CARF-F-032

Style Analysis Based on a General State Space Model and Monte Carlo Filter

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April 2005

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Abstract

This paper proposes a new approach to style analysis by utilizing a general state space model and Monte Carlo filter. In particular, We regard coefficients of style indices as state variables in the state space model and apply Monte Carlo filter as estimation method. Moreover, an empirical analysis using actual funds' data confirms the validity of our approach.

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1 Introduction

We propose a new framework for style analysis based on a general state space model and Monte Carlo filter. To our knowledge, this work is the first application of a general state space model and Monte Carlo filter to the estimation of mutual funds' styles while the application to finance was successfully implemented in the term structure models by Takahashi and Sato(2001).

The state space model consists of the system model describing the processes of state variables and the observation model representing the functional relation between state variables and the observational data in the real world. It is also possible to introduce measurement errors in the general way without any bias. In style analysis, we assume that a mutual fund can be approximatedly regarded as a portfolio of given style indices with their long positions; the coefficients of style indices represent the weights of the indices in the portfolio. Moreover, the coefficients are often varying over time. Hence, we need a model capturing dynamic change in non-negative weights of style indices.

We propose an approach based on the general state space model and Monte Carlo filter to satisfy those features. Once we regard the coefficients of styles as state variables and regard a fund's return and style indices as observations, our approach can be naturally applied to style analysis.

The paper is organized as follows. In the next section, we will briefly explain the exsisting methods of style analysis. In section three, first we will summarize the outline of the state space model. Then, we will explain style analysis in the framework of state space model. Third, we show several concrete examples. In section four, we will test the validity of our method by using actual data of Japanese mutuals fund and style indices. In section five, we will make concluding remarks. Appendix shows an algorithm of Monte Carlo filter.

2 Style Analysis by Exsisting Methods

We assume that there are n style indices and a return r_t of a portfolio is expressed as

$$r_t = \sum_{i=1}^{n} \beta_{it} I_{it} + u_t. \tag{1}$$

In the equation, β_{it} represents the coefficient of style index i, I_{it} denotes a return of a style index i and u_t is a residual. Estimaitng the coefficients β_{it}, $i = 1, \cdots, n$ by using observational data is main objective for style analysis. If each β_{it} is supposed to be invariant over time that is $\beta_{it} = \beta_i$, the problem is reduced to a regression analysis under constraints that all β_i are non-negative and the sum of β_i $i = 1, \cdots, n$ is equal to one; that is $\beta_i \geq 0$ for all $i = 1, \cdots, n$ and $\sum_{i=1}^{n} \beta_i = 1$. These constraints correspond to the assumption that the fund can be regarded as a portfolio of n style indices with their long positions. Then, the coefficients can be estimated by a least square method with constraints. This approach was initiated by Sharpe(1992). His framework is widely used in the practical world as well as in academic research; for instance, see Busse(1999), Fung and Hsieh(1997) and Chan, Chen and Lakonishok(2002). From a different point of view, Brown and Goetzmann(1997) proposed a new clustering method for style analysis.

It is difficult to apply these models with time-invariant weights to actual funds' data because a portfolio manager dynamically change the portfolio weights as argued by Grinblatt, Titman and Wermers(1995) and Ferson and Schadt(1996). Although Sharp(1992) tried to estimate the coefficients by using a window regression in order to capture dynamic variation of the weights of indices it is hard task to determine a optimal width of the window. Swinkels and van der Sluis (2002) presented an application of Kalman filter for the estimation of time-varing weights. However, they neglect non-negative constraint of weights. Moreover, it is almost impossible to trace sudden changes in the weights by those existing methods.

In order to overcome these problems, we propose a new framework for style analysis based on a general state space model and present an original estimation method by utilizing Monte Calo filter.

3 Style Analysis Based on a General State Space Model

In this section, we first introduce a general state space model and Monte Carlo filter as an estimation method. Then, we explain style analysis in the framework of the general state space model and present several examples.

3.1 State Space Modeling

First, we give the general form of state space models. (See Kitagawa and Gersh(1996) for the detail.) A state space model consists of the following system model and the observation model. That is,

$$\begin{cases} X_t = & F(X_{t-\Delta t}, v_t) \quad \text{system model} \\ Z_t = & H(X_t, u_t) \quad \text{observation model} \end{cases} \tag{2}$$

where X_t, Z_t and Δt denote a N dimensional state vector, a M dimensional observation vector at time t and the time interval of observational data respectively while v_t and u_t denote the system noise and the observational noise whose density functions are given respectively by $q(v)$ and $\psi(u)$. F and H are in general non-linear functions of $R^N \times R^N \mapsto R^N$ and $R^N \times R^M \mapsto R^M$, and the initial state vector X_0 is assumed to be a random variable whose density function is given by $p_0(X)$. We asuume that there exists the inverse function $H^{-1} : R^M \times R^N \to R^M$ such that $u_t = H^{-1}(Z_t, X_t)$.

Further, in order to handle the cases that explicit functional relations such as F and H are not obtained, we can introduce a general state space model based on conditional distributions:

$$\begin{cases} X_t \sim & \mathbf{F}|X_{t-\Delta t} \quad \text{system model} \\ Z_t \sim & \mathbf{H}|X_t \quad \text{observation model} \end{cases} \tag{3}$$

where $\mathbf{F}|X_{t-\Delta t}$ and $\mathbf{H}|X_t$ denote conditional distributions given $X_{t-\Delta t}$ and given X_t respectively. Examples in this class will appear in subsections 3.2 and 3.3 below. Next, we consider the estimation of unobservable state variables X through observable variables Z. We note that the standard Kalman filter can not be applied to the estimation as both the system model and the observation model described above are generally non-linear, and hence we should utilize Monte Carlo filter. While several approaches are proposed for Monte Carlo filter (see Doucet, Barat, and Duvaut(1995), Durbin, and Koopman(1997), Gordon, Salmond, and Smith(1993), Tanizaki(1993) for instance.), we take the approach developed by Kitagawa(1996). In Monte Carlo filter, we approximate the conditional distribution of X_t by many particles which can be considered to be realizations from the distribution $\mathbf{F}|X_{t-\Delta t}$. Given m particles of a state vector $\{\xi_{t-\Delta}^{[1]}, \cdots, \xi_{t-\Delta}^{[m]}\}$, we can obtain one step ahead predictor from the system model as a set of particles;

$$\{p_t^{[1]}, \cdots, p_t^{[m]}\} \sim \mathbf{F}|\{\xi_{t-\Delta}^{[1]}, \cdots, \xi_{t-\Delta}^{[m]}\}$$

Then we have a filter distribution of X_t through resampling from prediction distribution $\{p_t^{[1]}, \cdots, p_t^{[m]}\}$; higher weight is assigned on $p_t^{[k]}$ which with higher probability, generates a given observation Z_t. The resulting particles denoted by $\{\xi_t^{[1]}, \cdots, \xi_t^{[m]}\}$ are regard as the filtered estimation of X_t. We repeat these steps up to T. In Appendix, we will provide a typical algorithm of the Monte Carlo filter.

3.2 Style Analysis in the State Space Model

In this subsection, we consider an application of state space modeling to style analysis. First, we consider a system model. Essentially, we regard the coefficients of style indices denoted by $\beta_{it},\ i = 1, \cdots, n$ as state variables which follow stochastic processes with constraints. Moreover, we do not model β directly, but introduce more fundamental state variables Y behind β which determines the dynamics of β. First, we define $\mathbf{R}^{n+k}$-valued state variables $X_t = (Y_t, \beta_t)'$. The $\mathbf{R}^k$-valued state variables Y_t follow

$$Y_t = f(Y_{t-\Delta t}, \beta_{t-\Delta t}, t) + v_t$$

where

$$\begin{aligned} Y_t &= (Y_{1t}, \cdots, Y_{kt}) \\ \beta_t &= (\beta_{1t}, \cdots, \beta_{nt}), \end{aligned}$$

and the system noise v_t follows a distribution of which denstiy function is given by $q(v)$. The state variables $\beta_{it},\ i = 1, \cdots, n$ are determined based on Y_t so that β_{it} satisfy the constraints:

$$\begin{aligned} &\beta_{it} \geq 0 \text{ for all } i = 1, \cdots, n \\ &\sum_{i=1}^{n} \beta_{it} = 1. \end{aligned}$$

For instance, $\beta_{it},\ i = 1, \cdots, n$ are given by

$$\beta_{it} = h_i(Y_t, t),\ \ i = 1, \cdots, n$$

where $h_i(Y_t, t),\ i = 1, \cdots, n$ are $\mathbf{R}$-valued some functions of Y_t and t so that the constraints are satisfied. A *logit transformation* is an example: $\beta_{it},\ i = 1, \cdots, n$ are determined by

$$\beta_{it} = h_i(Y_t, t) = \frac{e^{Y_{it}}}{\sum_{i=1}^{n} e^{Y_{it}}}.$$

Hence, in this case the system equation is given by

$$\begin{aligned}
&Y_t = f(Y_{t-\Delta t}, \beta_{t-\Delta t}, t) + v_t \\
&\beta_{it} = h_i(Y_t, t),\ i = 1, \cdots, n \\
&\text{where } h_i,\ i = 1, \cdots, n \text{ are chosen so that} \\
&\beta_{it} \geq 0 \text{ for all } i = 1, \cdots, n \\
&\sum_{i=1}^{n} \beta_{it} = 1 \\
&\text{the system noise } v_t \text{ with the density function } q(v).
\end{aligned} \tag{4}$$

We note that the equation (4) corresponds to $F(\cdot)$ in the equations (2) of the state space model. This system model includes the model such that the current Y_t depends not only on $Y_{t-\Delta t}$ in the previous period, but also on $\beta_{t-\Delta t}$. The constraints in the equation (4) reflect the assumption stated in the previous section that the fund is a portfolio of style indices with long positions. Those constraints can be captured in the functions $h_i(\cdot)$, $i = 1, \cdots, n$ such as *logit transformations*. We also note that β_t and Y_t are estimated by using Monte Carlo filter since they are not observable, the functions f and h are non-linear in general, and the system noise v_t may follow a non-normal distribution.

