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Tomohiro Hirano
The University of Tokyo
Masaru Inaba
Kansai University
Noriyuki Yanagawa
The University of Tokyo

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Asset Bubbles and Bailout*

Tomohiro Hirano, Masaru Inaba, and Noriyuki Yanagawa

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Abstract

This paper theoretically investigates the relationship between government bailout and asset bubbles. We show that even riskier bubbles are more likely to occur, the more bailout is anticipated. We also analyze to what extent is ex-post bailout desirable from ex-ante efficiency in production. We show that expansion of the bailout initially enhances ex-ante efficiency in production and then decreases it. Furthermore, we analyze how the anticipated bailout affects boom-bust cycles. We show that anticipated bailout ends up with increasing boom-bust cycles and requiring large amount of public funds in the bubbles' collapsing. Finally, we derive an optimal bailout policy for tax payers. We show that partial bailout is optimal, in the sense that no-bailout is not optimal and rescuing all is not optimal neither. In the case of riskier bubbles, government has to give up some efficiency in production, so that even unproductive entrepreneurs produce and boom-bust cycles become milder.

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1 Introduction

Many countries have experienced bubble-like dynamics. Associated with the bursting part of asset price bubbles are significant contractions in real economic activity. Notable examples include the recent U.S. experiences after the financial crisis of 2007/2008 as well as Japan's experiences in the 1990s. To mitigate severe contractions, government tends to take various types of bailouts such as recapitalization through buying equity or through the purchase of troubled assets at inflated prices. Although these policies may mitigate the contractions ex-post, what happens if these policies are anticipated ex-ante? In this paper, we ask the following questions.

- How does anticipated bailout affect the emergence of asset bubbles?
- To what extent is ex-post bailout desirable from ex-ante perspective?
- How does anticipated bailout affect boom-bust cycles?
- Finally, we derive an optimal bailout policy for tax payers.

For this purpose, we develop a macroeconomic model with stochastic bubbles.[1] The recent developments on rational bubbles have provided a theoretical framework to analyze asset price bubbles (Caballero and Krishnamurthy, 2006; Farhi and Tirole, 2009; Kocherlalota, 2009; Hirano and Yanagawa, 2010a, 2010b; Martin and Ventura, 2010a, 2010b; Aoki and Nikolov, 2011; Miao and Wang, 2011). We extend a rational bubble model to include bailout. In the present paper, since bubble assets are risky in the sense that bubbles may collapse, risk-averse entrepreneurs want to hedge themselves by investing in safe assets. As we show, the entrepreneurs' portfolio decision depends upon not only the bursting probability of bubbles, but also the expectations about the government policy.

What is new in our framework is that through the change in risk-taking behaviors of the entrepreneurs, the anticipated bailout affects the emergence of asset price bubbles, ex-ante efficiency in production, and boom-bust cycles. We show that even riskier bubbles are more likely to occur, the more bailout is anticipated. We also show that expansion of the bailout initially enhances ex-ante efficiency in production and then decreases it. Furthermore,

[1] Weil (1987) is the first study that analyzes stochastic bubbles in a general equilibrium model.

we show that anticipated bailout ends up with increasing boom-bust cycles and requiring large amount of public funds in the bubbles' collapsing.

Finally, we derive an optimal bailout policy for tax payers. We show that partial bailout is optimal, in the sense that no-bailout is not optimal and rescuing all is not optimal neither. This result has implications for boom-bust cycles. In the case of riskier bubbles, government has to give up some efficiency in production, so that even unproductive entrepreneurs produce and boom-bust cycles become milder.

Our paper is related to theoretical literature that examines government bailouts and risk-taking. For example, Chari and Kehoe (2010), Diamond and Rajan (2011), and Farhi and Tirole (2011) stress moral hazard consequences of bailouts and other credit market interventions in a three-period model. Our paper is mainly different from these papers in the point that we analyze bailout within a full blown dynamic macroeconomic model. In this respect, our paper is closely related to Gertler et al. (2011). Gertler et al. show (2011) that anticipated monetary poicy induces banks to adopt a riskier balance sheet ex-ante, which will in turn require a larger scale credit market intervention during a crisis. They also analyze regulations that mitigates moral hazard and improves welfare. On the other hand, we derive an optimal bailout policy for tax papers in a rational bubbles model.

2 The Model

2.1 Framework

Consider a discrete-time economy with one homogeneous good and a continuum of entrepreneurs and workers. A typical entrepreneur and a representative worker have the following expected discounted utility,

$$E_0\left[\sum_{t=0}^{\infty}\beta^t \log c_t^i\right], \tag{1}$$

where i is the index for each entrepreneur, and c_t^i is the consumption of him/her at date t. $\beta \in (0,1)$ is the subjective discount factor, and $E_0[a]$ is the expected value of a conditional on information at date 0.

Let us start with the entrepreneurs. At each date, each entrepreneur meets high productive investment projects (hereinafter H-projects) with probability p, and low productive ones (L-projects) with probability $1-p$.[2] The investment projects produce capital. The investment technologies are as follows:

$$k_{t+1}^i = \alpha_t^i z_t^i, \tag{2}$$

where $z_t^i(\geq 0)$ is the investment level at date t, and k_{t+1}^i is the capital at date $t+1$ produced by the investment. α_t^i is the marginal productivity of investment at date t. $\alpha_t^i = \alpha^H$ if the entrepreneur has H-projects, and $\alpha_t^i = \alpha^L$ if he/she has L-projects. We assume $\alpha^H > \alpha^L$. For simplicity, we assume that capital fully depreciates in one period.[3] The probability p is exogenous, and independent across entrepreneurs and over time. At the beginning of each date t, the entrepreneur knows his/her own type at date t, whether he/she has H-projects or L-projects. Assuming that the initial population measure of each type of the entrepreneur is p and $1-p$ at date 0, the population measure of each type after date 1 is p and $1-p$, respectively. Throughout this paper, we call the entrepreneurs with H-projects "H-entrepreneurs" and the ones with L-projects "L-entrepreneurs".

We assume that because of frictions in a financial market, the entrepreneur can pledge at most a fraction θ of the future return from his/her investment to creditors.[4] In such a situation, in order for debt contracts to be credible, debt repayment cannot exceed the pledgeable value. That is, the borrowing constraint becomes:

$$r_t b_t^i \leq \theta q_{t+1} \alpha_t^i z_t^i, \tag{3}$$

where q_{t+1} is the relative price of capital to consumption goods at date $t+1$.[5] r_t and b_t^i are the gross interest rate and the amount of borrowing at date t, respectively. The parameter $\theta \in (0,1]$, which is assumed to be exogenous, can be naturally taken to be the degree of imperfection of the financial market.

In this economy, there are bubble assets denoted by x. Aggregate supply of the assets is assumed to be constant over time X. As in Tirole (1985),

[2] A similar setting is used in Woodford (1990), Kiyotaki (1998), Kiyotaki and Moore (2008), Kocherlakota (2009).

[3] As in Kocherlakota (2009), we can consider a situation where some fraction of capital depreciate, and consumption goods can be converted one-for-one into capital at each date, and vice-versa. In this setting, we can also obtain the same results as in the present papar.

[4] See Hart and Moore (1994) and Tirole (2006) for the foundations of this setting.

[5] On an equilibrium path we consider, q_{t+1} is not affected by whether bubbles collapse or not. Hence, there is no uncertainty with regard to q_{t+1}.

we define bubble assets as the assets that produce no real return, i.e., the fundamental value of the assets is zero. Following Weil (1987), we consider stochastic bubbles, in the sense that they may collapse. In each period t, bubble prices become zero (bubbles burst) with probability $1-\pi$ conditional on survive at date $t-1$. Once they burst, they never arise again. This implies that bubbles persist with probability $\pi(<1)$ and their prices are positive until they switch to being equal to zero forever. Let P_t be the per unit price of bubble assets at date t on survive in terms of consumption goods.

The entrepreneur's flow of funds constraint is given by

$$c_t^i + z_t^i + P_t x_t^i = q_t \alpha_{t-1}^i z_{t-1}^i - r_{t-1} b_{t-1}^i + b_t^i + P_t x_{t-1}^i + m_t^i. \tag{4}$$

where x_t^i be the level of bubble assets purchased by a type i entrepreneur at date t. The left hand side of (4) is expenditure on consumption, investment, and the purchase of bubble assets. The right hand side is the available funds at date t, which is the return from investment in the previous period minus debts repayment, plus new borrowing, the return from selling bubble assets, and bailout money, m_t^i. We assume that the bailout is proportional of holdings of the bubble assets.

$$m_t^i = d_t x_{t-1}^i. \tag{5}$$

When bubbles collapse, a fraction $\lambda \in [0,1]$ of the entrepreneurs who hold bubble assets is rescued. $\lambda = 0$ means that no-entrepreneur is rescued. $\lambda = 1$ means that all are rescued. Thus a rise in λ means expansion of the bailout. From ex-ante perspective, each entrepreneur anticipates the government bailout with probability λ. When the entrepreneur is rescued, he/she gets d_t units of consumption goods per unit of bubble assets. Otherwise, he/she gets nothing. We define the net worth of the entrepreneur at date t as

$$e_t^i \equiv q_t \alpha_{t-1}^i z_{t-1}^i - r_{t-1} b_{t-1}^i + P_t x_{t-1}^i + m_t^i.$$

We also impose the short sale constraint on bubble assets:[6]

$$x_t^i \geq 0. \tag{6}$$

Let us now turn to the workers. There are workers with a unit measure.

[6] Kocherlakota (1992) shows that the short sale constraint plays an important role for the emergence of asset bubbles in an endowment economy with infinitely lived agents.

Each worker is endowed with one unit of labor endowment in each period, which is supplied inelastically in labor markets, and earns wage rate, w_t. The flow of funds constraint, the borrowing constraint, and the short sale constraint for them are given by

$$c_t^u + P_t(x_t^u - x_{t-1}^u) = w_t - r_{t-1}b_{t-1}^u + b_t^u - T_t, \tag{7}$$

$$r_t b_t^u \leq 0, \tag{8}$$

$$x_t^u \geq 0, \tag{9}$$

where ^{u} represents the workers. T_t is a lump sum tax.[7] When bubbles collapse, the government levies the lump sum tax on workers, and transfers those funds to entrepreneurs. This means that workers pay direct costs of the bubbles' collapsing. Thus we call workers "tax payers". $T_t > 0$ only when bubbles collapse. $T_t = 0$ if they survive. As in Farhi and Tirole (2012), the aim of this bailout policy is to boost the net worth of entrepreneurs. Equation (8) says that workers cannot borrow, since they do not have any collateralizable assets such as returns from investment projects.

