal{V}_{\mathcal{D},j}(\kappa\pi) - \kappa\mathcal{V}_{\mathcal{D},j}(\pi)\right]$ , where  $\kappa \in [0,1]$  indicates  $\mathcal{B}_{\mathcal{D},j}(\kappa)$  is evaluated on the entire spectrum. The scalar  $KS_{\mathcal{D}}$  statistic at replication j is calculated as the maximal absolute value of  $\mathcal{B}_{\mathcal{D},j}(\kappa)$ ,  $KS_{\mathcal{D},j} = Max_{\kappa \in [0,1]} \left|\mathcal{B}_{\mathcal{D},j}(\kappa)\right|$ . Thus, a  $KS_{\mathcal{P},j}\left(KS_{\mathcal{T},j}\right)$  statistic measures the distance between a  $SD_{\mathcal{P}}\left(SD_{\mathcal{T}}\right)$  and  $\widehat{SD}$ .

We engage the CIC to measure the fraction of  $\mathcal{J}$  elements of a  $KS_{\mathcal{T}}$  distribution that occupies an interval defined by lower and upper quantiles of the associated  $KS_{\mathcal{T}}$  distribution at a 1-p percent confidence level. Suppose a habit NKDSGE model generates a distribution of the permanent  $SD_{\mathcal{T},\Delta Y}$  that fills this interval with more than 30 percent of its elements (CIC > 0.3), but the corresponding non-habit model does not ( $CIC \leq 0.3$ ). In this case (as DeJong, Ingram, and Whiteman (1996) imply in their analysis of RBC models), the habit NKDSGE model is a more plausible match to the permanent  $SD_{\mathcal{T},\Delta Y}$ . Note that this match requires the  $SD_{\mathcal{T}}$  distribution to be "near" the  $SD_{\mathcal{T}}$  distribution at several frequencies for a  $KS_{\mathcal{T}}$  statistic distribution to cover more than 30 percent of the distribution of a  $KS_{\mathcal{T}}$  statistics. Thus, a CIC constitutes a 'joint test' of NKDSGE model fit.

The  $KS_{\mathcal{T}}$  and  $KS_{\mathcal{T}}$  statistics are computed on the entire spectrum (*i.e.*, the long- to the short-run) and the business cycle horizons of 8 to 2 years per cycle. By isolating the business cycle fluctuations, we build on an insight of Diebold, Ohanian, and Berkowitz (1998). They focus on the business cycle frequencies when model misspecification corrupts measurement of short- and long-run fluctuations. We address this issue by limiting  $\kappa$  to frequencies between 8 and 2 years per cycle,  $\kappa \in [0.064, 0.25]$ , when compiling  $KS_{\mathcal{T}}$  and  $KS_{\mathcal{T}}$  distributions. This mitigates discounting NKDSGE models that perform well at business cycle horizons, but poorly on lower growth and higher short-run frequencies.

# 4. HABIT AND NON-HABIT NKDSGE MODEL EVALUATION

Before evaluating the fit of the NKDSGE models to posterior permanent and transitory  $SD_{P,\Delta Y}$ s

<sup>15.</sup> As in DeJong, Ingram, and Whiteman (1996), the *CIC* of a  $KS_T$  statistic distribution is set to  $\frac{1}{1-p} \int_a^b KS_{T,j} \, dj$  for a 1-p percent confidence level, where a(b) is the lower 0.5p (upper 1-0.5p) quantile and is normalized by 1-p to equal  $\int_a^b KS_{T,j} \, dj$ . 16. If the *CIC* of a NKDSGE model with internal consumption habit is larger than a NKDSGE model without it, the implication is that Bayes factor supports the former model.

and  $SD_{\mathcal{P},\Delta C}$ s, we study the means of these posterior moments. Figure 2 plots mean posterior permanent and transitory  $SD_{\mathcal{P},\Delta Y}$  and  $SD_{\mathcal{P},\Delta C}$ . The mean  $SD_{\mathcal{P},\Delta Y}$  ( $SD_{\mathcal{P},\Delta C}$ ) decomposes variation in the average response of  $\Delta Y$  ( $\Delta C$ ) to permanent and transitory shocks frequency by frequency.<sup>17</sup> The top (bottom) panel of figure 2 contains mean permanent (transitory)  $SD_{\mathcal{P},\Delta Y}$  and  $SD_{\mathcal{P},\Delta C}$ . Mean  $SD_{\mathcal{P},\Delta Y}$  appear as solid (blue) lines in figure 2 and mean  $SD_{\mathcal{P},\Delta C}$  are depicted with (blue) ' $\blacklozenge$ ' symbols.

