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Abstract

This paper estimates a money demand function using Japanese data from 1985 to 2017, which includes the period of near-zero interest rates over the last two decades. We compare a log-log specification and a semi-log specification by employing the methodology proposed by Kejriwal and Perron (2010) on cointegrating relationships with structural breaks. Our main finding is that there exists a cointegrating relationship with a single break between the money-income ratio and the interest rate in the case of the log-log form but not in the case of the semi-log form. More specifically, we show that the substantial increase in the money-income ratio during the period of near-zero interest rates is well captured by the log-log form but not by the semi-log form. We also show that the demand for money did *not* decline in 2006 when the Bank of Japan terminated quantitative easing and started to raise the policy rate, suggesting that there was an upward shift in the money demand schedule. Finally, we find that the welfare gain from moving from 2 percent inflation to price stability is 0.10 percent of nominal GDP, which is more than six times as large as the corresponding estimate for the United States.

JEL Classification Numbers: C22; C52; E31; E41; E43; E52
Keywords: money demand function; cointegration; structural breaks; zero lower bound; welfare cost of inflation; log-log form; semi-log form; interest elasticity of money demand

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1 Introduction

There is no consensus about whether the interest rate variable should be used in log or not when estimating the money demand function. For example, Meltzer (1963), Hoffman and Rasche (1991), and Lucas (2000) employ a log-log specification (i.e., the log of real money balances is regressed on the log of the nominal interest rate), while Cagan (1956), Lucas (1988), Stock and Watson (1993), and Ball (2001) employ a semi-log form (i.e., the log of real money demand is regressed on the level of the nominal interest rate). The purpose of this paper is to specify the functional form of money demand using Japanese data covering the recent period with nominal interest rates very close to zero.

Specifying the functional form of money demand is important for the following reasons. First, it allows us to test alternative underlying theories from which money demand stems. Specifically, the inventory theory approach advocated by Baumol (1952), Tobin (1956), and Miller and Orr (1966) implies a log-log form. Similarly, the transaction time approach advocated by McCallum and Goodfriend (1989) implies a log-log form. On the other hand, the money in the utility function approach implies a log-log form if the utility function is of constant relative risk aversion (Lucas 2000) but a semi-log form if the utility function is quasi-linear (Cysne 2009). Second, the welfare cost of inflation may differ substantially depending on the functional form of money demand. Lucas (2000) extends Bailey's (1956) surplus analysis to show that the welfare cost of inflation is greater if money demand is of log-log form than if it is of semi-log form, and that there is a significant welfare cost of deviating just slightly from the Friedman rule (Friedman 1969) in the case of log-log money demand but not in the case of semi-log money demand. Third, the shape of the money demand function near the zero interest rate bound differs substantially depending on whether it is log-log or semi-log. Specifically, the demand for money remains finite even when the nominal interest rate is exactly zero in the case of the semi-log form, while it goes to infinity as the nominal interest rate approaches zero in the case of the log-log form. This difference has important implications for the conduct of monetary policy near the zero interest rate bound (see, for example, Rognlie 2016).

Previous studies based on US data seem to suggest that the log-log form performs better than the semi-log form. Specifically, using US long-term annual data covering the period from 1900 to 1994, Lucas (2000) finds that the log-log form fits better than the semi-log form. However, Ireland (2009) argues that the two specifications could differ substantially when interest rates are sufficiently close to zero, so that it is crucially important to employ more recent data with near-zero interest rates. Based on this argument, Ireland (2009) uses US quarterly data for the period 1980:Q1-2006:Q4 to find that the semi-log specification performs better than the log-log specification. More recently, Watanabe and Yabu (2018) conducted a similar exercise but using data that include observations from the period of near-zero interest rates following the global financial crisis. They show that the log-log specification fits better.

Motivated by the argument by Ireland (2009) mentioned above, this study focuses on Japan, which has experienced near-zero interest rates for a much longer period than any other country, including the United States. Ireland's (2009) argument implies that the data for Japan should be more suitable to empirically discriminate the two functional forms of the money demand function than data for any other country. Previous studies based on Japanese data, including Miyao (2002), Bae et al. (2006), Inagaki (2009) and Nakashima and Saito (2012), make use of observations with near-zero interest rates until the mid-2000s. However, interest rates in Japan have declined even more since then. Specifically, the Bank of Japan adopted quantitative easing in 2001-2006, followed by monetary easing immediately after the global financial crisis, and quantitative and qualitative easing initiated in 2013 to escape from deflation. Due to these monetary easing operations over the last two decades, interest rates, both short- and long-term, gradually declined towards the zero lower bound, with some interest rates even falling below zero following the start of the negative interest rate policy in 2016. The purpose of this paper is to make use of these recent observations to obtain a more accurate estimate of the shape of the money demand function at the zero interest rate bound.

Previous studies based on Japanese data report that the empirical result depends crucially on whether or not structural breaks are allowed in the cointegration tests. This is a notable difference from the US results, such as those reported in Lucas (2000), Ireland (2009), and Watanabe and Yabu (2018). For example, Miyao (2002) and Nakashima and Saito (2012) find a cointegrating relationship with no structural break between the log of real money balances measured in terms of M1 and the log of the policy rate (i.e., the overnight call rate), as well as a cointegrating relationship between the log of real money and the level of the policy rate with a single structural break when the policy rate fell below 1 percent in the third quarter of 1995. This paper follows these studies in paying careful attention to the possibility of structural breaks but differs from them in that it uses more recent data with near zero interest rates, and in that - closely following previous studies on US money demand - it uses market interest rates (specifically, negotiable certificate of deposit rates with a maturity of 90-180 days) rather than the policy rate as an indicator of the opportunity cost of holding money.[1]

Our empirical approach is based on the test for structural breaks in cointegrated regressions proposed by Kejriwal and Perron (2010), which we refer to as the KP test. The KP test is suitable for our analysis because it allows for both $I(1)$ and $I(0)$ regressors and compares the null hypothesis of no break in the cointegrated vector and the alternative of k breaks.[2] Kejriwal and Perron (2010) additionally propose a sequential testing procedure to obtain a consistent estimate of the number of breaks. The approach we take in this paper is as follows: (1) we check for the presence of structural breaks using the KP test; (2) if structural breaks exist, we estimate the number of breaks; and finally, (3) we implement cointegration tests to compare the performance of the log-log and semi-log specifications taking the detected breaks into consideration.

The rest of the paper is organized as follows. Section 2 explains the data we use and conducts a visual comparison of the log-log and semi-log specifications using annual data. Section 3 conducts cointegration tests with no breaks using quarterly data, followed by similar cointegration tests but allowing for structural breaks. Section 4 conducts various robustness checks. Section 5 discusses potential causes of the upward shift of the money demand function in 2006. Section 6 calculates the welfare cost of inflation based on the estimated money demand function, with some discussion on the policy implications. Section 7 concludes the paper. Appendix A provides detailed explanations on how the log-log and semi-log money demand functions are derived from households' utility maximization, as well as how they change in an economy in which the storage cost of holding money is not negligible. Finally, Appendix B provides a detailed discussion of Mulligan and Sala-i-Martin's (1996, 2000) money demand model.

[1] Note that Inagaki (2009) uses the three-month rate in the Japanese interbank money market rather than the overnight rate. His study shows the presence of a cointegrating relationship between the log of real money balances and the log of the three-month rate but does not make a comparison between the log-log and semi-log specifications.

[2] Note that, as highlighted by Perron (2006), most break tests may have nonmonotonic power should the actual number of breaks be greater than the number explicitly accounted for during the construction of the tests. This means that as the magnitude of change increases, the test power plummets such that we cannot identify any structural breaks using these types of tests. Therefore, single break tests such as the Hansen (1992) test lose power when the number of breaks is larger than one and do even more so when the magnitude of the breaks is also large.

2 Data overview

In this section, we conduct a visual examination of the relationship between money demand and the nominal interest rate for Japan. The nominal interest rate we use is the interest rate on newly issued negotiable certificates of deposit (CDs) with a maturity of 90 to 180 day, which is provided by the Bank of Japan (BOJ). For nominal money balances, we use M1, with data obtained from the FRED data service. The constituent elements of M1 are currency held by the public, non-interest-bearing demand deposits, and interest-bearing ordinary deposits. The money-income ratio is calculated by dividing M1 by nominal GDP, which is provided by the Cabinet Office of Japan. The observation period is 1985-2017.[3] Note that Japan has experienced near-zero interest rates for much longer than other countries including the United States. Specifically, the three-month US TB rate, which Ireland (2009) and Watanabe and Yabu (2018) used as the interest rate variable, has been below 1 percent in 24 quarters over the last two decades and below 0.5 percent in 21 quarters. The corresponding figures for the interest rate for Japan (i.e., the negotiable CD rate with a maturity of 90-180 days) are 90 quarters below 1 percent and 66 quarters below 0.5 percent.

