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Role of Relative and Absolute Performance Evaluations in Intergroup Competition[1]

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Abstract

We investigate the moral hazard problem in which a principal delegates multiple tasks to multiple workers. The principal imperfectly monitors their action choices by observing the public signals that are correlated with each other through a macro shock. He divides the workers into two groups and makes them compete with each other. We show that when the number of tasks is sufficiently large, relative performance evaluation between the groups accompanied with absolute performance evaluation results in eliminating unwanted equilibria. In this case, any approximate Nash equilibrium nearly induces the first-best allocation.

Keywords: Multitask Agency, Moral Hazard, Group Incentives, Relative and Absolute Performance Evaluations, Uniqueness

JEL Classification Numbers: D20, D80, J33, L23

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1. Introduction

We investigate the agency problem in which a principal delegates multiple tasks to multiple workers. The principal is faced with a moral hazard problem as he is unable to directly observe the workers' action choices. He can only imperfectly monitor them by observing the public signal for each task that is drawn randomly and is dependent on the action choice for this task. The public signals are correlated with each other through a randomly drawn macro shock; this shock is realized neither by the workers nor by the principal. Moreover, the public signals depend not only on this common macro shock but also on their respective private shocks; these shocks, too, are observed neither by the workers nor by the principal. We assume conditional independence in that given the occurrence of a macro shock, the public signals are drawn randomly and independently.

In order to incentivize the workers to make desirable action choices, the principal divides them into two groups and makes these groups compete with each other. We assume that the members of each group agree to jointly make their action choices within this group and maximize the sum of their expected payoffs. The principal regards these groups, and not the individual workers, as the agents with whom he makes contracts. This paper shows that it is easier to incentivize groups rather than make contracts with individual workers; the establishment of competing groups is an effective method that enables the principal to resolve the moral hazard problem.

We specify a contract based on a combination of the concepts of relative and absolute performance evaluations, which is dependent on observed public signals as a punishment rule for each group. Each group's performance is measured by the proportion of its tasks for which "good" public signals are observed. If a group's performance is unsatisfactory as compared with that of the other group, the principal will fine the former based on relative performance evaluation. If the group's performance is almost identical to that of the other group but not sufficiently satisfactory in the absolute sense, the principal will fine this group based on absolute performance evaluation. In this case, it is important to note that even though a group's performance is unsatisfactory in the absolute sense, the principal will not fine this group if it performs sufficiently better than the other group.

The relative and absolute performance evaluation concepts are used in combination, particularly when the number of tasks is sufficiently large. According to the law of large numbers, private shocks for tasks delegated to a group can be cancelled out. Thus, if no macro shocks exist, the principal can determine if a group has deviated from the desirable action choices just by evaluating its performance in an absolute sense independently. However, if an unobservable macro shock exists, the principal needs to evaluate performances in a relative sense through intergroup comparison. On the basis of relative performance evaluation, the principal can detect, almost perfectly, if a group has deviated from desirable action choices, as long as the other group makes such choices. Thus, the groups are incentivized to make the desirable action choices as an approximate Nash equilibrium, where each group's gains from deviation are either negligible or less than zero.

Significantly, the establishment of competing groups in this manner makes it easy to eliminate unwanted equilibria. Suppose that both groups deviate from the desirable action choices for a non-negligible number of tasks. In this case, each group has an incentive to perform slightly better than the other group; the group can almost certainly escape punishment based on absolute performance evaluation. Crucially, this property relies on the fact that, unlike in the case of an individual worker with a single task, a group with a sufficient number of tasks can clearly inform the principal of its better performance merely by completing a slightly greater proportion of tasks with desirable action choices than the other group. Consequently, a group can help the principal to detect the other agent's deviation. This implies that unique implementation is virtually possible; that is, any approximate Nash equilibrium would induce the groups to make desirable action choices in almost all tasks.

