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Fuzzy Logic-based Portfolio Selection with Particle Filtering and Anomaly Detection *

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Abstract

This paper proposes a new knowledge-based system (KBS) featuring fuzzy logic (FL) with particle filtering and anomaly detection to create high-performance investment portfolios. In particular, our FL system selects a portfolio with fine risk-return profiles from a number of candidates by integrating multilateral performance measures. The candidates consist of various portfolios based on multiple time-series models estimated by a particle filter with anomaly detectors. In an out-of-sample numerical experiment with a dataset of international financial assets, we demonstrate our KBS successfully generates a series of selected portfolios with satisfactory investment records.

Keywords: knowledge-based system, expert system, fuzzy logic, investment portfolio, particle filtering, anomaly detection

*forthcoming in "Knowledge-Based Systems"
1 Introduction

It is well-recognized both in academia and industry that financial markets are highly complex and non-linear systems with large noise, which are affected by economic, political, geopolitical and psychological factors. Therefore, in the analysis of financial investment problems, machine learning (ML) techniques based on sophisticated computer science are widely used since they are effective to deal with non-linearity and uncertainty (e.g., [1]).

Fuzzy logic (FL) is considered to be one of the ML techniques, which is applied with great success in financial investment problems. Especially, fuzzy set theory, introduced by Zadeh [2], is utilized in portfolio optimization problems since it enables to represent imperfect knowledge or ambiguity for the future asset return. For example, although a mean-variance (MV) portfolio [3] has been one of the most famous strategies, there is a well-known serious problem that the direct MV optimization amplifies the effects of estimation errors (e.g., [4]). Consequently, many researchers introduce the fuzziness in portfolio optimization problems from various perspectives (e.g., [5–22]).

On the other hand, fuzzy logic systems (e.g., [23, 24]) are effective to construct knowledge-based systems (KBS) for trading strategies with technical and fundamental analysis (e.g., [25–32]). In trading practice, domain expert knowledge is often expressed in the linguistic form, which can be incorporated into trading strategies through IF-THEN rules of FL systems.

This paper proposes a new knowledge-based system (KBS), particularly expert system (ES) featuring fuzzy logic (FL) to create a high-performing portfolio. Specifically, our ES consists of three stages: estimation, portfolio simulation and FL-based selection. That is, we first estimate expected return and volatility with several time-series models, and then calculate MV optimal portfolio weights based on these predictors with several types of anomaly detectors and different levels of risk-averseness. Finally, our FL system, by combining multiple investment criteria, evaluates the historical performances of each simulated MV portfolio and selects the best one. Here, we employ a variety of performance measures which are practically well-known to investment experts such as hedge fund managers.

Although each previous research has presented a state-of-the-art investment scheme, it may have some difficulties in adapting to frequent or/and drastic changes in market conditions. Differently, our approach does not rely on a single scheme, but selects the most appropriate one under the current market environment among various options. Consequently, our proposed system creates high performance over a period that includes different market conditions.

In the estimation step, we introduce state space models to obtain the estimates of expected return and volatility for MV portfolio construction. State space models are commonly used in various fields to represent the dynamic dependence between the latent state and observed variables. Their dynamics are described as stochastic processes called system and observation models. Moreover, applying filtering methods, we can partially observe latent state variables with noises through the observation model.

In the current work, we regard expected return and volatility as state variables, which are observed with noise as asset return. For model estimation, we resort to a particle filtering (PF) method applicable to non-linear and non-Gaussian models (e.g., [33, 34]). PF is an on-line estimation algorithm to a time-series data under the state-space representation of models, which takes much less computational time than repeated implementation of off-line algorithms with sliding windows.

In particular, we assume an exponential moving average (EMA) and generalized EMA as system models. The former is often used both in investment practice and previous ML researches (e.g., [35–38]), and the latter is introduced in Nakano, Takahashi, and Takahashi [39] to obtain marginally better predictions based on EMA.
In addition, we exploit three anomaly detection (AD) methods to refine investment universe ([39]). Since we can obtain the model likelihoods or the distributions of state variables for each time step, PF is easily combined with on-line detection schemes. In our investment problem, realized asset returns sometimes largely deviate from the models, whence it is inappropriate to implement predictions. Hence, by excluding the assets for which anomalies are detected from investment universe (i.e. trading assets), we can enhance portfolio performance.

Our investment universe consists of international equities, REITs and bonds with cash. Besides, we employ various investment criteria, i.e., compound return (CR), standard deviation (SD), downside deviation (DD), maximum drawdown (MDD), Sharpe ratio (ShR), Sortino ratio (SoR) and Sterling ratio (StR), which enable multilateral assessment.

Lastly, we briefly summarize the motivation, contribution and implication of our work.

(i) Motivation:

The motivation of our work is how to select the best investment decision from numerous possible options under the situation that we cannot know it in advance. For instance, in the current work, we employ a mean-variance portfolio among various investment strategies. Then, we must estimate means and variances of asset returns, which involves choosing a statistical model and its parameters from rich variations. Besides, it is necessary to decide investment assets before portfolio construction. In addition, we cannot always determine a crisp parameter value (risk aversion parameter) in mean-variance analysis, which further diversifies the possible options.

(ii) Contribution:

Our contribution is to propose an effective solution to this problem by introducing a fuzzy system at the final stage of an investment process, which is a new perspective in FL-based portfolio construction. In particular, we develop an ES featuring FL system which is able to integrate each investor’s performance measures to select the best investment decision from various possible options. Importantly, since performance measures are the most essential and critical investment objectives for investors, this specification is expected to directly link to high performance.

(iii) Implication:

The applicability of our FL system is quite broad. Namely, we use a FL system at the last stage of investment decision processes, i.e. performance evaluation, which is the reason why our approach is extensively applicable. Especially, although the current paper focuses on a mean-variance portfolio, our FL system can incorporate any strategies into the possible options. For instance, we are able to employ the investment strategies appearing in the previous works, that is, fuzzy rule-based technical/fundamental trading (Section 2.2) or extension of Markowitz model with fuzzy logic (Section 2.3). We will show an example in Section 4.3.

The remainder of this paper is organized as follows. Section 2 summarizes related works. Section 3 presents composition of our ES: PF-based estimation with AD scheme, MV portfolio simulation and FL-based selection. Section 4 shows the results of out-of-sample numerical experiments. Finally, Section 5 concludes.

2 Related works

In this section, we shortly review previous works for application of fuzzy set theory to three financial investment topics, that is, time-series prediction, technical/fundamental trading and modern portfolio theory.
2.1 Application to financial time-series prediction

FL is frequently used to develop KBSs for financial time-series prediction due to its general applicability. For example, Korol [40] builds a fuzzy system for forecasting exchange rates based on various economic factors such as GDP and inflation, which achieves lower mean absolute percentage error than other statistical models and artificial neural network approaches.

Cai, Zhang, Zheng, and Leung [41] develop a new fuzzy time series forecasting model. Particularly, they exploit ant colony optimization to promote the forecasting performance. Further, the auto-regression method is adopted to make better use of historical information. The new model combined with these techniques is shown to be more effective than existing models through the application to Taiwan capitalization weighted stock index.

Hadavandi, Shavandi, and Ghanbari [42] construct a stock price forecasting expert system based on genetic fuzzy systems and artificial neural networks. More precisely, after step-wise regression analysis determines factors having most influence on stock prices, they divide raw data into multiple clusters by self-organizing map neural network. Then, each cluster is fed into genetic fuzzy systems with the ability of rule base extraction and database tuning.

Singh and Borah [43] develop a new high-order fuzzy time-series model, where artificial neural network based architecture is exploited for defuzzification. In particular, they discuss the importance on "lengths of intervals" for time-series and introduce a repartitioning discretization approach. Their methodology is validated with daily temperature data and stock price data.

2.2 Application to technical/fundamental trading

Since fuzzy IF-THEN rules are helpful to quantitatively express expert knowledge for technical and fundamental trading, various researchers have applied FL systems to this field. As a pioneering work, in 1991, Kosaka, Mizuno, Sasaki, Someya, and Hamada [27] propose a framework for fuzzy rule-based technical trading, which is illustrated by single stock data.


Lam [28] proposes a fuzzy expert system for stock trading, which is optimized by genetic algorithm, where inputs are twelve technical indicators and outputs are buy/sell signals. The numerical test with Hong-Kong stock market data illustrates that the system performs better than buy-and-hold strategies while individual technical indicator seems unreliable itself.