Mean permanent  $SD_{\mathcal{P},\Delta Y}$  and  $SD_{\mathcal{P},\Delta C}$  display the greatest power at frequency zero in the top panel of figure 2. Subsequently, these SDs decay from the growth (*i.e.*, more than 8 years per cycle) into the highest frequencies (*i.e.*, less than 2 years per cycle), which indicate these frequencies matter less and less for variation in the permanent components of  $\Delta Y$  and  $\Delta C$ . Compared to the  $SD_{\mathcal{P},\Delta C}$ , the mean permanent  $SD_{\mathcal{P},\Delta Y}$  exhibits about five times the power at the long run. This is consistent with the PIH and other business cycle theories that predict greater volatility in  $\Delta Y$  than in  $\Delta C$ . However, the PIH is not supported by the mean permanent  $SD_{\mathcal{P},\Delta C}$  because it is not flat across the entire spectrum.

The lower panel of figure 2 presents mean transitory  $SD_{\mathcal{P},\Delta Y}$  and  $SD_{\mathcal{P},\Delta C}$  with disparate shapes. The latter SD peaks around six years per cycle. Rather than a peak, the mean transitory  $SD_{\mathcal{P},\Delta C}$  plateaus from the growth frequencies to four years per cycle before dropping off in the high frequencies. Thus, the transitory or "output gap" component of  $\Delta Y$  exhibits greater periodicity in the business cycle frequencies compared to the transitory component of  $\Delta C$ .

Figure 2 summarizes the challenge for NKDSGE models. The models must produce permanent  $SD_{\mathcal{T},\Delta Y}$  and  $SD_{\mathcal{T},\Delta C}$  that are not white noise, but instead have most power at the lowest frequencies, to match the permanent  $SD_{\mathcal{T}}$ s displayed in the top panel of figure 2. The lower panel of figure 2 challenges NKDSGE models to generate transitory  $SD_{\mathcal{T},\Delta Y}$  ( $SD_{\mathcal{T},\Delta C}$ ) with the greatest power at the business cycle (growth) frequencies. As a result, NKDSGE models need empirically and economically meaningful propagation and monetary transmission mechanisms to match permanent and transitory  $SD_{\mathcal{T}}$ .

#### 4.1 Quantifying NKDSGE model fit: CIC

Table 2 presents *CIC* that measures the overlap of distributions of  $KS_{\mathcal{T}}$  and  $KS_{\mathcal{T}}$  statistics. The *CIC* is calculated at a 95 percent confidence level, p = 0.05. The overlap of these distributions gauges the fit of 12 NKDSGE models to permanent and transitory  $SD_{\mathcal{P},\Delta Y}$  and  $SD_{\mathcal{P},\Delta C}$  distributions.

The 12 NKDSGE models are defined by different combinations of nominal frictions as well as by monetary policy rule. The first NKDSGE model is our baseline that includes sticky prices and wages.  $\overline{17}$ . A mean  $SD_P$  is computed across an ensemble of  $SD_{P,j}$ ,  $j = 1, ..., \mathcal{J}$ , pointwise or frequency by frequency.

From this baseline, two more NKDSGE models are created by stripping out one or the other nominal rigidity. Baseline, sticky price (SPrice), and sticky wage (SWage) NKDSGE models have household preferences with either no consumption habit, h = 0, or internal consumption habit,  $h \in [0.05, 0.95]$ . These six NKDSGE models are doubled by defining monetary policy with the MGR (8) or the TR (9).

The *CIC* of baseline, SPrice, and SWage non-habit NKDSGE models is listed in the top panel of table 2. The lower panel contains *CIC* of habit NKDSGE models. The NKDSGE-MGR and NKDSGE-TR models define monetary policy with the MGR (8) and the TR (9), respectively. Columns titled  $[0, \pi]$  and [8, 2] report *CIC* quantifying the overlap of distributions of  $KS_{\mathcal{T}}$  and  $KS_{\mathcal{T}}$  statistics on the entire frequency domain (*i.e.*, the long-run or frequency zero to the short-run) and the business cycle frequencies that run from 8 to 2 years per cycle, respectively.

Table 2 contains *CIC* that indicates internal consumption habit improves the fit of NKDSGE models. Habit NKDSGE models generate 18 *CIC* > 0.3 (or 37.5 percent) out of a possible 48 in the bottom half of table 2, but the top half of the table shows that non-habit NKDSGE models are responsible for only 9 *CIC* > 0.3 (or 18.75 percent). Thus, internal consumption habit contributes to permanent and transitory  $SD_{\mathcal{T},\Delta Y}$  and  $SD_{\mathcal{T},\Delta C}$  that better replicate the posterior moments.

A striking feature of table 2 is the impact of the monetary policy rules on the fit of NKDSGE models to transitory  $SD_{\mathcal{P},\Delta Y}$  and  $SD_{\mathcal{P},\Delta C}$ . In these models, the MGR (8) has few successes at transmitting its shock innovation  $\mu_t$  into  $\Delta Y$  and  $\Delta C$  fluctuations that resemble the posterior moments. According to the *CIC* of table 2, only the SPrice habit NKDSGE-MGR model accomplishes this task and only on the business cycle frequencies of 8 to 2 years per cycle.