Several previous works, such as Holmstrom (1982), Lazear and Rosen (1981), Green and Stokey (1983), and Nalebuff and Stiglitz (1983), have studied relative performance evaluation in the context of the moral hazard problem. These studies generally investigated cases in which each agent is delegated a single task.[3] They showed that, in comparison with independent absolute evaluation, relative performance evaluation provides for better risk sharing when there exists an unobservable macro

3 An exception is Franckx, D'Amato, and Brose (2004), which extended Lazear and Rosen (1981) to a multitask setting. See also Battaglini (2005).

shock. However, the presence of private shocks in addition to the macro shock generally prevents relative performance evaluation from approximately achieving the first-best allocation. In contrast, this paper shows that relative performance evaluation can nearly achieve the first-best allocation as an approximate Nash equilibrium when the number of tasks is sufficiently large; this is because private checks can be cancelled out. This result is robust with respect to limited liability constraints; whenever the upper bound of the monetary fine is large enough to incentivize the agents such that the principal can perfectly monitor their action choices, then the principal can generally incentivize them even when faced with monitoring imperfection.[4]

Several works such as Mookherjee (1984), Demski and Sappington (1984), Ma (1988), and Battaglini (2006 have investigated the uniqueness in implementation with a moral hazard; when each agent is delegated only a single task, relative performance evaluation accompanied with absolute performance evaluation does not necessarily function for unique implementation; these papers demonstrated alternative concepts of mechanism design to eliminate unwanted equilibria.[5] In contrast, this paper shows that if an agent (as a collection of many workers) is delegated a sufficient number of tasks, relative performance evaluation can compel each agent to blow the whistle with regard to the other agent's deviation in exchange for an exemption from punishment based on absolute performance evaluation. Thus, this can be the driving force behind relative and absolute performance evaluations and would enable the principal to eliminate any unwanted equilibria.

In agency literature, several studies such as Varian (1990), Holmstrom and Milgrom (1990), and Itoh (1993) have analyzed cases in which there exist multiple workers and have demonstrated the superiority of group decisions over individual decisions. In these studies, it is assumed that the members of each group mutually observe their action choices and design a side contract contingent on these choices; this contract is enforceable through non-judicial channels such as word of honor. Tirole (1992)

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[4] Legros and Matsushima (1991) investigated the moral hazard problem in general partnerships with limited liability.

[5] Some of these works are related to the concepts of mechanism design, explored in the adverse selection literature on the implementation of social choice functions. See Abreu and Matsushima (1992), Moore (1992), Palfrey (1992), Osborne and Rubinstein (1994, Chapter 10), and Maskin and Sjöström (2002).

explains the manner in which hidden side-contracting technology can be specified. Generally, these works studied only the behavior of a single group that includes all workers; nevertheless, if a macro shock occurs, even this group would have an incentive to deviate. In contrast, this paper examines a case in which there exist two separate groups that compete with each other; herein, the method of relative and absolute performance evaluations serves well, particularly when a macro shock occurs.

We also refer to previous works on multitask incentives that used the law of large numbers to cancel out private shocks. Bundling of goods by a monopolist (Armstrong, 1999), multimarket contacts (Matsushima, 2001), and linking mechanisms (Jackson and Sonnenschein, 2007 Matsushima, Miyazaki, and Yagi, 2010) are some examples.

This paper is organized as follows. Section 2 presents the model, and section 3 specifies the punishment rule. While section 4 presents the main theorem and its logical core, section 5 provides the complete proof of the theorem.

