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A Decision Rule to Minimize Daily Capital Charges in Forecasting Value-at-Risk*

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Abstract

Under the Basel II Accord, banks and other Authorized Deposit-taking Institutions (ADIs) have to communicate their daily risk estimates to the monetary authorities at the beginning of the trading day, using a variety of Value-at-Risk (VaR) models to measure risk. Sometimes the risk estimates communicated using these models are too high, thereby leading to large capital requirements and high capital costs. At other times, the risk estimates are too low, leading to excessive violations, so that realised losses are above the estimated risk. In this paper we analyze the profit maximizing problem of an ADI subject to capital requirements under the Basel II Accord as ADI's have to choose an optimal VaR reporting strategy that minimizes daily capital charges. Accordingly, we suggest a dynamic communication and forecasting strategy that responds to violations in a discrete and instantaneous manner, while adapting more slowly in periods of no violations. We apply the proposed strategy to Standard & Poor's 500 Index and show there can be substantial savings in daily capital charges, while restricting the number of violations to within the Basel II penalty limits.

Key words and phrases: Daily capital charges, endogenous violations, frequency of violations, optimizing strategy, risk forecasts, value-at-risk.

JEL Classifications: G32, G11, G17, C53.

1. Introduction

The Value-at-Risk (VaR) concept has become a standard tool in the exploding area of risk measurement and management. In brief, VaR is defined as an estimate of the probability and size of the potential loss to be expected over a given period. This concept has become especially important following the 1995 amendment to the Basel Accord, whereby banks and other Authorized Deposit-taking Institutions (ADIs) were permitted to use internal models to calculate their VaR thresholds (see Jorion (2000) for a detailed discussion of VaR). Consequently, the last few years have witnessed a growing literature comparing modelling approaches and implementation procedures to answer the question of how to measure VaR, with many research studies arguing in favour or against various VaR models.

The amendment to the Basel Accord was designed to reward institutions with superior risk management systems. A back-testing procedure, whereby the realized returns are compared with the VaR forecasts, was introduced to assess the quality of the internal models. In cases where internal models lead to a greater number of violations than could reasonably be expected, given the confidence level, the ADI is required to hold a higher level of capital (see Table 1 in the Appendix for the penalties imposed under the Basel II Accord). If an ADI's VaR forecasts are violated more than 10 times in any financial year, the ADI may be required to adopt the 'Standardized' approach. The imposition of such a penalty is severe as it affects the profitability of the ADI directly through higher capital charges, has a damaging effect on the ADI's reputation, and may lead to the imposition of a more stringent external model to forecast the ADI's VaR thresholds. That is why financial managers tend to prefer following strategies that are passive and conservative.

Excessive conservatism has a negative impact on the profitability of ADIs as higher capital charges are subsequently required. Academics and practitioners should ask the question if there is room to minimise the capital charges not only using different models to forecast VaR but also through a communication strategy given the Basel II Accord. ADIs are not allowed to violate more than 10 times in any financial year, but any number less than 10 is permitted. Therefore, the decision maker should seek a strategy that allows an endogenous decision as to how many times ADIs should violate in any financial year.

In this paper we formulate the profit maximizing problem of an ADI subject to capital requirements under the Basel II Accord. It is suggested that ADI's may choose an optimal reporting policy that can strategically under-report or over-report their VaR forecasts in order to minimize daily capital charges. Accordingly, we characterize a strategic market risk disclosure policy meant to reduce daily capital charges and to manage the number of violations. We suggest that the decision maker should take some actions in each state based on the trade-off between expected capital requirements and the expected number of violations. Financial managers could adopt a different strategy in favourable situations (a small number of violations) than in unfavourable situations (a large number of violations). The amount of expected risk that the manager should report to the monetary authority (namely, a fraction of the VaR estimated using a given procedure) should increase with the number of violations.

In a favourable situation, the decision maker could take more risk (perhaps reporting an expected risk lower than the one suggested by the model used to forecast volatility). In cases of a small number of violations, communication of a low amount of risk allows profiting from the lower capital requirement, subject to having an acceptable trade-off with the upside-risk of increasing the number of violations. If the capital requirement is lower, more funds can be invested in assets at the cost of a marginal increase in the probability of violation. In a situation of a high number of violations, the decision maker must take less risk, and reporting high expected risk (even higher than the forecast) is needed to decrease the probability of violations.

The remainder of the paper is as follows. In Section 2 we present the problem faced by ADI's subject to the Basel II Accord Amendment. Section 3 reviews some of the most frequently used univariate VaR forecasting models. In Section 4 we present the new market risk disclosure strategy. Section 5 gives some experimental results, and Section 6 summarizes the main conclusions.

2. Maximizing Profits, Value-at-Risk and Daily Capital Charges

We consider an ADI that invests an amount *A* in a portfolio of risky assets. At the beginning of the period, the return on the ADI's portfolio, $r_{A,}$ is random. For purposes of modelling market risk, we assume that $r_A \sim N\left(\mu_A, \sigma_A^2\right)$. The ADI's portfolio is financed by deposits (*D)* and equity capital (*E*), where r_E denotes the cost of holding equity.

A simplified balance sheet of an ADI is at each point of time:

ADI balance sheet

Assets, A	Deposits, D
	Equity, E

Equity is held by shareholders and necessarily $E = A - D$

Consider a competitive ADI which faces the risky returns, $r_{A,}$ on its assets and the market rate for deposits $r_{D.}$ Thus the ADI profit for day *t* can be stated as follows

$$\Pi_t = r_{At} A_t - r_{Dt} D_t - r_{Et} E_t \qquad (1)$$

Given the volume of assets and deposits, an increase in *E* reduces expected profits, so the ADI is interested in using the lowest possible equity.