The TR (9) improves NKDSGE model fit compared to the MGR (8). Of the 27 CIC > 0.3 found in table 2, the TR (9) is tied to 22. In the top half of table 2, 8 of the 9 CIC > 0.3 are generated by non-habit NKDSGE-TR models. Similarly, habit NKDSGE-TR models yield 14 of the 18 CIC > 0.3 in the bottom half of table 2. Table 2 also indicates that, on the business cycle frequencies of 8 to 2 years per cycle, habit and non-habit NKDSGE-TR models match transitory  $SD_{\mathcal{P},\Delta Y}$  and  $SD_{\mathcal{P},\Delta C}$  with CIC > 0.3 in all 12 possible cases. Thus, table 2 gives evidence that habit and non-habit NKDSGE models have empirically useful monetary transmission mechanisms when initiated by the TR shock innovation  $v_t$ . However, only habit NKDSGE-TR models are able to replicate the transitory  $SD_{\mathcal{P}}$ s on the entire spectrum because

<sup>18.</sup> The sticky wage NKDSGE model requires the degenerate prior  $\mu_P=0$  with fixed markup  $\phi=(\xi-1)/\xi$ . When the nominal wage is flexible, households set their optimal wage period by period in sticky price NKDSGE models. In this case, the markup in the labor market is fixed at  $(\theta-1)/\theta$ , which equals  $n^{-1/\gamma}$ , given  $\mu_W=0$ .

these models are responsible for 11 of 12 possible CIC > 0.33 in the bottom half of table 2.

The propagation mechanisms of the NKDSGE models are not held in similar regard by the permanent  $SD_{\mathcal{T},\Delta Y}$  and  $SD_{\mathcal{T},\Delta C}$ . These  $SD_{\mathcal{T}}$ s are matched only when permanent  $SD_{\mathcal{T},\Delta Y}$  and  $SD_{\mathcal{T},\Delta C}$  are generated by SPrice habit and non-habit NKDSGE-MGR and -TR models. The bottom half of table 2 shows that a CIC > 0.3 appears in four of the four possible cases when SPrice habit NKDSGE-MGR and -TR models are asked to propagate the TFP innovation shock  $\varepsilon_t$  into the business cycle frequencies of 8 to 2 years per cycle. The SPrice non-habit NKDSGE-MGR and -TR models are less successful at this task. Only the permanent  $SD_{\mathcal{T},\Delta Y}$  is duplicated by these models. When the propagation mechanisms of these models are examined using the entire spectrum, a CIC > 0.3 is produced only for the permanent  $SD_{\mathcal{T},\Delta Y}$ . Nontheless, this shows that measuring model fit on the entire spectrum leads to otherwise empirically relevant NKDSGE models being undervalued.

In summary, internal consumption habit confers a superior fit on NKDSGE models according to the *CIC* of table 2. The improved fit of habit NKDSGE models is predicated, in part, on the TR (9), but non-habit NKDSGE-TR models possess empirically and economically credible monetary transmission mechanisms when asked to match transitory  $SD_T$ s only on the business cycle frequencies of 8 to 2 years per cycle. Limiting the analysis to the business cycle frequencies improves the fit of SPrice habit NKDSGE-MGR and -TR models when the match is to permanent  $SD_{T,\Delta Y}$  and  $SD_{T,\Delta C}$ . <sup>19</sup>

#### 4.2 NKDSGE model dynamics: Internal consumption habit

The habit NKDSGE models provide a superior fit to the permanent and transitory  $SD_{\mathcal{T},\Delta Y}$ s and  $SD_{\mathcal{T},\Delta C}$ s, according to the CIC of table 2. Although moment matching is useful for assessing the fit of the NKDSGE models, this evaluation process is not informative about the propagation and monetary transmission mechanism dynamics of NKDSGE models. This section and the next explores NKDSGE model dynamics by comparing mean permanent and transitory  $SD_{\mathcal{T}}$ s and  $SD_{\mathcal{T}}$ s.

We summarize evidence about NKDSGE model dynamics in figures 3, 4, and 5. The figures consist of four rows and two columns of panels containing means of  $SD_{\mathcal{T}}$ s and  $SD_{\mathcal{T}}$ s. From top to bottom, the rows of figures 3–5 plot mean permanent  $SD_{\Delta Y}$ s, transitory  $SD_{\Delta Y}$ s, permanent  $SD_{\Delta C}$ s, and transitory  $SD_{\Delta C}$ s. The NKDSGE-MGR models are responsible for  $SD_{\mathcal{T}}$ s that appear in the left column of