Dourra and Siy [25] present a trading system using technical analysis and fuzzy logic, which features a convergence module mapping technical indicators into new inputs for a Mamdani type fuzzy system. Their trading system is applied to stock price data of four companies with investment strategies based on two types of buy/sell trigger identification, which outperforms buy-and-hold of S&P 500.

Dymova, Sevastianov, and Bartosiewicz [26] integrate fuzzy logic and methods of the Dempster-Shafer theory (the so-called rule-base evidential reasoning) to build expert systems for investment with technical indicators as inputs and buy/hold/sell signals as outputs. The system is tested by a Warsaw stock index futures contract and performs well.

Lincy and John [29] construct a fuzzy inference system for daily stock trading, in which inputs are mean and standard deviation of historical returns as well as earnings per share (EPS) while outputs are buy, hold and sell signals. Their experiment based on dataset of 25 stocks in NASDAQ stock exchange shows that the ES performs better than simple application of famous technical indicators and several existing models.

In Yunusoglu and Selim [31], the proposed ES provides an optimal portfolio in terms of rating
point which is created by integrating technical and fundamental evaluation based on various fuzzy rules. The system incorporates investor’s risk preference through the predetermined weight constraints. Using the data of 61 stocks in Istanbul Stock Exchange National-100 Index during 2002-2010, they demonstrate its validity by comparison with the benchmark index.

Chourmouziadis and Chatzoglou [32] design a short-term trading ES which tells how much amount should be invested in a risky asset based on fuzzy rules. One of its important features is the use of rare technical indicators, i.e. Parabolic SAR and GANN-HiLo. In the numerical example with the daily data of Athens Stock Exchange General Index over a period of more than 15 years, the strategy with their ES is shown to be superior to the buy-and-hold strategy.

Finally, we remark that most of the works in this subsection are also related to portfolio construction.

2.3 Extension of Markowitz model with fuzzy logic

The concept of fuzziness including vagueness of human reasoning is also applied to the modern portfolio theory, introduced by Markowitz [3]. As a pioneering work, in 1997, Watada [5] introduces vague targets for portfolio expected return and risk in the Markowitz model. As well, in 1998, Ramaswamy [6] introduces fuzzy decision theory (Bellman and Zadeh [44]) into portfolio selection to represent the situation that an investor allows target rate of return is not necessarily attained if his/her market scenario turns out to be incorrect. Moreover, Fang, Lai, and Wang [9] consider a fuzzy portfolio rebalancing model with transaction costs, where the required levels of portfolio liquidity in addition to return and risk are regarded as fuzzy numbers.

At the same time, asset returns are also regarded as fuzzy numbers in portfolio selection problems. For instance, Tanaka, Guo, and Türksen [7] assume that returns of securities follow possibility distributions (Zadeh [45]) in a framework of mean-variance analysis.

In these formulation, various risk measures are studied and advanced to express different investors’ risk preferences which is not necessarily represented by existing approaches. Huang [10] proposes a fuzzy portfolio selection model using a new definition of investment risk based on credibility theory, which is solved by genetic algorithm. Also, Liu [15] considers a fuzzy portfolio optimization problem with mean-absolute deviation function and Zadeh’s extension principle, where two level mathematical programs are transformed into a pair of ordinary one-level linear ones. Likewise, Zhou, Li, and Pedrycz [22] introduce a concept of fuzzy semientropy to quantify the down side uncertainty, which formulates two mean-semi-entropy portfolio selection models with simulation-based genetic algorithm. Moreover, Nguyen, Gordon-Brown, Khosravi, Creighton, and Nahavandi [19] introduce a new portfolio risk measure and fuzzy Sharpe ratio. Correspondingly, two portfolio optimization problems are formulated and solved by fuzzy approach or genetic algorithm, where an experimental result shows their approach is more effective than the existing one.

Since the higher moment information of asset return gets more important, many researchers also incorporate it into the fuzzy framework. For example, Wang, Wang, and Watada [21] develop a VaR-based fuzzy portfolio selection model, which is solved by an improved particle swarm optimization algorithm. Also, Nguyen and Gordon-Brown [18] apply constrained fuzzy analytic hierarchy process methods to incorporate higher moment information such as skewness and kurtosis as well as volatility into portfolio selection. Further, Li, Guo, and Yu [12] propose a new mean-variance-skewness fuzzy portfolio model, which is shown to be more diversified than an existing credibilistic model.

In turn, for better portfolio performances, it seems important to employ different kinds of criteria including qualitative ones. From this perspective, Li and Xu [14] address a multi-objective fuzzy portfolio selection model with genetic algorithm, where investors take into consideration various elements including historical price data, their own investment attitudes and experts’
opinions in addition to return, risk and liquidity. As well, Mehlawat [16] presents a credibilistic mean-entropy model for multi-objective multi-period portfolio selection, whose major criteria are wealth, risk, transaction cost, liquidity, and number of investment assets. Besides, Pai [20] discusses a metaheuristic portfolio optimization with multiple objective and constraints, where constructed portfolios are actively rebalanced based on simulated future market scenarios and interval type-2 fuzzy sets. Furthermore, Mehlawat and Gupta [17] address fuzzy chance-constrained multi-objective portfolio optimization problem with a hybridization of fuzzy simulation and real-coded genetic algorithm, where famous financial criteria, i.e. return, risk and liquidity, are characterized by two measure representing short- and long-term variants.

Additionally, there are several works to focus on different important perspectives for portfolio investment. For instance, Jalota, Thakur, and Mittal [11] focus on automatic process of fitting parameters for several types of multi-objective credibilistic portfolio selection problems with L-R fuzzy numbers, which are solved by entropy-cross entropy algorithm. Differently, Chen and Huang [8] present a fuzzy portfolio optimization scheme for numerous equity mutual funds using a cluster analysis based on rates of return, standard deviation, turnover rate, and Treynor index. Elsewhere, Li and Xu [13] introduce the concepts of $\lambda$-mean variance efficient portfolios and frontiers in order to take into account investors’ different forecasts about future returns of securities.
3 Expert system

Our expert system (ES) is summarized by the following flowchart.

![Flowchart of our expert system](image)

Here, we shortly describe the composition of our ES. Firstly, we estimate expected returns and volatilities with particle filtering, where anomaly detectors are applied to select investment universe (i.e., trading assets) in real-time. Secondly, at each time, we simulate MV portfolios based on each estimation result for several risk aversion parameters. At this stage, we run \( L \) kinds of MV portfolio simulations. Thirdly, we assess these simulated MV portfolios with \( M \) kinds of investment performance measures commonly used in practice. Lastly, we exploit a FL system for integrating these evaluations, which gives us the final portfolio.
3.1 Estimation

3.1.1 State space modeling

In constructing mean-variance (MV) portfolios, we need to estimate expected return and volatility. For estimation, this paper employs a state space model that consists of the following system and observation model.

\[
\begin{align*}
Y_t &= H(Z_t, u_t), \quad \text{[observation model]} \\
Z_t &= F(Z_{t-1}, v_t), \quad \text{[system model]} \\
\end{align*}
\]

(1)

where \(Z_t\) and \(Y_t\) denote a \(n\)-dim state and \(m\)-dim observation vector at time \(t\), respectively. In state space modeling, we suppose that there exist unobservable latent/state variables \(Z_t\) driven by the system model \(Z_t = F(Z_{t-1}, v_t)\), which are observed with noise as \(Y_t = H(Z_t, u_t)\). Here, \(H : \mathbb{R}^n \times \mathbb{R}^m \rightarrow \mathbb{R}^m\) and \(F : \mathbb{R}^n \times \mathbb{R}^m \rightarrow \mathbb{R}^n\) are non-linear functions, and the observation noise \(u_t\) and system noise \(v_t\) are random variables which do not necessarily follow Gaussian distributions.

In general, unobservable state variables are estimated by the statistical on-line algorithm called filtering. Hence, we can sequentially estimate expected return and volatility by regarding them as state variables, which are observed with noise as realized asset return.

3.1.2 Particle filtering

The objective of filtering is to sequentially estimate the unobservable state variables \(Z_t\) through the observation variables \(Y_{1:t} \equiv (Y_1, \ldots, Y_t)\) until time \(t\), that is, to estimate the density \(p(Z_t|Y_{1:t})\) called the filtering density.