Moreover, when we try to apply models such that it is difficult to capture the required constraints as explicit functions, we can utilize a general state space framework introduced in the equations (3). Note first that the condition $\sum_{i=1}^{n} \beta_{it} = 1$ allow us to reduce the dimension of $\beta = (\beta_1, \cdots, \beta_n)$ from n to $n-1$. Next we fix some $j \in \{1, 2, \cdots, n\}$ and introduce the notation $x^{(j)}$ as a vector of which the j-th element is removed from a vector x. For example, $\beta_t^{(j)}$ is defined as

$$\beta_t^{(j)} \equiv (\beta_{1t}, \cdots, \beta_{j-1t}, \beta_{j+1t}, \cdots, \beta_{nt}).$$

Given information at $t - \Delta t$ that is, $Y_{t-\Delta t}$, $\beta_{t-\Delta t}$, Y_t is generated according to the equation;

$$Y_t = f(Y_{t-\Delta t}, \beta_{t-\Delta t}, t) + v_t$$

where Y_0 and β_0 are given. Define a set $A_t^{(j)}$ as

$$A_t^{(j)} = \{0 \leq \hat{h}_1(Y_t, t), \cdots, 0 \leq \hat{h}_{j-1}(Y_t, t), 0 \leq \hat{h}_{j+1}(Y_t, t), \cdots, 0 \leq \hat{h}_n(Y_t, t), \sum_{i \neq j} \hat{h}_i(Y_t, t) \leq 1\}$$

where $\hat{h}_i(Y_t, t)$ is some $\mathbf{R}$-valued function of Y_t and t. Then, $\beta_t^{(j)}$ is generated according to the distribution function $G(y^{(j)})$ which is defined by

$$G(y^{(j)}) = Pr(\{\hat{h}^{(j)}(Y_t, t) \leq y^{(j)}\} | A_t^{(j)}) = \frac{Pr(\{\hat{h}^{(j)}(Y_t, t) \leq y^{(j)}\} \cap A_t^{(j)})}{Pr(A_t^{(j)})}$$

where $y^{(j)} \in \mathbf{R}^{n-1}$ and

$$\hat{h}^{(j)}(Y_t, t) = (\hat{h}_1(Y_t, t), \cdots, \hat{h}_{j-1}(Y_t, t), \hat{h}_{j+1}(Y_t, t), \cdots, \hat{h}_n(Y_t, t)).$$

Finally, β_j is determined by

$$\beta_{jt} = 1 - \sum_{i \neq j} \beta_{it}.$$

Thus, β_{it}, $i = 1, \cdots, n$ are modeled satisfying the constraints; $\beta_{it} \geq 0$ and $\sum_{i=1}^{n} \beta_{it} = 1$. Further, in order to avoid any bias caused by some particular j being fixed we can introduce a model so that $j \in \{1, \cdots, n\}$ are randomly chosen with prbability $\frac{1}{n}$ at each time point t. In this way, we can express various types of the system model, some of which are shown in the following subsection.

Next, consider the observation equation. A fund return r_t is determined by

$$r_t = \sum_{i=1}^{n} \beta_{it} I_{it} + u_t$$

where I_{it}, $i = 1, \cdots, n$ represent style indices and the observation noise $(u_t)_t$ follows of which density function is given by $\psi(u)$. Here, we note that the fund return r_t and style indices X_{it}, $i = 1, \cdots, n$ are obtained as observations. Hence, the observation equation is given by

$$r_t = \sum_{i=1}^{n} \beta_{it} I_{it} + u_t \tag{5}$$

the observation noise $(u_t)_t$ with the density function $\psi(u)$.

Note that the equation (5) corresponds to $H(\cdot)$ in the equations (2) of the state space model.

3.3 Examples

In this subsection, we show several concrete examples of the system model.

1. (i) Let $k = n$, and hence $Y_t = (Y_{1t}, \cdots, Y_{nt})$. Let some $j \in \{1, \cdots, n\}$ *fixed*. For each $i = 1, \cdots, n$, $i \neq j$, Y_{it} is generated by the equation;

$$Y_{it} = f_i(Y_{t-\Delta t}, \beta_{t-\Delta t}, t) + v_{it} = \beta_{i,t-\Delta t} + v_{it} \tag{6}$$

where each system noise v_{it} follows

$$\begin{aligned} &N(0, \sigma_i^2) \text{ with probability } \alpha_i \in (0,1) \\ &N(0, c_i\sigma_i^2) \text{ with probability } 1 - \alpha_i \end{aligned}$$

where c_i is a positive constant. That is, each Y_i, $i \neq j$ is generated around β_i in the previous period by adding the system niose. We also note that this formulation of the system noise capture sudden changes in the weights of style indices. We specify $\hat{h}_i(Y_t, t)$, $i = 1, \cdots, n$, $i \neq j$ as

$$\hat{h}_i(Y_t, t) = Y_{it}.$$

Then, $\beta_t^{(j)}$ is generated according to the distribution function $G(y^{(j)})$ which is defined by

$$G(y^{(j)}) = Pr(\{Y_t^{(j)} \leq y^{(j)}\} | A_t^{(j)}) = \frac{Pr(\{Y_t^{(j)} \leq y^{(j)}\} \cap A_t^{(j)})}{Pr(A_t^{(j)})} \tag{7}$$

where

$$A_t^{(j)} = \{0 \le Y_{1t}, \cdots, 0 \le Y_{j-1t}, 0 \le Y_{j+1t}, \cdots, 0 \le Y_{nt}, \sum_{i \ne j} Y_{it} \le 1\}. \tag{8}$$

Finally, β_{jt} is determined by

$$\beta_{jt} = 1 - \sum_{i \ne j} \beta_{it}.$$

In sum, β_{it}, $i \ne j$ is modeled as a random walk with constraints in this example.

(ii) In above example, in order to avoid any bias caused by j being fixed we randomly choose $j \in \{1, \cdots, n\}$ with prbability $\frac{1}{n}$ at each time point t. The other scheme is the same as above.

2. (i) $k = n$, and hence $Y_t = (Y_{1t}, \cdots, Y_{nt})$. Let some $j \in \{1, \cdots, n\}$ *fixed.* For each $i \ne j$, Y_{it} is generated by the equation;

$$Y_{it} = f_i(Y_{t-\Delta t}, \beta_{t-\Delta t}, t) + v_{it} = \log \beta_{i,t-\Delta t} + v_{it}$$

where each system noise v_{it} follows

$$\begin{array}{l} N(0, \sigma_i^2) \text{ with probability } \alpha_i \in (0,1) \\ N(0, c_i\sigma_i^2) \text{ with probability } 1 - \alpha_i \end{array}$$

where c_i is a positive constant. That is, each Y_i, $i \ne j$ is generated around $\log \beta_i$ in the previous period by adding the system niose. Next, we set $\hat{h}_i(Y_t, t)$, $i = 1, \cdots, n$, $i \ne j$ as

$$\hat{h}_i(Y_t, t) = \hat{h}(Y_{it}) = e^{Y_{it}}.$$

Then, $\beta_t^{(j)}$ is generated according to the distribution function $G(y^{(j)})$ which is defined by

$$G(y^{(j)}) = Pr(\{\hat{h}^{(j)}(Y_t, t) \le y^{(j)}\} | A_t^{(j)}) = \frac{Pr(\{\hat{h}^{(j)}(Y_t, t) \le y^{(j)}\} \cap A_t^{(j)})}{Pr(A_t^{(j)})}$$

where

$$\hat{h}^{(j)}(Y_t, t) = (e^{Y_{1t}}, \cdots, e^{Y_{j-1t}}, e^{Y_{j+1t}}, \cdots, e^{Y_{nt}}).$$

Note also that in this case $A_t^{(j)}$ is simplified to

$$A_t^{(j)} = \{\sum_{i \ne j} e^{Y_{it}} \le 1\}$$

because non-negative constrains are automatically satisfied. Finally, β_{jt} is determined by

$$\beta_{jt} = 1 - \sum_{i \ne j} \beta_{it}.$$

(ii) In example 2(i) above, we randomly choose $j \in \{1, \cdots, n\}$ with prbability $\frac{1}{n}$ at each time point t in stead of some j being fixed. The other scheme is the same as above.