<sup>19.</sup> The appendix reports results of Bayesian Monte Carlo experiments that substitute the Cramer-von Mises goodness of fit statistic for the KS statistic to quantify NKDSGE model fit, estimate VAR(4)s instead of VAR(2)s, and replace the uniform prior,  $h \sim U(0.05, 0.95)$ , with either  $h \sim U(0.05, 0.499)$ ,  $h \sim U(0.50, 0.95)$ , or  $h \sim \beta(0.65, 0.15)$ . The latter prior implies a 95 percent coverage interval for h of [0.38, 0.88]. These experiments reinforce the message table 2 has for the impact of internal consumption habit on NKDSGE model fit as well the vulnerabilities in the fit of NKDSGE models under different combinations of the nominal frictions and monetary policy rules.

figures 3–5. The right column of these figures present  $SD_{\mathcal{T}}$ s that are generated by NKDSGE-TR models. Figures 3–5 display mean  $SD_{\mathcal{T}}$ s with solid (blue) lines, mean  $SD_{\mathcal{T}}$ s created by non-habit NKDSGE models with (green) dotted lines, and mean  $SD_{\mathcal{T}}$ s tied to habit NKDSGE models with (red) dot-dashed lines. The mean  $SD_{\mathcal{T}}$ s of baseline, SPrice, and SWage NKDSGE models are depicted in figures 3, 4, and 5, respectively.

Figures 3–5 display mean permanent and transitory  $SD_{\mathcal{T},\Delta Y}$ s and  $SD_{\mathcal{T},\Delta C}$ s, with (green) dotted plots in figures 3–5, produced by non-habit NKDSGE models that often exhibit greater power than the (red) dot-dash mean  $SD_{\mathcal{T}}$ s created by habit NKDSGE models. As a result, habit NKDSGE models generate  $SD_{\mathcal{T}}$ s that are either closer to or intersect mean  $SD_{\mathcal{T}}$ s, especially at the business cycle frequencies.

These mean  $SD_{\mathcal{T}}$ s reproduce an important feature of the  $SD_{\widetilde{\Delta C}}$  plotted in the bottom window of figure 1. Remember the  $SD_{\widetilde{\Delta C}}$  are computed using the linear approximate Euler equation (2) and from top to bottom are indexed by h=0 to 0.85. The bottom window of figure 1 shows the  $SD_{\widetilde{\Delta C}}$ s converge pointwise to the horizontal axis as h increases. Since the mean permanent and transitory  $SD_{\mathcal{T},\Delta Y}$ s and  $SD_{\mathcal{T},\Delta C}$ s of figures 3–5 often have the same ordering, the utility adjustment costs inherent in internal consumption habit are responsible, in part, for propagation and monetary transmission in NKDSGE models that replicate the mean permanent and transitory  $SD_{\mathcal{T},\Delta Y}$ s and  $SD_{\mathcal{T},\Delta C}$ s of figure 2.

# 4.3 NKDSGE model dynamics: Mixing nominal frictions and monetary policy rules

This section explores the impact that different combinations of sticky prices, sticky wages, and monetary policy rule have on propagation and monetary transmission in NKDSGE models. For example, the first and third rows of figures 3–5 establish that there is no mix of nominal rigidities and monetary policy rule in either habit or non-habit NKDSGE models that produce a credible facsimile of the mean permanent  $SD_{\mathcal{T},\Delta Y}$  and  $SD_{\mathcal{T},\Delta C}$ . The large gaps between these posterior moments and mean permanent  $SD_{\mathcal{T},\Delta Y}$  and  $SD_{\mathcal{T},\Delta C}$  are not because the NKDSGE models lack powerful propagation mechanisms. Instead, the top row of windows of figures 3–5 reveal the 12 NKDSGE models propagate the TFP shock innovation  $\varepsilon_t$  into  $SD_{\mathcal{T},\Delta Y}$  that often have greatest variation in the business cycle frequencies between 8 and 4 years per cycle. In comparison, the mean permanent  $SD_{\mathcal{T},\Delta Y}$  has greatest power at the long run before dropping off in the business cycle frequencies. Similarly, the third row of figures 3–5 displays large gaps between the  $SD_{\mathcal{T},\Delta C}$  and  $SD_{\mathcal{T},\Delta C}$  and  $SD_{\mathcal{T},\Delta C}$  s.

The mean transitory  $SD_{\mathcal{T},\Delta Y}$  and  $SD_{\mathcal{T},\Delta C}$  of figures 3–5 show that the impact of inflation smoothing implied by the optimal forward-looking price setting of equation (7) contributes, along with internal

consumption habit and the TR (9), to economically credible monetary transmission. This is the SPrice habit NKDSGE model whose mean transitory  $SD_{\mathcal{T},\Delta Y}$  and  $SD_{\mathcal{T},\Delta C}$  are depicted with (red) dot-dash plots in the even numbered rows of figure 4's right-hand column. Observe in the second row of figure 4 that the mean  $SD_{\mathcal{T},\Delta Y}$  peaks between 8 and 4 years per cycle, which echoes the shape of the mean transitory  $SD_{\mathcal{T},\Delta Y}$ . The SPrice habit NKDSGE model also generates a mean transitory  $SD_{\mathcal{T},\Delta C}$  in the bottom right window of figure 4 that reproduces the plateau in the growth frequencies of the mean transitory  $SD_{\mathcal{T},\Delta C}$ . The left-hand column of Figure 4 shows that dropping internal consumption habit or swapping the MGR (8) for the TR (9) pushes mean transitory  $SD_{\mathcal{T}}$ s away from the mean posterior moments.