Especially, particle filtering (PF) resorts to Monte Carlo simulations for state estimation, whereby it is applicable to non-linear and non-Gaussian settings. We first explain the PF algorithm in the case that all parameters are known (e.g., [33, 34]).

First of all, Bayes formula shows

\[
p(Z_t|Y_{1:t}) \propto p(Y_t|Z_t)p(Z_t|Y_{1:t-1})
\]

(2)

Therefore, we exploit a sampling importance resampling (SIR) method to obtain the samples \(\{Z_t^{(r)}\}_{r=1,\ldots,L}\) from the posterior \(p(Z_t|Y_{1:t})\). That is, by regarding the prior \(p(Z_t|Y_{1:t-1})\) as an importance function, we first draw \(\{Z_t^{(r)}\}_{r=1,\ldots,L}\) from the distribution \(p(Z_t|Y_{1:t-1})\), then resample from them with weights proportional to the likelihood \(p(Y_t|Z_t)\), which gives us \(\{Z_t^{(r)}\}_{r=1,\ldots,L}\).

Here, we get the samples \(\{Z_t^{(r)}\}_{r=1,\ldots,L}\) from the prior \(p(Z_t|Y_{1:t-1})\) by the following way. Note that

\[
p(Z_t|Y_{1:t-1}) = \int \int p(Z_t, Z_{t-1}, v_t|Y_{1:t-1})dZ_{t-1}dv_t
\]

\[
= \int \int p(Z_t|Z_{t-1}, v_t, Y_{1:t-1})p(v_t|Z_{t-1}, Y_{1:t-1})p(Z_{t-1}|Y_{1:t-1})dZ_{t-1}dv_t
\]

\[
= \int \int p(Z_t|Z_{t-1}, v_t)p(v_t)p(Z_{t-1}|Y_{1:t-1})dZ_{t-1}dv_t
\]

\[
= \int \delta(Z_t - F(Z_{t-1}, v_t))p(v_t)p(Z_{t-1}|Y_{1:t-1})dZ_{t-1}dv_t,
\]

(3)

where \(\delta(z)\) is Dirac delta function. Then, for given the samples of \(\{v_t^{(r)}\}_{r=1,\ldots,R}\) and \(\{Z_{t-1}^{(r)}\}_{r=1,\ldots,R}\) from \(p(v_t)\) and \(p(Z_{t-1}|Y_{1:t-1})\), we obtain the samples \(\{Z_t^{(r)}\}_{r=1,\ldots,R}\) from \(p(Z_t|Y_{1:t-1})\) by setting \(Z_t^{(r)} = F(Z_{t-1}^{(r)}, v_t^{(r)})\).
1. Generate the initial state vector \( \{ \hat{Z}_0^{(1)}, \ldots, \hat{Z}_0^{(R)} \} \).
2. Apply the following steps (a)~(d) to each time \( t = 1, \ldots, T \).
   (a) Generate system noise \( v_t^{(r)} \), \( r = 1, \ldots, R \).
   (b) Compute for each \( r = 1, \ldots, R \) \( Z_t^{(r)} = F(\hat{Z}_{t-1}^{(r)}, v_t^{(r)}) \).
   (c) Evaluate the weights of particles \( \{ Z_t^{(1)}, \ldots, Z_t^{(R)} \} \) as \( \delta_t^{(r)} \equiv p(Y_t|Z_t^{(r)}), \ r = 1, \ldots, R \) by using the likelihood function.
   (d) Resample \( \{ \hat{Z}_t^{(1)}, \ldots, \hat{Z}_t^{(R)} \} \) from \( \{ Z_t^{(1)}, \ldots, Z_t^{(R)} \} \). More precisely, resample each \( \hat{Z}_t^{(r')}, \ r' = 1, \ldots, R \) from \( \{ Z_t^{(1)}, \ldots, Z_t^{(R)} \} \) with the probability given by
   \[
   \text{Prob.}(\hat{Z}_t^{(r')} = Z_t^{(r')}|Y_t) = \frac{\delta_t^{(r')}}{\sum_{k=1}^R \delta_t^{(k)}}, \quad r = 1, \ldots, R.
   \]

Here, the likelihood at time \( t \), \( p(Y_t|Z_t) \), is approximately calculated by
\[
p(Y_t|Z_t) \approx \frac{1}{R} \sum_{k=1}^R \delta_t^{(k)}. \quad (4)
\]

Next, we discuss the case that there exist unknown parameters. The unknown parameters’ vector \( \theta_t \) is sequentially estimated by augmenting the state vector as \( \tilde{Z}_t = (Z_t, \theta_t) \). If the transition of \( \theta_t \) follows
\[
\theta_t = \theta_{t-1}, \quad (5)
\]
this algorithm will degenerate in the sense that almost all of the particles quickly reach zero weight. Besides, parameter estimation does not work when the true values are not included in particles generated by initial distributions. Then, we add an artificial noise \( \zeta_t \) to Eq. (5):
\[
\theta_t = \theta_{t-1} + \zeta_t. \quad (6)
\]
This framework is called "self-organizing state space model" by Kitagawa [46], which enables us to estimate states and parameters simultaneously.

For implementing the self-organizing method, it is necessary to specify the distribution of the artificial noise \( \zeta_t \) conditioned on \( \theta_{t-1} \). In this paper, we use a kernel smoothing (KS) method developed by Liu and West [47]. In the KS method, the distribution of \( \zeta_t \) conditioned on \( \theta_{t-1} \) is given by
\[
p(\zeta_t|\theta_{t-1}) \sim N((a-1)(\theta_{t-1}-\bar{\theta}_{t-1}), (1-a^2)s_{t-1}^2), \quad (7)
\]
i.e. the conditional distribution of \( \theta_t \) is
\[
p(\theta_t|\theta_{t-1}) \sim N(a\theta_{t-1} + (1-a)\bar{\theta}_{t-1}, (1-a^2)s_{t-1}^2), \quad (8)
\]
where \( a = (3d-1)/2d \). \( \bar{\theta}_t \) and \( s_{t}^2 \) represent the mean and variance of the particles \( \{ \theta_t^{(r)} \}_{r=1, \ldots, R} \), respectively. \( d \) is a shrinkage factor which usually takes a value between 0.95 and 0.99. Here, we set \( d = 0.98 \).

In this paper, we execute PF individually for each asset using 1,000,000 particles. In other words, we do not take into account the correlations among asset returns, which reduces estimation errors and computational complexity.
3.1.3 Asset return model

In the current work, we assume that the dynamics of observed asset returns (rates of returns) \( y = \{ y_t; t = 0, 1, \cdots, T \} \) are represented by the following observation model:

\[
y_t = \mu_t + \sigma_t \epsilon_t, \quad \epsilon_t \sim i.i.d. \ N(0, 1), \quad t \geq 0,
\]

where \( \mu = \{ \mu_t; t = 0, \cdots, T \} \) and \( \sigma = \{ \sigma_t; t = 0, \cdots, T \} \) are state variables which stand for expected return and volatility processes, respectively.

With regard to system models, we use the next two time-series models for \( \mu \).

- **EMA model:**
  \[
  \mu_t = \beta_\mu y_{t-1} + (1 - \beta_\mu) \mu_{t-1}, \quad t \geq 1,
  \mu_0 = \alpha_\mu \in \mathbb{R},
  \]

- **generalized EMA (GEMA) model:**
  \[
  \mu_t = \beta_\mu y_{t-1} + (1 - \beta_\mu) \mu_{t-1} + \sigma_\mu \eta_t, \quad t \geq 1,
  \mu_0 \sim N(\alpha_\mu, \sigma_\mu^2/(1 - \beta_\mu^2)), \quad \eta_t \sim i.i.d. \ N(0, 1), \quad Cov(\epsilon_t, \eta_t) = 0,
  \]

where \( \sigma_\mu(>0) \) is a constant unknown parameter and \( \beta_\mu \in (0,1) \) is the so-called smoothing factor that represents the degree of weighting decrease in EMA. Note that the GEMA model is a stochastic model which aims to obtain marginally better estimation than EMA.

As well, we assume EMA and GEMA model for volatility process \( \sigma_t \).