At any given time, ADI's are required to satisfy capital requirement (CRq_t) according to the associated risks. Specifically, this means:

$$E_t \geq E\left(\delta A_t\right) = CRq_t A_t \quad with\ E'\left(\delta A_t\right) > 0\ and\ \delta > 0 \qquad (2)$$

The Basel II Accord stipulates that daily CRq_t for market risk must be set at the higher of the previous day's VaR and the average *VaR* over the previous 60 business days, multiplied by a factor *m* = [3+*k*], where *k* is a penalty in the form of a higher multiplicative factor when there are more than 4 violations in the preceding financial year.

A violation occurs when actual (negative) returns are worse than the predicted VaR, that is, $r_{At} < -VaR_t$. Therefore, the capital requirement for market risk can be written as:

$$CRq_t = Max\left[-VaR(t-1), [3+k]\frac{1}{60}\sum_{p=1}^{60} -VaR(t-p)\right] \tag{3}$$

k is to be set within a range of 0 to 1 (see (4) below) depending on the supervisor's assessment of the ADI's risk management practices and on the results of a simple back test (Basel Committee on Banking Supervision (1996)). The multiplication factor is determined by the number of times losses exceed the day's VaR figure (Basel Committee on Banking Supervision (1996)). The minimum multiplication factor of 3 is in place to compensate for a number of errors that arise in model implementation: simplifying assumptions, analytical approximations, small sample biases and numerical errors will tend to reduce the true risk coverage of the model (Stahl (1997)). The increase in the multiplication factor is then designed to scale up the confidence level implied by the observed number of exceptions to the 99 per cent confidence level desired by the regulators.

$$k = \begin{cases} 0 & if \quad nov \le 4 \\ 0.40 & if \quad nov = 5 \\ 0.50 & if \quad nov = 6 \\ 0.65 & if \quad nov = 7 \\ 0.75 & if \quad nov = 8 \\ 0.85 & if \quad nov = 9 \\ 1 & if \quad nov \ge 10 \end{cases} \tag{4}$$

Where *nov* is the number of violations during the previous financial year.[1] In calculating the *nov*, ADIs will be required to compare the forecasted VaR numbers with realised profit and loss figures for the previous 250 trading days. In 1995, the 1988 Basel Accord (Basel Committee on Banking Supervision (1988) was amended to allow ADIs to use internal models to determine their VaR thresholds

[1] Penalties of the Basel II Accord

(Basel Committee on Banking Supervision (1995)). However, ADIs wishing to use internal models must demonstrate that their models are sound.

Value-at-Risk refers to the lower bound of a confidence interval for a (conditional) mean. If interest lies in modelling the random variable, Y_t, it could be decomposed as follows:

$$Y_t = E(Y_t \mid F_{t-1}) + \varepsilon_t. \tag{5}$$

This decomposition suggests that Y_t is comprised of a predictable component, $E(Y_t \mid F_{t-1})$, which is the conditional mean, and a random component, ε_t. The variability of Y_t, and hence its distribution, is determined entirely by the variability of ε_t. If it is assumed that ε_t follows a distribution such that:

$$\varepsilon_t \sim D(\mu_t, \sigma_t^2) \tag{6}$$

where μ_t and σ_t are the unconditional mean and standard deviation of ε_t, respectively, these can be estimated using a variety of parametric and/or non-parametric methods. The VaR threshold for Y_t can be calculated as:

$$VaR_t = E(Y_t \mid F_{t-1}) - \alpha\sigma_t \tag{7}$$

where α is the critical value from the distribution of ε_t to obtain the appropriate confidence level. It is possible for σ_t to be replaced by alternative estimates of the conditional variance in order to obtain an appropriate VaR (for a useful review of recent theoretical results for conditional volatility models, see Li et al. (2002), while McAleer (2005) reviews a variety of univariate and multivariate, conditional, stochastic and realized. volatility models). The next section describes several models that are widely used to forecast the 1-day ahead conditional variances and VaR thresholds.

Summarizing, the ADI is assumed to maximize expected profit with respect to the composition of its financial assets and the forecasted *VaR*. Thus, the standard optimization problem is given as:

$$\underset{A_t,\, VaR_t}{Max} \Pi = r_{At} A_t - r_{Dt} D_t - r_{Et} E_t \tag{8}$$

Subject to (2), (3), (4) and (7)

A necessary condition to maximize 錯誤! 找不到參照來源。 in a given period of (say) 250 days, is to minimize the total CRq_t for the period. The standard approach in the literature is to report in (3) the estimate of VaR obtained from a given model (see, for example, Sarma et al. (2003)). In this paper, we incorporate the additional flexibility of modifying the forecast of VaR from a given model.

The leading econometric models for forecasting VaR are modified to accommodate alternative risk strategies. We propose a simple and intuitive rule for modifying the values of the forecasts from the models. The rule becomes more conservative as the number of violations increases, and more aggressive in periods of few or no violations, by incorporating penalties and rewards that are based on performance. The new approach contains the standard rule as a special case.

3. Models for Forecasting VaR

As discussed previously, ADIs can use internal models to determine their VaR thresholds. There are alternative time series models for the conditional volatility, σ_t. In what follows, we present several conditional volatility models to evaluate our strategic market risk disclosure, namely GARCH, GJR and EGARCH, with both normal and *t* distribution errors. For an extensive discussion of the theoretical properties of several of these models, see Ling and McAleer (2002a, 2002b, 2003a). As an alternative to estimating the parameters, we use the exponential weighted moving average (EWMA) method by *Riskmetrics*^{TM} (1996) that calibrates the unknown parameters. The models are presented in increasing order of complexity.