There are competitive firms which produce final consumption goods using capital and labor.[8] The production function of each firm is

$$y_t = k_t^{\sigma} n_t^{1-\sigma}. \tag{10}$$

Factors of production are paid their marginal product:

$$q_t = \sigma K_t^{\sigma-1} \text{ and } w_t = (1-\sigma)K_t^{\sigma}, \tag{11}$$

where K is the aggregate capital stock.

[7] In order to focus on a transfer policy from workers to entrepreneurs as in Farhi and Tirole (2012), we do not consider a tax on entrepreneurs. Even if we consider the capital income tax, we can obtain the same results as in the main text, although the borrowing constraint for entrepreneurs and the investment function would be complicated as analyzed in Aoki et al. (2009).

[8] We assume that each firm is operated by the workers. Since the final goods market is competitive, the net profit from operating the firm is zero, so that the flow of funds constraint of the workers is unchanged as equation (6) in equilibirum.

2.2 Equilibrium

Let us denote the aggregate consumption of H-and L-entrepreneurs and workers at date t as $\sum_{i\in H_t} c_t^i \equiv C_t^H$, $\sum_{i\in L_t} c_t^i \equiv C_t^L$, C_t^u, where H_t and L_t mean a family of H-and L-entrepreneurs at date t. Similarly, let $\sum_{i\in H_t} z_t^i \equiv Z_t^H$, $\sum_{i\in L_t} z_t^i \equiv Z_t^L$, $\sum_{i\in H_t} b_t^i \equiv B_t^H$, $\sum_{i\in L_t} b_t^i \equiv B_t^L$, B_t^u, $\sum_{i\in H_t\cup L_t} k_t^i \equiv K_t$, $(\sum_{i\in H_t\cup L_t} x_t^i + X_t^u) \equiv X_t$ be the aggregate investment, the aggregate borrowing, the aggregate capital stock, and the aggregate demand for bubble assets. Then the market clearing condition for goods, credit, labor, and bubble assets are

$$C_t^H + C_t^L + C_t^u + Z_t^H + Z_t^L = Y_t, \tag{12}$$

$$B_t^H + B_t^L + B_t^u = 0, \tag{13}$$

$$N_t = 1, \tag{14}$$

$$X_t = X. \tag{15}$$

The competitive equilibrium is defined as a set of prices $\{r_t, w_t, P_t\}_{t=0}^{\infty}$ and quantities $\left\{C_t^H, C_t^L, C_t^u, B_t^H, B_t^L, B_t^u, Z_t^H, Z_t^L, G_t, X_t, K_{t+1}, Y_t\right\}_{t=0}^{\infty}$, such that (i) the market clearing conditions, (12)-(15), are satisfied in each period, and (ii) each entrepreneur chooses consumption, borrowing, investment, and the amount of bubble assets, $\{c_t^i, b_t^i, z_t^i, x_t^i\}_{t=0}^{\infty}$, to maximize his/her expected discounted utility (1) under the constraints (2)-(6), taking the bursting probability of bubbles and the bailout probability into consideration. (iii) each worker chooses consumption, borrowing, and the amount of bubble assets, $\{c_t^u, b_t^u, x_t^u\}_{t=0}^{\infty}$, to maximize his/her expected discounted utility (1) under the constraints (7)-(9), taking the bursting probability into consideration.

2.3 Optimal Behavior of Entrepreneurs and Workers

We now characterize the equilibrium behavior of entrepreneurs and workers. We consider the case

$$q_{t+1}\alpha^L \le r_t < q_{t+1}\alpha^H.$$

In equilibrium, interest rate must be at least as high as $q_{t+1}\alpha^L$, since nobody lends to the projects if $r_t < q_{t+1}\alpha^L$.

For workers, both the borrowing constraint and the short sale constraint become binding in equilibrium. Thus, they consume all the wage income in each period. We later verify this in Appendix. That is,

$$c_t^u = w_t - T_t.$$

$T_t > 0$ only when bubbles collapse.

For H-entrepreneurs at date t, both the borrowing constraint and the short sale constraint simultaneously become binding. Since the utility function is log-linear, each entrepreneur consumes a fraction $1-\beta$ of the net worth in each period, that is, $c_t^i = (1-\beta)e_t^i$.[9] Then, by using (3), (4), and (6), the investment function of H-entrepreneurs at date t can be written as

$$z_t^i = \frac{\beta e_t^i}{1 - \dfrac{\theta q_{t+1}\alpha^H}{r_t}}. \tag{16}$$

This is a popular investment function under financial constraint problems, except that the presence of bubble assets and government bailout affect the net worth.[10] We see that the investment equals the leverage, $1/\left[1-(\theta q_{t+1}\alpha^H/r_t)\right]$, times a fraction β of the net worth. From this investment function, we understand that for the entrepreneurs who purchased bubble assets in the previous period, they are able to sell those assets at the time they encounter H-projects. As a result, their net worth increases (compared to the bubbleless case), which relaxes the borrowing constraint and boosts their investments. That is, bubbles generate balance sheet effects. Moreover, the expansion level of the investment is more than the direct increase of the net worth because of the leverage effect. In our analysis, the entrepreneurs buy risky bubble assets when they have L-projects, and sell those assets when they have opportunities to invest in H-projects.

For L-entrepreneurs at date t, both the borrowing constraint and the short sale constraint do not become binding. Instead, they decide optimal portfolio choices. Since bubble assets deliver no return with probability $1-\pi$, risk-averse L-entrepreneurs may want to hedge themselves by investing in L-projects as well as lending to other entrepreneurs. Since $c_t^i = (1-\beta)e_t^i$, the budget constraint (4) becomes

$$z_t^i + P_t x_t^i - b_t^i = \beta e_t^i. \tag{17}$$

[9] See, for example, chapter 1.7 of Sargent (1988).

[10] See, for example, Bernanke and Gertler (1989), Bernanke et al. (1999), Holmstrom and Tirole (1998), Kiyotaki and Moore (1997), and Matsuyama (2007, 2008).

Each L-entrepreneur allocates his/her savings, βe_t^i, into z_t^i, $P_t x_t^i$, and b_t^i. Since investing in L-projects and lending are both safe assets, $z_t^i \geq 0$ if $r_t = q_{t+1}\alpha^L$, and $z_t^i = 0$ if $r_t > q_{t+1}\alpha^L$. That is, the following conditions must be satisfied:

$$(r_t - q_{t+1}\alpha^L)z_t^i = 0,\ \ z_t^i \geq 0,\ \text{and}\ r_t - q_{t+1}\alpha^L \geq 0.$$

Each L-entrepreneur chooses optimal amounts of b_t^i, x_t^i, and z_t^i, so that the expected marginal utility from investing in three assets is equalized. The first order conditions with respect to b_t^i and x_t^i are

$$(b_t^i):\ \frac{1}{c_t^i} = \pi\beta\frac{r_t}{c_{t+1}^{i,\pi}} + (1-\pi)\lambda\beta\frac{r_t}{c_{t+1}^{i,(1-\pi)\lambda}} + (1-\pi)(1-\lambda)\beta\frac{r_t}{c_{t+1}^{i,(1-\pi)(1-\lambda)}}, \tag{18}$$

$$(x_t^i):\ \frac{1}{c_t^i} = \pi\beta\frac{1}{c_{t+1}^{i,\pi}}\frac{P_{t+1}}{P_t} + (1-\pi)\lambda\beta\frac{1}{c_{t+1}^{i,(1-\pi)\lambda}}\frac{d_{t+1}}{P_t}, \tag{19}$$

where $c_{t+1}^{i,\pi} = (1-\beta)(q_{t+1}\alpha^L z_t^i - r_t b_t^i + P_{t+1}x_t^i)$ is the date $t+1$ consumption when bubbles survive. The first term of the right hand side in equation (18) and (19) represents the expected marginal utility from lending a unit and from buying a unit of risky bubble assets in this case. $c_{t+1}^{i,(1-\pi)\lambda} = (1-\beta)(q_{t+1}\alpha^L z_t^i - r_t b_t^i + m_{t+1}^i)$ is the date $t+1$ consumption when bubbles collapse and the government rescues the entrepreneur. The second term of equation (18) and (19) represents the expected marginal utility from lending a unit and from buying a unit of risky bubble assets in this case. $c_{t+1}^{i,(1-\pi)(1-\lambda)} = (1-\beta)(q_{t+1}\alpha^L z_t^i - r_t b_t^i)$ is the date $t+1$ consumption when bubbles collapse and the government does not rescue the entrepreneur. The third term of equation (18) represents the expected marginal utility from lending a unit in this case.[11] P_{t+1}/P_t is the rate of return of bubbles on survive. In this analysis, we consider the bailout that fully recovers the net worth of rescued entrepreneurs, i.e., $d_{t+1} = P_{t+1}$.[12]

[11] Since the entrepreneur consumes a fraction $1-\beta$ of the current net worth in each period, the optimal consumption level at date $t+1$ is independent of the entrepreneur's type at date $t+1$. It only depends on whether bubbles collapse and whether government rescues the entrepreneur.

[12] In this model, on the saddle path, the following condition is satisfied.

$$\frac{P_{t+1}}{P_t} = \frac{K_{t+1}^{\sigma}}{K_t^{\sigma}}.$$

Since P_t, K_t, and K_{t+1} are predetermined variables at the beginning of date $t+1$, the

From (17), (18), and (19), we can derive the demand function for risky bubble assets of a type i L-entrepreneur:

$$P_t x_t^i = \frac{\delta(\lambda)\frac{P_{t+1}}{P_t} - r_t}{\frac{P_{t+1}}{P_t} - r_t}\beta e_t^i, \tag{20}$$

where $\delta(\lambda) \equiv \pi + (1-\pi)\lambda$.

The remaining fraction of savings is split across z_t^i and b_t^i :

$$z_t^i - b_t^i = \frac{[1-\delta(\lambda)]\frac{P_{t+1}}{P_t}}{\frac{P_{t+1}}{P_t} - r_t}\beta e_t^i.$$

From (20), we see that an entrepreneurs's portfolio decision depends upon its perceptions of risk, which in turn depends upon both the probability of bursting of bubbles (π) and expectations about the government bailout (λ). We obtain the following Proposition.

Lemma 1 *x_t^i is an increasing function of λ.*

Lemma 1 means that when λ rises, the type i L-entrepreneur is willing to buy more bubble assets. That is, the anticipated bailout induces L-entrepreneurs to take on more risk, even though the potential probability of the bubble bursts remain unchanged (π is unchanged).