There is no NKDSGE with nominal wage smoothing generated by the optimal nominal wage equation (6) that provides an economically meaningful monetary transmission mechanism. Figures 3 and 5 are clear that habit and non-habit NKDSGE models with sticky wages have difficulties reproducing mean transitory  $SD_{\mathcal{T},\Delta Y}$  and  $SD_{\mathcal{T},\Delta C}$ . The baseline and SWage habit and non-habit NKDSGE models often generate power in mean transitory  $SD_{\mathcal{T},\Delta Y}$ s and  $SD_{\mathcal{T},\Delta C}$ s at the business cycle frequencies from 8 to 2 years a cycle, which is excessive compared to the mean transitory  $SD_{\mathcal{T},\Delta Y}$ s and  $SD_{\mathcal{T},\Delta C}$ s. The distance between  $SD_{\mathcal{T}}$ s and  $SD_{\mathcal{T}}$ s is repaired only in part by switching from the MGR (8) to the TR (9) in baseline and SWage NKDSGE models. However, the bottom right window of figure 3 reports that the baseline habit NKDSGE-TR model generates a mean transitory  $SD_{\mathcal{T},\Delta C}$  with less power in the business frequencies, which moves it closer to the  $SD_{\mathcal{T},\Delta C}$ s.

# 5. Conclusion

This paper studies the business cycle implications of internal consumption habit for NKDSGE models. We examine the fit of 12 NKDSGE models that have different combinations of internal consumption habit, Calvo-staggered prices, and nominal wages, along with several other real rigidities. The NKDSGE models are confronted with posterior *SD*s of output and consumption growth identified by permanent TFP and transitory monetary policy shocks.

The fit of NKDSGE models with and without internal consumption habit is explored by comparing posterior population moments to theoretical prior population moments. Our analysis shows that the fit of NKDSGE models with consumption habit is susceptible to (1) changing the mix of nominal rigidities, (2) identifying *SD*s on permanent TFP shocks instead of transitory monetary policy shocks, and (3) evaluating *SD*s on the entire spectrum rather than the business cycle frequencies.

These results indicate that there are vulnerabilities in the specification of NKDSGE models. Not unexpectedly, the new Keynesian monetary transmission mechanism is not the issue. There are combinations of sticky prices, sticky wages, and monetary policy rule that matches the posterior *SD*s of output and consumption growth. Nonetheless, only when internal consumption habit, sticky prices, and a Taylor rule are included in a NKDSGE model, does it transmit monetary policy shocks into empirically plausible mean *SD*s of output and consumption growth. The vulnerabilities in NKDSGE model fit are tied to the mix of nominal rigidities and judging fit on the entire spectrum when the moment matching exercise is identified with the permanent TFP shock. Thus, the economic and empirical relevance of the propagation mechanisms of NKDSGE models remains open to more research. We hope this paper plays a part in inspiring further research into the role real and nominal rigidities play in propagation as well as monetary transmission in NKDSGE models.

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TABLE 1: BAYESIAN CALIBRATION OF NKDSGE MODELS

		Prior Distribution	Mean	Standard Deviation	95 Percent Cover Interval
h	Internal Consumption Habit	Uniform	_	_	[0.0500, 0.9500]
β	H'hold Subjective Discount	Beta	0.9930	0.0020	[0.9886, 0.9964]
γ	Labor Supply Elasticity	Normal	1.5500	0.5360	[0.4995, 2.6005]
$\delta$	Depreciation Rate	Beta	0.0200	0.0045	[0.0122, 0.0297]
α	Deterministic Growth Rate	Normal	0.0040	0.0015	[0.0011, 0.0064]
$\overline{\omega}$	Capital Adjustment Costs	Normal	4.7710	1.0260	[2.7601, 6.7819]
ψ	Capital's Share of Output	Beta	0.3500	0.0500	[0.2554, 0.4509]
$\sigma_{arepsilon}$	TFP Growth Shock Std.	Uniform	_	_	[0.0070, 0.0140]
ξ	Final Good Dmd Elasticity	Normal	12.0000	4.0820	[3.9994, 20.0006]
$\mu_P$	No Price Change Probability	Beta	0.5500	0.0500	[0.4513, 0.6468]
$\theta$	Labor Demand Elasticity	Normal	15.0000	3.0800	[8.9633, 21.0367]
$\mu_W$	No Wage Change Probability	Beta	0.7000	0.0500	[0.5978, 0.7931]
$m^*$	$\Delta \ln M$ Mean	Beta	0.0114	0.0030	[0.0063, 0.0180]
$ ho_m$	$\Delta \ln M$ AR1 Coef.	Beta	0.6278	0.0800	[0.4653, 0.7767]
$\sigma_{\mu}$	$\Delta \ln M$ Shock Std.	Beta	0.0064	0.0012	[0.0043, 0.0090]
$a_{\varsigma}$	Taylor Rule $\mathbf{E}_{t} \varsigma_{t+1}$ Coef.	Normal	1.8250	0.2300	[1.3742, 2.2758]
$a_{\hat{Y}}$	Taylor Rule $\hat{Y}_t$ Coef.	Normal	0.1000	0.0243	[0.0524, 0.1476]
$ ho_R$	Taylor Rule AR1 Coef.	Beta	0.6490	0.0579	[0.5317, 0.7578]
$\sigma_v$	Taylor Rule Shock Std.	Beta	0.0051	0.0016	[0.0025, 0.0087]

