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Group Provision Against Adversity: Security By Insurance vs. Protection

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Group Provision Against Adversity: Security By Insurance vs. Protection

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Abstract

We investigate the structure of interactions among countries exercising voluntary uncoordinated choice but sharing a common "risk profile" --- a vector comprised of chance of adversity/emergency and magnitude of loss under adversity/emergency. We use the term "emergency costs" to refer to the vector and/or its components. Countries strive to reduce emergency costs (a) by providing mutual self-insurance against loss and (b) mutual self-protection against risk. Because of their common risk profile each country's security spending whether on self-insurance or on self-protection provides a public good (pure or impure) to all in the group --- hence our term "mutual." We show that under expected utility maximization the normality or inferiority of such public goods depends crucially on hitherto unrecognized interactions between preference functions and status quo risks. Moreover we discover that these interactions differ systematically between insurance and protection with important policy implications for comparing the two instruments. Furthermore, we demonstrate that configurations where security is inferior are not at all unlikely. In such case the provision of international public goods can easily face an endogenous obstacle, an “inferior good barrier,” under Nash-Cournot behavior and under Leader-Follower behavior with Stackelberg outcomes. (These, however, display novel and desirable properties even when the public good is inferior.)

When improvements in risk profile generate not pure public goods, but instead imperfect, ambiguous, or even negative benefits among partners, who is an ally and who an adversary, itself becomes ambiguous. For this configuration where the emergency risk profile differs among allies, we show how spillovers from emergency cost reduction and their effects on welfare will be depend critically on the sign of income effects.

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1. Introduction

Among the first to examine how individuals can manage risks to their well-being were Ehrlich and Becker (EB, 1972). EB identified several types of preparation available to expected utility maximizing individuals faced with what we will call "costs of emergency." These consist of any profile of (a) probability of loss and (b) magnitude of loss (hereafter together referred to as "risk profile"). Among such preparations were "self insurance" to compensate for or reduce the magnitude of loss and "self-protection" to reduce the probabilities of loss. Similarly for entire societies their governments need not passively accept risks that production and hence consumption decline in unwelcome situations. Several measures exist to manage national adversity. Moreover, as Olson and Zeckhauser (1966) propounded, security benefits may spill over becoming a public good among countries so that they interact in their allocations of national income to international (or regional) safety. The EB distinctions apply here also. Mutual self-insurance and self-protection can be available to groups of countries, for example, by formation of international organizations, collective military preparedness, active international diplomacy, and foreign aid that reduces the probabilities of regional and international tension. And special trading agreements or collaborative stockpiling or common strategic defenses may provide mutual self-insurance to reduce or offset emergency losses.

These economic features, where a public good is voluntarily provided by allied governments, are in reality crucial to today's international security alliances involving Japan, other Asian countries, the European Community, and the United States. Applications of the voluntary public good (VPG) model to issues of international cooperation have proven informative (e.g. Ihori, 1994; Alesina and Perotti, 1995; Alesina and Spolaore, 1995; Boadway and Hayashi, 1999). But utilization of EB to model collective improvements of the "risk profile" as an international public good (as defined above) is sparse, except for some work on terrorism such as Lapan and Sandler (1988), Sandler (1992, 1997, 2005), or Arce, Daniel, and Sandler, (2005). In particular, economists' VPG models with many agents have not been well extended to understand the consequences of differences in risk aversion in this risk management context.

Using standard Nash and standard Stackelberg equilibrium models we will incorporate such security spending among allies into the utility function in these two alternative ways --- as mutual insurance and mutual protection --- and thereby will clarify the differential effects of risk and loss on security spending, and national welfare[1]. As we show, crucial to these relations among welfare, and the nature of international hazard are the income effects of changes in risk profile.

Specifically we show that the sign of the income effect under quite ordinary circumstances can be inferior just as well as normal both for self-protection and self-insurance. As we demonstrate, this allows us to derive important general properties of expected utility optimization. Once we have isolated these income effects we can then explore group behavior including how the size of group and costs of emergency affect the voluntary provision of security spending among allies and their national welfares in a perilous world. For example if the pubic good is normal, the effects upon optimal self insurance of an increase in risk are qualitatively just the opposite of the effects of an increased magnitude of loss upon optimal expenditures for self protection. This implies, as we also show, that if security becomes inferior then groups will face a difficulty in providing international public goods because of an endogenous "inferior good barrier".

This paper consists of six sections. First, we formulate a basic analytical framework. Section 2 develops characteristics of optimization under mutual self insurance and alternatively mutual self protection. Section 3 considers Nash and Stackelberg equilibria for provision of (1) mutual insurance and (2) mutual protection individually[2]. Then, in section 4 extending Ihori and McGuire (2006), we demonstrate an inherent potential for an "inferior good barrier" which endogenously limits a group's collective security outlays on mutual protection or mutual insurance and results in "immiserizing growth". Derivation of this endogenous barrier uses the Cornes-Hartley (2000, 2003) replacement function. Section 5 generalizes the replacement function, extends the analysis to impure public goods and differential country-specific risk, and thus explores the resulting ambiguity in the

[1] . Security spending among allies is very common in the real world. For example, Japan contributed an additional $9 billion, on top of the $4 billion it offered in 1990, to the allied effort on the Gulf war in 1991. After the event of September 11, 2001 many countries raised their security spending to cope with the threat from international terrorists.

[2] Combined provision of self insurance and self protection is not addressed.

identification of ally vs. adversary. Finally section 6 concludes the paper.

2. Analytical Framework

2.1 Risk Profiles and Emergency Cost

As in Ihori and McGuire (2006), let the world consist of two countries; country 1 and country 2, and have two states, a good state "1" and a bad state, "0". Ignoring all insurance and compensation possibilities, expected utility for a single country i (i = 1, 2) is given as:

$$W_i = pU^1(C_i) + (1-p)U^0(C_i - L_i) \tag{1}$$

$$\text{or} \qquad W_i = W_i(C_i, p) \tag{2}$$

where W_i is expected utility for Country i, C_i is i's private consumption, L_i is i's loss in the bad state, and p is the chance of a good state. Subscript i denotes country i. (If both countries are identical, we may delete subscript i unless doing so creates confusion.) Our analysis will focus on two types of Ehrlich-Becker (EB) modality of defense; (i) EB's "self-protection;" which raises p and reduces (1- p), (ii) EB's "self-insurance" which reduces L_i. The variable "p" might be risk of war, shared indivisibly by two coalition members. Here we assume p_i, the probability of good state for country i, is the same for all. So we delete subscript i; $p_1=p_2=p$. In section 5 we shall consider a more general case where p_1 is not necessarily the same as p_2. Utility function U( ) is assumed to be the same whether luck is good or bad. U^1 denotes realized utility if the good event happens, and U^0 if the bad event happens. We assume that $U_Y \equiv \partial U / \partial Y > 0, U_{YY} \equiv \partial^2 U / \partial Y^2 < 0$.

Introducing collective risk/loss management an individual country's budget constraint is

$$Y_i = C_i + m_i \tag{3}$$

where Y_i is a fixed national income and m_i denotes allocations to risk or loss reduction; that is, m_i gives the voluntary input to the public good by Country i. With "summation finance" and assuming two countries, (i = 1, 2), the international public good M is naturally proxied (as argued below) by

$$M = m_1 + m_2 . \tag{4a}$$

Protective expenditures by countries 1 and 2 are assumed to be equally effective in reducing the common emergency costs (L and/or 1-p) in this basic model. Specifically, m_1 and m_2 are perfect substitutes for each other. In section 5, we will consider country-specific emergency costs and impure public goods, where m_1 and m_2 are not perfect substitutes.

2. 2 Mutual Self -Insurance

When countries provide collective or mutual self insurance only we have

$$L = L(M) \tag{4b}$$

So that with p taken to be a parameter eq. (1) can be written

$$\tilde{W}_i = \tilde{W}_i(C_i, M) \tag{5}$$

Eq (5) shows --- as argued in Ihori and McGuire (2006) for the case of self-protection --- how it is natural and helpful to consider M rather than L to be the public good. To support an interior solution it is assumed that the marginal productivity of national security expenditure is decreasing; that is - L' > 0, - L" < 0 (or equivalently L' < 0, L" > 0). If M is very productive, L may even be negative.

