# SEQUENTIAL COSTLY STATE VERIFICATIONS UNDER TWO-STATE MARKOV CHAIN SHOCKS: A MODEL OF DYNAMIC LOAN-DEFAULT NEGOTIATION

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Abstract. This paper studies sequential costly state verification as a formal representation of loan-default negotiation in an infinite horizon principal-agent model with privately observed two-state Markov income shocks. The main result is as follows: With some particular parametric assumptions, there exists a trade equilibrium where costly state verifications occur recurrently only when lender believes that high state is probable while borrower knows that the true state is low. In the trade equilibrium, default plays a positive role in ex ante contingent agreement. Also, as an application to international finance, this paper gives some insights into autarkic-financing features in the world's poorest economies.

## 1. Introduction

This paper studies sequential costly state verification (or CSV) as a formal representation of loan-default negotiation in an infinite horizon principal-agent model with privately observed first-order Markov income shocks. This model has three main characteristics as compared to the previous defaultable contract literature. First, default means that a borrower (1) breaks his payment promise, and negotiates for a restructuring plan by incurring some positive, finite costs, then (2) reorganizes under the agreed plan, where he is protected from the lender's intervention and is given some payment allowance during a planned period of time, and (3) continues the relationship with the same lender beyond a default. We may call this type of default as reorganization default. The default may occur recurrently in the model. Then, a borrower's default decision is not specifically anticipated as a response to the terms of the agreement, and is averse to a lender. In the previous default literature, a default is often defined as a terminal event. However, the presumption of default seems unrealistic. A defaulting borrower often keeps accessing to the loan markets after a default. In addition, a borrower may keep the relationship with the same lender beyond a default. Even a liquidation (or repudiation) is a result after a consideration of reorganization. In practice, this type

Version: August 24, 2005.

*JEL Classifications.* C73, D82, G33.

*Key words and phrases.* Inertia, Loan-default negotiation, Recurrent costly state verification, Payment allowance.

This is an extension of my disseratation. I would like to thank Ed Green, Lars Peter Hansen, Milton Harris, Andrew Levin, and Takashi Ui for their advice and comments. I would also like to thank all the seminar participants at the Federal Reserve Bank of Atlanta, the Federal Reserve Board, Hitotsubashi University, Kyoto University, the Contract and mechanism design II session at the Japan Economic Association meetings (fall 2004), and Yokohama National University for valuable comments. All errors are my own.

of default is associated with sovereign debt in international finance and Chapter 11 bankruptcy in corporate finance.[1] Our interest is in that type of default. The second characteristic is that a privately-observed shock process is Markov. This is more realistic than most previous models with no intertemporal exogenous links of the private variables. So, we study the inertial effect of reorganization default in equilibrium. The third characteristic is that the lender, as well as the borrower, faces a time-consistency problem. Most contracting literature models the agent's time-consistency constraint but assumes that the principal can commit in advance to its future treatment of the agent. In contrast, this model assumes limited commitment not only on the part of the borrower but also on the part of the lender. That is, we study the subgame perfectness of the equilibrium in a non-cooperative dynamic Bayesian game. This paper provides a positive analysis of the strategic loan-default negotiations, that is, what conditions under which both lender and borrower choose reorganization default over autarky enforcement after an interim negotiation. Our main result is: If lender's autarky utility level is state-dependent (that is, lender's and borrower's utility are positively correlated), and with some particular parametric assumptions, then there exists a trade equilibrium where costly disclosures and defaults occur recurrently only when lender believes that high state is probable while borrower knows that the true state is low. The default plays a positive role in ex ante contingent agreement. Also, in equilibrium, for several subsequent dates after a default, borrower is allowed to make only low payments, no matter what the true states are. We may call the consecutive dates as an episode of inertia or as a reorganization episode with payment allowance. In equilibrium, state verification occurs when, and only when, a default occurs. The trade equilibrium does not stipulate autarky after a default, even temporarily.

This model is basically a dynamic version of Townsend [27]'s CSV environment. There are risk-neutral lender and borrower, who consume single non-storable consumption goods in infinite horizon discrete-time ($t = 0, 1, 2, \ldots$). The stage environment is similar to Townsend's. Lender has one unit of indivisible physical input, while borrower has no physical input. There is no capital depreciation and accumulation. If lender delivers the input to borrower, then borrower produces the goods with his investment project and the input. Borrower has private information about his current income and stochastic prospects for future income. State verification (or disclosure) technology is available. Using the technology causes one-period non-pecuniary cost, and makes current private information of borrower's output known to lender with complete accuracy. Before date 0, the lender agrees to let the borrower use the capital for an indefinite period. The investment occurs only at the outset of the

[1]Notice that in the previous Chapter 11 bankruptcy literature, loan-default negotiation among multi-creditors is one of the main concerns (for example, seniority problems). However, my model is a simple principal-agent model, and so does not deal with multi-creditor negotiation problems.

game. Subsequently, from date 0, they share output from borrower's project unless the agreement is terminated. Precisely, at each date, first, output is produced from borrower's project with the input, and provides accurate knowledge of the state to borrower. Then, borrower decides whether or not to disclose current state to lender. If disclosure occurs, then disclosure cost is imposed on borrower, and lender sees the true state. At the same time, borrower makes a payment to lender. Then, they consume the allocated goods. Next, lender decides whether or not to terminate the agreement. If lender terminates the agreement, then lender repossesses the input. The game ends. Subsequently, each lives in autarky. On the other hand, if lender commits to the agreement, then they move on to the next-date stage game.

This model differs from standard CSV models in several crucial points. First, this model is an infinite horizon model, in contrast to most CSV models, which are 2-date models.[2] Secondly, the income shock process is two-state first-order Markov: high and low income levels $\{L, H\}$ with $0 < L < H$. In particular, both shocks are persistent, and high-income shock is more persistent than low-income shock. Then, the players' continuation utilities are not independent of their history. That is, the dynamic game is a Markov game, not a repeated game, in the sense that a stage game has a physical link to the stage games played in the past. Thirdly, there exists no explicit, direct communication. That is, borrower cannot make any report to lender in the game. Still, information flows from borrower to lender because some actions or outcomes are feasible only in particular states. Then, this game is characterized as a dynamic signaling game induced by observable outcomes and physical actions. Fourthly, as is discussed above, most importantly, there exists opportunities of loan-default negotiations under the two-sided limited commitment. Autarky enforcement and default (or disclosure) decisions are distinguished explicitly. Finally, before addressing the optimal contracting, this paper takes a game approach where a fixed dichotomous payment choice set is imposed: high and low payment levels $\{x_l, x_h\}$ with $0 < x_l < x_h$. This approach intends to, as a positive analysis, stress an equilibrium loan-negotiation performance in a game in a intuitive way. In addition, for simplicity, the game approach imposes a feasibility constraint on the payment choice set: $0 < x_l < L < x_h < H$, that is, $x_h$ is infeasible in $L$. This approach is more restrictive in some respects than the parametric specification of most CSV models. However, there are trade-offs between the two approaches. In fact, the game approach has some advantages over the optimal contracting approach, making up for the limitations. First, the optimal contracting approach presumes a continuum of payments choices, and so could be complicated, especially in a dynamic model, to induce strategic implications in equilibrium. In contrast, the

[2] Wang [28] and Monnet-Quintin [21] are exceptions.

dichotomous-payment game approach enables us to use a game tree and to analyze the loan-default negotiation dynamics in a very intuitive way. Second, since borrower faces a disclose-or-not decision at each date, a model with the dichotomous payment choice set is a simplest way to analyze the state contingency of the equilibrium loan negotiation. However, a model with three or more payment choices might be interesting. For example, suppose that in addition to $\{x_l, x_h\}$, there is another choice that borrower pays nothing. Then, we can conjecture that in equilibrium, lender might allow borrower to make zero payment without enforcing autarky right after a disclosure. Then, after a little while, positive payments would be required to let the relationship be kept. In fact, from a dynamic viewpoint, a time-dependent dichotomous payment rule could be Pareto improving, as shown in the optimal contracting part later. Still, at each component game, borrower would face no larger than dichotomous choices in equilibrium, even though three or more payment choices are prepared. So, the dichotomous choice structure grasps the role of loan-default negotiation in a dynamic CSV environment very well. Third, in general, the restriction that $x_h$ is infeasible in $L$ might be dominated in the optimal contract. In fact, even if $x_h$ is feasible in $L$, and with some parametric assumptions, then high payment would be state-revealing. However, if only an appropriate fixed choice set is chosen, then the feasibility restriction might be reasonably true in equilibrium. In addition, the restriction simplifies the game structure drastically, because high payment itself would reveal the current true state $H$. In summary, the dichotomous payment game approach taken here seems adequate to provide a qualitative, positive theory of how reorganization default operates in practice.

In these environments, this paper focuses on a stationary pure-strategy weak sequential equilibrium where strategies are restricted to Markovian ones.[3] It, first, proves the existence of a stationary equilibrium, and, then, characterizes the equilibrium default performance. Precisely, it shows that, with some relevant parametric assumptions, there exists a trade equilibrium where costly disclosure (or default) occurs recurrently only when there is some conflict of economic perspectives between lender and borrower, that is, when lender believes that high state is probable while borrower knows that the true state is low. More precisely, in a trade equilibrium, two episodes evolve alternatively: pooling and separating episodes. While lender believes that high state is less probable, lender prefers to commit to the agreement for any borrower's action, because the expected autarky utility level is low. Then, borrower is allowed to make low payments without disclosing his true states. It is a pooling episode, during which time there occurs inertia due to the cost. On the other hand, once

[3]The philosophical backgrounds of the Markov strategies are: (1) No bootstrapping, (2) "Bygones are bygones" property, and (3) "Minor causes lead to minor effects" property (Maskin-Tirole [20]). The Markovian property has the simplest form of the behaviors that are consistent with rationality in dynamic models.

lender's posterior belief of high state becomes sufficiently high after a while, high payment is paid in high state while the conflict of their economic perspectives brings about default in low state. It is a separating episode. The costly default is a trigger into payment allowance. Therefore, in a trade equilibrium, there co-exist a no-monitoring pooling episode and a conditional-monitoring separating episode. Behind the equilibrium default performance, there is the following logic. When both players know that borrower's income will recover in future, lender has incentive to keep the relationship beyond a default, even though the current low state is revealed. Meanwhile, borrower chooses the default by incurring disclosure cost to enjoy payment allowance for several subsequent dates and expects his own recovery during the periods. The model induces two theoretical predictions: (1) Defaultable debt-type contract is enforceable in a trade equilibrium in costly verifiable information environments, and (2) In comparing across dynamic CSV environments, higher disclosure cost implies lower equilibrium default probability.

1.1. **Related literature.** In the related literature, there exist several versions of dynamic contract models with default. They can be categorized into two groups: symmetric information models and asymmetric information models. As to symmetric information models, we can refer to debt-constrained markets models.[4] There, a defaulting borrower is necessarily excluded from the loan markets forever. Since the models assume symmetric information in asset markets, there occurs no bankruptcy in equilibrium. However, the exclusion assumption seems unrealistic. Endogenous default literature replaces the exclusion assumption with the assumption of finite, positive default penalty/cost. Dubey-Geanakoplos-Shubik [3] and Zame [29] study default in finite-period general equilibrium models with incomplete security markets. In their models, a security is defined as a promise to pay, and default means that agent does not -or cannot- keep some of these promises and incurs default penalty. The default is ascribed to the impossibility of perfectly foreseeing all possible contingencies. In the environments, they focus on perfect competition in the given security markets where some kind of market incompleteness is equivocally stimulated. However, they avoid game-theoretic treatments of default. Also, Lustig [18] studies default by presuming collateral in Lucas-type asset markets in infinite horizon. That is, if agent opts for a default, then all his assets are seized as collateral, but the agent can keep his private endowment from the seizure, and subsequently continue to access to the security markets beyond the default. However, he does not study a role of asymmetric information in loan-default negotiation.