3.1 GARCH

For a wide range of financial data series, time-varying conditional variances can be explained empirically through the autoregressive conditional heteroskedasticity (ARCH) model, which was proposed by Engle (1982). When the time-varying conditional variance has both autoregressive and moving average components, this leads to the generalized ARCH(p,q), or GARCH(p,q), model of Bollerslev (1986). It is very common to impose the widely estimated GARCH(1,1) specification in advance.

Consider the stationary AR(1)-GARCH(1,1) model for daily returns, y_t:

$$y_t = \phi_1 + \phi_2 y_{t-1} + \varepsilon_t, \qquad |\phi_2| < 1 \tag{9}$$

for $t = 1,\ldots,n$, where the shocks to returns are given by:

$$\begin{aligned} \varepsilon_t &= \eta_t \sqrt{h_t}, \quad \eta_t \sim iid(0,1) \\ h_t &= \omega + \alpha\varepsilon_{t-1}^2 + \beta h_{t-1}, \end{aligned} \tag{10}$$

and $\omega > 0, \alpha \geq 0, \beta \geq 0$ are sufficient conditions to ensure that the conditional variance $h_t > 0$. The stationary AR(1)-GARCH(1,1) model can be modified to incorporate a non-stationary ARMA(p,q) conditional mean and a stationary GARCH(r,s) conditional variance, as in Ling and McAleer (2003b).

3.2 GJR

In the symmetric GARCH model, the effects of positive shocks (or upward movements in daily returns) on the conditional variance, h_t, are assumed to be the same as the negative shocks (or downward movements in daily returns). In order to accommodate asymmetric behaviour, Glosten, Jagannathan and Runkle (1992) proposed a model (hereafter GJR), for which GJR(1,1) is defined as follows:

$$h_t = \omega + (\alpha + \gamma I(\eta_{t-1}))\varepsilon_{t-1}^2 + \beta h_{t-1}, \tag{11}$$

where $\omega > 0, \alpha \geq 0, \alpha + \gamma \geq 0, \beta \geq 0$ are sufficient conditions for $h_t > 0$, and $I(\eta_t)$ is an indicator variable defined by:

$$I(\eta_t) = \begin{cases} 1, & \varepsilon_t < 0 \\ 0, & \varepsilon_t \geq 0 \end{cases} \tag{12}$$

as η_t has the same sign as ε_t. The indicator variable differentiates between positive and negative shocks, so that asymmetric effects in the data are captured by the coefficient γ. For financial data, it is expected that $\gamma \geq 0$ because negative shocks have a greater impact on risk than do positive shocks of similar magnitude. The asymmetric effect, γ, measures the contribution of shocks to both short run persistence, $\alpha + \gamma/2$, and to long run persistence, $\alpha + \beta + \gamma/2$.

3.3 EGARCH

An alternative model to capture asymmetric behaviour in the conditional variance is the Exponential GARCH, EGARCH(1,1), model of Nelson (1991), namely:

$$\log h_t = \omega + \alpha \left| \frac{\varepsilon_{t-1}}{h_{t-1}} \right| + \gamma \frac{\varepsilon_{t-1}}{h_{t-1}} + \beta \log h_{t-1}, \quad |\beta| < 1 \tag{13}$$

where the parameters α, β and γ have different interpretations from those in the GARCH(1,1) and GJR(1, 1) models.

As noted in McAleer et al. (2007), there are some important differences between EGARCH and the previous two models, as follows: (i) EGARCH is a model of the logarithm of the conditional variance, which implies that no restrictions on the parameters are required to ensure $h_t > 0$; (ii) moment conditions are required for the GARCH and GJR models as they are dependent on lagged unconditional shocks, whereas EGARCH does not require moment conditions to be established as it depends on lagged conditional shocks (or standardized residuals);

EGARCH captures asymmetries differently from GJR. The parameters α and γ in EGARCH(1,1) represent the magnitude (or size) and sign effects of the standardized residuals, respectively, on the conditional variance, whereas α and $\alpha+\gamma$ represent the effects of positive and negative shocks, respectively, on the conditional variance in GJR(1,1).

3.4 Exponentially Weighted Moving Average (EWMA)

The three conditional volatility models given above are estimated under the following distributional assumptions on the conditional shocks: (1) normal, and (2) *t*. As an alternative to estimating the parameters of the appropriate conditional volatility models, *Riskmetrics*^{TM} (1996) developed a model which estimates the conditional variances and covariances based on the exponentially weighted moving average (EWMA) method, which is, in effect, a restricted version of the ARCH(∞) model. This approach forecasts the conditional variance at time t as a linear combination of the lagged conditional variance and the squared unconditional shock at time $t-1$. The EWMA model calibrates the conditional variance as:

$$h_t = \lambda h_{t-1} + (1-\lambda)\varepsilon_{t-1}^2 \tag{14}$$

where λ is a decay parameter. *Riskmetrics*™ (1996) suggests that λ should be set at 0.94 for purposes of analysing daily data. As no parameters are estimated, there is no need to establish any moment or log-moment conditions for purposes of demonstrating the statistical properties of the estimators.

4. A Dynamic Decision Rule for Strategic Market Risk Disclosure

Recent empirical studies (see, for example, Berkowitz and O'Brien (2001) and Gizycki and Hereford (1998)) indicate that some financial institutions overestimate their market risks in disclosures to supervisory authorities. This implies a costly restriction to the banks trading activity. ADIs may prefer to report high VaR numbers to avoid the possibility of regulatory intrusion. This conservative risk reporting suggests that efficiency gains may be feasible. It may be possible to increase profits by embedding this VaR report problem in a more general profit maximization framework. Therefore, as ADIs already have effective tools for the measurement of market risk, while satisfying the qualitative requirements, ADI managers could wilfully attempt to reduce the daily capital charges by implementing a context-dependent market risk disclosure policy. For a discussion of alternative approaches to optimize VaR and daily capital charges, see McAleer (2008).