3. $k = n$, and hence $Y_t = (Y_{1t}, \cdots, Y_{nt})$. For each $i = 1, \cdots, n$, Y_{it} is generated by the equation;

$$Y_{it} = f_i(Y_{t-\Delta t}, \beta_{t-\Delta t}, t) + v_{it} = Y_{i,t-\Delta t} + v_{it}.$$

Then, β_{it}, $i = 1, \cdots, n$ are determined by a logit trasformation:

$$\beta_{it} = h_i(Y_t, t) = \frac{e^{Y_{it}}}{\sum_{i=1}^{n} e^{Y_{it}}}.$$

4 An Empirical Analysis

In this section, we explain the results of an empirical analysis using actual data of Japanese mutual funds and style indices. We select data of three funds and six style indices from Dec. 26, 1997 to Dec. 10, 2003. Figures 1 and 2 show these data and the correlation matrix respectively. We note that there exist high correlations among the style indices. A significant feature in this analysis is that we have monthly true weights of the style indices for each fund that cannot be available in most cases, and hence we are able to evaluate our method precisely.

We apply the model introduced in Example 1(ii) of 3.3 in the anaylsis; we generate $\beta_t^{(j)}$ which follows a conditional distribution expressed as the equation (7) by selecting samples that satisfy the condition (8). We also assume that the observation noise follows a *i.i.d.* normal distribution in the equation (5), and estimate state variables as well as parameters by Monte Carlo filter.

One of the reason to take this model is that it is a natural extension of a random walk which is neutral in model selection in a sense that it has no bias for the changes in the weights from the current period to the next period as long as the constraints are satisfied; the weights in the next period is modeled around those in the current period by adding the noise which is able to caputure sudden changes in the weights. We also apply a more specific model such as in Example 3 of 3.3, which shows similar performance to that of Example 1(ii). Hence, in the following we concentrate on explaining the results for the estimation of β_{it} based on the model in Example 1(ii) and leave detailed discussions on the model selection as the subsequent research.

Figures 3-12 show the results. For comparative purpose, we also implement a window regression with moving average to smooth the estimated coefficients. Figures 3,4 and 5 show the result for actual monthly returns. Comparing the estimated weights with true ones, we can conlude that the estimates by the Monte Carlo fiter are better than those by the window regression though the estimates are not satisfactory especially for Fund B. Figures 6,7 and 8 show the result for the daily returns by using Monte Carlo filter. We notice that the results are better than those in the monthly case.

However, in Fund B there is still significant difference between actual weights and estimated weights. Therefore, in order to examine the difference further we create the returns by using true weights and style indices and compare them with those of Fund B. Figure 9 shows that there exists explicit difference between them. Judging from this, we should conclude that the assumption that the fund can be approximated by a portfolio of style indices is not appropriate for this fund.

Finally, we generate the time series of returns by using the true weights and the monthly stye indices and estimate the styles' weights based on those series. Figures 10,11 and 12 show the results. We see that the Monte Calro filter is able to trace the dynamics of the true weights quite well and that the result by the method is better than that by the window regression. In particular, we notice that the sudden shifts of the weights can be captured by the Monte Carlo filter while those can not by the window regression. Hence, we can conclude that our method is valid for estimating time-varing coefficients of style indices under non-negative constraints.

Although we introduce only a few examples, we can handle a broad class of models with various types of functional forms of $h_i(\cdot)$ in this framework and can estimate state variables as well as parameters in a unified way by Monte Carlo filter. These generarity is not easily achieved by the other methods and hence is the biggest advantage of the proposed approach.

5 Concluding Remarks

We develop a new framework for style analysis based on the general state space model and Monte Carlo filter. As an example, we apply the method to the time series of actual Japanese mutual funds' returns and style indices, and confirm the validity of our method. Further researches include how to utilize available information such as periodical(for instance, yearly) report of true weights, the estimation of the hedge funds' styles as well as detailed investigation of model selection.

Appendix: An Algorithm of Monte Carlo Filter

In this appendix, we describe the outline of a standard algorithm of the Monte Carlo filter. See Kitagawa(1996) for more detail of an algorithm.

First, we summarize the notation following Kitagawa(1996). $p(X_t|Z_{t-\Delta t})$, called "one step ahead prediction" denotes the conditional density function of X_t given $Z_{t-\Delta t}$ where Δt is the interval of time series. $p(X_t|Z_t)$, called "filter" denotes the conditional density function of X_t given Z_t. $\{p_t^{[1]}, \cdots, p_t^{[m]}\}$ and $\{\xi_t^{[1]}, \cdots, \xi_t^{[m]}\}$ represent the vectors of the realization of m trials of Monte Carlo from $p(X_t|Z_{t-\Delta t})$ and $p(X_t|Z_t)$, respectively. Then, if we set $\{\xi_0^{[1]}, \cdots, \xi_0^{[m]}\}$ as the realization of Monte Carlo from $p_0(X)$, the density function of the initial state vector X_0, an algorithm of Monte Carlo filter is as follows.

[The summary of an algorithm of Monte Carlo filter]

1. Generate the initial state vector $\{\xi_0^{[1]}, \cdots, \xi_0^{[m]}\}$.

2. Apply the following steps (a)∼(d) to each time $t = 0, \Delta t, 2\Delta t, \cdots, (T_* - \Delta t), T_*$ where T_* denotes the final time point of the data.

   - (a) Generate the system noise $v_t^{[k]}$, $k = 1, \cdots, m$ according to the density function $q(v)$.
   - (b) Compute for each $k = 1, \cdots, m$

     $$p_t^{[k]} = F(\xi_{t-\Delta t}^{[k]}, v_t^{[k]})$$

     or

     $$p_t^{[k]} \sim \mathbf{F}|\xi_{t-\Delta t}^{[k]}.$$

   - (c) Evaluate the density function of $\psi(u)$ at $u = H^{-1}(Z_t, p_t^{[k]})$, $k = 1, \cdots, m$ and define those as $\alpha_t^{[k]}$, $k = 1, \cdots, m$.
   - (d) Resample $\{\xi_t^{[1]}, \cdots, \xi_t^{[m]}\}$ from $\{p_t^{[1]}, \cdots, p_t^{[m]}\}$. More precisely, resample each $\xi_t^{[k]}$, $k = 1, \cdots, m$ from $\{p_t^{[1]}, \cdots, p_t^{[m]}\}$ with the probability

     $$\text{Prob.}(\xi_t^{[k]} = p_t^{[i]}|Z_t) = \frac{\alpha_t^{[i]}}{\sum_{k=1}^m \alpha_t^{[k]}},\ k = 1, \cdots, m,\ i = 1, \cdots, m.$$

The estimation of unknown parameters is based on the maximum likelihood method. If μ denotes the vector representing whole unknown parameters, the likelihood $L(\mu)$ is given by

$$\begin{aligned} &L(\mu) = g(Z_{\Delta t}, \cdots, Z_{T_*}|\mu) = \Pi_{i=1}^{\frac{T_*}{\Delta t}} g_i(Z_{i\Delta t}|Z_{\Delta t}, \cdots, Z_{(i-1)\Delta t}, \mu); \\ &g_1(Z_{\Delta t}|Z_0) = p_0(Z_{\Delta t}) \end{aligned}$$

where $g(Z_{\Delta t}, \cdots, Z_{T_*}|\mu)$ and $g_i(Z_{i\Delta t}|Z_{\Delta t}, \cdots, Z_{(i-1)\Delta t}, \mu)$ denote the joint density function of $Z_{\Delta t}, \cdots, Z_{T_*}$ with parameter vector μ and the condtional density fuction of $Z_{i\Delta t}$ given $Z_{\Delta t}, \cdots, Z_{(i-1)\Delta t}$ with μ, respectively. The log-likelihood $l(\mu)$ is computed approximately within the framework of the Monte Carlo filter by

$$l(\mu) = \sum_{i=1}^{\frac{T_*}{\Delta t}} \left(\log \sum_{k=1}^m \alpha_{i\Delta t}^{[k]} \right) - \frac{T_*}{\Delta t} \log m.$$

Then, maximize $l(\mu)$ with respect to μ to obtain the maximum likelihood estimator $\hat{\mu}$. For optimization, grid search and a self-organizing method are applied. (See Kitagawa(1998) for details of a self-organizing state-space model.) Finally, we utilize AIC(Akaike's Information Criterion) as a criterion to select a model if there are several candidates. That is, the model with the smaller AIC can be regarded as the better model.

$$\text{AIC} = -2l(\hat{\mu}) + 2(\text{the number of parameters})$$

In the empirical analysis of section 4, we first select $j \in \{1, \cdots, n\}$ with probability $\frac{1}{n}$ for each sample $k = 1, \cdots, m$ in 2.(b) above. Next, we generate $p_t^{[k]}$ correponding to Y_{it}, $i \neq j$ from the equation (6) and generate $p_t^{[k]}$ correponding to $\beta^{(j)}$ from the conditional distribution (7), $G(y^{(j)})$ by selecting samples that satisfy the condition (8):

$$A_t^{(j)} = \{0 \leq Y_{1t}, \cdots, 0 \leq Y_{j-1t}, 0 \leq Y_{j+1t}, \cdots, 0 \leq Y_{nt}, \sum_{i \neq j} Y_{it} \leq 1\}.$$