[4] For example, Alvarez-Jermann [2], Kehoe-Perri [11] [12], Kehoe-Levine [13] [14], Kocherlakota [16].

On the other hand, as to asymmetric information models, default is ascribed explicitly to incomplete information. This paper belongs to the category. In this category, first, we can refer to the incomplete contracting literature, which formulates a default as an unforeseen sudden event, and allows renegotiation to occur ex post (Hart-Moore [9]). In particular, the literature assumes inability to write a contingent contract due to some bounded rationality, and restricts the discussions to simple institutions (ownership contract, authority problem, short-term contract, etc.). However, rational investor might anticipate default dynamically, even though it is not ex ante describable specifically due to costly disclosure, and try to enclose the anticipation in the ex ante agreement when ex post actions and payoffs are describable (Maskin-Tirole [19]). Contrary to the incomplete contracting models, this paper focuses on state-contingent aspects in loan-default negotiations, and particularly shows that the existence of the inertia leads to the irrelevance of disclosure cost from an ex-ante viewpoint. In terms of dynamic CSV models, Wang [28] is most closely related to this paper in the sense that they study a dynamic CSV with deterministic monitoring and 2-state income shocks in infinitely horizontal discrete-time.[5] That paper presumes IID technology shocks of privately observed incomes. Then, there is no intertemporal link between recurrent monitorings in equilibrium. In fact, their main concern is not regarding equilibrium default behaviors. In their monitoring equilibrium, a monitoring occurs in and only in low state. In that sense, the default behavior is static to shock. By presuming a risk-averse borrower with CARA one-time utility function, their study focuses on a comparative static relationship between the ARA coefficient and the cut-off monitoring cost level below which a conditional monitoring policy only in low state is optimal. Due to the time-separable utility structure, the ARA coefficient works as substitution not only across states but also over time. So, there is a non-monotonic relationship between the cut-off monitoring cost and the ARA coefficient.[6] In contrast, this paper focuses on equilibrium loan-default negotiation performance under Markov technological structure. In most dynamic situations, intertemporal exogenous links of privately observed incomes are realistic. Especially, individual income process is likely to be serially correlated in actual economies. As pointed out in Fernandes-Phelan [7], with the assumption of intertemporal exogenous links of private variables, at the beginning of a given date, a borrower's forward looking utility that follows a given strategy for a given contract is not independent of past histories. In general, such time-dependence of privately-observed variable causes some pertinent effect on equilibrium contractual performance. Tchistyi [26] studies the optimality

[5]Monnet-Quintin [21] study stochastic monitoring in a finite number of periods.

[6]A recursive utility formulation such as a Kihlstrom-Mirman type [15] might induce interesting implications.

of a credit line with performance pricing under the circumstances where privately-observed variable follows two-state Markov process and voluntary reports are possible. On the line, this paper elaborates time-dependent equilibrium default behaviors, especially the existence of inertia after a default and the co-existence of a no-monitoring pooling episode and a conditional-monitoring separating episode.

The empirical research is beyond the scope of this dissertation, although the results proved here provide a basis for such research. Duffie-Singleton [5] and Duffie-Pedersen-Singleton [4] study defaultable bond models where default time arrives based on an exogenously-given intensity probability distribution. Such exogeneity of default may be a good approximation under some parametric environments but is unrealistic in terms of loan-default negotiation. This paper may give a structural-form interpretation to those defaultable bond models.

This paper is organized as follows. Next section describes our physical and informational environments under a dichotomous payment game approach. Section 3 formalizes a game-theoretic representation of the loan-default negotiations, and defines equilibrium notion. Section 4 characterizes the equilibrium and solves. Section 5 generalizes the game formulation into an optimal contracting problem. Final section concludes with some discussions of possibilities of future extensions.

## 2. Environment

2.1. **Players.** There are two infinitely-lived players: borrower and lender (denoted by player $i = 1, 2$, respectively) in discrete-time: $t \in T := \{0, 1, \dots, \infty\}$. Each consumes non-storable consumption goods (or output) and has a linear one-period utility with a constant discount rate $0 < \beta < 1$. Precisely, for some sequences $\{c_{1t} \geq 0, c_{2t} \geq 0\}_{t=0}^{\infty}$ where $c_{it}$ denotes one-period consumption level of player $i$ at date $t$, lifetime utility levels are defined as $\sum_{t=0}^{\infty} \beta^t c_{1t}$ and $\sum_{t=0}^{\infty} \beta^t c_{2t}$. However, player 2's autarky lifetime utility levels are lower-bounded by positive values, which may depend on the termination-date state of nature (See below).

2.2. **Technology.** Player 2 has one unit of physical input before date 0, while player 1 has no input. If player 2 invests in player 1' project, then the project at date $t$ produces $S_t$ units of consumption goods: $S_t \in \mathbb{S} = \{L, H\}$ with $0 < L < H$ and $S_0 = L$. Call $S_t$ as state of nature (or state) at date $t$ and $\mathbb{S}$ as state-of-nature space. State-of-nature (or income shock) process $S_t$ is first-order Markov: $S_{t+1} = S_t$ with probability $\pi_{S_t} > \frac{1}{2}$; $S_{t+1} \neq S_t$ with probability $1 - \pi_{S_t}$ where $\frac{1}{2} < \pi_L < \pi_H$. In words, both shocks are persistent, and in addition, high-income shock is more persistent than low-income shock.[7] If player 2 does not invest in player 1's project, then no production occurs in the economy.

State verification (or disclosure) technology is available and provides accurate information of current output of player 1 to player 2. For each $t$, let disclosure result be denoted by $m_t \in \{-,+\}$ where $m_t = +$ means use of disclosure and $m_t = -$ means no disclosure. $S_t$ is common knowledge if disclosure occurs at $t$ (or if $t = 0$), while $S_t$ is private information of player 1 otherwise. The disclosure cost is non-pecuniary, that is, if disclosure occurs, then the non-pecuniary cost $z$ is subtracted from player 1's utility level at the date.

2.3. **Game form and information structure.** Before date 0, player 2 agrees to let player 1 use the capital for an indefinite period. The agreement is characterized by a dichotomous set $\{x_l, x_h\}$. In this model, we impose a certain choice set $\{x_l, x_h\}$ as given. In the sense, this model does not deal with mechanism designs of the agreement, but studies an equilibrium under a given game form (or agreement). In particular, we assume $0 < x_l < L < x_h < H$.[8] At the same time, player 2 invests his input in player 1's project. From date 0 onwards, player 1 produces output with the input, and makes a payment to player 2 under the agreement. Timing of events at the stage game at each date from date 0 is as follows. At each date $t$, output is produced from player 1's project with the input, and provides accurate knowledge of current state to player 1. Then, player 1 decides whether or not to disclose the state to player 2. If disclosure occurs, then disclosure cost is imposed on player 1, and player 2 sees the state. At the same time, player 1 makes a payment level $x(t)$ to player 2, choosing it from the given set $\{x_l, x_h\}$. Then, they consume the allocated goods. Next, player 2 chooses whether or not to terminate the agreement ($A$ means player 2's agreement termination decision and induces autarky from then onwards forever, while $C$ means his continuation decision). On the one hand, if player 2 decides to commit to the agreement, then the game moves on to the next-date stage game. On the other hand, if player 2 decides to terminate the agreement, then each lives in autarky subsequently forever. Player 1 has no chance of having positive consumption from the date on forever, while player 2 receives $V_{\text{-}L}$ (or $V_{\text{-}H}$, respectively) units of lifetime utility at the termination date when the termination-date state is $L$ (or $H$). In particular, we assume that $\frac{\beta x_l}{1-\beta} < V_{\text{-}L} \leq V_{\text{-}H} < \bar{V}_{\text{-}H}$ where $\bar{V}_{\text{-}H} = \frac{\beta[(1-\beta\pi_L)\{\pi_H x_h + (1-\pi_H)x_l\} + (1-\beta\pi_H)\{\pi_L x_l + (1-\pi_L)x_h\}]}{(1-\beta\pi_H)(1-\beta\pi_L) - \beta^2(1-\pi_H)(1-\pi_L)}$. The lower bound $\frac{\beta x_l}{1-\beta}$ implies player 2's worst future scenario under the agreement, that is, his discounted continuation utility level when low payment is made from the next period onwards

[7]The assumption may insure that the borrower's income tends to be sufficiently high in probability to cause lender's incentive to keep the contractual relationship, even though low shock is persistent.

[8]This assumption is restrictive, but makes clear the effect of loan-default negotiations in the Markovian technological structure.

forever. So, the lower bound insures player 2's autarky threat to be credible. As to the upper bound $\bar{V}_{\text{-}H}$, suppose that high payment $x_h$ is necessarily made in state $H$. Then, player 2's discounted expected continuation utility levels after current state $H$ and $L$ respectively are:

$$\bar{V}_{\text{-}H} = \beta \left\{ \pi_H \left( x_h + \bar{V}_{\text{-}H} \right) + (1 - \pi_H) \left( x_l + \bar{V}_{\text{-}L} \right) \right\},$$

$$\bar{V}_{\text{-}L} = \beta \left\{ \pi_L \left( x_l + \bar{V}_{\text{-}L} \right) + (1 - \pi_L) \left( x_h + \bar{V}_{\text{-}H} \right) \right\}.$$

Directly, the parametric result is achieved. Also, $\bar{V}_{\text{-}H} > \bar{V}_{\text{-}L}$. So, the upper bound implies player 2's best future scenario after current state $H$ when high payment $x_h$ is made in state $H$ under the agreement, that is, his discounted continuation utility level after current state $H$ when high payment $x_h$ is necessarily made in state $H$. So, the upper bound keeps player 2 from exploiting too much rent. In summary, the inequalities imply that player 2 has sufficiently beneficial, but not so beneficial, outside options relative to the agreement characterized by the dichotomous payment choice set $\{x_l, x_h\}$. Note that we assume that at the time of player 2's agreement termination decision, the true state is revealed without any cost. During the game, player 1 cannot make any report – even an unverified one – to player 2 under the agreement. By the assumption of $x_h > L > x_l > 0$, $x_h$ is not feasible in state $L$. Hence, the high payment itself would reveal the high state $H$. Therefore, information flows from player 1 to player 2 either if high payment, determined by player 1 and observed by player 2, is made or if disclosure takes place. Player 1's information consists of calendar date, current and past outcomes, and disclosed outcomes, current and past payments, while player 2's information consists of calendar date, initial state $L$, and when disclosure has occurred in the past, what was the outcome, and current and past payments. Notationally, define $p_t \in [0, 1]$ as player 2's date-$t$ posterior belief that date-$t$ state would be $H$ after player 1's date-$t$ actions. Also, $V_{\text{-}}(p_t) := (1 - p_t) V_{\text{-}L} + p_t V_{\text{-}H}$. The timing of the events in the game is shown in Table 2.1. Also, the game tree in an extensive-form stage game is shown in Figure 1.

| Timing of events |
|---|
| ♢ Before date 0: |
| 1. Two players make a lending contract. |
| 2. Player 2 transfers his input to player 1. |
| ♢ Stage game at each date from date 0: |
| 1. Output is produced from player 1's project with the input, and provides |
| accurate knowledge of current state to player 1. |
| 2. Player 1 decides whether or not to disclose the state to player 2. |
| 3. If disclosure occurs, then disclosure cost is imposed on player 1, and player 2 |
| sees the state. |
| At the same time, player 1 makes a payment to player 2. They consume |
| the allocated goods. |
| 4. Player 2 decides whether or not to terminate the contract. |
| 5. If player 2 terminates the contract, then player 2 repossesses the input. |
| The game ends. Then, each lives in autarky subsequently. |
| On the other hand, if player 2 continues the contract, then they move on to |
| next-date stage game. |