As we saw in section 2, a necessary condition to maximize profits is to minimize daily capital charges while the number of violations remains below 10. Accordingly, in this section, as a solution to the general ADI's profit maximization problem, we propose to calculate forecasts of VaR based on the models described in the previous section and reporting a modified value of it, called Market Risk Disclosure (MRD) Policy, as follows:

$$Report_t = MRD_t = P_t \ VaR_t \tag{15}$$

where P_t varies with the number of violations to communicate risk to the monetary authority. The variable P_t is a measure of how conservative or aggressive the MRD is in comparison with the estimated risk: $P_t < 1$ corresponds to an aggressive strategy because the MRD is below the estimated risk, whereas $P_t > 1$ represents a conservative approach. Notice that when $P_t = 1$, the standard reporting strategy is a special case of the new strategy for forecasting and communication.

Dynamic Learning Strategy (DYLES)

In (15), P_t is given by

$$P_t = P_0 + \theta^P * nov_{t-1} - \theta^R \sum_{s=1}^{t} I_{25,s}, \tag{16}$$

The dynamic learning function, P_t, consists of three additive terms:

(1) P_0 is an initial condition and, as time passes, has a decreasing effect on DYLES.

(2) θ^P is the penalty for each violation: any additional violation should be penalized, thereby increasing the market risk disclosure and making our strategy more conservative.

(3) nov_{t-1} is the number of violations up to period *t-1*.

(4) θ^R is the reward (that is, the reduction in the penalty) for each 25-day period without any violations.

(5) $I_{25,t}$ indicates whether there has been a violation in a given period. We divide the 250-day testing period, into 10 fixed periods of 25 days. At the end of each 25-day interval, we check whether there have been any violations during the period. If there have been no violations, the reward consists of decreasing the penalty by θ^R.

The indicator function, $I_{25,t}$, performs the counting: it takes the value one when there have been no violations during a fixed 25-day period, and zero otherwise. It can only change value at the end of each 25-day period. At that point, the reward is either given or it is not:

$$I_{25,t} = \begin{cases} 1 & if\ t = 25j, \quad j = 1, 2, ..., 10 \quad and \quad nov_t - nov_{t-25} = 0 \\ 0 & otherwise \end{cases}$$ **(17)**

By using DYLES in (16), we gain flexibility by considering past information in reaching a decision for any given day. The flexible learning strategy proposed is in the spirit of Benjaafar et al. (1995, p. 438), who formalize the notion of flexibility in sequential decision making, and conclude:

"... a flexible, or reversible, position is preferred when the decision maker is uncertain about the future and/or expects to learn more with the passage of time. A flexible position gives decision makers the possibility to change their minds upon the receipt of new information. In this sense, flexibility limits the risks of an early commitment ... expected value should not decrease with an increase in flexibility ..."

Thus the optimization problem may now be formalized as:

$$\underset{\substack{\Theta=\left[P_0,\theta^P,\theta^R\right] \\ VaR\ \mathrm{model},}}{Min} \frac{1}{250}\sum_{p=j}^{250} Max\left[-P_t\ VaR(t-1), \left[3+k\right]\frac{1}{60}\sum_{p=1}^{60} -P_t VaR(t-p)\right]$$
$$s.t.$$
$$nov < 10,$$
$$Equations \quad (4)\ and\ (16) \qquad (18)$$

In the above formulation, the solutions to (18) belong to the parametric class implied by (16). This is a simplification of the more general problem given in 錯誤! 找不到參照來源。 and (15) (see, for example, Mulvey (1995), Hirano (2008) and Topaloglu (2008) for discussions of simplifying decision problems).

The new rule in (16) can be interpreted as the second and subsequent stages in the optimization problem, in the sense of Sahinidis (2004, p. 972):

> "Traditionally, the second-stage variables are interpreted as corrective measures or recourse against any infeasibilities arising due to a particular realization of uncertainty. However, the second-stage problem may be also an operational-level decision problem following a first-stage plan and the uncertainty realization".

For the optimization approach proposed in this paper, we optimize over the parameters of the second-stage problem jointly with the choice of VaR model at the first stage.

In the proposed approach, *MRD* policies are assessed by comparing mean capital requirements during the last 250 days. A *MRD* strategy that minimizes (18), while restricting *nov* to be less than 10 is said to be optimal in terms of minimum daily *CRq*, as compared with the leading alternative strategies.

Choosing a market risk disclosure policy implies choosing both a VaR model and a parameter value for the communication rule. Thus, the proposed decision rule evaluates the performance of ADIs under alternative combinations of parameter values and VaR models, bearing in mind that there is not always a single parameter combination, Θ^*, that dominates uniformly over all VaR models. For a related general discussion of multi-stage stochastic programming, see Möller et al. (2008).

We find that when the ADI manager uses DYLES to market risk disclosures, taking into account the number of violations, the average daily capital charges during the 260 trading days of 2007 can decrease by up to 14%. A special case of the endogenous violations discussed above, in which the number of violations is not a choice variable but is exogenously determined, is analysed in McAleer and da Veiga (2008a, 2008b).

DYLES is designed to decrease the capital requirements of a MRD policy based on a given VaR, while restricting the number of violations in a given period within the limits of the Basel II Accord.

DYLES works in a complementary manner with the volatility model, but behaves quite differently from the volatility models:

- It operates in a **discrete and fast** manner when there is a violation, whereas the volatility models adjust smoothly to violations.

- It is **context sensitive** as it takes different values depending on the history of the violations. It is more conservative when there have been many violations, and is less conservative when there have been fewer violations.