Figure 1 shows a scatter plot of the money-income ratio and the nominal interest rate. Observations for the first twenty years, 1985 to 2005, are represented by circles, while observations for more recent years, i.e., 2006-2017, are represented by x-marks. The figure clearly shows that there is an almost monotonic decline in the money-income ratio the lower the nominal interest rate falls, but the decline is not linear. Also, there is no indication that the money-income ratio approaches a finite value as the interest rate comes closer to the zero lower bound. These observations suggest that a log-log specification provides a better fit than a semi-log specification.

To conduct a more detailed comparison between the log-log and semi-log specifications, we plot the data into a semi-log graph in Figure 2(a) and into a log-log graph in Figure 2(b). Figure 2(a) shows that there seems to exist a linear relationship between the log of the money-income ratio and the interest rate as long as the interest rate is above 1 percent, but this linear relationship disappears once the interest rate falls below 1 percent. Clearly, the semi-log specification does not account for the substantial increase in the money-income ratio near the zero lower bound. In contrast, Figure 2(b) shows the presence of a linear

[3] We chose to start our observation period in 1985 since money markets in Japan were highly regulated up until the mid-1980s, so that interest rates associated with market instruments such as CDs were relatively distorted.

relationship between the two variables for the first two decades (i.e., 1985-2005), and another linear relationship for the more recent period (i.e., 2006-2017). However, the lines associated with the two subperiods are not the same, although it looks as though they are almost parallel to each other.

To take a closer look at what happened between the two subperiod, we show in Figure 3 how the money-income ratio and the CD rate fluctuated in 2004:Q1-2010:Q4. The BOJ ended quantitative easing in March 2006 and started to raise the policy rate in July 2006 by 0.25 percentage points, so that the CD rate increased by about 0.7 percentage points. In response, the money-income ratio declined to some extent, but that decline in the money-income ratio was small compared to the increase in the CD rate, resulting in an upward shift of the money demand function. We will come back to this issue later in Section 4.

3 Cointegration tests

3.1 Cointegration with no breaks

In this section, we conduct cointegration tests to more rigorously examine the findings in the previous section based on a casual examination of the data. Our approach is the same as that taken by Ireland (2009) and Watanabe and Yabu (2018) for the United States, who conduct residual-based cointegration tests using quarterly data to see which of the two specifications is supported by the data. Specifically, for the semi-log specification, we run a regression of the form

$$\ln m_t = \alpha + \beta r_t + u_t \tag{1}$$

where m_t denotes the ratio of M1 divided by nominal GDP and r_t the CD rate for a maturity of 90-180 days. We use seasonally adjusted data for M1 and nominal GDP but not for the CD rate. If the residual from this regression is stationary, this means that the two variables are cointegrated and that the semi-log specification is supported by the data. On the other hand, for the log-log specification, we run a regression of the form

$$\ln m_t = \alpha + \beta \ln r_t + u_t \tag{2}$$

to see whether the residual from this regression is stationary or not. If it is stationary, the log-log specification is accepted.

We start by conducting unit root tests – i.e., the augmented Dickey-Fuller (ADF) and

Phillips-Perron (PP) tests – for $\ln(m)$, $\ln(r)$, and r with a constant term included.[4] We find that the null hypothesis of a unit root is not rejected for all three variables. Given this result, we examine in the rest of this section whether there exists a cointegrating relationship between the variables. Specifically, we employ three different cointegration tests: the ADF test proposed by Engle and Granger (1987); the PP test proposed by Phillips and Ouliaris (1990) with test statistics given by Z_t; and the same PP test but with test statistics given by Z_α.

Table 1 shows the test statistics associated with the ADF, Z_t, and Z_α tests together with the static OLS estimates of the cointegrating vector. In the ADF test, the lag length is chosen based on the modified Akaike information criterion (see Ng and Perron 2001). In the PP tests, we compute the long-run variance based on the quadratic spectral kernel and choose the truncation parameter based on Andrews's (1991) plug-in method. For the semi-log specification, the test statistics are all close to zero, indicating that the null hypothesis of no cointegration is not rejected. On the other hand, for the log-log specification, the test statistics are slightly larger in absolute values than those for the semi-log specification but still not that different from zero, so that none of the three tests rejects the null of no cointegration. In this sense, the semi-log and log-log specifications are both rejected by the data.

As we saw in Figure 2(a), the semi-log specification is clearly inconsistent with the data, so that it is not very surprising to find that the null is not rejected for the semi-log specification. For the log-log specification, however, we saw in Figure 2(b) a linear relationship between the two variables, so that the result of no cointegration is somewhat surprising. Also, this is different from the results for the United States obtained by Watanabe and Yabu (2018), which did not reject the null for the semi-log specification but rejected that for the log-log specification. The failure to reject the null for the log-log specification may be due to the upward shift in the money demand function in 2006 seen in the last section. As pointed out by Gregory et al. (1996) and Gregory and Hansen (1996), if a simple cointegration test is applied when some variables are actually cointegrated with structural breaks, the result will be biased toward accepting the null of no cointegration. Given this, in the rest of this section, we check whether there exist structural changes in the cointegrating vector based on the KP test and, if structural breaks do exist, repeat the cointegration tests but take the detected structural breaks into consideration.

[4] We obtain the same results when including a time trend.

3.2 Structural break tests

In this subsection, we adopt the test proposed by Kejriwal and Perron (2010) to check whether there exist any changes in the intercept and the slope coefficient of the money demand function, i.e., α and β in eqs. (1) and (2). We consider the following money demand function with k breaks (that is, there are $k+1$ different regimes):

$$\ln m_t = \alpha_i + \beta_i z_t + \sum_{j=-\iota_T}^{\iota_T} \delta_j \Delta z_{t-j} + u_t \qquad \text{for} \quad T_{i-1} < t \leq T_i \tag{3}$$

where i represents the regime ($i = 1, \ldots, k+1$). By convention, $T_0 = 0$ and $T_{k+1} = T$, where T represents the sample size. The explanatory variable z_t is either r_t in the case of the semi-log form or $\ln r_t$ in the case of the log-log form. We add leads and lags of Δz_t as auxiliary variables to correct for potential endogeneity between z_t and u_t. The number of leads and lags is set to 2 (i.e., $\iota_T = 2$).

Let $\lambda = \{\lambda_1, \ldots, \lambda_k\}$ denote the vector of break fractions with $\lambda_i = T_i/T$. Note that λ is an element of the set $\Lambda_\epsilon = \{\lambda : |\lambda_{i+1} - \lambda_i| \geq \epsilon, \lambda_1 \geq \epsilon, \lambda_k \leq 1 - \epsilon\}$ for some $\epsilon > 0$. Therefore, each regime contains at least as many observations as $[\epsilon T]$, where $[\cdot]$ denotes the greatest integer that is less than or equal to its argument. The trimming parameter ϵ is set to 0.15.

Kejriwal and Perron (2010) employ the sup Wald statistic to test the null of no break against the alternative of k breaks in cointegrated models that allow for both $I(1)$ and $I(0)$ regressors. The test statistic is defined as follows:

$$\sup F_T^*(k) = \sup_{\lambda \in \Lambda_\epsilon} \frac{\mathrm{SSR}_0 - \mathrm{SSR}_k}{\widehat{\sigma}^2} \tag{4}$$

where SSR_0 and SSR_k are the sum of squared residuals under the null of no break and the alternative of k breaks; $\widehat{\sigma}^2$ is the long-run variance computed using the residuals from the model estimated under the null of no break. Based on the sup Wald test shown above, Kejriwal and Perron (2010) propose a double-maximum test in which the alternative hypothesis contains an unknown number of breaks between 1 and the upper bound M:

$$\mathrm{UDmax}(M) = \max_{1 \leq k \leq M} \sup F_T^*(k) \tag{5}$$

This is known as the most useful test to determine the presence of any structural changes. We set the upper bound to 5 ($M = 5$).

The other test proposed by Kejriwal and Perron (2010) is a test of the null hypothesis of k breaks against the alternative of $k+1$ breaks. This test makes it possible to identify the number of breaks. The sequential test procedure is as follows: We start by testing if there are any structural breaks by applying either the $\sup F_T^*(1)$ test or the UDmax(5) test. If we reject the null hypothesis of no break, we then test for one versus two breaks. We continue this process until we fail to reject the null. The number of breaks estimated in this way, which is equal to the number of rejections, is a consistent estimate of the true number of breaks.

Table 2 presents the test statistics $\sup F_T^*(k)$ and UDmax(5) for the semi-log and log-log specifications. The table shows the number of breaks we detect as well as the break dates estimated by minimizing the sum of squared residuals based on eq. (3). The results Table 2 in show that for the semi-log specification, none of the test statistics indicate the presence of structural breaks. On the other hand, for the log-log specification, the null of no breaks is rejected for $\sup F_T^*(1)$ and UDmax(5), suggesting the presence of a single break.[5] We also conduct the sequential procedure to confirm that the number of breaks is one. The date of the single structural break estimated based on the minimization of the sum of squared residuals is the third quarter of 2005.