- **EMA (IGARCH(1,1)) model:**
  \[
  \sigma_t^2 = \beta_\sigma (y_{t-1} - \mu_{t-1})^2 + (1 - \beta_\sigma) \sigma_{t-1}^2,
  \sigma_0 = \alpha_\sigma \in \mathbb{R}
  \]

- **GEMA model:**
  \[
  \log \sigma_t^2 = \log \{ \beta_\sigma (y_{t-1} - \mu_{t-1})^2 + (1 - \beta_\sigma) \sigma_{t-1}^2 \} + \sigma_\xi_t, \quad t \geq 1,
  \log \sigma_0^2 \sim N(\log \sigma_0^2, \sigma_0^2), \quad \xi_t \sim i.i.d. \ N(0, 1), \quad Cov(\epsilon_t, \xi_t) = Cov(\eta_t, \xi_t) = 0,
  \]

where \( \beta_\sigma \in (0,1) \) is the so-called smoothing factor of EMA and \( \sigma_\sigma(>0) \) is a constant unknown parameter.

Now, let us summarize the system models, i.e. the combinations of \( \mu \) and \( \sigma \) processes. Namely, “EMA+GEMA”, “GEMA+EMA” and “GEMA+GEMA” are stochastic models for improving estimation of simple EMA in volatility, expected return and both expected return & volatility, respectively. Here, we employ the notation “model I + model II” representing model I for \( \mu \) and model II for \( \sigma \).

Let us remark that as the estimates of expected return and volatility for time \( t \), we take averages of one-step-ahead predictions at time \( t - 1 \) for state variables \( \mu_t \) and \( \sigma_t \), that is the particles \( \{ Z_t^{(r)} \}_{r=1,\cdots,R} \) obtained from the step (b) of PF algorithm, described in Section 3.1.2.

With regard to correlation, we employ the standard EMA model, i.e.,

\[
\rho_{t,i,j} = \frac{\sigma_{t,i,j}}{\sigma_{t,i}\sigma_{t,j}},
\]

where \( \sigma_{t,i,j} = \beta_\rho (y_{t-1,i} - \mu_{t-1,i})(y_{t-1,j} - \mu_{t-1,j}) + (1 - \beta_\rho)\sigma_{t-1,i,j} \).
Here, we describe the setting of model parameters. First, we notice that in the above models the smoothing factors of EMA/GEMA are fixed. In detail, we consider the case $\beta_\mu = \beta_\sigma = \beta_p (\equiv \beta)$ for simplicity and set $\beta = 0.2, 0.4, 0.6, 0.8$ in advance. Although this parameter strongly affects the investment results, it is very difficult to know the optimal level in advance. It may seem natural to estimate it by a self-organizing framework, but its estimates are optimal in terms of likelihood, not investment performance. Therefore, we do not estimate $\beta$ in this stage. Instead, our FL system, introduced later, selects the best one in terms of investment record.

Second, as for constant $\alpha_\mu$ and $\alpha_\sigma$ in EMA/GEMA, we set as the sample mean and standard deviation of asset returns in the first two years, $\{y_t\}_{t=0\ldots 23}$, respectively. With regard to an unknown parameter $\sigma_\mu$ in GEMA, we guess the value by using the data during the training period. That is, we use 0.5 times the sample standard deviation of EMAs in the first two years, $\sigma_{1,2}$. Differently from $\sigma_\mu$, it is difficult to guess a reasonable value of $\sigma_\sigma$. Therefore, we estimate it with a self-organizing approach. For their initial distributions, we draw $\sigma_\sigma$ from the uniform distribution $U(0, 1)$ following the previous research [39].

3.1.4 Anomaly detection

We also adopt three types of anomaly detectors to judge whether the models really capture asset returns’ dynamics, as introduced in Nakano et al. [39]. If an anomaly is detected for an asset at time $t$, we exclude it from our investment assets before time-$t$ portfolio construction, which makes it possible to enhance investment performances.

The first type utilizes the log-likelihoods. Here we define the log-likelihood at time $t$ by $\ell(y_t) \equiv \log p(y_t|Z_t)$ which is approximately calculated with taking log of the right hand side of Eq. (4). In our PF algorithm, we are able to obtain $\{\ell(y_t)\}_t$ for each asset. Then, if $\ell(y_t)$ takes a lower value than a predetermined threshold at time $t$, we regard an asset return $y_t$ as an anomaly.

In the second type, we employ the traditional Hotelling approach since the observational noise $\epsilon_t$ follows the standard normal distribution. In this approach, we define an anomaly indicator $a(y_t)$ by the negative log-likelihood $-\ell(y_t)$. We rewrite $a(y_t)$ by using only the term which includes $y_t$ as follows.

$$a(y_t) = \left( \frac{y_t - \mu_t}{\sigma_t} \right)^2.$$  \hspace{1cm} (15)

Note that in our models $a(y_t)$ coincides with the square of observation error, $\epsilon_t^2$. Again, this can be approximately calculated in our estimation algorithm. Then the remaining task is to determine a threshold as in the first approach. Here, we make use of the fact that this $a(y_t)$ asymptotically follows a chi-squared distribution with one degrees of freedom $\chi^2(1)$. Although the number of our data may not be large enough to utilize this asymptotic property, we approximately use the 95 percentile of $\chi^2(1)$ as a threshold.

In the third type, we test a method by the one-step-ahead predictive distribution of asset returns, $p(y_t|y_{1:t-1})$ where $y_{1:t-1} \equiv (y_1, \ldots, y_{t-1})$. In PF algorithm, we can obtain the approximation of predictive distribution $p(y_t|y_{1:t-1})$ based on Monte Carlo simulation. Then, we are able to detect an anomaly by calculating the 2.5 and 97.5 percentiles of $p(y_t|y_{1:t-1})$. If the realized return $y_t$ of some asset is outside these percentiles, we exclude it from our investment assets.

In the following, we call those three AD methods AD1, AD2 and AD3, respectively. Note that all the methods are implemented in our PF algorithm quite easily. In contrast to Nakano et al. [39] not offering a useful way for deciding in advance which AD is the best, our ES proposes a method for its decision.
Let us remark that although we exclude the assets for which anomalies are detected from our investment assets before portfolio construction at each time, all data including anomaly ones are used in the PF estimation stage.

Indeed, a previous work [48] for PF with AD regards anomaly data as uninformative ones and skips the resampling step 2.(d) in our algorithm (Section 3.1.2). However, since we focus on financial time-series data, which is characterized by significant non-stationarity in general, anomaly data may reflect important structural changes. Hence, we incorporate these irregular data into our estimation stage including the resampling step 2.(d).

3.2 Mean-variance portfolio and performance measure

Suppose that there exists a risk-free asset in the financial market and the risk-free rate is zero. We also put no-short-sale constraint. Then, the weight of MV portfolio at each time step \( t = 0, 1, \cdots, T - 1 \) is given by the solution of the following optimization problem for given estimates of an expected return vector \( \mu_t = (\mu_{t,i})_{i=1,\ldots,N} \) and a covariance matrix \( \Sigma_t = (\sigma_{t,i,j})_{i,j=1,\ldots,N} \).

\[
\max_{\omega_t} \omega_t'\mu_t - \frac{1}{2}\omega_t'\Sigma_t\omega_t, \\
\text{s.t. } \omega_{t,i} \geq 0, \quad \sum_{i=1}^{N} \omega_{t,i} \leq 1,
\]

where \( N \) denotes the number of risky assets composing the investment universe, and \( \omega_t = (\omega_{t,1}, \cdots, \omega_{t,N})' \) is a portfolio weight vector of risky assets in the time interval \([t, t+1)\). Besides, the parameter \( \gamma \) indicates degree of risk aversion and we test \( \gamma = 1, 5, 10, 20, 50 \). Although the investment result of MV portfolio largely depends on risk aversion parameter, our ES can select its optimal value in terms of portfolio performance as well as the smoothing factor. We notice that there are sometimes abnormal periods when the investment universe is not large enough to construct MV portfolio due to the anomaly detectors, that is \( N = 0, 1 \). In this case, we do not invest risky asset at all. Remark that the weight of risk-free asset \( \omega_{t,N+1} \) is determined by \( \omega_{t,N+1} = 1 - \sum_{i=1}^{N} \omega_{t,i} \).