Table 1. Timing of events

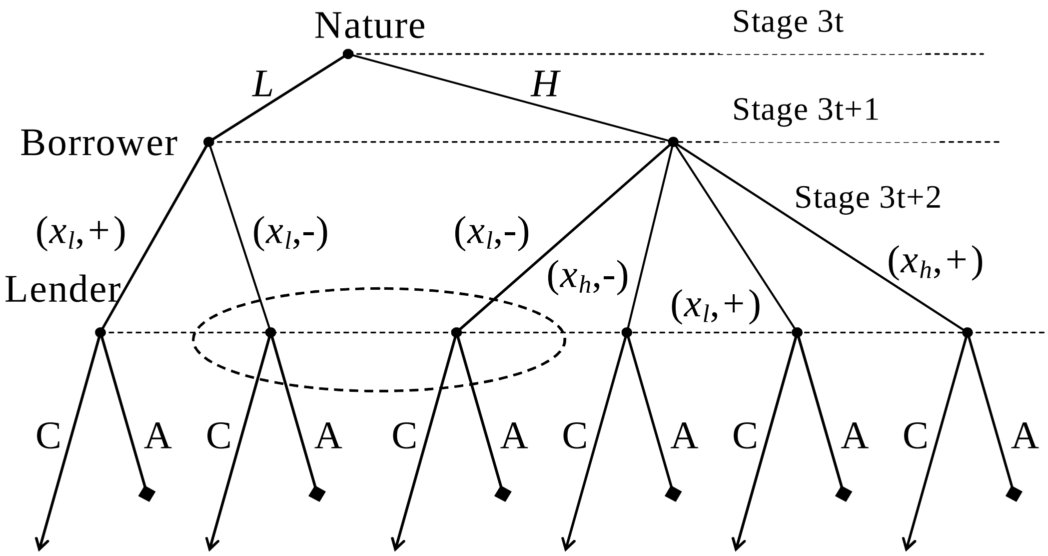


Figure 1. Date-$t$ extensive-form stage game

## 3. Strategies and equilibrium notion

The game form is characterized by $\{x_l, x_h\}$. This game is essentially an indirect communication game in the sense that by construction a standard direct communication is impossible, and so the message space consists of physical actions and outcomes. Player set is expanded to involve player 0 that represents nature who controls state of nature at chance nodes. Hence, $i \in I := \{0,1,2\}$. The stage index of the extensive-form game is defined as $k = k(t,j) = 3t + j$ for each $t \in T$ and $j = 0,1,2$. Also, $t(k)$ is integer part of $\frac{k}{3}$ and $j(k) = k - 3t(k)$. That is, each date $t \in T$ consists of three extensive-form game stages $\{3t, 3t+1, 3t+2\}$. Let $K$ denote the set of all the stages. Let $n$ (or $n(k)$) and $N_k$ respectively denote a node (or in particular, a node at stage $k$) and the set of nodes at stage $k$. $N := \bigcup_{k\in K} N_k$. Let $\mathcal{N}$ (or $\mathcal{N}_k$ resp.) denote all the subsets of $N$ (or $N_k$) where $\mathcal{N}_k \subset \mathcal{N}$. Let $E$ denote the set of terminal nodes in the game form. Then, every node $n \in N \diagdown E$ is characterized by two labels: a player who controls the node, $i(n) \in I$, and $i(n)$'s information set $s(n)$. Also, we may write $n = i(n).s(n)$. In particular, 0.0 denotes the root of the whole game at date 0, that is, $N_0 = \{0.0\}$. Note that for $n \in N_{3t+j}$, $i(n) = j \in I$.

Next, formalize moves. Let $M$ denote the set of all possible moves in the game form, $M := \{L,H\} \cup \{\{-,+\} \times \{x_h, x_l\}\} \cup \{A, C\}$. In particular, define move operator as a single-valued selection mapping $\theta : N \cup \{-1\} \to M \cup \{-1\}$ as follows.[9] First of all, $\theta(n) \in M$ for $n \in N \diagdown E$. More precisely, if $n \in N_{3t} \diagdown E$ for each $t$, then $\theta(n) \in \{L,H\}$. If $n \in N_{3t+1}$ for each $t$, then $\theta(n) \in \{-,+\} \times \{x_h, x_l\}$. For convenience, let player 1's (compound) actions be denoted by $\sigma_l^- := (-, x_l)$, $\sigma_l^+ := (+, x_l)$, $\sigma_h^- := (-, x_h)$, and $\sigma_h^+ := (+, x_h)$. $\Sigma := \left\{\sigma_l^-, \sigma_l^+, \sigma_h^-, \sigma_h^+\right\}$.[10] If $n \in N_{3t+2}$ for each $t$, then $\theta(n) \in \{A, C\}$. Secondly, if $n \in E$, then $\theta(n) = -1$. Thirdly, $\theta(-1) = -1$. Notice that, after any terminal node, this mapping has the image $-1$ as a character, not as a move. Let $\Theta$ denote the set of all possible move functions. Define a recursive operator $\langle \cdot, \cdot \rangle : \{N \cup \{-1\}\} \times \Theta \to N \cup \{-1\}$, that is, a mapping from a node or $-1$ and its move to its immediately-following node or $-1$. If $k = 3t$ for each $t$ and $n \in N_k$, then $n$ is a chance node (i.e., $i(n) = 0$) and $\theta(n) \in \{L,H\}$ and so $\langle n, L\rangle, \langle n, H\rangle \in N_{k+1}$. If $k = 3t+1$ for each $t$ and $n \in N_k$ and $n_{-1} \in N_{k-1}$, then $i(n) = 1$, $\theta(n) \in \Sigma$, and either $n = \langle n_{-1}, L\rangle$ or $n = \langle n_{-1}, H\rangle$ holds. Since by the assumption of $x_l < L < x_h$, $x_h$ is not feasible in state $L$. If the true state is $L$ (that is, $n = \langle n_{-1}, L\rangle$), then there are two possible actions. So, $\left\langle n, \sigma_l^+\right\rangle, \left\langle n, \sigma_l^-\right\rangle$ in $N_{k+1}$. On the other hand, if the true state is $H$ (that is, $n = \langle n_{-1}, H\rangle$), then there are four possible actions. So, $\left\langle n, \sigma_l^+\right\rangle, \left\langle n, \sigma_l^-\right\rangle, \left\langle n, \sigma_h^+\right\rangle, \left\langle n, \sigma_h^-\right\rangle \in$

[9] This represents strategies in a reduced form. Later, we will define the corresponding strategy mappings in a structural form.

[10] Notice that by the construction, $\sigma_h^+$ is always dominated.

$N_{k+1}$. If $k = 3t + 2$ for each $t$ and $n \in N_k$, then $i(n) = 2$, $\theta(n) \in \{A, C\}$, and $\langle n, A\rangle, \langle n, C\rangle \in N_{k+1}$. Then, $n' = \langle n, A\rangle \in N_{k+1}$ is a terminal node. Then, $\langle n', \theta(n')\rangle = \langle n', -1\rangle = -1$. Hence, $E = \{n' : n' = \langle n, A\rangle$ for any $n \in N_{3t+2} \forall t\}$, that is, the set of terminal nodes at the whole game. Further, $\langle -1, \theta(-1)\rangle = -1$. On the other hand, if $n' = \langle n, C\rangle$, then $n' \in N_{3t+3}$ and $i(n') = 0$. Then, $\langle n', L\rangle, \langle n', H\rangle \in N_{3t+4}$. Therefore, the character $-1$ implies that it follows the terminal node. Define player 1's stage-$k+1$ payoff as a mapping $\bar{u} : \{N \cup \{-1\}\} \times \Theta \to \mathbb{R}$ as follows. Fix $k$, $n \in N_k \cup \{-1\}$, and $n_{-1} \in N_{k-1} \cup \{-1\}$. If $n = \langle n_{-1}, L\rangle$, then $\bar{u}(n, \sigma_l^-) = L - x_l$ and $\bar{u}(n, \sigma_l^+) = L - x_l - z$. If $n = \langle n_{-1}, H\rangle$, then $\bar{u}(n, \sigma_l^-) = H - x_l$, $\bar{u}(n, \sigma_l^+) = H - x_l - z$, $\bar{u}(n, \sigma_h^-) = H - x_h$, and $\bar{u}(n, \sigma_h^+) = H - x_h - z$. If $n \neq \langle n_{-1}, L\rangle$ and $n \neq \langle n_{-1}, H\rangle$, then $\bar{u}(n, \theta(n)) = 0$. Also, define player 2's stage-$k+1$ payoff as a mapping $\bar{v} : \{N \cup \{-1\}\} \times \Theta \to \mathbb{R}$ as follows. Fix $k$, $n \in N_k \cup \{-1\}$, $n_{-1} \in N_{k-1} \cup \{-1\}$, and $n_{-2} \in N_{k-2} \cup \{-1\}$. If either $n = \left\langle n_{-1}, \sigma_l^-\right\rangle$ or $n = \left\langle n_{-1}, \sigma_l^+\right\rangle$ and if $n_{-1} = \langle n_{-2}, L\rangle$, then $\bar{v}(n, C) = x_l$ and $\bar{v}(n, A) = x_l + V_{\text{-}L}$. If either $n = \left\langle n_{-1}, \sigma_l^-\right\rangle$ or $n = \left\langle n_{-1}, \sigma_l^+\right\rangle$ and if $n_{-1} = \langle n_{-2}, H\rangle$, then $\bar{v}(n, C) = x_l$ and $\bar{v}(n, A) = x_l + V_{\text{-}H}$. If either $n = \left\langle n_{-1}, \sigma_h^-\right\rangle$ or $n = \left\langle n_{-1}, \sigma_h^+\right\rangle$, then $\bar{v}(n, C) = x_h$ and $\bar{v}(n, A) = x_h + V_{\text{-}H}$. For all other node $n \in N \cup \{-1\}$, $\bar{v}(n, \theta(n)) = 0$. Note that $\bar{u}(n(3t+3), \theta(n(3t+3)))$ and $\bar{v}(n(3t+3), \theta(n(3t+3)))$ are rewarded at a node $n \in N_{3t+3}$ but are discounted by $\beta^t$ as we interpret as if payoffs occur during date $t$. Also, we may write $\bar{u}(n) = \bar{u}(n, \theta(n))$ and $\bar{v}(n) = \bar{v}(n, \theta(n))$ unless they cause any confusion.

Information set at each node is formalized as follows. Player 1's information set $s(n)$ at a node $n \in N_{3t+1}$ for each $t$ are singletons because he is fully informed. On the other hand, player 2's information set $s(n)$ is not singleton because of informational asymmetry. Yet, by the assumption of $x_l < L < x_h$, the high payment itself would reveal the high state $H$. Therefore, player 2's information set $s(n)$ at a node $n \in N_2$ are $\left\{\left(0.0, L, \sigma_l^+\right)\right\}$, $\left\{\left(0.0, H, \sigma_l^+\right)\right\}$, $\left\{\left(0.0, L, \sigma_l^-\right), \left(0.0, H, \sigma_l^-\right)\right\}$, $\left\{\left(0.0, H, \sigma_h^-\right)\right\}$, and $\left\{\left(0.0, H, \sigma_h^+\right)\right\}$. Then, fix a node $n \in N_{3t+2}$ equipped with $s(n)$ for some $t \geq 1$. For $y \in N_{3t+5}$, the information set $s(y)$ are: $\left\{\left(n, L, \sigma_l^+\right) \mid s(n)\right\}$, $\left\{\left(n, L, \sigma_l^-\right) \mid s(n)\right\} \cup \left\{\left(n, H, \sigma_l^-\right) \mid s(n)\right\}$, $\left\{\left(n, H, \sigma_h^-\right) \mid s(n)\right\}$, $\left\{\left(n, H, \sigma_l^+\right) \mid s(n)\right\}$, $\left\{\left(n, H, \sigma_h^+\right) \mid s(n)\right\}$. Hence, by induction, the information set sequence is defined recursively.