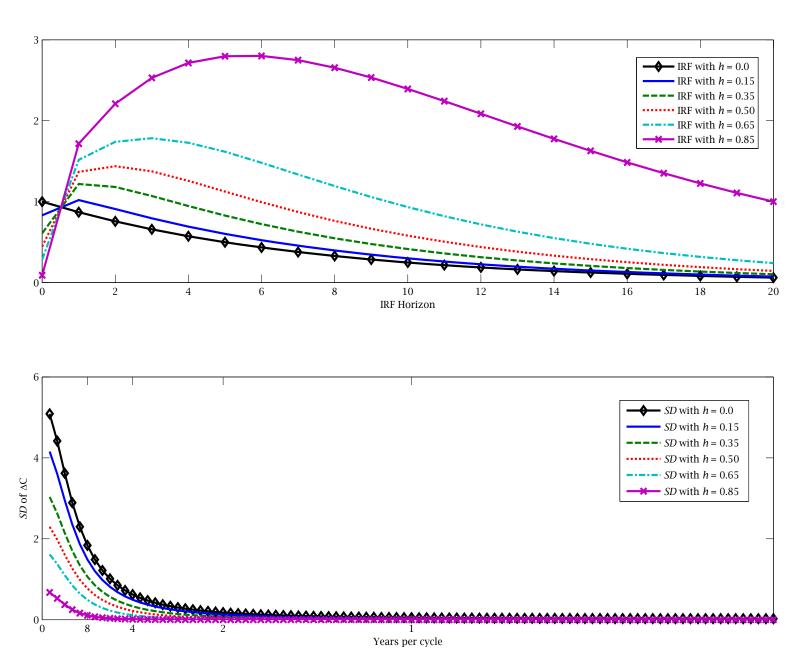
The calibration relies on existing DSGE model literature; see the text for details. For a non-informative prior, the right most column contains the lower and upper end points of the uniform distribution. When the prior is based on the beta distribution, its two parameters are  $a = \overline{\Gamma}_{i,n} \left[ (1 - \overline{\Gamma}_{i,n}) \overline{\Gamma}_{i,n} / STD(\Gamma_{i,n})^2 - 1 \right]$  and  $b = a(1 - \overline{\Gamma}_{i,n}) / \overline{\Gamma}_{i,n}$ , where  $\overline{\Gamma}_{i,n}$  is the degenerate prior of the ith element of the parameter vector of model  $n = 1, \ldots, 4$ , and its standard deviation is  $STD(\Gamma_{i,n})$ .

TABLE 2: CIC OF KOLMOGOROV-SMIRNOV STATISTICS

	$\Delta Y$ w/r/t		$\Delta Y \text{ w/r/t}$		$\Delta C$ w/r/t		$\Delta C$ w/r/t	
Model	Trend Sh'k $[0, \pi]$ $[8, 2]$		Transitory Sh'k $[0, \pi]$ $[8, 2]$		Trend Sh'k $[0, \pi]$ $[8, 2]$		Transitory Sh'k $[0, \pi]$ [8, 2]	
Model	[0, 11]	[0, 2]	[0, 11]	[0, 2]	[0, 11]	[0, 2]	[0, 11]	[0, 2]
Non-Habit NKDSGE								
Baseline								
MGR	0.02	0.03	0.00	0.01	0.00	0.00	0.00	0.00
TR	0.01	0.00	0.12	0.71	0.00	0.00	0.08	0.68
SPrice								
MGR	0.03	0.47	0.00	0.23	0.01	0.17	0.00	0.04
TR	0.40	0.57	0.00	0.76	0.01	0.16	0.00	0.49
SWage								
MGR	0.00	0.00	0.00	0.06	0.00	0.00	0.00	0.06
TR	0.00	0.00	0.21	0.37	0.00	0.00	0.02	0.81
Habit NKDSGE								
Baseline								
MGR	0.00	0.04	0.16	0.18	0.02	0.16	0.13	0.18
TR	0.00	0.03	0.64	0.52	0.03	0.14	0.53	0.85
SPrice								
MGR	0.14	0.64	0.11	0.59	0.09	0.44	0.29	0.49
TR	0.43	0.74	0.29	0.65	0.15	0.46	0.33	0.76
SWage								
MGR	0.00	0.01	0.18	0.23	0.01	0.10	0.13	0.23
TR	0.00	0.05	0.55	0.45	0.03	0.13	0.44	0.77

