CARF Working Paper

CARF-F-268

Asset Bubbles and Bailouts

Tomohiro Hirano
The University of Tokyo
Masaru Inaba
Kansai University/The Canon Institute for Global Studies
Noriyuki Yanagawa
The University of Tokyo

First version: January 2012
Current version: April 2013

CARF is presently supported by Bank of Tokyo-Mitsubishi UFJ, Ltd., Dai-ichi Mutual Life Insurance Company, Meiji Yasuda Life Insurance Company, Nomura Holdings, Inc. and Sumitomo Mitsui Banking Corporation (in alphabetical order). This financial support enables us to issue CARF Working Papers.

Asset Bubbles and Bailouts*

Tomohiro Hirano† Masaru Inaba ‡ Noriyuki Yanagawa§

First Version, April 2011

This Version, April 2013

*This is a revised version of our previous working paper, Hirano, Tomohiro and Noriyuki Yanagawa. (2012 January) "Asset Bubbles and Bailout". We have benefited greatly from discussions with Harald Uhlig, Nobuhiro Kiyotaki, Albert Martin, Jaume Ventura, Fumio Hayashi, Kiminori Matsuyama, and Joseph Stiglitz. We also thank Kosuke Aoki, Jeffrey Campbell, Hugo Hopenhayn, Katsuhito Iwai, Tatsuro Iwaisako, Jianjun Miao, Kalin Nikolov, Fabrizio Perri, Vincenzo Quadrini, Penfei Wang, and seminar participants at 2012 AFR Summer Institute of Economics and Finance, CIGS Conference on Macroeconomic Theory and Policy 2012, 2012 Econometric Society North American Summer Meeting, European Economic Association Meeting, 2012 Midwest Macroeconomics Meeting, 2012 saet conference, and 2012 Royal Economic Society.

†The University of Tokyo

‡Kansai University/The Canon Institute for Global Studies

§The University of Tokyo

Abstract

This paper investigates the relationship between bubbles and government bailouts. It shows that bailouts for bursting bubbles may positively influence ex-ante production efficiency and relax the existence condition of stochastic bubbles. The level of bailouts has a non-monotonic relationship with production efficiency and not full bailouts but a "partial bailout" policy realizes production efficiency. Moreover, it examines the welfare effects of bailout policies rigorously. The welfare of rescued entrepreneurs is an increasing function of bailout level, but the welfare of taxpayers (workers) shows a non-monotonic relation with bailout level. It shows that even non-risky bubbles may be undesirable for taxpayers.

Key words: Asset Bubbles, Anticipated Bailouts, Production Efficiency, Boom-Bust Cycles, Welfare Effects of Anticipated Bailouts

JEL Classification Numbers: E32, E44, E61

1 Introduction

Many countries have experienced bubble-like dynamics, notably the recent U.S. experiences after the global financial crisis of 2007—2009 as well as Japan's experiences in the 1990s. The bursting part of asset bubbles is generally followed by significant contractions in real economic activity. To mitigate these contractions, the government tends to provide various bailouts, such as by purchasing legacy assets at inflated prices or proposing a capital injection policy. Although such bailout initiatives are becoming more frequent, the effects of these policies have thus far been underexamined in the theoretical literature, especially in full blown macroeconomic models. For example, although bailouts may mitigate the adverse ex-post effects of the bubble bursting, it remains unclear what happens if bailouts are anticipated ex-ante. Do they affect boom-bust cycles? Do they change the emergence conditions of bubbles? More generally, to what extent are ex-post bailouts efficient from an ex-ante perspective? Further, can we derive an optimal bailout policy? In this paper, we theoretically investigate these questions by using a simple infinite horizon general equilibrium model with financial imperfection and stochastic bubbles.

The first notable contribution of this paper is that we explore that bailouts in the wake of bursting bubbles may positively influence ex-ante production efficiency. The recent theoretical literature on bailout policies tends to investigate the moral hazard consequences of bailouts (e.g., Diamond and Rajan, 2012; Farhi and Tirole, 2009, 2012), finding that moral hazard negatively affects ex-ante efficiency. This paper, however, shows that the effects of bailouts provided after bursting bubbles are quite the opposite. An intuitive reason for this finding comes from a crowd-in effect of bubbles. If the financial market is imperfect, the existence of bubbles may be able to crowd in investments, because the bubbles have a positive wealth effect and relax borrowing constraints. By contrast, ex-post bailouts make bubbles safer and more profitable assets, and demand for bubbles subsequently rises. This higher demand raises the price of bubbles and increases the crowd-in effect. Hence, bailouts positively influence ex-ante production efficiency. In order to explain this point clearly, we extend the approach taken by Kiyotaki (1998) by developing a macroeconomic model with heterogeneous investments, financial market imperfection, asset bubbles, and bailouts.

As we show herein, anticipated bailouts induce low-productivity entrepreneurs to buy risky bubble assets. By encouraging such risk-taking behavior, anticipated bailouts also affect the existing conditions of asset bubbles. We show that bubbles that have a high probability of bursting cannot occur in the absence of government guarantees. However, if bailouts are guaranteed by the government, then even those riskier bubbles can arise.

The second contribution of this paper is that it examines the possibility of partial bailouts. In reality, the provision of bailouts is not comprehensive. For example, in the recent global financial crisis, AIG was rescued, while Lehman Brothers was not. In this paper, we consider such possibility. Financial safety net is provided by the government following the collapse of bubbles. We focus on a bailout in which the government guarantees bubble investments against losses derived from the collapse of bubbles. The aim of bailouts is to recapitalize the net worth of entrepreneurs and mitigate economic contractions. An important assumption in our model is that not all entrepreneurs who suffer losses from bubble investments are necessarily rescued, that is, some entrepreneurs are rescued while the others are not. The government can choose the percentage of entrepreneurs rescued. This assumption captures the possibility of partial bailouts, and we show that partial bailouts have a superior aspect. In order to realize ex-ante production efficiency, not full bailouts but partial bailouts are desirable. This point comes from a crowd-out effect of bubbles. Although bubbles have the crowd-in effect, it is well known in the literature on bubbles, for example, Tirole (1985), that they also have the crowd-out effect on investments. In this paper, we show that the crowd-out effect dominates the crowd-in effect if the bubble becomes sufficiently large. In other words, too generous financial safety net such as full bailouts creates too large bubbles and decreases investment.

The interesting point is that bailouts have non-monotonic impacts on ex-ante production efficiency, which is the production level until the bubble bursts when bailout policies are anticipated. We show that expansions in the government guarantees initially crowds in productive projects, thereby increasing production efficiency as long as bubbles do not burst. Too generous guarantees, however, lead to strong crowd-out effects, thereby decreasing ex-ante production efficiency. This non-monotonic impact on ex-ante production efficiency suggests that there is a certain bailout level at which ex-ante production efficiency is maximized.

Under the bailout policy, the output level in each period is increased by improving production efficiency. This, however, implies that the economy experiences a sharp drop in output when bubbles collapse. In other words, such a bailout policy may increase boom-bust cycles and require large amounts of public funds following the collapse of bubbles. This finding suggests a trade-off between economic stability and efficient resource allocation, which leads onto our third contribution.

The third contribution of this paper is that we derive the effects of bailouts on economic welfare rigorously. Since there are heterogeneous agents in this economy, it is difficult to examine total welfare directly. Instead, we examine the welfare of each type of agent and discuss that actual bailout policies may change depending on various objectives of the government or conflicts of interest between taxpayers and rescued entrepreneurs. For this consideration, the non-monotonic impact on production efficiency is important. Given the fact that wage rate is positively correlated with production efficiency, the welfare of workers has a non-monotonic relation with the broader provision of bailouts. Moreover, workers may have to pay tax to rescue bubble holders. Hence, in the case of riskier bubbles, the optimal bailout level for taxpayers is lower than the level at which production efficiency is maximized, which has an important implication for boom-bust cycles. In order to maximize taxpayers' welfare, the government must sacrifice some production efficiency in order to reduce the size of bubbles and soften boom-bust cycles. However, a no-bailout policy is not optimal, even for taxpayers. By contrast, an entrepreneur's welfare monotonically increases with broader bailout provision, because entrepreneurs receive a higher transfer from such an expansion and enjoy the wealth effect of consumption. Thus, there are conflicts of interest between taxpayers and rescued entrepreneurs about a desirable bailout level.

Finally, we discuss the welfare effects of bailout policies that make stochastic bubbles non-stochastic ones (e.g., government debt as a bailout tool). We show that stochastic bubbles can be better than non-stochastic ones from the perspective of taxpayers' welfare, suggesting that increasing the fragility of bubbles might actually enhance the welfare of taxpayers.

The rest of this paper is organized as follows. In subsection 1.1, we discuss the related works in the literature. In section 2, we present our basic model with stochastic bubbles and government bailouts. In section 3 and 4, we examine dynamics of rational bubbles and analyze how the government's financial safety net affects the existence conditions of rational bubbles. In section 5, we investigate how expansions in government guarantees affect ex-ante production efficiency and boom-bust cycles. In section 6, we conduct a welfare analysis of anticipated bailouts and show an optimal bailout policy. In section 7, we analyze whether a bailout policy that makes stochastic bubbles non-stochastic ones is the best from a welfare perspective. In section 8, we conclude our argument.

1.1 Related Literature

Recent examinations about rational bubbles have provided a theoretical framework to analyze the macroeconomic effects of asset bubbles. In particular, seminal works, such as Farhi and Tirole (2012), Martin and Ventura, (2012), and Woodford (1990), have enriched the argument about the consequences of asset bubbles by showing that they may have a crowd-in effect. Adding to these papers, a growing body of literature is examining asset bubbles and macro dynamics (e.g., Aoki and Nikolov, 2011; Caballero and Krishnamurthy, 2006; Jovanovic, 2012; Kamihigashi, 2011, 2012; Kocherlalota, 2009; Hellwig and Lorenzoni, 2009; Hirano and Yanagawa, 2010; Miao and Wang, 2011). The present paper discusses the results of the above mentioned papers in order to examine in depths both the crowd-in effect and the crowd-out effect of bubbles. Specifically, the main original contribution of this paper is exploring the effects of bailouts within a rational bubbles framework, and analyzing desirable bailout policies from a welfare perspective.

In this vein, Uhlig (2010) models a systemic bank run in the light of the recent financial crisis. His analysis supports for the argument that the outright purchase of troubled assets by the government at above current market prices can both alleviate financial crises as well as provide taxpayers with returns above those for safe securities. Similarly, Diamond and Rajan (2012) and Farhi and Tirole (2009, 2012) examine the moral hazard consequences of bailouts and welfare analysis in order to derive optimal regulations or bailout policies. Our paper lends support to Uhlig's (2010) results by developing a rigorous welfare analysis and builds on the findings of the other three papers by proposing optimal bailout policies following the collapse of bubbles. Moreover, rather than using a three-period model with an endowment economy, we examine the effects of bailouts in a full blown dynamic macroeconomic model with a production economy.

Regarding previous works that have used dynamic macroeconomic models, Brunnermeier and Sannikov (2011), Gertler and Kiyotaki (2010), and Kiyotaki and Moore (2008) examine government bailouts (i.e., credit market interventions) in a liquidity crisis, while Gertler and Karadi (2011) and Roch and Uhlig (2012) adopt dynamic macroeconomic models in order to analyze the welfare effects of bailouts. Roch and Uhlig (2012), for example, provide a theoretical framework to analyze the dynamics of a sovereign debt crisis and bailouts. Their paper, based on an endowment economy, characterizes the minimal actuarially fair bailouts that restore the good equilibrium. In contrast, our model is based on production economy. Hence, the anticipated bailouts greatly affect welfare through the change in production. Moreover, Gertler and Karadi (2011) analyze whether government's interventions in a crisis (i.e., direct lending by the central banks) can improve post-crisis welfare. By contrast, our paper takes into account the anticipated effects of government policy, and computes welfare from an ex-ante perspective.

Gertler et al. (2011) examine the welfare effects of a government's credit policy in a crisis by considering the anticipated effects, and computing welfare from an ex-ante perspective. In their model, the anticipated credit policy induces the ex-ante risk-taking of intermediaries, while they also show that ex-ante regulations reduce risk-taking and improve welfare. By contrast, the model presented herein suggests that, anticipated bailouts induce risk-taking ex-ante and that, such risk-taking can improve welfare; however, we also conclude that too much risk-taking reduces welfare by creating large bubbles.

2 The Model

2.1 Framework

Consider a discrete-time economy with one homogeneous good and a continuum of entrepreneurs and workers. A typical entrepreneur and a representative worker have the following expected discounted utility,

$$E_0 \left[\sum_{t=0}^{\infty} \beta^t \log c_t^i \right], \tag{1}$$

where i is the index for each entrepreneur, and c_t^i is the consumption of him/her at date t. $\beta \in (0,1)$ is the subjective discount factor, and $E_0 [a]$ is the expected value of a conditional on information at date 0.

Let us start with the entrepreneurs. At each date, each entrepreneur meets high productive investment projects (hereinafter H-projects) with probability p, and low productive ones (L-projects) with probability $1-p$. The investment projects produce capital. The investment technologies are as follows:

$$k_{t+1}^i = \alpha_t^i z_t^i, \tag{2}$$

where $z_t^i (\geq 0)$ is the investment level at date t, and k_{t+1}^i is the capital at date $t+1$ produced by the investment. α_t^i is the marginal productivity of investment at date t. $\alpha_t^i = \alpha^H$ if the entrepreneur has H-projects, and $\alpha_t^i = \alpha^L$ if he/she has L-projects. We assume $\alpha^H > \alpha^L$. For simplicity, we assume that capital fully depreciates in one period.[1] The probability p is exogenous, and independent across entrepreneurs and over time. The entrepreneur knows his/her own type of date t, whether he/she has

[1] As in Kocherlakota (2009), we can consider a case where only a fraction η of capital depreciates, and consumption goods can be converted one-for-one into capital, and vice-versa. In this setting, we can also obtain the same results as in the present paper.

H-projects or L-projects. Assuming that the initial population measure of each type is p and $1-p$ at date 0, the population measure of each type after date 1 is p and $1-p$, respectively. Throughout this paper, we call the entrepreneurs with H-projects "H-types" and the ones with L-projects "L-types".

We assume that because of frictions in a financial market, the entrepreneur can pledge at most a fraction θ of the future return from his/her investment to creditors.[2] In such a situation, in order for debt contracts to be credible, debt repayment cannot exceed the pledgeable value. That is, the borrowing constraint becomes:

$$r_t b_t^i \leq \theta q_{t+1} \alpha_t^i z_t^i, \tag{3}$$

where q_{t+1} is the relative price of capital to consumption goods at date $t+1$.[3] r_t and b_t^i are the gross interest rate and the amount of borrowing at date t. The parameter $\theta \in (0, 1]$, which is assumed to be exogenous, can be naturally taken to be the degree of imperfection of the financial market.

In this economy, there are bubble assets denoted by x. The aggregate supply of bubble assets is assumed to be constant over time X. As in Tirole (1985), we define bubble assets as those assets that produce no real return, i.e., the fundamental value of the assets is zero. However, under some conditions, the prices of bubble assets become positive, which means that bubbles arise in equilibrium. Here, following Weil (1987), we consider stochastic bubbles, in the sense that they may collapse. In each period, bubble prices become zero (i.e., bubbles burst) at a probability of $1-\pi$ conditional on survival in the previous period. A lower π means riskier bubbles, because the bursting probability is higher. In line with the literature in this regard,

[2] See Hart and Moore (1994) and Tirole (2006) for the foundations of this setting.

[3] On an equilibrium path, q_{t+1} is not affected by the collapse of bubbles. Hence, there is no uncertainty with regard to q_{t+1}.

burst bubbles do not arise again unless agents change their expectations about their formation through, for example, unexpected shocks. This implies that bubbles persist with a probability $\pi(< 1)$ and that their prices are positive until they switch to being equal to zero. Let P_t be the per unit price of bubble assets at date t on survival in terms of consumption goods.

The entrepreneur's flow of funds constraint is given by

$$c_t^i + z_t^i + P_t x_t^i = q_t \alpha_{t-1}^i z_{t-1}^i - r_{t-1} b_{t-1}^i + b_t^i + P_t x_{t-1}^i + m_t^i. \tag{4}$$

where x_t^i be the level of bubble assets purchased by a type i entrepreneur at date t. The left hand side of (4) is expenditure on consumption, investment, and the purchase of bubble assets. The right hand side is the available funds at date t, which is the return from investment in the previous period minus debts repayment, plus new borrowing, the return from selling bubble assets, and bailout money, m_t^i.

When bubbles collapse at the beginning of date t, all the wealth invested in bubble assets is wiped out. This decreased wealth and the resulting net worth of entrepreneurs lead to severe contractions during the bursting of bubbles. Although the government bails out entrepreneurs in order to mitigate these contractions, not all entrepreneurs are necessarily rescued. To formulate the possibility of so-called "partial bailouts", we assume that only a certain proportion $\lambda \in [0, 1]$ of the entrepreneurs who suffer losses from bubble investments are rescued. $\lambda = 0$ means that no-entrepreneurs are rescued, while $\lambda = 1$ means that all are rescued. A rise in λ means expansions in the government's financial safety net. This bailout scheme suggests that from an ex-ante perspective, each entrepreneur anticipates government bailouts with a probability λ. When entrepreneur i is rescued, we assume that the government guarantees bubble investments against losses and that the bailout is proportional to the entrepreneur's holdings of bubble assets:

$$m_t^i = d_t^i x_{t-1}^i. \tag{5}$$

Here, we specifically consider those bailouts that fully guarantee bubble investments against losses. Hence, $d_t^i = P_t > 0$ if the agent i is rescued when bubbles collapse at t. Otherwise $m_t^i = d_t^i = 0$.