Information evolution is formalized as follows. Define a measure that is assigned to a subset of $N$ as a mapping $\Delta : \mathcal{N} \to [0, 1]$ such that $\Delta(N) = 1$.[11] In addition, let the corresponding marginal measure with respect to branch mappings following $n \in N_{k-1}$ with $i(n) = 1, 2$ for each $k$ be denoted by $\delta_{t(k)}^{i(n)} : \mathcal{N}_k \to [0, 1]$. We may also write this as $\delta$ unless it causes any confusion. $\left\{\Delta_t^1, \Delta_t^2\right\} := \left\{\delta_s^1, \delta_s^2\right\}_{s \geq t}^{\infty}$. For each stage $k = 3t \; \forall t \in T$, let true chance probability be denoted by a measure $\pi : \mathcal{N}_{k+1} \to [0, 1]$ such that if $\theta\left(n\left(3t+1\right)\right) = \theta\left(n\left(3t+4\right)\right) = L$, then $\pi\left(n\left(3t+4\right)\right) = \pi_L$,

[11]The measure is well-defined by the Kolmogorov extension theorem.

if $\theta(n(3t+1)) = L$ and $\theta(n(3t+4)) = H$, then $\pi(n(3t+4)) = 1-\pi_L$, if $\theta(n(3t+1)) = H$ and $\theta(n(3t+4)) = L$, then $\pi(n(3t+4)) = 1-\pi_H$, and if $\theta(n(3t+1)) = \theta(n(3t+4)) = H$, then $\pi(n(3t+4)) = \pi_H$. Next, for each stage $k = 3t+1$ for each $t$, define player 2's posterior belief of which node he is located in. Assume that the player 2's belief over current state evolves based on Bayesian updating while not revealing the true states. Precisely, if state $L$ is revealed at date $t \geq 0$, then $p_t = 0$, while if state $H$ is revealed at date $t$, then $p_t = 1$. Afterwards, so long as true state is not revealed either via disclosure or high payment, $p_t = \pi_H p_{t-1} + (1-\pi_L)(1-p_{t-1})$. Define $\lambda(p_{t-1}, \sigma_t)$ as 1 if state $L$ is revealed at date $t$ given $p_{t-1}, \sigma_t$ (otherwise 0) and $\eta(p_{t-1}, \sigma_t)$ as 1 if state $H$ is revealed at date $t$ given $p_{t-1}, \sigma_t$ (otherwise 0). Also let the posterior in not revealing true states at date $t$ given $p_{t-1}$ be denoted by $\mu(p_{t-1}) = \pi_H p_{t-1} + (1-\pi_L)(1-p_{t-1})$. Then, $p_t = \psi(p_{t-1}, \sigma_t) := \lambda(p_{t-1}, \sigma_t) \cdot 0 + \eta(p_{t-1}, \sigma_t) \cdot 1 + \{1 - \lambda(p_{t-1}, \sigma_t) - \eta(p_{t-1}, \sigma_t)\} \cdot \mu(p_{t-1})$. $p_{t+k}$ monotonically converges to $P := \frac{1-\pi_L}{(1-\pi_H)+(1-\pi_L)}$ as $k$ goes to infinity during no state-revealing. Suppose that low state is revealed at a certain date. Then, his posterior belief of high state is monotonically, concavely increasing as time goes unless the true state is revealed, like $p = 0, 1-\pi_L, (\pi_H + \pi_L)(1-\pi_L), \ldots$, and converges to a posterior that high shock is probable (i.e., $P > \frac{1}{2}$). An intuitive logic behind that belief evolution is as follows. Since low state is persistent (i.e., $\pi_L > \frac{1}{2}$), player 2 believes that low state is probable (i.e., $p < \frac{1}{2}$) for a while after low state is revealed. But he also knows that once high state occurs even with a small chance (i.e., $(1-\pi_L)$), high state is repeated more persistently at the subsequent dates (i.e., $\pi_L < \pi_H$). Due to the higher persistence of high state, his current belief of high state is increasing date-by-date. After a while, it exceeds $\frac{1}{2}$. The increment is getting smaller as time goes since the probability is, by definition, upper-bounded. Then, the posterior $p$ converges to $P > \frac{1}{2}$. Parallel to this belief evolution, after low state is revealed at a date, the expected autarky utility (i.e., $V_{\text{-}}(p_t) = (1-p_t) V_{\text{-}L} + p_t V_{\text{-}H}$) is monotonically, concavely increasing in $t$ unless the true state is revealed. The set of the posterior sequences is denoted by $\Pi \subset [0,1]^{\mathbb{N}\times\Sigma}$. Now, for each stage $k = 3t$ for each $t$, define player 2's posterior belief of which node he is located in as a measure $\bar{\pi} : \mathcal{N}_{k+1} \to [0,1]$ such that if $\theta(n(3t+1)) = L$, then $\bar{\pi}(n(3t+1)) = 1 - p_t$ and if $\theta(n(3t+1)) = H$, then $\bar{\pi}(n(3t+1)) = p_t$. Hence, $\pi_s$ is recursively induced as functions of $\bar{\pi}, \Delta^1, \Delta^2$.

Now, define player 1's expected discounted continuation utility $U(3t+1, \Delta_t^1, \Delta_t^2, \bar{\pi} \mid n)$ at a node $n \in N_{3t+1}$ for each $t$ as:

$$U(3t+1, \Delta_t^1, \Delta_t^2, \bar{\pi} \mid n(3t+1) \in N_{3t+1})$$

$$= \sum_{n(3t+2)\in N_{3t+2}} \delta(n(3t+2)) \cdot \left[ \begin{array}{c} \bar{u}\left(n\left(3t+2\right)\right) + \sum_{n(3t+3)\in N_{3t+3}} \delta\left(n\left(3t+3\right)\right) \cdot \\ \beta \left[ \begin{array}{c} \sum_{n(3t+4)\in N_{3t+4}} \pi\left(n\left(3t+4\right)\right) \cdot \\ U(3t+4, \Delta^1_{t+1}, \Delta^2_{t+1}, \bar{\pi} \mid n(3t+4) \in N_{3t+4}) \end{array} \right] \end{array} \right]$$

Also, define player 2's expected discounted continuation utility $V(3t+2, \Delta^1_{t+1}, \Delta^2_t, \bar{\pi} \mid n)$ at a node $n \in N_{3t+2}$ for each $t$ as:

$$V(3t+2, \Delta^1_{t+1}, \Delta^2_t, \bar{\pi} \mid n(3t+2) \in N_{3t+2})$$

$$= \sum_{y\in s(n(3t+2))} \pi_s(y) \cdot$$

$$\sum_{n(3t+3)\in N_{3t+3}} \delta(n\left(3t+3\right)) \left[ \begin{array}{c} \bar{v}\left(n\left(3t+3\right)\right) + \beta \sum_{n(3t+4)\in N_{3t+4}} \bar{\pi}\left(n\left(3t+4\right)\right) \cdot \\ \left[ \begin{array}{c} \sum_{n(3t+5)\in N_{3t+5}} \delta\left(n\left(3t+5\right)\right) \cdot \\ V(3t+5, \Delta^1_{t+2}, \Delta^2_{t+1}, \bar{\pi} \mid n(3t+5) \in N_{3t+5}) \end{array} \right] \end{array} \right]$$

The assessment $\left(\Delta^1_0, \Delta^2_0, \bar{\pi}\right)$ is said to be a *weak sequential equilibrium* (or wSE) iff for each $t$ and every $\tilde{\Delta}^1_t$ and $\tilde{\Delta}^2_t$, $U(3t+1, \Delta^1_t, \Delta^2_t, \bar{\pi} \mid n \in N_{3t+1}) \geq U(3t+1, \tilde{\Delta}^1_t, \Delta^2_t, \bar{\pi} \mid n \in N_{3t+1})$ and $V(3t+2, \Delta^1_{t+1}, \Delta^2_t, \bar{\pi} \mid n \in N_{3t+2}) \geq V(3t+2, \Delta^1_{t+1}, \tilde{\Delta}^2_t, \bar{\pi} \mid n \in N_{3t+2})$. In particular, a wSE is said to be a *trade equilibrium* iff it is strictly preferred to financial autarky for each time.

## 4. Equilibrium

4.1. **Stationary pure-strategy trade equilibrium.** This section studies the long-run property of a trade equilibrium. In particular, we focus on a stationary trade equilibrium in which players 1 and 2 play strategies of a particular form in the following senses. First, we focus on pure-strategies of both players. Assume that the game has a pure-strategy wSE. Second, player 1's strategies are assumed to be stationary and Markovian in the sense that they depend only on the fixed-dimensional, current coarsest information set. Specifically, player 1's pure-strategy is characterized by a selection mapping $\tilde{\sigma} : [0,1] \times \{L, H\} \to \Sigma$, that is, $\tilde{\sigma}$ consists of disclosure and payment pure strategies, each of which maps player 2's stage-$3t+1$ posterior belief $p_{t-1}$ (i.e., before player 1's action at date $t$) and player 1's privately-observed current true state $S_t$ to either player 1's disclosure/payment single-valued selection. Let $\tilde{m}$ and $\tilde{x}$ respectively denote the disclosure and payment pure-strategies, that is, $\tilde{\sigma} = (\tilde{m}, \tilde{x})$. In particular, $\tilde{x}$ is said to be *feasible* if $0 \leq \tilde{x}(p_{t-1}, H) \leq H$, and $0 \leq \tilde{x}(p_{t-1}, L) \leq L$ for each $p_{t-1}$. Let $\tilde{X}$ denote the set of feasible $\tilde{x}$. By the assumption of $x_h > L$, $x_h$ is not feasible in state $L$. Let $\tilde{M}$ denote the set of $\tilde{m}$. Then, $\tilde{\Sigma}$ represents the set of feasible disclosure/payment pure-strategies. $\sigma_t = (m_t, x_t)$ denotes player 1's stage-$3t+1$ action pair. On the other hand, player 2's autarky enforcement pure-strategy is characterized by a mapping: $\tilde{a} : [0,1] \times \Sigma \to \{A, C\}$, that is, a mapping from player 2's current posterior belief $p \in [0,1]$, and player 1's $\sigma \in \Sigma$ to player 2's autarky enforcement decision $a \in \{A, C\}$. Also, $a_t$ denotes player 2's stage-$3t+2$ action. Directly from the previous section's equilibrium representation, the pure-strategy wSE is well-defined. Let the optimized action sequences be denoted by $\{\sigma_t^*, a_t^*\}_{t=0}^{\infty}$ where $\sigma_t^* = (x_t^*, m_t^*)$ for each $t$. Let player 1's and player 2's date-$t$ optimized utility levels respectively be denoted by $u^*(t, p_{t-1}, S_t)$ and $v^*(t, p_t)$ induced by the optimized action sequences from date $t$ on.