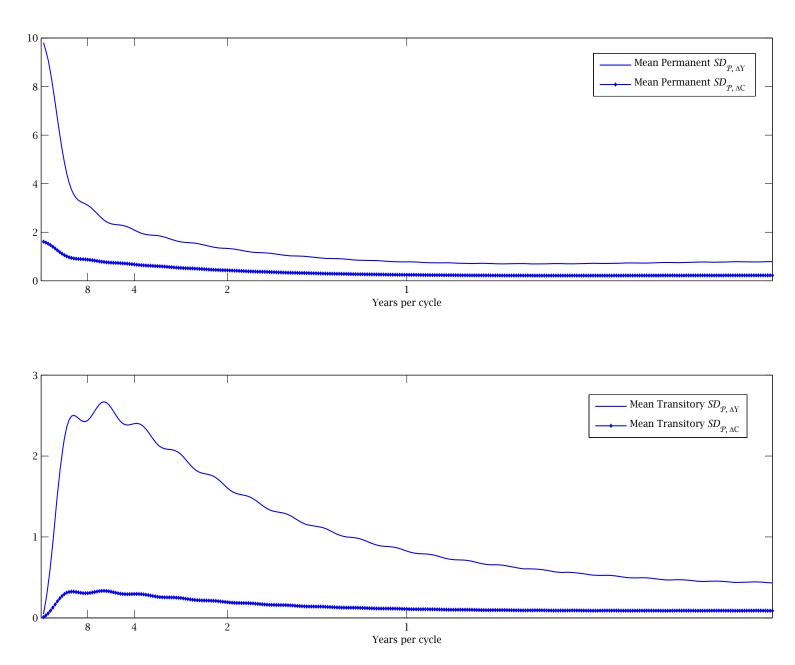
Baseline NKDSGE models include sticky prices and sticky wages. The acronyms SPrice and SWage represent NKDSGE models with only sticky prices or sticky nominal wages, respectively. The money growth and Taylor rules of equations (8) and (9) are denoted by MGR and the TR, respectively. The column headings  $[0, \pi]$  and [8, 2] indicate that the *CIC* quantifies the intersection of  $KS_{\mathcal{P}}$  and  $KS_{\mathcal{T}}$  statistic distributions computed from permanent and transitory SDs of  $\Delta Y$  and  $\Delta C$  with domains on the entire spectrum (*i.e.*, from frequency zero or the long run to the short run), and from 8 to 2 years per cycle, respectively.

Figure 1:  $\Delta C$  and a Real Interest Rate Shock: Impulse Response Functions and SDs



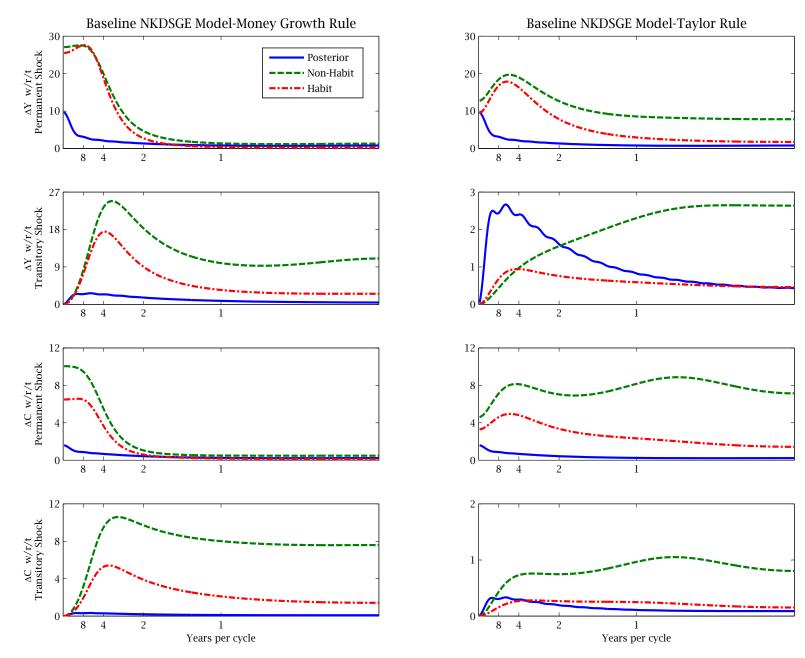
The top (bottom) window plots the impulse response functions (SDs) of  $\Delta C$  generated from the solved linearized Euler (2) given a 1 percent shock to the forecast innovation of the AR(1) of the real rate,  $q_t$ .

