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Asymmetry and Leverage in Realized Volatility

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Abstract

A wide variety of conditional and stochastic variance models has been used to estimate latent volatility (or risk). In both the conditional and stochastic volatility literature, there has been some confusion between the definitions of asymmetry and leverage. In this paper, we first show the relationship among conditional, stochastic, integrated and realized volatilities. Then we develop a new asymmetric volatility model, which takes account of small and large, and positive and negative, shocks. Using the new specification, we examine alternative volatility models that have recently been developed and estimated in order to understand the differences and similarities in the definitions of asymmetry and leverage. We extend the new specification to realized volatility by taking account of measurement errors. As an empirical example, we apply the new model to the realized volatility of Standard and Poor's 500 Composite Index using Efficient Importance Sampling to show that the new specification of asymmetry significantly improves the goodness of fit, and that the out-of-sample forecasts and VaR thresholds are satisfactory.

1   Introduction

The accurate specification and modelling of risk are integral to optimal portfolio selection and risk management using high frequency and ultra high frequency data. In this context, a wide variety of conditional and stochastic variance models has been used to estimate latent volatility (or risk) using high frequency data, while the availability of tick data has led to alternative models of realized volatility to estimate integrated volatility in analysing ultra high frequency data (see McAleer (2005) for a comprehensive review of univariate and multivariate, and symmetric and asymmetric, conditional and stochastic volatility models, and Asai, McAleer and Yu (2006) for a detailed review of alternative specifications and estimation algorithms for multivariate stochastic volatility models).

In the framework of diffusion processes, the daily variance of stock return is expressed as an integral of the intraday variance, which is called the integrated variance. If the microstructure noise is ignored, we may estimate it by the sum of squared returns of ultra high frequency data. Such an estimator is called the realized variance, which corresponds to an estimate of the integrated variance, namely the true daily variance. In the paper, we refer to the square root of the realized variance as the Realized Volatility (RV). For a recent extensive review of the RV literature, see McAleer and Medeiros (2008).

Upon estimating RV by using ultra high frequency data, one of the major problems that arises is microstructure noise. Several authors have proposed alternative methods for removing the microstructure noise (see, for example, Bandi and Russell (2006), Barndorff-Nielsen, Hansen, Lunde and Shephard (2007), Zhang, Mykland and Aït-Sahalia (2005), and Hansen, Large and Lunde (2008)). Some methods provide bias-corrected and consistent estimators of the integrated variance, but other methods do not. Recently, Asai, McAleer and Medeiros (2008) have shown that, even when a bias-corrected and consistent estimator is used, non-negligible measurement errors remain in estimating and forecasting RV.

The purpose of the paper is to propose a new specification of the asymmetric RV model. Before doing so, we need to clarify three points: (i) the relationship among conditional, stochastic, integrated and realized volatility; (ii) the confusion between the definitions of asymmetry and leverage; and (iii) the way in which to estimate a model of integrated

volatility using RV data.

Regarding the second point, this paper proposes a general asymmetric volatility model. Based on the new specification, we examine alternative univariate volatility models that have recently been developed and estimated in order to understand the differences and similarities in the definitions of asymmetry and leverage. We focus on standardized and/or unstandardized leverage and size effects, and analyse five univariate SV models, namely the basic SV model, SV model with standardized leverage, and three different types of asymmetric SV models (see Harvey and Shephard (1996), Danielsson (1994), and Asai and McAleer (2005, 2008)). Then we evaluate the differential impacts of positive and negative shocks to returns of equal magnitude on future volatility, namely symmetry, asymmetry, type I asymmetry (or leverage), type II asymmetry, type III asymmetry and type IV asymmetry. The general model proposed here is classified as type IV asymmetry.

With respect to the third point, we propose a new asymmetric model for RV by extending the general asymmetric volatility model, with an additional term in order to capture RV errors. It should be noted that introducing a correction for measurement error in the RV process renders the true volatility process unobservable. In order to estimate the proposed model, we employ the efficient importance sampling (EIS) ML method proposed by Liesenfeld and Richard (2003, 2006). The EIS evaluates the log-likelihood function of the model, including the latent process, by using simulations, such as the Monte Carlo Likelihood (MCL) technique of Durbin and Koopman (1997). Compared with the MCL method, the EIS method is applicable to various kinds of latent models (see also the discussion in Liesenfeld and Richard (2003)).

The remainder of the paper is organized as follows. Section 2 shows the connection among conditional, stochastic, integrated and realized volatility. Section 3 develops a general asymmetric volatility model. We examine five kinds of asymmetric SV models, and propose classifications for symmetric and/or asymmetric effects. By using the structure of asymmetric effects, Section 4 proposes a new model for RV based on correcting for RV errors. Section 5 discusses the EIS-ML method, while Section 6 presents the empirical results for the RV model using Standard and Poor's 500 Composite Index, and evaluates the new specification of asymmetry with respect to goodness of fit, out-of-sample forecasts and VaR thresholds. Section 7 gives some concluding remarks.

2   Instantaneous, Integrated and Realized Volatility

Let $p(t+\tau)$ is the logarithmic price of a given asset at time τ $(0 \leq \tau \leq 1)$ on day t $(t=1,2,\ldots)$. We assume that $p(t+\tau)$ follows a continuous time diffusion process,

$$dp(t+\tau) = \mu(t+\tau)d\tau + \sigma(t+\tau)dW(t+\tau), \tag{1}$$

where $\mu(t+\tau)$ is the drift component, $\sigma(t+\tau)$ is the instantaneous volatility (or standard deviation), and $W(t+\tau)$ is a standard Brownian motion. Let r_t be the daily returns, defined as $r_t = p(t) - p(t-1)$. Conditionally on

$$\Im_t \equiv \Im\{\mu(t+\tau-1), \sigma(t+\tau-1)\}_{\tau=0}^{\tau=1},$$

which is the σ-algebra (information set) generated by the sample paths of $\mu(t+\tau-1)$ and $\sigma(t+\tau-1)$ $(0 \leq \tau \leq 1)$, we have

$$r_t \Big| \Im_t \sim N\left(\int_0^1 \mu(t+\tau-1)d\tau, \int_0^1 \sigma^2(t+\tau-1)d\tau\right).$$

The term $V_t^2 = \int_0^1 \sigma^2(t+\tau-1)d\tau$ is known as the *integrated variance*, which is a measure of the day-*t* ex post volatility. The integrated variance is typically the object of interest as a measure of the true daily volatility.