Expected utility ((1) or (5)) is then maximized with respect to m_i subject to constraints (3), (4a) and (4b). This gives (6a) and (7) as the first and the second order conditions:

$$\text{FOC:} \quad [pU_Y^1 + (1-p)U_Y^0] + (1-p)U_Y^0 L' = 0 \tag{6a}$$

Eq. (6a) shows the marginal cost of providing L, $[pU_Y^1 + (1-p)U_Y^0]$, equal to the marginal benefit of providing L, $-(1-p)U_Y^0 L'$. If this necessary condition is rewritten as in (6b) then its actuarial meaning becomes clear. There (-L'-1) gives the net marginal insurance receipt under adversity for the last dollar of premium paid in good times

$$\frac{U_Y^1}{U_Y^0} = \frac{(1-p)}{p}(-L'-1) \tag{6b}$$

If self insurance is actuarially fair at the margin then

$$p/(1-p) = (-L'-1) \tag{6c}$$

and therefore

$$U_Y^1 / U_Y^0 = 1 \tag{6d}$$

Thus, actuarially fair insurance equalizes marginal utility across contingencies, a well known principle (see Ehrlich and Becker, 1972, or McGuire and Becker, 2006 for extensions of this theme.)[3] When marginal utility is equalized then consumption also must be equalized for state independent utility so that L = 0. Equation (6b), therefore, informs us of the optimal degree of insurance protection.[4]

$$\text{SOC:}\quad D = [pU_{YY}^1 + (1-p)(1+L')^2 U_{YY}^0] - (1-p)U_Y^0 L'' < 0 \tag{7}$$

For comparative static results it is the sign of the income effect on security spending we must investigate. Specifically, we want to determine whether M is an inferior or a normal “good”. To decide this taking total differentiation of FOC (6a) gives:

$$\frac{\partial M}{\partial Y^*} \equiv M_Y = \frac{[pU_{YY}^1 + (1-p)U_{YY}^0] + (1-p)U_{YY}^0 L'}{D} \tag{8a}$$

where Y* is the individual effective “full” income that obtains at an interior solution. $Y_i^* = Y_i + m_j$ (i, j = 1, 2) The numerator of (8a) is actually the <u>difference</u> between the effects of greater income on marginal cost $\Delta MC = MC_{MY} = [pU_{YY}^1 + (1-p)U_{YY}^0] < 0$ and marginal benefit $\Delta MB = MB_{MY} = -(1-p)U_{YY}^0 L' < 0$. Note for later use that whereas MC_{MY} has elements in both states of the world, MB_{MY} refers exclusively to the bad state.

From the SOC we know

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3 The recent paper "Self-Insurance and Self-Protection as Public Goods," Tim Lohse, Julio R. Robledo, and Ulrich Schmidt, School of Economics and Management, University of Hannover, e-mail: lohse@fiwi.uni-hannover.de, presented at the same APET conference as this utilizes a similar framework but has a different focus.

4 Could insurance proceed so far that outcome in "war" is preferred to the outcome in "peace," i.e. so that $U^0 > U^1$? Conceivably yes, if loss-compensation activity L(M) is sufficiently productive (L < 0) and the chances of war sufficiently great that at the margin (-1-L') > p/(1-p) or equivalently -L' > 1/(1-p), then

$$U_Y^1 / U_Y^0 = (1-p)/p\,(-L' - 1) > 1 \tag{6e}$$

and therefore

$$U_Y^1 > U_Y^0 \Rightarrow U^1 < U^0 \quad (\text{or } L < 0) \tag{6f}$$

For more on this see McGuire and Becker (2006).

$$[pU^1_{YY} + (1-p)U^0_{YY}] + (1-p)U^0_{YY}L' < -(1-p)U^0_{YY}[2L' + (L')^2] + (1-p)U^0_Y L'' \gtreqless 0 \quad (8b)$$

Hence the sign of the numerator is ambiguous and M as self insurance may be inferior[5]. Specifically, if the numerator is positive, given the SOC, the sign of (8a) is negative, and M becomes inferior. That is, when M is inferior, and greater income lowers marginal cost --- $\Delta MC = MC_{MY} = [pU^1_{YY} + (1-p)U^0_{YY}] < 0$ by more than it reduces marginal benefit --- $-\Delta MB = -MB_{MY} = (1-p)U^0_{YY}L' > 0$.

This required relationship between MC_{MY} and MB_{MY} can be derived from the risk aversion properties of the utility function, with absolute risk aversion R defined as

$$R = -U_{YY}/U_Y \qquad \text{or} \qquad -U_{YY} = RU_Y \quad (8c)$$

Then the numerator of (8a) can be rewritten as

$$H = -(pR_1U^1_Y + (1-p)R_0U^0_Y) + (1-p)R_0U^0_Y L') \quad (8d)$$

Considering the FOC, we obtain

$$H = pU^1_Y(R_0 - R_1) = (1-p)U^0_Y(R_0 - R_1)(-L'-1): \; -L' > 1 \quad (8e)$$

Hence, if risk aversion is increasing ($R_1 > R_0$), H < 0 and the sign of (8a) is positive, and vice versa; that is, if risk aversion is increasing (decreasing), M is normal (inferior)[6]. Generally, we may expect absolute risk aversion to

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[5] Note that if self-insurance is actuarially fair and therefore (1+L')(1-p) = - p, then the numerator of (8a) becomes $p(U^1_{YY} - U^0_{YY})$.

[6] With the numerator of eq. (8a) written as $MC_{MY} - MB_{MY}$ it is understood that both MC_{MY} and MB_{MY} are negative. If $|MC_{MY}| < |MB_{MY}|$ then $M_Y < 0$ and M spent on self insurance is inferior. Using this notation to write (8d) gives

$$H = [MC_{MY} - MB_{MY}]$$

And

$$MC_{MY} = -R_1pU^1_Y - R_0(1-p)U^0_Y$$
$$-MB_{MY} = +R_0pU^1_Y + R_0(1-p)U^0_Y$$

In MC_{MY} the terms p and R interact just as in the case of M spent for self protection considered below. However, for self insurance when M and, therefore, -L are optimized this interaction is washed out of the sum of MC_{MY} and $-MB_{MY}$. The change in marginal benefit of more insurance when Y is increased depends only on its impact in one contingency, i.e. on. $(1-p)R_0U^0_YL'$ When M for insurance is optimized, as shown in eq 6b U_Y^0 and U_Y^1 are balanced as per eq. 6b so that the independent effect of U_Y^0 and (1-p) are all absorbed in U_Y^1 and p or vice versa. When similar analysis is performed on self protection, the FOCs do not cancel out the interdependence between R and p, so that our conclusions for the two cases decrease with wealth, so that the amount of insurance purchase declines. It follows that M may well be inferior. In fact this is what we should expect: public good inferiority for many plausible scenarios.

Now we desire to understand the effect of a shift in the parameter p on the optimal value of choice variable M (and therefore of L, or -L). From the first order condition we know

$$\frac{\partial M}{\partial p} \equiv M_p = -\frac{(-U_Y^1 + U_Y^0) + U_Y^0 L'}{D} \tag{9a}$$

From the FOC,

$$U_Y^0 + U_Y^0 L' = -\frac{p}{1-p} U_Y^1 < 0 \tag{9b}$$

meaning that the numerator of (9a) is negative. Hence, assuming the SOC obtain so that D < 0, the sign of (9a) is always negative. An increase in probability of the good state reduces the demand for M and, therefore, for -L(M)--- that is +p and -L are substitutes. Less risk leads to less insurance as is plausible and expected[7].

2. 3. Mutual Self Protection

Ihori and McGuire (2006), developed the expected utility model for m_i (i = 1, 2) spent on mutual self-protection to reduce the chance of a bad event, 1-p, or decrease what we call "baseline risk of [1-p(0)]," increasing "p" the probability of a good event for both parties. Here we account for the collective summation technology quality of the inputs to p by writing

$$p = p(M), \tag{10a}$$

Now "p" is the public good (as conventionally defined) for countries 1 and 2; i.e. we assume that protection "p" benefits both countries in a non-rival non-excluded fashion (until section 5). *A priori*, many risk reduction or self protection functions are plausible: quadratic, exponential, logistic etc. One expects p' > 0 throughout for all of

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differ.

[7] An increase in p will shift consumption from the bad state (C-L) to the good state (C) as a substitution effect. Since for the same budget constraint (3) the benefit from good-state consumption rises, m must be reduced. Note that eqs. (9a, b) would tell us the slope of the reaction curve of a country that provides only L (spending say m_L on insurance) to the provision of p from a country that provides only p (and spends m_p on p). In other words (9a, b) tells us the slope of L-providing country's reaction curve m_L to changes in m_p. We discuss the equilibrium implications of this fact for a two country equilibrium in section 5.

these; while p" may vary, we assume here that $p'' < 0$ throughout. Whatever the form of p(M) it becomes natural and useful to re-write Eq. (2) as

$$\widehat{W}_i = \widehat{W}_i(C_i, M) \tag{10b}$$

where L is now taken to be a parameter. [8]

The first and the second order conditions with respect to m_i are:

$$\text{FOC} \qquad p'(U^1 - U^0) - [pU_Y^1 + (1-p)U_Y^0] = 0 \tag{11}$$

$$\text{SOC} \quad E = p''(U^1 - U^0) - 2p'(U_Y^1 - U_Y^0) + [pU_{YY}^1 + (1-p)U_{YY}^0] < 0 \tag{12}$$

Note that the SOC need not always hold for mutual self protection under the assumptions made so far. However, we assume the SOC is satisfied; then taking total differentiation of FOC (11) gives:

$$\frac{\partial M}{\partial Y^*} = -\frac{p'(U_Y^1 - U_Y^0) - [pU_{YY}^1 + (1-p)U_{YY}^0]}{E} \tag{13}$$

Condition (12), the SOC, if it actually obtains, determines the sign of the denominator in (13) as negative at an optimum. But again the sign of the numerator is ambiguous, and the normality or inferiority of M depends on this numerator, just as in the self insurance model. The numerator of (13) can be written $MB_{MY} - MC_{MY}$, where $MB_{MY} = p'(U_Y^1 - U_Y^0) < 0$ and $MC_{MY} = pU_{YY}^1 + (1-p)U_{YY}^0) < 0$ Now, as shown in Ihori and McGuire (2006) there is an interaction between R and baseline probability, so the normality/inferiority of M is more involved than in the case of self-insurance. Here if absolute risk aversion is increasing and (1-p) is initially low, G will be normal, while if absolute risk aversion is decreasing and (1-p) is low, M becomes inferior.[9]

Again to derive the response of optimal choice of M (and therefore of p(M)) to a shift in parameter L we use

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[8] Functions $\tilde{W}_i(\cdot)$ *and* $\widehat{W}_i(\cdot)$ are different, but the difference is implicit in each model, so the notational distinction will be omitted henceforth.