2.4 Aggregation

We are now in a position to consider the aggregate economy. The great merit of the expressions for each entrepreneur's investment and demand for bubble assets, z_t^i and x_t^i, is that they are linear in period-t net worth, e_t^i. Hence aggregation is easy: we do not need to keep track of the distributions.

From (16), we can derive the aggregate H-investments:

$$Z_t^H = \frac{\beta p A_t}{1 - \dfrac{\theta q_{t+1}\alpha^H}{r_t}}, \tag{21}$$

government can know the bubble prices whey they survive at date $t+1$, P_{t+1}.

where $A_t \equiv q_t K_t + P_t X$ is the aggregate wealth of entrepreneurs at date t, and $\sum_{i \in H_t} e_t^i = pA_t$ is the aggregate wealth of H-entrepreneurs at date t. Recall that the probability of meeting H-projects is independently distributed. From this investment function, we see that the aggregate investments of H-entrepreneurs depend upon asset prices, P_t, as well as cash flows from the investment projects in the previous period, $q_t K_t$. In this respect, this investment function is similar to the one in Kiyotaki and Moore (1997). There is a significant difference. In the Kiyotaki-Moore model, the investment function depends upon land prices which reflect fundamentals (cash flows from the present to the future), while in our model, it depends upon bubble prices.

Aggregate L-investments depend upon the level of the interest rate.

$$Z_t^L = \begin{cases} \beta A_t - \dfrac{\beta p A_t}{1 - \dfrac{\theta \alpha^H}{\alpha^L}} - P_t X & \text{if } r_t = q_{t+1}\alpha^L, \\ \\ 0 & \text{if } r_t > q_{t+1}\alpha^L. \end{cases} \tag{22}$$

When $r_t = q_{t+1}\alpha^L$, L-entrepreneurs may invest. In this case, aggregate L-investments are determined by the goods market clearing condition (12). They are equal to aggregate savings of the economy minus aggregate H-investments minus aggregate value of bubbles. When $r_t > q_{t+1}\alpha^L$, L-entrepreneurs do not invest.

The aggregate counterpart to (20) is

$$P_t X_t = \frac{\delta(\lambda)\frac{P_{t+1}}{P_t} - r_t}{\frac{P_{t+1}}{P_t} - r_t} \beta(1-p)A_t, \tag{23}$$

where $\sum_{i \in L_t} e_t^i = (1-p)A_t$ is the aggregate net worth of L-entrepreneurs at date t. (23) is the aggregate demand function for bubble assets at date t.

2.5 Dynamics

Using (21) and (22), we can derive the evolution of aggregate capital stock:

$$K_{t+1} = \begin{cases} \alpha^H \frac{\beta p A_t}{1 - \frac{\theta \alpha^H}{\alpha^L}} + \alpha^L \left(\beta A_t - \frac{\beta p A_t}{1 - \frac{\theta \alpha^H}{\alpha^L}} - P_t X \right) & \text{if } r_t = q_{t+1}\alpha^L, \\ \\ \alpha^H \left[\beta A_t - P_t X \right] & \text{if } r_t > q_{t+1}\alpha^L. \end{cases} \tag{24}$$

The first term and the second term in the first line represent the capital stock at date $t+1$ produced by H-and L-entrepreneurs. When $r_t > q_{t+1}\alpha^L$, only H-entrepreneurs invest. From the goods market clearing condition, we know $Z_t^H = \beta A_t - P_t X$. $(-P_t X)$ in (24) captures a traditional crowd-out effect of bubbles (Tirole, 1985), i.e., L-entrepreneurs buy bubbles, which crowds investments out.

The aggregate wealth of entrepreneurs evolves over time as

$$A_{t+1} = q_{t+1}K_{t+1} + P_{t+1}X. \tag{25}$$

Defining $\phi_t \equiv P_t X/\beta A_t$ as the size of bubbles (the share of the value of bubbles), ϕ_t evolves over time as

$$\phi_{t+1} = \frac{\frac{P_{t+1}}{P_t}}{\frac{A_{t+1}}{A_t}}\phi_t. \tag{26}$$

The evolution of the size of bubbles depends upon the relation between wealth's growth rate (denominater) and bubbles' growth rate (numerator). In order that bubbles do not explode, the following condition must be satisfied:

$$\phi_t < 1. \tag{27}$$

If this condition is violated, the size of bubbles explodes and the economy cannot sustain bubbles.

Using ϕ_t, (23) can be solved for required rate of return on bubble assets, P_{t+1}/P_t,

$$\frac{P_{t+1}}{P_t} = \frac{r_t(1-p-\phi_t)}{\delta(\lambda)(1-p)-\phi_t}. \tag{28}$$

It follows that P_{t+1}/P_t is a decreasing function of λ. $(1-p-\phi_t)/[\delta(\lambda)(1-p)-\phi_t]$ captures risk premium on bubble assets. When λ rises, ceteris paribus, the entrepreneur's required rate of return on bubble assets becomes lower, because risk premium declines.

When $r_t > q_{t+1}\alpha^L$, interest rate is determined by the credit market clearing condition (13). (13) can be written as

$$\frac{\beta p A_t}{1-\dfrac{\theta q_{t+1}\alpha^H}{r_t}} + P_t X = \beta A_t.$$

Aggregate savings of entrepreneurs (βA_t) flow to aggregate H-investments and aggregate value of bubbles. When we solve for the interest rate, we get

$$r_t = \frac{q_{t+1}\theta\alpha^H(1-\phi_t)}{1-p-\phi_t}.$$

Thus equilibrium interest rate is

$$r_t = q_{t+1}Max\left[\alpha^L, \frac{\theta\alpha^H(1-\phi_t)}{1-p-\phi_t}\right]. \tag{29}$$

Note that when $r_t > q_{t+1}\alpha^L$, r_t is an increasing function of ϕ_t, reflecting the tighteness of the credit markets.

When we arrange (26) by using (25), (28), and (29), we get

$$\phi_{t+1} = \begin{cases} \dfrac{\frac{(1-p-\phi_t)}{\delta(\lambda)(1-p)-\phi_t}}{(1+\frac{\alpha^H-\alpha^L}{\alpha^L-\theta\alpha^H}p)\beta+\frac{[1-\delta(\lambda)](1-p)}{\delta(\lambda)(1-p)-\phi_t}\beta\phi_t}\phi_t & \text{if } r_t = q_{t+1}\alpha^L, \\ \\ \frac{\theta}{\beta}\frac{1}{\delta(\lambda)(1-p)-(1-\theta)\phi_t}\phi_t & \text{if } r_t > q_{t+1}\alpha^L. \end{cases} \tag{30}$$

Using ϕ, the evolution of the aggregate capital stock (24) can be written as

$$K_{t+1} = \begin{cases} \dfrac{\left[(1+\frac{\alpha^H-\alpha^L}{\alpha^L-\theta\alpha^H}p)\beta\alpha^L-\alpha^L\beta\phi_t\right]}{1-\beta\phi_t}\sigma K_t^\sigma & \text{if } r_t = q_{t+1}\alpha^L, \\ \\ \frac{\alpha^H\beta[1-\phi_t]}{1-\beta\phi_t}\sigma K_t^\sigma & \text{if } r_t > q_{t+1}\alpha^L. \end{cases} \tag{31}$$

The dynamical system of this economy can be characterized by (30) and (31).

Proposition 2 *There is a saddle point path on which the economy converges toward a stochastic stationary state before bubbles collapse (a state where all variables ($K_t, A_t, q_t, r_t, w_t, P_t, \phi_t$) become constant over time).*

Proof. See Appendix. ■

When the economy gets on the saddle path, ϕ_t becomes constant over time. From (30), we obtain

$$
\phi(\lambda) = \begin{cases} \dfrac{\delta(\lambda) - \frac{1-\delta(\lambda)\beta(1-p)}{\left[1+(\frac{\alpha^H-\alpha^L}{\alpha^L-\theta\alpha^H})p\right]\beta-\beta(1-p)}}{1-\frac{1-\delta(\lambda)\beta(1-p)}{\left[1+(\frac{\alpha^H-\alpha^L}{\alpha^L-\theta\alpha^H})p\right]\beta-\beta(1-p)}}(1-p) & \text{if } r_t = q_{t+1}\alpha^L, \\ \\ \frac{\delta(\lambda)\beta(1-p)-\theta}{\beta(1-\theta)} & \text{if } r_t > q_{t+1}\alpha^L. \end{cases} \tag{32}
$$

From (32), we observe the following property on the size of bubbles.

Proposition 3 *ϕ is an increasing function of λ. That is, the size of bubbles increases as government bailout is expected with higher probability.*

By inserting (32) into (31), we get dynamic equations of the aggregate capital stock on the saddle path.

$$
K_{t+1} = \begin{cases} \frac{\left\{\left[1+(\frac{\alpha^H-\alpha^L}{\alpha^L-\theta\alpha^H})p\right]\beta\alpha^L-\beta\alpha^L(1-p)\right\}}{1-\delta(\lambda)\beta(1-p)}\sigma K_t^\sigma & \text{if } r_t = q_{t+1}\alpha^L, \\ \\ \alpha^H\frac{\beta[1-\delta(\lambda)(1-p)]+(1-\beta)\theta}{1-\delta(\lambda)\beta(1-p)}\sigma K_t^\sigma & \text{if } r_t > q_{t+1}\alpha^L. \end{cases} \tag{33}
$$

When we solve $P_tX/\beta A_t = \phi(\lambda)$ for P_t, we get bubble prices on the saddle path.

$$
P_t = \frac{\beta\phi(\lambda)}{X[1-\beta\phi(\lambda)]}\sigma K_t^\sigma. \tag{34}
$$

We see that the bubble prices rise together with aggregate capital stock. We obtain the following property.

Proposition 4 *P_t is an increasing function of λ. That is, when the bailout is expected with higher probability, the current bubble prices jump up instantaneously.*

As long as bubbles persist, the economy runs according to the above equations, and converges to the stochastic stationary state.[13] A feature of

[13] From (26), (33), and (34), we observe that on the saddle path, aggregate wealth of entrepreneurs, output and the bubble prices grow at the same rate.

$$
\frac{A_{t+1}}{A_t} = \frac{Y_{t+1}}{Y_t} = \frac{P_{t+1}}{p_t}.
$$

bubbly dynamics is that there is a two-way interaction between bubble prices and output. An increase in cash flows in period t raises period t bubble price, which improves the net worth of H-entrepreneurs and expands their investments, thereby increases cash flows and bubble prices in period $t+1$ even further. These knock-on effects will continue not only in period $t+1$, but also in period $t+2, t+3, \cdots$. Moreover, this anticipated increase in the bubble prices is reflected in period t bubble prices, which improves the period t net worth again. In equilibrium, all these feedback effects occur simultaneously, and capital stock and the bubble prices run according to (33) and (34).