2. Model

A principal hires two agents (agents 1 and 2) and delegates $n \geq 1$ number of tasks, that is, tasks $(i,1)$, $(i,2)$, …, (i,n), to each agent $i \in \{1,2\}$, where we regard each agent as a collection of n individual workers (i.e., a group), as will be explained later. Each agent $i \in \{1,2\}$ selects a *strategy* $a_i = (a_{i,h})_{h=1}^{n}$, where $a_{i,h}$ implies the *action* for task (i,h). Let $A_{i,h} \equiv \{0,1\}$ denote the set of all actions for task (i,h). With regard to task (i,h), $a_{i,h} = 1$ implies the *desirable* action, whereas $a_{i,h} = 0$ implies the *undesirable* action. Let $A_i = \underset{h\in\{1,...,n\}}{\times} A_{i,h}$ denote the set of all strategies for agent i. Let $A = A_1 \times A_2$ denote the set of all strategy profiles. Let $a^n = a = (a_1, a_2) \in A$ denote a strategy profile. The *desirable strategy profile* is defined as $a^{n*} = a^* = (a_1^*, a_2^*) \in A$, where

$$a_{i,h}^* = 1 \text{ for all } i \in \{1,2\} \text{ and all } h \in \{1,...,n\} .$$

The principal is faced with a moral hazard problem in which he cannot observe the action choices but can only imperfectly monitor them by observing the public signal

$\omega_{i,h}$ for each task (i,h). Let $\Omega_{i,h} \equiv \{0,1\}$ denote the set of possible public signals for task (i,h). The public signal $\omega_{i,h} \in \Omega_{i,h}$ for each task (i,h) is randomly drawn after the agents make their action choices, the realization of which is dependent on the action choice $a_{i,h}$ for this task; $\omega_{i,h} = 1$ implies the *good* signal, whereas $\omega_{i,h} = 0$ implies the *bad* signal. Let $\Omega_i \equiv \underset{h \in \{1,\ldots,n\}}{\times} \Omega_{i,h}$ denote the set of possible public signal profiles for agent $i's$ tasks. Let $\Omega \equiv \Omega_1 \times \Omega_2$ denote the set of possible public signal profiles.

The public signals are imperfectly correlated across all tasks; there exists a macro shock θ that is observed by neither the agents nor the principal. It is randomly drawn according to the probability density distribution $f(\theta)$ in the interval $[0,1]$, where $f(\theta) > 0$ for all $\theta \in [0,1]$ and $\int_{\theta=0}^{1} f(\theta) d\theta = 1$. We assume that there exist a real number $\alpha \geq 1$ and an increasing and continuous function $p : [0, 1+\alpha] \to [0,1]$ such that for each $(i,h) \in \{1,2\} \times \{1,\ldots,n\}$, $p(a_{i,h} + \alpha\theta)$ is the probability that the good signal $\omega_{i,h} = 1$ for task (i,h) will be observed, provided the macro shock θ occurs and agent i selects action $a_{i,h}$ for this task. Hence, the public signals are correlated with each other through this macro shock. Since p is increasing with respect to θ, it follows that the stronger the macro shock θ, the better it is for each task in the business. Since $p(1+\alpha\theta) > p(\alpha\theta)$ for all $\theta \in [0,1]$, it follows that when a desirable action is chosen the probability of occurrence of a good signal for the task would be greater than if an undesirable action were chosen. Since the principal is unable to observe the occurred macro shock and the chosen strategy profile, he is unable to verify whether the occurrence of good signals for the tasks was due to the agent’s choice of desirable actions or the occurrence of a strong macro shock.[6] We assume conditional independence in that given the occurrence of a macro shock the public signals are drawn randomly and independently of each other. This implicitly assumes that there exists some private shock for each task that is drawn randomly and independently of each

[6] The assumption of $\alpha \geq 1$ makes the incentive problem of this paper non-trivial. If $\alpha < 1$, then for every $\theta \in [0,1]$, there exists no $\theta' \in [0,1]$ such that $p(1+\alpha\theta) = p(\alpha\theta')$; the principal can detect whether or not an agent deviated just by verifying the probability that the good signal will occur. This makes the incentive problem much easier to solve.

other and that influences the realization of the public signal.