[9] Actually Ihori and McGuire (2006) demonstrate that there is a critical-crossover probability p* such that if risk aversion is increasing M switches from normal to inferior while if risk aversion is decreasing M switches from inferior to normal as this value p* is crossed. Note that this interdependence between p and risk aversion is determined entirely by or reflected solely in $MC_{MY} = [pU^1_{YY} + (1\text{-}p)\, U^0_{YY}] < 0$ just as in the case of self insurance. The interdependence between risk aversion and risk is absent for self insurance because insurance allows transfer of resources across contingencies, and optimal insurance equates the properly weighted values of U^1_{YY}

$$\frac{\partial M}{\partial L} = -\frac{p'U_Y^0 + (1-p)U_{YY}^0}{E} \tag{14}$$

From (13) and (14), we derive

$$\frac{\partial M}{\partial Y} = -\frac{\partial M}{\partial L} - \frac{p'U_Y^1 - pU_{YY}^1}{E} \tag{15a}$$

Given the second order condition, the denominator E is negative and the second term $-[p'U_Y^1 - pU_{YY}^1]/E$ therefore is positive. It follows that

if $M_L \equiv \partial M/\partial L \leq 0$, then $M_Y \equiv \partial M/\partial Y$ >0: (15b)

if $M_Y \leq 0$, then M_L >0. (15c)

although $M_L > 0$ is consistent with positive or negative M_Y . Now an increase in the value of parameter L effectively causes a reduction of "real income," so that demand for M (and for p(M)) will decline if M is normal (or increase if M is inferior)[10].

2.4 Mutual Self-Insurance and Protection as Public Goods

Although the benefit of M is different for the two types of risk profile improvement, both models may be summarized in the same way as maximizing the following utility function

$$\hat{W}_i(C_i, M) \tag{2a}$$

subject to (3) (4a) and (4b) or (10a). Thus, we can use similar analytical methods and diagrams to investigate the structure of public goods provision for reducing either risk or magnitude of loss. For such comparisons of insurance vs. protection two features of the models stand out.

First, depending on absolute risk aversion (in the state-independent utility function), M may be normal or inferior in both models. This is because M enters into the utility function (2a) generally with marginal

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10. Note that eqs. (14, 15a, b, c) tell us the sensitivity of a country that provides only p (spending m_P solely on protection) to the provision of L from a country that provides only L (and spends m_L on L). In other words (14, 15a, b, c) tells us the slope of one single p-providing country's reaction curve m_P to changes in m_L. We discuss the equilibrium implications of this fact for a two country equilibrium in section 5.

productivities/utilities of insurance and of protection interdependent. [11] Thus inferiority of M arises in the most general models and requires no special assumptions.

Second, the main difference between these two approaches to defense occurs when the baseline risk increases. Specifically, when M finances self-insurance if the probability of bad state (1-p) rises, M ---spent for L(M) --- is always stimulated. But if M finances self-protection when the loss in the bad state (L) rises, M --- spent for p(M) --- is not always stimulated. If M (allocated to p(M) for self-protection) is normal M may well be depressed, and therefore p(M) may decline if L shifts up.

3. Alternative Equilibria in Provision for Common Security

3.1 Nash vs. Stackelberg Equilibria

Under voluntary uncoordinated behavior, all interactions among members of a public good group are transmitted by income effects. Accordingly for gauging equilibrium in security management we will focus on the sign of income effects. First, reaction functions in space (m_1, m_2) show a Nash equilibrium as in Figure 1 with curve $N^1(m_2)$ for country 1 and $N^2(m_1)$ for 2. As with any good, if M is normal ($dM/dY^* > 0$), the absolute value of the slope of N^1 with respect to the m_2-axis is less than 1 ($-1 < dm_1/dm_2 < 0$, (Cornes and Sandler, 1984, 1996)). Therefore, Nash point N is stable. But if the public good M is inferior $(dM/dY^* < 0)$ then $dm_1/dm_2 < -1$, the (absolute) slope of country 1's reaction curve (with respect to the m_2-axis) is greater than 1, and the interior Nash equilibrium point is unstable (Figure 2**).**

Next consider Stackelberg equilibrium. Suppose country 1 is the leader and country 2 is the follower, country 1 can choose its highest utility point on 2's reaction curve, N^2. In Figure 1, where M is normal for both

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11 If, as a special case we assumed that marginal productivities/utilities of M spent on insurance and on protection are independent then it is easy to show that M for insurance is always normal, as is well known from early work of Gorman (1959) in utility theory. In this case M enters into the original utility function like (1) additively , so we may write

$$W_i = pU^1(C_i) + (1-p)U^0(C_i - L_i) + V(M) \tag{1a}$$

$$\frac{dM}{dY^*} = \frac{pU^1_{YY} + (1-p)U^0_{YY}}{pU^1_{YY} + (1-p)U^0_{YY} + V_{MM}} > 0$$

countries, S gives the Stackelberg equilibrium. Here S is an interior point where both countries provide public goods; $m_1, m_2 > 0$. Comparing N and S, m_1 is lower, m_2 is higher, and M is lower at S than at N --- a well known result.

But now let M be inferior for both countries as in Figure 2; then the Stackelberg point is given at a corner N_C where Follower-country 2's reaction curve meets the abscissa and $m_1 = 0$. Now $M = m_2$ and country 2 alone provides all of the public goods while Leader-country 1 completely free-rides. Here we have multiple (three) Nash equilibria, N_A, N_B, and N_C . And the Stackelberg point when country 1(2) is the leader is the same as Nash point N_C (N_B) at $m_1=0$ ($m_2=0$). Note that security spending at N_B or N_C is greater than at N_A in Figure 2. This new result follows, in brief, from the fact that the effect on aggregate public good provision of a switch from Nash to Stackelberg equilibrium depends crucially on income effects. When income effects are normal (inferior) aggregate Nash (Stackelberg) provision is greater than Stackelberg (Nash) provision. But whether inferior or normal, a Stackelberg leader always reduces his contribution of m_i to M in any switch from Nash to Stackelberg equilibrium.[12] Since inferiority of M is entirely possible in the provision of mutual self-insurance and/or mutual self-protection as public goods, this result applies directly to alliance behavior.

This analysis demonstrates, therefore, that if M is normal for any move from S to N the value of m_1 increases, m_2 decreases and M increases. On the other hand, if M is inferior, then for any move from (N_B or N_C) to N_A, M decreases, and the system becomes unstable. Hence, if M is normal, then such a move raises M and both countries benefit from a reduction of emergency costs. But, if M is inferior, then such a switch from Stackelberg to (unstable) Cournot/Nash would not raise M and due to a higher risk of war/adversity <u>both countries would not always gain</u>. Moreover, if unstable Nash behavior obtains at N_A (rather than N_B or N_C) with spending on national security inferior, the "inferior goods barrier" identified above and described in section 4.2 may still create an obstacle to provision of international public goods compared with the Stackelberg solution.

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[12] Note also that if M is inferior, N_A is unstable but N_B and N_C are stable in a non-cooperative Nash game. And N_B and N_C remain stable under a two-stage dynamic game like Stackelberg. N_B or N_C and are sub-game perfect equilibrium points.

3.2 An Application of Stackelberg vs. Nash: Interactions between the US and Japan

For the world of real international conflict the Stackelberg leader-follower model merits consideration. In this model the leader may choose its own optimal point on the reaction curve of the follower. Such equilibrium is sub-game perfect and should be stable. In contrast, remember, in a Nash game if the public good is inferior, the interior equilibrium is unstable --- a comparison of particular relevance, we think, to the US-Japan relationship with respect to provision of international public goods.

Here a natural conjecture is that the US is the leader and Japan the follower. During the cold war era when the US was dominant the Stackelberg model assumedly would apply. There, even if the public good is inferior, we still have a stable equilibrium. But after the Cold War Japan's presence became stronger so the Nash model should be more relevant. Should the structure of interaction change to a non-cooperative Nash game, when the public good is inferior then the interior equilibrium becomes unstable. Hence, we could conjecture that if the public good is inferior, the system of mutual security between Japan and the US has become unstable in recent years. Similar changes are plausible for the US-EU relation.