In this paper, we examine this type of partial bailouts, but we may be able to consider other types of partial bailouts. We can easily imagine, for example, a bailout policy in which government guarantees only a part of bubble investments against losses for all bubble holders. The main reason for our approach is analytical tractability. In our setting, we can solve dynamics analytically and derive analytical solutions explicitly. Even if we consider more general bailout policies, our qualitative results which will be explained below remain unaffected. We will discuss about this point more in section 7.

We define the net worth of the entrepreneur at date t as

$$e_t^i \equiv q_t \alpha_{t-1}^i z_{t-1}^i - r_{t-1} b_{t-1}^i + P_t x_{t-1}^i + m_t^i.$$

We also impose the short sale constraint on bubble assets:[4]

$$x_t^i \geq 0. \tag{6}$$

Let us now turn to the maximization problem of workers. There are workers with a unit measure.[5] Each worker is endowed with one unit of labor endowment in

[4]Kocherlakota (1992) shows that the short sale constraint plays an important role for the emergence of asset bubbles in an endowment economy with infinitely lived agents.

[5]Even if we consider workers with N measure, all the results in our paper hold.

each period, which is supplied inelastically in labor markets, and earns wage rate, w_t. Workers do not have investment opportunities, and cannot borrow against their future labor incomes. The flow of funds constraint, and short sale constraint for them are given by

$$c_t^u + P_t(x_t^u - x_{t-1}^u) = w_t - r_{t-1}b_{t-1}^u + b_t^u - T_t^u, \tag{7}$$

$$x_t^u \geq 0, \tag{8}$$

where ^{u} represents workers. T_t^u is a lump sum tax. When bubbles collapse, government levies a lump sum tax on workers and transfers those funds to entrepreneurs who suffer losses from bubble investments. This means that workers are taxpayers and incur the direct costs of bubbles' collapsing. Thus, $T_t^u > 0$ only when bubbles collapse, while $T_t^u = 0$ if they survive. As in Farhi and Tirole (2012), the aim of this transfer policy (i.e., bailout policy) is to boost the net worth of entrepreneurs. In our model, this increased net worth can mitigate the adverse effects of bubbles' collapsing.

Let us mention the main reason behind the transfer policy from workers to entrepreneurs. In our model, as long as the government transfers resources among entrepreneurs, the aggregate wealth of entrepreneurs does not increase. As a result, economic contractions following the collapse of bubbles are not mitigated. The transfer policy from workers to entrepreneurs, however, increases the aggregate wealth of entrepreneurs and mitigates such contractions. We explain this point more in depth in a later section 7.

Lastly, we explain the production technology. There are competitive firms which produce final consumption goods using capital and labor. The production function of each firm is

$$y_t = k_t^{\sigma} n_t^{1-\sigma}. \tag{9}$$

Factors of production are paid their marginal product:

$$q_t = \sigma K_t^{\sigma-1} \text{ and } w_t = (1-\sigma) K_t^{\sigma}, \tag{10}$$

where K is the aggregate capital stock.

2.2 Equilibrium

Let us denote the aggregate consumption of H-and L-entrepreneurs and workers at date t as $\sum_{i \in H_t} c_t^i \equiv C_t^H$, $\sum_{i \in L_t} c_t^i \equiv C_t^L$, C_t^u, where H_t and L_t mean a family of H-and L-entrepreneurs at date t. Similarly, let $\sum_{i \in H_t} z_t^i \equiv Z_t^H$, $\sum_{i \in L_t} z_t^i \equiv Z_t^L$, $\sum_{i \in H_t} b_t^i \equiv B_t^H$, $\sum_{i \in L_t} b_t^i \equiv B_t^L$, B_t^u, $(\sum_{i \in H_t \cup L_t} x_t^i + X_t^u) \equiv X_t$ be the aggregate investments of each type, the aggregate borrowing of each type, and the aggregate demand for bubble assets. Then, the market clearing condition for goods, credit, capital, labor, and bubble assets are

$$C_t^H + C_t^L + C_t^u + Z_t^H + Z_t^L = Y_t, \tag{11}$$

$$B_t^H + B_t^L + B_t^u = 0, \tag{12}$$

$$K_t = \sum_{i \in H_t \cup L_t} k_t^i, \tag{13}$$

$$N_t = 1, \tag{14}$$

$$X_t = X. \tag{15}$$

The competitive equilibrium is defined as a set of prices $\{r_t, w_t, P_t\}_{t=0}^{\infty}$ and quantities $\left\{C_t^H, C_t^L, C_t^u, B_t^H, B_t^L, B_t^u, Z_t^H, Z_t^L, X_t, K_{t+1}, Y_t\right\}_{t=0}^{\infty}$, such that (i) the market clearing conditions, (11)-(15), are satisfied in each period, and (ii) each entrepreneur chooses consumption, borrowing, investment, and the amount of bubble assets, $\{c_t^i, b_t^i, z_t^i, x_t^i\}_{t=0}^{\infty}$, to maximize his/her expected discounted utility (1) under the constraints (2)-(6), taking into consideration the bursting probability of bubbles and the bailout probability. (iii) each worker chooses consumption, borrowing, and the amount of bubble assets, $\{c_t^u, b_t^u, x_t^u\}_{t=0}^{\infty}$, to maximize his/her expected discounted utility (1) under the constraints (7)-(8), taking the bursting probability into consideration.

2.3 Optimal Behavior of Entrepreneurs and Workers

We now characterize the equilibrium behavior of entrepreneurs and workers. We focus on the equilibrium where

$$q_{t+1}\alpha^L \leq r_t < q_{t+1}\alpha^H.$$

In equilibrium, interest rate must be at least as high as $q_{t+1}\alpha^L$, since nobody lends to the projects if $r_t < q_{t+1}\alpha^L$. Moreover, if the interest rate is higher than the rate of return of H-projects, nobody borrows[6]. Hence, this assumption is not restrictive at all.

Since the utility function is log-linear, each entrepreneur consumes a fraction $1 - \beta$ of the net worth in each period, that is, $c_t^i = (1 - \beta)e_t^i$.[7] For H-types at date t, the borrowing constraint (3) is binding since $r_t < q_{t+1}\alpha^H$ and the investment in

[6]When $r_t = q_{t+1}\alpha^H$, bubbles cannot exist as explored in the traditional literature about rational bubbles. Hence, we exclude this case from our consideration.

[7]See, for example, chapter 1.7 of Sargent (1988).

bubbles is not attractive, that is, (6) is also binding. We will verify this result in the Technical Appendix. Then, by using (3), (4), and (6), the investment function of H-types at date t can be written as

$$z_t^i = \frac{\beta e_t^i}{1 - \dfrac{\theta q_{t+1} \alpha^H}{r_t}}. \tag{16}$$

This is a popular investment function under financial constraint problems[8], except for the fact that the presence of bubble assets and bailout money affect the net worth. We see that the investment equals the leverage, $1/\left[1-(\theta q_{t+1}\alpha^H/r_t)\right]$, times a fraction β of the net worth. From this investment function, we understand that for the entrepreneurs who purchased bubble assets in the previous period, they are able to sell those assets at the time they encounter H-projects. As a result, their net worth increases, which boosts their investments. That is, bubbles generate balance sheet effects. Moreover, the expansion level of the investment is more than the direct increase of the net worth because of the leverage effect. In our model, the entrepreneurs buy bubble assets when they have L-projects, and sell those assets when they have opportunities to invest in H-projects.

For L-types at date t, since $c_t^i = (1-\beta)e_t^i$, the budget constraint (4) becomes

$$z_t^i + P_t x_t^i - b_t^i = \beta e_t^i. \tag{17}$$

Each L-type allocates his/her savings, βe_t^i, into three assets, i.e., z_t^i, $P_t x_t^i$, and $(-b_t^i)$. Each L-type chooses optimal amounts of b_t^i, x_t^i, and z_t^i so that the expected marginal utility from investing in three assets is equalized. By solving the utility maximization problem explained in the Technical Appendix, we can derive the demand function

[8]See, for example, Bernanke and Gertler (1989), Bernanke et al. (1999), Holmstrom and Tirole (1998), Kiyotaki and Moore (1997), and Matsuyama (2007, 2008).

for bubble assets of a L-type:

$$P_t x_t^i = \frac{\delta(\lambda)\frac{P_{t+1}}{P_t} - r_t}{\frac{P_{t+1}}{P_t} - r_t}\beta e_t^i, \tag{18}$$

where $\delta(\lambda) \equiv \pi + (1-\pi)\lambda$. From (18), we learn that an entrepreneurs's portfolio decision depends on its perceptions of risk, which in turn depends on both the bursting probability of bubbles (π) and expectations about government bailouts (λ). A rise in λ encourages entrepreneur's risk-taking to buy more bubble assets.

The remaining fraction of savings is split across z_t^i and $(-b_t^i)$:

$$z_t^i + (-b_t^i) = \frac{[1-\delta(\lambda)]\frac{P_{t+1}}{P_t}}{\frac{P_{t+1}}{P_t} - r_t}\beta e_t^i.$$

Since investing in L-projects (z_t^i) and secured lending to other entrepreneurs ($-b_t^i$) are both safe assets, $z_t^i \geq 0$ if $r_t = q_{t+1}\alpha^L$, and $z_t^i = 0$ if $r_t > q_{t+1}\alpha^L$. That is, the following conditions must be satisfied:

$$(r_t - q_{t+1}\alpha^L)z_t^i = 0, \;\; z_t^i \geq 0, \;\text{ and } r_t - q_{t+1}\alpha^L \geq 0.$$

Moreover, when $r_t = q_{t+1}\alpha^L$, investing in L-projects and secured lending to other entrepreneurs are indifferent for L-types, aggregate investment level of L-types, Z_t^L, is determined from (11).

Next, we examine the optimal behavior of workers. Since the equilibrium interest rate becomes relatively low because of the borrowing constraint, saving is not an attractive behavior for workers. Thus, we can prove that they consume all the wage income in each period unless there is no bailout policy. On the other hand, workers might save to smooth their consumption if a government uses a bailout policy. This is because if bubbles collapse, workers have to pay a lump sum tax to rescue entrepreneurs, which lowers their consumption, while if bubbles do not collapse, they do not. So, consumption will be more volatile compared with the case without a bailout policy. However, we can verify that under certain reasonable parameter values, workers do not save even if there is a bailout policy. We will verify this in the Technical Appendix. In this paper, we focus on the parameter ranges where workers do not save. That is,

$$c_t^u = w_t - T_t^u.$$

$T_t^u > 0$ only when bubbles collapse.

2.4 Dynamics

From (16) and

$$Z_t^H + Z_t^L + P_t X = \beta A_t, \tag{19}$$

we have the evolution of aggregate capital stock:

$$K_{t+1} = \begin{cases} \alpha^H \dfrac{\beta p A_t}{1 - \frac{\theta \alpha^H}{\alpha^L}} + \alpha^L \left(\beta A_t - \dfrac{\beta p A_t}{1 - \frac{\theta \alpha^H}{\alpha^L}} - P_t X \right) & \text{if } r_t = q_{t+1} \alpha^L, \\ \\ \alpha^H \left[\beta A_t - P_t X \right] & \text{if } r_t > q_{t+1} \alpha^L. \end{cases} \tag{20}$$

where $A_t \equiv q_t K_t + P_t X$ is the aggregate wealth of entrepreneurs at date t, and $\sum_{i \in H_t} e_t^i = p A_t$ is the aggregate wealth of H-types at date t. (More details about aggregation of each variable will be explained in the Technical Appendix). When $r_t = q_{t+1}\alpha^L$, both types of entrepreneurs may invest. The first term and the second term of the first line represent the capital stock at date $t+1$ produced by H-and L-types, respectively. When $r_t > q_{t+1}\alpha^L$, only H-types invest. From (19), we know

$Z_t^H = \beta A_t - P_t X$. $(-P_t X)$ in (20) captures a traditional crowd-out effect of bubbles analyzed in Tirole (1985), i.e., the presence of bubble assets crowds savings away from investments.

As long as $r_t > q_{t+1}\alpha^L$, the interest rate is determined by the credit market clearing condition (12), which can be written as

$$\frac{\beta p A_t}{1 - \dfrac{\theta q_{t+1}\alpha^H}{r_t}} + P_t X = \beta A_t.$$

That is, the aggregate savings of entrepreneurs, βA_t, flow to aggregate H-investments and bubbles. By defining $\phi_t \equiv P_t X / \beta A_t$ as the size of bubbles (the share of the value of bubbles), we can rewrite the above relation as

$$r_t = \frac{q_{t+1}\theta\alpha^H(1-\phi_t)}{1-p-\phi_t}.$$

It follows that r_t increases with ϕ_t, reflecting the tightness of the credit markets.

Thus, the equilibrium interest rate is determined as

$$r_t = q_{t+1} Max\left[\alpha^L, \frac{\theta\alpha^H(1-\phi_t)}{1-p-\phi_t}\right]. \tag{21}$$

In other words, $r_t = q_{t+1}\alpha^L$ and $Z_t^L \geq 0$ if $\phi_t \leq \phi^* \equiv \frac{\alpha^L(1-p)-\theta\alpha^H}{\alpha^L-\theta\alpha^H}$, and $r_t > q_{t+1}\alpha^L$ and $Z_t^L = 0$ if $\phi_t > \phi^*$.

Hence, by using ϕ_t, (20) can be written as

$$K_{t+1} = \begin{cases} \dfrac{\left[(1+\frac{\alpha^H-\alpha^L}{\alpha^L-\theta\alpha^H}p)\beta\alpha^L - \alpha^L\beta\phi_t\right]}{1-\beta\phi_t}\sigma K_t^\sigma & \text{if } \phi_t \leq \phi^*, \\ \\ \dfrac{\alpha^H\beta\left[1-\phi_t\right]}{1-\beta\phi_t}\sigma K_t^\sigma & \text{if } \phi_t > \phi^*. \end{cases} \tag{22}$$

The dynamical system of this economy is mainly characterized by this (22). As described in Figure 1, the dynamics of $K_{t+1}/K_t^{\sigma} = K_{t+1}/Y_t$ is an increasing function of ϕ_t as long as $\phi_t \le \phi^*$ and it becomes a decreasing function of ϕ_t if $\phi_t > \phi^*$. In other words, $\phi^* \equiv \frac{\alpha^L(1-p)-\theta\alpha^H}{\alpha^L-\theta\alpha^H}$ is the bubble size that maximizes the capital stock. This non-linear relationship shows that small bubbles increase capital accumulation but overly large bubbles are harmful for capital accumulation. An intuitive reason for this finding is simple. As long as the bubble size is small, bubbles crowd in H-projects according to the balance sheet effect, whereas they crowd out L-projects. Thus, bubbles increase K_{t+1}/Y_t. If bubbles become larger, however, all L-projects are crowded out, and even some H-projects are crowded out by overinvestment in bubbles, meaning K_{t+1}/Y_t decreases.

Thus far, the price of bubbles, P_t, has been exogenously given and we have not assumed rationality about bubble prices. Hence, the dynamics of the capital stock, (22), are satisfied even if bubbles are not rational. Even when bubbles exist for an irrational reason, the dynamics are characterized by (22).

3 Dynamics of Rational Bubbles

Next, we examine the dynamics of rational bubbles. Since we assume that rational bubbles are stochastic, that is, bubbles persist with probability $\pi(< 1)$, here, we focus on the dynamics of bubbles until bubbles collapse.

From the definition of $\phi_t \equiv P_t X/\beta A_t$, ϕ_t evolves over time as

$$\phi_{t+1} = \frac{\frac{P_{t+1}}{P_t}}{\frac{A_{t+1}}{A_t}}\phi_t. \tag{23}$$

The evolution of the size of bubbles depends on the relation between the growth rate of wealth and the growth rate of bubbles. When we aggregate (18), and solve for P_{t+1}/P_t, then we obtain the required rate of return on bubble assets:

$$\frac{P_{t+1}}{P_t} = \frac{r_t(1-p-\phi_t)}{\delta(\lambda)(1-p)-\phi_t}. \tag{24}$$

$(1-p-\phi_t)/[\delta(\lambda)(1-p)-\phi_t]$ captures the risk premium on bubble assets. It follows that if other things being equal, the risk premium is a decreasing function of λ. Using (21), (24), and the definition of aggregate wealth of entrepreneurs, (23) can be written as

$$\phi_{t+1} = \begin{cases} \dfrac{\dfrac{(1-p-\phi_t)}{\delta(\lambda)(1-p)-\phi_t}}{\left(1+\dfrac{\alpha^H-\alpha^L}{\alpha^L-\theta\alpha^H}p\right)\beta + \dfrac{[1-\delta(\lambda)](1-p)}{\delta(\lambda)(1-p)-\phi_t}\beta\phi_t}\phi_t & \text{if } \phi_t \leq \phi^*, \\[2em] \dfrac{\theta}{\beta}\dfrac{1}{\delta(\lambda)(1-p)-(1-\theta)\phi_t}\phi_t & \text{if } \phi_t > \phi^*. \end{cases} \tag{25}$$

Using this (25), we examine the sustainable dynamics of ϕ_t. In order for bubbles to be sustainable, the following condition must be satisfied for any t:

$$\phi_t < 1. \tag{26}$$

Violation of this condition means explosion of bubbles.