This paper examines the case in which the principal delegates a large number of tasks to each agent; in order that desirable action choices are made for all the tasks, the principal hires $2n$ workers, divides them into two *groups* with the same number of workers, and regards each group as an agent with which he makes a contract. In this case, the members of each group enter into a binding agreement to jointly make action choices for the n tasks that the principal delegates and to jointly maximize the sum of their expected payoffs. On the basis of this setting, the *payoff* for each agent $i \in \{1,2\}$ when this agent selects a strategy $a_i \in A_i$ and receives a monetary transfer $t_i \in R$ is given by

$$u(t_i) - \frac{1}{n}\sum_{h=1}^{n} a_{i,h} ,$$

where $u : R \to R$ is an increasing function, and $\frac{1}{n} a_{i,h}$ implies the cost of selecting the action for task (i,h); the desirable action choice is costlier than the undesirable action choice. Without loss of generality, we assume that

$$u_i(0) = 0 \text{ for each } i \in \{1,2\} .$$

In order to incentivize the agents to select the desirable strategy profile a^*, the principal will make a contract with each agent $i \in \{1,2\}$ as a *punishment rule* $x_i : \Omega \to [-H, 0]$, where $H > 0$ implies the upper bound of the monetary fine; agent i is fined a monetary amount $-x_i(\omega) \in [0, H]$ when the public signal profile $\omega \in \Omega$ occurs. Given that a strategy profile $a \in A$ is selected, the expected payoff for agent i is defined by

$$v_i(a; x_i) \equiv \sum_{\omega \in \Omega} u_i(x_i(\omega)) p(\omega \mid a) - \frac{1}{n}\sum_{h=1}^{n} a_{i,h} ,$$

where $p(\omega \mid a)$ denotes the probability that a public signal profile ω occurs when a strategy profile a is chosen; thus,

$$p(\omega \mid a) \equiv \int_{\theta=0}^{1} \prod_{\substack{(i,h)\in\{1,2\}\times\{1,\dots,n\}: \\ \omega_{i,h}=0}} \{1 - p(a_{i,h} + \alpha\theta)\} \prod_{\substack{(i,h)\in\{1,2\}\times\{1,\dots,n\}: \\ \omega_{i,h}=1}} p(a_{i,h} + \alpha\theta) f(\theta) d\theta .$$

Let $x = (x_1, x_2)$ denote a punishment rule profile.

The solution concept used in this paper is the *approximate Nash equilibrium*; for each positive real number $\varepsilon > 0$, a strategy profile $a \in A$ is said to be an $\varepsilon -$*Nash equilibrium* for a punishment rule profile x if, for every $i \in \{1,2\}$ and every $a_i' \in A$,

$$v_i(a;x_i) \geq v_i(a_i', a_j; x_i) - \varepsilon ,$$

where $j \neq i$; for each agent, the gain from deviating from an $\varepsilon -$Nash equilibrium is less than or equal to ε, provided the other agent follows this $\varepsilon -$Nash equilibrium.

This paper aims to design a punishment rule profile for which the desirable strategy profile a^* is an approximate Nash equilibrium. We also show the uniqueness-like property such that every approximate Nash equilibrium induces the desirable action choices for almost all tasks and rarely fines the agents.

3. Relative and Absolute Performance Evaluations

Arbitrarily fix a positive integer n and a positive integer $\lambda = \lambda(n) \in \{1,...,n\}$. According to a combined concept of relative and absolute performance evaluations, we specify a punishment rule profile $x^n = x = (x_1, x_2)$ as follows; for every $i \in \{1,2\}$,

(1) $\quad x_i(\omega) = -H \quad$ if $\sum_{h=1}^{n} \omega_{i,h} \leq \sum_{h=1}^{n} \omega_{j,h} - \lambda$,