But was it really the US which acted as a Stackelberg leader in public good provision during the Cold War? Contrary to the conventional wisdom, our model suggests maybe not! Of course, indisputably the US provided a lot of defense during the Cold War. Japan's spending was constrained by its Constitution. But since Japan could not commit much to spending on defense, the US had no choice but to provide a huge amount of international public good in order to cope with security risks in the Pacific area. Paradoxically then, in terms of our model, Japan could have been regarded as the leader and the US as the follower during this period. But now, Japan has grown much and its Constitutional constraint has become softer. So for describing Japan-US relations in the current era the Nash non-cooperative game may be superior.

4. Replacement Functions and Income Effects: Resulting Inferior Goods Barrier to Alliances Provision

4.1 The Replacement Function

Cornes and Hartley (2000, 2003) have suggested an elegant construction that allows a direct visualization of which members of a public good group will make positive contributions in Nash equilibrium, how much will be supplied, and how this outcome changes with group size, group and individual income, and composition. We can use this method to demonstrate the consequences of public good inferiority for group provision and its stability, and thus our claim that there is an "inferior goods barrier" which limits group size and aggregate provision, and induces instabilities in group behavior. They define a "replacement function" (or more generally replacement correspondence) as follows. Let the Nash reaction function be given as

$$m_i = N^i(Y_i, M_{-i}) = N^i(Y_i, M - m_i); \tag{16a}$$

$$M_{-i} \equiv \Sigma_{j \neq i} m_j; \tag{16b}$$

$$N^i_{M-i} \equiv \partial m_i / \partial M_{-i} \tag{16c}$$

where M_{-i} indicates public goods provided by all agents except agent i. Then the replacement function is:

$$m_i = r^i(Y_i, M) = Y_i - C_i(M) \tag{17a}$$

where C_i (M) denotes country i's optimal choice of C_i as a function of M. The geometric derivation of this function in the space of (M, m_i) is shown in Figure 3. It follows that

$$r^i_M \equiv \partial r^i / \partial M \quad = N^i_{M-i} / (1 + N^i_{M-i}) \tag{17b}$$

This immediately yields the Cournot-Nash equilibrium where the aggregate of individual replacement functions Σr = *R* crosses the 45°through the origin, i.e. at

$$\Sigma m_i = M = \Sigma[r_i(M)] \equiv R \tag{18}$$

To use r^i and *R,* it will be helpful first to relate their properties to the underlying Nash reaction functions. If M is a normal good, then -1< N^i_{M-i} < 0, and hence r^i_M < 0. In this case the individual replacement function is decreasing with M and we designate the function as "Normal." On the other hand, as shown in Ihori and McGuire

(2006) if M is an inferior good, then $1+N^i_{M-i} < 0$, whence $N^i_{M-i} < -1$ and hence $r^i_M > 1 > 0$. Now the individual replacement function is increasing in M with its slope greater than 1, and we call this function "Inferior."

Figure 4-1, describes the normal case. Assuming two identical countries N_2 shows the Nash equilibrium and S_2 is the Stackelberg equilibrium points. N_2 is given by the point where the aggregate replacement curve (r^1+r^2) intersects the 45 degree line. 2BD = BN_2. S_2 is given where country 2's replacement curve (r^2) is tangent to country 1's indifference curve, I. Note that country 1's indifference curve is defined as

$$\overline{W}_1 = W_1(c_1, M) = W_1(Y_1 + m_2 - M, M) \qquad (19)$$

The indifference curve is downward sloping if M is low, while it is upward sloping when M is high.

4.2 The Inferior Good Barrier to Group Size and Public Good Provision

Theory

The replacement function easily shows the consequences of income growth within an alliance, but it is especially useful for deriving the impact of an increase in the number of alliance members, n. Just to illustrate, without loss of generality, suppose country 3 joins a pre-existing alliance of countries 1 and 2. For simplicity, let country 3 be identical to countries 1 and 2. Then Figure 4-1 shows a new Nash point, N_3. Enlarging the membership size of the alliance will increase M if M is normal although m_i declines as compared to the old point N_2. EC = m_i for n=3 (< DB = m_i for n=2). In contrast at a new Stackelberg point, S_{23}, in Figure 4-1 M (assumed normal) increases even though $m_1 = m_2$ declines compared to the old Stackelberg point, S_2. Here we assume that 1 is the leader and 2 and 3 are followers. S_{23} is on the aggregate replacement curve for countries 2 and 3 (r^2+r^3).

Now in contrast let M be inferior for low incomes but normal at high incomes. Figure 4-2 pictures this case. The individual replacement function for either agent is then shown in Figure 4-2 by curve AC; section AB applies when M is normal and BC when it is inferior. In Figure 4-2, N_2 is the Nash equilibrium point and S_2 is the Stackelberg equilibrium point. When the good is inferior, Stackelberg equilibrium is given by a corner solution where the leader provides no public good at all. Note that this point is the same as the (stable) Nash point at the corner solution. If country 3 joins a pre-existing alliance of countries 1 and 2, N_2 is the old Nash and S_2 the old

Stackelberg point. Now in this inferior good case at a new Nash point, N_3, M, and $m_2 = m_3$ all decrease when membership grows. Similarly at the new Stackelberg point S_{23}, (the same point as N_2), M, and $m_2 = m_3$ all decline. (In all these cases of Stackelberg equilibrium in Figure 4-2 m_1=0.)

Thus when the public good is inferior, enlarging the membership of a group does not lead to an increase in the total provision --- demonstrating an "inferior good barrier" to voluntary risk reduction. Ihori and McGuire (2006) have developed the details of a similar inferior good barrier in the Nash equilibrium when M is used for self-protection to improve p. Thus our present analysis reflects the generality of the inferior good barrier and a *prima facie* assumption for its existence. It applies more generally to a variety of cases, specifically in Stackelberg equilibrium (and/or in the self-insurance model) as well. We could show a similar result when the income of allies grows. That is, remarkably if M is inferior, economic growth (an increase in Y) would depress spending on national security, diminishing economic welfare. If this negative effect dominates the positive welfare effect of growth itself, we have the paradoxical outcome of "immiserizing growth"[13]. Similarly, we could show this effect when baseline risk shifts exogenously --- following the argument of section 2.3 above --- this can cause the good of self protection to switch from normal to inferior or from inferior to normal[14] In all cases the paradox follows from differences in individual behaviors and group outcomes when a good is or becomes inferior.

Empirical Plausibility?[15]

Some may find the plausibility of inferior good case and hence the existence of an "inferior goods barrier" contrary to casual observation. And of course, the only way to really confirm or falsify the hypothesis would be through precise and rigorous empirical test. Here, we can only propose some components of a more rough and ready procedure.

With respect to self-insurance eq. (8e) implies a connection between risk aversion, baseline hazard, and

[13] Results are available on request.

[14] As mentioned Ihori and McGuire 2006, derived a crucial switching probability at which the change from inferior to (normal or vice versa) depends on risk aversion parameters of individual agents.

[15] We are indebted to Robin Boadway for raising these questions.

inferiority/normality. In this case, the inferiority of public good M (and thus of -L) follows from the risk aversion properties of U with sign independent of "baseline risk." But the strength of the effect should depend powerfully on the product of baseline risk, spread in the risk aversion parameter R, and income level or effective marginal utility of income, U_Y As far as self protection is concerned, basically our theory claims that a cross-over value of p = p * exists such a that as security risk moves from p < p* to p = p* and on to p > p*, depending on whether risk aversion is decreasing (or increasing) security as a good changes from normal (inferior) to inferior (normal) and the degree, magnitude, or quality of free riding among group members increases (decreases). We make the same assertion for the case of self insurance.

To test this idea one needs data relating to (a) the quality of risk aversion of countries taken as a whole or of their governments, (b) an empirical measure of total or average free riding (and security spending) in the relevant groups, and (c) some indication of whether group provision is characterized by instabilities and corner solutions, by Nash or Stackelberg outcomes. With this information we could construct an empirical test when the test environment is constant.[16]

5. Impure Public Goods and Country-Specific Risk/Loss

We now will extend our analysis from mutual insurance and protection as pure public goods, to the case where spillovers among countries are imperfect or even negative.[17] The pure public good assumption is heroic even among the closest of allies such as Japan, Canada, Britain or the US. But to change it is a big step because the details hidden within the assumption of pure publicness are so restrictive. To see this consider the case of self-protection. Our formulation, i.e., p= $p_1 = p_1(\Sigma m_i) = p_2(\Sigma m_i)... = p_n(\Sigma m_i)$ is a drastic simplification of the

[16] For security spending on self protection or insurance, expenditure patterns should be sensitive to exogenous shocks to the baseline security level p or emergency loss L. For exogenous shocks the collapse of the USSR could provide nicely. Here was a sudden reduction in the likelihood of war in NATO. Was this soon followed by more or less free riding by alliance member in the aggregate? But can one identify, and collect or construct independent indices of risk aversion for various NATO countries? Similarly, the terror attacks of 9/11/2001 could provide a natural experiment with an exogenous increase in p or L.