As examined in the literature (Tirole 1985; Farhi and Tirole 2012), dynamics of bubbles take three patterns. The first one is that bubbles become too large and explode to $\phi_t \geq 1$. This dynamic path cannot be sustained by this economy and thus, bubbles cannot exist in this pattern. The second pattern is that ϕ_t becomes smaller over time and converges to zero. This path is called asymptotically bubbleless. In this dynamic path, the effects of bubbles converge to zero. Hence, we exclude this path from our consideration as usual in the literature. The third pattern is that ϕ_t converges to a positive value as long as the bubbles do not collapse. In this paper, we focus on this third pattern as usual in the literature, for example, Farhi and Tirole (2012), and derive the dynamics of ϕ_t.

The dynamic system of this economy is characterized by (22) and (25). However, (25) is independent from K_t and the dynamics of ϕ_t is derived only by (25). From (25), we can derive that ϕ_t must be constant over time unless ϕ_t is asymptotically bubbleless. This means that on the saddle path, wealth of entrepreneurs and bubbles grow at the same rate. More precisely, under the existence condition of bubbles which wil be explained below, $\phi_t = \phi$ for any t and ϕ is a function of λ :

$$\phi(\lambda) = \begin{cases} \dfrac{\delta(\lambda) - \dfrac{1-\delta(\lambda)\beta(1-p)}{\left[1+(\frac{\alpha^H-\alpha^L}{\alpha^L-\theta\alpha^H})p\right]\beta - \beta(1-p)}}{1 - \dfrac{1-\delta(\lambda)\beta(1-p)}{\left[1+(\frac{\alpha^H-\alpha^L}{\alpha^L-\theta\alpha^H})p\right]\beta - \beta(1-p)}}(1-p) & \text{if } 0 \leq \lambda \leq \lambda^*, \\ \\ \dfrac{\delta(\lambda)\beta(1-p)-\theta}{\beta(1-\theta)} & \text{if } \lambda^* < \lambda \leq 1. \end{cases} \tag{27}$$

λ^* is the degree of bailouts which realizes the bubble size, ϕ^*. More precisely, $\lambda^* = Max[0,\ \hat{\lambda}]$, where $\hat{\lambda}$ is the value of λ which realizes $\phi(\hat{\lambda}) = \phi^*$, and it is explicitly written as

$$\hat{\lambda} = \frac{1}{1-\pi}\frac{\alpha^L\left[\beta(1-p)+(1-\beta+p\beta)\theta\right]-\theta\alpha^H\left[\beta+(1-\beta)\theta\right]}{\beta(1-p)(\alpha^L-\theta\alpha^H)} - \frac{\pi}{1-\pi}.$$

(The deriving process about $\phi(\lambda)$ and $\hat{\lambda}$ is explained in Appendix A.) In later sections, this λ^* becomes important for considering bailout policies. Let us add a few remarks concerning the value of $\hat{\lambda}$. The value of $\hat{\lambda}$ is a decreasing function of the survival rate of bubbles, π, the productivity of H-projects, α^H, and the efficiency of the financial market, θ. Thus, unless government bailouts are sufficiently guaranteed, L-types are more likely to invest if bubbles are riskier, the productivity of H-projects is lower, and the financial markets are less efficient.[9] Here we have not examined whether $\phi(\lambda)$ is positive or not, in other words, whether bubbles can exist or not. We examine this point in the next section.

4 Stochastic Stationary Equilibrium with Bubbles

Next, we examine the existence conditions of stochastic bubbles. In other words, we investigate whether a dynamic path with bubbles does not explode. As we show below, expectations about government guarantees affect the prevailing conditions. (Proofs of all the Propositions and Lemmas are in Appendix).

Proposition 1 *Stochastic bubbles can exist if and only if*

$$\theta < \delta(\lambda)\beta(1-p) \equiv \theta_1,$$

and

$$\pi > \frac{\alpha^L - \theta\alpha^H}{\beta(\alpha^L - \theta\alpha^H) + p\beta(\alpha^H - \alpha^L)} \frac{1}{1-\lambda} - \frac{\lambda}{1-\lambda} \equiv \pi_1,$$

are satisfied. Under the conditions, $\phi_t = \phi$ for any t and ϕ is a function of λ as (27).

This Proposition means that stochastic bubbles can arise if and only if bubbles are not too risky and when financial market imperfection is sufficiently severe.[10]

[9] If π, or α^H or θ is sufficiently high, then $\lambda^* = 0$, i.e., there is no region where L-types invest positive amount.

[10] In our model, if $\phi(\lambda) \leq 0$, no equilibrium with bubbles can exist. Not only stochastic stationary bubbles cannot exist, but also asymptotically bubbleless paths cannot exist.

Intuitively, when π is too low (i.e., the bursting probability is too high) and lower than the critical value π_1, the risk premium on bubble assets becomes too high, because the required rate of return is sufficiently high. As a result, the growth rate of bubbles is too high for the economy to sustain growing bubbles and thereby too risky bubbles cannot occur. Moreover, in high θ regions where financial markets are sufficiently efficient, the interest rate becomes sufficiently high in the credit market and so does the rate of return on bubbles. Bubbles then grow so fast that the economy cannot sustain them. Thus, if θ is greater than θ_1, bubbles cannot occur.

The important point is that both π_1 and θ_1 depend on λ. In other words, expectations about government guarantees affect the existence conditions. From the existence conditions, we learn that π_1 is a decreasing function of λ, and θ_1 is an increasing function of λ, i.e., bubble regions become wider with an increase in λ. This means that even too risky bubbles can arise once government guarantees are expected. In other words, the more government bailouts are guaranteed, the more likely riskier bubbles can occur. Intuition is that when bailouts are expected, the risk premium declines because bubble assets become safer assets. As a result, the growth rate of bubbles is sufficiently low that the economy can support growing bubbles.

Since ϕ_t is constant over time, the dynamics of K_t, (22), is very simple. From (22), and (27), we have

$$K_{t+1} = H(\lambda) K_t^{\sigma} \tag{28}$$

with

$$H(\lambda) = \begin{cases} \dfrac{\left(1 + \frac{\alpha^H - \alpha^L}{\alpha^L - \theta\alpha^H} p\right) \beta\alpha^L - \beta\alpha^L(1-p)}{1 - \delta(\lambda)\beta(1-p)} \sigma & \text{if } 0 \leq \lambda \leq \lambda^*, \\ \\ \alpha^H \dfrac{\beta\left[1 - \delta(\lambda)(1-p)\right] + (1-\beta)\theta}{1 - \delta(\lambda)\beta(1-p)} \sigma & \text{if } \lambda^* \leq \lambda \leq 1. \end{cases}$$

As long as bubbles persist, the economy runs according to (28) and converges toward the stochastic stationary state. $H(\lambda)$ represents aggregate investment efficiency. An important point is that this $H(\lambda)$ function is independent of time t. From this property, we can characterize the dynamics of K very simply. We see that bubbly dynamics depends on aggregate investment efficiency, which in turn depends on expectations about the government's financial safety net, λ.

Hence, by defining the stochastic stationary state as the state where all variables $(K_t, A_t, q_t, r_t, w_t, P_t, \phi_t)$ become constant over time as long as bubbles persist, we can derive the following Proposition.

Proposition 2 *There exists a saddle point path on which the economy converges toward the stochastic stationary state as long as bubbles persist.*

As we will explain in the next section, by using the result of this Proposition, we can derive the effects of bailouts on ex-ante production efficiency and boom-bust cycles.

Moreover, according to (27), we can derive the following Proposition on the size of bubbles, ϕ.

Proposition 3 *ϕ increases with λ. That is, the size of bubbles increases as more government bailouts are guaranteed.*

An intuition of this Proposition is natural. The change in ϕ by λ is realized through the change in P_t. That is, when government bailouts are expected at a higher probability, bubbles become safer assets and current bubble prices jump up instantaneously. This instant rise in bubble prices reflects not only the future transfer from the government but also future changes in output. An increase in bubble prices improves the net worth of H-types and expands their investments, thereby increasing future total output and bubble prices in period $t+1$, $t+2$, $t+3, \cdots$. These changes in future bubble prices must then be feedbacked into current bubble prices, which in turn affects current net worth of H-types once again. By reflecting on these effects, the $\phi(\lambda)$ function is complicated as in (27).

5 Macro Effects of Bailouts

5.1 Effects on Ex-ante Production Efficiency

Bailouts may mitigate the adverse effects of bubbles' collapsing. However, once bailouts are expected, they may produce inefficiency ex-ante. To what extent are ex-post bailouts desirable from an ex-ante perspective? In this subsection, we analyze how expansions in government guarantees affect ex-ante production efficiency, which is defined as the production level at any date before the bubble bursts.

The following Lemma summarizes the property on $H(\lambda)$.

Lemma 1 *$H(\lambda)$ increases with λ in the region of $\lambda \in [0, \lambda^*)$, while it decreases with λ in the region of $\lambda \in (\lambda^*, 1]$.*

This lemma reflects the fact that in the region of $\lambda \in [0, \lambda^*)$, where L-types as well as H-types invest, a rise in λ crowds in H-projects, while it crowds out L-projects, thereby increasing aggregate investment efficiency, $H(\lambda)$. By contrast, in the region of $\lambda \in (\lambda^*, 1]$, where only H-types invest, a rise in λ ends up crowding out H-projects, thereby decreasing aggregate investment efficiency.

From Lemma 1 and the dynamics of K, (28), we can show that expansions in government guarantees have a non-linear relation with ex-ante production efficiency. When bailouts are expected, L-types are willing to buy more bubble assets instead of investing in their own L-projects. Thus, L-projects are crowded out. By contrast, H-projects are crowded in, because bubble prices rise together with demand for bubble assets, which increases the net worth of H-types and their investments. Thus, in the region of $\lambda \in [0, \lambda^*)$, expansions in government guarantees enhance ex-ante production efficiency. When λ equals λ^*, all L-projects are completely crowded out and ex-ante production efficiency is maximized. If the government increases bailouts furthermore, i.e., beyond λ^*, then, even H-projects are crowded out. In other words, in the region of $\lambda \in (\lambda^*, 1]$, the more bailouts are guaranteed, the more H-projects are crowded out and the less productive activity is created. In this region, expansions in government guarantees generate overinvestment in bubbles and ex-ante production becomes inefficient.

To summarize, from the perspective of ex-ante production efficiency, no-bailouts ($\lambda = 0$) and overly generous bailouts ($\lambda \in (\lambda^*, 1]$) are undesirable. Partial bailouts are desirable. Figure 2 illustrates the relationship between ex-post bailouts and ex-ante production efficiency.

5.2 Effects on Boom-Bust Cycles

In this subsection, we discuss how anticipated bailouts affect boom-bust cycles. Suppose that at date 0 (initial period), bubbles occur. Here, at date -1, the economy is assumed to be in the steady state of a bubbleless economy. In Figure 3, the lines with $\lambda = \lambda^*$ are the impulse responses when bailouts are λ^*, while the lines with $\lambda = 0$ are impulse responses of the economy with no bailouts. These charts in Figure 3 represent qualitative solutions, because we can work with the model analytically. Figure 3 shows that boom-bust cycles are larger when $\lambda = \lambda^*$. When government bailouts are expected with a probability λ^* at date 0, L-types are willing to buy more bubble assets. Thus, bubble prices jump up in the initial period. Because of this increase in bubble prices, the net worth of H-types improves and their investments jump up too in the initial period, while the share of L-investments over aggregate savings ($Z_0^L/\beta A_0$) falls to zero. That is, production efficiency improves. As a result, both output and the wage rate also rise in the next period (date 1). Moreover, the aggregate consumption of entrepreneurs jumps up in the initial period through the wealth effect of bubbles (i.e., the aggregate wealth of entrepreneurs rises together with the increase in bubble prices). All these macroeconomic variables continue to increase until the bubble bursts. Since this is an asset pricing model, expected future increases in output are reflected in bubble prices in the initial period. Thus, bubble prices jump up largely at date 0, which in turn improves the net worth of H-types and their investments substantially. A two-way feedback between bubble prices and output thus operates, which leads to a bubbly boom. Once bubbles collapse, all those macroeconomic variables begin to fall and converge toward a stationary steady state of the bubbleless economy.

Figure 3 shows that once bailouts are anticipated ex-ante, it ends up destabilizing the economy and requiring large amounts of public funds following the collapse of bubbles. We should mention that this instability comes from an improvement in resource allocation, namely, L-projects are crowded out and H-projects are crowded in. Thus, there might be a trade-off between the improvement in resource allocation and stability of the economy. In the next section, we carry out welfare analysis by accounting for this trade-off in order to examine optimal bailouts.

Here let us add a few remarks concerning impulse responses when $\lambda \in (\lambda^*, 1]$. The more bailouts are guaranteed, the more H-projects are crowded out, and the less productive activity is created. A rise in λ therefore dampens both investment booms and output booms, and lowers the wage rate, but raises bubble prices more, increasing the consumption booms of entrepreneurs. These asymmetric impulse responses in the wage rate and entrepreneurs' consumption suggest that in the region of $\lambda \in (\lambda^*, 1]$, a rise in λ leads to increased inequality in average consumption between bubble holders (entrepreneurs) and non-bubble holders (workers).[11] As we will see in the next section, inequality in welfare also widens.

6 Welfare Analysis

In this section, we conduct a welfare analysis of anticipated bailouts to derive optimal bailouts for workers (i.e., taxpayers) and rescued entrepreneurs. We then discuss how to design desirable bailout policies depending on the various objectives of the government.

6.1 Welfare Effects for Taxpayers

Let us first examine whether bailouts are good for taxpayers after bubbles burst. Suppose that at date t, the bubble collapses (i.e., after date t, the economy is bubbleless). Whether the government decides to bail out entrepreneurs at date t depends on costs and benefits. For instance, when bubbles collapse, workers have to pay a lump sum tax to rescue entrepreneurs, which lowers their consumption and welfare. However, bailouts improve the net worth of the rescued entrepreneurs and their investments expand at date t compared with the no-bailout case. This thereby increases wage income and workers' consumption after date $t+1$ by expanding output, improving workers' welfare. Which of these effects dominates determines workers' welfare.

Let $V_t^{BL}(K_t)$ be the value function of taxpayers at date t when bubbles collapse.

[11] We get this asymmetric impulse response as long as β is sufficiently larger than σ (for example, $\beta = 0.99$ and $\sigma = 0.3$).

Given the optimal decision rules, the Bellman equation can be written as

$$V_t^{BL}(K_t) = \log c_t + \beta V_{t+1}^{BL}(K_{t+1}),$$

with

$$c_t = w_t - T_t^u,$$

$$c_t = w_t \text{ after date } t+1.$$

$T_t^u = \lambda P_t X$ is bailout money per unit of workers. Solving the value function yields (see the Technical Appendix for derivation.)

$$V_t^{BL}(K_t) = M(\lambda) + \frac{\sigma}{1-\beta\sigma}\log K_t, \tag{29}$$

with

$$\begin{aligned} M(\lambda) = \log\left[1 - \sigma - \lambda\frac{\beta\phi(\lambda)}{1-\beta\phi(\lambda)}\sigma\right] + \frac{\beta\sigma}{1-\beta\sigma}\log\left[1 + \lambda\frac{\beta\phi(\lambda)}{1-\beta\phi(\lambda)}\right] \\ + \frac{\beta\sigma}{1-\beta\sigma}\frac{1}{1-\beta}\log\left[\left(1+\frac{\alpha^H-\alpha^L}{\alpha^L-\theta\alpha^H}p\right)\beta\alpha^L\sigma\right] + \frac{\beta}{1-\beta}\log(1-\sigma). \end{aligned}$$

The first term in $M(\lambda)$ captures the costs of the bailouts, while the second term captures the benefits.[12] From (29), we obtain the following Lemma.[13]

Lemma 2 *Suppose that a bubble collapses at date t. Then, we have $\frac{dV_t^{BL}}{d\lambda} = \frac{dM(\lambda)}{d\lambda} < 0$, i.e., after bubbles' collapsing, bailout expansions reduce taxpayers' welfare monotonically. Thus, from an ex-post perspective, no-bailouts are optimal for taxpayers.*

We are now ready to compute the value function of taxpayers in the initial period

[12] $\left(1+\frac{\alpha^H-\alpha^L}{\alpha^L-\theta\alpha^H}p\right)\beta\alpha^L\sigma$ in $W(\lambda)$ is replaced with $\alpha^H\beta\sigma$ if $\alpha^H \geq \alpha^L(1-p)/\theta$. In our numerical examples, since we consider the case where $\lambda^* = \hat{\lambda}$ holds, $\alpha^H < \alpha^L(1-p)/\theta$ is satisfied.

[13] We should mention that Lemma 2 holds true irrespective of whether the bailout policy is anticipated or not (whether ϕ is dependent upon λ or not).