The portfolio values \( \{V_t\}_{t=0,\ldots,T} \) and portfolio returns \( \{R_t\}_{t=1,\ldots,T} \) are defined as follows.

\[
V_{t+1} = V_t \left( 1 + \sum_{i=1}^{N} \omega_{t,i}y_{t+1,i} \right) - \sum_{i=1}^{N} c_i |\omega_{t,i}V_t - \omega_{t-1,i}V_{t-1}(1 + y_{t,i})|, \quad V_0 = 1, \\
R_{t+1} = \frac{V_{t+1}}{V_t} - 1,
\]

where \( c_i \) and \( y_{t,i} \) denote a transaction spread and a return of \( i \)-th risky asset, respectively.

The penalty term \( \sum_{i=1}^{N} c_i |\omega_{t,i}V_t - \omega_{t-1,i}V_{t-1}(1 + y_{t,i})| \) of Eq. (17) is the total transaction cost arising from the portfolio re-balance at time \( t \). Since \( \omega_{t-1,i} \) and \( \omega_{t,i} \) are portfolio weights of the \( i \)-th risky asset during \([t-1, t)\) and \([t, t+1)\), \( \omega_{t,j}V_t \) and \( \omega_{t-1,j}V_{t-1}(1 + y_{t,i}) \) indicate the values of the \( i \)-th risky asset before and after the position change at time \( t \), respectively. That is, \( |\omega_{t,i}V_t - \omega_{t-1,i}V_{t-1}(1 + y_{t,i})| \) represents the necessary amount of money for the position change of asset \( i \) at time \( t \). Hence, the total transaction cost at time \( t \) equals to the summation of \( c_i |\omega_{t,i}V_t - \omega_{t-1,i}V_{t-1}(1 + y_{t,i})| \) for all \( i \). In this paper, we set \( c_i = 10 \text{ bps} \) for all risky assets.

In the following, we briefly describe the well-known performance measures used in our fuzzy inference system.

- **Compound Return (CR):** We define CR as the annualized geometric average of the portfolio
returns \( \{R_t\} \) defined in Eq. (17), which is a standard measure of investment returns.

\[
CR \equiv \left( \prod_{t=1}^{T} (1 + R_t) \right)^{12/T} - 1.
\]  

(18)

- Standard Deviation (SD), Downside Deviation (DD): SD is a well-known investment risk measure defined as the annualized standard deviation of \( \{R_t\} \), while DD only regards negative returns as risk.

\[
SD \equiv \left( \frac{12}{T} \sum_{t=1}^{T} (R_t - \bar{R})^2 \right)^{1/2}, \quad DD \equiv \left( \frac{12}{T} \sum_{t=1}^{T} \min(0, R_t)^2 \right)^{1/2}, \quad \bar{R} \equiv \frac{1}{T} \sum_{t=1}^{T} R_t.
\]  

(19)

- Maximum Drawdown (MDD):

\[
MDD \equiv \max_{1 \leq t \leq T} \frac{M_t - V_t}{M_t}, \quad M_t \equiv \max_{0 \leq s \leq t} V_s.
\]  

(20)

The drawdown is the decline from the past peak value \( M_t \) to the present value \( V_t \). In general, portfolio performance depends on the investment timing. The MDD contributes to the performance analysis because it is independent of the investment timing given the horizon \([0, T]\).

- Sharpe Ratio (ShR): ShR is usually defined as portfolio excess average returns divided by portfolio standard deviation. Since interest rates on cash are assumed to be zero, we define ShR as follows.

\[
\text{ShR} \equiv \frac{AR}{SD}, \quad AR \equiv 12\bar{R}.
\]  

(21)

Here, AR denotes the annualized arithmetic average of \( \{R_t\} \), which corresponds to a simple return.

- Sortino Ratio (SoR): SoR does not regard upside volatility as a risk while ShR penalizes both upside and downside volatility, which is often pointed out as a weakness of ShR.

\[
\text{SoR} \equiv \frac{AR}{DD}.
\]  

(22)

- Sterling Ratio (StR): StR is a measure of risk-adjusted return that uses drawdown measures as denominator. We adopt the following definition:

\[
\text{StR} \equiv \frac{AR}{MDD}.
\]  

(23)

### 3.3 Fuzzy logic system

In the above sections, there exists much variety for time-series modeling, that is EMA+GEMA, GEMA+EMA and GEMA+GEMA for each \( \beta = 0.2, 0.4, 0.6, 0.8 \) and anomaly detectors, AD1, AD2 and AD3. Moreover, we assume three cases of risk aversion parameter \( \gamma = 1, 5, 10, 20, 50 \). In practice, it is important to select the ”best” case from these 180 candidates (= \( 3 \times 4 \times 3 \times 5 \)), though we cannot know in advance which is the best.

One of the effective approaches to this problem is to choose a portfolio whose past performance is the best in terms of some measures introduced in Section 3.2. Then, we apply a FL system to integrating the practically well-known performance measures, which is meaningful since a unified evaluation is difficult due to the variety of these measures.
Suppose that there are $L$ kinds of portfolios, which are evaluated with $M$ kinds of performance measures. Our fuzzy system is a nonlinear mapping from $\mathbb{R}^{M \times L}$ into $\mathbb{R}^L$, that is, the inputs are $M$ well-known performance measures $\{x_{t,l,m}\}_{l=1,\ldots,L, \ m=1,\ldots,M}$ and the output is a new integrated performance measure $\{\hat{x}_{t,l}\}_{l=1,\ldots,L}$. More concretely, our fuzzy system implements the following procedures at each investment time $t = t_s, \ldots, t_e$, where $t_s > 0$, $t_e = T$.

(i) Firstly, performance measures $\{x_{t,l,m}\}_{l=1,\ldots,L, \ m=1,\ldots,M}$ are calculated by time-series of the past portfolio values $\{V_s,l; 0 \leq s \leq t\}$ for each portfolio $l = 1, \ldots, L$ as the inputs.

(ii) Secondly, these inputs are fuzzified by the following triangular membership functions associated with three kinds of fuzzy sets $X_k, k = 1, 2, 3$, i.e. High, Medium and Low.

\[
MF_1(x_{t,l,m}) = \max \left\{ \frac{x_{t,l,m} - a_{t,m,2}}{a_{t,m,1} - a_{t,m,2}}, 0 \right\}, \\
MF_2(x_{t,l,m}) = \min \left\{ \frac{x_{t,l,m} - a_{t,m,2}}{a_{t,m,1} - a_{t,m,2}}, \frac{x_{t,l,m} - a_{t,m,2}}{a_{t,m,2} - a_{t,m,3}} \right\} + 1, \\
MF_3(x_{t,l,m}) = \max \left\{ -\frac{x_{t,l,m} - a_{t,m,2}}{a_{t,m,2} - a_{t,m,3}}, 0 \right\},
\]  

(24)

\[
a_{t,m,1} = \max_{l=1,\ldots,L} \{x_{t,l,m}\}, \\
a_{t,m,3} = \min_{l=1,\ldots,L} \{x_{t,l,m}\}, \\
a_{t,m,2} = (a_{t,m,1} + a_{t,m,3})/2,
\]

for each $m = 1, \ldots, M$ and $t = t_s, \ldots, t_e$.

(iii) Thirdly, we employ the following form of IF-THEN rule (e.g. [24,49]):

- The case of return or risk-adjusted return measures (CR, ShR, SoR, StR):
  * IF $x_{t,l,m}$ is $X_1$ (High), THEN $\hat{x}_{t,l,m,k} = 2$.
  * IF $x_{t,l,m}$ is $X_2$ (Medium), THEN $\hat{x}_{t,l,m,k} = 1$.
  * IF $x_{t,l,m}$ is $X_3$ (Low), THEN $\hat{x}_{t,l,m,k} = 0$.

- The case of risk measures (SD, DD, MDD):
  * IF $x_{t,l,m}$ is $X_1$ (High), THEN $\hat{x}_{t,l,m,k} = 0$.
  * IF $x_{t,l,m}$ is $X_2$ (Medium), THEN $\hat{x}_{t,l,m,k} = 1$.
  * IF $x_{t,l,m}$ is $X_3$ (Low), THEN $\hat{x}_{t,l,m,k} = 2$.

Here, the strength of each IF-THEN rule is evaluated by using grades of the membership function:

\[
g_k(x_{t,l,m}) = \frac{MF_k(x_{t,l,m})}{\sum_{k=1,2,3, \ m=1,\ldots,M} MF_k(x_{t,l,m})}.
\]  

(25)

Note that we use $3 \times M$ number of IF-THEN rules.