[17] Still dealing only with the provision of either insurance or protection, we ignore cases where individual countries may mix their provisions of p and of L or where alliances may involve specialization of provision among different members.

more general and more realistic

$$\begin{aligned} p_1 &= p_1(m_1, m_2...) \\ p_2 &= p_2(m_1, m_2...) \\ &\vdots \\ p_n &= p_n(m_1, m_2.....m_n) \end{aligned} \qquad (20)$$

A scheme like (20) underlies much more than just an allocation problem. Rather it summarizes an entire strategic geopolitical situation. Following Hirshleifer and Riley (1975) in the context of two countries consider how p_1 and p_2 may be related. They may be independent and uncorrelated. They may show perfect positive correlation, and if identical yield the public good model we considered above --- a so called "common risk." They may have perfect negative correlation so that $p_1 = (1 - p_2)$ --- a "social risk." Or p_1 and p_2 may lie between these extremes in any way whatsoever. From this perspective, our model shows only the tip of an iceberg of daunting complexity. We have assumed not only perfect equivalence between p_1 and p_2 but also perfect equivalence and substitutability among inputs m_1 and m_2. That is, we have assumed that "summation finance" characterizes this social composition function (terminology of Hirshleifer, 1983).

Here we can take a first step to modify these heroic assumptions and thereby nudge our model toward greater realism. Specifically rather than pure public good and summation finance let us permit differences in effectiveness of providing the public goods among allies. Then, in place of (4) we have

$$M_i = m_i + \sum_{j \neq i} \varepsilon_{ij} m_j \qquad (21)$$

where ε_{ij} denotes the effectiveness of country j's contribution to country i's public good (compared to $\varepsilon_{ii} = 1$). This change makes risk and loss country specific --- which now resemble imperfection or impurity of public good provision in conventional single contingency analyses[18]. In our multi-contingency framework we will see that such imperfect spillovers equate to differences in country specific risk, and allow for risk transfer as well as risk reduction. This innovation thus extends the practical relevance of this model to a larger universe of strategic

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[18] Hirshleifer's (1983) "social composition function" and Cornes and Sandler (1996) generalizes such differential spillovers thus $M_i = \Sigma_{\forall j}[m_j]^{\kappa}$

situations beyond "perfect alliances" with perfectly correlated national interests.

Thus for mutual self insurance in place of (4b) we would now have

$$L_i = L(M_i) \tag{22a}$$

And for mutual self protection model in place of (10a) we have

$$p_i = p(M_i) \tag{22b}$$

We will illustrate the effects of this modification for a two country model where

$$M_1 = m_1 + \varepsilon_{12} m_2 \tag{23a}$$

$$M_2 = \varepsilon_{21} m_1 + m_2 \tag{23b}$$

Suppose ε_{12} is high, while ε_{21} is low. Then an increase in m_2 would greatly contribute to improving Country 1's risk profile by reducing L_1 or $(1\text{-}p_1)$ a lot, but an increase in m_1 would not contribute much to reducing Country 2's risk profile (L_2 or $(1\text{-}p_2)$). And ε_{ij} could be negative, in which a case security outlays shift risks and losses rather than reduce them. This even could make the two countries enemies rather than allies. In short we can model a variety of international relations, conflicts, and ambiguities by changing the sign and value of ε_{ij} .

5.1 Comparative Statics When Effectiveness ε_{ij} Varies Across Countries: The Generalized Replacement Function

For analysis of the effect of imperfect spillovers on equilibria, we will first extend and generalize the Cones-Hartley replacement function. Introducing ε_{ij} the Nash reaction function, instead of Eq. (16a) becomes

$$m_i = N^i(Y_i, \varepsilon_{ij} m_j; p\, or\; L) \qquad \text{(for i, j=1, 2).} \tag{16a*}$$

And, therefore, rather than Eq. (17a) country i's "generalized replacement function" becomes

$$m_i = r^i(Y_i, M_i\, ; p\, or\, L) = Y_i - C_i(M_i; p\, or\, L) \tag{17a*}$$

Here, the condition "p or L" indicates that the replacement function refers *only* to provision of one *or* the other, not both mixed together. Function r_i can now be used again to show the effects of differential technology or effectiveness ε_{ij} on group Nash and Stackelberg equilibria. r_i is independent of ε_{ij}. Again, as in the pure public good case, Eq. (17a*) will contain p (or L) as shift parameters in the generalized self-insurance (or protection) replacement function. Substituting the new replacement functions into (23a) and (23b), gives

$$r^1(M_1; p\, or\, L) + \varepsilon_{12} r^2(M_2; p\, or\, L) = M_1 \tag{24a}$$

$$\varepsilon_{21} r^1(M_1; p\, or\, L) + r^2(M_2; p\, or\, L) = M_2 \tag{24b}$$

To see the effect of improved efficiency by one partner total differentiation of this system gives

$$\begin{bmatrix} r_M^1 - 1, & \varepsilon_{12} r_M^2 \\ \varepsilon_{21} r_M^1, & r_M^2 - 1 \end{bmatrix} \begin{bmatrix} dM_1 \\ dM_2 \end{bmatrix} = \begin{bmatrix} -r^2 \\ 0 \end{bmatrix} d\varepsilon_{12} + \begin{bmatrix} 0 \\ -r^1 \end{bmatrix} d\varepsilon_{21} \tag{25}$$

So greater efficiency by country 2 alone leads to:

$$\frac{dM_1}{d\varepsilon_{12}} = \frac{1}{\Lambda}(-r^2)(r_M^2 - 1) \tag{26a}$$

$$\frac{dM_2}{d\varepsilon_{12}} = \frac{1}{\Lambda} r^2 \varepsilon_{21} r_M^1 \tag{26b}$$

where $\Lambda \equiv (r_M^1 - 1)(r_M^2 - 1) - \varepsilon_{12}\varepsilon_{21} r_M^1 r_M^2$.

Assuming M to be normal, $r_M^i < 0$; then since $\varepsilon_{12}\varepsilon_{21} < 1$, in this case $\Lambda > 0$, the sign of (26a) is positive, and of (26b) negative. An increase in ε_{12} (the effectiveness of country 2's public good provision on country 1's security) will raise M_1, improving country 1's risk profile, while reducing M_2, lowering country 2's risk profile. The intuition for this effect is as follows: an increase in ε_{12} raises the effective income of country 1, thereby raising M_1 and improving the risk profile of Country 1. Hence, country 1 can increase private consumption by reducing m_1 and free riding on country 2's higher productivity. But this reduction of m_1 would diminish M_2, and country 2 would suffer from an increase in emergency costs (higher 1-p_2 and/or L_2). Eq. (26b) also implies quite plausibly that if $\varepsilon_{21} = 0$, then (26b) is zero, and M_2 is independent of m_1, so that any change in ε_{12} does not affect country 2.

Such an increase in ε_{12} (effectiveness of country 2's public good provision of on country 1's security) might realistically occur when the structure of country 1's emergency risk changes. This could happen when an outside enemy country (country 3) threatens country 1, and 1's ally, country 2, develops a technology, which can reduce the threat more efficiently. For example, suppose North Korea (country 3) has missiles, which mainly threaten Japan (country 1), and the US (country 2) develops a technology to neutralize this threat more efficiently --- a case to be analyzed as above as an increase in ε_{12}.[19]

5.2 Comparative Statics When Emergency Costs Differ Among Countries

The foregoing generalized/extended replacement function also leads us to greater insight into the effects of a parametric change in any one single country's p_i (or L_i), on group provision (respectively) of L (p). in equilibrium.[20] Thus we can identify the spillover effects of exogenous change in one country on its own welfare and that of its partners. Would deterioration in p_k (or L_k) actually hurt or help country $s \neq k$? An answer to this indicates the degree to which individual country's interests of nominal partners are actually aligned or not. Here we can learn how the universal incentive to free ride works itself out in an expected-utility Cournot-or-Stackelberg world. Specific results vary with assumptions as to normality vs. inferiority, and Nash vs. Stackelberg behavior. But we show, for the most part that "allied" countries benefit from a deterioration in a partner's position. This is mainly because a country's response to an exogenous decline in its security is to increase its security spending. This in turn has positive spillover effects on its allied partners who can then can free-ride all the more.

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[19] The analysis readily extends to Stackelberg behavior and to public good inferiority. For Stackelberg equilibrium (with M normal) analytical results are qualitatively the same as the foregoing. On the other hand, for Stackelberg equilibrium (with M inferior) or Nash equilibrium (where M is inferior) analytical results are not the same. For instance, suppose Country 1 is the leader (or Country 1 contributes nothing at all at a stable Nash solution), then an increase in ε_{12} does not affect m_2. Since $m_1 = 0$ at the corner solution, a shift in ε_{12} would not affect m_1 either. Thus, (for M normal or inferior), neither m_1 nor m_2 change when ε_{12} increases but M_1 would rise and Country 1 gain. Alternatively if country 1 is the follower (or country 2 does not contribute at all at a stable Nash solution), then after an increase in ε_{12} M_1 rises and m_1 declines while m_2 remains equal to zero at the corner solution. In this case both Country 2 as well as Country 1 can gain from an increase in ε_{12}.