With respect to the model of the instantaneous volatility, there are several specifications, which are called "continuous-time Stochastic Volatility (SV)" models (see Ghysels, Harvey and Renault (1996), for example). Hull and White (1987) allow the squared-volatility to follow a diffusion process,

$$d\sigma^2 = \alpha\sigma^2 d\tau + \omega\sigma^2 dB \,, \tag{2}$$

where B is a second Brownian motion, and α and ω are parameters. Here, we have omitted $(t+\tau)$ in order to simplify the notation. Hull and White (1987) assume a negative correlation between W and B, thereby incorporating leverage effects. The model in (2) is closely related to the GARCH diffusion, which is derived as the diffusion limit of a sequence of GARCH(1,1) models (see Nelson (1990)).

Wiggins (1987) assumes that the log-volatility follows a Gaussian Ornstein-Uhlenbeck (OU) process,

$$d\log\sigma^2 = \alpha\left(\mu - \log\sigma^2\right)\sigma^2 d\tau + \omega dB \,. \tag{3}$$

In the specification, we may introduce leverage effects by assuming a negative correlation between W and B. The asymmetric SV model of Harvey and Shephard (1996) is considered to be an Euler-Maruyama approximation of the continuous-time model (3), with negative correlation. Three major extensions of such diffusion-based SV models incorporate jumps to volatility process (Eraker, Johannes and Polson (2003)), model volatility as a function of a number of factors (Chernov et al. (2003)), and allow the log-volatility to follow a long memory process (Comte and Renault (1998)).

If the underlying process of the instantaneous volatility is a continuous-time SV model, the resulting integrated variance is still a stochastic process. At this stage, it may be useful to distinguish the differences and similarities among the conditional variance, stochastic variance, and integrated variance. As shown in Nelson (1990), it is possible to consider the diffusion limits of typical conditional variance models, such as the GARCH model and the exponential GARCH model of Nelson (1991). Hence, conditional variance models are considered to be proxies or approximations of continuous-time SV models. Alternative approximations are the (discrete-time) SV models of Taylor (1982) and Harvey and Shephard (1996), which are obtained by the Euler-Maruyama discretization of the continuous-time SV models. Compared with the class of GARCH models, discrete-time SV models give better approximations in the sense that the latter is straightforwardly derived by the continuous-time SV models. Therefore, the conditional and (discrete-time) stochastic variance can be considered as approximations of the integrated variance obtained by the continuous-time SV models.

There are numerous extensions of GARCH models, and extensions of SV models are still being developed. There are many cases where it is not easy to consider a continuous-time SV model which corresponds to such an extension. For this reason, in the following section we consider asymmetric models of the integrated volatility directly.

Although the integrated variance is unobservable, it is possible to estimate it using high frequency data. Such estimates are called "Realized Volatility (RV)". Zhang, Mykland and Aït-Sahalia (2005) and Barndorff-Nielsen, Hansen, Lunde and Shephard (2007) propose consistent estimator of the integrated variance, under the existence of microstructure noise. For an extensive review of the realized volatility literature, see Bandi and Russell (2006) and McAleer and Medeiros (2008).

3   Structure of Asymmetric Volatility Models

In both the conditional and stochastic variance literature, there has been some confusion between the definitions of asymmetry and leverage. This section proposes a new asymmetric volatility model, and examines alternative asymmetric volatility models that have recently been developed and estimated, in order to understand the differences and similarities in alternative definitions of asymmetry and leverage.

3.1   Asymmetric Volatility Model

For the return process, we may write

$$r_t = m_t + V_t z_t, \quad z_t \sim \text{i.i.d.}(0,1),$$

where m_t and V_t are the time-varying mean and volatility processes, and z_t is the standardized disturbance. From the definition in the previous section, we have $m_t = \int_0^1 \mu(t+\tau-1)d\tau$, $V_t^2 = \int_0^1 \sigma^2(t+\tau-1)d\tau$ and $z_t \sim N(0,1)$. Here, we consider a more general model for empirical analysis in a later section. We assume that the log-volatility follows an ARFIMA(p,d,q) process,

$$\ln V_{t+1} = \alpha + (1-L)^{-d} \Phi^{-1}(L) \Theta(L) \xi_t, \tag{4}$$

where L is the lag-operator, $\Phi(L)$ and $\Theta(L)$ are the lag polynomials for the AR and

MA coefficients, and $(1-L)^d$ is the fractional difference operator. As discussed in the

previous section, the innovation term in the volatility equation plays an important role
in considering asymmetry and leverage effects. We suggest a generalized error, such
that

$$\xi_t = \xi_t^* - E(\xi_t^*) + \eta_t, \quad \eta_t \sim N(0, \sigma_\eta^2),$$

$$\tag{5}$$

$$\xi_t^* = \gamma_1 z_t + \gamma_2 |z_t| + \gamma_3 z_t I(0 \le z_t < \delta) + \gamma_3 \delta I(\delta \le z_t),$$

where γ_1, γ_2 and γ_3 are parameters, and $I(0 \le z < \delta)$ is the indicator function,

which takes the value of one if $0 \le z < \delta$, and zero otherwise. The first two terms in

ξ_t^* play similar roles as in the EGARCH model. Figure 1 shows the relationship

between ξ and z, and implies that negative shocks and large positive shocks increase
future volatility via ξ, but small positive shocks decrease volatility. Such a
phenomenon has recently been observed in Chen and Ghysels (2007) by their
semi-parametric method using realized volatility.

Consider an AR(1) model of log-volatility, namely, with $d=0$, $\Phi(L)=1-\phi L$, and

$\Theta(L)=1$, such that

$$\ln V_{t+1} = (1-\phi)\alpha + \phi \ln V_t + \xi_t. \tag{6}$$

Equation (6) is convenient for comparison with existing SV models:

(i)   **SV without Leverage**: Equations (5) and (6), with restrictions $\gamma_1 = \gamma_2 = \gamma_3 = 0$.