(iv) Lastly, the output $\hat{x}_{t,l}$ is defuzzified as follows:

\[
\hat{x}_{t,l} = \sum_{k=1,2,3, \ m=1,\ldots,M} g_k(x_{t,l,m})\hat{x}_{t,l,m,k} \\
= \sum_{m=1,\ldots,M} \sum_{k=1,2,3} g_k(x_{t,l,m})\hat{x}_{t,l,m,k}.
\]  

(26)
Then, by comparing the values of this integrated measure \( \{ \hat{x}_{t,l} \}_{l=1,\ldots,L} \), the final investment portfolio at time \( t \) is created. Specifically, we use the MV portfolio which attains the highest performance in terms of the new measure at each time.

Remark that since this FL system enables to evaluate a number of portfolios in a unified framework, it is applicable to fund of funds investment. For instance, by assessing various investable funds instead of the MV portfolios, we can construct a new portfolio of the hedge funds with weights proportional to the new integrated measures \( \{ \hat{x}_{t,l} \}_{l=1,\ldots,L} \).

Fig. 2: Membership function (CR)

In addition, we explain the reason for the specification of our membership functions Eq. (24) whose typical example is described in Fig. 2 for a performance measure CR. Namely, it is desirable to hold the following property for the membership functions: If a portfolio \( l \) shows higher performance than a portfolio \( l' \) in terms of a measure \( m \), the former is evaluated better than the latter in FL-based evaluation, which is represented as follows.

- The case of return or risk-adjusted return measures (CR, ShR, SoR, StR):

  \[
  x_{t,l,m} > x_{t,l',m} \Rightarrow \sum_{k=1,2,3} g_k(x_{t,l,m})\hat{x}_{t,l,m,k} > \sum_{k=1,2,3} g_k(x_{t,l',m})\hat{x}_{t,l',m,k}.
  \]  
  (27)

- The case of risk measures (SD, DD, MDD):

  \[
  x_{t,l,m} < x_{t,l',m} \Rightarrow \sum_{k=1,2,3} g_k(x_{t,l,m})\hat{x}_{t,l,m,k} > \sum_{k=1,2,3} g_k(x_{t,l',m})\hat{x}_{t,l',m,k}.
  \]  
  (28)

One of the specification to preserve these relations is the above three triangular membership functions with the parameters \( a_{t,m,1}, a_{t,m,2}, a_{t,m,3} \) in Eq. (24).

Here, we dynamically adjust the parameters \( a_{t,m,1}, a_{t,m,2}, a_{t,m,3} \) because the investment environment changes over time. In other words, since the levels of performance measures \( x_{t,l,m} \) may largely differ over \( t \), it seems inappropriate to apply constant values to these parameters.
4 Numerical experiment

4.1 Data

We use monthly total returns of 8 indexes corresponding to stocks, bonds, and REITs as listed in Table 1. Hereafter we employ the abbreviations of the index names in this table. The time period of the return data is 156 months, from April 2003 to March 2016. As the PF estimation during the first several periods are strongly affected by the initial distribution, we discard the estimation results over the first 24 periods, which means $t = 0$ is April 2005. Moreover, $t = t_s$ is set to be April 2007 for the FL system that requires historical portfolio value processes, as described in Section 3.3. Further, the threshold of AD1 is set to be the 5 percentile of $\{\ell(y_t)\}_{t=6,\ldots,47}$. We omit the first six months of the log-likelihoods because this period seems to be strongly affected by initial distributions.

An asset monthly return $y_t$ is given by $y_t = 100 \times (P_t/P_{t-1} - 1)$ where $P_t$ denotes the asset price at time $t$. Our data are downloaded from Bloomberg in JPY (Japanese yen)-denominated form so that we consider the global investment with no currency hedging, where the initial investment is made in JPY. Table 2 shows the descriptive statistics of the asset returns.

Table 1: Data

<table>
<thead>
<tr>
<th>Index name</th>
<th>Ticker (Bloomberg)</th>
<th>Abbreviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tokyo Stock Price Index</td>
<td>TPXDDVD.Index</td>
<td>JPE</td>
</tr>
<tr>
<td>Tokyo Stock Exchange REIT Index</td>
<td>TPXDREIT.Index</td>
<td>JPR</td>
</tr>
<tr>
<td>S&amp;P500 Index</td>
<td>SPTR.Index</td>
<td>USE</td>
</tr>
<tr>
<td>Morgan Stanley REIT Index</td>
<td>RMS.G.Index</td>
<td>USR</td>
</tr>
<tr>
<td>FTSE Developed ex North America Net Tax (US RIC) Index</td>
<td>TGPVAN33.Index</td>
<td>DE</td>
</tr>
<tr>
<td>FTSE Emerging Total Return Index</td>
<td>FTS5ALEM.Index</td>
<td>EE</td>
</tr>
<tr>
<td>Barclays US Treasury 10 Year TERM Index</td>
<td>BCEY4T_INDEX</td>
<td>USB</td>
</tr>
<tr>
<td>JPMorgan Emerging Market Bond Index</td>
<td>JPEIGBL_INDEX</td>
<td>EB</td>
</tr>
</tbody>
</table>


REIT, or Real Estate Investment Trust, is a security that invests in real estate through property or mortgages, and often trades on major exchanges like equities. REITs provide investors with an extremely liquid stake in real estate.
Table 2: Descriptive statistics

<table>
<thead>
<tr>
<th></th>
<th>Mean</th>
<th>Std. Dev.</th>
<th>Skew</th>
<th>Kurtosis</th>
</tr>
</thead>
<tbody>
<tr>
<td>JPE</td>
<td>0.629</td>
<td>5.273</td>
<td>-0.407</td>
<td>0.855</td>
</tr>
<tr>
<td>JPR</td>
<td>0.953</td>
<td>5.721</td>
<td>-0.206</td>
<td>3.531</td>
</tr>
<tr>
<td>USE</td>
<td>0.853</td>
<td>5.279</td>
<td>-0.724</td>
<td>1.764</td>
</tr>
<tr>
<td>USR</td>
<td>1.193</td>
<td>7.338</td>
<td>-0.941</td>
<td>6.002</td>
</tr>
<tr>
<td>DE</td>
<td>0.825</td>
<td>5.903</td>
<td>-0.933</td>
<td>2.384</td>
</tr>
<tr>
<td>EE</td>
<td>1.216</td>
<td>7.188</td>
<td>-0.833</td>
<td>2.598</td>
</tr>
<tr>
<td>UB</td>
<td>0.430</td>
<td>2.572</td>
<td>-0.181</td>
<td>0.720</td>
</tr>
<tr>
<td>EB</td>
<td>0.726</td>
<td>3.486</td>
<td>-1.528</td>
<td>8.993</td>
</tr>
</tbody>
</table>

Skew = \( \frac{1}{T} \sum_{t=1}^{T} \left( \frac{y_t - \bar{y}}{\sigma} \right)^3 \), Kurtosis = \( \frac{1}{T} \sum_{t=1}^{T} \left( \frac{y_t - \bar{y}}{\sigma} \right)^4 - 3 \), where \( \bar{y} \) and \( \sigma \) denote Mean and Std. Dev., respectively.

4.2 Out-of-sample investment result

Table 3 compares the investment results of our proposed ES with the traditional strategies such as buy-and-hold (BH) and equally-weighted (EW) strategy.