Comparing our results with the results reported in previous studies on Japan such as Miyao (2002) and Nakashima and Saito (2012) shows the following. First, they fail to detect a break for the log-log specification. However, the observation periods used by Miyao (2002) and Nakashima and Saito (2012) ended in 2001 and 1999 respectively, so that our result, which detects a break in 2005, is not inconsistent with their results. Second, Miyao (2002) and Nakashima and Saito (2012) detect a single structural break for the semi-log specification. As a simple exercise to identify what is responsible for this difference, we repeat the KP test using a shorter observation period (namely,1979:Q1-2001:Q4) to find that there exists a single break even for the semi-log specification. This suggests that the difference comes mainly from the observation period used rather than from the estimation method employed[6] or from the interest rate variable used.

To investigate this issue in more detail, we conduct a rolling regression to see how the interest rate elasticity changed over time. The solid line shown in Figure 4 is the semi-elasticity obtained from regressing $\ln m_t = \alpha + \beta r_t$. The rolling regression is conducted using the observations for the past 20 quarters. The figure shows that the semi-elasticity obtained from the rolling regression is stable up until the first half of the 1990s but exhibits a number of

[5]Note that the other test statistics (i.e., $\sup F_T^*(2)$, ..., $\sup F_T^*(5)$) indicate that the null is not rejected even for the log-log specification. These results can be interpreted as resulting from the lower test power due to the fact that the number of breaks under the alternative is greater than the true number.

[6]Note that Miyao (2002) and Nakashima and Saito (2012) both use Hansen's (1992) single break test.

sizable changes afterward. The semi-elasticity starts to decline from 1995 onward and reaches -30 in 2000:Q1 and -360 in 2006:Q1, followed by a sharp increase in 2006:Q2 back toward the original level. However, it started to decline again and reached -200 in 2017:Q4. These fluctuations in the semi-elasticity estimated from the rolling regression suggest that there were multiple structural breaks of considerable size and that the number of such breaks is greater than five, which is the maximum number of breaks we set for conducting the KP test. It is likely that the power of the KP test is substantially lowered by the size and the frequency of structural breaks, so that the null of no breaks cannot be rejected. Overall, the additional evidence from the rolling regression suggests that the semi-log specification is not appropriate for the money demand function. Note that the estimate of the semi-elasticity obtained from the log-log regression,[7] which is depicted by the dotted line in Figure 3, successfully reproduces the developments in the semi-elasticity estimated from the rolling regression. This can be seen as another piece of evidence that the log-log specification performs better than the semi-log specification.

3.3 Cointegration tests with structural breaks

In this subsection, we conduct cointegration tests taking the structural breaks detected in the previous subsection into consideration. As the number of breaks detected in the previous subsection was one for the log-log specification, we adopt the methodology proposed by Gregory and Hansen (1996), which conduct a cointegration test allowing for a single structural break in the intercept and slope.

The results are presented in Table 3 and all three test statistics indicate that the null of no cointegration is rejected at the 1 percent significance level for the log-log specification. Together with the result in Section 3.1, this shows that the log of the money-income ratio and the log of the interest rate are cointegrated with a single break. In contrast, for the semi-log specification, the three test statistics are close to zero, so that the null of no cointegration is not rejected even when the possibility of a single break is taken into consideration.

Given the above results, we proceed to estimating the cointegrating vector, α and β in eq. (3), by running a dynamic OLS with $z_t = \ln r_t$. Note that as long as the variables are cointegrated, dynamic OLS estimates are asymptotically efficient and conventional t-statistics obtained from dynamic OLS estimation have conventional normal asymptotic distributions

[7] Specifically, we run a log-log regression (which is *not* a rolling regression) taking a single break that occurs in 2005:Q3 into consideration and then convert the coefficient on $\ln r_t$ into the semi-elasticity by dividing it by the moving average of r_t over the last 20 quarters.

(see, for example, Stock and Watson 1993 and Saikkonen 1991). The regression result is given by

$$
\begin{aligned}
\widehat{\ln m_t} = & \underset{(0.042)}{[-1.623} - \underset{(0.005)}{0.150} \ln r_t] (1 - D_t) + \underset{(0.027)}{[-0.404} - \underset{(0.005)}{0.063} \ln r_t] D_t + \underset{(0.014)}{0.009} \Delta \ln r_{t-2} \\
& + \underset{(0.012)}{0.015} \Delta \ln r_{t-1} + \underset{(0.017)}{0.043} \Delta \ln r_t - \underset{(0.016)}{0.071} \Delta \ln r_{t+1} - \underset{(0.009)}{0.050} \Delta \ln r_{t+2} \qquad (6)
\end{aligned}
$$

where the figures in parentheses are robust standard errors and the dummy variable D_t is defined as $D_t = 1$ for $t > 2005{:}\text{Q3}$ but $D_t = 0$ otherwise.

Comparing the coefficients before and after the structural break date, the intercept increases from -1.623 to -0.404, indicating that the money demand function shifts upward after the break date. On the other hand, the coefficient on $\ln r_t$ decreases in absolute value from -0.150 to -0.063, indicating that money demand responds less elastically to changes in the interest rate. Turning to the coefficients on the lead and lag terms of $\ln r$, the lag terms, i.e., $\Delta \ln r_{t-1}$ and $\Delta \ln r_{t-2}$, are both not significantly different from zero. However, the lead terms, i.e., $\Delta \ln r_{t+1}$ and $\Delta \ln r_{t+2}$, are both negative and significantly different from zero, indicating that a more rapid rise of the nominal interest rate in the future periods is associated with a lower demand for money in the current period and vice versa.

To look more closely at the shift in the money demand function, we plot $\ln m$ and $\ln r$ using quarterly data in Figure 5, where the quarters before and including the break date are represented by circles and the quarters after the break date are represented by x-marks. We see that following the BOJ's decision in March 2006 to end quantitative easing, the CD rate rose from 0.016 in 2006:Q1 to 0.088 in 2006:Q2 and 0.3343 in 2006:Q3. This raises the question why the break was detected in 2005:Q3, when quantitative easing was still ongoing and the CD rate remained at a low level (the CD rate was 0.0060 in 2005:Q3 and 0.0067 in 2005:Q4). According to the result of the dynamic OLS estimation, the coefficients on the lead terms are negative and significantly different from zero. Therefore, the rapid increase in the CD rate in the first and second quarters of 2006 should have been associated with a lower demand for money in the fourth quarter of 2005. In practice, the decline in money demand did *not* occur in the fourth quarter of 2005, implying that the cointegrating vector changed in that quarter.

Finally, using a graph, let us check how well the log-log specification captures the relationship between the two variables. The solid and dotted lines in Figure 6 respectively represent the fitted values for the log-log and semi-log specifications calculated using the estimates of α and β, which are taken from Table 1 for the semi-log specification and from eq. (6) for the log-log specification. It can be clearly seen that money demand increases substantially as the interest rate approaches the zero lower bound. However, the semi-log specification fails to capture this. In contrast, the log-log specification performs well both in the high and the near-zero interest rate periods.

4 Robustness checks

In this section, we check the robustness of the results obtained in the previous section. We check, first, how the results change when we use alternative measures for the opportunity cost of holding money. Second, we use M1 data adjusted for the money shift from time deposits, which are outside M1, to settlement deposits and cash in response to the change in deposit insurance that was implemented in April 2002. Third, we relax the constraint that the income elasticity of money demand is unity. Fourth, we decompose M1 into cash and demand deposits and conduct the same exercise.

4.1 Alternative measure for the opportunity cost of holding money

In the previous section, we used M1 as the measure of money, but M1 contains ordinary deposits, i.e., interest-bearing deposits held for settlement purposes by households and firms (especially small firms) at commercial banks. However, as highlighted by Ericsson (1998), when estimating the opportunity cost of holding money, it is important to take the interest rates associated with assets into account in the definition of money. Given this, a more precise measure for the opportunity cost of holding money is the difference between the CD rate and the interest rate on ordinary deposits. A study that has taken this approach using Japanese data is that by Inagaki (2009), who estimated the money demand function based on the opportunity cost of holding money calculated as the interest rate differential between inside and outside money.

We follow Lucas and Nicolini (2015) and define the opportunity cost of holding money as

$$\sum_j \omega_j \left(r_t - r_{t,j}\right) \tag{7}$$

where ω_j is the share of asset j in the monetary aggregate and $r_{t,j}$ is the interest rate paid by asset j. Japanese M1 consists of cash ($j = 1$), current account deposits ($j = 2$), and ordinary deposits ($j = 3$). Cash and current account deposits bear no interest (i.e., $r_{t,1} = r_{t,2} = 0$), but ordinary deposits do. The share of cash in M1 is about 20 percent ($\omega_1 = 0.2$), and the share of ordinary deposits in the sum of current and ordinary deposits is about 80 percent. We set the ω's at $\omega_1 = 0.2$, $\omega_2 = 0.16$ ($= 0.8 \times 0.2$), and $\omega_3 = 0.64$ ($= 0.8 \times 0.8$).