- It operates **asymmetrically over time**, as it reacts immediately when there is a violation, but moves discretely when there is a period without any violations.

The parameters of the penalty function would need to be calibrated for a given asset and for each model to calculate VaR. In the next section, we provide some insights as to how well this function works for a given portfolio and for different models of the conditional variance.

5. Experimental Results

When we have proposed a new MRD policy based on a dynamic learning penalty function, we are interested in assessing how well it performs in terms of daily capital costs and the number of violations compared with the alternative strategy (that is, no strategy) of not responding to either violations or the absence of violations.

Owing to the dearth of theoretical results in this area, we examine the behaviour of our penalty function using calibration. As the basis for comparison we use Standard and Poor's Composite 500 index from 1 January 2007 to 31 December of 2007. The parameters in the vector $\Theta = \left[P_0 > 0, \theta^P > 0, \theta^R > 0 \right]$ in (17) have to be positive, and calibration suggests the following intervals:

$$P_0 \in [0.6, ..., 1.2],$$

$$\theta^P \in [0.06, ..., 0.12],$$

$$\theta^R \in [0.1, ..., 0.4].$$

$P_0 < 0.6$ (aggressive strategies) would imply numbers of violations in excess of 10, and P_0 greater than 1.2 (conservative risk reporting) would lead to high daily capital requirements.

The calibration procedure is as follows

:

1. We assume the models described in Section 3 to be the internal ADI's models that are used to forecast the 1-day ahead conditional variances and VaR thresholds.
2. For all possible parameter combinations of Θ, we calculate P_t , as given in (17).
3. Given the VaR calculated in step 1 and P_t in step 2, calculate the market risk disclosure using (11).
4. The number of violations (NoV) and the average capital requirements (AvCRq) for the whole period are reported.
5. Finally, we compare NoV and the AvCRq requirements with the no strategy policy.

5.1 Data and volatility measures

The data used for the calibration of DYLES are the closing daily prices for Standard and Poor's Composite 500 Index. Data were obtained from the Ecowin Financial Database for the period 3 January 2000 to 31 December 2007.

The returns at time *t* (R_t) are defined as:

$$R_t = \log\left(P_t / P_{t-1}\right), \tag{19}$$

where P_t is the market price.

[Insert Figure 1 around here]

Figure 1 shows the Standard and Poor's returns. The series exhibit clustering, which could be captured by an appropriate time series model. The descriptive statistics for the index returns are given in Table 2. The mean is close to zero, and the range is between – 6% and 5.57%. The Jarque-Bera Lagrange multiplier test for normality rejects the null hypothesis of normally distributed returns. As the series displays a high kurtosis, this would seem to indicate the existence of extreme observations, which is not surprising for financial returns data.

[Insert Table 2 around here]

Several measures of volatility are available in the literature. In order to gain some intuition, we adopt the measure proposed in Franses and van Dijk (1999), where the true volatility of returns is defined as:

$$V_t = \left(R_t - E\left(R_t \mid F_{t-1}\right)\right)^2, \tag{20}$$

where F_{t-1} is the information set at time t-1. Figure 2 shows the S&P volatility defined as in (20). The series exhibit clustering, which needs to be captured by an appropriate time series model. The volatility of the series appears to be high during the early 2000's, followed by a quiet period from 2003 to the beginning of 2007. The volatility appears to increase dramatically around 2007, due in large part to the worsening global credit environment. This increase in volatility persists until the end of the period, and continues during 2008.

[Insert Figure 2 around here]

[Insert Table 3 around here]

5.2 Results

In Table 3 we see the differences between using DYLES or a passive strategy for the S&P and several VaR models. After calibration, we have chosen the set of parameters $P_0 = 1.2$, $\theta^P = 0.12$, $\theta^R = 0.3$, which is the combination which seems to be most widely optimal across the various models.

Figure 1

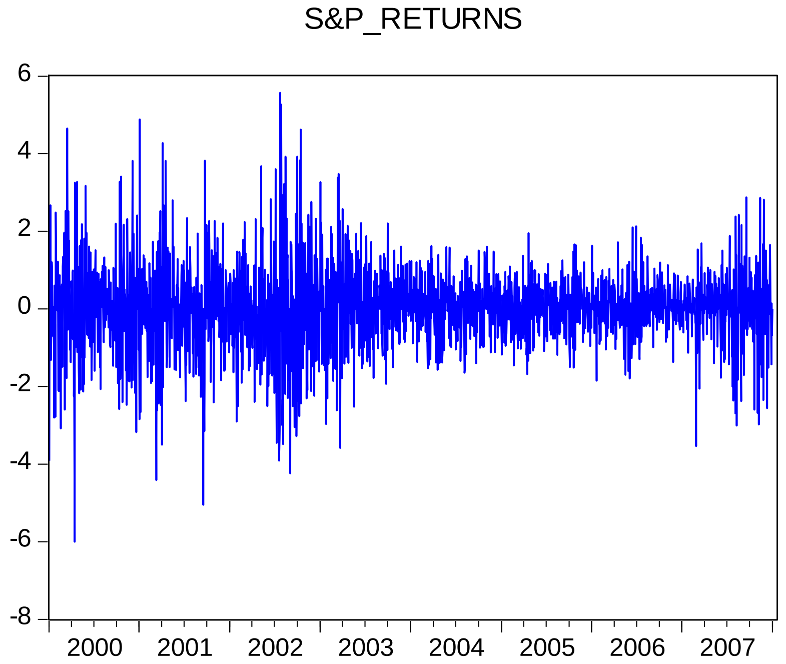
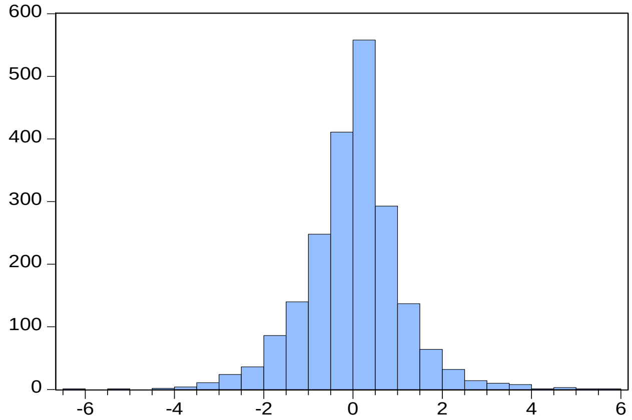