Then, $\xi_t^{[k]}$ corresponding to Y_{it}, $i \neq j$ and $\beta^{(j)}$ are determined by 2.(c) and (d) above. Finally, $\xi_t^{[k]}$ corresponding to β_j is determined by the constraint;

$$\beta_j = 1 - \sum_{i \neq j} \beta_i.$$

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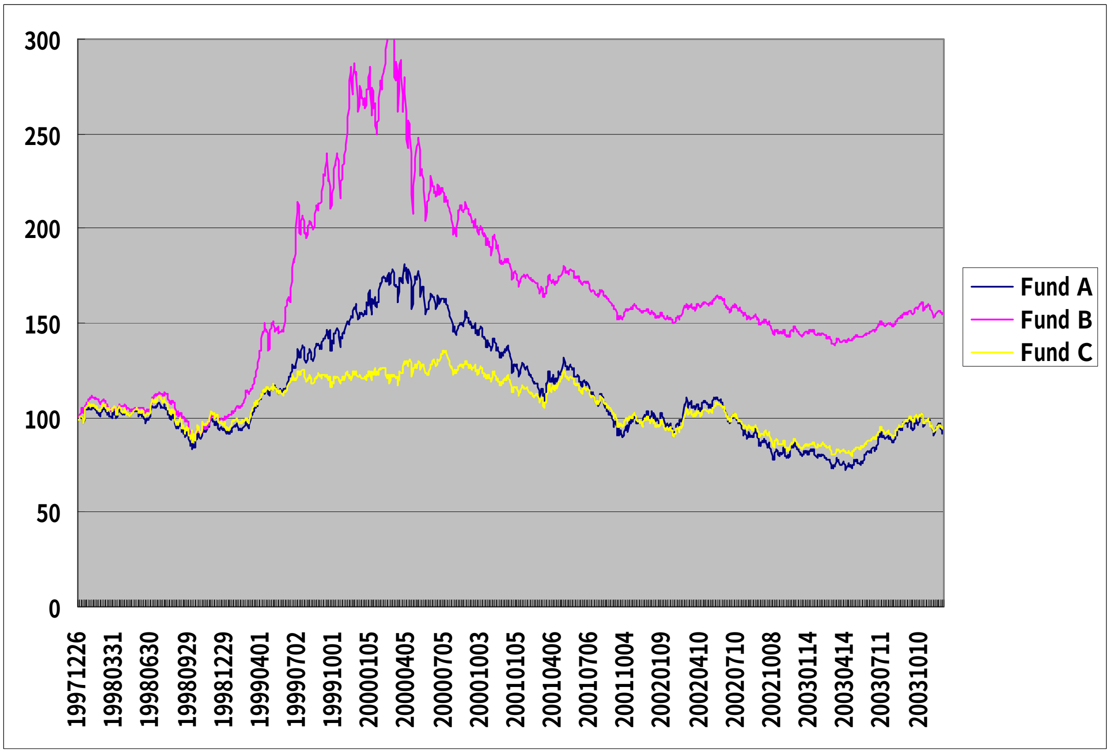
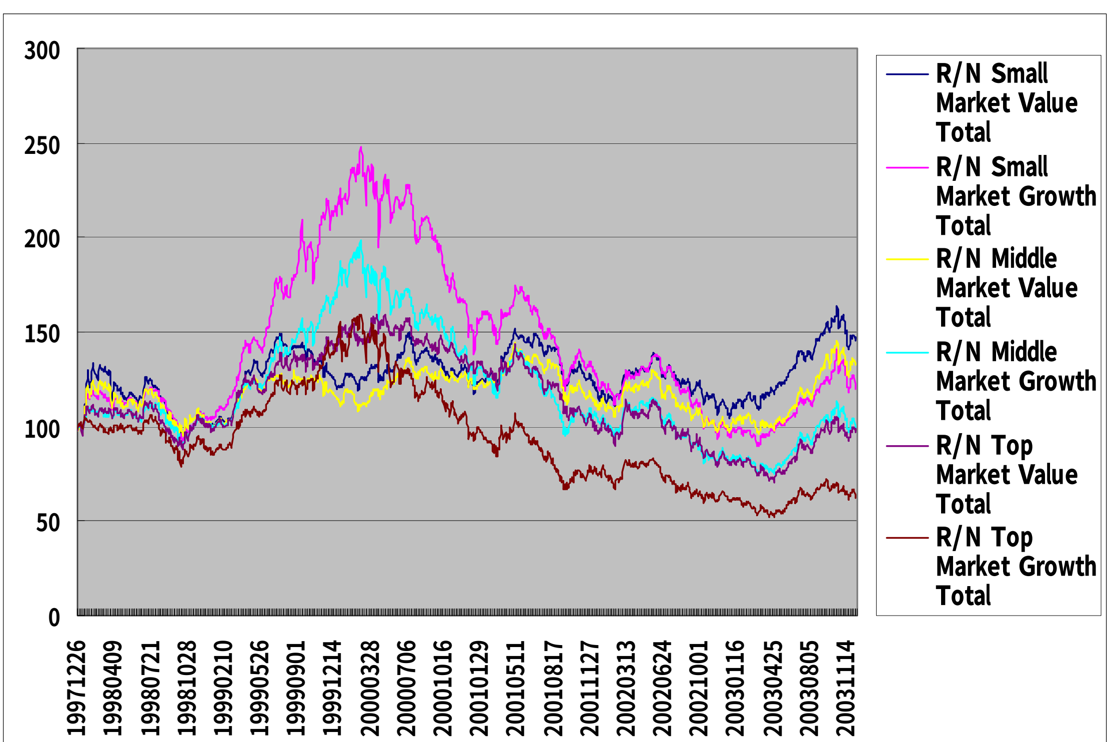
Figure 1 : Observation Data



Fund Series

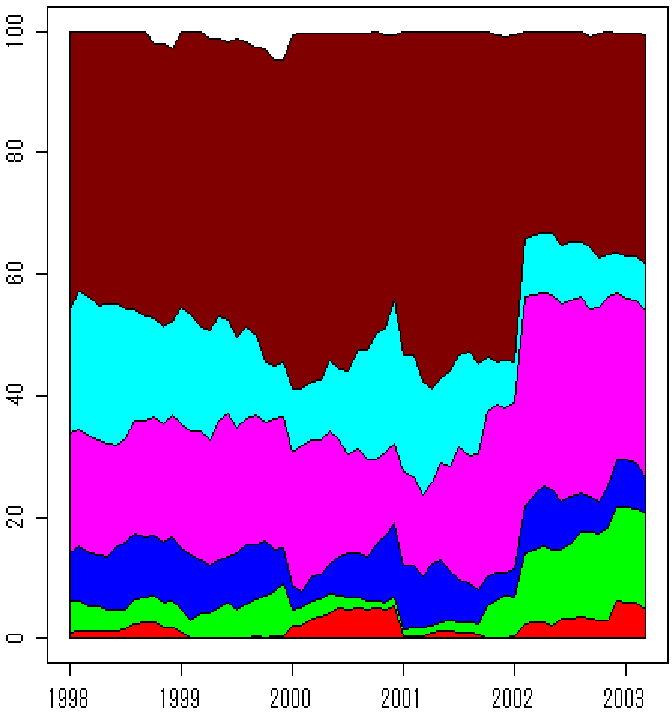


Style Index

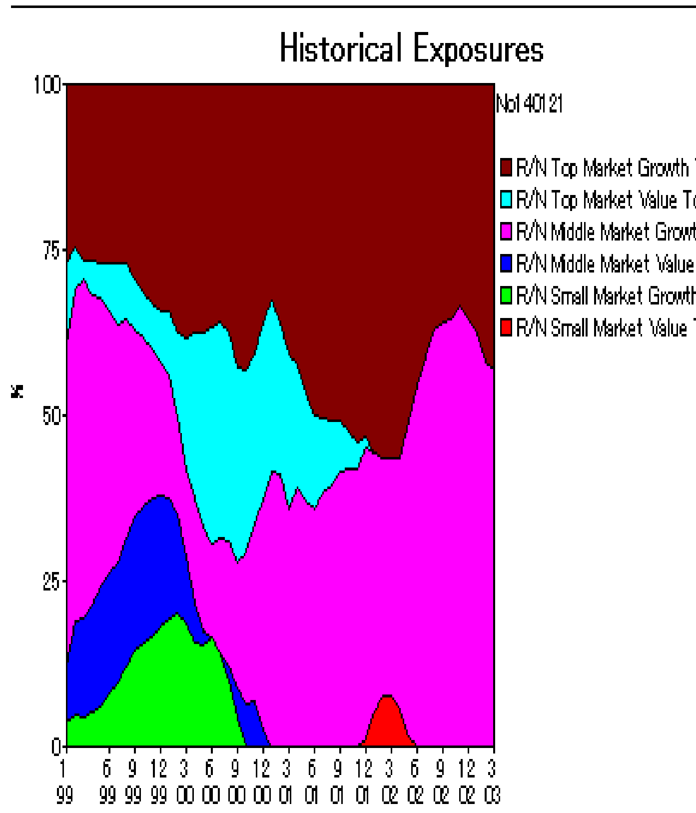
Figure 2: Correlation matrix for fund returns and style index

	SV	SG	MV	MG	TV	TG	A	B	C
SV	1	0.82	0.89	0.78	0.77	0.68	0.74	0.48	0.81
SG	0.82	1	0.71	0.89	0.71	0.77	0.84	0.76	0.77
MV	0.89	0.71	1	0.77	0.87	0.71	0.77	0.39	0.88
MG	0.78	0.89	0.77	1	0.81	0.89	0.94	0.74	0.87
TV	0.77	0.71	0.87	0.81	1	0.84	0.87	0.48	0.90
TG	0.68	0.77	0.71	0.89	0.84	1	0.95	0.64	0.89
A	0.74	0.84	0.77	0.94	0.87	0.95	1	0.69	0.91
B	0.48	0.76	0.39	0.74	0.48	0.64	0.69	1	0.55
C	0.81	0.77	0.88	0.87	0.90	0.89	0.91	0.55	1