[20] As an example for differential risks, p_i consider the risk of tsunami. The actual loss due to tsunami could be reduced by mutual insurance $L(m_1+m_2)$, but the probability of tsunami or earthquake could be different for two countries and the benefits of warning diverse. Then, to assume that p_1 only changes could be realistic

We begin with identification of the incentives an exogenous shift creates for individual countries and their corresponding individual m_i -responses to such shifts. Table 1 shows how a country's replacement function for the provision of insurance (or protection), shifts when it experiences a shock to its risk p (or its loss L) respectively.

Table 1

Type/Purpose of Security Expenditure	Effect of Shift in p (L) on Own Expenditures on L (p)		Income Effect in Country's Expected Utility: Normal	Income Effect in Country's Expected Utility: Inferior
Self Insurance L [21]	$\frac{\partial m_i}{\partial p_i} \equiv r_p^i = \frac{\partial N^i / \partial p_i}{1+\partial N^i / \partial M_{-i}} \equiv \frac{N_p^i}{1+N_{M-i}^i}$	(27)	-	+
Self Protection p [22]	$\frac{\partial m_i}{\partial L_i} \equiv r_L^i = \frac{\partial N^i / \partial L_i}{1+\partial N^i / \partial M_{-i}} \equiv \frac{N_L^i}{1+N_{M-i}^i}$	(28)	likely -	-

Now we can use the information of Table 1 to derive the effect of a country-specific shift in parameter p_i (or L_i) on the Nash equilibrium solution for M-allocated-to-L_i (or to p_i) respectively. These results include cases where baseline risk or loss differs among countries (e.g. $p_1 \neq p_2$ *or* $L_1 \neq L_2$), but for simplicity we will assume public goods are pure: $\varepsilon_{12} = \varepsilon_{21} = 1$ and hence $M_1 = M_2 = M$. The results of this procedure are summarized in Table 2. Very briefly, if M is normal, the spillover welfare effect of a country-specific increase in the emergency cost, 1-p_1 or L_1, in country 1 on country 2 is are the opposite between self protection and insurance. On the other

[21] When M finances self insurance if risk aversion is increasing and therefore M is normal r_{iP} is negative; as shown by (9a, 9b), its numerator is negative and denominator positive. This relationship between the notation between (27) and (9a, 9b) cans be seen as follows. For simplicity, suppose $\varepsilon_{ij} = 1$. Then $M = N^i(Y_i, m_j, p \text{ or } L) - m_j$ Hence, $\partial M/\partial p = \partial N^i/\partial p$ and $\partial M/\partial L = \partial N^i/\partial L$. But if risk aversion is decreasing and M, therefore, inferior, both the numerator and denominator are negative so (27) would then be positive.

[22] Now, as in (14), if M is inferior, the numerator of (28) becomes positive the denominator negative, and hence, (28) is negative. With positive denominator (28) is likely to be negative if M is normal, since the numerator is likely to be negative. That is, as derived in Ihori and McGuire 2006, if absolute risk aversion is increasing and (1-p) initially low, or if absolute risk aversion is decreasing and (1-p) is high M is normal. But if absolute risk aversion is decreasing and (1-p) initially low or increasing and (1-p) is initially high, M becomes inferior. From (15) if M is normal and this normality is strong enough to make M_L positive then (28) becomes negative

hand, if M is inferior, the spillover welfare effect is always positive.

Table 2: The Effects of Increases in Country-Specific Emergency Cost On Voluntary Public Good Group Equilibrium Outcomes

Case	A	B	C	D
	Self-Insurance		Self-Protection	
	M is Normal	M is Inferior	M is Normal	M is Inferior
m_1	+	0* +**	-	0* +**
m_2	-	0	+	0
M	+	0* +**	-	0* +**
W_2: Welfare of Country 2	+	0* +**	-	0* +**

* Assuming Country 1 is the leader (or country 1 does not contribute at all at a stable Nash solution)
**Assuming Country 1 is the follower (or country 2 does not contribute at all at a stable Nash solution)
Case A: Effect of an increase in 1-p_1 in country 1 on aggregate M for self insurance, m_1, m_2, W_2: M is normal
Case B: Effect of an increase in 1-p_1 in country 1 on aggregate M for self insurance, m_1, m_2, W_2: M is inferior
Case C Effect of: an increase in L_1 in country 1 on aggregate M for self protection, m_1, m_2, W_2: M is normal
Case D: Effect of an increase in L_1 in country 1 on aggregate M for self protection, m_1, m_2, W_2: M is inferior

Effect of a Country-Specific Shift in p on M provided for Self-insurance L (Columns A and B)

For M allocated to self-insurance, L(M), in place of (26a) and (26b), Eq. (29), shows the effect of an increase in p_1 on M at a Nash solution to be:

$$\frac{dM}{dp_1} = \frac{1}{\Lambda} r_p^1 \tag{29}$$

A: M Normal (See Column A of Table 2),

Now if M is normal, then (29) is negative and an increase in the probability-of-the good-state (solely in Country 1), p_1, depresses the total provision of self insurance, M, for both countries. Put another way, an increase in the probability of war, for Country 1 alone, (i.e. (1-p_1)), will stimulate aggregate self insurance spending for both countries, M. This increase in 1-p_1 will raise m_1 and reduce m_2, but if M is normal, the increase in m_1 dominates the decrease in m_2, resulting in greater total M Stackelberg solutions give similar analytical results if M is normal. Thus, greater 1-p_1 has a positive spillover effect on country 2's welfare. But the effects on m_i and M change if M is inferior --- a configuration we consider next.

B: M Inferior (See Column B of Table 2)

Suppose M is inferior and Country 1 is the leader (or contributes nothing at all at a stable Nash solution), then for higher 1-p_1 neither m_1 nor m_2 would change. But if Country 2 is the leader (or contributes nothing at all at a stable Nash solution), then m_1 increases and m_2 remains zero for greater p_1. Again, an increase in 1-p_1 has a positive spillover effect on country 2's welfare.

Effect of a Country-Specific shift in L on M provided for self-protection, p.(Columns C and D).

When m_1 and m_2 and M finance self protection the effect of a parametric shift in L_i may be analyzed in a similar way by considering (30).

$$\frac{dM}{dL_1} = \frac{1}{\Lambda} r_L^1 \tag{30}$$

C: M Normal. (See Column C of Table 2)

With M normal, (30) is always negative. An increased war-loss solely in Country 1 depresses its defense spending for self-protection. Here an increase in L_1 reduces m_1 and raise m_2, and since the decrease in m_1 dominates the increase in m_2, total M will decline creating a negative spillover effect on country 2's welfare

D. M Inferior (See Column D of Table 2).

But where M (spent for protection) is inferior (entirely plausible as argued earlier), if country 1 is the leader (or contributes nothing at a stable Nash solution), then neither m_1 nor m_2 change with an upward shift (i.e. an increased loss) in L_1. If, however, Country 2 is leader (or contributes nothing at a stable Nash solution), then for an increase in $L_{1,}$ (greater loss in Country 1) m_1 rises and m_2 remains zero In this latter case, an increase in the loss in emergency in one country (Country 1) will raise group total expenditure on self protection and paradoxically raises (has a positive spillover effect on) the other country‘s (Country 2's) welfare.

Summary:

We can summarize by saying when emergency costs such as the probability of war or loss in the bad state increase solely in one country (1), that country loses and creates a spillover for its partners. Table 2 derives the ultimate implications, showing the effects of such spillovers on the allied country (2). As shown in most of cases (case A, B, D) the partner country 2 will gain (or not lose). This is mainly because an increase in security spending by country 1 has a positive spillover effect on its ally, and hence country 2 can free-ride on this increase in m_1. When M is inferior and country 1 is the leader, m_1 does not change, so that country 2 does not lose from the higher emergency cost of country 1 only. Obviously this misalignment of incentives can create a potential for instability or conflict among allies since each partner country may not be much concerned with reducing its own country-specific emergency risk. Only in case C (where country 1 reduces its spending in response to greater loss, L_1) is there a negative spillover effect on country 2. Here both countries suffer from an increase in L_1. But this is the only configuration where countries interests are actually aligned such that both are concerned with reducing country-specific increase in L.

Remark

The comparative statics results in this section may be relevant when country 1 provides only protection as a public good, $m_1 = M_1$, while country 2 provides only insurance $m_2 = M_2$ as a public good. Suppose that the two countries interact according to Nash Cournot behavior. Then when 1 provides m_1 it serves to shift p as a parameter in 2's utility function. This shift may stimulate or curtail 2's expenditure on m_2. The possibilities of normality or inferiority were shown in section 5.2. When 2 provides m_2 it serves to shift L as a parameter in 1's utility function. This serves to stimulate or curtail 1's provision of m_1.These possibilities of inferiority or normality also were shown in section 5.2. Section 5.2 thus describes the 2 partner system and shows whether the Nash equilibrium will be stable or unstable when one country provides only m_1 and another country provides only m_2.

6. Conclusion

This paper has extended the conventional model of voluntary public good provision into the realm of multi-contingency expected utility. We have demonstrated that the entire structure of expected utility improvement via "security" effort --- efficiency, burden distribution and stability --- depends crucially on which security measures are employed.