A stationary wSE is categorized into three episodes from a dynamic perspective. Precisely, since this program involves informational asymmetry, the equilibrium allocation may take one of three episodes for each $t$: (1) a full-disclosure episode (disclosure occurs for either state at date $t$), (2) a disclosure-contingent separating episode (disclosure occurs when and only when state $L$ occurs at date $t$), and (3) a no-disclosure pooling episode (disclosure never occurs for either state at date $t$). Over time, the wSE may evolve over the three episodes. They are categorized via screening conditions, which are induced directly by player 1's disclosure/payment actions. In particular, since the technological structure is Markovian, the screening conditions are history-dependent. Therefore, the optimization procedure is forward-looking, and has difficulty with solving for the stationary solutions via standard dynamic programming method as in Stokey-Lucas [25]. Against the difficulty, as in Fernandes-Phelan [7]00), let $w(p_t)$ denote player 2's date-$t+1$ continuation utilities that player 1 would insure to player 2 just at stage $3t+2$ by taking a particular action $\sigma_t$ given $p_{t-1}$ and $\psi(p_{t-1}, \cdot)$ at stage $3t+1$ (i.e., $p_t = \psi(p_{t-1}, \sigma_t)$) when assuming that both players 1 and 2 behave optimally from stage $3t+2$ on given $\sigma_t$. Crucially, $\sigma_t$ might not be an equilibrium. Then, the continuation utilities $w(p_t)$ are classified into three cases by particular values of $p_t = \psi(p_{t-1}, \sigma_t)$, each of which is induced by player 1's particular action $\sigma_t$:

$$w(1) = E_t^2 \left[ v^* \left( t+1, \psi \left( 1, \sigma_{t+1}^{**} \right) \right) \right]$$

$$w(0) = E_t^2 \left[ v^* \left( t+1, \psi \left( 0, \sigma_{t+1}^{**} \right) \right) \right]$$

$$w(\mu(p_{t-1})) = E_t^2 \left[ v^* \left( t+1, \psi \left( \mu(p_{t-1}), \sigma_{t+1}^{**} \right) \right) \right]$$

where $E_t^2$ denotes player 2's expectation operator over information state at his decision stage $3t+2$, and $\left\{ \sigma_{s+1}^{**}, a_s^{**} \right\}_{s=t}^{\infty}$ denotes the optimized action sequences given particular $\sigma_t$. Among the above three particular actions, one is in equilibrium while the others are out of equilibrium. Hence,

$\{\sigma_{s+1}^{**}, a_s^{**}\}_{s=t}^{\infty}$ may be different from $\{\sigma_{s+1}^{*}, a_s^{*}\}_{s=t}^{\infty}$ if $\sigma_t$ is out of equilibrium. The equilibrium continuation utility formulates promise-keeping, while the out-of-equilibrium continuation utility formulates threat-keeping. Let $\sigma(p_t) = (x(p_t), m(p_t))$ denote the action pair $\sigma_t = (x_t, m_t)$ that induces $w(p_t)$. Also, let player 1's expected next-date discounted utility level when promising expected next-date (i.e., date-$t+1$) discounted utility level $w(p_t)$ to player 2 after stage-$3t+1$ action given $S_t$ when assuming that both players 1 and 2 behave optimally from stage $3t+2$ on given $\sigma_t$ be denoted by:

$$y(S_t, w(p_t)) := E_t^1 \left[u^* \left(t+1, S_{t+1}, p_t\right) \mid S_t\right]$$

where $E_t^1 [\cdot \mid S_t]$ denotes player 1's expectation operator over next-date state conditional on $S_t$ at his decision stage $3t+1$. Now, we characterize conditions that classify the above three episodes. First, given $p_{t-1} \neq 1$, if player 2's expected autarky utility is relatively low (i.e., $\beta w(\mu(p_{t-1})) \geq V_{\text{-}}(\mu(p_{t-1})))$, then player 2 commits to the agreement, regardless of player 1's action. So, player 1 takes a low payment/no disclosure action. That is, it is a pooling episode. On the other hand, given $p_{t-1} \neq 1$, if player 2's expected autarky utility is sufficiently high (i.e., $\beta w(\mu(p_{t-1})) < V_{\text{-}}(\mu(p_{t-1})))$, then the true state may be revealed. That is, either episode (1) or (2) occurs. By the surjectiveness of the strategy mappings, a direct-revelation principle holds. So, we confine our attention to a dynamic signaling problem. Notice, however, that this model is different from standard dynamic signaling problems in the sense that due to the disclosure cost, there are dates during which player 2 does not enforce autarky if player 1 makes the low payment without revealing his true state. We call the dates an inertia time. Importantly, the inertia is caused endogenously in equilibrium and does not disturb both players' making a state-contingent contract ex ante. Define a date-$t$ screening condition as follows:

(4.1) If $p_{t-1} = 1$ or if $\beta w(\mu(p_{t-1})) < V_{\text{-}}(\mu(p_{t-1}))$ for $p_{t-1} \neq 1$, then

$$\beta y(L, w(0)) \geq z \text{ and } \beta y(H, w(1)) \geq \min\{x_h - x_l, z\}.$$

In words, suppose that high state is not revealed at the previous date (i.e., $p_{t-1} \neq 1$), and that if the true state were concealed at this date as well, lender's expected autarky utility would be higher than the continuation utility under low-payment/no disclosure action at the current date (i.e., $\beta w(\mu(p_{t-1})) < V_{\text{-}}(\mu(p_{t-1})))$. Then, $\beta y(L, w(0)) \geq z$ implies that in state $L$, borrower takes a disclosure action as his best response (Figure 2.1). Then, if lender commits to the agreement subsequently, then borrower could renege on high payment that would be required for lender's commitment if he were not to reveal the true state. So, disclosing the low state would induce payment allowance. This incident is interpreted as a default in our terminology. On the other hand, $\beta y(H, w(1)) \geq \min\{x_h - x_l, z\}$ implies that in state $H$, borrower reveals the true state by taking either a disclosure action or a high payment action as his best response (Figure 1). The same logic holds when the high state was revealed at the previous date (i.e., $p_{t-1} = 1$). So, the above condition (4.1) characterizes borrower's date-$t$ action such that if the high state was revealed at the previous date, or if lender's expected autarky utility would be higher than the continuation utility under low-payment/no-disclosure action when the true state were concealed at the current date, then borrower reveals the true state in either state as his best response. Contrary to standard signalling models, because of the disclosure cost, the truth-revealing is not required in equilibrium when the condition $\beta w(\mu(p_{t-1})) \geq V_{\text{-}}(\mu(p_{t-1}))$ holds. In fact, if $\beta w(\mu(p_{t-1})) \geq V_{\text{-}}(\mu(p_{t-1}))$, then borrower's low-payment/no-disclosure action could cause lender's commitment to the agreement. We show below that the disclosure cost could be irrelevant to a state-contingent agreement in equilibrium. Call the date-$t$ condition (4.1) as date-$t$ *temporary screening condition* after the terminology of Green [8]. Applying the first part of Theorem 2.1 of Fernandes-Phelan [7],

**Lemma 4.1.** *A trade equilibrium is achieved only if Condition (4.1) is satisfied for all $t$.*

Now, we prove the existence of a stationary wSE. The proof consists of two steps. First, we impose several assumptions, some of which put restrictions on endogenous variables, and show there exists a stationary wSE under the assumptions. Second, we show that conversely, if there is a trade equilibrium, then it satisfies all the assumptions. Assumptions are as follows. First,

**Assumption 4.1.** $V_{\text{-}L} < V_{\text{-}H}$.

This implies that the autarky threats are state-dependent (that is, player 1 and 2's utility are positively correlated). That economic meaning is discussed in the next subsection. Next, assume that either episode (1) or (2) occurs in equilibrium:

**Assumption 4.2.** $\exists t : \beta w(p_t) \geq V_{\text{-}}(p_t)$ *and* $\beta w(\mu(p_t)) < V_{\text{-}}(\mu(p_t))$.

Without this assumption, player 1 may make only low payment in no-disclosure pooling situation throughout the game. Then, player 2 cannot commit to the agreement. Third, as to the screening conditions,

**Assumption 4.3.** *Condition (4.1) is satisfied for all $t$.*

Due to Lemma 4.1, this is required to achieve a trade equilibrium. In addition, the following assumptions are imposed:

**Assumption 4.4.** $\beta\{y(L, w(0)) - y(L, w(p_{t-1}))\} \leq z$ *for any* $t$ *when* $p_{t-1} < 1$.

**Assumption 4.5.** $x_h - x_l \geq \beta\{y(H, w(1)) - y(H, w(p_{t-1}))\}$ *for any* $t$ *when* $p_{t-1} < 1$.

These two assumptions imply that both disclosure cost and high payment are sufficiently high to make room for player 1 to choose a low-payment/no-disclosure action when player 2 does not enforce autarky for either disclosure decision. Further,

**Assumption 4.6.** $\beta w(1) \geq V_{\text{-}H}$.

This excludes the possibility of autarky in high state. Finally,

**Assumption 4.7.** $x_h - x_l \leq z$.

If this assumption is not true, the disclosure is so costless that player 1 has incentive to use disclosure technology and make a low payment in state $H$. So, player 2 always receives low payment. This is time-inconsistent. So, in the case, episode (1) does not occur. In terms of dynamic programming methods, current-date disclosure and payment actions have influence on the next-period decision space via the promise-keeping and screening conditions in the Markovian structure. So, generally, the equilibrium correspondence $\Gamma : \Pi \times \mathbb{S} \to \Pi$ is not convex, that is, for some $p_{t-1}, S_t$, $p_t \notin \Gamma(p_{t-1}, S_t)$. Now, $w(p_{t-1})$ and $V_{\text{-}}(\mu(p_{t-1}))$ are added to state variables. Let the extended state variable set be denoted by $Z_t := (p_{t-1}, S_t, w(p_{t-1}), V_{\text{-}}(\mu(p_{t-1})))$. Next, define an one-step recursion operator $\mathcal{T} : C(Z) \to C(Z)$ regarding player 1's optimization program given player 2's optimal actions, that is, a mapping from a continuous function of the extended state variables $Z$ to a continuous function of the extended state variables $Z$ as follows:[12]

$$\mathcal{T}(u)(Z) = \sup_{(x,m)\in\Sigma} S - x - \mathbf{1}_m z + \beta E_t^1 [u(Z') \mid S]$$

s.t. (i) $p = \psi(p_-, \sigma) = \lambda(p_-, \sigma) \cdot 0 + \eta(p_-, \sigma) \cdot 1 + \{1 - \lambda(p_-, \sigma) - \eta(p_-, \sigma)\} \cdot \mu(p_-)$

(ii) $S' = S$ with probability $\pi_S > \dfrac{1}{2}$ and

$S' \neq S$ with probability $1 - \pi_S$ where $\dfrac{1}{2} < \pi_L < \pi_H$

(iii)(a) $w(0) \leq x(\mu(0)) + \beta w(\mu(0))$.

[12]For any time-dependent variable $y$, $y$ denotes its current value, $y'$ denotes its next-date value, $y_-$ denotes its previous-date value. Define difference operator $\Delta$ such that $\Delta y = y - y_-$ and $\Delta y' = y' - y$.

(b) For $0 < p_- < 1$, if $\beta w(\mu(p_-)) \geq V_-(\mu(p_-))$, then

$w(p_-) \leq x(\mu(p_-)) + \beta w(\mu(p_-))$, or else

$w(p_-) \leq (1 - \mu(p_-)) \{x(0) + \beta w(0)\} + \mu(p_-) \{x(1) + \beta w(1)\}$,

(c) Or else $w(1) \leq (1 - \pi_H) \{x(0) + \beta w(0)\} + \pi_H \{x(1) + \beta w(1)\}$,

where $\mu(p_-) = (\pi_L + \pi_H - 1)p_- + (1 - \pi_L)$ and $V_-(p) = (1 - p) V_{-L} + pV_{-H}$. Restriction (i) denotes the evolution of player 2's posterior belief. Restriction (ii) denotes the evolution of player 1's income shock. Restriction (iii) denotes the evolution of player 2's continuation utility subject to the screening conditions characterized by Condition (4.1). Specifically, Restriction (iii-a) means that after low state is disclosed at the previous date, player 1 makes a low payment without a disclosure at current date. Then, a pooling episode (i.e., episode (3)) begins. By Assumption 4.2, such date may exist. Restriction (iii-b) means that when player 2's expected autarky utility is relatively low, player 1 keeps making a low payment without a disclosure. If it turns out that a low payment action without a disclosure would lead to autarky at current date, then the true state is revealed in either state by Condition (4.1). This is the start of episode (2). Restriction (iii-c) means that after high state is revealed at the previous date, the true state is revealed in either state at current date as well. The monotonic structure of player 2's continuation utility evolution is induced by the monotonically increasing evolution of player 2's posterior belief of high state after low state is disclosed. Let $\Phi(p_-, S)$ denote the set of $w(p_-)$ and $V_-(\mu(p_-))$ satisfying Restriction (iii) (i.e., player 2's continuation utility evolution) given $p_-, S$. By a straightforward application of the self-generation operation of Abreu-Pearce-Stacchetti [1],

**Lemma 4.2.** *Suppose that Assumptions 4.1-4.7 are satisfied. Then, $\Phi(p_-, S)$ is non-empty and compact for each $p_-, S$.*

In other words, $Z$ is sufficient to describe wSE agreement recursively (Figure 2).