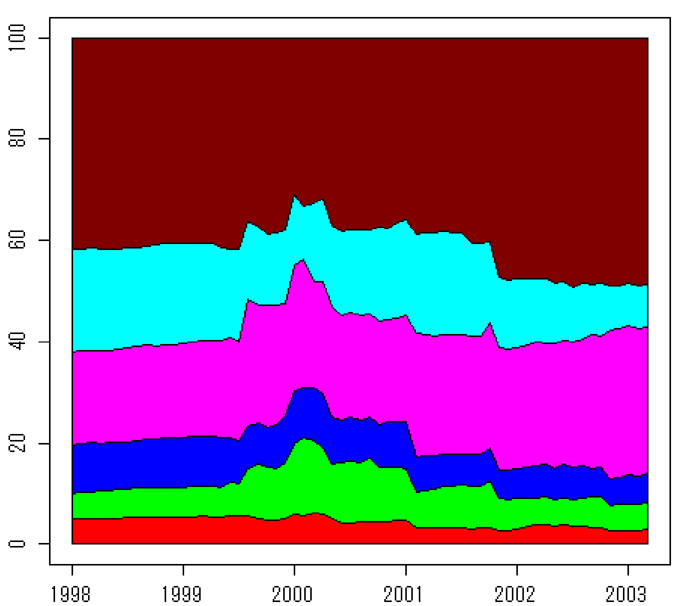
Figure3: Estimating results for real monthly returns of fund A



True weight

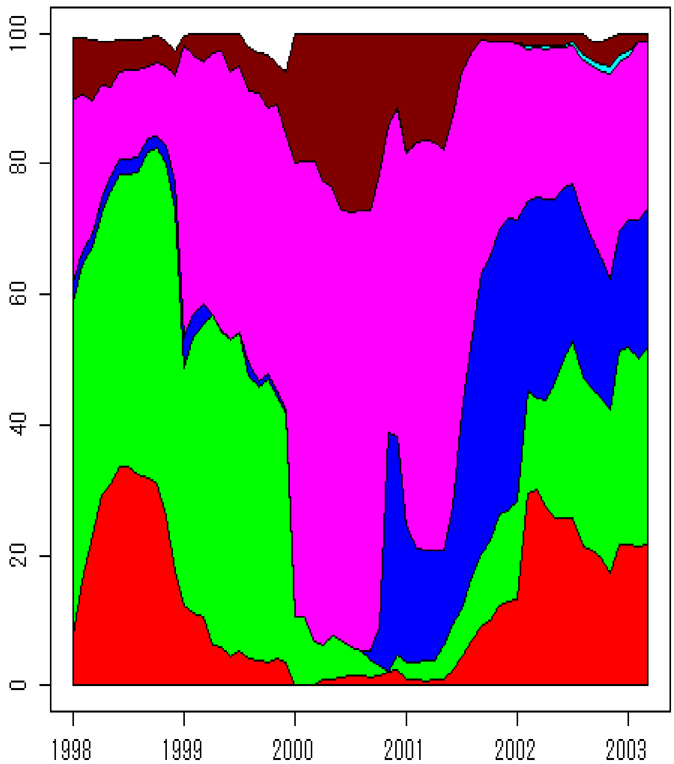


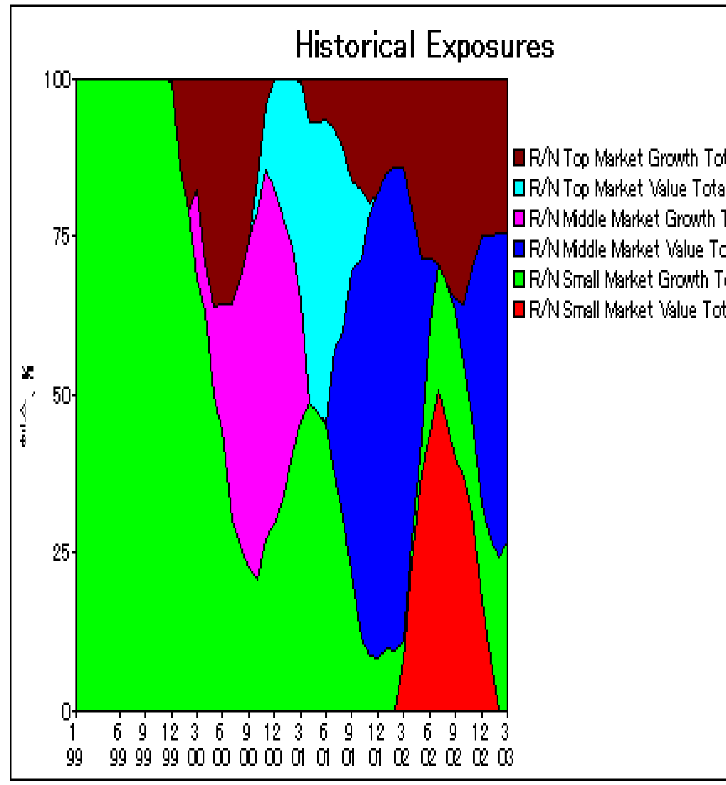
Result by window regression +moving average



Result by Monte Carlo filter

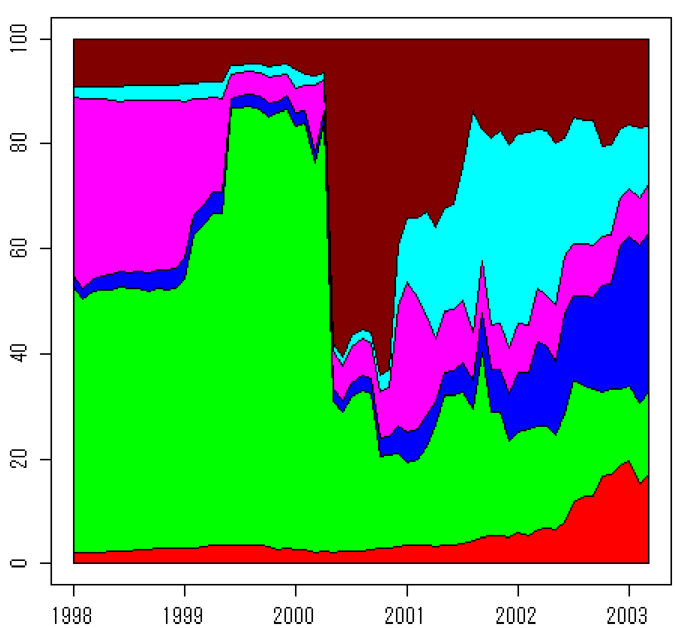
Figure 4: Estimating results for real monthly returns of fund B