(ii)   **SV with Standardized Leverage**: Equations (5) and (6), with restrictions $\gamma_1 < 0$ and $\gamma_2 = \gamma_3 = 0$.

(iii) **SV with Unstandardized Leverage and Unstandardized Size Effects**: Equations (5) and (6), with $\gamma_3 = 0$ and with the replacement of z_t by $r_t - m_t$.

(iv)   **SV with Standardized Leverage and Unstandardized Size Effects**: Equations (5) and (6), with $\gamma_3 = 0$ and with the replacement of $|z_t|$ by $|r_t - m_t|$.

(v)   **SV with Standardized Leverage and Standardized Size Effects**: Equations (5) and (6), with $\gamma_3 = 0$.

Model (i) is the basic SV model of Taylor (1982), which is symmetric as positive and negative shocks to returns have identical effects on future volatility. Model (ii) was suggested by Harvey and Shephard (1996), with different notation. Model (iii) was proposed by Danielsson (1994), and was estimated in Asai and McAleer (2005). Model (iv) was suggested by Asai and McAleer (2005) to capture both leverage and asymmetric effects. Model (v) adapts the EGARCH model of Nelson (1991) to the SV literature, and was suggested and estimated by Asai and McAleer (2008). In contrast to Model (iii), Model (v) uses the standardized returns in forecasting future volatility, and can capture various types of asymmetric and leverage effects.

As compared with existing models, the new model in (4) and (5) allows log-volatility to follow the ARFIMA process, and incorporates more flexible asymmetric effects.

3.2 Types of Asymmetry

Given the various models presented above, it is useful to consider the following categories of symmetric and asymmetric volatility models, conditional on a negative shock leading to an increase in volatility:

(a)   **Symmetry:** Positive and negative shocks to returns of equal magnitude have identical effects on future volatility.

(b)   **Asymmetry**: Positive and negative shocks to returns of equal magnitude have different effects on future volatility.

(c)   **Type I Asymmetry (Leverage)**: A negative correlation exists between current shocks to returns and future volatility.

(d)   **Type II Asymmetry**: Positive and negative shocks to returns increase future volatility, but a negative shock has a larger effect than does a positive shock of equal magnitude.

(e)   **Type III Asymmetry**: Positive and negative shocks to returns increase future volatility, but a positive shock has a larger effect than does a negative shock of equal magnitude.

(f)   **Type IV Asymmetry**: Negative shocks and large positive shocks increase future volatility, but small positive shocks decrease future volatility. As for Type II Asymmetry, a negative shock has a larger effect on future volatility than does a positive shock of equal magnitude.

Type I Asymmetry, or leverage, is based on the original framework of Black (1976) and Christie (1982), and is also consistent with the definition of leverage for continuous time SV models.

In the conditional volatility literature, the empirical results based on the GJR model of Glosten, Jagannathan and Runkle (1992) and the EGARCH model of Nelson (1991) typically fall into the Type II Asymmetry category. Given the specification, leverage effects are not possible for the GJR model, whereas leverage is possible, though it is frequently not observed, for the EGARCH model.

For the new model in (4) and (5), the basic model arises when $\gamma_1 = \gamma_2 = \gamma_3 = 0$. Setting $\gamma_1 = \gamma_3 = 0$ provides a Symmetric model. Type I Asymmetry is given by $\gamma_1 < 0$, $|\gamma_2| < -\gamma_1$ and $\gamma_3 = 0$. Restrictions on the parameters for Type II Asymmetry are given by $\gamma_1 < 0$, $\gamma_1 + \gamma_2 > 0$ and $\gamma_3 = 0$. For the case of Type III Asymmetry, the restrictions are $\gamma_2 < \gamma_1 < 0$ and $\gamma_3 = 0$. Regarding Type IV Asymmetry, the restrictions are $\gamma_1 < 0$, $\gamma_1 + \gamma_2 > 0$, $\gamma_3 < 0$ and $\delta > 0$.

4   Model Specification for Realized Volatility

In this section, we consider the model for realized volatility (RV), based on the discussions in the previous section.

Recently, Asai, McAleer and Medeiros (2008) showed that, even if a bias-corrected and consistent estimator of integrated volatility is used, RV will still include measurement errors, called 'Realized Volatility errors', which are not negligible in estimating volatility models.

Let y_t be the daily RV, which is a consistent estimate of integrated volatility (IV). The new asymmetric model for RV to be analysed in the paper is given by

$$y_t = \ln V_t + U_t,$$

$$\ln V_{t+1} = \alpha + (1-L)^{-d} \Phi^{-1}(L) \Theta(L) \xi_t$$

$$\xi_t = \xi_t^* - E(\xi_t^*) + \eta_t, \quad \eta_t \sim N(0, \sigma_\eta^2), \tag{7}$$

$$\xi_t^* = \gamma_1 z_t + \gamma_2 |z_t| + \gamma_3 z_t I(0 \le z_t < \delta) - \gamma_3 \delta I(\delta \le z_t),$$

$$z_t = r_t / V_t,$$

where z_t is the standardized return and follows the standard normal distribution. This specification enables U_t to capture the measurement errors in RV. We will refer to this

model as the "RV-ARFIMA(p,d,q)-AS $(\gamma_1, \gamma_2, \gamma_3)$-noise" model. The model allows various types of symmetric and/or asymmetric effects, long-memory property, and takes account of the realized volatility errors. If the measurement errors are neglected, we will have a special case with $\sigma = 0$. It should be noted that we consider the mean subtracted return, r_t, instead of return.

5   EIS-ML Estimation

The likelihood function for the asymmetric model in equation (7) includes high-dimensional integration, which is difficult to calculate numerically. We employ the Efficient Importance Sampling (EIS) method developed by Liesenfeld and Richard (2003, 2006) for evaluating the log-likelihood.