(2) $\quad x_i(\omega) = -H \quad$ if $\left| \sum_{h=1}^{n} \omega_{i,h} - \sum_{h=1}^{n} \omega_{j,h} \right| < \lambda$ and $\sum_{h=1}^{n} \omega_{i,h} \leq np(1)$,

and

(3) $\quad x_i(\omega) = 0 \quad$ otherwise,

where $j \neq i$. If the absolute value of difference between the numbers of good signals for agent $i's$ and agent $j's$ tasks is greater than or equal to λ, that is,

$$\left| \sum_{h=1}^{n} \omega_{1,h} - \sum_{h=1}^{n} \omega_{2,h} \right| \geq \lambda ,$$

then the principal will assess each agent's performance by *relative performance evaluation*; if the good signals for agent $i's$ tasks are fewer than those for agent $j's$ tasks such that

$$\sum_{h=1}^{n}\omega_{j,h}-\sum_{h=1}^{n}\omega_{i,h}\geq\lambda,$$

then agent i, but not agent j, is fined a monetary amount H.

If the absolute value of difference between the number of good signals for agent $i's$ tasks and that of agent $j's$ tasks is less than λ, that is,

$$\left|\sum_{h=1}^{n}\omega_{1,h}-\sum_{h=1}^{n}\omega_{2,h}\right|<\lambda,$$

then the principal will evaluate each agent's performance by *absolute performance evaluation*; consider $np(1)$ as the *threshold* to determine whether an agent should be fined or not in the absolute sense, where $p(1)$ implies the probability that good signal will occur for a task, provided the desirable action is chosen and the weakest macro shock $\theta=0$ occurs. If the number of good signals for agent $i's$ tasks is less than or equal to this threshold, that is,

(4) $$\sum_{h=1}^{n}\omega_{i,h}\leq np(1),$$

then agent i is fined a monetary amount H; if the number of good signals for agent $i's$ tasks is greater than this threshold, he is not fined. It is important to note that although the inequality of (4) holds, agent i is never fined if the number of his tasks that send good signals is relatively larger than the corresponding number pertaining to agent j, that is,

$$\sum_{h=1}^{n}\omega_{i,h}-\sum_{h=1}^{n}\omega_{j,h}\geq\lambda.$$

4. Theorem

We assume that $u(-H)<u(0)-\frac{1}{n}\sum_{i=1}^{n}a_i^*$. That is,

(5) $$u(-H)<-1.$$

Clearly, the assumption of (5) is a necessary condition for the principal to resolve the incentive problem. For instance, let us consider a situation in which the principal can monitor the agents' action choices almost perfectly, and he will fine any agent a monetary amount H if he detects the agent making undesirable action choices for all the tasks. In this case, in order to incentivize them, it is necessary to require that the payoff induced by desirable action choices for all the tasks with no fines (i.e., $u(0)-\frac{1}{n}\sum_{i=1}^{n}a_i^*$) be greater than the payoff induced by the undesirable action choices for all the tasks with a fine of H, that is, $u(-H)$. This implies the inequality of (5).

The following theorem shows that the assumption of (5) is not only necessary but also sufficient for unique implementation.

Theorem: *With the assumption of (5), there exists an infinite sequence of positive integers* $(\lambda(n))_{n=1}^{\infty}$ *that satisfies the following properties:*

(i) $\lambda(n)\in\{1,...,n\}$ *for all* $n\geq 1$*;*

(ii) *For every* $\varepsilon>0$, *there exists* $\bar{n}$ *such that for every* $n\geq\bar{n}$ *and every* $i\in\{1,2\}$, *whenever* a^{n*} *is selected, the probability of* $x_i^n(\omega)=-H$ *is less than* ε*;*

(iii) *For every* $\varepsilon>0$, *there exists* $\bar{n}$ *such that for every* $n\geq\bar{n}$, a^{n*} *is a* $\varepsilon-$*Nash equilibrium for* x^n*;*

(iv) *For every* $\eta>0$, *there exist* $\varepsilon>0$ *and* $\bar{n}$ *such that for every* $n\geq\bar{n}$, *there exists no* $\varepsilon-$*Nash equilibrium* $a^n=a$ *for* x^n *that satisfies*

$$\frac{1}{n}\sum_{h=1}^{n}a_{i,h}\leq 1-\eta \text{ for some } i\in\{1,2\}.$$

The theorem states that if the number of tasks is sufficiently large, then a^* is an approximate Nash equilibrium; moreover, every approximate Nash equilibrium induces agents to make desirable action choices for almost all tasks and rarely fines them. Hence, the principal succeeds in achieving desirable action choices for all tasks.