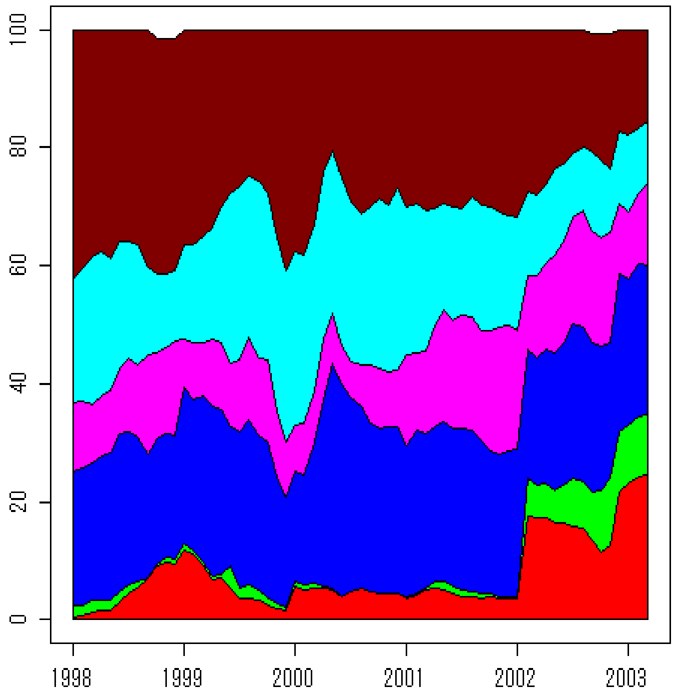
True weight

Result by window regression +moving average

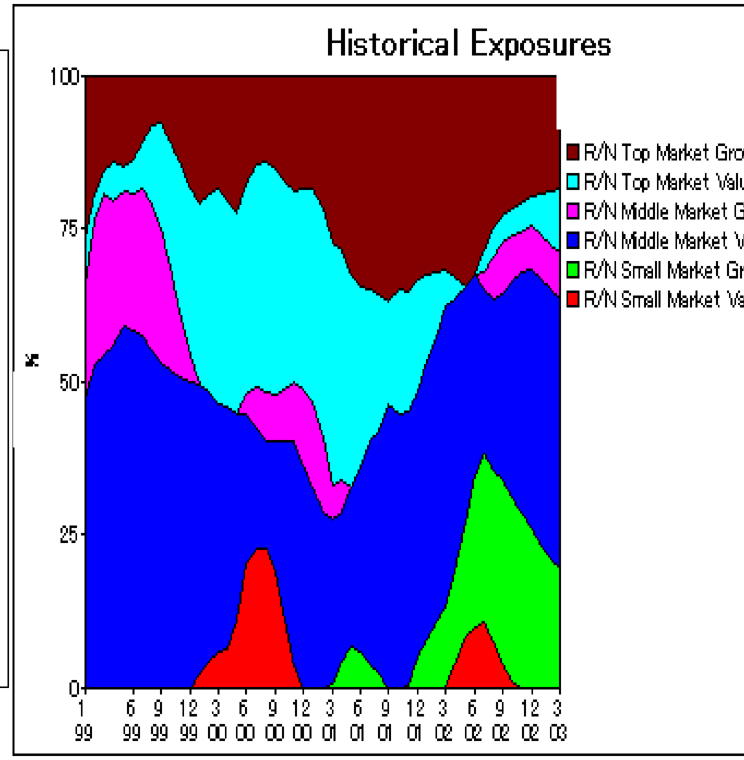


Result by Monte Carlo filter

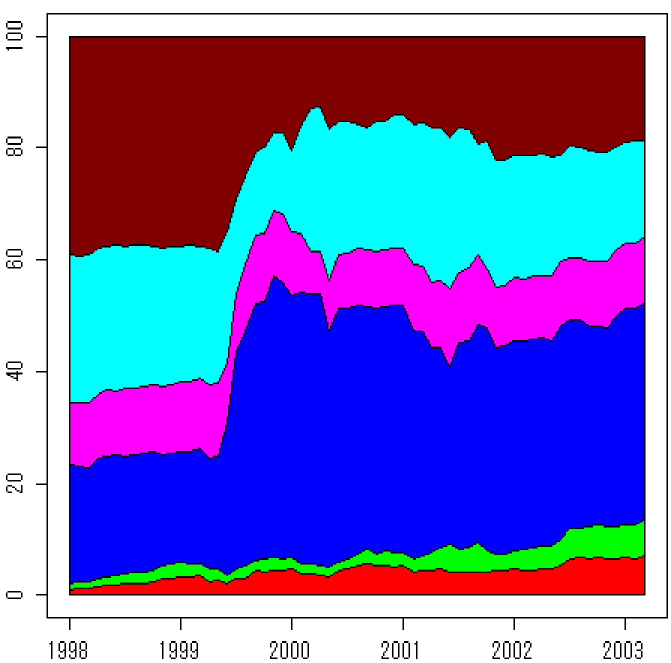
Figure 5: Estimating results for real monthly returns of fund C