The pilot method for the EIS is the Accelerated Gaussian Importance Sampling (AGIS) approach, as developed in Danielsson and Richard (1993). The AGIS approach is designed to estimate dynamic latent variable models, where the latent variable follows a linear Gaussian process. While the AGIS technique has limited applicability, the EIS is applicable to models with more flexible classes of distributions and specifications for the latent variables. As in the case of AGIS, EIS is a Monte Carlo technique for the evaluation of high-dimensional integrals. The EIS relies on a sequence of simple low-dimensional least squares regressions to obtain a very accurate global approximation of the integrand. This approximation leads to a Monte Carlo sampler, which produces highly accurate Monte Carlo estimates of the likelihood.

5.1   Likelihood Evaluation via EIS

Let y_t be an observable variable and $h_t = \ln V_t$ be a latent variable. We denote the joint density of $Y_T = \{y_t\}_{t=1}^{T}$ and $H_T = \{h_t\}_{t=1}^{T}$ as $f(Y_T, H_T; \theta)$, indexed by the unknown parameter vector θ. In dynamic latent variable models, the joint density is typically formulated as:

$$f(Y_T, H_T; \theta) = \prod_{t=1}^{T} f(y_t, h_t | Y_{t-1}, H_{t-1}, \theta) = \prod_{t=1}^{T} g(y_t | h_t, Y_{t-1}, \theta) p(h_t | H_{t-1}, Y_{t-1}, \theta),$$

where $g(\square)$ denotes the conditional density of y_t given (h_t, y_{t-1}), and $p(\square)$ the conditional density of h_t given (H_{t-1}, Y_{t-1}). For ease of notation, it is assumed that the initial conditions are known constants, but EIS can easily accommodate alternative (stochastic) assumptions.

The likelihood function is given by the T-dimensional integral:

$$L(\theta; Y_T) = \int f(Y_T, H_T; \theta) dH_T,$$

and a natural MC estimate of $L(\theta; Y_T)$ is given by

$$\hat{L}(\theta; Y_T) = \frac{1}{N} \sum_{i=1}^{N} \left[\prod_{t=1}^{T} g\left(y_t \middle| \tilde{h}_t^{(i)}, Y_{t-1}, \theta \right) \right], \tag{8}$$

where $\left\{ \tilde{h}_t^{(i)}(\theta) \right\}_{t=1}^{T}$ denotes a trajectory drawn from the sequence of T densities. Each $\tilde{h}_t^{(i)}(\theta)$ is drawn from the conditional density $p\left(h_t \middle| \tilde{H}_{t-1}^{(i)}(\theta), Y_{t-1}, \theta \right)$.

In order to understand the EIS, we first note that EIS searches for a sequence of samplers that exploits the sample information on h_t conveyed by y_t. Let $\left\{ m\left(h_t \middle| H_{t-1}, x_t \right) \right\}_{t=1}^{T}$ denote a sequence of auxiliary samplers, indexed by the auxiliary parameters $X_n = \{x_t\}_{t=1}^{T}$. Regardless of the values of the auxiliary parameters, the likelihood function, $L(\theta; Y_T)$, is rewritten as

$$L(\theta; Y_T) = \int \prod_{t=1}^{T} \left[\frac{f\left(y_t, h_t \middle| Y_{t-1}, H_{t-1}, \theta \right)}{m\left(h_t \middle| H_{t-1}, x_t \right)} \right] \prod_{t=1}^{T} m\left(h_t \middle| H_{t-1}, x_t \right) dH_T,$$

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and the corresponding importance sampling MC estimate of the likelihood is given by

$$\tilde{L}\left(\theta;Y_T,X_T\right)=\frac{1}{N}\sum_{i=1}^{N}\left[\prod_{t=1}^{T}\left\{\frac{f\left(y_t,\tilde{h}_t^{(i)}\left(x_t\right)\middle|Y_{t-1},\tilde{H}_{t-1}^{(i)}\left(x_{t-1}\right),\theta\right)}{m\left(\tilde{h}_t^{(i)}\left(x_t\right)\middle|\tilde{H}_{t-1}^{(i)}\left(x_{t-1}\right),x_t\right)}\right\}\right], \tag{9}$$

where $\left\{\tilde{h}_t^{(i)}\left(x_t\right)\right\}_{t=1}^{T}$ denotes a trajectory drawn from the sequence of auxiliary importance samplers, m.

The EIS chooses a sequence of m densities by selecting values of the auxiliary parameters, X_T, which provide a good match between the product in the numerator and that in the denominator in equation (9) to minimize the MC sampling variance of $\tilde{L}\left(\theta;Y_T,X_T\right)$. In order to implement EIS, it requires constructing a positive functional approximation, $k\left(H_t;x_t\right)$, for the density $f\left(y_t,h_t\middle|Y_{t-1},H_{t-1},\theta\right)$, with the requirement that it be analytically integrable with respect to h_t. In Bayesian terminology, $k\left(H_t;x_t\right)$ plays a role of a density kernel for $m\left(h_t\middle|H_{t-1},x_t\right)$, which is then given by

$$m\left(h_t\middle|H_{t-1},x_t\right)=\frac{k\left(H_t;x_t\right)}{\chi\left(H_{t-1},x_t\right)}, \tag{10}$$

where $\chi\left(H_{t-1},x_t\right)=\int k\left(H_t;x_t\right)dh_t$. Then, the EIS requires solving a back-recursive sequence of low-dimensional least squares problems of the form:

$$\begin{aligned}\hat{x}_t=\arg\min_{x_t}\sum_{i=1}^{N}\Big[&\ln\left\{f\left(y_t,\tilde{h}_t^{(i)}\left(\theta\right)\middle|Y_{t-1},\tilde{H}_{t-1}^{(i)}\left(\theta\right),\theta\right)\xi\left(\tilde{H}_t^{(i)}\left(\theta\right),x_{t+1}\left(\theta\right)\right)\right\}\\&-c_t-\ln k\left(\tilde{H}_t^{(i)}\left(\theta\right);x_t\right)\Big]^2,\end{aligned} \tag{11}$$

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for $t : T \to 1$, with $\xi(H_T, x_{T+1}) \equiv 1$. As in equation (8), $\left\{ \tilde{h}_t^{(i)}(\theta) \right\}_{t=1}^{T}$ denotes a trajectory drawn from the p densities, and the c_t are unknown constants to be estimated jointly with x_t. If the density kernel $k(H_t; x_t)$ is chosen within the exponential family of distributions, the EIS least squares problems become linear in x_t under the canonical representation of exponential kernels.