Although the complete proof of this theorem is presented in the next section, a brief outline is presented here. Consider a sufficiently large n. The law of large numbers implies that when each agent $i\in\{1,2\}$ selects a_i^*, it is almost certain that irrespective of θ, the average of the signals relevant to agent i (i.e., $\frac{1}{n}\sum_{h=1}^{n}\omega_{i,h}$) is approximated by $p(1+\theta)$; when the agents select a^*, it is almost certain that

$\frac{1}{n}\sum_{h=1}^{n}\omega_{1,h}$ and $\frac{1}{n}\sum_{h=1}^{n}\omega_{2,h}$ are nearly the same. Hence, it is almost certain that these agents will not be fined according to relative performance evaluation. Moreover, it is almost certain that $\frac{1}{n}\sum_{h=1}^{n}\omega_{i,h}$ is greater than $p(1)$ for each $i \in \{1,2\}$; given that $\frac{1}{n}\lambda(n)$ is close to zero, the agents will not be fined according to absolute performance evaluation, either. Hence, with a sufficiently large n, $a^* = a^{n*}$ almost certainly induces $x_i(\omega) = 0$ for each $i \in \{1,2\}$; that is, property (ii) holds.

Let us arbitrarily fix $\varepsilon \in (0,1)$. When each agent $i \in \{1,2\}$ selects the undesirable action 0 for approximately $n\varepsilon$ tasks, it is almost certain that irrespective of θ, $\frac{1}{n}\sum_{h=1}^{n}\omega_{i,h}$ is close to the value of $p(1+\theta) - \varepsilon\{p(1+\theta) - p(\theta)\}$, which is less than $p(1+\theta)$ by the positive value $\varepsilon\{p(1+\theta) - p(\theta)\}$. Hence, relative performance evaluation almost certainly detects agent $i's$ deviation, as long as the other agent $j \neq i$ adopts a_j^*. Based on this observation, along with the inequality of (5), we can show that a^* is an approximate Nash equilibrium; that is, property (iii) holds.

Property (iv) is the main contribution of this paper. Let us consider any strategy profile a, according to which an agent selects undesirable actions for a non-negligible number of tasks. If a sufficiently weak macro shock occurs, it is almost certain that

$$\sum_{h=1}^{n}\omega_{i,h} < np(1).$$

Hence, some agents are fined on the basis of absolute performance evaluation with a positive probability. On the other hand, the law of large numbers implies that if an agent can alter the proportion of desirable action tasks performed by the agent so that such tasks slightly outnumber similar tasks by the other agent, then the former can almost certainly avoid being fined on the basis of absolute performance evaluation. This contradicts the approximate Nash equilibrium concept. Hence, we can show that any approximate Nash equilibrium induces agents to make desirable action choices for almost all tasks, and they are rarely fined. This is property (iv).

This paper did not consider the possibility that agents overwork. However, it is easy to resolve this issue by modifying the specification of the punishment rule profile such that each agent is fined whenever the proportion of his tasks for which good public signals occur is sufficiently greater than $p(1+\alpha)$. Moreover, we must note that the theorem depends on the implicit assumption that the range of possible macro shocks θ has an upper bound. In this case, for any macro shock θ that is close to the upper bound, there exists no θ' such that $p(1+\alpha\theta) = p(\alpha\theta')$. The proof of the theorem does use this property.