True weight

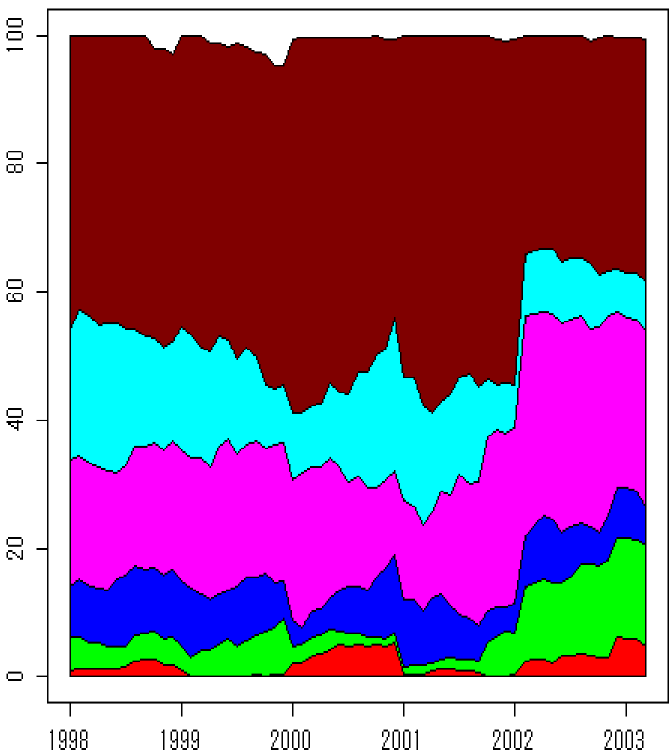


Result by window regression +moving average

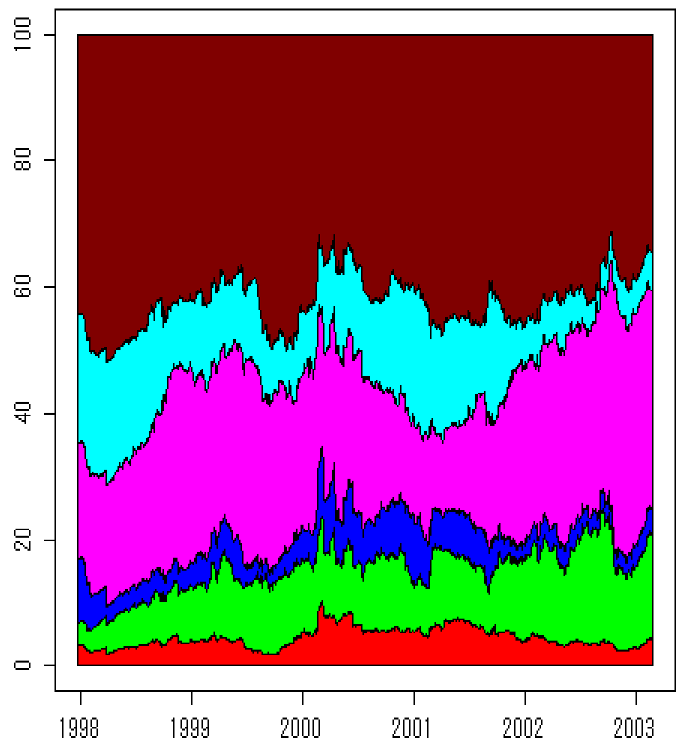


Result by Monte Carlo filter

Figure 6: Result for real daily returns
of fund A

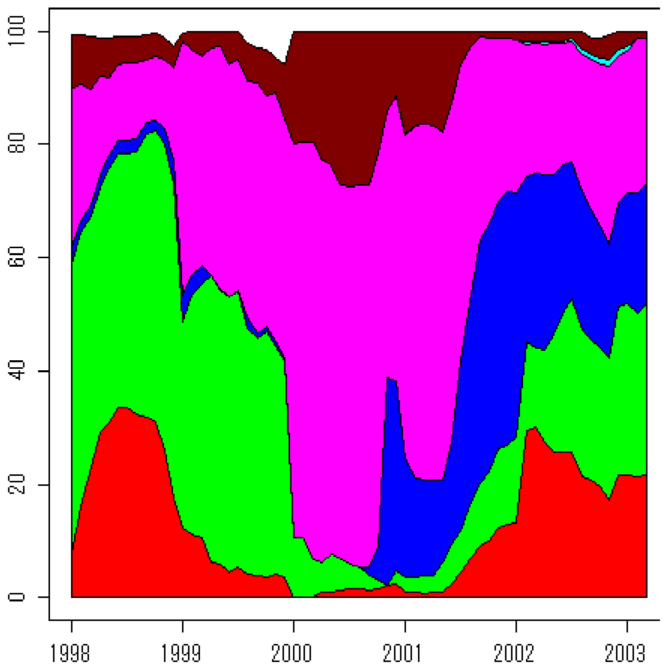


True weight

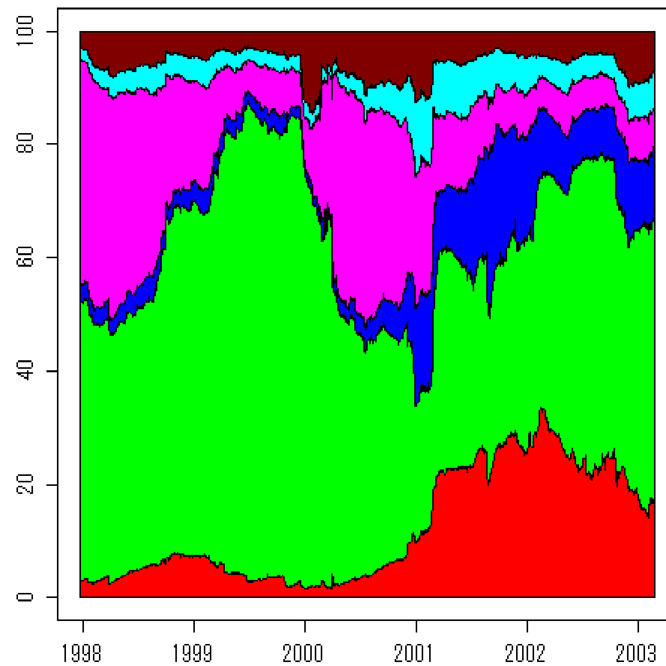


Result by Monte
Carlo filter

Figure 7: Result for real daily returns of fund B

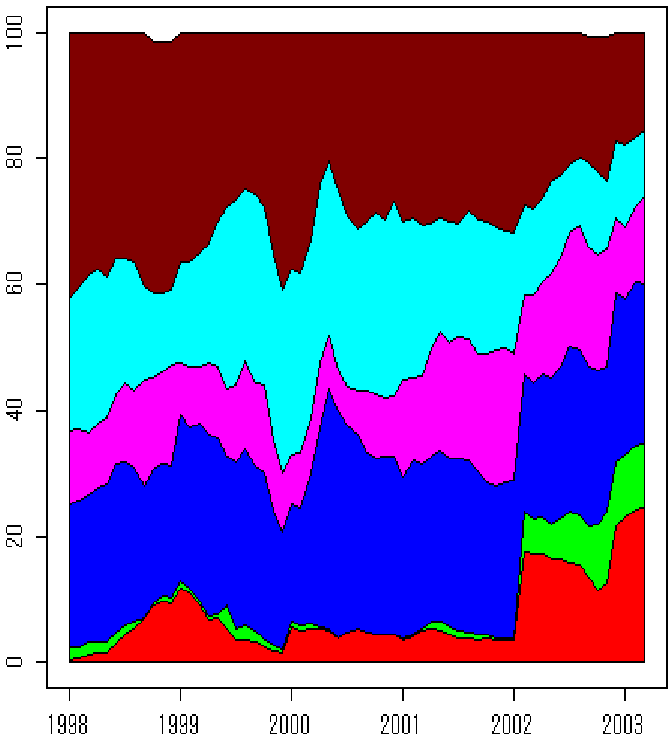


True weight

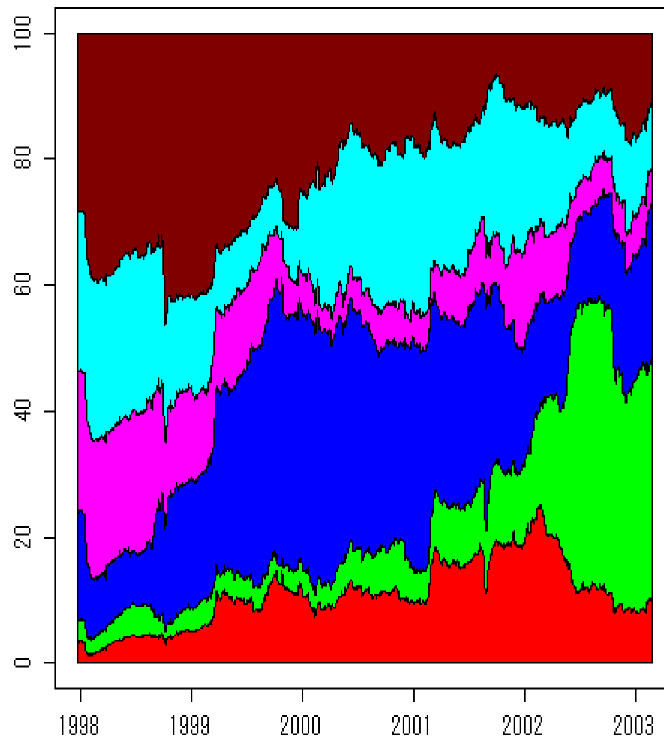


Result by Monte Carlo filter

Figure 8: Result for real daily returns of fund C



True weight



Result by Monte Carlo filter

Figure 9: comparison between real return and style index for Fund B

Scatter plot of real monthly returns (X_t) of Fund B vs. returns (Y_t) of portfolio of style indices by actual weights

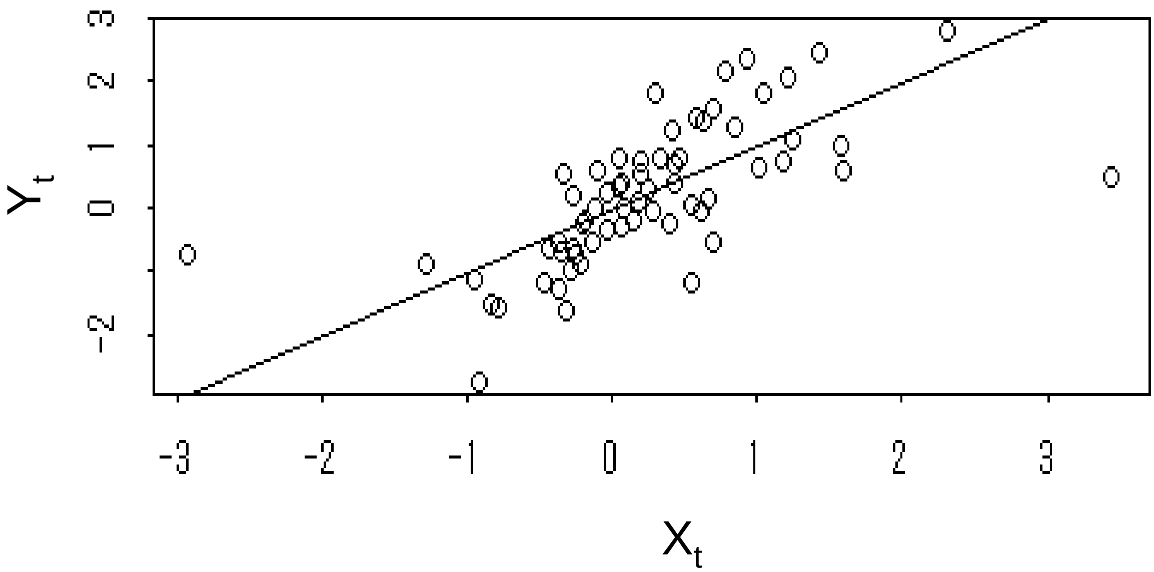
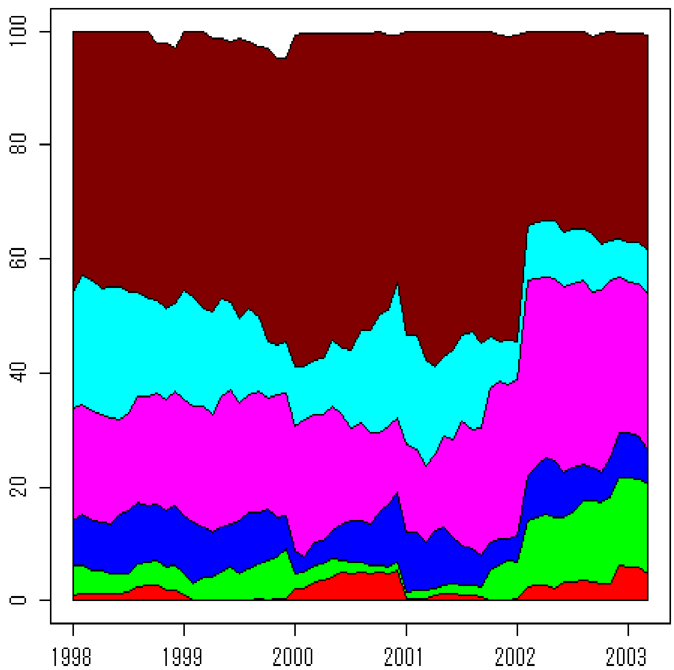
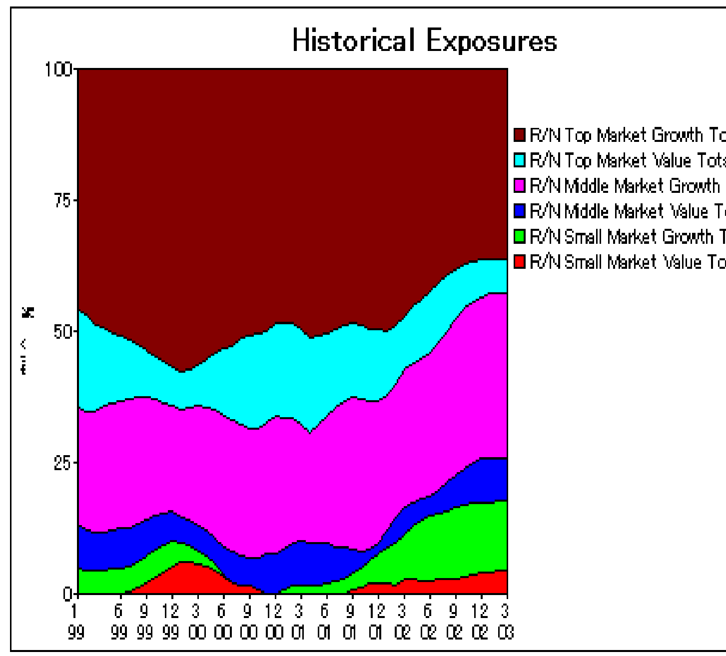


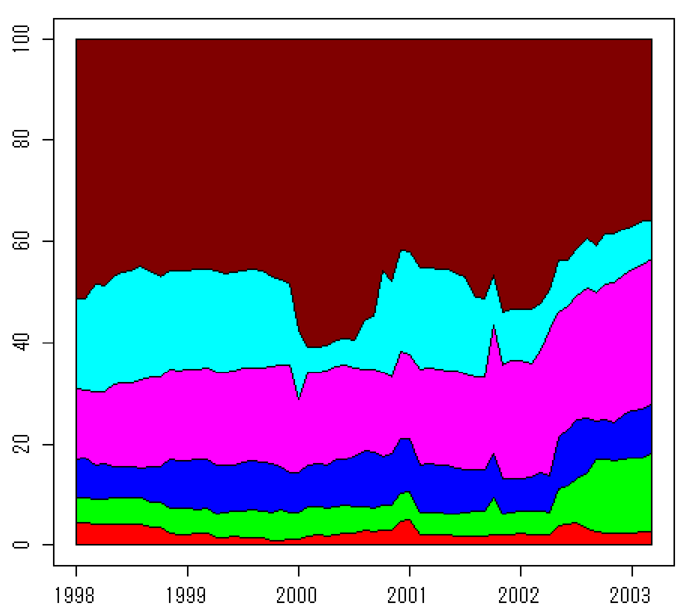
Figure 10: Results for simulation data based on monthly true weights of Fund A