The EIS estimate of the likelihood function for a given value of θ is obtained by substituting $\left\{ \hat{x}_t(\theta) \right\}_{t=1}^{T}$ for $\left\{ x_t \right\}_{t=1}^{T}$ in equation (9). In order to obtain maximally efficient importance samplers, a small number of iterations of the EIS algorithm is required, where the natural samplers p are replaced by the previous stage importance samplers. For such iterations to converge to fixed values of the auxiliary parameters, $\hat{x}_t$, which are expected to produce optimal importance samplers, it is necessary to apply the technique of Common Random Numbers (CRNs).

5.2 Implementation Issues

As we consider the nonlinear ARFIMA(p,d,q) process, it is not easy to incorporate it in the likelihood function. Hence, we suggest to employ an AR(J) approximation of the AR(∞) representation of the ARFIMA part. It is similar to the MA(J) approximation of the FIEGARCH model by Bollerslev and Mikkeslen (1996), in the sense that the coefficient of the J-th lagged term is almost zero and negligible for large J, such as J=1000.

Based on the above truncation, we have the distributions of y_t and h_t. The RV-ARFIMA(p,d,q)-AS$(\gamma_1, \gamma_2, \gamma_3)$-noise model in equation (7) assumes that RVs, y_t, given the latent log-volatility, h_t, follows the normal distribution,

$$g(y_t | h_t, \theta) \propto \exp\left\{ -\frac{1}{2\sigma^2}(y_t - h_t)^2 \right\}.$$

Conditional on $\left(H_{t-1}, r_{t-1} \right)$, the log-volatility, $h_t \left(= \ln V_t \right)$, follows the normal distribution,

$$p\left(h_t | H_{t-1}, r_{t-1}, \theta \right) \propto \exp\left\{ -\frac{1}{2\sigma_t^2}\left(h_t - \alpha - l_t \right)^2 \right\},$$

where

$$l_t = \begin{cases} 0 & \text{for} \quad t=1 \\ \sum_{i=1}^{t-1} \lambda_i h_{t-i} + \xi_t^* - E\left(\xi_t^* \right) & \text{for} \quad t=2,\ldots,J \\ \sum_{i=1}^{J} \lambda_i h_{t-i} + \xi_t^* - E\left(\xi_t^* \right) & \text{for} \quad t=J+1,\ldots,T \end{cases}$$

and

$$\sigma_t^2 = \begin{cases} \Lambda'_{J,J} C_J \Lambda_{J,J} + \sigma_\xi^2 & \text{for} \quad t=1 \\ \Lambda'_{t,J} C_{J-t+1} \Lambda_{t,J} + \sigma_\eta^2 & \text{for} \quad t=2,\ldots,J \\ \sigma_\eta^2 & \text{for} \quad t=J+1,\ldots,T \end{cases}$$

with $\xi_t^* = \gamma_1 r_t e^{-h_t} + \gamma_2 |r_t| e^{-h_t} + \gamma_3 r_t e^{-h_t} I\left(0 \le r_t e^{-h_t} < \delta \right) + \gamma_3 \delta I\left(\delta \le r_t e^{-h_t} \right)$, σ_ξ^2 is the variance of ξ_t determined by $\left(\gamma_1, \gamma_2, \gamma_3, \delta, \sigma_\eta^2 \right)$, $\Lambda_{t,J} = \left(\lambda_t, \lambda_{t+1}, \ldots, \lambda_J \right)'$ and C_j is the unconditional covariance matrix of $\left(h_1, \ldots, h_j \right)'$.

We chose m as the parametric extensions of the natural samplers, p. Hence, the parameterization for k is given by

$$k\left(H_t; x_t, r_{t-1} \right) = p\left(h_t | H_{t-1}, r_{t-1}, \theta \right) \zeta\left(h_t, x_t \right),$$

where the auxiliary function $\zeta(h_t, x_t)$ is itself a Gaussian density kernel. Under this parameterization, the natural sampler, p, cancels out in the least squares problem in equation (11), to the effect that $\ln \zeta(h_t, x_t)$ serves to approximate $\ln g(y_t | h_t, Y_{t-1}, r_{t-1}, \theta)$ $+ \ln \chi(H_t, x_{t+1}, r_t)$. In particular, the appropriate auxiliary function for the asymmetric model is given by $\ln \zeta(h_t, x_t) = \exp(x_{1t} h_t + x_{2t} h_t)$, with $x_t = (x_{1t}, x_{2t})$, and the density kernels of the importance samplers have the form

$$k(H_t; x_t, r_{t-1}) \propto \exp\left[-\frac{1}{2}\left\{ \left(\frac{\lambda_t}{\sigma_t}\right)^2 - 2\left(\frac{\alpha + l_t}{\sigma_t^2} + x_{1t}\right) h_t + \left(\frac{1}{\sigma_t^2} - 2x_{2t}\right) h_t^2 \right\} \right].$$

Accordingly, the conditional mean and variance of h_t on m are given by

$$\mu_{m,t} = \sigma_{m,t}^2 \left(\frac{\alpha + l_t}{\sigma_t^2} + x_{1t} \right), \quad \sigma_{m,t}^2 = \frac{\sigma_t^2}{1 - 2\sigma_t^2 x_{2t}}, \tag{12}$$

respectively. Integrating $k(H_t; x_t, r_{t-1})$ with respect to h_t, and omitting irrelevant multiplicative factors, leads to the following expression for the integrating constant:

$$\chi(H_{t-1}, x_t, r_{t-1}) \propto \exp\left\{ \frac{\mu_{m,t}^2}{2\sigma_{m,t}^2} - \frac{(\alpha + l_t)^2}{2\sigma_t^2} \right\}. \tag{13}$$

Based upon these functional forms, the computation of an EIS estimate of the likelihood for the asymmetric model requires the following steps:

Step (0): Use the natural samplers, p, to draw N trajectories of the latent variable,

$$\left\{ \tilde{h}_t^{(i)}(\theta) \right\}_{t=1}^{T}.$$

Step (t): $(t : T \to 1)$: Use these random draws to solve the back-recursive sequence of least squares problems, as defined in equation (11). The step t least squares problem is characterized by the following linear auxiliary regression:

$$-\frac{1}{2\sigma^2}(y_t - h_t)^2 + \ln \chi\left(\tilde{h}_t^{(i)}(\theta), \hat{x}_{t+1}(\theta)\right)$$
$$= \text{constant} + x_{1t}\tilde{h}_t^{(i)}(\theta) + x_{2t}\left\{\tilde{h}_t^{(i)}(\theta)\right\}^2 + u_t^{(i)}, \quad i : 1 \to N,$$

where $u_t^{(i)}$ denotes the regression error term. The initial condition for the integrating constant (in equation (13)) is given by $\chi(h_T, x_{T+1}, r_T) \equiv 1$.

Step (T + 1): The EIS samplers, $\left\{ m\left(h_t \big| H_{t-1}, \hat{x}_t(\theta)\right) \right\}_{t=1}^{T}$, which are characterized by the conditional mean and variance given in equation (12), are used to draw N trajectories $\left\{ \tilde{h}_t^{(i)}\left(\hat{a}_t(\theta)\right) \right\}_{t=1}^{T}$, from which the EIS estimate of the likelihood is calculated according to equation (9).

We set $N = 50$, as Liesenfeld and Richard (2003) reported that 50 is sufficient for univariate and nonlinear latent variable models, such as SV. After 7-10 iterations, $\tilde{L}(\theta; Y_T, X_T, R_T)$ converged for each θ. The next section gives the EIS-ML estimates for the asymmetric model of RV.

For the case of neglecting measurement errors (that is, $\sigma = 0$), h_t is observable, so it is possible to perform maximum likelihood estimation without simulations. By comparing the log-likelihood with the EIS log-likelihood above, we have the conventional likelihood ratio test statistics, which follows the $\chi^2(1)$ distribution under

the null hypothesis that $\sigma = 0$.

6   Empirical Results

The empirical analysis focuses on the RV of Standard and Poor's 500 Composite Index. In order to estimate the daily realized volatility, we use the two time scales estimator (TTSE) of Zhang, Mykland and Aït-Sahalia (2005) with five-minute grids, which is a consistent estimator of the daily realized volatility. The sample period is Jan/3/1996 to March/29/2007, giving T=2796 observations of RV.

As a preliminary analysis, we consider the new Fractional Integrated EGARCH-t models given in Section 3 as

$$r_t = \sigma_t z_t, \quad z_t \sim St(v),$$

$$\ln \sigma_{t+1}^2 = \alpha + (1-L)^{-d} \Phi^{-1}(L) \Theta(L) \xi_t,$$

(14)

$$\xi_t = \xi_t^* - E(\xi_t^*),$$

$$\xi_t^* = \gamma_1 z_t + \gamma_2 |z_t| + \gamma_3 z_t I(0 \le z_t < \delta) - \gamma_3 \delta I(\delta \le z_t),$$

where $St(v)$ denotes the standardized t distribution, with degrees of freedom given by

v. Note that this model implicitly specifies that $\sigma_\eta = 0$, so that σ_t is determined by

the   past   information.   We   denote   the   above   model   as   the

FIEGARCH(p,d,q)-t-AS $(\gamma_1, \gamma_2, \gamma_3)$ model and, for the case where d=0, as the

EGARCH(p,q)-t-AS$(\gamma_1, \gamma_2, \gamma_3)$ model.

We   estimated   two   kinds   of   models,   EGARCH(1,1)-t-AS $(\gamma_1, \gamma_2, \gamma_3)$ and

FIEGARCH$(1,d,1)$-t-AS$(\gamma_1,\gamma_2,0)$. Table 1 shows the ML estimates of these models.

For the former model, all the estimated parameters, except for α and γ_3, are significant at the five percent level. The estimate of ϕ is close to 0.99, showing high persistence in volatility. The estimate of γ_1 is negative, while that of γ_2 is positive. The estimate of $1/\nu$ is 0.11, indicating that the estimate of ν is close to 9. The results are typical for the EGARCH-t specification. For the long memory model, all the estimated parameters, except for γ_1 and $1/\nu$, are significant. This specification shows the lack of importance of asymmetric effects and heavy-tailed conditional distributions. Similar results are also found in the literature with the FIEGARCH-t specification. Regarding the type of asymmetry, the estimates of the former model indicate Type II asymmetry, while those of the latter model display Symmetry.

In the following, we will show that the empirical results are completely different in the case of RV. It should be noted that it is inadequate to compare the log-likelihood of the EGARCH models with that of RV models as the former is based on r_t while the latter is based on the RV, y_t. Furthermore, the fat tails of the conditional distribution of r_t are irrelevant for the RV model.

Table 2 shows the EIS-ML results of the RV-AR(1)-AS$(\gamma_1,\gamma_2,\gamma_3)$-noise model.

Regarding asymmetry, we consider four specifications, namely AS$(0,0,0)$, AS$(\gamma_1,0,0)$, AS$(\gamma_1,\gamma_2,0)$, and AS$(\gamma_1,\gamma_2,\gamma_3)$. All the estimated parameters are significant at the five percent level. The the AS$(\gamma_1,\gamma_2,\gamma_3)$ model has the smallest AIC and BIC, we report the results only for this specification.

The estimate of σ is close to 0.4, showing that the RV errors are not negligible. The estimate of ϕ is 0.986, while that of σ_η is 0.11, which are typical of SV models. The estimate of γ_1 is negative, while that of γ_2 is positive. Unlike the estimates of the EGARCH model, the estimate of γ_3 is negative and significant. Figure 2 gives the news impact from z_t to $\ln V_{t+1}$, showing that negative shocks and large positive shocks increase future volatility, but small positive shocks decrease volatility. In other

words, Figure 2 displays Type IV Asymmetry.