(period 0). Let $V_t^{BB}(K_t)$ be the value function of taxpayers at date t in the bubble economy. Given the optimal decision rules, the Bellman equation can be written as

$$V_t^{BB}(K_t) = \log c_t + \beta\left[\pi V_{t+1}^{BB}(K_{t+1}) + (1-\pi)V_{t+1}^{BL}(K_{t+1})\right].$$

Solving the value function yields (see the Technical Appendix for derivation.)

$$\begin{aligned} V_t^{BB}(K_t) = \frac{1}{1-\beta\pi}\frac{\beta\sigma}{1-\beta\sigma}\log H(\lambda) + \frac{\beta(1-\pi)}{1-\beta\pi}M(\lambda) \\ + \frac{1}{1-\beta\pi}\log(1-\sigma) + \frac{\sigma}{1-\beta\sigma}\log K_t. \quad (30) \end{aligned}$$

The first term in equation (30) captures the effects of anticipated bailouts on welfare before bubbles' collapse, which are influenced by changes in aggregate investment efficiency, $H(\lambda)$. The second term captures the effects on welfare after bubbles' burst, which are influenced by the changes in $M(\lambda)$. Since we consider expected discounted welfare, both terms are weighted by the survival rate of bubbles. Thus, by setting $t = 0$, we can understand how a change in λ affects taxpayers' welfare in the initial period.[14] Differentiating (30) with respect to λ yields

$$\frac{dV_0^{BB}}{d\lambda} = \frac{1}{1-\beta\pi}\frac{\beta\sigma}{1-\beta\sigma}\frac{d\log H(\lambda)}{d\lambda} + \frac{\beta(1-\pi)}{1-\beta\pi}\frac{dM(\lambda)}{d\lambda}. \quad (31)$$

In order to check the sign of (31), let us first consider the region of $\lambda \in (\lambda^*, 1]$. In this region, we know from Lemma 1 that aggregate investment efficiency decreases with λ. That is,

$$\frac{d\log H(\lambda)}{d\lambda} < 0.$$

We also know from Lemma 2 that after bubbles' collapse, bailout expansions reduce

[14]When we compute how tax payers' welfare is affected in the initial period, we assume that bubbles arise in the initial period.

taxpayers' welfare. That is,

$$\frac{dM(\lambda)}{d\lambda} < 0.$$

Taken together, we have

$$\frac{dV_0^{BB}}{d\lambda} < 0 \text{ in } \lambda \in (\lambda^*, 1]\,.$$

This means that too generous bailouts reduce taxpayers' welfare. Thus, the optimal λ for taxpayers never exists in the region where ex-ante production efficiency decreases. The following lemma 3 summarizes this.

Lemma 3 *Let λ^{**} be the value of λ which maximizes V_0^{BB}. Then, $\lambda^{**} \notin (\lambda^*, 1]\,.$*

Let us next consider the region of $\lambda \in [0, \lambda^*)\,.$ In this region, we know that aggregate investment efficiency increases with λ. That is,

$$\frac{d\log H(\lambda)}{d\lambda} > 0.$$

Thus, in this region, expansions in the government's financial safety net generate two competing effects. One is the welfare-enhancing effect captured by the first term of (31). The other is the welfare-reducing effect captured by the second term of (31). Whether expansions in bailout guarantees increase taxpayers' welfare thus depends on which of these effects dominates.

Here let us assume

$$(1-p)[1-\beta\phi(\lambda=0)](1-\sigma) > \phi(\lambda=0)(1-\beta)[1-\pi\beta(1-p)]. \tag{A1}$$

This assumption ensures that the slope of V_0^{BB} evaluated at $\lambda = 0$ is positive. Since $1-p > \phi$ and $1-\beta\phi > 1-\beta$, this assumption is more likely to be satisfied if σ is small enough. The reason is that when σ is small enough, the share of wage income over output is large relative to the share of total bailout money. As a result, the welfare-reducing effect becomes sufficiently small and the welfare-enhancing effect dominates.

Using Lemma 3, we obtain the following Proposition.

Proposition 4 *If (A1) holds, $\lambda^{**} \neq 0$. That is, partial bailouts are optimal for taxpayers, wheres no-bailouts ($\lambda = 0$) and overly generous bailouts ($\lambda \in (\lambda^{*}, 1]$) are not optimal for taxpayers.*

From the property of $\hat{\lambda}$, Lemma 3, and Proposition 4, we learn the following implications. In the economies where the survival rate of bubbles, π, the productivity of H-projects, α^{H}, and efficiency of the financial market, θ, are relatively low, then partial bailouts are optimal. This is because in those economies, without bailouts, resource allocation is inefficient. Increasing financial safety net can improve resource allocation by encouraging risk-taking, thereby increasing welfare, but too generous financial safety net induces too much risk-taking and reduces welfare. By contrast, in the economies where those variables are relatively high, no-bailouts are optimal. In those economies, even without bailouts, only H-types invest. In such a situation, expansions in financial safety net reduce welfare by crowding out productive investments.

Figure 4 illustrates numerical examples of Proposition 4 showing the relationship between V_0^{BB} and λ. When we compute V_0^{BB}, without loss of generality, we set an initial aggregate capital stock, K_0, to the steady-state value of the bubbleless economy. Other parameter values are shown in Table 1. The only difference between the four cases lies in the bursting probability of the bubbles. The lower π is, the riskier bubbles are. In the benchmark case and case 1, bubbles are therefore relatively risky compared with case 2 and case 3.

Figure 4 holds an important implication. In the benchmark case and case 1, taxpayers' welfare is maximized at λ, which is lower than λ^*. Thus, the equilibrium where even L-types invest is optimal from a welfare perspective for taxpayers. This finding suggests that in order to maximize taxpayers' welfare, the government must sacrifice some efficiency in production. Put simply, in the case of riskier bubbles, L-types do not want to invest a large proportion of their savings in bubble assets, because the bursting probability is high. Therefore, they end up investing more savings in their own L-projects in order to hedge their risk. In such a situation, in order to crowd out L-projects completely, the government needs to rescue greater fractions of entrepreneurs (i.e., $\lambda^* = \hat{\lambda}$ is a decreasing function of π.), which directly increases the total bailout money required. Moreover, when anticipated, such large-scale government bailouts create large bubbles, which increases the bailout money, too. These two effects require large amounts of public funds (i.e., taxpayers' money). Thus, welfare decreases with λ in the region of $\lambda \in (\lambda^{**}, 1]$.[15] By contrast, in case 2 and case 3, where the bursting probability of bubbles is lower (i.e., bubbles are relatively safer), ex-ante production efficiency is maximized by rescuing only smaller fractions of entrepreneurs, meaning that the total bailout money required is lower. The welfare-enhancing effect thus dominates the welfare-reducing one in all ranges of $\lambda \in (0, \lambda^*)$ and welfare is maximized at $\lambda = \lambda^*$.

The presented results show that the government faces a trade-off. When financial markets are imperfect, enough resources cannot be transferred to the productive sector and resource allocation is inefficient. Although the presence of the government's financial safety net can improve resource allocation by encouraging risk-taking, it

[15] In $\lambda \in [0, \lambda^{**}]$, the welfare-enhancing effect dominates the welfare-reducing one. In $\lambda \in (\lambda^{**}, 1]$, the welfare-reducing effect dominates the welfare-enhancing one.

also generates costs. Together with the improvement in resource allocation, large bubbles are created, which reduces welfare. In the case of riskier bubbles, the latter effect becomes too large. Thus, if the government aims to maximize taxpayers' welfare, it must sacrifice some production efficiency.

6.2 Welfare Effects for Entrepreneurs

We next examine the welfare effects for entrepreneurs. Let $W_t^{BB}(e_t, K_t)$ be the value function of the entrepreneur in the bubble economy who holds the net worth, e_t, at the beginning of period t before knowing his/her type of period t. When we compute the value function, we must take into account the survival probability of bubbles, the probability of becoming H-type and L-type, and the bailout probability. Given the decision rules, the Bellman equation can be written as

$$\begin{aligned} W_t^{BB}(e_t, K_t) = \log c_t &+ \beta\pi \left[pW_{t+1}^{BB}(R_t^H \beta e_t, K_{t+1}) + (1-p)W_{t+1}^{BB}(R_t^L \beta e_t, K_{t+1}) \right] \\ &+ \beta(1-\pi) \left[\begin{array}{c} pW_{t+1}^{BL}(R_t^H \beta e_t, K_{t+1}) + (1-p)\lambda W_{t+1}^{BL}(R_t^L \beta e_t, K_{t+1}) \\ +(1-p)(1-\lambda)W_{t+1}^{BL}(R_t^{LL} \beta e_t, K_{t+1}) \end{array} \right], \end{aligned}$$

where $R_t^H \beta e_t$, $R_t^L \beta e_t$, and $R_t^{LL} \beta e_t$ are the net worth of the entrepreneur at date $t+1$ in each state. Note that the net worth of entrepreneurs evolves as $e_{t+1} = R_t^j \beta e_t$, where $j = H$, L, LL. R_t^H, R_t^L, and R_t^{LL}, which are given in the Technical Appendix, are realized rate of return on savings from date t to date $t+1$.

Solving the value function yields (derivation is given in the Technical Appendix.)

$$W_t^{BB}(e_t, K_t) = m(\lambda) + \frac{\beta\sigma(\sigma-1)}{1-\beta\sigma}\frac{1}{1-\beta}\log K_t + \frac{1}{1-\beta}\log e_t(\lambda), \tag{32}$$

where $m(\lambda)$ is given in the Technical Appendix. The period t net worth, e_t, in the third term increases with λ. When bailouts are expected, bubble prices and entrepreneurs' net worth jump up instantaneously, which increases their consumption. Thus, the third term captures the wealth effect of consumption.

By setting $t = 0$, we can understand how a rise in λ affects entrepreneurs' welfare in the initial period. Since it is hard to provide full characterization analytically on the relationship between W_0^{BB} and λ, here, we show numerical examples. We will explain more details about procedures to derive numerical examples in the Technical Appendix. Figure 5 illustrates the relationship in the benchmark parameter case, highlighting that welfare increases monotonically with λ. That is, the more bailouts are guaranteed, the more entrepreneurs gain. This result holds in cases 1-3, too. Thus, in our numerical examples, $\lambda = 1$ is optimal for entrepreneurs. Intuition is the following. In the region of $\lambda \in [0, \lambda^*)$, a rise in λ increases production before and after bubbles' collapse, which increases entrepreneurs' consumption. Moreover, because of the increase in bubble prices, the wealth effect of consumption is enhanced. Thus, entrepreneurs' consumption increases with λ throughout the lifetime. In the region of $\lambda \in (\lambda^*, 1]$, a rise in λ reduces production before the bubble bursts, which lowers entrepreneurs' consumption, while the wealth effect of consumption operates. In our numerical examples, the latter effect dominates the former. Hence, entrepreneurs' consumption increases even in the region where ex-ante production efficiency worsens with λ.[16]

Furthermore, the fact that entrepreneurs' welfare monotonically increases with λ in $\lambda \in (\lambda^{**}, 1]$, wheres workers' welfare decreases, suggests that in $\lambda \in (\lambda^{**}, 1]$, the rise in λ leads to increased inequality in welfare between bubble holders and non-bubble holders as well as in average consumption.[17]

[16]Moreover, in the region of $\lambda \in (\lambda^*, 1]$, the rate-of-return difference in H-projects, bubbles, and lending becomes smaller with λ, which contributes to decreasing consumption volatility.

[17]Stiglitz (2012) discusses how speculative activity including bubbles affects inequality between

In light of the above welfare analysis, the government's financial safety net can be Pareto-improving as long as λ is relatively low. In our numerical analysis, the government's financial safety net makes all agents better off until $\lambda = \lambda^{**}$. Therefore, if the government chooses λ according to the Pareto dominance criteria, λ stops at λ^{**}. This holds an important implication for boom-bust cycles. Figure 3 compares three cases of boom-bust cycles, $\lambda = 0$, $\lambda = \lambda^{*}$, and $\lambda = \lambda^{**}$. The Figure 3 illustrates the case of $\lambda^{**} < \lambda^{*}$. These charts show that in the case of $\lambda = \lambda^{**}$, boom-bust cycles are milder than they are in the case of $\lambda = \lambda^{*}$, although production efficiency decreases compared with $\lambda = \lambda^{*}$.

Of course, an actual λ may change depending on the objectives of the government. For example, suppose that workers were median voters. The objective of the government would be to maximize workers' welfare by setting $\lambda = \lambda^{**}$. Alternatively, if the government aimed to maximize entrepreneurs' welfare for political reasons, then they would rescue all loss-suffering entrepreneurs by setting $\lambda = 1$. In this case, overinvestment in bubbles would occur. Moreover, if the government's objective were to maximize ex-ante production efficiency, setting $\lambda = \lambda^{*}$ would be optimal; however it may not choose this λ^{*} since a conflict of interest exists.

7 Discussion

7.1 How to Finance Bailouts

Thus far, we have considered the case that the total bailout money is financed by taxing workers. Here we should mention the reason why we focus on this policy. In our model, as long as the government transfers resources among entrepreneurs, for example, from entrepreneurs who do not suffer losses to those who do, neither the

speculators and non-speculators.

aggregate wealth of entrepreneurs nor the aggregate net worth of H-types increases.[18] This means that transferring resources among entrepreneurs does not mitigate economic contractions when bubbles burst. In our model, entrepreneurs' aggregate wealth and H-types' aggregate net worth increase only if the government transfers resources from workers to entrepreneurs, thereby mitigating such contractions.

To see this point clearly, we consider a case that in order to finance the total bailout fund, the government taxes not only workers, but also entrepreneurs who do not suffer losses from bubble investments. In this case, when the bubble collapses at date t, the bailout fund, $\lambda P_t X$, is financed through the aggregate tax revenues received both from workers, T_t^u, and from entrepreneurs who do not suffer losses, T_t^e:

$$\lambda P_t X = T_t^u + T_t^e, \tag{33}$$

with

$$T_t^e = \tau \left(q_t \alpha^H Z_{t-1}^H - r_{t-1} B_{t-1}^H \right)$$

where τ is the tax rate imposed at date t on the date t net worth of the non–loss-making entrepreneurs, (i.e., H-types in period $t-1$).[19]

The date t aggregate wealth of entrepreneurs after the transfer policy can be written as

$$A_t = (1-\tau)(q_t \alpha^H Z_{t-1}^H - r_{t-1} B_{t-1}^H) + q_t \alpha^L Z_{t-1}^L - r_{t-1} B_{t-1}^L + \lambda P_t X.$$

The aggregate wealth of entrepreneurs at date t is composed of two parts. The first

[18] Of course, this depends on the assumption that the arrival rate of H-projects is the same for every entrepreneur in every period. In effect, the identity of H-types and L-types is completely reshuffled in every period. As we have seen, this assumption greatly simplifies aggregation. At the aggregate level, then, distribution between H-types and L-types does not matter.

[19] For technical reasons, i,e., in order to derive entrepreneur's consumption function explicitly, we consider the case that the government taxes entrepreneur's net worth.

term is the date t aggregate net worth after tax of entrepreneurs who were H-types in period $t-1$. The second term is the date t aggregate net worth including the bailout money of entrepreneurs who were L-types in period $t-1$. By using (33), the above A_t can be rearranged as

$$A_t = q_t K_t + T_t^u.$$

It follows that T_t^u, namely, the transfer from workers to entrepreneurs, matters for the aggregate wealth of entrepreneurs at date t as well as the aggregate net worth of H-types in period t. This finding lends support to why we consider transfer policies from workers to entrepreneurs. In this case, welfare implications remains unaffected. In the Appendix G, We explain this point more in depth.

7.2 Is Non-Stochastic Bubbles the Best?

In this final subsection, we ask the following question: If the government could make stochastic bubbles non-stochastic ones with a bailout policy, would that be the best policy from a welfare perspective? Thus far, we have examined the effects of transfer policies from workers to loss-suffering entrepreneurs. However, instead of transfer policies, we could consider a different bailout policy that uses government debt as a bailout tool. For example, consider an entrepreneur who holds bubble assets when the bubble collapses at date t. The government promises to hand out that entrepreneur government bonds. The bond prices are pegged to bubble prices on survival at date t. This bailout means that the government fully guarantees bubble investments against losses for all loss–suffering entrepreneurs. After date $t+1$, the government then simply rolls over the debt.[20] If entrepreneurs are aware of this government's bailout

[20] Government debt can be a substitute for privately created bubble assets. See Caballero and Krishnamurthy (2006), Kocherlakota (2009), and Hirano and Yanagawa (2010b) for details.

plan, they expect bubble assets to deliver the same rate of return regardless of the realization of π. Bubble assets are thus considered to be risk-free, and so L-types no longer invest in their L-projects for risk-hedge. This policy restores entrepreneurs' net worth to what it would have been in the absence of the bubbles' bursting. The law of motion of the aggregate economy is thus the same as in the non-stochastic bubbly economy throughout the lifetime of the economy, not just after the bubble bursts, but also before the bursts. However, although this bailout policy has exactly the same effects as the government policy that makes stochastic bubbles non-stochastic ones, is this the best policy from a welfare perspective?

Moreover, the probability, π, might be affected by transfer policies from workers to loss-suffering entrepreneurs. In this paper, we have assumed that π is exogenously given and unaffected by policies in line with the traditional literature. However, if the government fully guaranteed bubble investments against losses for all loss-suffering entrepreneurs, (i.e., setting $\lambda = 1$), all agents might change their expectations about bubble bursts and might expect $\pi = 1$. Here, we have no intention to extend our argument to examine whether such a change in expectations is reasonable or not, or examine endogenous formulation of π. However, if agents were to change their expectations to $\pi = 1$, setting $\lambda = 1$ would make stochastic bubbles non-stochastic ones. If so, once again, is this the best policy?