Table 3: Performance comparison with traditional strategies (%)

<table>
<thead>
<tr>
<th></th>
<th>CR</th>
<th>SD</th>
<th>DD</th>
<th>MDD</th>
<th>ShR</th>
<th>SoR</th>
<th>StR</th>
</tr>
</thead>
<tbody>
<tr>
<td>our ES</td>
<td>12.29</td>
<td>11.58</td>
<td>4.59</td>
<td>8.76</td>
<td>106.12</td>
<td>267.68</td>
<td>140.27</td>
</tr>
<tr>
<td>EW strategy</td>
<td>3.91</td>
<td>17.26</td>
<td>12.61</td>
<td>53.63</td>
<td>31.25</td>
<td>42.76</td>
<td>10.05</td>
</tr>
<tr>
<td>BH strategy</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>JPE</td>
<td>-0.66</td>
<td>19.77</td>
<td>14.25</td>
<td>56.23</td>
<td>6.67</td>
<td>9.26</td>
<td>2.35</td>
</tr>
<tr>
<td>USE</td>
<td>5.96</td>
<td>20.58</td>
<td>14.56</td>
<td>59.79</td>
<td>38.77</td>
<td>54.80</td>
<td>13.35</td>
</tr>
<tr>
<td>USR</td>
<td>-0.05</td>
<td>23.07</td>
<td>17.19</td>
<td>63.04</td>
<td>11.80</td>
<td>15.83</td>
<td>4.32</td>
</tr>
<tr>
<td>DE</td>
<td>4.37</td>
<td>28.58</td>
<td>20.75</td>
<td>73.94</td>
<td>30.22</td>
<td>41.63</td>
<td>11.68</td>
</tr>
<tr>
<td>EE</td>
<td>1.37</td>
<td>27.17</td>
<td>19.67</td>
<td>67.67</td>
<td>19.29</td>
<td>26.65</td>
<td>7.75</td>
</tr>
<tr>
<td>USB</td>
<td>5.72</td>
<td>9.16</td>
<td>5.71</td>
<td>15.25</td>
<td>65.44</td>
<td>104.95</td>
<td>39.32</td>
</tr>
<tr>
<td>EB</td>
<td>6.18</td>
<td>13.30</td>
<td>9.68</td>
<td>33.00</td>
<td>52.12</td>
<td>71.63</td>
<td>21.00</td>
</tr>
</tbody>
</table>

First of all, Table 3 shows that our proposed ES substantially outperforms the traditional strategies in all performance measures other than SD, which is also confirmed by Fig. 3. Although our proposed ES is slightly worse than BH strategy of USB in terms of SD, this is not a problem at all because the result of DD suggests that the higher SD results from the upside deviation, i.e., the positive portfolio return.
We summarize the performances of 180 candidate and our proposed portfolios by using their quantiles in each performance measure, i.e., CR, SD, DD, MDD, ShR, SoR, StR and FL-based integrated measure. We also evaluate the final output portfolio compared to those 180 simulated MV portfolios, which are too many to present all the records. Therefore, we summarize those investment results as the quantiles for each performance measure in Table 4. It also shows ranking of the final portfolio.

From Table 4, it is clear that our ES can successfully construct a MV portfolio achieving fine risk-return profiles, though the investment performance substantially changes depending on the models, smoothing factors, anomaly detectors and risk aversion parameters. Particularly, all risk adjusted return measures (ShR, SoR and StR) of our proposed ES rank in top 10. Moreover, we can see from this table that our proposed ES in fact achieves a considerably high ranking in terms of FL-based measure. This is also clearly observed by the red line in the Fig. 4, which indicates high ranking of our portfolio in the histogram of FL-based measure.

<table>
<thead>
<tr>
<th>CR</th>
<th>SD</th>
<th>DD</th>
<th>MDD</th>
<th>ShR</th>
<th>SoR</th>
<th>StR</th>
<th>FL-based measure</th>
</tr>
</thead>
<tbody>
<tr>
<td>0th percentile</td>
<td>3.51</td>
<td>6.57</td>
<td>3.43</td>
<td>7.43</td>
<td>27.95</td>
<td>45.29</td>
<td>12.47</td>
</tr>
<tr>
<td>25th percentile</td>
<td>8.32</td>
<td>12.68</td>
<td>5.76</td>
<td>14.05</td>
<td>69.99</td>
<td>132.87</td>
<td>50.68</td>
</tr>
<tr>
<td>50th percentile</td>
<td>12.68</td>
<td>14.97</td>
<td>6.88</td>
<td>17.03</td>
<td>90.70</td>
<td>195.85</td>
<td>80.31</td>
</tr>
<tr>
<td>75th percentile</td>
<td>15.15</td>
<td>16.13</td>
<td>7.79</td>
<td>19.50</td>
<td>98.95</td>
<td>220.71</td>
<td>96.17</td>
</tr>
<tr>
<td>100th percentile</td>
<td>17.81</td>
<td>19.19</td>
<td>11.90</td>
<td>42.66</td>
<td>109.74</td>
<td>272.46</td>
<td>145.40</td>
</tr>
<tr>
<td>our ES</td>
<td>12.29</td>
<td>11.58</td>
<td>4.59</td>
<td>8.76</td>
<td>106.12</td>
<td>267.68</td>
<td>140.27</td>
</tr>
<tr>
<td>rank</td>
<td>95</td>
<td>34</td>
<td>22</td>
<td>9</td>
<td>10</td>
<td>3</td>
<td>5</td>
</tr>
</tbody>
</table>

We summarize the performances of 180 candidate and our proposed portfolios by using their quantiles in each performance measure, i.e., CR, SD, DD, MDD, ShR, SoR, StR and FL-based integrated measure.
Lastly, to test the validity of our FL system, we also construct portfolios by using only each performance measure (CR, SD, DD, MDD, ShR, SoR and StR), instead of our FL-based criteria.

Table 5: Comparison with single performance measure based evaluation

<table>
<thead>
<tr>
<th></th>
<th>CR</th>
<th>SD</th>
<th>DD</th>
<th>MDD</th>
<th>ShR</th>
<th>SoR</th>
<th>StR</th>
</tr>
</thead>
<tbody>
<tr>
<td>our ES</td>
<td>12.29</td>
<td>11.58</td>
<td>4.59</td>
<td>8.76</td>
<td>106.12</td>
<td>267.68</td>
<td>140.27</td>
</tr>
<tr>
<td>ranking (out of 181)</td>
<td>95</td>
<td>35</td>
<td>24</td>
<td>9</td>
<td>10</td>
<td>3</td>
<td>5</td>
</tr>
<tr>
<td>CR-based</td>
<td>11.25</td>
<td>16.47</td>
<td>8.61</td>
<td>26.54</td>
<td>72.94</td>
<td>139.50</td>
<td>45.26</td>
</tr>
<tr>
<td>ranking (out of 181)</td>
<td>107</td>
<td>144</td>
<td>167</td>
<td>166</td>
<td>133</td>
<td>134</td>
<td>144</td>
</tr>
<tr>
<td>ranking (out of 181)</td>
<td>180</td>
<td>3</td>
<td>7</td>
<td>48</td>
<td>166</td>
<td>166</td>
<td>174</td>
</tr>
<tr>
<td>DD-based</td>
<td>3.93</td>
<td>7.08</td>
<td>3.78</td>
<td>14.43</td>
<td>58.04</td>
<td>108.71</td>
<td>28.46</td>
</tr>
<tr>
<td>ranking (out of 181)</td>
<td>176</td>
<td>10</td>
<td>10</td>
<td>49</td>
<td>162</td>
<td>159</td>
<td>170</td>
</tr>
<tr>
<td>MDD-based</td>
<td>7.73</td>
<td>9.22</td>
<td>4.13</td>
<td>12.72</td>
<td>85.49</td>
<td>191.07</td>
<td>62.01</td>
</tr>
<tr>
<td>ranking (out of 181)</td>
<td>144</td>
<td>16</td>
<td>16</td>
<td>30</td>
<td>109</td>
<td>95</td>
<td>123</td>
</tr>
<tr>
<td>ShR-based</td>
<td>10.72</td>
<td>15.07</td>
<td>6.94</td>
<td>14.66</td>
<td>75.00</td>
<td>162.95</td>
<td>77.07</td>
</tr>
<tr>
<td>ranking (out of 181)</td>
<td>109</td>
<td>93</td>
<td>94</td>
<td>53</td>
<td>129</td>
<td>127</td>
<td>98</td>
</tr>
<tr>
<td>SoR-based</td>
<td>9.63</td>
<td>11.30</td>
<td>4.86</td>
<td>12.05</td>
<td>87.07</td>
<td>202.32</td>
<td>81.66</td>
</tr>
<tr>
<td>ranking (out of 181)</td>
<td>122</td>
<td>31</td>
<td>26</td>
<td>27</td>
<td>103</td>
<td>77</td>
<td>87</td>
</tr>
<tr>
<td>StR-based</td>
<td>11.35</td>
<td>11.68</td>
<td>4.72</td>
<td>12.86</td>
<td>98.06</td>
<td>242.50</td>
<td>89.04</td>
</tr>
<tr>
<td>ranking (out of 181)</td>
<td>103</td>
<td>36</td>
<td>24</td>
<td>30</td>
<td>50</td>
<td>12</td>
<td>63</td>
</tr>
</tbody>
</table>

Table 5 shows the investment records of single measure-based portfolios compared with our FL system. From Table 5, it is obvious that the single measure-based assessment does not work at all, especially in return measures and risk adjusted returns. That is, in the cases of CR, ShR, SoR and StR, the investment performances drastically get worse than our FL-based evaluation. With regard to the risk measures, i.e., SD, DD and MDD, even if portfolio risk becomes relatively low, investment return and risk-adjusted return (CR, ShR, SoR and StR) are much worse than our FL case. As a result, our FL system, which combines various investment criteria, substantially contributes to the selection of a well-performed MV portfolio.
4.3 Application of our framework to other investment strategies

As mentioned before, since achieving target performance measures is crucial, we set a FL system at the final stage in our ES (i.e., 3rd stage in Fig. 1 of Section 3). On the other hand, we can interpret that other works generally apply their fuzzy systems to the earlier stages. For instance, let us briefly explain Yunusoglu et al. [31] and Chourmouziadis and Chatzoglou [32] within our framework.