Table 3 presents the EIS-ML results for the RV-ARFIMA(1,d,0)-AS$\left(\gamma_1,\gamma_2,\gamma_3\right)$-noise model. As before, we consider four kinds of asymmetric effects. The AIC and BIC selected the AS$\left(\gamma_1,\gamma_2,\gamma_3\right)$ model, so we will concentrate the analysis on this model. All the estimated parameters are significant at the five percent level. The estimate of σ is close to 0.4, indicating that the RV errors are not negligible. The estimate of d is 0.47, showing that the log-volatility has long memory and is a stationary process. The estimate of ϕ is positive and close to 0.4, which is against the typical value of -0.1 in the RV literature. The difference can be explained by the existence of RV errors, $U_t = y_t - \ln V_t$. As shown in the Monte Carlo experiments of Asai, McAleer and Medeiros (2008), even minor RV errors cause bias in the estimates if the RV error is neglected in estimation. The signs of γ_1, γ_2 and γ_3 are the same as in the case of Table 2. Figure 3 shows the news impact from z_t to $\ln V_{t+1}$, and the curve is classified as Type IV asymmetry.

From Tables 2 and 3, we find that the RV-ARFIMA(1,d,0)-AS$\left(\gamma_1,\gamma_2,\gamma_3\right)$-noise model has the smallest AIC, while BIC chooses the RV-AR(1)-AS$\left(\gamma_1,\gamma_2,\gamma_3\right)$-noise model. These tables indicate that including the additional term, γ_3, significantly improves the goodness of fit of the model.

Regarding the RV-ARFIMA(1,d,0)-AS$\left(\gamma_1,\gamma_2,\gamma_3\right)$-noise model, we examine the performance of the out-of-sample forecasts. Fixing the sample size at 2,500, we re-estimated the model and computed one-step-ahead forecasts of log-volatility for the last 150 days. Our evaluation is based on the Mincer-Zarnowitz Regression, namely

$$x_t = a + b\hat{x}_{t|t-1} + e_t, \quad t = 1,2,\ldots,150$$

where x_t can be the observed RV or log-RV on day t, and $\hat{x}_{t|t-1}$ is the one-step-ahead

forecast of x_t on day t. If the model is correctly specified, then $a = 0$ and $b = 1$. Table 4(a) shows the estimates of the coefficients and the heteroskedasticity-consistent F test statistics for the joint null hypothesis. For both of the RV and log-RV series, the F test does not reject the null hypothesis. As our model is based on log-RV, the estimates for log-RV are very close to the values expected under the null hypothesis.

For the above out-of-sample analysis, we also calculated the VaR thresholds, accommodating the filtered historical simulation (FHS) approach, which is an effective method for predicting VaR thresholds (see Kuester et al. (2006) for some recent studies regarding the FHS approach). In short, the FHS approach estimates the empirical distribution of the standardized returns, then obtains the $100p$ percentiles to compute the $100p$ percent VaR thresholds. In our analysis, each time we estimated the model with 2,500 observations, we computed the $100p$ percentiles of the empirical distribution based on the last 500 observations, discarding the first 2,000 observations. Combined with the one-day-ahead forecasts of log-volatility, we computed the $100p$ percent VaR thresholds.

In order to assess the estimated VaR thresholds, we conducted the unconditional coverage and independence tests developed by Christoffersen (1998). Consider the "hit sequence" of VaR violations, which takes one if the loss is larger than the VaR threshold, while it takes zero if the VaR is not violated. If we could predict the VaR violations, then that information may help to construct a better model. Hence, the hit sequence of violations should be unpredictable, and should follow independent Bernoulli distribution with parameter p. This is the null hypothesis of the likelihood ratio test for unconditional coverage. The likelihood ratio test of independence is constructed against a first-order Markov alternative, and the tests have an asymptotic

χ_1^2 distribution. Table 4(b) shows the percentage of VaR violations and test results. For the 5% and 1% VaR thresholds, the tests do not reject the null hypothesis, which suggests that the estimated VaR thresholds are satisfactory.

7   Concluding Remarks

In both the conditional and stochastic volatility literature, there has been some confusion between the definitions of asymmetry and leverage. In order to clarify this

issue, the paper showed the relationship among conditional, stochastic, integrated and realized volatility. Then we proposed a new asymmetric volatility model, which sensitively capture the effects of large and small, and positive and negative, shocks. Based on the new specification, this paper examined alternative univariate volatility models that have recently been developed and estimated in order to understand the differences and similarities in the definitions of asymmetry and leverage.

We extended the specification of asymmetric volatility in order to model RV by taking account of the RV errors. This is a general model which includes not only various kinds of asymmetric effects, but also short and long memory specifications. We applied the EIS-ML method to estimate the model of RV.

The empirical results for the RV of Standard and Poor's 500 Composite Index showed the existence of RV errors. The estimates of the short and long memory models supported the Type IV asymmetry definition, which satisfies the following three conditions: (i) negative shocks to returns increase future volatility; (ii) large positive shocks to returns increase future volatility, but a negative shock has a larger effect on volatility than does a positive shock of equal magnitude; and (iii) small positive shocks to returns decrease future volatility. Thus, the new specification of asymmetry significantly improved the goodness of fit, and the out-of-sample forecasts and VaR thresholds were satisfactory.

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Table 1: ML Estimates of the New EGARCH Class

Parameters	New EGARCH-t		FIEGARCH(1,d,0)-t	
d			0.4067	(0.0256)
ϕ	0.9856	(0.0034)	-0.2651	(0.0450)
α	0.1937	(0.2013)	0.3101	(0.0777)
γ_1	-0.1004	(0.0122)	0.0115	(0.0441)
γ_2	0.1025	(0.0142)	2.4407	(0.0776)
γ_3	-0.0295	(0.0211)		
δ	0.5049	(0.0438)		
$1/\nu$	0.1109	(0.0165)	0.0004	(0.0410)
Log-Like	-3872.87		--	
AIC	7759.74		--	
BIC	7801.30		--	

Note: Standard errors are in parentheses. With respect to FIEGARCH, the log-likelihood and other statistics are not reported, as they are not comparable because of 1000 initial values.