Figure 7 shows a scatter plot of the money-income ratio and the opportunity cost of holding money. This is similar to Figure 2, but the horizontal axis is now the opportunity cost of holding money given by eq. (7) rather than the CD rate itself. Figure 7(a) shows that switching to the interest rate differential does not make a big difference: as we saw in the last section, the substantial increase in money demand during the near-zero interest period is not captured by the semi-log specification. On the other hand, Figure 7(b) shows the presence of two (almost) parallel straight lines, suggesting that there exists a cointegrating relationship between the log of the money-income ratio and the log of the opportunity cost of holding money with a single structural break.

Part (a) of Table 4 shows the results from the residual-based cointegration tests, as well as the Gregory-Hansen tests. The test statistics associated with the residual-based cointegration tests (i.e., ADF, Z_t, and Z_α) indicate that the null of no cointegration with *no* break cannot be rejected for both the semi-log and the log-log specification. However, the test statistics associated with the Gregory-Hansen tests (i.e., Inf-ADF, Inf-Z_t, and Inf-Z_α) show that the null of no cointegration is not rejected for the semi-log specification even when the possibility of a single break is taken into account, but rejected for the log-log specification. These results confirm that the findings obtained in the previous section are robust to changes in the definition of the opportunity cost of holding money, suggesting that the upward shift of the money demand curve we detected in the previous section is not due to the use of the wrong indicator of the opportunity cost of holding money.

4.2 Eliminating the impact of the deposit insurance reform in 2002 on M1

Previous studies on money demand in Japan have often argued that money demand increased substantially before the implementation of a deposit insurance reform in April 2002, in which it was decided that (1) from April 2002 onward time deposits exceeding 10 million yen would no longer be protected in full; (2) interest-bearing demand deposits would be protected in full until March 2005, but from April onward deposits exceeding 10 million yen would no longer be protected in full; (3) non-interest-bearing demand deposits would continue to be protected in full even after March 2005. In response to this reform, depositors, in particular households, shifted money from time deposits, which are outside M1, to ordinary deposits, which are inside M1, before April 2005. The upward shift in the money demand function detected in the previous section could to some extent be related to this shift in households' money. In this subsection, we eliminate the effect of the deposit insurance reform on M1 and repeat the exercise in the previous section.

We employ the following procedure to eliminate the impact of the deposit insurance reform on M1. According to the monetary aggregates statistics published by the Bank of Japan, the inflow of money to M1 from time deposits, or "quasi-money" in the monetary aggregates statistics, started to increase substantially in the first quarter of 1999 and continued to increase until the second quarter of 2002, when the deposit insurance reform was implemented.[8] We eliminate this inflow of money to M1 by assuming that the shares of currency and demand deposits in the sum of M1 and quasi-money continued the trend observed in 1997-98.[9] Figure 8 compares the adjusted and the unadjusted M1. The difference between the two increased quickly in 2001:Q2, just before the full protection of time deposits ended in April 2002, followed by a gradual increase over time to reach 120 trillion yen in 2005.

Next, we plot in Figure 9 the money-income ratio using the adjusted M1 on the horizontal axis and the CD rate on the vertical axis. We see no significant difference from Figure 2, suggesting that although the inflow of money into M1 is not trivial in size, it does not change our main findings. Finally, we conduct the same cointegration tests as in the last section to find again that the null of no cointegration with no breaks cannot be rejected for both the semi-log and the log-log specification, but that the null of no cointegration is rejected for the log-log specification even when the possibility of a single break is taken into account (see part (b) of Table 4). These results confirm that the findings obtained in the previous section remain unchanged even when eliminating the effect of the deposit insurance reform on M1.

4.3 No restriction on income elasticity

The cointegration tests we conducted in Section 3 are based on the assumption that income elasticity is unity. In this subsection, we relax this assumption and examine the cointegrating relationship among the following three variables: real money balances, real income, and nominal interest rates.

Let M denote M1, Y the nominal GDP, P the GDP deflator, and r the CD rate. We

[8]The share of quasi-money in the sum of M1 (currency and demand deposits) and quasi-money declined from 71 percent in 1998:Q4 to 55 percent in 2002:Q4.

[9]The share of currency in the sum of M1 and quasi-money increased by 0.5 percentage points from 5.1 percent in 1996:Q4 to 5.6 percent in 1998:Q4, while the share of demand deposits in the sum of M1 and quasi-money increased by 2.1 percentage points from 20.6 percent in 1996:Q4 to 22.7 percent in 1998:Q4.

conduct cointegration tests for M/P, Y/P, and r. Specifically, we regress $\ln(M/P)$ on a constant, $\ln(Y/P)$, and $\ln(r)$ or r, and check whether the residual is stationary or not. Part (c) of Table 4 shows the test statistics based on the residual-based cointegration tests (i.e., ADF, Z_t, and Z_α), as well as those based on the Gregory-Hansen tests (i.e., Inf-ADF, Inf-Z_t, and Inf-Z_α). The results are the same as in Section 3. Specifically, the null of no cointegration with no structural breaks is not rejected for both the semi-log and the log-log specification. However, the null of cointegration is rejected for the log-log specification if we allow for a single break. On the other hand, the null is not rejected for the semi-log specification even when allowing for a single break.

Next, we run a dynamic OLS regression for the log-log specification considering a single break in 2005:Q3 to obtain the following result:

$$\widehat{\ln \frac{M_t}{P_t}} = \big[\underset{(3.068)}{1.603} + \underset{(0.204)}{0.786} \ln \frac{Y_t}{P_t} - \underset{(0.009)}{0.157} \ln r_t\big](1 - D_t) + \big[\underset{(5.597)}{-9.365} + \underset{(0.366)}{1.584} \ln \frac{Y_t}{P_t} - \underset{(0.009)}{0.056} \ln r_t\big] D_t \tag{8}$$

where the figures in parentheses are robust standard errors and the estimates on the leads and lags of $\Delta \ln Y_t/P_t$ and $\Delta \ln r_t$ are not reported. As we saw in Section 3, the intercept is larger and the slope is smaller in absolute value in and after 2005:Q4 than before. The estimates for the income elasticity before and after the break date are 0.786 and 1.584 respectively, neither of which is significantly different from unity.[10] Turning to the interest rate elasticity, this is -0.157 before the break date and -0.056 after the break date. Both these values are quite close to the estimates obtained in Section 3. Overall, the results are essentially the same as those in Section 3, indicating that the findings do not depend on the assumption of a unitary income elasticity.

4.4 Cash and demand deposits

Recent studies on monetary policy in an economy with negative interest rates such as Japan's are based on the assumption that the cost of storing cash is not negligible (see Rognlie 2016 and Eggertsson et al. 2019). For example, Eggertsson et al. (2019), assuming that the marginal storage cost is positive and constant, show that the opportunity cost of holding cash is the sum of the interest rate associated with assets outside money, such as government bonds,

[10] We conduct an additional dynamic OLS regression assuming that there is a structural break in the intercept and interest rate elasticity, as before, but not in the income elasticity. We find that the estimate for the income elasticity is 0.815 with a standard error of 0.207, indicating again that the null of a unitary income elasticity is not rejected.

and the marginal storage cost. An important upshot of this is that even if the interest rate is zero, the opportunity cost of holding cash could be positive, so that people would willingly hold such assets. Put differently, the demand for cash remains finite even at the zero lower bound.[11] On the other hand, demand deposits do not incur any storage costs, so that the demand for deposits would become infinite at the zero lower bound. These considerations suggest that the demand functions for cash and demand deposits take a different shape. To examine how the demand functions differ and to what extent the observations regarding money demand obtained in the previous section apply to the individual components of M1, we decompose M1 into cash and deposits.[12]

Specifically, we decompose M1 into the following three components: currency, demand deposits held by households, and demand deposits held by non-financial firms. Figure 10 presents log-log scatter plots of the money-income ratio and the CD rate, where money is defined as currency in Figure 10(a), demand deposits held by households in Figure 10(b), and demand deposits held by non-financial firms in Figure 10(c). The observation period is 1998:Q2 to 2017:Q4. All three graphs show two (almost) parallel straight lines, suggesting that there exists a cointegrating relationship between the log of the money-income ratio and the log of the interest rate with a single structural break.

Part (d) of Table 4 shows the results from the residual-based cointegration tests, as well as the Gregory-Hansen tests. For each of the three components of M1, the null of no cointegration with no break cannot be rejected for both the semi-log and the log-log specification. However, the null of no cointegration with a single break is not rejected for the semi-log specification but is rejected for the log-log specification. These results indicate that the results obtained in the previous section apply to each of the three components of M1.

5 Discussion of the causes of the upward shift of the money demand function in 2006

In the previous sections, we showed that the money demand schedule exhibited an upward shift in 2006 in the following sense. First, the demand for money did *not* show any significant decline in 2006:Q2 to 2007:Q4, when the BOJ terminated quantitative easing and started to

[11] This is clearly different from the traditional view, as expressed, for example, by Hicks (1937), that interest rates cannot go below zero in equilibrium.