Chourmouziadis and Chatzoglou [32] employ raw return data as KBS inputs. Then, using technical indicators (corresponding to our time-series models), they develop a fuzzy system to output the investment amount into the single risky asset, the general index of Athens Stock Exchange (the 1st & 2nd stage in Fig. 1).

Yunusoglu et al. [31] employ raw time-series data obtained from financial statements as KBS inputs, in addition to raw return data. Then, using technical indicators, as well as relative values and ROC (rate of change momentums) of FR (fundamental ratio) based on raw financial statement data, they apply their own fuzzy system to creating time-series of rating for each stock (the 1st stage in Fig. 1). Finally, they construct optimal portfolios based on the ratings under appropriate constraints (the 2nd stage in Fig. 1).

In the following numerical example, we apply a simple FL system for technical trading to the 1st stage (Fig. 1) as in the previous works, and further employ our FL system at the final stage to demonstrate the effectiveness of our approach.

Particularly, we design the following fuzzy system to generate investment candidates. Firstly, we get access to daily closing price data for the assets in Table 1 and calculate technical indicators at each time \( t \) (the end of a month), that is, price momentum, MACD-based signal, moving SD and William’s \( \%R \) with standard parameter sets.

- Price momentum of \( n \)-day is the last closing price minus a closing price \( n \) days ago. Here, we set \( n = 10, 20, 60 \).
- MACD is defined by subtracting the \( n \)-day EMA from \( m \)-day EMA \( (n > m) \). In technical analysis, MACD minus its \( \ell \)-day EMA is used as a trend-following indicator. Then, we use this value as MACD-based signal with \( (n, m, \ell) = (26, 12, 9), (52, 24, 9), (26, 12, 2), (52, 24, 2) \).
- Moving SD is a standard deviation over recent \( n \)-days closing prices. Here, we put \( n = 20, 60 \).
- William’s \( \%R \) is usually specified as \((\text{the last closing price} - \text{highest high})/(\text{highest high} - \text{lowest low})\). Here, since there does not exist high/low price data for some assets, we use the highest/lowest closing price over the recent \( n \) days instead of highest high and lowest low, where \( n = 14, 70 \).

In other words, there are 48 \((= 3 \times 4 \times 2 \times 2)\) types of technical indicator sets. Besides, we note that there are two trend-following indicators (price momentum and MACD-based signal), one range-based indicator (William’s \( \%R \)), and one risk indicator (moving SD).

Secondly, as in Section 3.3, we apply the following fuzzy IF-THEN rules to the technical indicators for each asset to compute its score at time \( t \) (the end of month).

- IF price momentum is High/Medium/Low, THEN the asset’s score is 2/1/0.
- IF MACD-based signal is High/Medium/Low, THEN the asset’s score is 2/1/0.
- IF moving SD is High/Medium/Low, THEN the asset’s score is 0/1/2.
- IF William’s \( \%R \) is High/Medium/Low, THEN the asset’s score is 0/1/2.
Lastly, by using the calculated scores, we construct 6 (= 2 × 3) kinds of portfolios ω_t. Specifically, we invest (buy) top one or two assets with three leverage levels according to the following rules:

- For single asset investment,
  \[ \omega_{t,i} = \begin{cases} c_1, & i = i_{max}, \\ 0, & \text{otherwise}, \end{cases} \]
  where \( i_{max} \) denotes an asset index whose score is the highest. We test the cases of leverage levels \( c_1 = 0.5, 1, 2 \).

- For two asset investment,
  \[ \omega_{t,i} = \begin{cases} c_2, & i = i_{max1}, i_{max2}, \\ 0, & \text{otherwise}, \end{cases} \]
  where \( i_{max1} \) and \( i_{max2} \) denote an asset index whose score is the highest and the second highest, respectively. We test the cases of \( c_2 = 0.25, 0.5, 1 \), that is leverage levels are \( c_2 \times 2 \).

In the above investment policies, position to cash, i.e. \( \omega_{t,N+1} \), is set to be \( 1 - c_1 \) (\( =1 - c_2 \times 2 \). For instance, in the case of single asset investment with \( c_1 = 2 \) (or two asset investment with \( c_2 = 1 \)), the cash position becomes minus one (\( \omega_{t,N+1} = -1 \)), which denotes a twice leveraged portfolio.

Since 288 (= 48 × 6) kinds of portfolios are generated, we can apply our FL system described in Section 3.3 by setting \( L = 288 \), whose result is shown in the following Table 6.

| Performance Comparison with Simulated Fuzzy Rule-Based Portfolios (%) |
|-----------------------------|-------------|---------|-------------|---------|----------------|---------|-------------|----------------|
|                            | CR          | SD      | DD         | MDD     | ShR     | SoR     | StR     | FL-based measure |
| 0th percentile             | -9.56       | 7.01    | 4.39       | 12.78   | -4.06   | -6.03   | -1.81   | 0.00           |
| 25th percentile            | 1.84        | 9.43    | 5.95       | 25.79   | 23.33   | 36.17   | 9.10    | 0.87           |
| 50th percentile            | 3.73        | 16.65   | 10.43      | 39.81   | 37.88   | 58.34   | 15.34   | 1.11           |
| 75th percentile            | 6.60        | 30.68   | 19.61      | 62.30   | 48.05   | 78.02   | 22.32   | 1.34           |
| 100th percentile           | 17.87       | 41.64   | 29.43      | 90.58   | 69.21   | 120.31  | 46.55   | 1.83           |
| our proposal               | 5.05        | 9.18    | 4.90       | 21.77   | 58.25   | 109.22  | 24.57   | 1.61           |
| rank (out of 288)          | 104         | 62      | 24         | 48      | 28      | 10      | 51      | 13             |

We summarize the performances of 288 candidate and our proposed portfolios by using their quantiles in each performance measure, i.e., CR, SD, DD, MDD, ShR, SoR, StR and FL-based integrated measure.

It is observed that as in the previous subsection for the case of MV portfolio, the investment performance drastically changes depending on the parameters of technical indicators and investment strategies, which causes substantial difficulty for selecting a portfolio in advance. Table 6 demonstrates that in this situation, our approach successfully chooses the well-performed portfolio from a number of candidates. For example, the risk-adjusted returns of our proposal are relatively high as well as DD and MDD, which leads to the high ranking in terms of FL-based measure.

5 Conclusion

In this paper, we have developed a novel knowledge-based system (KBS), particularly expert system (ES) featuring fuzzy logic (FL) to select a portfolio with fine risk-return profiles from
a number of investment candidates. In particular, our ES directly links to high performance because it chooses the best portfolio in terms of various evaluation criteria frequently used in practice.

Concretely, our ES consists of three stages: estimation, portfolio construction and selection with a FL system. First, we assume exponential moving average (EMA)-based models for multiple smoothing factors to prevent model misspecification, which are estimated by particle filtering with anomaly detectors. Then, we calculate mean-variance (MV) portfolio weights with regard to various models, anomaly detectors and risk aversion parameters. Lastly, we assess historical records for each portfolio based on a FL system through integrating well-known performance measures, and select the best portfolio. Further, an out-of-sample numerical experiment has confirmed that our ES generates a satisfactory investment record.

Although the current paper has considered the limited cases, i.e. only EMA-based models and MV portfolios, the proposed ES is effective in more general settings such as different time-series models and investment strategies, which will be shown in our future researches. For example, our ES is applicable to fund of funds investment, since it provides a unified scheme to comprehensive portfolio evaluation.

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