2.6 Anticipated Bailout and Asset Bubbles

In order that stochastic bubbles can exist, the following condition must be satisfied:[14]

$$\phi(\lambda) > 0.$$

If this condition is violated, any equilibrium with bubbles cannot exist. From (32), we obtaint the following Proposition.

Proposition 5 *The existence condition of stochastic bubbles when government bailout is anticipated with probability λ is*

$$\theta < \delta(\lambda)\beta(1-p),$$

and

$$\alpha^H > \alpha^L \frac{1-\delta(\lambda)(1-p)\beta}{[1-\delta(\lambda)\beta]\theta + p\beta\delta(\lambda)} \equiv \alpha_1^H. \tag{35}$$

Proof. See Appendix. ■

Figure characterizes bubble regions with θ and α^H.[15] The first condition is that in order that stochastic bubbles can arise, financial market imperfection must be sufficiently high. The other condition is that α^H, productivity

[14] If $\phi \leq 0$, even the other equilibrium path with bubbles except for the saddle one cannot exist (see footnote 13). Thus, no equilibrium path with bubbles can exist if $\phi \leq 0$.

[15] Note that if

$$\alpha^H < \frac{\alpha^L}{\delta\beta},$$

then, stochastic bubbles cannot exist. In other words, if productivity is too low, bubbles never arise for any θ.

of the economy, must be sufficiently high. Intuitively, in high θ regions, interest rate becomes so great and bubbles' growth rate also becomes too high compared to the economy's growth rate. As a result, the economy cannot sustain growing bubbles. Thus, in high θ regions, bubbles cannot occur. On the other hand, in low θ regions, interest rate becomes low and so is the bubbles' growth rate. As long as α^H is sufficiently high, wealth's growth rate becomes sufficiently high that the economy can sustain growing bubbles.[16] From these observations, we learn that bubbles cannot occur in low-productivity economies or in economies with more efficient financial markets.

Interesting point is that expectations about the government bailout affects the bubble regions. When bailout is expected, required rate of return of bubbles declines, which lowers their growth rate. Thus, even economies with lower productivity or economies with more efficient financial markets can support bubbles. This suggests that bubble regions become wider and wider, the more bailout is anticipated.

Moreover, our model predicts that even riskier bubbles are more likely to occur because of the government bailout. When we solve (35) for π, we get

$$\pi > \frac{\alpha^L - \theta\alpha^H}{\alpha^H\beta(p-\theta) + \alpha^L\beta(1-p)} \frac{1}{1-\lambda} - \frac{\lambda}{1-\lambda} \equiv \pi_1.$$

This condition means that in order that stochastic bubbles can arise, π must be sufficiently high. Intuitively, when π is too low and the bursting probability is too high, risk premium on bubble assets and bubbles' growth rate become too high. As a result, the economy cannot sustain bubbles. Thus, bubbles with high bursting probability cannot occur. However, once the bailout is anticipated, even those bubbles can arise.

Proposition 6 *π_1 is a decreasing function of λ. Bubbles with higher bursting probability can arise as government bailout is expected with higher probability.*

[16] We can use the structure of the bubbleless economy to characterize the existence condition. The existence condition also says that the interest rate is sufficiently lower than the economy's growth rate in the bubbleless economy. This condition is similar to the existence condition in Tirole(1985). In our model, since we consider stochastic bubbles, interest rate must be sufficiently low in the bubbleless economy. Otherwise, stochastic bubbles cannot arise.

Now we are in a position to describe how interest rate is determined in equilibrium. Let us define $\hat{\alpha}^H$ such that $Z_t^L = 0$ when $r_t = q_{t+1}\alpha^L$.[17]

$$Z_t^L = \left[1 - \frac{p}{1 - \frac{\theta\hat{\alpha}^H}{\alpha^L}} - \frac{\delta\beta(1-p) - \theta}{\beta(1-\theta)}\right]\beta A_t = 0.$$

From this, we know

$$\begin{aligned} Z_t^L &> 0 \text{ in } \alpha_1^H < \alpha^H < \hat{\alpha}^H, \\ Z_t^L &= 0 \text{ in } \alpha^H \geq \hat{\alpha}^H. \end{aligned}$$

When we solve for $\hat{\alpha}^H$, we get

$$\hat{\alpha}^H = \frac{\alpha^L\left\{\beta(1-\delta)(1-p) + (1-\beta+p\beta)\theta\right\}}{\theta\left\{\beta[1-\delta(1-p)] + (1-\beta)\theta\right\}}. \tag{36}$$

Thus, we see that equilibrium interest rate in the bubble economy depends upon α^H.

$$r_t = \begin{cases} q_{t+1}\alpha^L & \text{if } \alpha_1^H < \alpha^H \leq \hat{\alpha}^H, \\ \theta q_{t+1}\alpha^H\gamma & \text{if } \alpha^H \geq \hat{\alpha}^H, \end{cases} \tag{37}$$

where $\gamma \equiv \frac{\beta[1-\delta(1-p)]+(1-\beta)\theta}{\beta(1-p)(1-\delta)+(1-\beta+p\beta)\theta}$. In Figure, we characterize interest regions with θ and α^H.

From (33) and (37), we obtain

$$K_{t+1} = \begin{cases} \frac{\left\{\left[1+(\frac{\alpha^H-\alpha^L}{\alpha^L-\theta\alpha^H})p\right]\beta\alpha^L - \beta\alpha^L(1-p)\right\}}{1-\delta\beta(1-p)}\sigma K_t^\sigma & \text{if } \alpha_1^H < \alpha^H \leq \hat{\alpha}^H, \\ \alpha^H\frac{\beta[1-\delta(1-p)]+(1-\beta)\theta}{1-\delta\beta(1-p)}\sigma K_t^\sigma & \text{if } \alpha^H \geq \hat{\alpha}^H. \end{cases} \tag{38}$$

[17] $1 - \frac{p}{1-\frac{\theta\hat{\alpha}^H}{\alpha^L}} - \frac{\delta\beta(1-p)-\theta}{\beta(1-\theta)} \gtreqless 0$ is equivalent to $\alpha^L \gtreqless \frac{\theta\alpha^H(1-\phi)}{1-p-\phi}$ in (29).

3 Macroeconomic Effects of Anticipated Bailout

In this section, we examine macroeconomic effects of the anticipated bailout.

3.1 Effects on Ex-ante Efficiency in Production

The bailout may mitigate adverse effects of the bubbles' collapsing on output. In this sense, the bailout may be desirable from ex-post perspective. However, when the bailout is anticipated, it may produce inefficiency ex-ante. To what extent is ex-post bailout desirable from ex-ante perspective? In this subsection, we analyze how expanding bailout affects ex-ante efficiency in production.

For this purpose, let us define α_2^H such that $Z_t^L = 0$ when $r_t = q_{t+1}\alpha^L$ and $\lambda = 0$. From (36), we get

$$\alpha_2^H = \frac{\alpha^L \left\{\beta(1-\pi)(1-p) + (1-\beta+p\beta)\theta\right\}}{\theta \left\{\beta[1-\pi(1-p)] + (1-\beta)\theta\right\}}.$$

We proceed to analyze the effects of the anticipated bailout depending upon the level of α^H.

Let us first analyze the case of $\alpha^H \geq \alpha_2^H$. In this case, as long as bubbles persist, the evolution of aggregate capital stock follows

$$K_{t+1} = \alpha^H \frac{\beta\left[1-\delta(1-p)\right] + (1-\beta)\theta}{1-\delta\beta(1-p)} \sigma K_t^\sigma.$$

Thus, we have the following Proposition.

Proposition 7 *In $\alpha^H \geq \alpha_2^H$, Y_t is a decreasing function of λ in the bubble economy.*

Let τ be the time bubbles collapse. This Proposition means that given a same initial K_0, Y_t becomes lower for any $1 \leq t \leq \tau$ under high λ. In other words, ex-ante efficiency in production decreases as bailout expands. Intuition is that when productivity of the economy is high, only H-entrepreneurs produce. In this situation, expansion of bailout crowds savings away from H-projects. Thus if the government wants to maximize ex-ante production efficiency, then $\lambda = 0$ is desirable. Figure shows the relationship between ex-ante efficiency in production (Y_t in $1 \leq t \leq \tau$) and λ in $\alpha^H \geq \alpha_2^H$.

Next we examine the case of $\alpha_1^H < \alpha^H < \alpha_2^H$. In this region, when $\lambda = 0$, L-entrepreneurs invest positive amount. Bailout changes this by crowding L-projects out.

Proposition 8 *In $\alpha_1^H < \alpha^H < \alpha_2^H$, there is a critical value of $\lambda \equiv \lambda^*$ under which $Z_t^L = 0$ when $r_t = q_{t+1}\alpha^L$. $Z_t^L > 0$ in $0 \leq \lambda < \lambda^*$ and $Z_t^L = 0$ in $\lambda^* \leq \lambda \leq 1$. λ^* satisfies*

$$\lambda^* = \frac{1}{1-\pi}\frac{\alpha^L\left[\beta(1-p) + (1-\beta+p\beta)\theta\right] - \theta\alpha^H\left[\beta + (1-\beta)\theta\right]}{\beta(1-p)(\alpha^L - \theta\alpha^H)} - \frac{\pi}{1-\pi}.$$

It follows that λ^* is a decreasing function of π. This means that in the case of riskier bubbles, in order to maximize ex-ante efficiency in production, the government needs to rescue a greater fraction of entrepreneurs. Intuition is the following. When the bursting probability of bubbles is higher, risk-averse L-entrepreneurs do not want to invest a lot of their savings in risky bubble assets. They end up with investing more part of their savings in their own projects with low returns for risk-hedge. In this situation, in order to crowd L-projects out completely, the government needs to rescue a greater fraction of entrepreneurs.

From Proposition, we learn that when $\alpha_1^H < \alpha^H < \alpha_2^H$, the evolution of aggregate capital stock follows

$$K_{t+1} = \begin{cases} \frac{\left(1+\frac{\alpha^H-\alpha^L}{\alpha^L-\theta\alpha^H}p\right)\beta\alpha^L - \beta\alpha^L(1-p)}{1-\delta\beta(1-p)}\sigma K_t^\sigma & \text{if } 0 \leq \lambda \leq \lambda^*, \\ \\ \alpha^H\frac{\beta[1-\delta(1-p)]+(1-\beta)\theta}{1-\delta\beta(1-p)}\sigma K_t^\sigma & \text{if } \lambda^* \leq \lambda \leq 1. \end{cases}$$

Given a same initial K_0, K_t is an increasing function of λ in $\lambda \in [0, \lambda^*)$, and a decreasing function of λ in $\lambda \in [\lambda^*, 1]$. In other words, expansion in the bailout initially increases ex-ante efficiency in production and then decreases it. W summarize this result in the following Proposition.