Table 2



Series: SP_RETURNS
Sample 3/01/2000 18/01/2008
Observations 2086

Mean	0.000431
Median	0.001107
Maximum	5.573610
Minimum	-6.004513
Std. Dev.	1.094369
Skewness	0.048622
Kurtosis	5.760783
Jarque-Bera	663.2940
Probability	0.000000

We observe in the last column that the capital requirement decreases in all cases between 0.4% (3 basis points) and 14.01% (96 basis points) when using DYLES, except when the GARCH-t model is used. Moreover, the number of violations decreases when they were above 10 and increases, within the limits, when there are few violations. The exception in Table 4 in the Appendix shows that the best result for the GARCH-t model and DYLES is for Θ = [1.0, 0.11, 0.3] with 9 violations and 5.80% as AvCRq, that is, 90 basis points less than the no strategy policy.

We conclude that DYLES always beats the passive strategy when properly calibrated. Due to its context-sensitive behaviour, DYLES tends to concentrate the distribution of the number of violations. In cases where there is conservative behaviour, it tends to increase the number of violations, whereas when the number of violations is large, it tends to reduce it to below the limit of 10.

When properly used, DYLES can decrease the daily capital requirements substantially, up to 96 basis points in the case analyzed above, while restricting the number of violations to within the limits of the Basel II Accord.

[Insert Figure 3 around here]

In order to gain some intuition, in Figure 3 we present a comparison of DYLES with the results for *Riskmetrics*^{TM}.

a. The returns data are for the Standard and Poor's index during the last 100 days of 2007.
b. The stepwise line corresponds to the values of DYLES (P_t) during the period for which **Θ = [1.2, 0.12, 0.3]**.
c. Of the two bottom lines, the one that starts higher is the VaR calculated by *Riskmetrics*^{TM}
d. Of the two bottom lines, the one (line with symbols) that starts lower (with greater risk) is the VaR of *Riskmetrics*^{TM} + DYLES, which is our RMD.

Figure 2

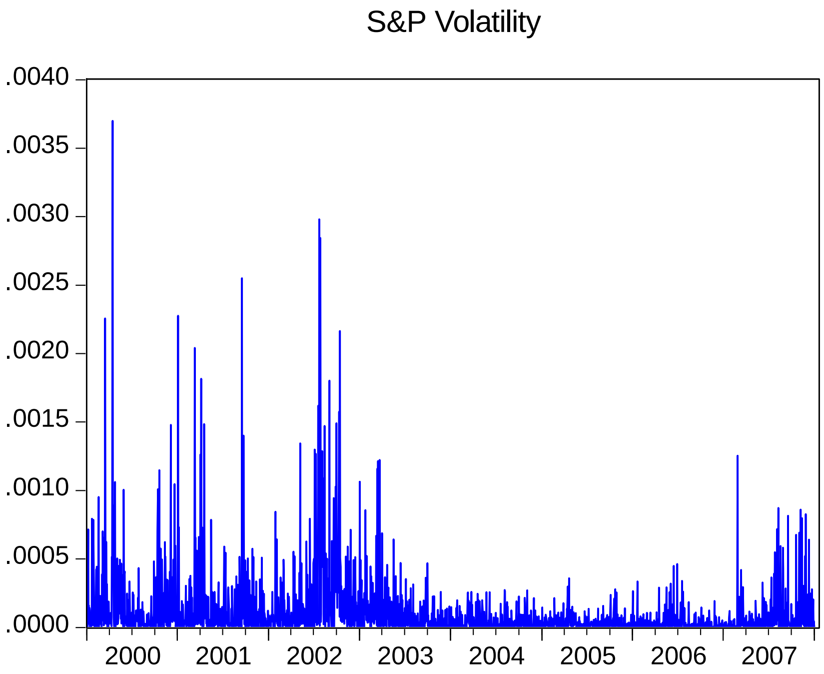

S&P Volatility
.0040
.0035
.0030
.0025
.0020
.0015
.0010
.0005
.0000
2000
2001
2002
2003
2004
2005
2006
2007

Table 3

Benefits from DYLES, Standard and Poor's, θ = [1.2, 0.12, 0.3]

Model	DYLES		Passive		Improvement DYLES vs Passive		
	NoV	AvCRq	NoV	AvCRq	NoV	AvCRq	Basic points
EGARCH	9	5.94%	13	6.24%	-4	4.8%	30
EGARCH-t	7	5.86%	9	6.82%	-2	14.1%	96
GARCH	8	6.00%	11	6.54%	-3	8.3%	54
GARCH-t	7	7.00%	3	6.70%	4	-4.5%	-30
GJR	8	6.00%	9	6.75%	-1	11.1%	75
GJR-t	8	7.01%	4	7.04%	4	0.4%	3
Riskmetrics^{TM}	8	5.98%	12	6.61%	-4	9.5%	63

Note that when the *Riskmetrics*^{TM} + DYLES line is above the *Riskmetrics*^{TM} line, it implies less risk and leads to lower daily capital requirements. The improvement in capital requirements is given by the difference between the red (dark) and blue (light) lines in Figure 3. This is attributable exclusively to the penalty function of the dynamic learning function strategy (DYLES).