Now, our main result of this section is achieved.

**Theorem 4.1.** *Suppose that Assumptions 4.1-4.7 are satisfied. Then there exists a trade equilibrium characterized by a stationary value function $u$ such that $u = u^*$. A transition function on the Markov shocks converges weakly to invariant probability distributions that have a cycle. Conversely, if there is a trade equilibrium, then it satisfies Assumptions 4.1-4.7.*

*Proof.* Suppose that Assumptions 4.1-4.7 are satisfied. State space is compact. Because the shock is first-order Markov, and because $p_\tau$ is monotonically convergent to $P$ (i.e., $\frac{1}{2} < \frac{1-\pi_L}{(1-\pi_L)+(1-\pi_H)} < 1$)

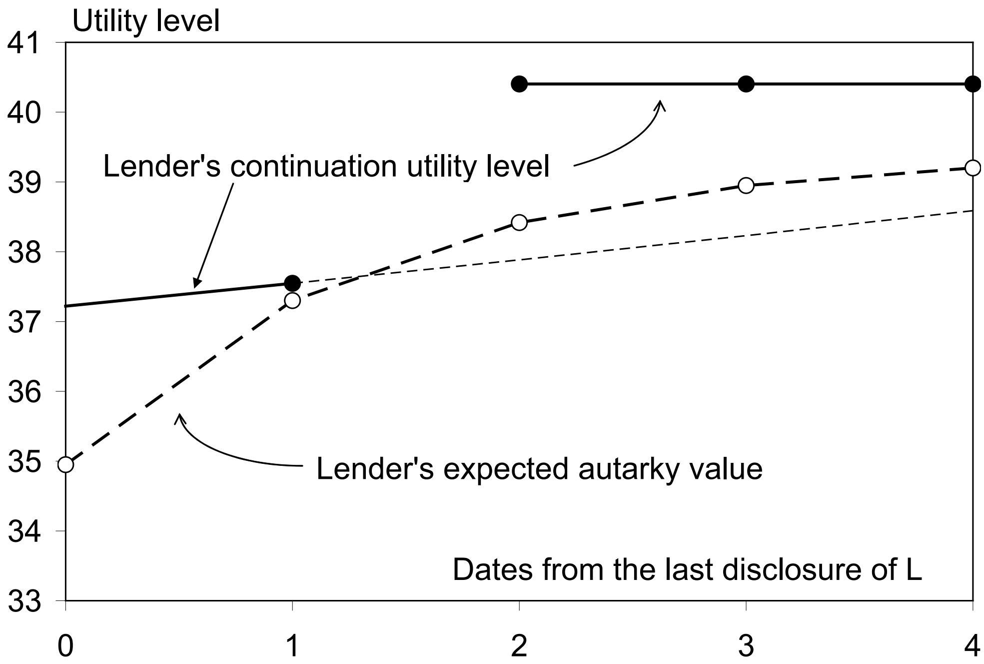


FIGURE 2. Lender's continuation utility evolution

as $\tau$ goes to infinity unless the true state is revealed, by Lemmas 4.1,4.2 above and Theorem 9.6 in Stokey-Lucas ([25], p.263), $\mathcal{T}$ has a fixed point in $C(Z)$. Therefore, $u$ is achieved. As to the shock variable $S$, since the shock space is compact and a transition function on the Markov process has the Feller property and is monotone, there exist invariant probability distributions, to which the transition function converges weakly. Due to the time-dependent screening conditions, however, the transition function does not satisfy the mixing condition. Because of the monotonic belief evolution, the invariant distributions have cycles after each disclosure. By the principle of optimality, $w(p_-)$ is monotone in $p_-$. Next, conversely, we examine the sufficiency of the claim. Suppose that the value function exists. Directly, Assumptions 4.2, 4.4-4.7 are satisfied. Suppose that Condition (4.1) is not satisfied for some date. By Lemma 4.1, it leads to autarky almost surely. It contradicts. Hence, Assumption 4.3 is satisfied. Next, suppose that $V_{\text{-}L} = V_{\text{-}H}$ (say $V_{\text{-}}$). $\frac{\beta x_l}{1-\beta} < V_{\text{-}} < \frac{\beta\{\pi_H x_h + (1-\pi_H) x_l\}}{1-\beta}$. Suppose that $w(p_-)$ is non-decreasing in $p_-$. If $\beta w(0) \geq V_{\text{-}}$, then by the hypothesis, player 1's best response is always $\sigma_l^-$. Hence, $w = \frac{\beta x_l}{1-\beta}$. By $\frac{\beta x_l}{1-\beta} < V_{\text{-}}$, player 2 enforces autarky at stage 2. Therefore, $\beta w(0) < V_{\text{-}}$. Then, there must be some date $t^*$ such that for all $t \geq t^*$, $\beta w(p_{t^*}) \geq V_{\text{-}}$. Then, during a disclosure-contingent separating episode (i.e., $t \geq t^*$), player 1's best response causes $w(1) = \frac{\beta x_l}{1-\beta}$. It contradicts. On the other hand, suppose that $w(p_-)$ is strictly decreasing in $p_-$. Then, $\beta w(0) > V_{\text{-}}$. Therefore, $w(1) = \frac{\beta(1-\pi_H)}{1-\beta\pi_H} w(0) +$

$\frac{\pi_H x_h+(1-\pi_H)x_l}{1-\beta\pi_H} < w(0)$. That is, $w(0) > \frac{\pi_H x_h+(1-\pi_H)x_l}{1-\beta}$. It contradicts with the assumption of $V_{\text{-}} < \frac{\beta\{\pi_H x_h+(1-\pi_H)x_l\}}{1-\beta}$. Hence, Assumption 4.1 is satisfied. Also, $w(p_-)$ is strictly increasing in $p_-$. The desired result is achieved. □

Call the weakly-convergent characteristic of the stationary equilibrium as Markov stationarity. Define $\tau(t)$ as number of dates elapsed since the last disclosure of state $L$ at or prior to date $t$. In addition, let $\tau_H(\tau(t))$ denote number of dates elapsed from the last high payment after the last disclosing of state $L$ at or prior to date $t$ where we assume that $\tau_H(\tau(t))$ is not finite until high payment is made after disclosing state $L$. Immediately,

**Corollary 4.1.** *With respect to $\tilde{x}$, either the number of dates elapsed from the last disclosure $\tau$ or player 2's posterior belief of high state $p_{\tau-1}$ is a sufficient statistic.*

Therefore, there exists a mapping $f : [0,1] \to \mathbb{R}_+$, which maps the posterior $p_{\tau-1}$ to the output transfer amount $x_\tau$ such that $f(p_{\tau-1}) = \tilde{x}(p_{\tau-1}, S_\tau)$ for $\tau \geq 1$. Now, redefine time scale by $\tau$. Also, redefine value functions as follows: $V(p_\tau)$: Player 2's value function evaluated at the beginning of date $\tau \geq 0$ and $U(S_{\tau+1}, p_\tau)$: Player 1's value function for $\tau + 1 \geq 1$.

4.1.1. *Application: International finance in the world's poorest economies.* This subsection studies an application of the above model to international finance. In the model, a trade equilibrium would require player 2's outside options to be state-dependent, because Condition (4.1) in the Bayesian game would hold almost surely only if the autarky threats are state-dependent (that is, player 1 and 2's utility are positively correlated).[13] More precisely, if the autarky threats are not state-dependent, then player 1 could only pay low payment throughout the game. Then, it would lead to autarky.

**Corollary 4.2.** *If $V_{\text{-}L} = V_{\text{-}H}$, then autarky is the only equilibrium.*

This result gives some insights into autarkic-financing features in the world's poorest economies. Very poor agricultural economies tend to be independent of world business cycles caused mainly by manufacturing economies. Then, an external investor faces difficulty with sorting out privately-observed outcomes in state-independent agricultural economies.[14] Thus, the external investor could

[13] In general, if lender's autarky utility level is state-independent, there might exist a trade equilibrium only if $V_{\text{-}H} \geq \frac{\beta\{\pi_H x_h+(1-\pi_H)x_l\}}{1-\beta}$. By the model construction, borrower necessarily commits to the agreement except for choosing reorganization defaults. So, lender could exploit much rent (i.e., $V_{\text{-}H} \geq \frac{\beta\{\pi_H x_h+(1-\pi_H)x_l\}}{1-\beta} > \bar{V}_{\text{-}H}$). Also, in the equilibrium, the continuation utility tends to be negatively correlated between lender and borrower dynamically. Such trade equilibrium is not interesting from an economic perspective. Therefore, this paper excludes the case by imposing the restriction on lender's autarky utility levels. that is, $V_{\text{-}H} < \bar{V}_{\text{-}H}$.

[14] Note again that we impose the parametric restriction on lender's autarky utility.

not threaten the borrower in the economies credibly, and so would have no incentive to commit to the agreement, even if the borrower's production level is not so low. Thus, the very poor agricultural economies tend to live in financial autarky. This result might be of some relevance to many of the world's poorest economies such as Sudan.

4.2. **Solutions.** The solutions are as follows. While player 2's posterior belief of high state is low after state $L$ occurs and it is verified by disclosure, player 2 commits to the agreement no matter what disclosure action is taken, because player 2's expected autarky lifetime utility is not sufficiently high. Therefore, player 1's best response is $\sigma_l^-$. $\{\sigma_l^-, C\}$ is a pooling equilibrium for $\tau < \tau^*$ (Figure 3).

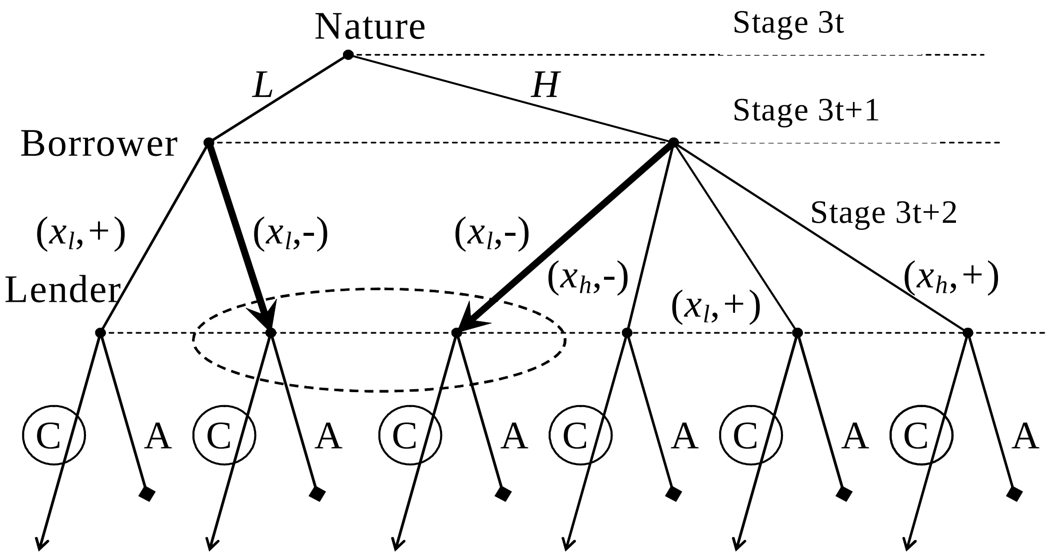


FIGURE 3. Stage game: Pooling epsiode

When player 2's posterior belief of high state hits a threshold $p^*$ after a while, player 2's expected autarky lifetime utility becomes sufficiently high to threaten the low payment. As a result, if $x_l$ is paid, then player 2 imposes autarky unless state $L$ is verified by disclosure. On the other hand, if $x_h$ is paid, then player 2 commits to the agreement no matter what disclosure action is taken. Therefore, player 1's best responses are $\sigma_l^+$ in state $L$ and $\sigma_h^-$ in state $H$. $\{\{\sigma_l^+, C\}, \{\sigma_h^-, C\}\}$ is a separating equilibrium. Hence, in the trade equilibrium, costly disclosures occur recurrently only when player 2 believes that high state is probable while player 1 knows that true state is low (Figure 4).