Proposition 9 *In $\alpha_1^H < \alpha^H < \alpha_2^H$, Y_t is an increasing function of λ in $\lambda \in [0, \lambda^*)$, and a decreasing function of λ in $\lambda \in [\lambda^*, 1]$ in the bubble economy.*

Intuitively, when government bailout is expected ex-ante, L-entrepreneurs are willing to buy more risky bubble assets instead of investing in their own L-projects. L-projects are crowded out. On the other hand, together with more demand for bubble assets, bubble prices increase, which improves the net worth of H-entrepreneurs, thereby crowding H-projects in. Thus ex-ante efficiency in production improves in $\lambda \in [0, \lambda^*)$ as λ rises. When λ becomes equal to λ^*, all L-projects are completely crowded out. That is, ex-ante efficiency in production is maximized. If the government expands the bailout beyond λ^*, even H-projects starts being crowded out. Too much bailout generates inefficiency in production. Thus, production decreases in $\lambda \in [\lambda^*, 1]$ as λ rises. In this region, choosing λ^* is desirable from the perspective of ex-ante production efficiency. Figure shows the relationship between ex-ante efficiency in production (Y_t in $1 \leq t \leq \tau$) and λ in $\alpha_1^H < \alpha^H < \alpha_2^H$.

3.2 Effects on Boom-Bust Cycles

In this subsection, we discuss how the anticipated bailout affects macro-dynamics before and after the bubbles' bursts.

Suppose that at date 0 (initial period), bubbles occur. Here at date -1, the economy is assumed to be in the steady-state of the bubbleless economy. The line with $\lambda = 0$ is impulse response of the economy when no-bailout is expected. The line with $\lambda = \lambda^*$ is impulse response when the bailout is expected with probability λ^*. These charts in Figure represent qualitative solutions. The Figure shows that boom-bust cycles become larger in $\lambda = \lambda^*$. Intuition is the following. When government bailout is expected at date 0, L-entrepreneurs are willing to buy more risky bubble assets. Bubble prices jump up in the initial period. Because of this increase, the net worth of H-entrepreneurs improves and their investments jump up too, while the share of L-investments becomes zero, $s_0 = 0$. That is, production efficiency improves. As a result, output as well as wage rate also jumps up in the next period (date 1). Consumption of the entrepreneurs jumps up in the initial period together with the increase in aggregate wealth of the entrepreneurs. All macroeconomic variables continue to increase until bubbles collapse. Since this is an asset pricing model, when output is expected to increase in the future, that is reflected in the bubble prices of the initial period. Thus in the initial period, the bubble prices jump up largely, which in turn increases the net worth of H-entrepreneurs and their investments substantially.

These charts show interesting features. When we think about the aim of the bailout, the aim is to stabilize the economy during bubbles' collapsing. However, once the bailout is anticipated ex-ante, it ends up with destabilizing the economy and results in requiring large amounts of public funds in the collapsing of bubbles. We should mention that this instability comes from improvement in resource allocations. Thus, there is a trade-off between improvement in resource allocations and stability of the economy.

Here let us add a few remarks concerning the case of $\alpha^H \geq \alpha_2^H$. In this case, the anticipated bailout reduces H-projects, thereby dampening output booms.

4 Optimal Bailout Policy for Tax Payers

In this section, we derive an optimal bailout policy for tax payers.[18]

4.1 Optimal Bailout Policy from Ex-post Perspective

Let us first examine whether bailout is desirable for tax payers from ex-post perspective. Suppose that at date τ, bubbles collapse. After date τ, the economy enters the bubbleless economy. The question here is should the government bail entrepreneurs out at date τ? The answer depends upon cost and benefit of the bailout. The cost is that when bubbles collapse, workers have to pay the lump sum tax to rescue entrepreneurs. This lowers their welfare. On the other hand, the benefit is that thanks to the bailout, the net worth of the rescued entrepreneurs improves and their borrowing constraint becomes relaxed. As a result, aggregate investments expand at date τ, which has persistent positive effects on capital stock and wage rate after date $\tau+1$. This improves workers' welfare. Whether tax payers' welfare improves from ex-post perspective depends upon which one of these effects dominates.

The value function for tax payers at date τ denoted by $V_\tau^{ex-post}$ can be written as[19]

$$\begin{aligned} V_\tau^{ex-post} = & \log\left[1-\sigma-\lambda\frac{\beta\phi(\lambda)}{1-\beta\phi(\lambda)}\sigma\right] + \frac{\beta\sigma}{1-\beta\sigma}\log\left[1+\lambda\frac{\beta\phi(\lambda)}{1-\beta\phi(\lambda)}\right] \\ & + \frac{\beta\sigma}{1-\beta\sigma}\frac{1}{1-\beta}\log D + \frac{\beta}{1-\beta}\log(1-\sigma) + \frac{\sigma}{1-\beta\sigma}\log K_\tau, \quad (39) \end{aligned}$$

[18] We assume that workers (tax payers) are median voters. Thus, the objective of the government is to maximize their welfare.

[19] See Appendix for derivation of the value function.

with

$$D = \begin{cases} \left(1 + \frac{\alpha^H - \alpha^L}{\alpha^L - \theta\alpha^H} p\right) \beta\alpha^L\sigma & \text{if } \alpha_1^H < \alpha^H \leq \frac{\alpha^L}{\theta}(1-p), \\ \\ \alpha^H \beta\sigma & \text{if } \alpha^H \geq \frac{\alpha^L}{\theta}(1-p). \end{cases}$$

The first term in the first line in equation (39) captures the cost of the bailout while the second term captures the benefit. From (39), we can understand how expanding bailout affects ex-post tax payers' welfare.

By differentiating equation (39) with respect to λ, we obtain

$$\begin{aligned} \frac{dV_\tau^{ex-post}}{d\lambda} &= \frac{-\sigma}{1-\sigma-\lambda\frac{\beta\phi(\lambda)}{1-\beta\phi(\lambda)}\sigma} \left\{ \frac{\beta\phi(\lambda)}{1-\beta\phi(\lambda)} + \frac{\lambda\beta}{[1-\beta\phi(\lambda)]^2} \frac{d\phi(\lambda)}{d\lambda} \right\} \\ &+ \frac{\beta\sigma}{1-\beta\sigma} \frac{1}{1+\lambda\frac{\beta\phi(\lambda)}{1-\beta\phi(\lambda)}} \left\{ \frac{\beta\phi(\lambda)}{1-\beta\phi(\lambda)} + \frac{\lambda\beta}{[1-\beta\phi(\lambda)]^2} \frac{d\phi(\lambda)}{d\lambda} \right\} < 0 \quad (40) \end{aligned}$$

The first line captures the marginal cost associated with the expansion of the bailout, while the second line captures the marginal benefit. Equation (40) says that the marginal cost dominates the marginal benefit. Expanding bailout reduces tax payers' welfare monotonically. Thus from ex-post perspective, no-bailout is optimal for tax payers. We summarize this result in the following Proposition.

Proposition 10 *From ex-post perspective, $\lambda = 0$ is optimal, i.e., no-bailout is optimal for tax payers.*

So far we have examined the welfare effects when the bailout is anticipated. Here let us discuss welfare impacts when the bailout is not anticipated. In this case, ϕ becomes independent of λ. Then (40) becomes

$$\frac{dV_\tau^{ex-post}}{d\lambda} = \frac{-\sigma}{1-\sigma-\lambda\frac{\beta\phi}{1-\beta\phi}\sigma} \frac{\beta\phi}{1-\beta\phi} + \frac{\beta\sigma}{1-\beta\sigma} \frac{1}{1+\lambda\frac{\beta\phi}{1-\beta\phi}} \frac{\beta\phi}{1-\beta\phi} < 0.$$

Expansion in the bailout reduces welfare monotonically. We summarize this result in the following Proposition.

Proposition 11 *Suppose that the bailout policy is not anticipated. From ex-post perspective, $\lambda = 0$ is optimal, i.e., no-bailout is optimal for tax payers.*

4.2 Optimal Bailout Policy from Ex-ante Perspective

Now we are in a position to derive an optimal bailout policy from ex-ante perspective. When the bailout is anticipated, ex-ante efficiency in production change, which in turn affects welfare before the bubbles' collapsing. When we compute ex-ante welfare, we need to take this effect into consideration.

The value function for tax payers in an initial period (at date 0) denoted by $V_0^{ex-ante}$ can be written as[20]

$$V_0^{ex-ante} = \frac{1}{1-\beta\pi}\frac{\beta\sigma}{1-\beta\sigma}\log H(\lambda) + \frac{\beta(1-\pi)}{1-\beta\pi}W(\lambda) + \frac{1}{1-\beta\pi}\log(1-\sigma) + \frac{\sigma}{1-\beta\sigma}\log K_0, \quad (41)$$

with

$$W(\lambda) = \log\left[(1-\sigma) - \lambda\frac{\beta\phi(\lambda)}{1-\beta\phi(\lambda)}\sigma\right] + \frac{\beta\sigma}{1-\beta\sigma}\log\left[1+\lambda\frac{\beta\phi(\lambda)}{1-\beta\phi(\lambda)}\right] + \frac{\beta\sigma}{1-\beta\sigma}\frac{1}{1-\beta}\log D + \frac{\beta}{1-\beta}\log(1-\sigma),$$

If $\alpha_1^H < \alpha^H < \alpha_2^H$,

$$H(\lambda) = \begin{cases} \frac{\left(1+\frac{\alpha^H-\alpha^L}{\alpha^L-\theta\alpha^H}p\right)\beta\alpha^L - \beta\alpha^L(1-p)}{1-\delta\beta(1-p)}\sigma & \text{if } 0 \leq \lambda < \lambda^*, \\ \alpha^H\frac{\beta[1-\delta(1-p)]+(1-\beta)\theta}{1-\delta\beta(1-p)}\sigma & \text{if } \lambda^* \leq \lambda \leq 1. \end{cases} \quad (42)$$

If $\alpha^H \geq \alpha_2^H$,

$$H(\lambda) = \alpha^H\frac{\beta\left[1-\delta(1-p)\right]+(1-\beta)\theta}{1-\delta\beta(1-p)}\sigma.$$

The first term in the first line in equation (41) represents the effects of the bailout on welfare before the bubbles' collapsing. $H(\lambda)$ captures welfare impacts through production. The second term represents the effects on ex-post welfare from ex-ante perspective.