[12] Lucas and Nicolini (2015) extend the Baumol-Tobin model to the case of multiple means of payment (i.e., cash, checks, and money market deposit accounts) to show that a new monetary aggregate introduced by them, which is referred to as NewM1, performs well.

raise the policy rate. Second, the demand for money *did* increase in response to the decline in the interest rate from 2009:Q1 onward, when the BOJ restarted monetary easing in response to the onset of the global financial crisis. More importantly, a log-log relationship between money and the interest rate reemerged, taking almost the same slope as observed before 2006. Third, the money demand schedule has not shown any sign of shifting back to the original one (i.e., the money demand schedule observed before 2006) for a decade since the start of monetary easing in 2009, suggesting that the upward shift in the money demand schedule is not a temporary but a permanent phenomenon. These observations remain unchanged even when alternative measures of money and interest rates are adopted, as seen in the previous section. The purpose of the present section is to further investigate the causes of the upward shift of the money demand schedule in 2006.

5.1 High switching costs due to irreversible investments during very low interest periods

The three findings mentioned above indicate that the demand for money responded asymmetrically to the rise and decline in interest rates. The presence of such asymmetry has been pointed out and examined further in previous studies on the demand for money in high inflation economies like Chile, Argentina, and Israel. Specifically, Piterman (1988) shows that the demand for money in those countries declined substantially during high inflation periods but did not recover after high inflation subsided, which she referred to as the "ratchet effect" in the demand for money. Piterman (1988) argues that this occurred because, during high inflation periods, banks, firms, and households invested resources in developing ways to reduce the money (i.e., cash and demand deposits) they hold, for example by rapidly switching between saving and demand deposits and using overdrafts. These innovations incurred non-trivial fixed costs, but they were available at a low marginal cost even after high inflation ended, so that the demand for money did not revert to the level before inflation began to rise. In this sense, investment into these financial innovations during high inflation periods had a permanent effect on the demand for money.[13]

Although the ratchet effect we have detected in the Japanese data is of the opposite

[13] This type of hysteresis in the demand for money has been studied further in the context of currency substitution in Latin American countries (see Melnick 1990, Kamin and Ericsson 2003). In these countries, the demand for money (i.e., domestic currency) exhibited a substantial decline in high inflation periods due to dollarization and did not recover after high inflation had subsides and interest rates had returned to a normal level. To explain these phenomena, Uribe (1997) focuses on the role of fixed costs due to network externalities when switching from the use of foreign currency for transactions to the use of local currency. As a result of these costs, small declines in domestic interest rates do not induce a switch back to local currency.

direction to that observed in high inflation economies, these studies suggest that investments in financial innovation during very low interest periods may have played an important role in creating hysteresis in the demand for money. One possible candidate in terms of financial innovation that may have brought about such hysteresis is the diffusion of ATMs. ATMs are ubiquitous in Japan. In particular, ATMs run by retailers including convenience stores first appeared in 2001, increased rapidly in number afterwards, and now account for a quarter of the total number of ATMs in operation.[14] With more ATMs nearby, households and firms find it less time-consuming and costly to make a cash withdrawal from demand deposits, so that the attractiveness of demand deposits relative to other financial assets such as cash and saving deposits has increased.[15] It therefore seems safe to say that the increase in demand deposits over the last two decades has at least partially been driven by the investment in ATM networks, especially by retailers, although it is difficult to measure the extent to which this is the case. More importantly, given that a large number of ATMs have already been installed and can be used at a relatively low marginal cost, households and firms have less incentive than before to switch from demand deposits to interest-bearing assets like saving deposits and bonds even if interest rates goes up to some extent.

5.2 High switching costs due to households' insufficient knowledge on financial technology

The second potential explanation for the shift in money demand is based on the lack of knowledge on financial technology by Japanese households. Mulligan and Sala-i-Martin (2000) propose a money demand theory based on the assumption that households that decide to hold interest-bearing assets must incur a fixed cost, which they refer to as the "adoption cost." This adoption cost could include resources spent on traveling to or learning about a securities market and its participants, or resources spent complying with government regulations. They point out that, according to the 1989 *US Survey of Consumer Finances*, 59 percent of US households do not hold any interest-bearing assets, and show that a substantial part of fluctuations in US money demand can be accounted for by changes in the fraction of such

[14] According to the Japanese Bankers Association, there were 137,000 ATMs run by banks, credit associations, and Japan Post Bank in Japan at the end of September 2016. The number of ATMs run by retailers was 55,000, so the total is over 190,000 ATMs in operation across the country.

[15] Paroush and Ruthenberg (1986) construct a theoretical model to show that, with more ATMs nearby, households and firms make more frequent but smaller cash withdrawals. As a result, households' average cash balances decline while demand deposits balances increase. See Attanasio et al. (2002) and Daniels and Murphy (1994a) for empirical evidence on the effect of ATM diffusion on cash holdings for Italy and the United States, and Columba (2009) and Daniels and Murphy (1994b) for evidence on the effect on demand deposits holdings.

households with no interest-bearing assets.

In their model, an increase in interest rates reduces the demand for money through the following two channels. First, households that have already incurred the adoption cost and have knowledge on financial technology raise the share of interest-bearing assets in their wealth in response to an increase in interest rates. Consequently, the demand for money from such households declines. Second, households with no knowledge on financial technology (and that therefore hold all of their assets in the form of money) decide to incur the adoption cost to acquire knowledge on financial technology and start to hold interest-bearing assets. This also leads to a decline in money demand. Mulligan and Sala-i-Martin (2000) refer to the two channels as the "intensive margin" and the "extensive margin," respectively.

The red line in Figure 11 shows the fraction of Japanese households with no interest-bearing assets, which is taken from the *Survey of Household Finances* - an annual survey conducted by the Central Council for Financial Services Information.[16] According to the survey, the fraction of households with no interest-bearing assets was about 8 percent in 1989, which is much lower than the corresponding figure for the United States mentioned above. An interpretation of this figure based on Mulligan and Sala-i-Martin (2000) is that, as of 1989, most Japanese households had already acquired knowledge on financial technology available at that time. However, as shown in Figure 11, the fraction of households with no interest-bearing assets started to increase gradually in the mid-1990s and continued to increase in the 2000s even at an accelerated rate. It reached 31 percent in 2013 and has been at a high level since then. The rise of the fraction of households with no interest-bearing assets itself is not that surprising given that interest rates declined consistently during this period, approaching the zero lower bound. However, this implies that about one-third of households are not familiar with recent financial technology needed to hold interest-bearing assets, which is probably quite different from the financial technology available in the late 1980s.

Is the extensive margin an important determinant of the demand for money in Japan, as it is in the United States? The blue solid line in Figure 11 represents the amount of interest-bearing assets per household divided by per-capita nominal GDP. The line peaks

[16] The survey covers 8,000 households with two or more family members. The survey asks households about the amount of financial assets and liabilities they hold as well as their financial portfolios, including currency, current deposits and time deposits, life insurance, non-life insurance, personal annuity insurance, bonds, stocks and investment trusts, workers' asset formation savings, and other financial products. The survey also asks whether respondents hold any interest-bearing assets other than bank account deposits for the purpose of payment.

in 2000 and exhibits a substantial decline since then. This is simply the flip side of the fact we observed in the previous sections that the demand for money increased significantly in response to the consistent decline in interest rates during this period. We decompose this decline in money demand into the extensive margin (i.e., the fraction of households with interest-bearing assets) and the intensive margin (i.e., the amount of interest-bearing assets per household for households with interest-bearing assets), which is represented by the dotted blue line in the figure. We can clearly see that the decline in interest-bearing assets per household is mainly due to changes in the extensive margin. In fact, about two-thirds of the decline in interest-bearing assets per household during this period is accounted for by changes in the extensive margin.[17]

What happened in 2006 to the fraction of households with no interest-bearing assets? As shown in the figure, the fraction of such households declined in 2006, but only slightly. This could be interpreted as implying that the majority of households with no interest-bearing assets, who were not familiar with financial technology at that time, refused to incur the adoption cost and continued to hold all of their assets in the form of money. This could account, at least partially, for the upward shift in the money demand schedule in 2006.[18]

6 Welfare cost of inflation

In this section, we calculate the welfare cost of inflation using the parameter estimates obtained in Section 3 and compare our results with those reported in previous studies such as Lucas (2000), Ireland (2009), and Watanabe and Yabu (2018).

Table 5 shows the estimation results. Bailey (1956) defines the welfare cost of inflation in terms of how much lower welfare is in an economy with a positive nominal interest rate than in an economy with a zero interest rate (i.e., the Friedman equilibrium). The welfare cost of inflation, $w(r)$, is given by $w(r) = e^{\alpha}\left(-\beta/(1+\beta)\right) r^{1+\beta}$ for the log-log specification and

[17]Interest-bearing assets per household declined by 30 percent from 2000 to 2017. The extensive margin (i.e., the fraction of households with non-zero interest-bearing assets) declined by 21 percent during the same period, while the intensive margin (i.e., the amount of interest-bearing assets per household for households with non-zero interest-bearing assets) declined only by 11 percent.