5. Proof of the Theorem

The law of large numbers implies that irrespective of θ, with a sufficiently large n, it is almost certain that for each $i \in \{1,2\}$, the average of the signals for agent $i's$ tasks, (i.e., $\frac{1}{n}\sum_{h=1}^{n}\omega_{i,h}$) is approximated by

$$\frac{1}{n}\{p(1+\alpha\theta)\sum_{h=1}^{n}a_{i,h} + p(\alpha\theta)(n-\sum_{h=1}^{n}a_{i,h})\},$$

provided agent i selects a_i and macro shock θ occurs. Hence, it is almost certain that the difference between the numbers of good signals for agent $i's$ and agent $j's$ tasks (i.e., $\frac{1}{n}\sum_{h=1}^{n}\omega_{i,h} - \frac{1}{n}\sum_{h=1}^{n}\omega_{j,h}$) is approximated by

$$\frac{1}{n}\{p(1+\alpha\theta) - p(\alpha\theta)\}(\sum_{h=1}^{n}a_{i,h} - \sum_{h=1}^{n}a_{j,h}),$$

provided the agents select a and macro shock θ occurs. Hence, we can select $(\lambda(n))_{n=1}^{\infty}$ that satisfies property (i) and the following properties.

(iv) $\lim_{n\to\infty}\frac{1}{n}\lambda(n) = 0$, and

(v) the probability of $\left|\sum_{h=1}^{n}\omega_{1,h} - \sum_{h=1}^{n}\omega_{2,h}\right| < \lambda(n)$ when the agents select a^{n*} converges to unity as n increases.

Moreover, for each $i \in \{1,2\}$,

(vi) the probability of $\sum_{h=1}^{n}\omega_{i,h} > np(1)$ when agent i selects a_i^{n*} converges to unity as n increases.

Property (v) implies that it is almost certain that agents are never fined according to relative performance evaluation. Property (vi) implies that it is almost certain that agents are never fined according to absolute performance evaluation, either. Hence, for a sufficiently large n, a^{n*} almost certainly induces $x_i(\omega)=0$ for each $i\in\{1,2\}$; that is, property (ii) holds.

Let us arbitrarily fix $\varepsilon > 0$. We prove property (iii) as follows. Suppose that there exists $a_i \neq a_i^*$ such that

(6) $$v_i(a_i, a_j^*) > v_i(a^*) + \varepsilon .$$

In this case, the number of tasks for which agent i makes undesirable action choices must be greater than $n\varepsilon$; that is,

$$n - \sum_{h=1}^{n} a_{i,h} > n\varepsilon .$$

Hence, irrespective of θ, it is almost certain that $\frac{1}{n}\sum_{h=1}^{n}\omega_{j,h} - \frac{1}{n}\sum_{h=1}^{n}\omega_{i,h}$ is approximated by

$$\frac{1}{n}\{p(1+\alpha\theta) - p(\alpha\theta)\}(n - \sum_{h=1}^{n} a_{i,h}),$$

which is greater than

$$\{p(1+\alpha\theta) - p(\alpha\theta)\}\varepsilon > 0 .$$

This, along with the fact that $\frac{1}{n}\lambda(n)$ is close to zero, implies that it is almost certain that

$$\frac{1}{n}\sum_{h=1}^{n}\omega_{j,h} - \frac{1}{n}\sum_{h=1}^{n}\omega_{i,h} \geq \frac{1}{n}\lambda(n),$$

and, therefore, agent i is almost certainly fined according to relative performance evaluation. Hence, $u_i(a_i, a_j^*) - u_i(a^*)$ is approximated by

$$u_i(-H) - \frac{1}{n}\sum_{h=1}^{n} a_{i,h} - \{u_i(0) - 1\},$$

which is negative, because of inequality (5). This contradicts (6). Hence, we have proved that with a sufficiently large n, a^{n*} is an $\varepsilon-$Nash equilibrium.