Let us first derive the optimal bailout in the case of $\alpha^H \geq \alpha_2^H$. By differ-

[20] See Appendix for derivation of the value function.

entiating (41) with respect to λ, we obtain

$$\frac{dV_0^{ex-ante}}{d\lambda} = \frac{1}{1-\beta\pi}\frac{\beta\sigma}{1-\beta\sigma}\frac{d\log H(\lambda)}{d\lambda} + \frac{\beta(1-\pi)}{1-\beta\pi}\frac{dW(\lambda)}{d\lambda} < 0. \tag{43}$$

The first term represents the marginal effect on welfare before the bursts. The second one represents the marginal effect on ex-post welfare. When productivity of the economy is relatively high ($\alpha^H \geq \alpha_2^H$), a rise in λ crowds H-projects out and reduces wage rate, thereby decreasing welfare before the bursts monotonically. Thus the sign of the first term is negative. Moreover, as we showed in Propostion, ex-post welfare decreases by the expansion of the bailout. That is, the second term is negative. Taken together, expanding bailout reduces ex-ante welfare for tax payers monotonically. Thus, no-bailout is optimal. Figure shows the relationship between $V_0^{ex-ante}$ and λ in $\alpha^H \geq \alpha_2^H$. We summarize this result in the following Proposition.

Proposition 12 *Let λ^{**} be the value of λ which maximizes ex-ante welfare for tax payers. $\lambda^{**} = 0$ in $\alpha^H \geq \alpha_2^H$. That is, no-bailout is optimal.*

Next let us derive the optimal bailout in the case of $\alpha_1^H < \alpha^H < \alpha_2^H$. From (42), we see

$$\begin{aligned} \frac{d\log H(\lambda)}{d\lambda} &> 0 \text{ in } \lambda \in [0, \lambda^*)\,, \\ \frac{d\log H(\lambda)}{d\lambda} &< 0 \text{ in } \lambda \in [\lambda^*, 1]\,. \end{aligned}$$

That is, expansion in the bailout initially increases ex-ante efficiency in production and then decreases it.

In $\lambda \in [0, \lambda^*)$, there are two competing effects on ex-ante welfare. One is welfare-enhancing effect. The other is welfare-reducing effect. On one hand, increasing bailout crowds L-projects out and crowds H-projects in, thereby increasing ex-ante efficiency in production and wage rate. This improves welfare before the bubbles' collapsing. On the other hand, the increase in the bailout reduces ex-post welfare. Thus, in this region, whether ex-ante welfare increases or decreases depends upon which one of these effects dominates.

In $\lambda \in [\lambda^*, 1]$, expanding bailout decreases ex-ante efficiency in production. Therefore, it reduces welfare before and after the bubbles' collapsing. Thus we get

$$\frac{dV_0^{ex-ante}}{d\lambda} < 0 \text{ in } \lambda \in [\lambda^*, 1]. \tag{44}$$

This means that too much bailout reduces ex-ante welfare.

Here let us assume

$$(1-p)(1-\beta\phi)(1-\sigma) > \phi(1-\beta)[1-\pi\beta(1-p)], \tag{A2}$$

with

$$\phi = \frac{\pi - \frac{1-\pi\beta(1-p)}{\left[1+(\frac{\alpha^H-\alpha^L}{\alpha^L-\theta\alpha^H})p\right]\beta-\beta(1-p)}}{1-\frac{1-\pi\beta(1-p)}{\left[1+(\frac{\alpha^H-\alpha^L}{\alpha^L-\theta\alpha^H})p\right]\beta-\beta(1-p)}}(1-p).$$

Since $1-p > \phi$ and $1-\beta\phi > 1-\beta$, this assumption is more likely to be satisfied if σ is small enough. This assumption ensures that the slope of $V_0^{ex-ante}$ evaluated at $\lambda = 0$ is positive.[21]

From (43) and (44) together with the assumption (A2), optimal bailout is

$$\lambda^{**} \in (0, \lambda^*].$$

In other words, partial bailout is optimal. We summarize this result in the following Proposition.

Proposition 13 *If (A2) holds, then $\lambda^{**} \in (0, \lambda^*]$ in $\alpha_1^H < \alpha^H < \alpha_2^H$. Partial bailout is optimal for tax payers from ex-ante perspective.*

This proposition means that no-bailout and rescuing all are not optimal neither. When $\lambda = 0$, the government rescues too less. When $\lambda = 1$, the government rescues too much.

Here let us show you numerical examples. Figure are numerical examples showing the relationship between ex-ante welfare for tax payers and λ. Parameter values are shown in Table. The difference between each of four cases lies in the bursting probability of bubbles. The lower π is, the higher the bursting probability is.

[21] Smaller σ means that the share of the wage income over aggregate output is larger relative to the share of the total bailout money. As a result, the marginal cost by expansion of the bailout becomes small and the marginal benefit is more likely to dominate the marginal cost. This is the reason the slope of $V_0^{ex-ante}$ evaluated at $\lambda = 0$ is positive if σ is small enough.

These Figures show very interesting features. In the case of riskier bubbles, ex-ante welfare is maximized below the value of λ which maximizes ex-ante efficiency in production, i.e., $\lambda^{**} < \lambda^{*}$. This suggests that if the government wants to maximize ex-ante welfare for tax payers, it has to give up some efficiency in production, allowing L-entreprenurs to produce. Intuition is the following. In the case of riskier bubbles, L-entrepreneurs do not want to invest a lot of their savings in risky bubble assets. They end up with investing more part of their savings in their own L-projects for risk-hedge. In this situation, in order to crowd L-projects out completely, the government needs to rescue a greater fraction of entrepreneurs (remember that λ^{*} is an decreasing function of π.), which directly increases bailout money. Moreover, when entrepreneurs anticipate that a greater fraction of them is rescued, this generates a large increase in bubble prices and creates large size bubbles. These two effects produce large costs for tax payers, requiring large amounts of public funds when bubbles collapse. Thus ex-ante welfare starts decreasing when $\lambda > \lambda^{**}$. In $\lambda \in [0, \lambda^{*})$, ex-ante welfare increases because the welfare-enhancing effect dominates the welfare-reducing one. On the other hand, in the case of safer bubbles, ex-ante efficiency in production is achieved by rescuing a smaller fraction of entrepreneurs. This means that total bailout money does not become too large, requiring small amount of public funds during the bubbles' collapsing. The welfare-enhancing effect dominates the welfare-reducing one until $\lambda = \lambda^{*}$. Thus ex-ante welfare is maximized at $\lambda = \lambda^{*}$.

Our results show that the government faces dilemma. When financial markets are imperfect, enough resources cannot be transferred to the productive sector. The presence of asset bubbles and government bailout improve this situation, helping to smooth resource allocations from L-entrepreneurs to H-entrepreneurs. However, if the government tries to improve resource allocations by expanding bailout, on one hand, production efficiency enhances, which improves tax payers' welfare. But on the other hand, bailout money increases, which lowers their welfare. In the case of riskier bubbles, the latter effect becomes too large. Thus, from tax payers' point of view, the government cannot but give up some efficiency in resource allocations.

Moreover, our results also have interesting implications for boom-bust cycles. Figure compares boom-bust cycles in three cases, $\lambda = 0$, $\lambda = \lambda^{*}$, and $\lambda = \lambda^{**}$. We consider the case where $\lambda^{**} < \lambda^{*}$. The charts in the Figure show that boom-bust cycles become milder, because production is not efficient. This suggests that in order to maximize ex-ante welfare for tax payers, the government needs to mitigate boom-bust cycles.

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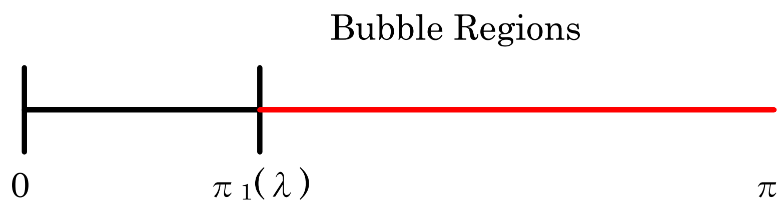
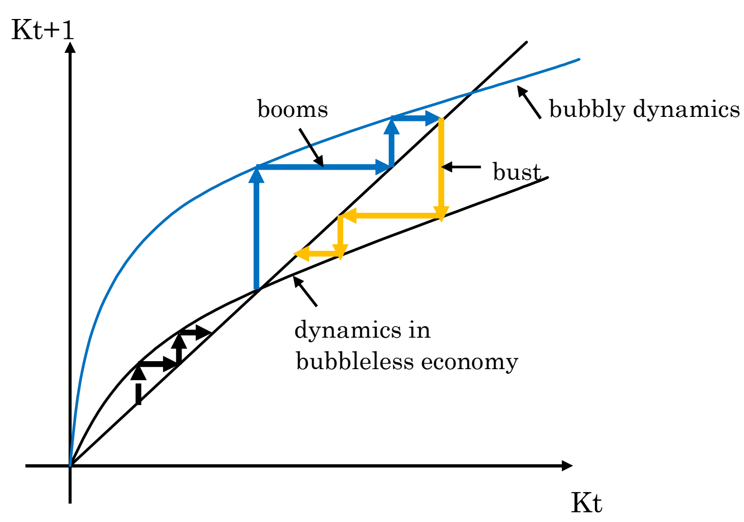
Given θ and α^H,



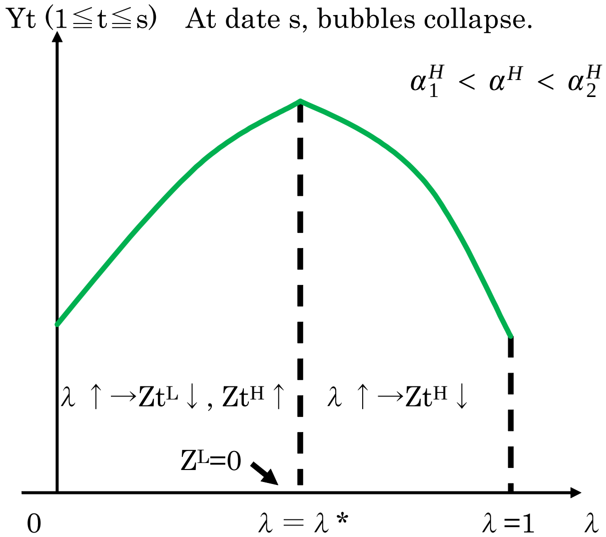
π_1 is a decreasing function of λ.