[18]Note that Mulligan and Sala-i-Martin's (2000) analysis is based on the assumption that households' knowledge about financial technology is *not* durable (i.e., it lasts only one period), and that households have to incur the adoption cost every period in order to get access to the financial technology available at that time. However, this assumption is inappropriate since an important component of the adoption cost could be the learning that needs to take place before one can use financial assets. Once learned, the technology can be used for a while, and, in this sense, households' knowledge on financial technology is durable. Mulligan and Sala-i-Martin (1996) provide a rough sketch of a model with durable adoption costs. See Appendix B for more on this.

$w(r) = -e^{\alpha}\left[1-(1-\beta r)e^{\beta r}\right]/\beta$ for the semi-log specification. In this calculation, we assume that the real interest rate at the steady state is 3 percent. For example, $r = 0.05$ means that the inflation rate at the steady state is 2 percent, which is the current target level set by the Bank of Japan.

Using annual data for the United States for the period 1900-1994, Lucas (2000) obtains $\alpha = -1.036$ and $\beta = -7.000$ for the semi-log specification and $\alpha = -3.020$ and $\beta = -0.500$ for the log-log specification. Given these parameter values, the welfare cost of 2 percent inflation is 0.25 percent of national income in the case of the semi-log specification and 1.09 percent in the case of the log-log specification. These results indicate that the welfare cost of inflation is not negligible even when the inflation rate is only 2 percent, especially in the case of the log-log specification. On the other hand, using quarterly data for the United States for the period 1980:Q1-2013:Q4, Watanabe and Yabu (2018) obtain $\alpha = -1.778$ and $\beta = -2.276$ for the semi-log specification and $\alpha = -2.089$ and $\beta = -0.055$ for the log-log specification. These coefficient estimates imply that the welfare cost of 2 percent inflation is only 0.04 percent of national income in both cases. These estimates are almost the same size as those obtained by Ireland (2009), who used quarterly data for the United States for the period 1980:Q1-2006:Q4.

The results for Japan presented in the bottom three rows of the table, which are based on the estimates of α and β presented in Table 1 for the semi-log specification and eq. (6) for the log-log specification, show that the welfare cost of 2 percent inflation is 0.99 percent of national income for the semi-log specification and 0.27 percent for the log-log specification. Our estimate based on the log-log specification is very similar to that by Shiratsuka (2001), who found that the welfare cost of 2 percent inflation is 0.18 percent of national income based on a log-log money demand estimate for the post war period (1950-1999). Note that, as shown in the table, the welfare cost estimates based on the log-log specification for the period before and the period after the break date are very similar, which may be surprising given that the money demand curve shifted upward after the break date, as we saw in Section 3. The intercept is indeed larger after the break date, but the effect of this increase on the welfare cost of inflation is cancelled out by the smaller interest rate elasticity in the period after the break date.

Comparing the result for Japan based on the log-log specification with the corresponding result for the United States presented by Watanabe and Yabu (2018) suggests that the welfare cost of 2 percent inflation in Japan is more than six times as large as in the United States, implying that the welfare gain from moving from 2 percent inflation to perfect price stability (i.e., zero percent inflation) is almost negligible for the United States but not for Japan.

Finally, Figure 12 compares the welfare cost of inflation based on the log-log and the semi-log specification, with the nominal interest rate on the horizontal axis and the welfare cost of inflation on the vertical axis. Our main focus here is on the result for the log-log specification, but we also show the result for the semi-log specification for comparison. The figure indicates that the welfare cost estimate based on the semi-log specification is a convex function of r, while that based on the log-log specification is a concave function of r. This difference in the shape implies that when r declines from 2 to 1 percent, $w(r)$ falls by 0.157 percentage points in the case of the log-log specification and by 0.055 percentage points in the case of the semi-log specification, so that the changes in $w(r)$ associated with a decline in r from 2 to 1 percent are much larger in the case of the semi-log specification. However, when r declines from 1 to 0 percent, $w(r)$ in the case of the log-log specification falls by 0.060 percentage points, which is slightly greater than the welfare improvement from 2 to 1 percent, but $w(r)$ falls only by 0.061 percentage points, which is less than half of the welfare improvement from 2 to 1 percent, in the case of the semi-log specification.

This difference between the log-log and semi-log specifications – that is, that the welfare improvement associated with a decline in interest rates towards zero decelerates for the semi-log specification but not for the log-log specification – has been highlighted in previous studies such as Lucas (2000) and Wolman (1997). This difference arises because in the log-log specification money demand increases substantially as the interest rate falls from 1 to zero percent, but such an increase in money demand does not occur in the case of the semi-log specification. An important implication of this difference is that if money demand follows a log-log form, which it does, as we showed in this paper, it would make more sense for the central bank to reduce inflation and thus the nominal interest rate from 1 to zero percent, which corresponds to the optimal rate of deflation under the Friedman rule, rather than try to raise it to 2 percent.

7 Conclusion

Identifying the proper specification of the money demand function has important implications for accurately quantifying the welfare cost of inflation and for the conduct of monetary policy. However, there is no consensus on whether the nominal interest rate as an independent variable should be used in linear or log form in regression analyses of money demand functions.

Ireland (2009) argues that the two specifications could differ substantially when interest rates are sufficiently close to zero, so that it is crucially important to employ more recent data with near-zero interest rates. Motivated by Ireland's (2009) argument, we focused in this paper on Japan, which has experienced near-zero interest rates for much longer than other countries including the United States.

Our empirical results based on data for Japan from 1985 to 2017 can be summarized as follows. First, comparing the log-log and the semi-log specification employing cointegration tests, we found that there exists a cointegrating relationship with a single break between the money-income ratio and the interest rate in the case of the log-log form but not in the case of the semi-log form. More specifically, we showed that the substantial increase in the money-income ratio during the period of near-zero interest rates is well captured by the log-log form but not by the semi-log form.

Second, we showed that the demand for money did *not* decline in 2006 when the Bank of Japan terminated quantitative easing and started to raise the policy rate, suggesting that there was an upward shift in the money demand schedule. In fact, our structural break test based on Kejriwal and Perron (2010) showed that there was a single break, which occurred in 2006, in the cointegrating relationship between money and the interest rate in log-log form. This is similar (but in the opposite direction) to the downward shift in the money demand schedule repeatedly observed in high inflation economies, where the demand for money does not increase even after inflation has subsided. The upward shift in the money demand schedule in 2006 points to the presence of high switching costs between money and interest-bearing assets. Based on Mulligan and Sala-i-Martin (2000), we provided the interpretation that Japanese households kept almost their entire wealth in the form of money (i.e., cash and demand deposits) over the two-decade-long near-zero interest period, and consequently failed to update their knowledge on recent financial technology to hold interest-bearing assets.

Third, we computed the welfare cost of inflation based on the estimated money demand function to find that the welfare gain from moving from 2 percent inflation to price stability is 0.10 percent of nominal GDP, which is more than six times as large as the corresponding estimate for the United States.

Our result on the shape of the money demand function has important implications for the conduct of monetary policy at the zero lower bound. A money demand function that takes a semi-log form implies that the marginal utility of money reaches zero at a finite value of real money balances and becomes negative beyond that level. In this case, the opportunity cost of holding money can go below zero, and in this sense there is no lower bound on nominal interest rates, as shown by Rognlie (2016). However, our results support the log-log form, implying that the marginal utility of money monotonically declines and approaches zero as the opportunity cost of holding money falls, but the marginal utility of money never goes below zero. This potentially poses a serious constraint on monetary policy. On the other hand, our finding regarding the upward shift in the money demand schedule points to an advantage of prolonged quantitative easing for the central bank not previously identified; namely, the central bank may be able to raise the policy rate after prolonged quantitative easing without being faced with a reduction in the demand for money.

A Derivation of the log-log and semi-log money demand functions

This appendix explains how we derive the log-log and semi-log money demand functions from households' utility maximization and how the functions change in an economy in which the storage cost of holding money is not negligible. Specifically, we derive the log-log and semi-log money demand functions closely following Lucas (2000) and Cysne (2009), respectively. We then add the storage cost of holding money closely following Wolman (1997), Rognlie (2016), and Eggertsson et al. (2019).