In order to answer these questions, we thus examine whether stochastic bubbles (riskier bubbles) can be better than non-stochastic ones from the welfare perspective of workers. In order to examine this, we set $\lambda = 0$ in (30). Then, from (30), we have

$$\frac{dV_0^{BB}}{d\pi}\Big|_{\pi=1} = -\frac{\beta^2\sigma}{1-\beta\sigma}\frac{1}{p\beta+(1-\beta)\theta}\frac{(1-p)(1-\theta)}{1-\beta+p\beta}$$
$$+\frac{1}{(1-\beta)^2}\frac{\beta^2\sigma}{1-\beta\sigma}\left\{\log\left[\alpha^H\frac{p\beta+(1-\beta)\theta}{1-\beta+p\beta}\sigma\right]-\log\left[\left(1+\frac{\alpha^H-\alpha^L}{\alpha^L-\theta\alpha^H}p\right)\beta\alpha^L\sigma\right]\right\}, \tag{34}$$

(34) says that rise in π produces two competing effects on workers' welfare. If the bursting probability of bubbles falls, workers can earn higher wage incomes and consume more for longer periods, improving their levels of welfare. The second line of (34) captures this effect. The term in the brackets in the second line reflects the difference in wage income between the bubble economy and the bubbleless one. On the other hand, with the increase in π, L-types are willing to buy more bubble assets, because bubble assets become safer. As a result, larger bubbles are created (ϕ increases with π), which strengthens the crowding out effect of bubbles. This, in turn, reduces production, wage income, and consumption before the bubble bursts, lowering workers' welfare. The first line of (34) captures this effect. Under certain parameter values (see the Appendix F), the expansionary effects of bubbles are relatively small, namely the wage-income difference becomes sufficiently small. As a result, the crowding-out effect dominates the first effect. Thus, the sign of (34) becomes negative. This all means that the following welfare ranking holds for workers:

$$V_0^{BB}(\pi=1) < V_0^{BB}(\pi<1 \text{ and } \lambda=0).$$

i.e., stochastic bubbles can be better than non-stochastic bubbles. This finding suggests that increasing the fragility of bubbles might actually enhance workers' welfare. The following Proposition summarizes this.

Proposition 5 *From welfare perspective of workers, stochastic bubbles can be better than non-stochastic ones, i.e.,* $\pi = 1$ *is not necessarily the best.*

Moreover, under some parameter values,[21] we have the following welfare ranking for workers:

$$V_0^{BB}(\pi = 1) < V_0^{BB}(\pi < 1 \text{ and } \lambda = 0) < V_0^{BB}(\pi < 1 \text{ and } \lambda > 0).$$

This says that welfare under stochastic bubbles without bailouts is better than that under non-stochastic bubbles. Moreover, welfare under stochastic bubbles with bailouts is better than that under stochastic bubbles without bailouts.

8 Conclusion

In this paper, we analyzed how anticipated bailouts affect the existence of stochastic bubbles, production efficiency, and boom-bust cycles. Moreover, we examined the welfare consequences of such anticipated bailouts, and considered optimal bailout policies for taxpayers and for rescued entrepreneurs. Based on the presented analysis, we can draw the following conclusions.

Firstly, bailouts affect the existence conditions of stochastic bubbles. Even riskier bubbles can occur because of the existence of government guarantees.

Secondly, bailouts initially improve ex-ante efficiency in production by crowding in productive investments, while crowding out unproductive ones, but too generous bailouts lead to overinvestment in bubbles, which leads to strong crowding-out effects even on productive investments, thereby decreasing the production efficiency. In other words, bailouts have non-monotonic effects on ex-ante production efficiency.

[21]For example, $\alpha^H = 1.065$, $\alpha^L = 1$, $\beta = 0.98$, $\sigma = 0.25$, $\theta = 0.1$, $p = 0.3$, $\pi = 0.99999999$, $\lambda = 0.0000001$.

This suggests that there is a certain bailout level at which ex-ante production efficiency is maximized. Under the bailout policy, although the production efficiency is maximized, it may increase boom-bust cycles and require large amount of public funds following the collapse of bubbles. This finding suggests a trade-off between economic stability and efficient resource allocation, which leads onto our third contribution.

Thirdly, we found that no-bailouts and full-bailouts are not optimal for taxpayers, i.e., partial bailouts are optimal for them. Moreover, in the case of riskier bubbles, in order to maximize taxpayers' welfare, the government must sacrifice some production efficiency to reduce the size of bubbles and soften boom-bust cycles. In contrast, welfare for rescued entrepreneurs monotonically increases with the provision of bailouts. Our finding from the welfare analysis suggests that bailouts can be Pareto-improving from a welfare perspective.

Lastly, stochastic bubbles can be better than non-stochastic ones from the welfare perspective of taxpayers. This finding suggests that increasing the fragility of bubbles might actually enhance taxpayers' welfare.

In this paper, we focused on optimal ex-post bailouts by taking ex-ante effects into consideration. Future work could extend the presented analysis in order to additionally consider ex-ante regulations such as leverage regulations or tax/subsidy policy on risky assets. It would also be interesting to examine the desirable policy mix from a welfare perspective.

Appendix A: Proof of Proposition 1

From (25),

$$\phi_{t+1} = \begin{cases} f(\phi_t) & \text{if } \phi_t \le \phi^*, \\ \\ g(\phi_t) & \text{if } \phi_t > \phi^*, \end{cases} \tag{A.1}$$

where $f(\phi_t) \equiv \frac{\frac{(1-p-\phi_t)}{\delta(\lambda)(1-p)-\phi_t}}{\left(1+\frac{\alpha^H-\alpha^L}{\alpha^L-\theta\alpha^H}p\right)\beta+\frac{[1-\delta(\lambda)](1-p)}{\delta(\lambda)(1-p)-\phi_t}\beta\phi_t}\phi_t$, $g(\phi_t) \equiv \frac{\theta}{\beta}\frac{1}{\delta(\lambda)(1-p)-(1-\theta)\phi_t}\phi_t$, and $\phi^* \equiv \frac{\alpha^L(1-p)-\theta\alpha^H}{\alpha^L-\theta\alpha^H}$. Note that if and only if $\theta < \frac{\alpha^L}{\alpha^H}(1-p)$, $\phi^* > 0$. Moreover, $f(\phi^*) = g(\phi^*)$. We can easily derive that $g' > 0$ and $g" > 0$. Hence, if and only if $g'(0) < 1 \Leftrightarrow \theta < \delta(\lambda)\beta(1-p)$, $g(\phi) = \phi$ has a unique strictly positive solution, $\phi^g = \frac{\delta(\lambda)\beta(1-p)-\theta}{\beta(1-\theta)} > 0$. $f(\phi_t)$ function is rather complicated, but, by solving $f(\phi) = \phi$ explicitly, we can find that this equation has only two solutions, 0 and $\phi^f = \frac{\delta(\lambda)-\frac{1-\delta(\lambda)\beta(1-p)}{\left[1+(\frac{\alpha^H-\alpha^L}{\alpha^L-\theta\alpha^H})p\right]\beta-\beta(1-p)}}{1-\frac{1-\delta(\lambda)\beta(1-p)}{\left[1+(\frac{\alpha^H-\alpha^L}{\alpha^L-\theta\alpha^H})p\right]\beta-\beta(1-p)}}(1-p)$. Furthermore $\phi^f > 0$ if and only if $f'(0) < 1 \Leftrightarrow \theta > \frac{(1-\delta\beta)\alpha^L-p\beta\delta(\alpha^H-\alpha^L)}{\alpha^H(1-\delta\beta)}$.

(i-1) Obviously, if $\phi^f \le 0$ and $\phi^g \le 0$, bubbles cannot exist.

(i-2) Next, we examine the case where $\phi^f \le 0$ and $\phi^g > 0$. In this case ϕ^g is a candidate to realize $\phi_{t+1} = \phi_t \equiv \phi$. However, $\phi^f \le 0$ means $f(\phi) > \phi$ for any positive ϕ and thus $f(\phi^*) = g(\phi^*) > \phi^*$. It follows that $\phi^g < \phi^*$ and $\phi^g \ne \phi$. In other words, bubbles cannot exist.

(i-3) When $\phi^f > 0$ and $\phi^g \le 0$, ϕ^f is a candidate of ϕ. In order that both $\phi^g \le 0$ and $\phi^* > 0$ are satisfied, $\delta\beta(1-p) < \theta < \frac{\alpha^L}{\alpha^H}(1-p)$ and $\delta\beta < \frac{\alpha^L}{\alpha^H}$ must be satisfied. However, when $\delta\beta < \frac{\alpha^L}{\alpha^H}$, $\theta < \frac{(1-\delta\beta)\alpha^L-p\beta\delta(\alpha^H-\alpha^L)}{\alpha^H(1-\delta\beta)}$ and ϕ^f cannot be strictly positive. Hence, there is a contradiction and bubbles cannot exist even in this case.

(i-4) Lastly, we examine the case where $\phi^f > 0$ and $\phi^g > 0$. This is the situation where $\frac{(1-\delta\beta)\alpha^L-p\beta\delta(\alpha^H-\alpha^L)}{\alpha^H(1-\delta\beta)} < \theta < \frac{\alpha^L}{\alpha^H}(1-p)$ and $\frac{\alpha^L}{\alpha^H}(1-p) < \delta\beta(1-p) \Leftrightarrow \phi^* > 0$ is satisfied from $\frac{(1-\delta\beta)\alpha^L-p\beta\delta(\alpha^H-\alpha^L)}{\alpha^H(1-\delta\beta)} < \theta$. By defining that $\hat{\lambda}$ is the value of λ that satisfies $\phi^g(\hat{\lambda}) = \phi^*$, we can obtain that $\phi^* = g(\phi^*) = f(\phi^*) = \phi^f(\hat{\lambda})$. Since ϕ^f and ϕ^g are increasing functions of λ, $\phi^f(\lambda) < \phi^*$ and $\phi^g(\lambda) < \phi^*$ under $\lambda < \hat{\lambda}$. This means $\phi^f(\lambda) = \phi(\lambda)$ under $\lambda < \hat{\lambda}$. Similarly, $\phi^f(\lambda) > \phi^*$ and $\phi^g(\lambda) > \phi^*$ under $\lambda > \hat{\lambda}$, and we obtain that $\phi^g(\lambda) = \phi(\lambda)$ under $\lambda > \hat{\lambda}$. If $\hat{\lambda} < 0$, $\phi^f(\lambda) > \phi^*$ and $\phi^g(\lambda) > \phi^*$ for any λ and $\phi^g(\lambda) = \phi(\lambda)$. From these results we can obtain (27).

Moreover, if and ony if $\frac{(1-\delta\beta)\alpha^L - p\beta\delta(\alpha^H - \alpha^L)}{\alpha^H(1-\delta\beta)} < \theta < \frac{\alpha^L}{\alpha^H}(1-p)$, $\phi(\lambda) > 0$ and stochastic bubbles can exist. The condition $\theta < \delta(\lambda)\beta(1-p) \equiv \theta_1$ is directly derived from the second inequality, and the condition $\pi > \frac{\alpha^L - \theta\alpha^H}{\beta(\alpha^L - \theta\alpha^H) + p\beta(\alpha^H - \alpha^L)} \frac{1}{1-\lambda} - \frac{\lambda}{1-\lambda} \equiv \pi_1$ can be derived from the first inequality.

Appendix B: Proof of Proposition 2

When we solve for the steady-state capital stock from (28), we learn that there are two steady-state values. One is $K = 0$. The other is $K(\lambda) > 0$. Since $\frac{dK_{t+1}}{dK_t} > 0$, $\frac{d^2K_{t+1}}{dK_t^2} < 0$, $\frac{dK_{t+1}}{dK_t}(K_t = 0) = \infty$, $\frac{dK_{t+1}}{dK_t}(K_t = K(\lambda) > 0) = \sigma < 1$, the stochastic steady state $K(\lambda) > 0$ is globally stable. Thus, for any positive initial capital stock level, $K_0 > 0$, the economy runs according to (28) and converges to the stochastic steady-state $K(\lambda) > 0$ until bubbles collapse. As long as the capital stock level converges to $K(\lambda)$, other variables, A_t, q_t, r_t, and w_t must converge to constant levels. Moreover, P_t converges to a positive value since A_t converges to a positive value and ϕ is constant over time.

Appendix C: Proof of Proposition 3

Differentiating (27) in $0 \leq \lambda \leq \lambda^*$ yields

$$\frac{d\phi(\lambda)}{d\lambda} = \frac{(1-p)(1-\pi)}{\left[\left(1+\frac{\alpha^H-\alpha^L}{\alpha^L-\theta\alpha^H}p\right)\beta - 1 - \beta(1-p)[1-\delta(\lambda)]\right]^2} \times \left[\left(1+\frac{\alpha^H-\alpha^L}{\alpha^L-\theta\alpha^H}p\right)\beta - 1\right]\left[\left(1+\frac{\alpha^H-\alpha^L}{\alpha^L-\theta\alpha^H}p\right)\beta - \beta(1-p)\right] > 0,$$

because in the bubble regions, since $\phi(\lambda) > 0$, the following inequality is satisfied:

$$\left(1+\frac{\alpha^H-\alpha^L}{\alpha^L-\theta\alpha^H}p\right)\beta > \frac{1}{\delta(\lambda)} > 1 > \beta(1-p) > 0.$$

In $\lambda^* < \lambda \leq 1$, differentiating (27) yields

$$\frac{d\phi(\lambda)}{d\lambda} = \frac{\beta(1-p)(1-\pi)}{\beta(1-\theta)} > 0.$$

Appendix D: Proof of Lemma 1

In $0 \leq \lambda < \lambda^*$, differentiating $H(\lambda)$ with respect to λ yields

$$\frac{dH(\lambda)}{d\lambda} = \frac{\left(1+\frac{\alpha^H-\alpha^L}{\alpha^L-\theta\alpha^H}p\right)\beta\alpha^L - \beta\alpha^L(1-p)}{[1-\delta(\lambda)\beta(1-p)]^2}\sigma\beta(1-p)(1-\pi) > 0,$$

because in the bubble regions, since $\phi(\lambda) > 0$, the following inequality is satisfied:

$$\left(1+\frac{\alpha^H-\alpha^L}{\alpha^L-\theta\alpha^H}p\right)\beta > \frac{1}{\delta(\lambda)} > 1 > \beta(1-p) > 0.$$

In $\lambda^* < \lambda \leq 1$, differentiating $H(\lambda)$ with respect to λ yields

$$\frac{dH(\lambda)}{d\lambda} = -\frac{\alpha^H \beta(1-p)(1-\pi)\sigma}{[1-\delta(\lambda)\beta(1-p)]^2}(1-\beta)(1-\theta) < 0.$$

Appendix E: Proof of Lemma 2

By differentiating equation (29) with respect to λ, we obtain

$$\begin{aligned}\frac{dV_t^{BL}}{d\lambda} &= \frac{-\sigma}{1-\sigma-\lambda\frac{\beta\phi(\lambda)}{1-\beta\phi(\lambda)}\sigma}\left\{\frac{\beta\phi(\lambda)}{1-\beta\phi(\lambda)} + \frac{\lambda\beta}{[1-\beta\phi(\lambda)]^2}\frac{d\phi(\lambda)}{d\lambda}\right\} \\ &\quad + \frac{\beta\sigma}{1-\beta\sigma}\frac{1}{1+\lambda\frac{\beta\phi(\lambda)}{1-\beta\phi(\lambda)}}\left\{\frac{\beta\phi(\lambda)}{1-\beta\phi(\lambda)} + \frac{\lambda\beta}{[1-\beta\phi(\lambda)]^2}\frac{d\phi(\lambda)}{d\lambda}\right\} < 0\end{aligned}$$

The first line is the marginal cost of bailout expansions, while the second line is the marginal benefit. The above equation says that the marginal cost dominates the marginal benefit.

Appendix F: Proof of Proposition 5

We need to prove that there exist parameter values under which stochastic bubbles can arise and the sign of (34) is negative.

First, we prove that there exist parameter values under which the sign of (34) is negative. With regard to the second term of (34), when we solve for α^H that satisfies $\alpha^H \frac{p\beta+(1-\beta)\theta}{1-\beta+p\beta}\sigma = \left(1+\frac{\alpha^H-\alpha^L}{\alpha^L-\theta\alpha^H}p\right)\beta\alpha^L\sigma$, then we obtain $\alpha^H = \frac{\alpha^L(1-\beta+p\beta)}{(1-\beta)\theta+p\beta}$ and $\frac{\alpha^L\beta(1-p)}{\theta}$. We focus on the case where $\theta < \frac{p\beta^2(1-p)}{(1-\beta)(1-\beta+p\beta)+p\beta}$. In this case, $\frac{\alpha^L(1-\beta+p\beta)}{(1-\beta)\theta+p\beta} < \frac{\alpha^L\beta(1-p)}{\theta}$. If we pick up $\alpha_1^H \in \left(\frac{\alpha^L(1-\beta+p\beta)}{(1-\beta)\theta+p\beta}, \frac{\alpha^L\beta(1-p)}{\theta}\right)$ that is sufficiently close to $\frac{\alpha^L(1-\beta+p\beta)}{(1-\beta)\theta+p\beta}$ or $\frac{\alpha^L\beta(1-p)}{\theta}$, then $\alpha^H\frac{p\beta+(1-\beta)\theta}{1-\beta+p\beta}\sigma$ is sufficiently close to $\left[1+\frac{\alpha^H-\alpha^L}{\alpha^L-\theta\alpha^H}p\right]\beta\alpha^L\sigma$. Thus, under α_1^H, the first term of (34) dominates the second term of (34), i.e., under α_1^H, the sign of (34) is negative.

Next, we prove that stochastic bubbles can arise under α_1^H. If we characterize bubble regions with α^H, stochastic bubbles can arise if and only if $\alpha^H > \frac{\alpha^L[1-\pi\beta(1-p)]}{(1-\pi\beta)\theta+p\beta\pi} \equiv \hat{\alpha}^H(\pi)$, where $\hat{\alpha}^H$ is a decreasing function of π in $0 < \pi \leq 1$ with $\hat{\alpha}^H = \frac{\alpha^L(1-\beta+p\beta)}{(1-\beta)\theta+p\beta}$ if $\pi = 1$ (Note that if $\pi = 1$, deterministic bubbles can arise in $\alpha^H \geq \frac{\alpha^L(1-\beta+p\beta)}{(1-\beta)\theta+p\beta}$). Thus, there exist a critical value of $\pi_1(\alpha_1^H) < 1$ where stochastic bubbles can arise under α_1^H in $\pi \in \big(\pi_1(\alpha_1^H), 1\big]$.