Figure 4 shows the four previous series for the whole sample, from 1 January 2007 to 31 December 2007. As we can see, the lines representing the *Riskmetrics*^{TM} VaR and the RMD policy move together when there is no violation. As soon as a violation occurs, the DYLES strategy becomes more conservative and moves away from zero until a period of no violations occurs, when it becomes more aggressive and moves toward zero.

[Insert Figure 4 around here]

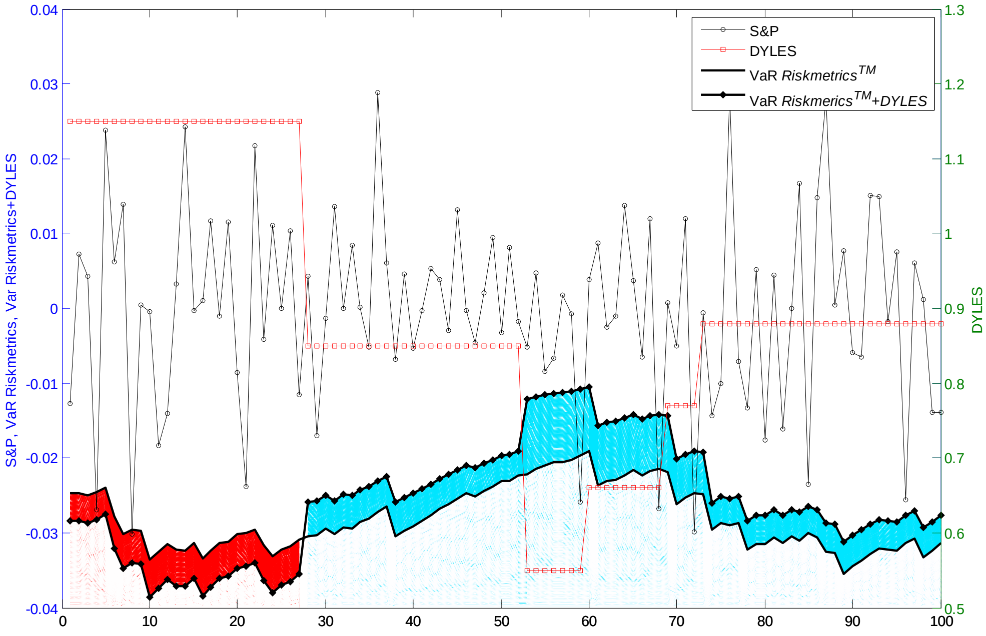
Overall, the capital requirements for DYLES are lower than when DYLES is not used by 9.5% (63 basis points). The number of violations also decreases from 12 to 8. This is a general pattern observed across different VaR models to varying degrees, as seen in Table 3 above.

This example suggests that, with proper calibration, the DYLES strategy can help decrease daily capital requirements while restricting the number of violations to within the desired limits.

Tables 4-7 in the Appendix show the NoV and AvCRq for values of Θ when two conditions are met: (i) NoV is less than 10; and (ii) AvCRq is less than 6.00%. The first criterion is consideredbecause we want our MRD to be sound for the monetary authority, while the second considers a reduction in the AvCRq by 9% (61 basis points) when compared with the *Riskmetrics*^{TM} procedure.

Figure 3. S&P, DYLES, VaR RiskMetrics^{TM}, VaR RiskMetrics^{TM} + DYLES

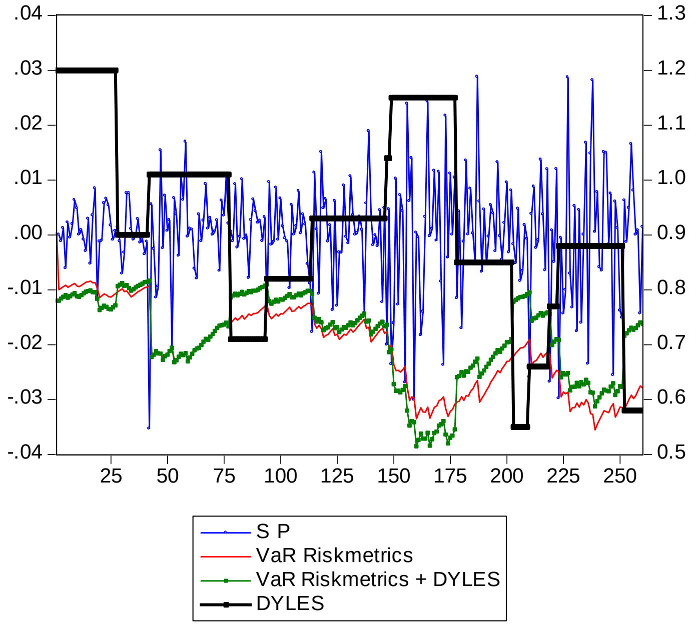
Last 100 Observations Θ = [1.2, 0.12, 0.3]



The red-area means *Riskmetrics^{TM} beats DYLES and the blue-area means DYLES beats no strategy*

Figure 4

Standard&Poor's / VaR *Riskmetrics*^{TM} / VaR *Riskmetrics*^{TM} + DYLES / DYLES
260 observations. Θ = [1.2, 0.12, 0.3].



Based on Tables 4-7 and the previous analysis, we conclude that there are combinations of parameters in Θ that can reduce the daily capital requirements compared with existing models and strategies, while producing an acceptable number of violations. It would seem to be straightforward to find a parameter vector Θ with a systematically smaller AvCRq and NoV below 10 for all the models, when compared with the no strategy policy.