We consider two generic threats and corresponding measures for raising expected utility. First, the risks of a bad outcome can be reduced; second, the magnitude of loss in the bad state can be curtailed. We have shown that the benefit from both of these instruments --- denominated respectively "self-protection" and "self-insurance" --- can easily be "inferior" under the classic definition of that term. Moreover, this novel and unexpected result that <u>security can be inferior</u> in a multi contingency context depends specifically on the risk aversion properties of the state-independent utility function.

The implications of this unanticipated income effect are far reaching. As our analysis highlights, if security is inferior the allocative behavior of individual members of an explicit or implicit alliance becomes problematic. The stable equilibria and a sharing of costs that should characterize alliances will vanish, replaced by dominance of corner solutions and instabilities. Comparisons between Nash and Leader-follower Stackelberg solutions also are overturned, when the latter may induce greater total security effort than the former --- just the opposite of the conventional result when income effects are normal.

In addition to these general features, our analysis reveals a new and novel interaction between management of risk and of magnitude of loss. When security expenditures finance self insurance if the probability of the bad state (1-p) rises exogenously, then security efforts toward loss reduction are normally stimulated. But if security efforts go toward improving risk, then when the magnitude of loss (L) in the bad state rises exogenously, these self-protection efforts are not always stimulated. Most paradoxically if security spending on risk improvement is normal, it may well be depressed should the magnitude of loss shift upwards.

We have also extended a theorem of Ihori and McGuire (2006). They showed that when security outlay goes toward collective self-protection, inferiority of this good (under Nash-Cournot behavior) can create an endogenous limit to group membership and collective provision. This means that enlarging the membership of an alliance may well not lead to an increase in the total provision of its public good First we have shown (section 4.2) that this result --- the "inferior good barrier" --- also obtains if security outlay finances loss reduction. And second we have also shown that the similar inferior good barrier occurs in the Stackelberg equilibrium as well.

Lastly the paper develops an extension of the pure public good version of collective risk improvement that can represent a wide variety of realistic cases. Specifically we consider differences among countries in effectiveness or technology of providing security. We modify the pure public goods model to include differences as to comparative advantage in risk management, including risk shifting to others. Further insights into the structure of economic behavior follow. For example if outlays finance self-protection and if M is normal, greater emergency loss in just one country alone will depress the total provision of M. In contrast, when security provides insurance an exogenous increase in the probability of the bad state will stimulate total provision of security spending (M assumed to be normal). Thus if M is normal, the spillover welfare effect is different between self protection and insurance. On the other hand, if M is inferior, the spillover welfare effect is always positive.

Finally, improvements in risk profile may generate non-pure public goods, but instead imperfect, ambiguous, or even negative benefits among partners. Then who is an ally and who an adversary itself becomes ambiguous. For this configuration where the emergency risk profile differs among allies, we show how spillovers and their effects on welfare and thus the alignment of interest among alliance partners will be depend critically on the sign of income effects.