Appendix G: Tax on entrepreneurs

In the main text, we have considered the case that the total bailout money is financed by taxing workers. In this Appendix G, we consider a case that in order to finance bailout money, the government taxes not only workers, but also entrepreneurs who do not suffer losses from bubble investments.

In this case, when bubbles collapse at date t, bailout money, $\lambda P_t X$, is financed through aggregate tax revenues from workers, T_t^u, and aggregate tax revenues from entrepreneurs who do not suffer losses, T_t^e:

$$\lambda P_t X = T_t^u + T_t^e,$$

with

$$\begin{aligned} T_t^e &= \tau \left(q_t \alpha^H Z_{t-1}^H - r_{t-1} B_{t-1}^H \right) \\ &= \begin{cases} \tau \dfrac{\alpha^H(1-\theta)p}{(\alpha^L - \theta\alpha^H)[1-\phi(\lambda)] + (\alpha^H - \alpha^L)p} \sigma K_t^\sigma & \text{if } 0 \leq \lambda \leq \lambda^*, \\ \\ \tau(1-\theta)\sigma K_t^\sigma & \text{if } \lambda^* \leq \lambda \leq 1, \end{cases} \end{aligned}$$

where τ is a tax rate imposed on the date t net worth of the non–loss-making entrepreneurs, (i.e., H-types in period $t-1$). For technical reasons, i,e., in order to derive entrepreneur's consumption function explicitly, we consider the case that the government taxes entrepreneur's net worth. T_t^e increases with λ in $0 \leq \lambda \leq \lambda^*$. This means that as λ rises, aggregate H-investments expand during bubbly periods, which increases tax revenues from the non–loss-making entrepreneurs when bubbles collapse. This increase in tax revenues reduces tax burden for workers. When we solve for tax burden per unit of workers, T_t^u (recall that there are workers with unit measure), we learn

$$T_t^u = \lambda P_t X - T_t^e = F(\lambda)\sigma K_t^\sigma, \tag{G.2}$$

with

$$F(\lambda) = \begin{cases} \lambda \dfrac{\beta\phi(\lambda)}{1-\beta\phi(\lambda)} - \tau \dfrac{\alpha^H(1-\theta)p}{(\alpha^L - \theta\alpha^H)[1-\phi(\lambda)] + (\alpha^H - \alpha^L)p} & \text{if } 0 \leq \lambda \leq \lambda^*, \\ \\ \lambda \dfrac{\beta\phi(\lambda)}{1-\beta\phi(\lambda)} - \tau(1-\theta) & \text{if } \lambda^* \leq \lambda \leq 1. \end{cases} \tag{G.3}$$

It follows that T_t^u is a decreasing function of τ.

By using (G.2) and (G.3), $W(\lambda)$ is replaced with

$$\begin{aligned} M(\lambda) \quad &= \quad \log\left[1-\sigma-\sigma F(\lambda)\right] + \frac{\beta\sigma}{1-\beta\sigma}\log\left[1+F(\lambda)\right] \\ &+ \frac{\beta\sigma}{1-\beta\sigma}\frac{1}{1-\beta}\log\left[\left(1+\frac{\alpha^H-\alpha^L}{\alpha^L-\theta\alpha^H}p\right)\beta\alpha^L\sigma\right] + \frac{\beta}{1-\beta}\log(1-\sigma). \end{aligned} \tag{G.4}$$

From (30) together with (G.4), we see how an increase in τ affects workers' welfare. We learn that (30) is an increasing function of τ, i.e., workers' welfare increases with τ. Intuition is very simple. If the government imposes higher tax rate on the non–loss-making entrepreneurs, tax burden per unit of workers decreases, which increases workers' consumption when bubbles collapse, thereby improving their welfare. We also learn from (30) how an increase in bailout guarantees affects workers' welfare in this case. We find that even in this case, if (A1) holds, then partial bailouts are optimal for workers. Moreover, when we compute (30) with (G.4) under the benchmark parameter case, then we find that λ^{**} increases with τ and approaches λ^{*}. This means that optimal bailouts for workers approach the bailout level that maximizes ex-ante output efficiency. When the government taxes the non–loss-making entrepreneurs, tax revenues from those entrepreneurs increase together with an increase in λ, since T_t^e is an increasing function of λ. This increase in tax revenues lowers tax burden for workers. As a result, the welfare-enhancing effect captured by the first term of (31) dominates the welfare-reducing effect captured by the second term of (31) even in greater values of $\lambda < \lambda^{*}$.

We can also compute welfare for entrepreneurs in this case. When computing it, we need to take into account the fact that entrepreneurs are taxed when bubbles collapse if they are H-types in one period before bubbles' collapsing (see the Technical Appendix for derivation of the value function in this case). We find that welfare for entrepreneurs monotonically increases with λ even in this case.

Technical Appendices

Appendix H: Derive the demand function for bubble assets of a L-entrepreneur

Each L-entrepreneur chooses optimal amounts of b_t^i, x_t^i, and z_t^i so that the expected marginal utility from investing in three assets is equalized. The first order conditions with respect to x_t^i and b_t^i are

$$(x_t^i): \ \frac{P_t}{c_t^i} = \pi\beta\frac{P_{t+1}}{c_{t+1}^{i,\pi}} + (1-\pi)\lambda\beta\frac{d_{t+1}}{c_{t+1}^{i,(1-\pi)\lambda}}, \tag{H.5}$$

$$(b_t^i): \ \frac{1}{c_t^i} = \pi\beta\frac{r_t}{c_{t+1}^{i,\pi}} + (1-\pi)\lambda\beta\frac{r_t}{c_{t+1}^{i,(1-\pi)\lambda}} + (1-\pi)(1-\lambda)\beta\frac{r_t}{c_{t+1}^{i,(1-\pi)(1-\lambda)}}, \tag{H.6}$$

where $c_{t+1}^{i,\pi} = (1-\beta)(q_{t+1}\alpha^L z_t^i - r_t b_t^i + P_{t+1}x_t^i)$, $c_{t+1}^{i,(1-\pi)\lambda} = (1-\beta)(q_{t+1}\alpha^L z_t^i - r_t b_t^i + m_{t+1}^i)$, and $c_{t+1}^{i,(1-\pi)(1-\lambda)} = (1-\beta)(q_{t+1}\alpha^L z_t^i - r_t b_t^i)$.[22] The RHS of (H.5) is the gain in expected discounted utility from holding one additional unit of bubble assets at date $t+1$. With probability π bubbles survive, in which case the entrepreneur can sell the additional unit at P_{t+1}, but with probability $1-\pi$ bubbles collapse, in which case with probability λ he/she is rescued and receives d_{t+1} units of consumption goods per unit of bubble assets, and with probability $1-\lambda$, he/she is not rescued and receives nothing. The denominators reflect the respective marginal utilities of consumption. The RHS of (H.6) is the gain in expected discounted utility from lending one additional unit. It is similar to the RHS of (H.5), except for the fact that lending yields r_t at date $t+1$, irrespective of whether or not bubbles collapse.

[22] Since the entrepreneur consumes a fraction $1-\beta$ of the current net worth in each period, the optimal consumption level at date $t+1$ is independent of the entrepreneur's type at date $t+1$. It only depends on whether bubbles collapse and whether government rescues the entrepreneur.

From (17), (H.5), and (H.6), we can derive the demand function for bubble assets of a type i L-entrepreneur in the main text.

Appendix I: Aggregation

The great merit of the expressions for each entrepreneur's investment and demand for bubble assets, z_t^i and x_t^i, is that they are linear in period-t net worth, e_t^i. Hence aggregation is easy: we do not need to keep track of the distributions.

From (16), we learn the aggregate H-investments:

$$Z_t^H = \frac{\beta p A_t}{1 - \dfrac{\theta q_{t+1}\alpha^H}{r_t}}, \tag{I.7}$$

where $A_t \equiv q_t K_t + P_t X$ is the aggregate wealth of entrepreneurs at date t, and $\sum_{i \in H_t} e_t^i = pA_t$ is the aggregate wealth of H-entrepreneurs at date t. From this investment function, we see that the aggregate H-investments are both history-dependent and forward-looking, because they depend on asset prices, P_t, as well as cash flows from the investment projects in the previous period, $q_t K_t$. In this respect, this investment function is similar to the one in Kiyotaki and Moore (1997). There is a significant difference. In the Kiyotaki-Moore model, the investment function depends on land prices which reflect fundamentals (cash flows from land), while in our model, it depends on bubble prices.

Aggregate L-investments depend on the level of the interest rate:

$$Z_t^L = \begin{cases} \beta A_t - \frac{\beta p A_t}{1 - \frac{\theta \alpha^H}{\alpha^L}} - P_t X & \text{if } r_t = q_{t+1}\alpha^L, \\ \\ 0 & \text{if } r_t > q_{t+1}\alpha^L. \end{cases} \tag{I.8}$$

When $r_t = q_{t+1}\alpha^L$, L-entrepreneurs may invest positive amount. In this case, we know from (19) that aggregate L-investments are equal to aggregate savings of the economy minus aggregate H-investments minus aggregate value of bubbles. When $r_t > q_{t+1}\alpha^L$, L-entrepreneurs do not invest.

The aggregate counterpart to (18) is

$$P_t X_t = \frac{\delta(\lambda)\frac{P_{t+1}}{P_t} - r_t}{\frac{P_{t+1}}{P_t} - r_t}\beta(1-p)A_t, \tag{I.9}$$

where $\sum_{i\in L_t} e_t^i = (1-p)A_t$ is the aggregate net worth of L-entrepreneurs at date t. (I.9) is the aggregate demand function for bubble assets at date t.

Appendix J: Worker's Behavior

We verify that workers do not save nor buy asset bubbles in equilibrium. First, we verify that workers do not save. When the borrowing constrained binds, workers do not save. The condition that the borrowing constraint binds is

$$\frac{1}{c_t^u} > \pi\beta\frac{r_t}{c_{t+1}^{u,\pi}} + (1-\pi)\beta\frac{r_t}{c_{t+1}^{u,1-\pi}}.$$

We know that $c_t^u = w_t$ and $c_{t+1}^{u,\pi} = w_{t+1}$ if workers do not save nor buy bubble assets. Then, the above can be written as

$$1 > \left[\pi + (1-\pi)\frac{1-\sigma}{1-\sigma-\lambda\frac{\beta\phi(\lambda)}{1-\beta\phi(\lambda)}\sigma}\right]\beta\frac{K_t^\sigma}{K_{t+1}^\sigma}r_t. \tag{J.10}$$

When $r_t = q_{t+1}\alpha^L$, (J.10) can be written as

$$\frac{H(\lambda)}{\beta\sigma\alpha^L} > \pi + (1-\pi)\frac{1-\sigma}{1-\sigma-\lambda\frac{\beta\phi(\lambda)}{1-\beta\phi(\lambda)}\sigma}. \tag{J.11}$$

Since $H(\lambda)/\beta\sigma\alpha^L > 1$ in the bubble regions and the right hand side of (J.11) is an increasing function of σ and equals one with $\sigma = 0$, (J.11) holds if σ is sufficiently small.

When $r_t = \theta q_{t+1}\alpha^H[1-\beta\phi(\lambda)]/[1-p-\phi(\lambda)]$, (J.11) can be written as

$$\frac{H(\lambda)[1-p-\phi(\lambda)]}{\theta\beta\sigma\alpha^H[1-\phi(\lambda)]} > \pi + (1-\pi)\frac{1-\sigma}{1-\sigma-\lambda\frac{\beta\phi(\lambda)}{1-\beta\phi(\lambda)}\sigma}. \tag{J.12}$$

Since $H(\lambda)[1-p-\phi(\lambda)]/\theta\beta\sigma\alpha^H[1-\phi(\lambda)] > 1$ in the bubble regions and the right hand side of (J.11) is an increasing function of σ and equals one with $\sigma = 0$, (J.12) holds if σ is sufficiently small. Under the reasonable parameter values in our numerical examples, both (J.11) and (J.12) hold.

Next, we verify that workers do not buy bubble assets. When the short sale constraint binds, workers do not buy bubble assets. The condition that the short sale constraint binds is

$$\frac{1}{c_t^u} > \pi\beta\frac{1}{c_{t+1}^{u,\pi}}\frac{P_{t+1}}{P_t}.$$

We know $c_t^u = w_t$ and $c_{t+1}^{u,\pi} = w_{t+1}$ if workers do not save nor buy bubble assets. Then, the above can be written as

$$1 > \pi\beta\frac{w_t}{w_{t+1}}\frac{P_{t+1}}{P_t} = \pi\beta,$$

which is true.

Appendix K: Behavior of H-types

We verify that H-types do not buy bubble assets in equilibrium. When the short sale constraint binds, H-types do not buy bubble assets. In order that the short sale constraint binds, the following condition must hold:

$$\frac{1}{c_t^i} > \beta E_t\left[\frac{1}{c_{t+1}^i}\frac{P_{t+1}}{P_t}\right]. \tag{K.13}$$

Since the borrowing constraint is binding for H-types, we have

$$\frac{1}{c_t^i} = \beta E_t\left[\frac{r_t}{c_{t+1}^i}\frac{q_{t+1}\alpha^H(1-\theta)}{r_t-\theta q_{t+1}\alpha^H}\right]. \tag{K.14}$$

We also know that $c_{t+1}^i = (1-\beta)\left[\frac{r_t\alpha^H(1-\theta)}{r_t-\theta q_{t+1}\alpha^H}\right]$ if (K.13) is true. Inserting (K.14) into (K.13) yields

$$\beta\frac{1}{c_{t+1}^i}\frac{r_t\left[q_{t+1}\alpha^H-\delta(\lambda)\frac{P_{t+1}}{P_t}\right]+\theta q_{t+1}\alpha^H\left[\delta(\lambda)\frac{P_{t+1}}{P_t}-r_t\right]}{r_t-\theta q_{t+1}\alpha^H} > 0. \tag{K.15}$$

If (K.15) holds, then the short sale constraint binds. We see that the second term in the numerator is positive as long as $\phi > 0$ and we know that $\phi > 0$ on the saddle path. Thus, if the first term is positive, (K.15) holds. The condition that the first term is positive is

$$q_{t+1}\alpha^H > \delta(\lambda)\frac{P_{t+1}}{P_t}.$$

On the saddle path, since P_t follows according to

$$P_t = \frac{\beta\phi(\lambda)}{X[1-\beta\phi(\lambda)]}\sigma K_t^\sigma, \tag{K.16}$$

Using (K.16), the above inequality condition can be written as

$$\sigma\alpha^H K_t^\sigma > \delta(\lambda)K_{t+1}. \tag{K.17}$$

First, we show that (K.17) holds in $0 \leq \lambda \leq \lambda^*$. In $0 \leq \lambda \leq \lambda^*$, aggregate capital stock follows (28). Thus, (K.17) can be written as

$$\alpha^H > \delta(\lambda) \left[(1 + \frac{\alpha^H - \alpha^L}{\alpha^L - \theta\alpha^H} p)\beta\alpha^L - \beta\alpha^L \phi(\lambda) \right] / \left[1 - \beta\phi(\lambda) \right], \tag{K.18}$$

which is equivalent to

$$\alpha^H > \delta(\lambda) \left[(1 + \frac{\alpha^H - \alpha^L}{\alpha^L - \theta\alpha^H} p)\beta\alpha^L - \beta\alpha^L (1-p) \right] / \left[1 - \delta(\lambda)\beta(1-p) \right]. \tag{K.19}$$

The right hand side of (K.19) is an increasing and convex function of λ in $0 \leq \lambda \leq \lambda^*$. Thus (K.17) holds in $0 \leq \lambda \leq \lambda^*$ if (K.17) is true at $\lambda = \lambda^*$. At $\lambda = \lambda^*$, we know $\phi = [\alpha^L(1-p) - \theta\alpha^H]/(\alpha^L - \theta\alpha^H)$. Inserting this relation into (K.18) yields

$$\alpha^H(1-\beta) + \frac{\alpha^L p \beta \alpha^H}{\alpha^L - \theta\alpha^H}[1 - \delta(\lambda^*)] > 0,$$

which is true.

Next, we show that (K.17) holds in $\lambda^* \leq \lambda \leq 1$. In $\lambda^* \leq \lambda \leq 1$, aggregate capital stock follows (28). Thus, (K.17) can be written as

$$1 - \beta\phi > \delta(\lambda)\beta(1-\phi),$$

which is true, since $1 - \beta\phi > 1 - \phi$ and $\delta(\lambda)\beta < 1$.