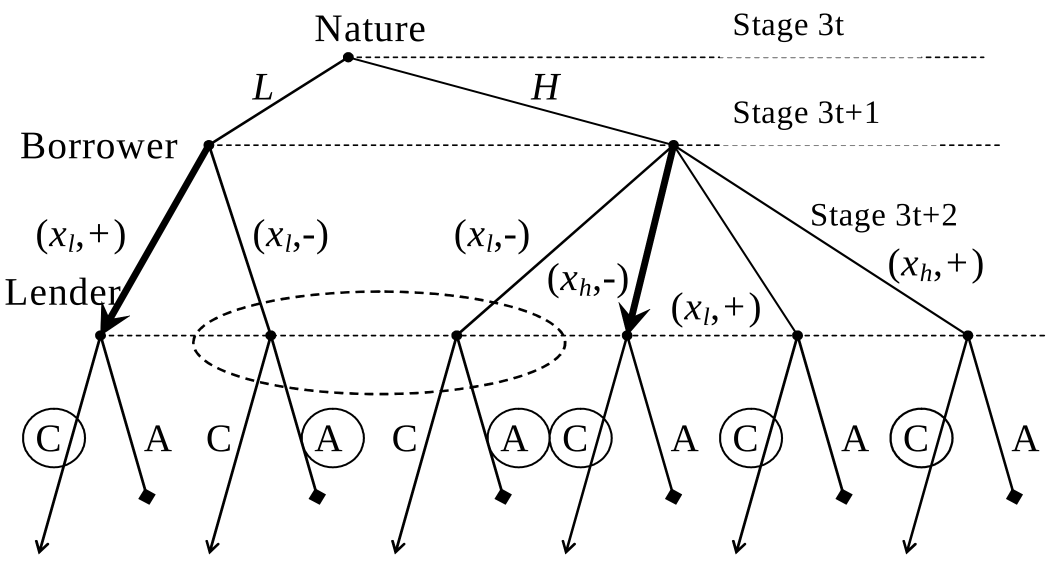


FIGURE 4. Stage game: Separating episode

**Proposition 4.1.** *In a trade equilibrium, there exists $\tau^*$ such that for $\tau < \tau^*$, $\left\{\sigma_l^-, C\right\}$ is (pooling) wSE in either state, while for $\tau \geq \tau^*$, $\left\{\sigma_l^+, C\right\}$ in state $L$ and $\left\{\sigma_h^-, C\right\}$ in state $H$ are (separating) wSE.*

Obviously, $p^* = p_{\tau^*}$. In addition, $\tau_H(\tau^*) = 0$. For $\tau < \tau^*$, the true state is not revealed from a realized profile of payment process in equilibrium. This is because the existence of disclosure cost interrupts state-revelation for low income level. In this model, player 2 cannot predict disclosures and defaults specifically, but since he can observe player 1's payment action each date, the existence of the cost does not disturb ex ante agreement in a payoff-relevant way. More precisely, player 2 allows player 1 to make only the low payments for several subsequent dates after a default while player 2 believes that the economy is in a recession. We may call the equilibrium situation as inertia. A costly disclosure is a trigger to the inertia. The inertia provides player 1 with a reorganization opportunity. The equilibrium default is a reorganization default. Then, once player 2 gets to believe that the economy is booming, high payment is required for player 2's commitment unless player 1 asks for a reorganization default by verifying his current low income level with the costly disclosure technology. The dynamic default performance is a contrast to standard CSV models.

Next, define a payoff-contingent agreement $\gamma := \mathbb{N} \times \Sigma \to \{A, C\}$ with a dated menu list $\mathbb{N} \times \Sigma$ that deterministically announced before date 0. There, $\mathbb{N} \times \Sigma$ is the dated message space, and the corresponding outcome function is well characterized as above.

**Corollary 4.3.** *A payoff-contingent agreement $\gamma$ with a date menu list: $\sigma_l^-$ for $\tau < \tau^*$ and $\{\sigma_l^+, \sigma_h^-\}$ for $\tau \geq \tau^*$ could enforce the same wSE outcomes as above.*

This type of agreement may be called as being of a dynamic defaultable-debt type in the senses that the payout processes are deterministically continuous on a.e. sample path, except for a countable, discrete set of discontinuities, and that player 1 has a right to default at any date by incurring disclosure cost and to continue the relationship beyond the default.

Finally, we simulate a trading equilibrium for a certain sequence of shock realization, and analyze comparative static with respect to structural parameters. Set $\beta = 0.97$, $(L, H) = (1, 2)$, $(V_{\text{-}L}, V_{\text{-}H}) = (35, 39)$, $(\pi_L, \pi_H) = (0.6, 0.96)$, $z = 0.45$, $(x_l, x_h) = (0.8, 1.2)$. We achieve a trade equilibrium as in Figure 5.

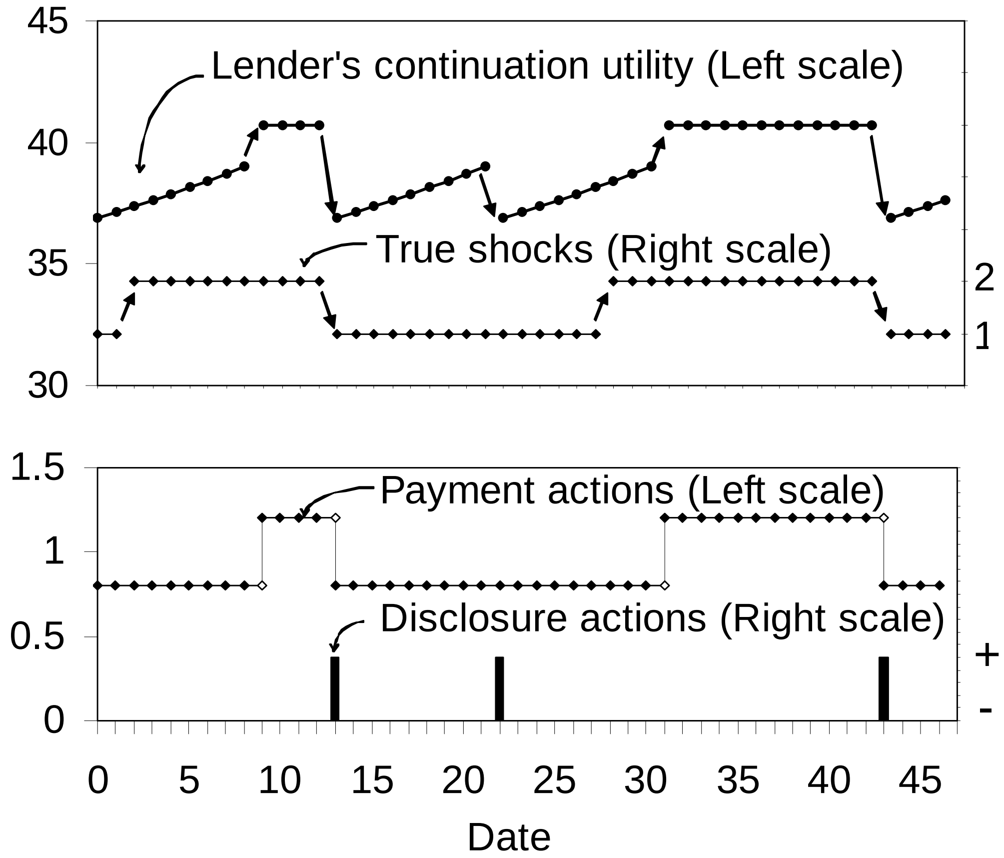


FIGURE 5. Simulation

Given the dichotomous payment choice set, Assumptions 4.3,4.7 induce the upper and lower bound $z_h, z_l$ of $z$ such that higher disclosure cost increases $\tau^*$ (that is, lowers bankruptcy probability) only if $z \in [z_l, z_h]$. If disclosure cost is too large to hold Assumption 4.3 (i.e., $z > z_h$), then no disclosure occurs in state $L$ even if the posterior is high, because the large disclosure cost would kill the agreement opportunity. On the contrary, if disclosure cost is too small to hold Assumptions 4.3 and 4.7 (i.e., $z < z_l$), then disclosure occurs in state $H$ as well as in state $L$ because player 2 prefers truth. Either case leads to autarky equilibrium, although the cost environments are totally the opposite. In particular, suppose that lowering disclosure cost causes the violation of Assumption 4.7. Then, $\sigma_l^+$ is the best response for either shock. Then, it does not save the disclosure cost in equilibrium. Player 1 does not commit to the game form, and enforces autarky. Intuitively, it means that more interim information revelation would kill the financing opportunity between the two players.

## 5. Optimal contract

This section generalizes the above dichotomous payment game formulation into an optimal contracting problem. The modifications are as follows. First, with respect to the game form, the dichotomous payment choice set $\{x_l, x_h\}$ is removed. So is the feasibility restriction $x_l < L < x_h < H$. Instead, during date-$t$ component game (each $t \in T$), at stage $3t+1$, player 1 decides whether or not to disclose his current state. Subsequently, at stage $3t + \frac{3}{2}$, player 1 makes a voluntary payment (See the modified component game form in Figure 6).

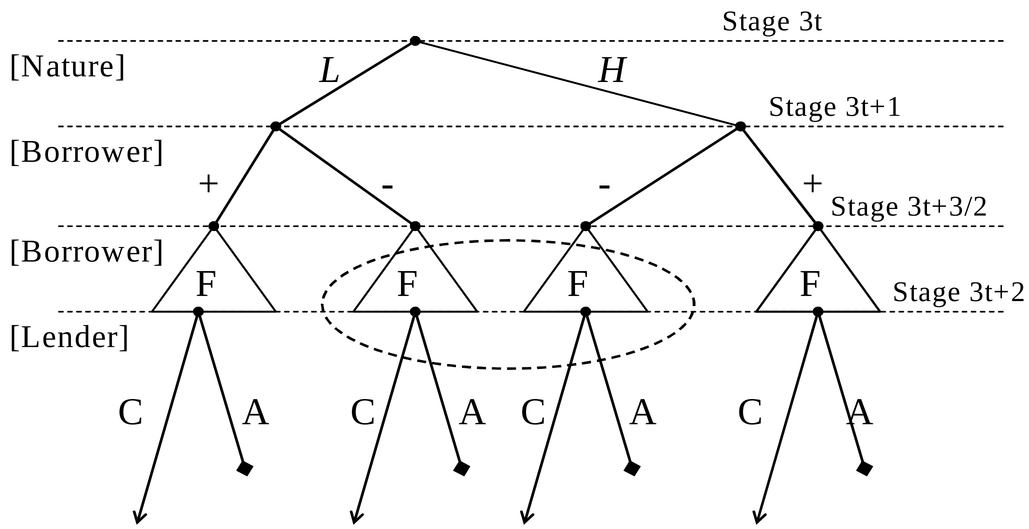


Figure 6. Modified Game Form

Therefore, with respect to stationary Markovian strategy mappings, player 1's disclosure strategy is represented by a mapping $\tilde{m} : [0,1] \times \{L, H\} \to \{-, +\}$, that is, $m_t = \tilde{m}\,(p_{t-1}, S_t)$. Player 1's voluntary payment strategy is defined as a mapping $F : [0,1] \times \{L, H\} \times \{-,+\} \to \mathbb{R}_+$, that is, $x_t = F(p_{t-1}, S_t, m_t)$. Payment must be feasible in the sense that for each $t$, $S_t - F(p_t, S_t, m_t) \geq 0$. Player 2's autarky enforcement strategy is formalized as a mapping $\tilde{a} : \{-,+\} \times \mathbb{R}_+ \times [0,1] \to \{A, C\}$, that is, $a_t = \tilde{a}(m_t, x_t, p_t)$, where $p_t$ is revised from $p_{t-1}$ after $m_t, x_t$ are realized.