When bailout is expected, even riskier bubbles can occur.

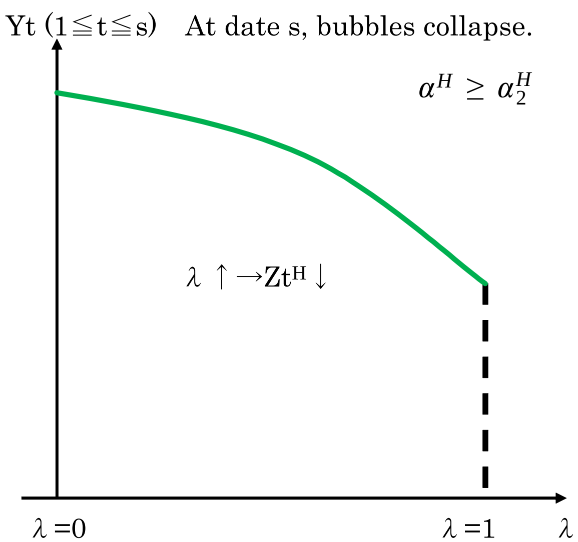
When α^H is relatively low



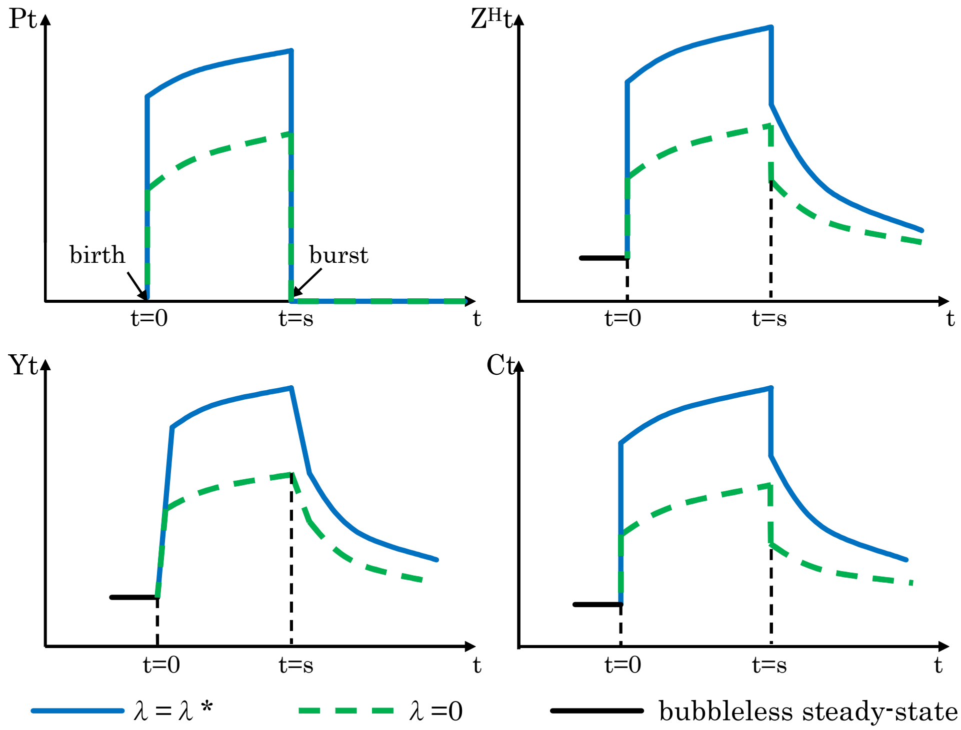
Dynamic Effect of Stochastic Bubbles on the saddle path

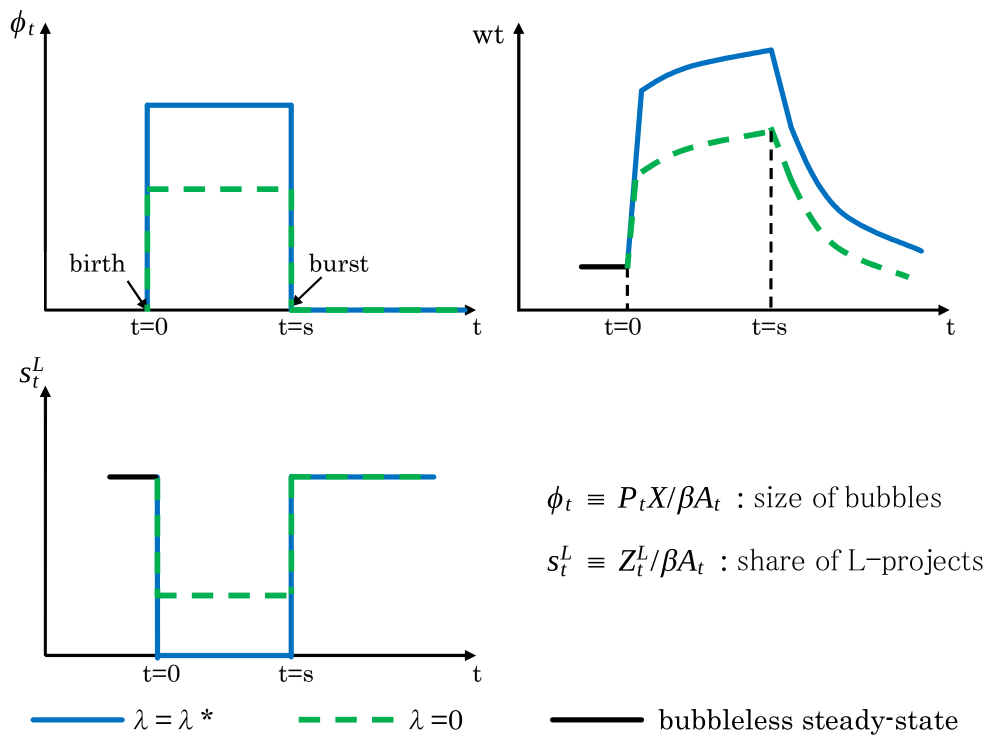


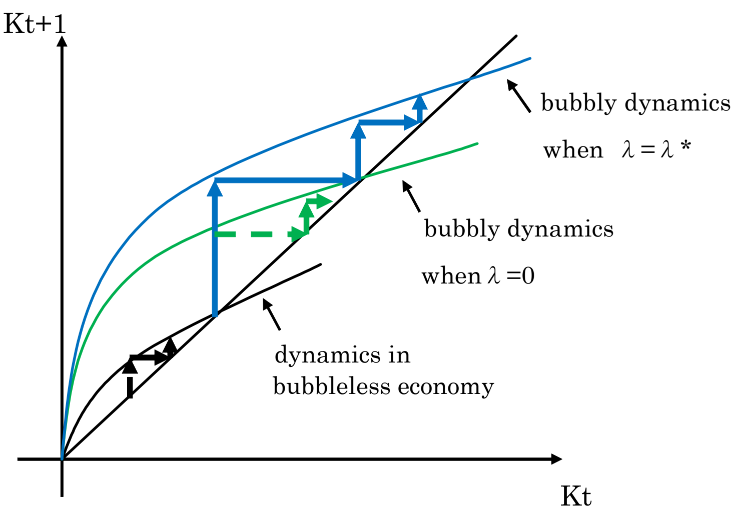
Value of λ where ex-ante efficiency is maximized.



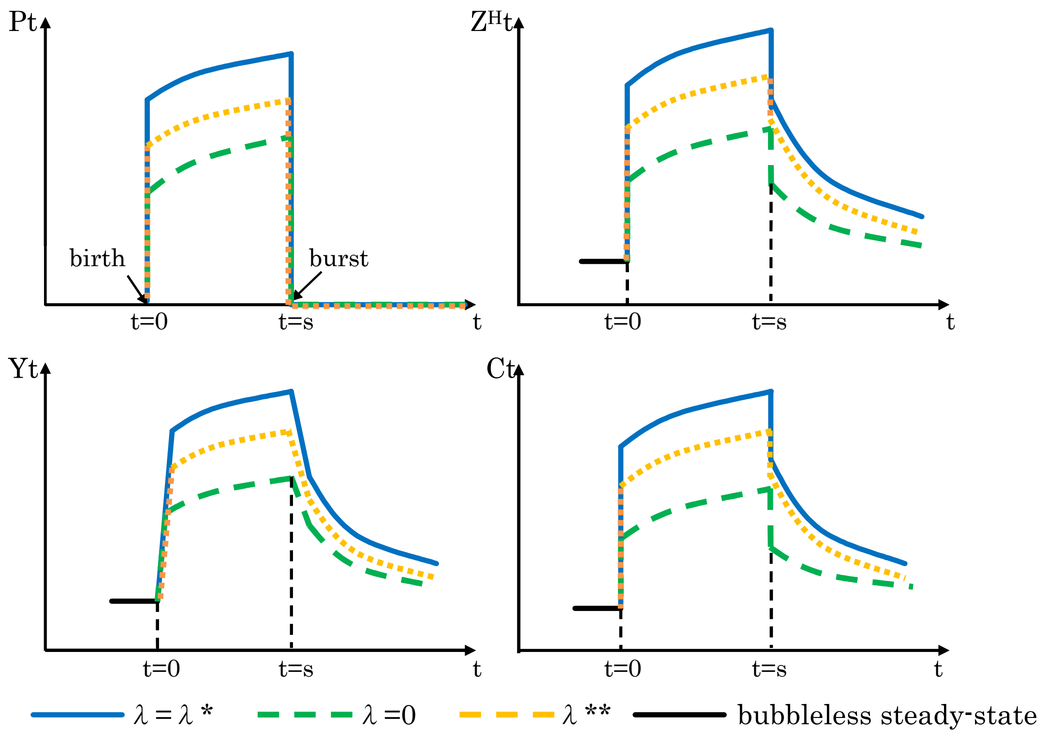
Value of λ where ex-ante efficiency is maximized.