Appendix L: Derivation of taxpayer's value function

Suppose that at date t, bubbles collapse. After the date t, the economy is in the bubbleless economy. Let V_t^{BL} be the value function of taxpayers at date t when bubbles collapse and the government bails out entrepreneurs. First, we solve V_{t+1}^{BL}. Given the optimal decision rules, the Bellman equation can be written as

$$V_{t+1}^{BL}(K_{t+1}) = \log c_{t+1} + \beta V_{t+2}^{BL}(K_{t+2}), \quad \text{after date } t+1, \tag{L.20}$$

with

$$\begin{cases} c_{t+1} = w_{t+1} & \text{after date } t+1, \\ K_{t+2} = \left[1 + \frac{\alpha^H - \alpha^L}{\alpha^L - \theta\alpha^H} p\right] \beta\alpha^L \sigma K_{t+1}^{\sigma} & \text{after date } t+1. \end{cases} \tag{L.21}$$

We guess that the value function is a linear function of $\log K$:

$$V_{t+1}^{BL}(K_{t+1}) = f + g \log K_{t+1} \quad \text{after date } t+1. \tag{L.22}$$

From (L.20)-(L.22), applying the method of undetermined coefficients yields

$$f = \frac{1}{1-\beta}\log(1-\sigma) + \frac{1}{1-\beta}\frac{\beta\sigma}{1-\beta\sigma}\log\left[\left(1 + \frac{\alpha^H - \alpha^L}{\alpha^L - \theta\alpha^H} p\right)\beta\alpha^L\sigma\right],$$

$$g = \frac{\sigma}{1-\beta\sigma}.$$

Thus, we have

$$V_{t+1}^{BL}(K_{t+1}) = \frac{1}{1-\beta}\log(1-\sigma) + \frac{1}{1-\beta}\frac{\beta\sigma}{1-\beta\sigma}\log\left[\left(1+\frac{\alpha^H-\alpha^L}{\alpha^L-\theta\alpha^H}p\right)\beta\alpha^L\sigma\right] + \frac{\sigma}{1-\beta\sigma}\log K_{t+1}, \text{ after date } t+1. \tag{L.23}$$

Next, we derive the value function of taxpayers at date t when bubbles collapse and the government bails out entrepreneur by taking into account the effects of bailouts on the date t consumption and the date $t+1$ aggregate capital stock. The value function of taxpayers at date t satisfies

$$V_t^{BL}(K_t) = \log c_t + \beta V_{t+1}^{BL}(K_{t+1}), \tag{L.24}$$

with

$$\begin{cases} c_t = w_t - \lambda P_t X = w_t - \lambda \frac{\beta\phi(\lambda)}{1-\beta\phi(\lambda)}\sigma K_t^\sigma, \\ K_{t+1} = \left[1+\frac{\alpha^H-\alpha^L}{\alpha^L-\theta\alpha^H}p\right]\beta\alpha^L\sigma\left[1+\lambda\frac{\beta\phi(\lambda)}{1-\beta\phi(\lambda)}\right]K_t^\sigma. \end{cases} \tag{L.25}$$

From (L.23), (L.24), and (L.25), we have (29) in the text.

Now, we are in a position to derive the value function at any date t in the bubble economy. Let $V_t^{BB}(K_t)$ be the value function of taxpayers at date t in the bubble economy. Given optimal decision rules, the Bellman equation can be written as

$$V_t^{BB}(K_t) = \log c_t + \beta\left[\pi V_{t+1}^{BB}(K_{t+1}) + (1-\pi)V_{t+1}^{BL}(K_{t+1})\right]. \tag{L.26}$$

with the optimal decision rule of aggregate capital stock until bubbles collapse:

$$K_{t+1} = H(\lambda)K_t^\sigma, \tag{L.27}$$

We guess that the value function is a linear function of $\log K$:

$$V_t^{BB}(K_t) = s + Q\log K_t, \tag{L.28}$$

From (29), and (L.26)-(L.27), applying the method of undetermined coefficients yields

$$s = \frac{1}{1-\beta\pi}\log(1-\sigma) + \frac{\beta(1-\pi)}{1-\beta\pi}M(\lambda) + \frac{1}{1-\beta\pi}\frac{\beta\sigma}{1-\beta\sigma}\log H(\lambda),$$

$$Q = \frac{\sigma}{1-\beta\sigma}.$$

Thus, we have (30) in the text.

Appendix M: Derivation of entrepreneur's value function

Appendix M:.1 the case where the government does not tax entrepreneurs

Suppose that at date t, bubbles collapse. After the date t, the economy is in the bubbleless economy. Let $W_t^{BL}(e_t, K_t)$ be the value function of the entrepreneur at date t who holds the net worth, e_t, at the beginning of the period t before knowing his/her type of the period t. First, we solve $W_{t+1}^{BL}(e_t, K_t)$. Given the optimal decision rules, the Bellman equation can be written as

$$W_{t+1}^{BL}(e_{t+1}, K_{t+1}) = \log c_{t+1}^i + \beta \begin{bmatrix} pW_{t+2}^{BL}(R_{t+1}^{\prime H}\beta e_{t+1}, K_{t+2}) \\ +(1-p)W_{t+2}^{BL}(R_{t+1}^{\prime L}\beta e_{t+1}, K_{t+2}) \end{bmatrix} \text{ after date } t+1, \tag{M.29}$$

where $R'^H_{t+1}\beta e_{t+1}$ and $R'^L_{t+1}\beta e_{t+1}$ are the date $t+2$ net worth of the entrepreneur when he/she was H-type and L-type at date $t+1$, respectively. R'^H_{t+1} and R'^L_{t+1} are realized rate of return on savings from date $t+1$ to date $t+2$ in the bubbleless economy, and they satisfy

$$\begin{cases} R'^H_t = \frac{q_{t+1}\alpha^H(1-\theta)}{1-\frac{\theta\alpha^H}{\alpha^L}} & \text{after date } t, \\ \\ R'^L_t = q_{t+1}\alpha^L & \text{after date } t. \end{cases} \tag{M.30}$$

Aggregate capital stock follows:

$$K_{t+2} = (1+\tfrac{\alpha^H-\alpha^L}{\alpha^L-\theta\alpha^H}p)\beta\alpha^L\sigma K^\sigma_{t+1} \quad \text{after date } t+1. \tag{M.31}$$

We guess that the value function are linear functions of $\log K$ and $\log e$:

$$W^{BL}_{t+1}(e_{t+1}, K_{t+1}) = f_1 + g_1 \log K_{t+1} + h_1 \log e_{t+1} \tag{M.32}$$

From (M.29)-(M.32), applying the method of undetermined coefficients yields

$$\begin{aligned} f_1 \;=\; & \frac{1}{1-\beta}\log(1-\beta) + \frac{\beta}{(1-\beta)^2}\log(1-\beta) + \frac{\beta}{(1-\beta)^2}\log\sigma \\ & + \frac{\beta}{(1-\beta)^2}\left[p\log\frac{\alpha^H(1-\theta)}{1-\frac{\theta\alpha^H}{\alpha^L}} + (1-p)\log\alpha^L\right] \\ & + \frac{\beta(\sigma-1)}{(1-\beta)^2}\frac{1}{1-\beta\sigma}\log\left[(1+\frac{\alpha^H-\alpha^L}{\alpha^L-\theta\alpha^H}p)\beta\alpha^L\sigma\right], \end{aligned} \tag{M.33}$$

$$g_1 = \frac{\beta\sigma}{1-\beta\sigma}\frac{\sigma-1}{1-\beta}, \tag{M.34}$$

$$h_1 = \frac{1}{1-\beta}. \tag{M.35}$$

Next, we derive the value function at date t when bubbles collapse and the government bails out entrepreneurs by taking into account the effects of bailouts on the date $t+1$ aggregate capital stock. Given the optimal decision rules, the value function at date t satisfies

$$W_t^{BL}(e_t, K_t) = \log c_t + \beta \left[p W_{t+1}^{BL}(R_t^{\prime H} \beta e_t, K_{t+1}) + (1-p) W_{t+1}^{BL}(R_t^{\prime L} \beta e_t, K_{t+1}) \right], \tag{M.36}$$

with

$$K_{t+1} = \left[1 + \frac{\alpha^H - \alpha^L}{\alpha^L - \theta \alpha^H} p \right] \beta \alpha^L \sigma \left[1 + \lambda \frac{\beta \phi(\lambda)}{1 - \beta \phi(\lambda)} \right] K_t^{\sigma}. \tag{M.37}$$

From (M.32)-(M.37), we obtain

$$W_t^{BL}(e_t, K_t) = f_1 + \frac{\beta(\sigma-1)}{1-\beta} \frac{1}{1-\beta\sigma} \log \left[1 + \lambda \frac{\beta \phi(\lambda)}{1 - \beta \phi(\lambda)} \right] + \frac{\beta\sigma}{1-\beta\sigma} \frac{\sigma-1}{1-\beta} \log K_t + \frac{1}{1-\beta} \log e_t. \tag{M.38}$$

Now, we are in a position to derive the value function at any date t in the bubble economy. $W_t^{BB}(e_t, K_t)$ is the value function of the entrepreneur at any date t in the bubble economy who holds the net worth, e_t, at the beginning of the period t before knowing his/her type of the period t. Given optimal decision rules, the Bellman equation can be written as

$$\begin{aligned} W_t^{BB}(e_t, K_t) = \log c_t &+ \beta\pi \left[p W_{t+1}^{BB}(R_t^H \beta e_t, K_{t+1}) + (1-p) W_{t+1}^{BB}(R_t^L \beta e_t, K_{t+1}) \right] \\ &+ \beta(1-\pi) \left[\begin{array}{c} p W_{t+1}^{BL}(R_t^H \beta e_t, K_{t+1}) + (1-p)\lambda W_{t+1}^{BL}(R_t^L \beta e_t, K_{t+1}) \\ +(1-p)(1-\lambda) W_{t+1}^{BL}(R_t^{LL} \beta e_t, K_{t+1}) \end{array} \right], \end{aligned} \tag{M.39}$$

where $R_t^H \beta e_t$, $R_t^L \beta e_t$, and $R_t^{LL} \beta e_t$ are the date $t+1$ net worth of the entrepreneur in each state. R_t^H, R_t^L, and R_t^{LL} are realized rate of return from savings from date t

to date $t+1$, and in $0 \leq \lambda \leq \lambda^*$, they satisfy

$$\begin{cases} R_t^H = \frac{q_{t+1}\alpha^H(1-\theta)}{1-\frac{\theta\alpha^H}{\alpha^L}}, \\ \\ R_t^L = \delta(\lambda)\frac{P_{t+1}}{P_t} = \delta(\lambda)\frac{q_{t+1}\alpha^L[1-p-\phi(\lambda)]}{\delta(\lambda)(1-p)-\phi(\lambda)}, \\ \\ R_t^{LL} = \frac{q_{t+1}\alpha^L[1-p-\phi(\lambda)]}{1-p}. \end{cases} \tag{M.40}$$

and in $\lambda^* \leq \lambda \leq 1$, they satisfy

$$\begin{cases} R_t^H = \frac{q_{t+1}\alpha^H(1-\theta)[1-\phi(\lambda)]}{p}, \\ \\ R_t^L = \delta(\lambda)\frac{P_{t+1}}{P_t} = \delta(\lambda)\frac{q_{t+1}\theta\alpha^H[1-\phi(\lambda)]}{\delta(\lambda)(1-p)-\phi(\lambda)}, \\ \\ R_t^{LL} = \frac{q_{t+1}\theta\alpha^H[1-\phi(\lambda)]}{1-p}. \end{cases} \tag{M.41}$$

Aggregate capital stock until bubbles collapse follows:

$$K_{t+1} = H(\lambda)K_t^\sigma. \tag{M.42}$$

We guess that the value function are linear functions of $\log K$ and $\log e$:

$$W_t^{BB}(e_t, K_t) = m + l\log K_t + n\log e_t. \tag{M.43}$$

From (M.38)-(M.43), and (M.39), applying the method of undetermined coefficients yields

$$\begin{aligned}
m &= \frac{1}{1-\beta\pi}\log(1-\beta) + \frac{1}{1-\beta\pi}\frac{\beta}{1-\beta}\log\beta + \frac{1}{1-\beta\pi}\frac{\beta}{1-\beta}\log\sigma \\
&+ \frac{\beta(\sigma-1)}{1-\beta\pi}\frac{1}{1-\beta\sigma}\frac{1}{1-\beta}\log H(\lambda) \\
&+ \frac{\beta(1-\pi)}{1-\beta\pi}\left\{f_1 + \frac{\beta(\sigma-1)}{1-\beta}\frac{1}{1-\beta\sigma}\log\left[1+\lambda\frac{\beta\phi(\lambda)}{1-\beta\phi(\lambda)}\right]\right\} \\
&+ \frac{1}{1-\beta\pi}\frac{\beta}{1-\beta}\left[\pi J_1 + (1-\pi)J_2\right],
\end{aligned}$$

$$l = \frac{\beta\sigma(\sigma-1)}{1-\beta\sigma}\frac{1}{1-\beta},$$

$$n = \frac{1}{1-\beta},$$

where in $0 \le \lambda \le \lambda^*$,

$$J_1 = p\log\frac{\alpha^H(1-\theta)}{1-\frac{\theta\alpha^H}{\alpha^L}} + (1-p)\log\left[\delta(\lambda)\frac{\alpha^L[1-p-\phi(\lambda)]}{\delta(\lambda)(1-p)-\phi(\lambda)}\right],$$

$$\begin{aligned}
J_2 &= p\log\frac{\alpha^H(1-\theta)}{1-\frac{\theta\alpha^H}{\alpha^L}} + (1-p)\lambda\log\left[\delta(\lambda)\frac{\alpha^L[1-p-\phi(\lambda)]}{\delta(\lambda)(1-p)-\phi(\lambda)}\right] \\
&+(1-p)(1-\lambda)\log\left[\frac{\alpha^L[1-p-\phi(\lambda)]}{1-p}\right].
\end{aligned}$$

and in $\lambda^* \le \lambda \le 1$,

$$J_1 = p\log\frac{\alpha^H(1-\theta)[1-\phi(\lambda)]}{p} + (1-p)\log\left[\delta(\lambda)\frac{\theta\alpha^H[1-\phi(\lambda)]}{\delta(\lambda)(1-p)-\phi(\lambda)}\right],$$

$$
\begin{aligned}
J_2 &= p\log\frac{\alpha^H(1-\theta)[1-\phi(\lambda)]}{p} + (1-p)\lambda\log\left[\delta(\lambda)\frac{\theta\alpha^H[1-\phi(\lambda)]}{\delta(\lambda)(1-p)-\phi(\lambda)}\right] \\
&\quad +(1-p)(1-\lambda)\log\left[\frac{\theta\alpha^H[1-\phi(\lambda)]}{1-p}\right].
\end{aligned}
$$

Thus, we have (32) in the text.

Appendix M:.2 the case where the government taxes entrepreneurs

When the government taxes entrepreneurs who do not suffer losses from bubble investments, m and J_2 change as follows:

$$
\begin{aligned}
m &= \frac{1}{1-\beta\pi}\log(1-\beta) + \frac{1}{1-\beta\pi}\frac{\beta}{1-\beta}\log\beta + \frac{1}{1-\beta\pi}\frac{\beta}{1-\beta}\log\sigma \\
&\quad +\frac{\beta(\sigma-1)}{1-\beta\pi}\frac{1}{1-\beta\sigma}\frac{1}{1-\beta}\log H(\lambda) \\
&\quad +\frac{\beta(1-\pi)}{1-\beta\pi}\left\{f_1 + \frac{\beta(\sigma-1)}{1-\beta}\frac{1}{1-\beta\sigma}\log\left[1+F(\lambda)\right]\right\} \\
&\quad +\frac{1}{1-\beta\pi}\frac{\beta}{1-\beta}\left[\pi J_1 + (1-\pi)J_2\right],
\end{aligned}
$$

in $0 \leq \lambda \leq \lambda^*$,

$$
\begin{aligned}
J_2 &= p\log\frac{(1-\tau)\alpha^H(1-\theta)}{1-\frac{\theta\alpha^H}{\alpha^L}} + (1-p)\lambda\log\left[\delta(\lambda)\frac{\alpha^L[1-p-\phi(\lambda)]}{\delta(\lambda)(1-p)-\phi(\lambda)}\right] \\
&\quad +(1-p)(1-\lambda)\log\left[\frac{\alpha^L[1-p-\phi(\lambda)]}{1-p}\right].
\end{aligned}
$$

in $\lambda^* \leq \lambda \leq 1$,

$$\begin{aligned} J_2 &= p \log \frac{(1-\tau)\alpha^H(1-\theta)[1-\phi(\lambda)]}{p} + (1-p)\lambda \log \left[\delta(\lambda) \frac{\theta\alpha^H[1-\phi(\lambda)]}{\delta(\lambda)(1-p)-\phi(\lambda)}\right] \\ &+ (1-p)(1-\lambda) \log \left[\frac{\theta\alpha^H[1-\phi(\lambda)]}{1-p}\right]. \end{aligned}$$

Appendix N: Procedures to derive numerical examples of entrepreneur's welfare

When we compute (32), we make the following assumptions: aggregate capital stock in the initial period is set to the steady-state value of the bubbleless economy; population measure of entrepreneurs is assumed to be equal to one; in the initial period, each entrepreneur is endowed with the same amount of capital, $k_t^i = k_t$, and one unit of bubble assets, and owes no debt. Under these assumptions, all entrepreneurs hold the same amount of net worth in the initial period, i.e., $e_0 = q_0 k_0 + P_0$. By using determination of equilibrium bubble prices (K.16), e_0 can be written as

$$e_0(\lambda) = \frac{1}{1-\beta\phi(\lambda)} \sigma K_0^\sigma.$$

Inserting the above relation into (32) yields

$$W_0^{BB}(K_0) = m(\lambda) + \frac{1}{1-\beta} \log \sigma + \frac{1}{1-\beta\sigma} \log K_0 - \frac{1}{1-\beta} \log \left[1-\beta\phi(\lambda)\right].$$

Figure 4 describes the relationship between W_0^{BB} and λ.

References

[1] Aoki, Kosuke, and Kalin Nikolov. 2011. "Bubbles, Banks and Financial Stability," University of Tokyo.

[2] Bernanke, Ben, and Mark Gertler. 1989. "Agency Costs, Net Worth, and Business Fluctuations," *American Economic Review*, 79(1): 14–31.