A.1 Log-log form

Let us start with a version of Sidrauski's (1967) model.[1] The representative household maximizes the present value of the sum of utilities,

$$U_t = \sum_{T=t}^{\infty} \beta^{T-t} U(c_T, z_T)$$

where c and z denote consumption and real money balances. Following Lucas (2000), we assume that the current period utility function is given by

$$U(c,z) = \frac{1}{1-\sigma}\left[c\varphi\left(\frac{z}{c}\right)\right]^{1-\sigma} \tag{A.1}$$

where $\sigma > 0$ and $\sigma \neq 1$, and $\varphi(\cdot)$ is a strictly increasing and concave function. We will specify $\varphi(\cdot)$ later. The household faces the following flow budget constraint:

$$M_t + (1 + r_{t-1})B_{t-1} = M_{t-1} + B_t + P_t y_t - P_t c_t \tag{A.2}$$

where B_t, r_t, P_t, and y_t denote a one period risk-free bond held by the household, the nominal interest rate associated with the bond, the price level, and income. The first order conditions for utility maximization imply that the optimal holding of money has to satisfy

$$\frac{U_z}{U_c} = \frac{\varphi'\left(\frac{z}{c}\right)}{\varphi\left(\frac{z}{c}\right) - \frac{z}{c}\varphi'\left(\frac{z}{c}\right)} = r \tag{A.3}$$

Following Lucas (2000), we consider an endowment economy characterized by a balanced growth equilibrium path, on which the money growth rate is constant, maintained by a

[1] Alternatively, the log-log and semi-log money demand functions can be derived using McCallum and Goodfriend's (1989) transaction time model. See Lucas (2000), Simonsen and Cysne (2001), and Cysne (2005) for a discussion of the welfare cost of inflation in a version of the transaction time model with a log-log money demand function.

constant ratio of transfers to income. In this setting, the money-income ratio, given by $m = z/y$, is also constant. Then eq. (A.3) can be rewritten as

$$\frac{\varphi'(m)}{\varphi(m) - m\varphi'(m)} = r$$

This implies that, if money demand is of log-log form, i.e.,

$$m(r) = Ar^{\alpha} \qquad \text{for} \quad r > 0 \tag{A.4}$$

with $A > 0$ and $\alpha < 0$, then the function $\varphi(\cdot)$ solves a differential equation of the form

$$\frac{\varphi'(m)}{\varphi(m)} = \frac{\psi(m)}{1 + m\psi(m)} = \frac{A^{-1/\alpha}m^{1/\alpha}}{1 + mA^{-1/\alpha}m^{1/\alpha}}$$

where $\psi(\cdot)$ is the inverse money demand function (i.e., $(\psi(\cdot) \equiv m^{-1}(\cdot))$. The solution to this differential equation is given by[2]

$$\varphi(m) = \left(1 + A^{-\frac{1}{\alpha}} m^{\frac{1+\alpha}{\alpha}}\right)^{\frac{\alpha}{1+\alpha}}$$

Conversely, if the utility function (A.1) is specified as

$$U(c, z) = \frac{1}{1-\sigma}\left[c\left(1 + A^{-\frac{1}{\alpha}}\left(\frac{c}{z}\right)^{\frac{1+\alpha}{\alpha}}\right)^{\frac{\alpha}{1+\alpha}}\right]^{1-\sigma} \tag{A.5}$$

then the money demand function derived from utility maximization is of log-log form.

When the money demand function takes a log-log form, Bailey's (1956) measure for the welfare gain achieved by lowering the interest rate (and inflation) from r to zero is given by

$$w(r) = \int_{m(r)}^{m(0)} \psi(x)dx = \int_0^r m(x)dx - rm(r) = A\left(-\frac{\alpha}{1+\alpha}\right)r^{1+\alpha} \tag{A.6}$$

This equation implies that $w'(r) > 0$ and $w''(r) < 0$ for $r > 0$; that is, the marginal welfare gain of lowering the interest rate from r toward zero is positive and *increases* as r comes closer to $r = 0$ (i.e., the Friedman rule). This property of the log-log money demand function has been extensively discussed by Lucas (2000) and Wolman (1997). It arises due to the following reasons. First, the marginal utility of money, U_z/U_c, stays strictly positive even when m takes an extremely large value, meaning that there exists no satiation of money if the money demand function is of the form given by (A.3), and thus the zero lower bound is an asymptotic one. Therefore, lowering the interest rate from a positive level toward zero always leads to an increase in utility. Second, the extent to which money demand responds to a reduction in the interest rate gets bigger as r comes closer to $r = 0$.

[2]Lucas (2000) shows that the solution is given by $\varphi(m) = \left(1 + A^2/m\right)^{-1}$ when $\alpha = -1/2$.

A.2 Semi-log form

Following Cysne (2009), we replace the current period utility function by

$$U(c,z) = g\left[c + \lambda(z)\right] \tag{A.7}$$

where $g^{'}(\cdot) > 0$, $g^{''}(\cdot) \leq 0$, $\lambda^{'}(\cdot) > 0$, and $\lambda^{''}(\cdot) < 0$. The first order conditions for utility maximization imply

$$\frac{U_z}{U_c} = \lambda'(z) = r \tag{A.8}$$

If the money demand function is of semi-log form, i.e.,

$$m(r) = B\exp(\alpha' r) \tag{A.9}$$

with $B > 0$ and $\alpha' < 0$, then the corresponding differential equation is given by

$$\lambda'(m) = \frac{1}{\alpha'}\left(\ln m - \ln B\right)$$

and the solution to this differential equation is

$$\lambda(m) = \frac{m}{\alpha'}\left[1 + \ln\left(\frac{B}{m}\right)\right]$$

Conversely, if the utility function is specified as

$$U(c,z) = g\left[c + \frac{z}{\alpha'}\left(1 + \ln\left(\frac{B}{z}\right)\right)\right] \tag{A.10}$$

then the money demand function derived from utility maximization is of semi-log form.

Bailey's (1956) measure for the welfare gain of lowering the interest rate from r to zero is given by

$$w(r) = \int_{m(r)}^{m(0)} \psi(x)dx = -\frac{B}{\alpha'}\left[1 - \left(1 - \alpha' r\right)\exp(\alpha' r)\right] \tag{A.11}$$

Note that $w'(r) > 0$ for $r > 0$, $w'(r) < 0$ for $r < 0$, and that $w'(0) = 0$ and $w''(0) > 0$, implying that the marginal welfare gain obtained by lowering the interest rate and inflation *decreases* as r comes down from positive to zero, and it is exactly zero when $r = 0$. This means that, in the case of the semi-log money demand function, moving from a positive to a zero interest rate does not result in a substantial welfare improvement when r is already close to zero, as pointed out by Lucas (2000) and Wolman (1997). This is in sharp contrast with the implications of the log-log money demand function. This difference arises because, in the semi-log case, the marginal utility of money reaches zero when m takes a finite value and turns negative when m exceeds that value. In other words, real money balances held by the household when $r = 0$ reach a finite satiation level. Reflecting this, the marginal welfare gain obtained by lowering the interest rate reaches zero when $r = 0$ and turns negative when r is below zero.[3]

A.3 The storage cost of money

Recent studies on negative interest rate policy, including Rognlie (2016) and Eggertsson et al. (2019), argue that the cost of holding cash is not negligible. In this subsection, we introduce the storage cost of money into Sidrauski's (1967) model closely following Eggertsson et al. (2019) to examine how the demand for money and the welfare gain of lowering interest rates differ with and without the storage cost of money.

The flow budget constraint now changes to

$$M_t + (1 + r_{t-1})B_{t-1} = M_{t-1} + B_t + P_t y_t - P_t c_t - S(M_{t-1}) \tag{A.12}$$

where $S(M_{t-1})$ denotes the storage cost of money. Note that $S(M_{t-1})$ represents nominal storage costs and depends on nominal (rather than real) money balances. The first order conditions for utility maximization imply

$$\frac{U_z}{U_c} = \frac{\varphi'\left(\frac{z}{c}\right)}{\varphi\left(\frac{z}{c}\right) - \frac{z}{c}\varphi'\left(\frac{z}{c}\right)} = r + S'(M)$$

Following Eggertsson et al. (2019), we assume that the marginal storage cost is positive and constant, so that $S'(M) = \theta > 0$.

Starting from the utility function given by (A.5), we end up with a money demand function of the following form:

$$\begin{aligned} m &= A\,(r+\theta)^{\alpha} \qquad\qquad (A.13)\\ \text{or} \qquad \ln m &= \ln A + \alpha \ln(r+\theta) \end{aligned}$$

which is close to a log-log form but differs from it in that a constant term, θ, is added to r before taking the logarithm. Note that m takes a finite value when $r = 0$, which is an important difference from the case of no storage costs. Turning to the welfare analysis, the

[3]Using a semi-log money demand function, Rognlie (2016) argues that negative interest rates yield an inefficiently high demand for money, thus leading to a deterioration in welfare. In other words, any deviation from the Friedman rule (i.e. $r = 0$), whether $r > 0$ or $r < 0$, results in a suboptimal outcome.

welfare gain of lowering the interest rate from r to zero is now given by

$$\begin{aligned} w(r) &= \int_0^r m(x)dx - rm(r) \\ &= \frac{A}{1+\alpha}\left[(r+\theta)^{1+\alpha} - \theta^{1+\alpha}\right] - rA(r+\theta)^{\alpha} \end{aligned} \tag{A.14}$$

implying that $w'(r) > 0$ for $r > 0$, $w'(r) < 0$ for $-\theta < r < 0$, and that $w'(0) = 0$ and $w''(0) > 0$. An important difference from the case without the storage cost of money is that there exists a finite satiation level of money even for the log-log money demand function, at which the marginal utility of money coincides with the marginal storage cost of money, and that the satiation level is achieved by setting $r = 0$ (i.e., the Friedman rule). Any deviation from the Friedman rule, whether $r > 0$ or $r < 0$, ends up with a suboptimal outcome.