True weight

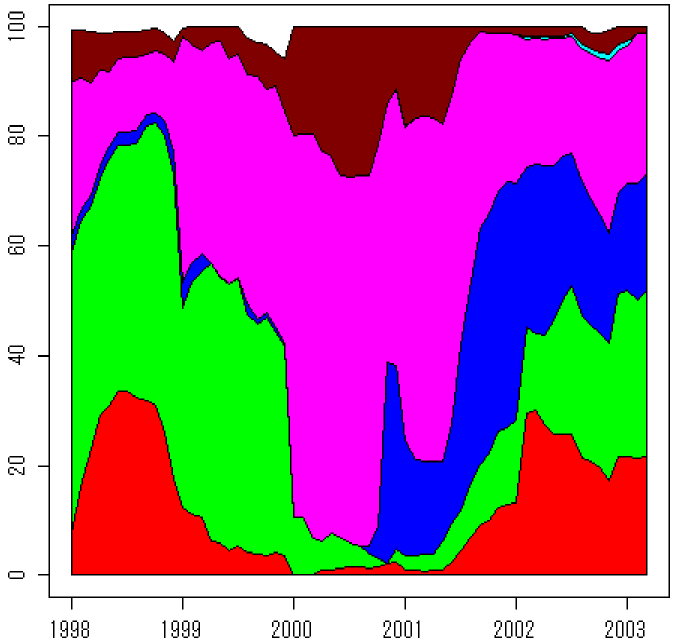


Result by WR+MA

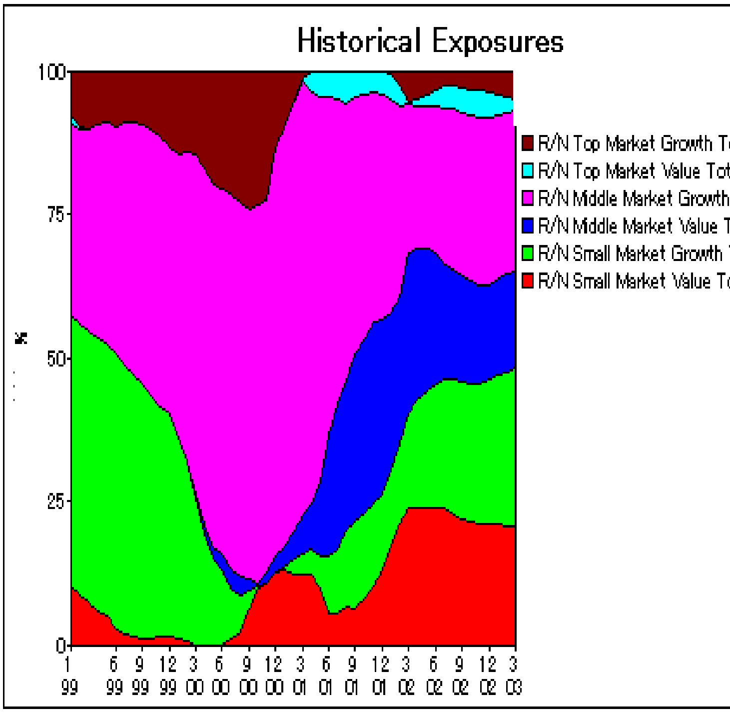


Result by MCF

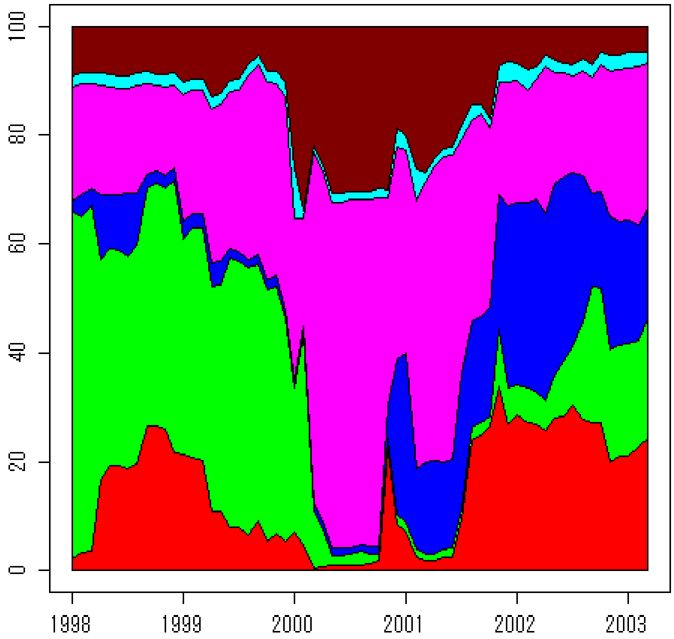
Figure 11: Results for simulation data based on monthly true weights of Fund B



True weight

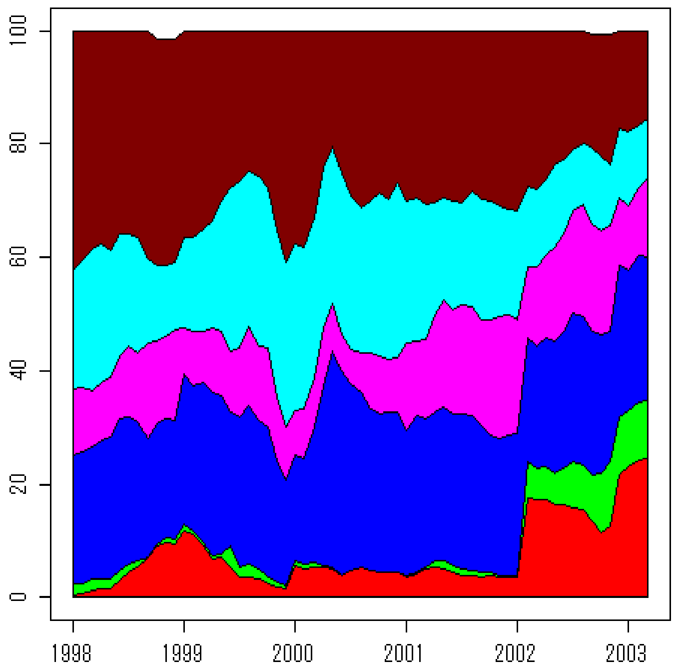


Result by WR+MA

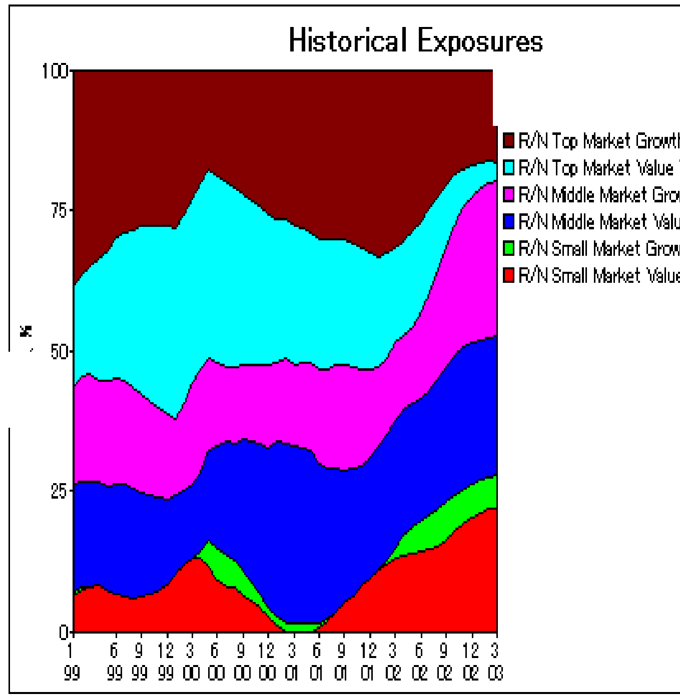


Result by MCF

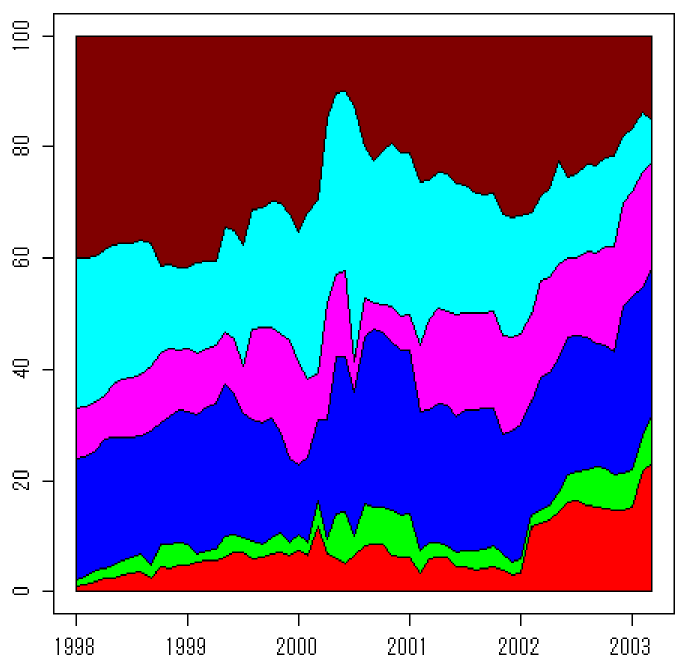
Figure 12: Results for simulation data based on monthly true weights of Fund C



True weight



Result by WR+MA



Result by MCF