Table 2: EIS Estimates of RV-AR(1)-AS$\left(\gamma_1,\gamma_2,\gamma_3\right)$-noise

Parameters	AS$(0,0,0)$	AS$(\gamma_1,0,0)$	AS$(\gamma_1,\gamma_2,0)$	AS$(\gamma_1,\gamma_2,\gamma_3)$
ϕ	0.9747	0.9728	0.9870	0.9856
	(0.0051)	(0.0040)	(0.0044)	(0.0044)
σ_η	0.1478	0.1111	0.1110	0.1103
	(0.0091)	(0.0075)	(0.0074)	(0.00720)
α	-0.3148	-0.1795	-0.8439	1.2246
	(0.1091)	(0.0788)	(0.3227)	(0.5127)
γ_1		-0.0681	-0.0649	-0.0418
		(0.0046)	(0.0043)	(0.0062)
γ_2			0.0424	0.0561
			(0.0074)	(0.0079)
γ_3				-0.1934
				(0.0471)
δ				0.4902
				(0.0605)
σ	0.4054	0.4092	0.4125	0.4116
	(0.0073)	(0.0067)	(0.0067)	(0.0067)
Log-Like	-1921.94	-1821.52	-1806.24	-1793.51
AIC	3851.88	3653.04	3624.48	3603.03
BIC	3875.63	3682.72	3660.10	3650.52

Note: Standard errors are in parentheses.

Table 3: EIS Estimates of RV-ARFIMA(1,*d*,0)-AS$(\gamma_1, \gamma_2, \gamma_3)$-noise

Parameters	AS$(0,0,0)$	AS$(\gamma_1,0,0)$	AS$(\gamma_1,\gamma_2,0)$	AS$(\gamma_1,\gamma_2,\gamma_3)$
d	0.4955	0.4987	0.4748	0.4727
	(0.0039)	(0.00089)	(0.0090)	(0.0076)
ϕ	0.3261	0.3676	0.4166	0.4373
	(0.0603)	(0.0438)	(0.0538)	(0.0291)
σ_η	0.2416	0.1750	0.1852	0.1739
	(0.0225)	(0.0147)	(0.0157)	(0.0080)
α	-0.5832	0.0021	0.0051	0.0046
	(0.2394)	(0.0026)	(0.0027)	(0.0020)
γ_1		-0.0865	-0.0827	-0.0275
		(0.0061)	(0.0063)	(0.0075)
γ_2			0.0226	0.0511
			(0.0077)	(0.0076)
γ_3				-0.2428
				(0.0287)
δ				0.8841
				(0.0196)
σ	0.3648	0.3844	0.3827	0.3858
	(0.01208)	(0.0081)	(0.0085)	(0.0067)
Log-Like	-1908.27	-1819.30	-1811.37	-1792.16
AIC	3826.54	3650.60	3636.75	3602.31
BIC	3856.22	3686.22	3678.30	3655.74

Note: Standard errors are in parentheses.

Table 4: One-Day-Ahead Forecasting Results

(a) Mincer-Zarnowitz Regression

$$x_t = a + b\hat{x}_{t|t-1} + e_t$$

Dependent variable	Constant	Forecast	F test
Volatility	-0.2348	1.7574	4.1797
	(0.1380)	(0.4207)	[0.1237]
Log-Volatility	-0.0044	0.9834	0.3065
	(0.1305)	(0.1109)	[0.8579]

Note: Heteroskedasticity-consistent standard errors are in parentheses. 'F test' denotes the value of the heteroskedasticity-robust F test for the null hypothesis $H_0 : a = 0, b = 1$. P-values are in brackets.

(b) Backtesting VaRs

VaR	% Violation	UC	ID
5%	0.0464	0.0432	0.6856
		[0.8354]	[0.4077]
1%	0.0132	0.1458	0.0541
		[0.7026]	[0.8162]

Note: '% Violation' is the percentage of days when returns are less than the VaR threshold. UC and IND are the likelihood ratio tests for unconditional coverage and independence (against a first-order Markov alternative) developed by Christoffersen (1998). P-values are in brackets.

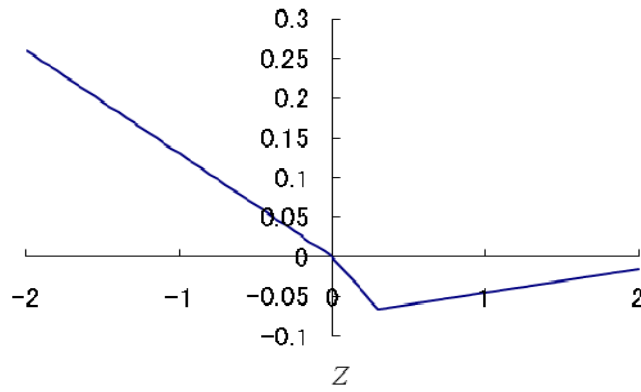Figure 1: News Impact from Z_t to ξ_t

Figure 2: News Impact from Z_t to $\ln V_{t+1}$;
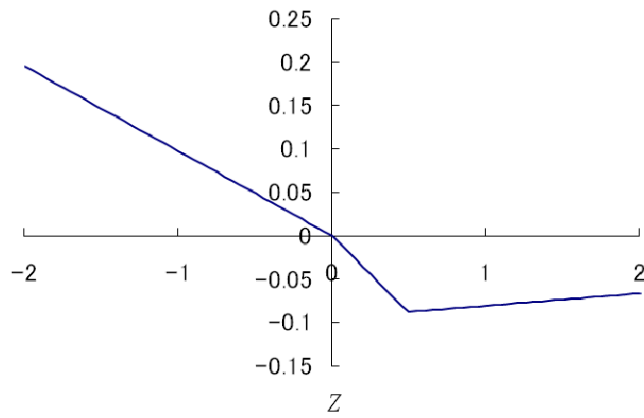RV-AR(1)-AS($\gamma_1,\gamma_2,\gamma_3$)-Noise Model

Figure 3: News Impact from Z_t to $\ln V_{t+1}$;
RV-ARFIMA(1,d,0)-AS($\gamma_1,\gamma_2,\gamma_3$)-Noise Model