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Figure 1

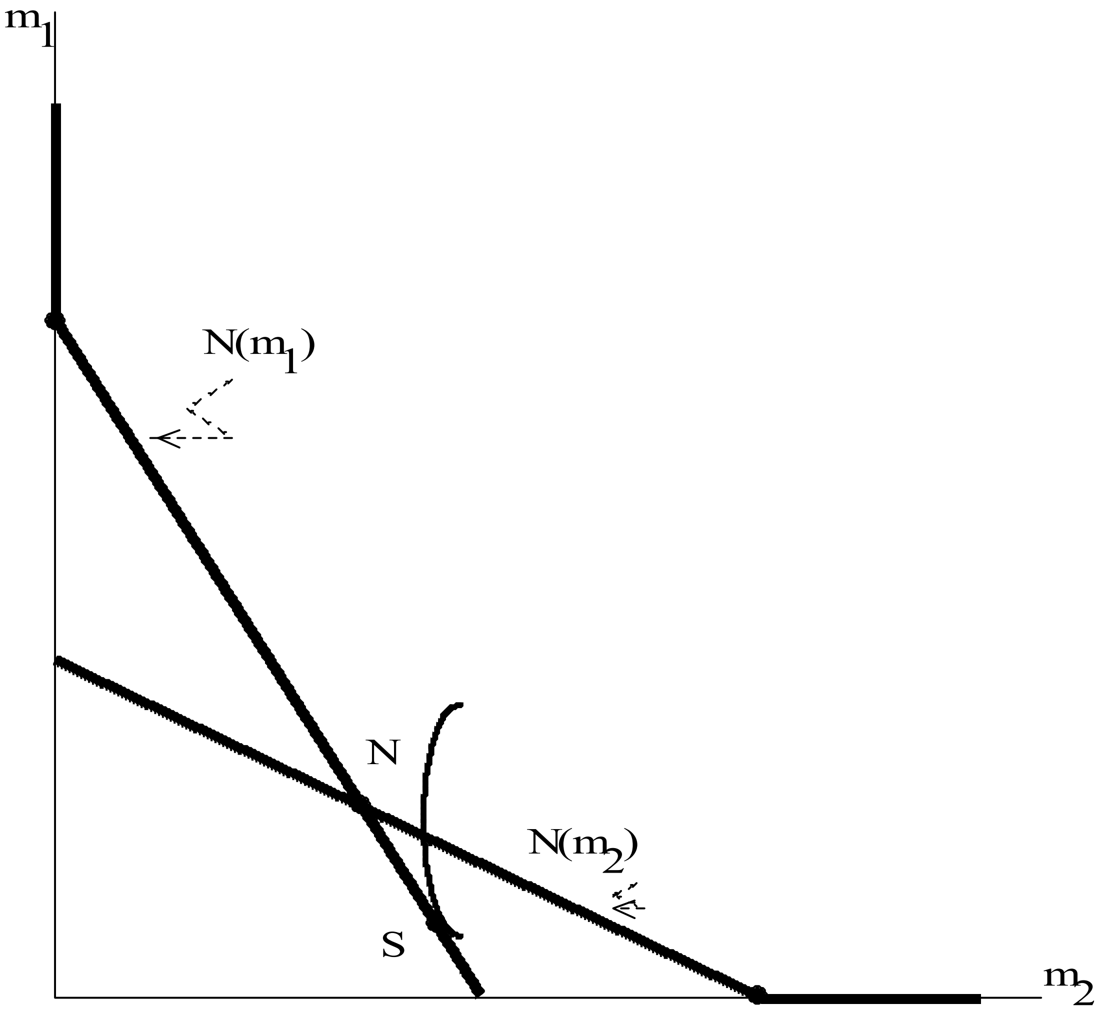


Figure 2

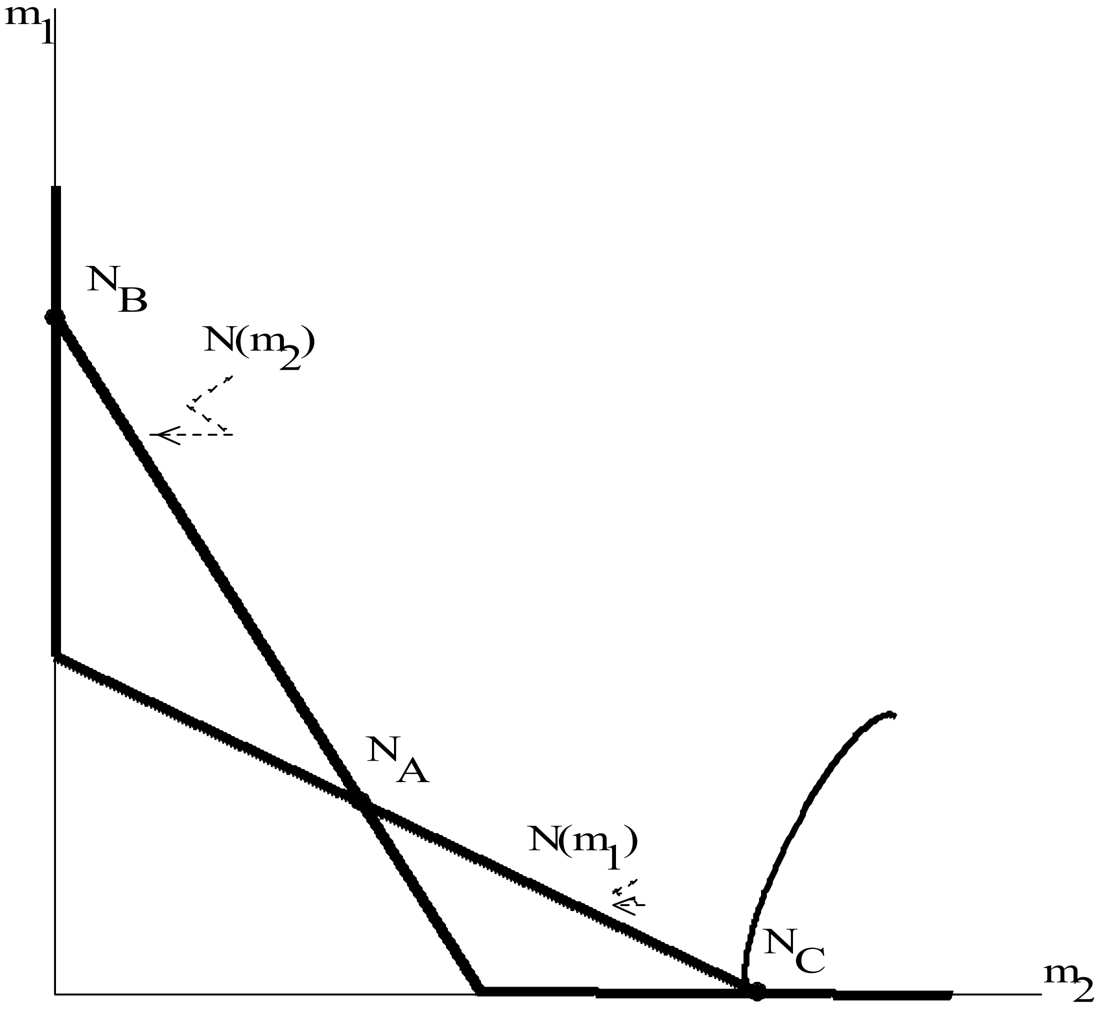
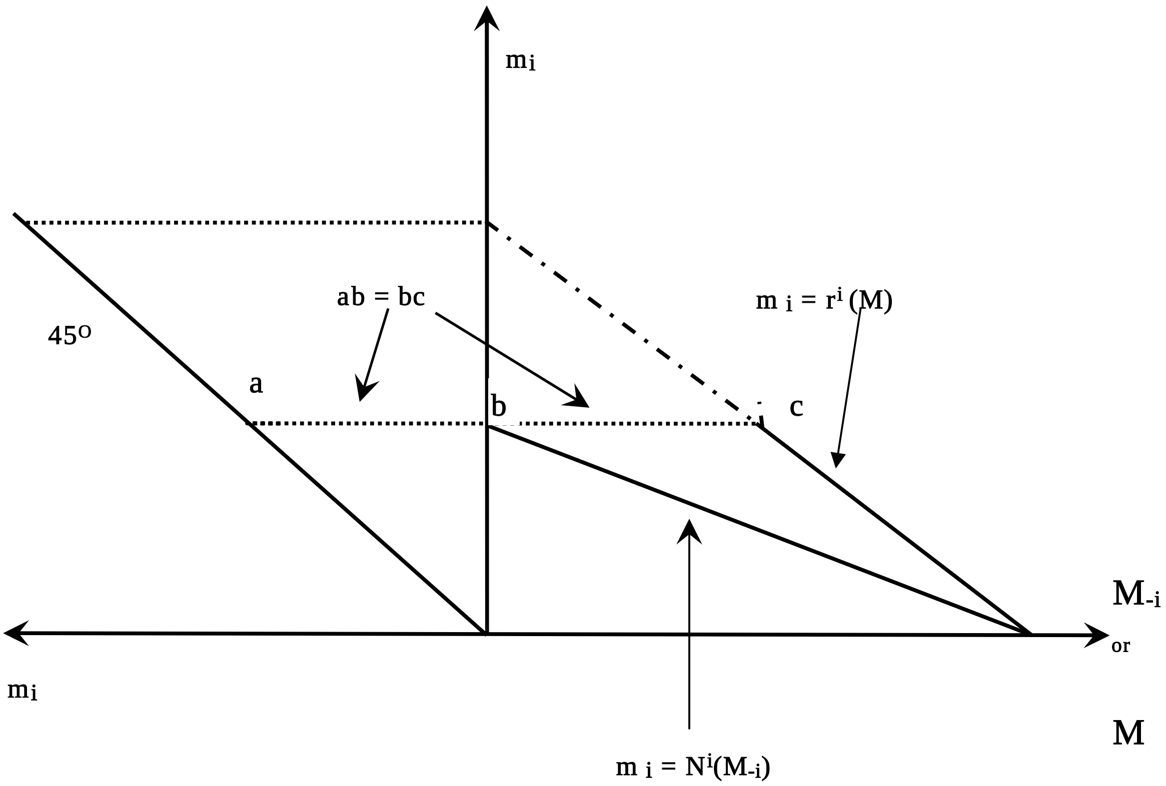


Figure 3



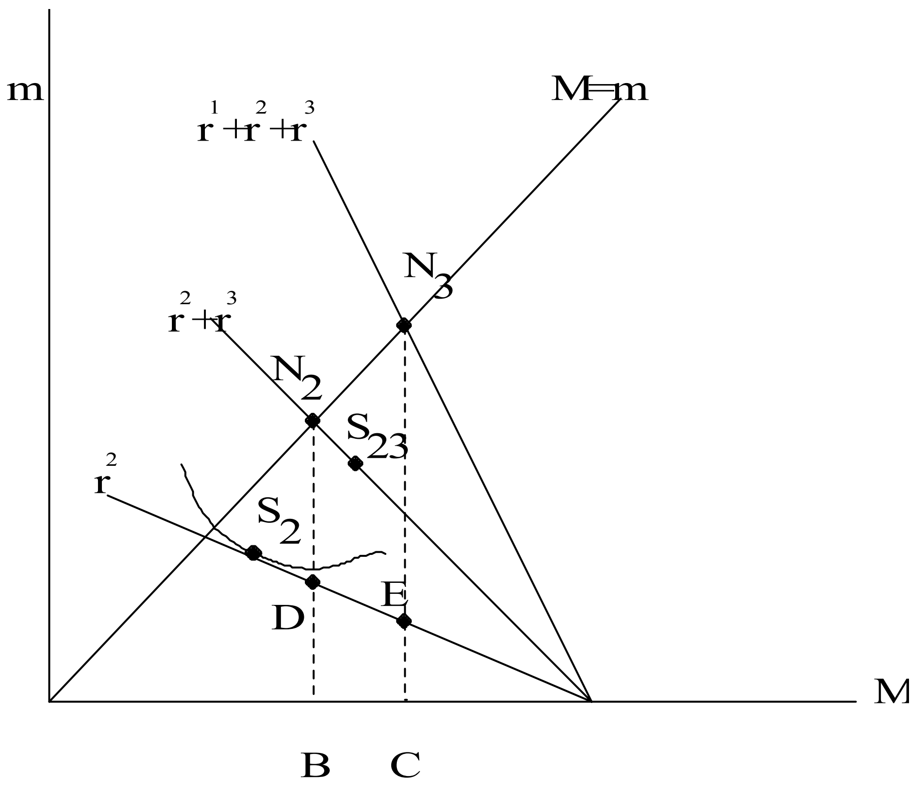


Figure 4-1

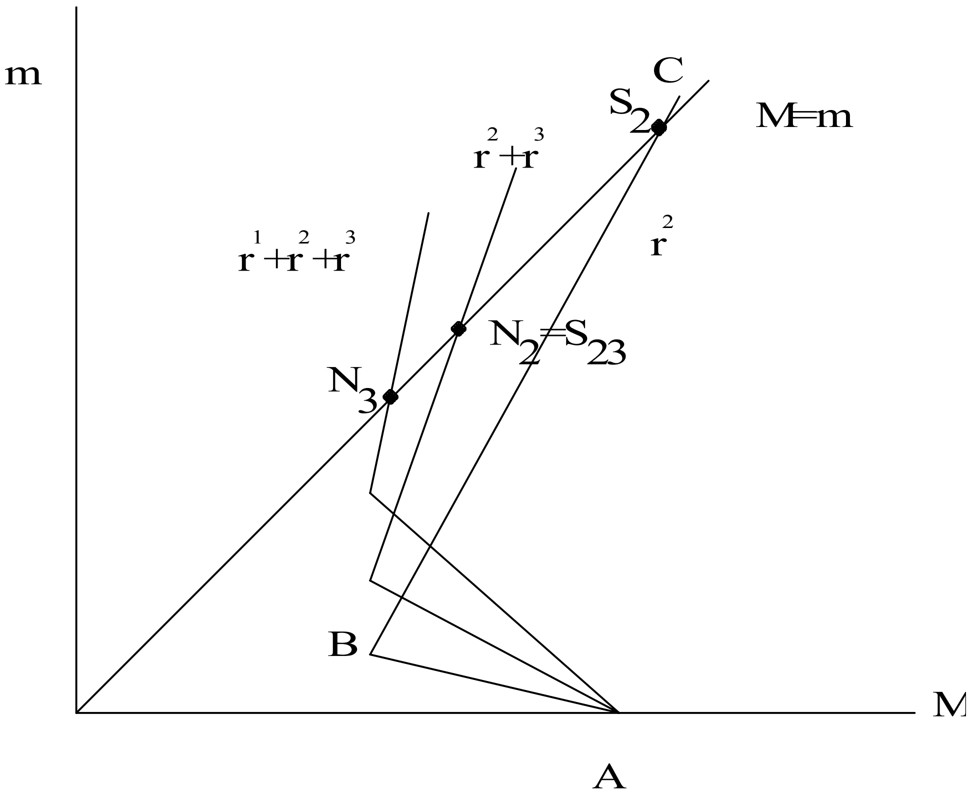


Figure 4-2