[3] Bernanke, Ben, Mark Gerlter, and Simon Gilchrist. 1999. "The Financial Accelerator in a Quantitative Business Cycle Framework," in J. Taylor and M. Woodford eds, *the Handbook of Macroeconomics*, 1341-1393. Amsterdam: North-Holland.

[4] Brunnermeier, Markus, and Yuliy Sannikov. 2011. "The I Theory of Money," mimeo, Princeton University.

[5] Caballero, Ricardo, and Arvind Krishnamurthy. 2006. "Bubbles and Capital Flow Volatility: Causes and Risk Management," *Journal of Monetary Economics*, 53(1): 35-53.

[6] Diamond, Douglas, and Raghuram Rajan. 2012. "Illiquid Banks, Financial Stability, and Interest Rate Policy," *Journal of Political Economy*, 120(3), 552-91.

[7] Farhi, Emmanuel, and Jean Tirole. 2009a. "Bubbly Liquidity," *Review of Economic Studies*, 79(2): 678-706.

[8] Farhi, Emmanuel, and Jean Tirole. 2009b. "Leverage and the Central Banker's Put," *American Economic Review Papers and Proceedings*, 99(2): 589-593.

[9] Farhi, Emmanuel, and Jean Tirole. 2012. "Collective Moral Hazard, Maturity Mismatch and Systemic Bailouts," *American Economic Review*, 102(1): 60-93

[10] Gertler, Mark, and Peter Karadi. 2011. "A Model of Unconventional Monetary Policy," *Journal of Monetary Economics*, 58(1): 17–34.

[11] Gerlter, Mark, and Nobuhiro Kiyotaki. 2010. "Financial Intermediation and Credit Policy in Business Cycle Analysis," In Friedman, B., and Woodford, M. eds, *Handbook of Monetary Economics*, 547-599. Elsevier, Amsterdam, Netherlands.

[12] Gertler, Mark, Nobuhiro Kiyotaki, and Albert Queralto. 2011. "Financial Crises, Bank Risk Exposure and Government Financial Policy," Princeton University.

[13] Hart, Oliver, and John Moore. 1994. "A Theory of Debt Based on the Inalienability of Human Capital," *Quarterly Journal of Economics*, 109(4): 841–879.

[14] Hellwig, Christian, and Guido Lorenzoni. 2009. "Bubbles and Self-Enforcing Debt," *Econometrica*, 77(4): 1137-1164.

[15] Hirano, Tomohiro, and Noriyuki Yanagawa. 2010a. "Asset Bubbles, Endogenous Growth, and Financial Frictions," Working Paper, CARF-F-223, University of Tokyo.

[16] Hirano, Tomohiro, and Noriyuki Yanagawa. 2010b. "Financial Institution, Asset Bubbles and Economic Performance," Working Paper, CARF-F-234, University of Tokyo.

[17] Hirano, Tomohiro, and Noriyuki Yanagawa. 2012(January). "Asset Bubbles and Bailout," CIRJE-F-838, CIRJE, University of Tokyo.

[18] Holmstrom, Bengt, and Jean Tirole. 1998. "Private and Public Supply of Liquidity," *Journal of Political Economy,* 106(1): 1-40.

[19] Jovanovic, Boyan. 2012. "Bubbles in Prices of Exhaustible Resources," *International Economic Review*, 54(1): 1-34.

[20] Kamihigashi, Takashi, 2009. "Asset Bubbles in a Small Open Economy," Kobe University.

[21] Kamihigashi, Takashi, 2011. "Recurrent Bubbles," *The Japanese Economic Review*, 62(1): 27-62.

[22] Kiyotaki, Nobuhiro, 1998. "Credit and Business Cycles," *The Japanese Economic Review*, 49(1): 18–35.

[23] Kiyotaki, Nobuhiro, and John Moore. 1997. "Credit Cycles," *Journal of Political Economy*, 105(2): 211-248.

[24] Kiyotaki, Nobuhiro, and John Moore. 2008. "Liquidity, Business Cycles and Monetary Policy," Princeton University.

[25] Kocherlakota, R. Narayana. 1992. "Bubbles and Constraints on Debt Accumulation," *Journal of Economic Theory*, 57(1): 245-256.

[26] Kocherlakota, R. Narayana. 2009. "Bursting Bubbles: Consequences and Cures," University of Minnesota.

[27] Martin, Alberto, and Jaume Ventura. 2011b. "Economic Growth with Bubbles," Forthcoming in *American Economic Review*.

[28] Matsuyama, Kiminori. 2007. "Credit Traps and Credit Cycles," *American Economic Review*, 97(1): 503-516.

[29] Matsuyama, Kiminori. 2008. "Aggregate Implications of Credit Market Imperfections," in D. Acemoglu, K. Rogoff, and M. Woodford, eds., *NBER Macroeconomics Annual 2007*, 1-60. University of Chicago Press.

[30] Miao, Jianjun, and Pengfei Wang. 2011. "Bubbles and Credit Constraints," mimeo, Boston University.

[31] Roch, Francisco, and Harald Uhlig. 2012. "The Dynamics of Sovereign Debt Crises and Bailouts," Chicago University.

[32] Stiglitz, Joseph. 2012. The Price of Inequality: How Today's Divided Society Endangers Our Future. W. W. Norton & Company.

[33] Tirole, Jean. 1985. "Asset Bubbles and Overlapping Generations," *Econometrica*, 53(6): 1499-1528.

[34] Tirole, Jean. 2005. *The Theory of Corporate Finance*. Princeton, New Jersey: Princeton University Press.

[35] Uhlig, Harald (2010), "A Model of a Systemic Bank Run," *Journal of Monetary Economics*, 57(1): 78-96.

[36] Weil, Philippe. 1987. "Confidence and the Real Value of Money," *Quarterly Journal of Economics*, 102(1): 1-22.

[37] Woodford, Michael. 1990. "Public Debt as Private Liquidity," *American Economic Review*, 80(2): 382-388.

Parameters		benchmark	case 1	case 2	case 3
α^H	productivity of high project	1.1	1.1	1.1	1.1
α^L	productivity of low project	1	1	1	1
β	discount factor	0.98	0.98	0.98	0.98
σ	capital share	0.25	0.25	0.25	0.25
θ	collateral ratio	0.1	0.1	0.1	0.1
π	survival rate of bubbles	0.99	0.995	0.997	0.998
p	probability for high project	0.3	0.3	0.3	0.3

Table 1: Parameters

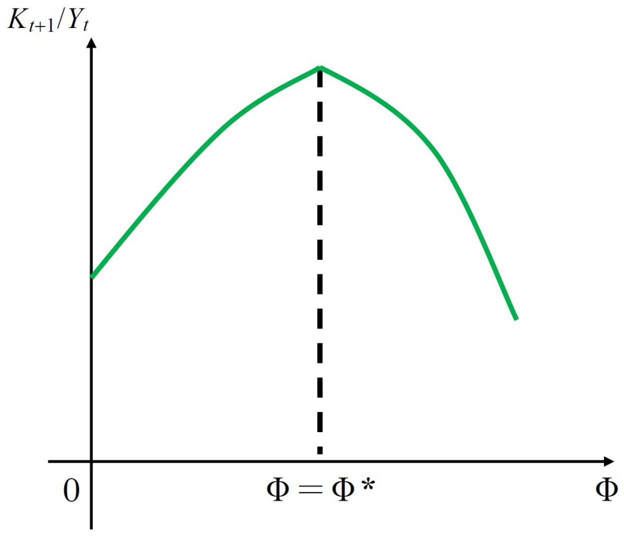

Figure 1: Effects on Capital Stock

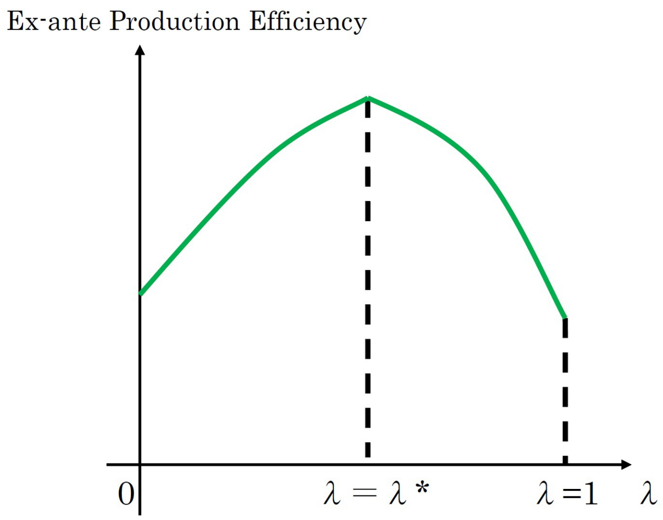

Figure 2: Effects on Ex-ante Production Efficiency

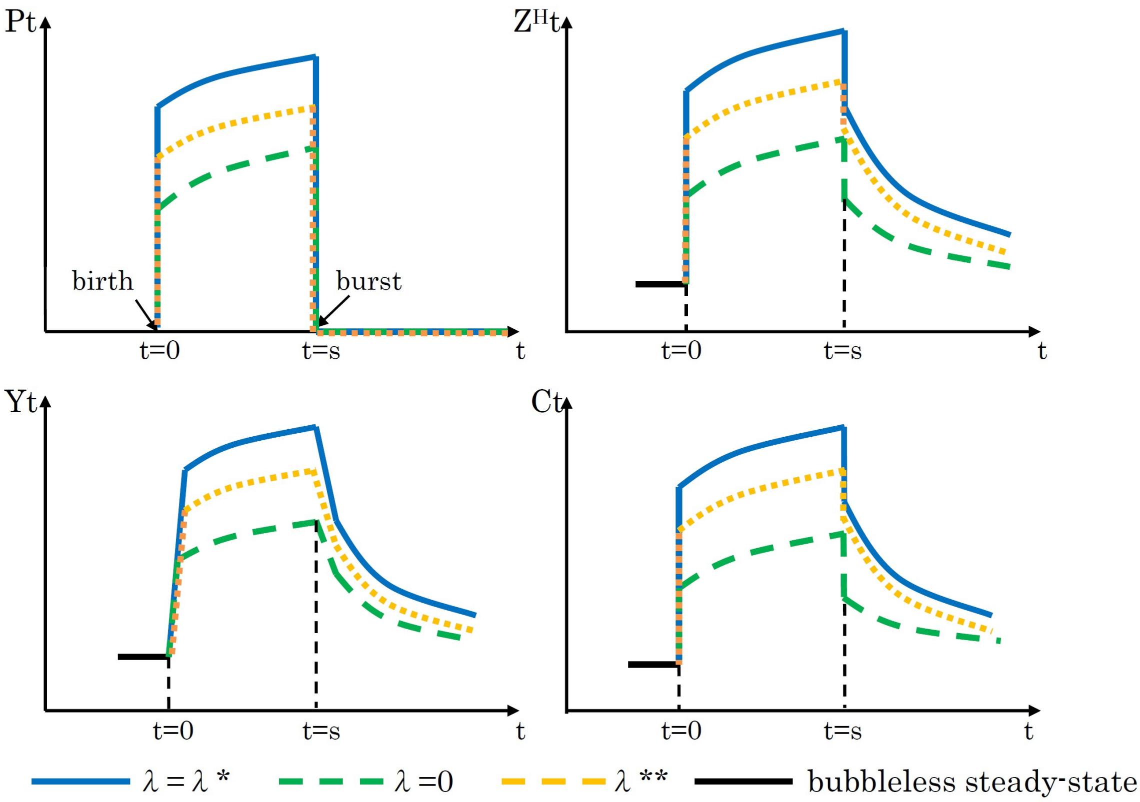

Figure 3: Anticipated Bailouts and Boom-Bust Cycles

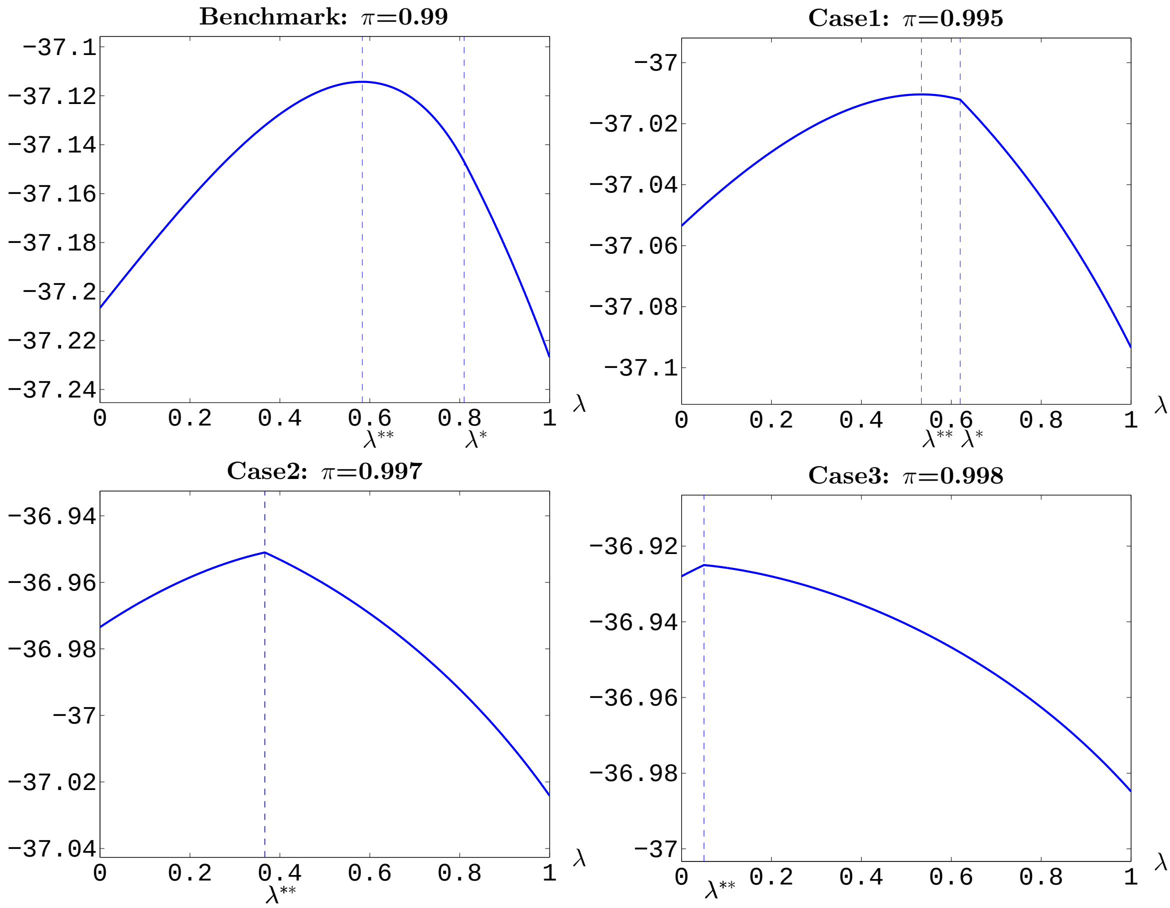

Figure 4: Taxpayer's Welfare

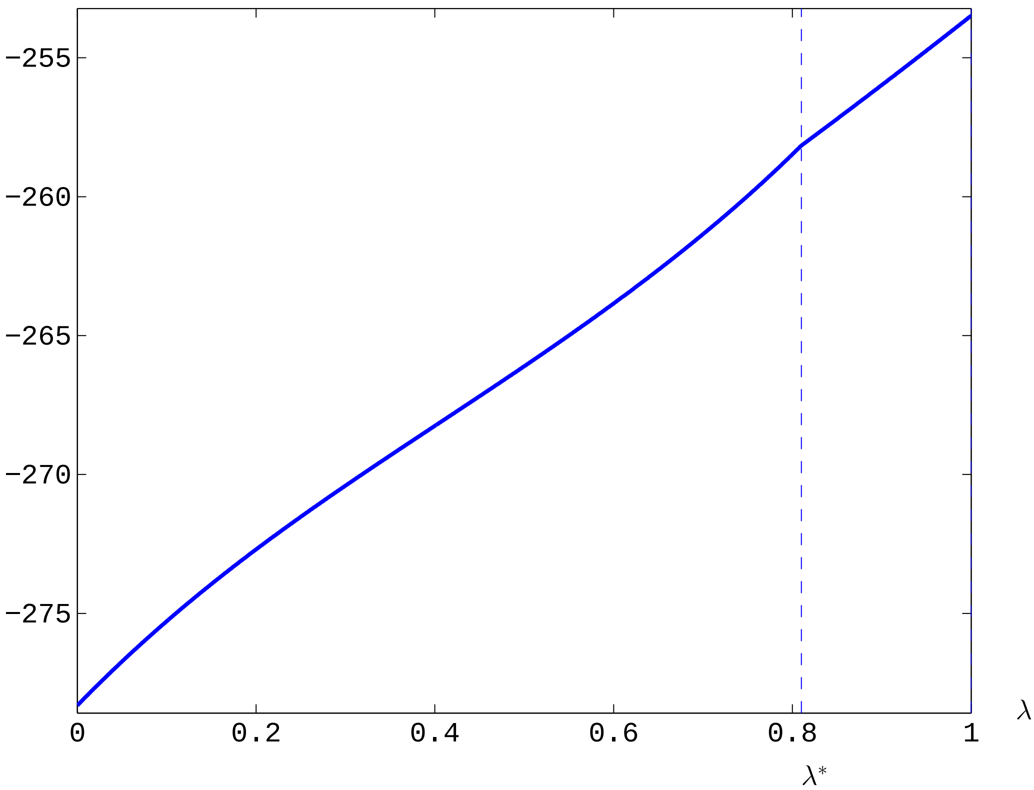

Figure 5: Entrepreneur's Welfare