We prove property (iv) as follows. Since $\alpha \geq 1$ and $p(\cdot)$ are continuous and increasing, it follows that for every $\eta > 0$, there exists $\theta = \theta^*(\eta) > 0$ such that

$$p(1) = \eta p(\alpha\theta) + (1-\eta) p(1+\alpha\theta).$$

Let us arbitrarily fix $\eta > 0$. From the inequality of (5), we can select $\varepsilon > 0$ such that

(7) $$\min[-u_i(-H)F(\theta^*(\eta)), -u_i(-H)-1] > 3\varepsilon,$$

where we denote $F(\theta) = \int_{\tau=0}^{\theta} f(\tau)d\tau$. For each $i \in \{1,2\}$, let us consider any strategy profile a such that

$$\frac{1}{n}\sum_{h=1}^{n} a_{i,h} \leq 1-\eta \text{ and } \sum_{h=1}^{n} a_{i,h} \leq \sum_{h=1}^{n} a_{j,h}.$$

From the definition of $\theta^*(\eta)$, it follows that when a macro shock weaker than $\theta^*(\eta)$ occurs, it is almost certain that, for a sufficiently large n,

$$\sum_{h=1}^{n} \omega_{i,h} \leq \min\left[np(1), \sum_{h=1}^{n} \omega_{j,h} + \lambda(n)\right].$$

This implies that the probability of agent i being fined is greater than $F(\theta^*(\eta))$ in approximation.

Suppose that

$$0 \leq \sum_{h=1}^{n} a_{j,h} - \sum_{h=1}^{n} a_{i,h} < n\varepsilon.$$

When agent i selects another strategy $\tilde{a}_i \neq a_i$ such that $\frac{1}{n}(\sum_{h=1}^{n} \tilde{a}_{i,h} - \sum_{h=1}^{n} a_{i,h})$ is approximated by 2ε, for a sufficiently large n, it is almost certain that $\sum_{h=1}^{n} \omega_{i,h} - \sum_{h=1}^{n} \omega_{j,h} \geq \lambda(n)$; therefore, agent i is almost certainly never fined. Hence, $u_i(\tilde{a}_i, a_j) - u_i(a)$ is at least approximated by

$$-u_i(-H)F(\theta^*(\tilde{\eta})) - 2\varepsilon,$$

which is greater than ε, because of inequality (7). This implies that a is not an $\varepsilon-$Nash equilibrium.

Next, suppose that

$$\sum_{h=1}^{n} a_{j,h} - \sum_{h=1}^{n} a_{i,h} \geq n\varepsilon .$$

Irrespective of θ, for a sufficiently large n, it is almost certain that $\frac{1}{n}\sum_{h=1}^{n}\omega_{j,h} - \frac{1}{n}\sum_{h=1}^{n}\omega_{i,h}$ is at least approximated by

$$\{p(1+\alpha\theta) - p(\alpha\theta)\}\varepsilon ,$$

which is a positive value. This, along with property (i), implies that it is almost certain that $\frac{1}{n}\sum_{h=1}^{n}\omega_{i,h} - \frac{1}{n}\sum_{h=1}^{n}\omega_{j,h} \leq -\lambda(n)$; therefore, agent i is almost certainly fined. When agent i selects a_i^* instead of a_i, for a sufficiently large n, it is almost certain that he is never fined. Hence, $v_i(a_i^*, a_j) - v_i(a)$ is approximated by $-u_i(-H) - 1 + \frac{1}{n}\sum_{h=1}^{n} a_{i,h}$, where

$$-u_i(-H) - 1 + \frac{1}{n}\sum_{h=1}^{n} a_{i,h} \geq -u_i(-H) - 1 ,$$

which is greater than ε because of inequality (7). This implies that a is not an $\varepsilon-$Nash equilibrium. Hence, we have proved that with a sufficiently large n, there exists no $\varepsilon-$Nash equilibrium a such that $\frac{1}{n}\sum_{h=1}^{n} a_{i,h} \leq 1-\eta$; that is, property (iv) holds.

Q.E.D.

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