FIGURE 2: MEAN PERMANENT AND TRANSITORY  $SD_{P}$ S



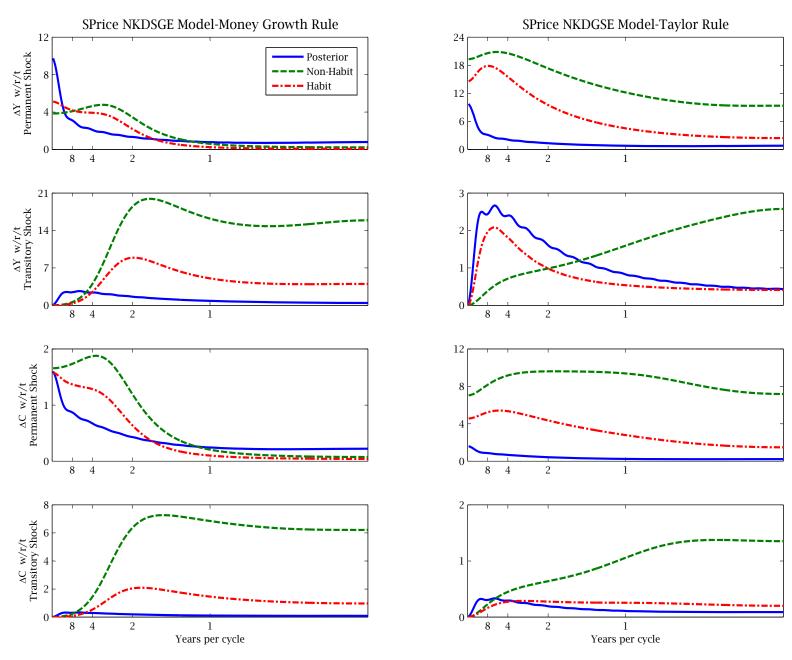
Mean permanent and transitory  $SD_{\mathcal{P},\Delta Y}$  and  $SD_{\mathcal{P},\Delta C}$  are averaged frequency by frequency across ensembles that consist of  $\mathcal{J}$  of these SDs. The SDs are constructed using  $SVMA(\infty)s$  that rely on LRMN, the BQ decomposition, unrestricted VAR(2)s.

FIGURE 3: MEAN STRUCTURAL  $SD_{\mathcal{P}}$ S AND  $SD_{\mathcal{T}}$ S OF BASELINE NKDSGE MODELS



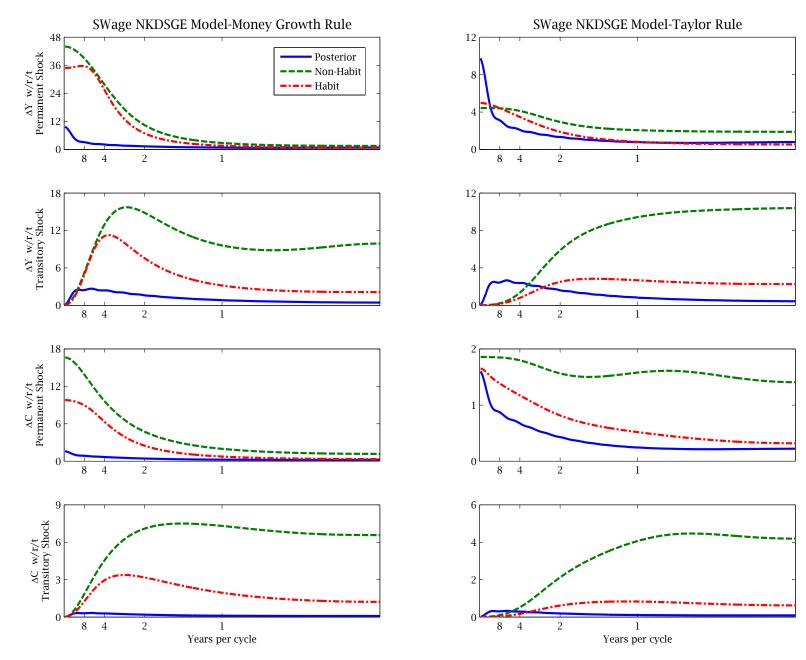
The habit and non-habit NKDSGE models generate ensembles of permanent and transitory  $SD_{\mathcal{T},\Delta Y}$ s and  $SD_{\mathcal{T},\Delta C}$ s that are averaged frequency by frequency to produce the mean  $SD_{\mathcal{T}}$ s. Otherwise, see the notes to figure 2.

FIGURE 4: MEAN STRUCTURAL  $SD_{\mathcal{T}}$ S AND  $SD_{\mathcal{T}}$ S OF SPRICE NKDSGE MODELS



See figure 3 for details.

FIGURE 5: MEAN STRUCTURAL  $SD_{\mathcal{T}}$ S AND  $SD_{\mathcal{T}}$ S OF SWAGE NKDSGE MODELS



See the notes to figure 3.