It is noteworthy that, while VaR is used by numerous financial institutions, it is not without shortcomings. The VaR measure can under or overestimate risk. There is even debate as to how best to model the behaviour of volatility in market returns. Relying on DYLES, which modulates market risk disclosure, can reduce the effects of these deficiencies, as DYLES concentrates NoV throughout the models tested. In some of the cases discussed above, DYLES can control the number of violations at low cost in terms of the daily capital requirements.

6. Conclusion

Under the Basel II Accord, ADIs have to communicate their risk estimates to the monetary authorities, and use a variety of VaR models to estimate risks. ADIs are subject to a back-test that compares the daily VaR to the subsequent realized returns, and ADIs that fail the back-test can be subject to the imposition of standard models that can lead to higher daily capital costs. Additionally, the Basel II Accord stipulates that the daily capital charge that the bank must carry as protection against market risk must be set at the higher of the previous day's VaR or the average VaR over the last 60 business days, multiplied by a factor *k.* An ADI's objective is to maximize profits, so they wish to minimize their capital charges while restricting the number of violations in a given year below the maximum of 10 allowed by the Basel II Accord.

Alternative VaR models currently in use can lead to high daily capital requirements or an excessive number of violations. In this paper we formulated the profit maximizing problem of an ADI subject to capital requirements under the Basel II Accord as ADI's are required to choose an optimal reporting strategy that may strategically under-report or over-report their VaR forecasts in order to minimize daily capital charges. We also proposed a new dynamic learning strategy, DYLES, designed to minimize the daily capital requirements, while restricting the number of violations to below the penalty limit. We designed a market risk disclosure strategy driven by the number of violations to communicate the risk to the monetary authority. The strategy is context sensitive, and depends on the history of violations. It is intended to penalize VaR models when a loss exceeds the reported VaR by increasing the risk for the following periods. On the other hand, after a given period with no violations, the criterion offers a reward by decreasing the reported risk.

In order to illustrate the practicability of DYLES, we applied it to the Standard and Poor 500 Index using seven different VaR models. After estimation of the VaR models and calibration of the parameters, we showed that it could lower the daily capital requirements substantially (by up to 14.3%, or 95 basis points, when we compared, for example, the GJR-*t* model + DYLES to the no strategy *Riskmetrics*^{TM} policy), while restricting the numbers of violations to within the Basel II Accord limits (9 for GJR-*t* + DYLES and 12 for the no strategy *Riskmetrics*^{TM} policy).

Simplicity would seem to have been the key to the popularity of VaR, particularly as a means of providing information to an ADI's senior management. DYLES is as simple and intuitive as VaR. When there is a violation, it increases immediately, thereby becoming more conservative and decreasing the risks of further violations, whereas after a period of no violations, it becomes less conservative, thereby allowing lower daily capital requirements.

The preceding arguments suggest that DYLES can be used profitably by ADIs to reduce their average daily capital requirements, while restricting the numbers of violations to the Basel II Accord penalty limits.

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Appendix 1

Table 1: Basel Accord Penalty Zones

Zone	Number of Violations	Increase in k
Green	0 to 4	0.00
Yellow	5	0.40
	6	0.50
	7	0.65
	8	0.75
	9	0.85
Red	10+	1.00

Note: The number of violations is given for 250 business days. The penalty structure under the Basel II Accord is specified for the number of penalties and not their magnitude, either individually or cumulatively.

Table 4

MRD results when NoV < = 10 and CRq < 6.1%

EGARCH					EGARCH-t				
NoV	**AvCRq**	**P_0**	**θ^P**	**θ^R**	**NoV**	**AvCRq**	**P_0**	**θ^P**	**θ^R**
7	5.97%	0.9	0.10	0.1	9	5.91%	0.8	0.11	0.3
9	5.91%	0.9	0.12	0.3	9	5.94%	1.1	0.06	0.2
9	6.00%	1.0	0.10	0.3	9	5.95%	1.1	0.11	0.3
9	5.70%	1.2	0.06	0.2	9	5.87%	1.2	0.09	0.3
9	5.91%	1.2	0.07	0.2	8	5.70%	1.2	0.11	0.3
7	5.93%	1.2	0.10	0.2	7	5.86%	1.2	0.12	0.3
9	5.94%	1.2	0.12	0.3					

Table 5

MRD results when NoV < = 10 and CRq < 6.1%

GARCH					GARCH-t				
NoV	**AvCRq**	**P_0**	**θ^P**	**θ^R**	**NoV**	**AvCRq**	**P_0**	**θ^P**	**θ^R**
9	5.89%	0.8	0.11	0.3	7	6.00%	0.9	0.11	0.2
9	5.93%	1.1	0.11	0.3	9	5.80%	1.0	0.11	0.3
8	5.85%	1.2	0.11	0.3	8	5.87%	1.1	0.11	0.3

Table 6

MRD results when NoV < = 10 and CRq < 6.1%

GJR					**GJR-t**				
NoV	**AvCRq**	**P_0**	**θ^P**	**θ^R**	**NoV**	**AvCRq**	**P_0**	**θ^P**	**θ^R**
9	5.85%	0.6	0.11	0.2	8	6.00%	0.6	0.10	0.2
9	5.92%	1.1	0.06	0.2	8	5.90%	1.0	0.07	0.2
9	5.92%	1.1	0.12	0.3	9	5.66%	1.0	0.12	0.3
9	5.87%	1.2	0.11	0.3	9	5.90%	1.1	0.10	0.3

Table 7

MRD results when NoV < = 10 and CRq < 6.1%

Riskmetrics™				
NoV	**AvCRq**	**P_0**	**θ^P**	**θ^R**
9	5.81%	0.8	0.11	0.3
8	5.76%	1.2	0.11	0.3
8	5.98%	1.2	0.12	0.3