Second, instead of imposing the exogenous threats $\{V_{\text{-}L}, V_{\text{-}H}\}$, we presume an endogenous threat formulation induced by player 2's self-production, which is less productive in either state than player 1's production. Precisely, if player 2 repossesses his capital, then he starts his self-production. Notice that, like the game approach, once he repossesses the capital, he must keep it forever. We presume that his income process is subject to the same Markov chain shocks as player 1's. This means that the Markov shock process is a common macro shock process. Let the outcome be denoted by $a_L$ in $L$ and $a_H$ in $H$ with $0 \leq a_L \leq a_H < H$ and $a_L < L$. Also, under the self-production, player 2 can observe his own outcome with perfect accuracy. Note that we have assumed that only producers can see the realized shocks. We still presume that player 2 decides autarky enforcement decision after receiving player 1's voluntary payment at the current date, and that if autarky is enforced, the current true state is revealed to player 2 costlessly. Then, let $V_L$ (or $V_H$) denote player 2's life-time self-production value in case the previous-date state is $L$ (or $H$). That is,

$$V_H = \frac{a_H \{(1-\beta)\pi_H + \beta(1-\pi_L)\} + a_L(1-\pi_H)}{(1-\beta\pi_H)(1-\beta\pi_L) - \beta^2(1-\pi_H)(1-\pi_L)},$$

$$V_L = \frac{a_H(1-\pi_L) + a_L\{(1-\beta)\pi_L + \beta(1-\pi_H)\}}{(1-\beta\pi_H)(1-\beta\pi_L) - \beta^2(1-\pi_H)(1-\pi_L)}$$

Therefore, $V_H \geq V_L$ with equality only if $a_H = a_L$. Let $V_A(p_t)$ denote player 2's expected lifetime self-production value from date $t+1$ onwards given his current posterior belief $p_t$ after observing player 1's current actions:

$$V_A(p_t) = (1-p_t)V_L + p_t V_H.$$

That is, $\beta V_A(p_t)$ works as expected autarky threat in date-$t$ component game for each $t \in T$.

Now, again, focus on a stationary Markovian wSE, in particular a stationary Markovian trade equilibrium. Induce a temporary screening condition for $t \in T$,

(5.1) If $F(\mu(p_{t-1}), L, 0) \geq F(0, L, 1) + z + \beta\{y(L, w(\mu(p_{t-1}))) - y(L, w(0))\}$,

then $F(1, H, 0)$ is paid in $H$ while default occurs in $L$.

In the similar way to the game approach,

**Lemma 5.1.** *A trade equilibrium is achieved only if Condition (5.1) is satisfied for all $t$.*

In a trade equilibrium, a disclosure can occur only in state $L$. In words, player 1 asks for payment allowance in a recession when payment has become too high while in an default inertia. Hence, we can guess straightforwardly that either player 2's posterior belief or the number of dates elapsed from the last disclosure of $L$ is a sufficient statistic for the voluntary payment. We may also write the voluntary payment function $F(p_t)$ in equilibrium. Therefore, we define players' continuation utility as in the game approach unless it causes any confusion.

Now, we solve for an optimal contract. The solution method consists of two steps in the similar way to the game approach: We first impose several assumptions, some of which put restrictions on endogenous variables, and show there exists a stationary Markovian trade equilibrium under the assumptions, and second conversely show that if there is a trade equilibrium, then it satisfies all the assumptions.

Assumptions are as follows. First,

**Assumption 5.1.** $a_L < a_H$.

This implies that the autarky threats are state-dependent. This is required for player 1 and 2's utility to be positively correlated in equilibrium. Without this assumption, player 2's continuation utility would be no higher than his utility level in a wSE equilibrium. Second, assume that episode (1) occurs in equilibrium:

**Assumption 5.2.** $F(\mu(p_{t-1}), L, 0) \geq F(0, L, 1) + z + \beta\{y(L, w(\mu(p_{t-1}))) - y(L, w(0))\}$ *for some $t$ almost surely.*

This corresponds to Assumption 4.2 in the game approach. Assumption 5.1 is necessary to this assumption. At least, if $F(\mu(p_{t-1}), L, 0)$ exceeds $L$, then the payment itself would reveal his current true state $H$. Physically, if $a_L, a_H$ are sufficiently high that Assumption 5.2 holds, then $V_A$ would become high enough to threaten player 1 to reveal his true state for some $t$ almost surely. Without this assumption, a trade equilibrium would not be achieved. Finally, as to the screening conditions,

**Assumption 5.3.** *Condition (5.1) is satisfied for all $t$.*

Due to Lemma 5.1, this assumption is required to achieve a trade equilibrium. In addition, it corresponds to Assumptions 4.4-4.7. Define a one-step recursion operator $\mathcal{T} : C(Z) \to C(Z)$

regarding player 1's best response program given player 2's optimal actions, that is, a mapping from a continuous function of the extended state variables $Z$ (i.e., $Z = \{p_-, S, w(p_-), V_A(\mu(p_-))\}$) to a continuous function of the extended state variables $Z$ as follows:

$$\mathcal{T}(u)(Z) = \sup_{(m,F)} S - F(p) - \mathbf{1}_m z + \beta E_t^1 [u(Z') \mid S]$$

s.t. (i) $p = \psi(p_-, \sigma) = \lambda(p_-, \sigma) \cdot 0 + \eta(p_-, \sigma) \cdot 1 + \{1 - \lambda(p_-, \sigma) - \eta(p_-, \sigma)\} \cdot \mu(p_-)$

(ii) $S' = S$ with probability $\pi_S > \dfrac{1}{2}$ and

$S' \neq S$ with probability $1 - \pi_S$ where $\dfrac{1}{2} < \pi_L < \pi_H$

(iii)(a) If $F(\mu(p_-), L, 0) < F(0, L, 1) + z + \beta\{y(L, w(\mu(p_-))) - y(L, w(0))\}$, then

$w(p_-) \leq F(\mu(p_-)) + \beta w(\mu(p_-))$ where $w(\mu(p_-)) = V_A(\mu(p_-))$.

(b) Or else $F(1, H, 0)$ is paid in $H$ while a default occurs in $L$.

where $\mu(p_-) = (\pi_L + \pi_H - 1)p_- + (1 - \pi_L)$ and $V_A(p) = (1-p)V_L + pV_H$. Therefore, in the similar to the proof of Theorem 4.1,

**Theorem 5.1.** *Suppose that Assumptions 5.1-5.3 are satisfied. Then there exists a trade equilibrium characterized by a stationary value function $u$ such that $u = u^*$. A transition function on the Markov shocks converges weakly to invariant probability distributions that have a cycle. Conversely, if there is a trade equilibrium, then it satisfies Assumptions 5.1-5.3. Further, the optimal contract is of a defaultable debt type.*

In words, in the optimal contracting, player 1 makes a payment insuring player 2's autarky utility level so long as the temporary screening condition (5.1) is slack. Once the screening condition (5.1) binds, player 1 reveals his true state. In particular, when he encounters state $L$, he discloses the current state to player 2 and asks for payment allowance. It shows a positive role of costly default. With respect to the optimal default and payment performance, a payment function $F(p(t))$ is increasing concavely for $0 \leq p(t) < 1$, while it is constant for $p(t) = 1$ (Figure 7). Then,

**Corollary 5.1.** *In a trade equilibrium, there exists a deterministic time $\tau^*$ such that for $\tau < \tau^*$, $\{F(p_\tau), C\}$ is a pooling wSE in either state, while for $\tau \geq \tau^*$, $\{F(0), C\}$ in state $L$ and $\{F(1), C\}$ in state $H$ are separating wSE. In particular, the equilibrium payment levels during the inertia episode are increasing concavely in the number of dates elapsed from the last disclosure of $L$. Also, there exists a corresponding payoff-contingent payment rule.*

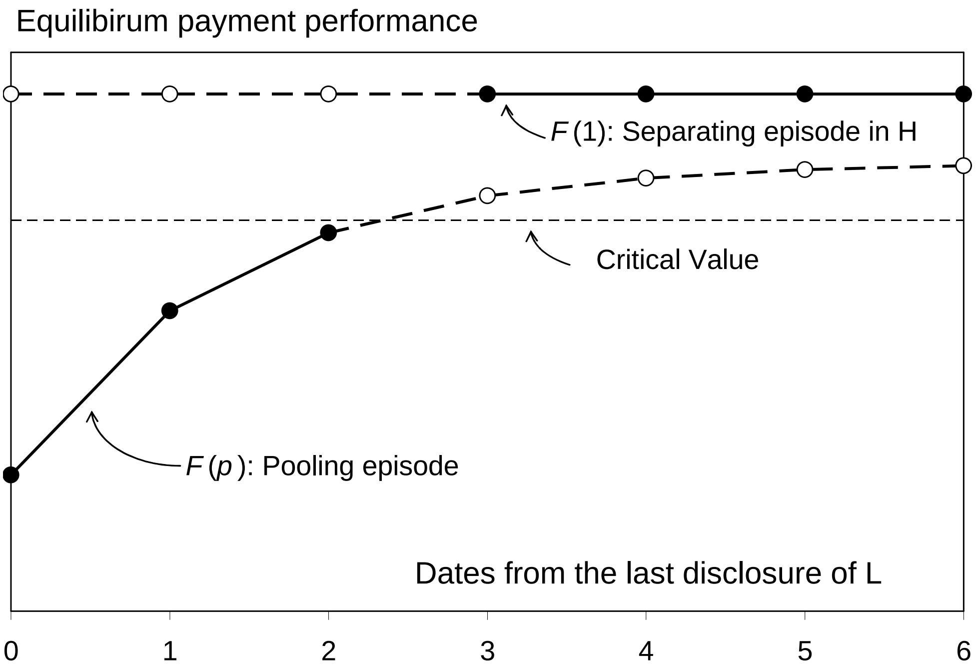


FIGURE 7. Equilibrium Payment Performance

Hence, essentially, both the equilibrium default and payment performance are grasped by the above dichotomous payment game approach, except for time-dependent payment profiles during inertia episodes.

## 6. Concluding remarks

This paper studied sequential costly state verifications in an infinite horizon principal-agent model with first-order Markov technology shocks. Two main results were obtained: (1) If lender's autarky utility level is state-independent, then no trade (i.e., autarky) is the only equilibrium, (2) If lender's autarky utility level is state-dependent, and with some particular parametric assumptions, then there exists a trade equilibrium where costly disclosures occur recurrently only when lender believes that high state is probable while borrower knows that true state is low.

I conclude by pointing out the model's limitations and possibilities of future extensions. First, this paper restricts the discussions to several specific physical and informational structures for stressing the effect of default on ex ante agreement in costly information environment with Markovian technological structure. However, the simple specifications seem to cause some limited applicability to actual financial economy. Precisely, first, the risk-neutrality assumption is restrictive from an empirical perspective. Also theoretically, as Hellwig [10] discusses, the risk-aversion of players might cause the suboptimality of a debt-type contract because a loan contract should play both the roles of insurance and financing. Second, more importantly, the simple principal-agent framework might be another limitation. Especially, the framework might cause difficulty with dealing with competitive security pricing. In actual financial contracts, competitive solutions, rather than autarky levels, might formulate credible threats. In addition, it is known that a competition among mechanism designers might lead to a failure of the standard revelation principle (Epstein-Peters [6]). Therefore, the extension to a multi-player competition framework could be non-trivial to study actual defaultable securities.[15]

Secondly, empirical research is beyond the scope of this paper, although the results proved here provide a basis for such research. There exists enormous empirical finance literature of defaultable bonds such as Duffie-Singleton [5]. It, however, has often presumed that a default time arrives based on exogenously-given stochastic processes such as Poisson processes, and has had difficulty with disentangling credit events and liquidity factors in high yield spreads in debt pricing. Our model may provide a better framework to elaborate analytically a role of loan-default negotiation in defaultable bond pricing (e.g. Nakamura [23] [24]).

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