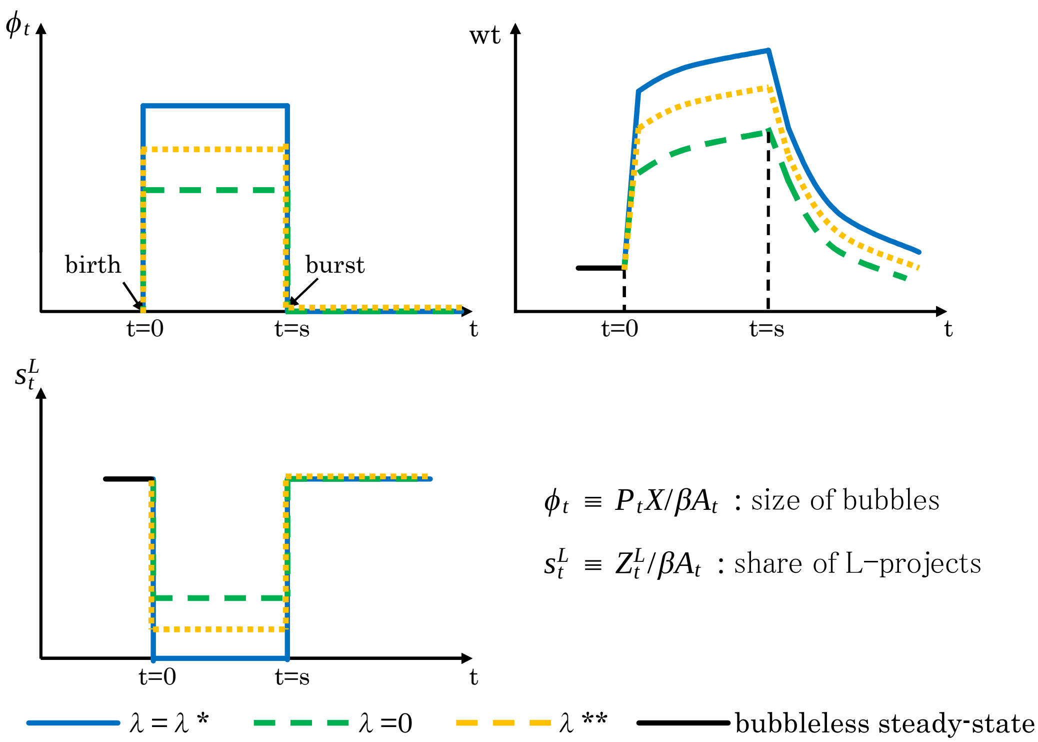

Pt
birth
burst
t=0
t=s
t
Z^Ht
t=0
t=s
t
Yt
t=0
t=s
t
Ct
t=0
t=s
t
λ = λ *
λ =0
bubbleless steady-state



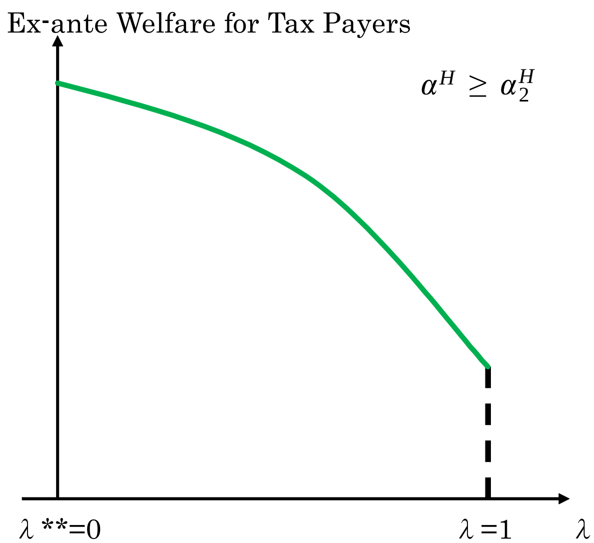


Anticipated Bailout and Bubbly Dynamics


Pt
birth
burst
t=0
t=s
t
Z^H t
t=0
t=s
t
Yt
t=0
t=s
t
Ct
t=0
t=s
t
λ = λ *
λ =0
λ **
bubbleless steady-state

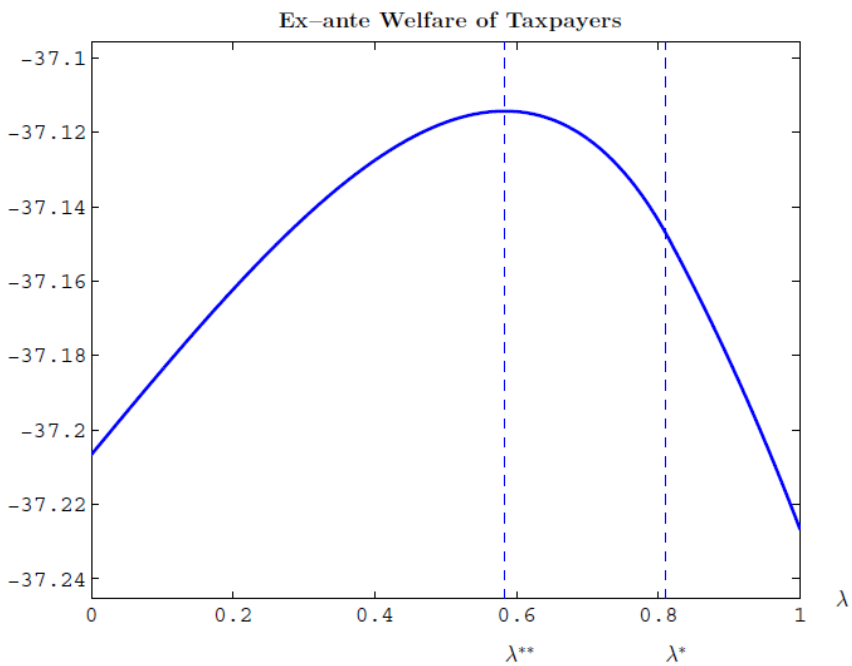


ϕ_t
birth
burst
t=0
t=s
t
wt
t=0
t=s
t
s_t^L
t=0
t=s
t
$\phi_t \equiv P_t X/\beta A_t$: size of bubbles
$s_t^L \equiv Z_t^L/\beta A_t$: share of L−projects
$\lambda = \lambda$ *
λ =0
λ **
bubbleless steady-state

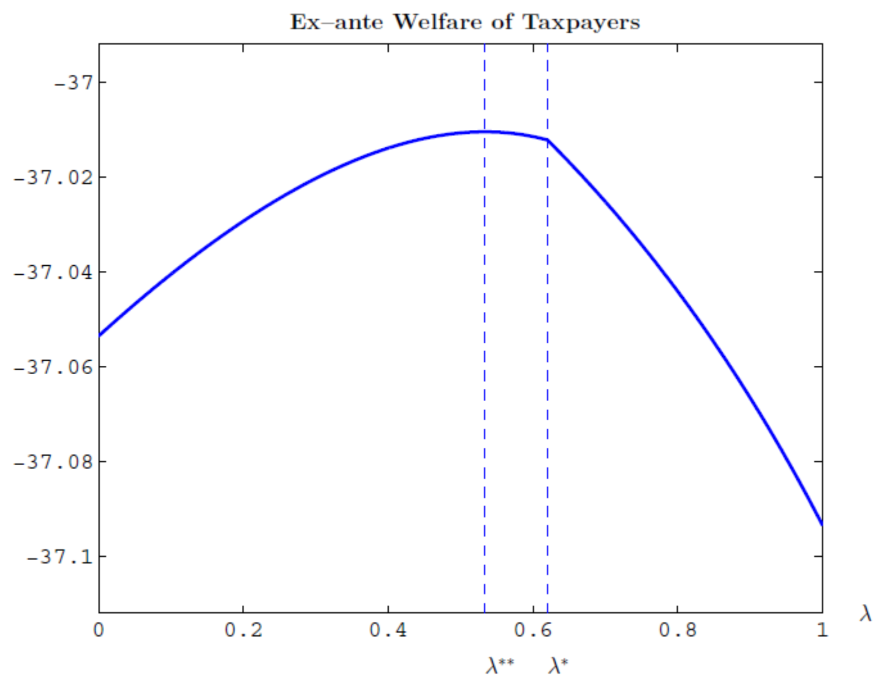


Value of λ where ex-ante welfare for tax payers is maximized.

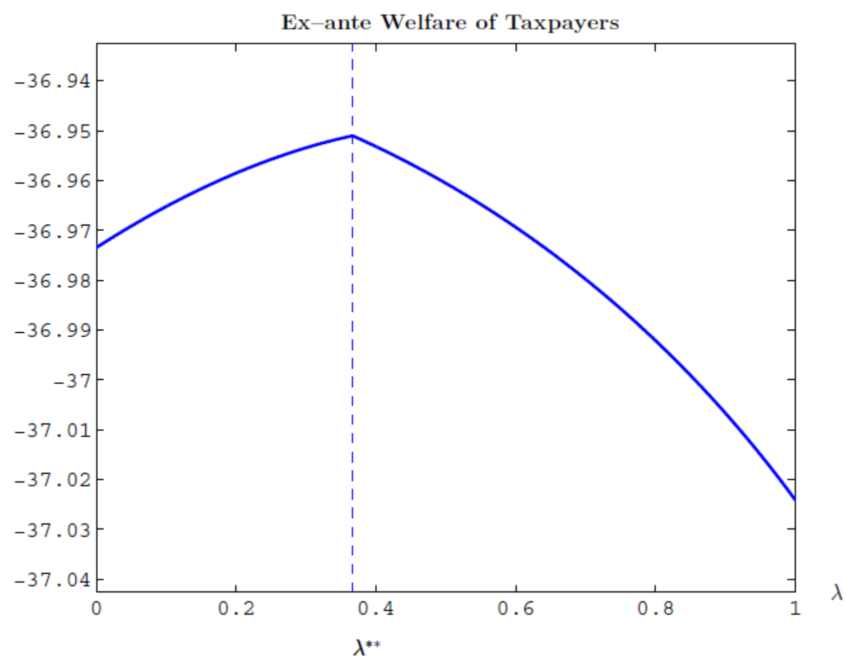
```
benchmark
p=0.3; % propability of H type
alphaH=1.1; % productivity of H
alphaL=1; % productivity of L > theta*alphaH
beta=0.98; % discount factor
sigma=0.25; % capital share
pi=0.99; % probability for bubble coutinue
theta=0.1; % collateral ratio
```



```
case 1
p=0.3; % propability of H type
alphaH=1.1; % productivity of H
alphaL=1; % productivity of L > theta*alphaH
beta=0.98; % discount factor
sigma=0.25; % capital share
pi=0.995; % probability for bubble coutinue
theta=0.1; % collateral ratio
```



```
case 2
p=0.3; % propability of H type
alphaH=1.1; % productivity of H
alphaL=1; % productivity of L > theta*alphaH
beta=0.98; % discount factor
sigma=0.25; % capital share
pi=0.997; % probability for bubble coutinue
theta=0.1; % collateral ratio
```



```
case 3
p=0.3; % propability of H type
alphaH=1.1; % productivity of H
alphaL=1; % productivity of L > theta*alphaH
beta=0.98; % discount factor
sigma=0.25; % capital share
pi=0.998; % probability for bubble coutinue
theta=0.1; % collateral ratio
```