Similarly, if we start from the utility function given by (A.10), we obtain a money demand function of the form

$$\begin{aligned} m &= B\exp\left[\alpha'(r+\theta)\right] && \text{(A.15)} \\ \text{or} \qquad \ln m &= \left(\ln B + \alpha'\theta\right) + \alpha' r \end{aligned}$$

which is a semi-log form quite similar to (A.9). The welfare gain of lowering the interest rate toward zero is given by

$$\begin{aligned} w(r) &= \int_0^r m(x)dx - rm(r) \\ &= \frac{B}{\alpha'}\left[\left(1-\alpha' r\right)\exp\left(\alpha'(r+\theta)\right) - \exp\left(\alpha'\theta\right)\right] \end{aligned} \tag{A.16}$$

implying that $w'(r) > 0$ for $r > 0$, $w'(r) < 0$ for $r < 0$, and that $w'(0) = 0$ and $w''(0) > 0$. Again, any deviation from the Friedman rule ($r = 0$), whether $r > 0$ or $r < 0$, leads to a suboptimal outcome.

B Mulligan and Sala-i-Martin's (1996) model on durable knowledge on financial technology

Mulligan and Sala-i-Martin (2000) make their model a static one by assuming that households' knowledge about financial technology is not durable (i.e., it lasts only one period), and that households have to incur an adoption cost every period in order to get access to financial technology. However, this assumption seems inappropriate if the cost of adoption entails a startup component. For example, an important component of the cost of adoption could be the learning that needs to take place before one can use financial assets. This learning process is probably done once. Once learned, the technology can be used without having to pay any further learning costs. In this sense, households' knowledge on financial technology is durable.

Mulligan and Sala-i-Martin (1996) provide a rough sketch of a model with startup costs. In this setting, the money demand function takes the following form:

$$m_t = \Phi\left(r_t^*, w_t^*\right) m^A\left(r_t\right) + \left[1 - \Phi\left(r_t^*, w_t^*\right)\right] w_t \tag{B.1}$$

where m_t represents money holdings per household divided by the nominal GDP per capita, $m^A(\cdot)$ is the money holding per household divided by nominal GDP per capita for households that have already incurred a startup cost in period t or before that ("adopters"), and w_t is wealth per household divided by nominal GDP per capita. An important determinant of m_t is the fraction of households with knowledge on financial technology, which is denoted by $\Phi\left(r_t^*, w_t^*\right)$. The fraction of such households is determined by households' expectations of the current and future levels of interest rates, as well as their expectations of the current and future levels of their wealth. Households' expectations formed in period t are summarized by a "permanent" level of the interest rate, denoted by r_t^*, and a permanent level of wealth, w_t^*. For example, if households expect in period t that the interest rate will remain high in and after period t, they have sufficient incentive to incur the startup cost, so that the fraction of households with knowledge on financial technology increases (i.e., $\partial\Phi/\partial r_t^* > 0$). Note that the fraction of such households depends only on the current interest rate level if knowledge on financial technology is not durable as assumed by Mulligan and Sala-i-Martin (2000). Similarly, if households expect that future wealth will be higher, they have more incentive to incur the startup cost (i.e., $\partial\Phi/\partial w_t^* > 0$). The first and second terms on the right-hand side of eq. (B.1) represent, respectively, the money demand of adopters and that of non-adopters.

Differentiating (B.1) with respect to r_t leads to

$$\frac{dm_t}{dr_t} = \underbrace{\Phi\left(r_t^*, w_t^*\right) \frac{dm^A}{dr_t}}_{\text{Intensive Margin}} \underbrace{- \left(\frac{\partial\Phi}{\partial r_t^*}\frac{dr_t^*}{dr_t} + \frac{\partial\Phi}{\partial w_t^*}\frac{dw_t^*}{dr_t}\right)\left[w_t - m^A(r_t)\right]}_{\text{Extensive Margin}} \tag{B.2}$$

The term we focus on here is dr_t^*/dr_t, which measures the extent to which a change in the current level of the interest rate, dr_t, is expected to be persistent. This measure was probably close to unity before 2006, when the interest rate consistently declined over time, so that households expected the interest rate decline in period t to be persistent. However, it is highly likely that households regarded the interest rate increase in 2006-2007 as temporary based on the prevailing view at the time that the economic recovery that had started in 2006 would be modest and short-lived. This means that dr_t^*/dr_t was likely to be very close to zero, so that the extensive margin effect associated with the interest rate increase in 2006-2007 was also close to zero.[4] This seems to be a plausible explanation of the shift in the money demand function in 2006.

[4] Mulligan and Sala-i-Martin (2000) highlight that the extensive margin effect crucially depends on the level of interest rates. In an economy with high interest rates, an increase in interest rates, even if it is small, provides households with a strong incentive to switch to interest-bearing assets, since such switching yields sufficiently large interest revenue to cover the fixed costs associated with switching. However, in an economy in which interest rates are close to zero, an interest rate increase does not yield sufficiently large benefits to compensate for the fixed costs associated with switching, so that the extensive margin effect is small.

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Table 1: Cointegration tests

	$\hat{\alpha}$	$\hat{\beta}$	ADF	Z_t	Z_α
Semi-log form $\ln m = \alpha + \beta r$	-0.220	-16.973	-1.029	-1.087	-3.154
Log-log form $\ln m = \alpha + \beta \ln r$	-1.561	-0.177	-1.664	-1.859	-6.718

Note: For the PP tests based on Z_t and Z_α, the log-run variance is computed based on the quadratic spectral kernel and Andrews's (1991) plug-in method. For the ADF test, the lag length is chosen based on the modified Akaike information criterion. ***, **, and * indicate that the null hypothesis of no cointegration can be rejected at the 1%, 5%, and 10% level, respectively.

Table 2: Structural break tests

	Semi-log form $\ln m = \alpha + \beta r$	Log-log form $\ln m = \alpha + \beta \ln r$
$\sup F^*(1)$	5.647	14.803***
$\sup F^*(2)$	3.957	6.613
$\sup F^*(3)$	5.825	4.438
$\sup F^*(4)$	5.188	4.749
$\sup F^*(5)$	4.781	4.778
UDmax(5)	5.825	14.803***
No. of breaks	0	1
Break dates	No breaks	2005:Q3

Note: The break dates are estimated by minimizing the sum of squared residuals based on eq. (3). ***, **, and * indicate that the null hypothesis of no cointegration can be rejected at the 1%, 5%, and 10% level, respectively.

Table 3: Gregory-Hansen tests

	Inf-ADF	Inf-Z_t	Inf-Z_α
Semi-log form			
$\ln m = \alpha + \beta r$	-2.748	-2.807	-16.348
Log-log form			
$\ln m = \alpha + \beta \ln r$	-6.198***	-7.152***	-76.867***

Note: The three test statistics (Inf-ADF, Inf-Z_t, and Inf-Z_α) are computed using the procedure proposed by Gregory and Hansen (1996). ***, **, and * indicate that the null hypothesis of no cointegration is rejected at the 1%, 5%, and 10% level, respectively.

Table 4: Robustness checks

(a) *Alternative measure for the opportunity cost of holding money*						
	ADF	Z_t	Z_α	Inf-ADF	Inf-Z_t	Inf-Z_α
Semi-log form	-1.263	-0.713	-1.691	-2.764	-2.786	-16.166
Log-log form	-1.671	-2.636	-13.083	-3.829	-7.898***	-88.775***

(b) *Adjustment to M1*						
	ADF	Z_t	Z_α	Inf-ADF	Inf-Z_t	Inf-Z_α
Semi-log form	-1.630	-0.926	-2.670	-2.622	-2.351	-11.518
Log-log form	-1.813	-1.959	-7.415	-4.584	-6.195***	-61.287***

(c) *No restriction on income elasticity*						
	ADF	Z_t	Z_α	Inf-ADF	Inf-Z_t	Inf-Z_α
Semi-log form	-1.678	-1.879	-3.873	-4.543	-4.009	-29.405
Log-log form	-1.821	-2.027	-5.884	-5.604**	-7.200***	-77.638***

(d) *Components of M1*						
	ADF	Z_t	Z_α	Inf-ADF	Inf-Z_t	Inf-Z_α
Log-log form						
Currency	-2.372	-1.926	-2.529	-5.965***	-6.083***	-52.117**
Demand deposits held by households	-2.512	-2.036	-2.329	-5.806***	-5.944***	-51.024**
Demand deposits held by non-financial firms	-1.656	-1.878	-3.318	-7.148***	-6.396***	-57.595***

Note: For the PP tests (Z_t and Z_α), the long-run variance is based on the quadratic spectral kernel (Andrews 1991 and Andrews and Monahan 1992). For the ADF test, the lag length is chosen based on the modified Akaike information criterion. For the Gregory-Hansen test, the three test statistics (Inf-ADF, Inf-Z_t, and Inf-Z_α) are computed using the procedure proposed by Gregory and Hansen (1996). ***, **, and * indicate that the null hypothesis of no cointegration is rejected at the 1%, 5%, and 10% level, respectively.

Table 5: Welfare cost of inflation

Country	Observation period	Functional	Parameter values		Welfare cost		
		form	α	β	$r = 0.13$	$r = 0.05$	$r = 0.03$
United States	1900-1994	semi-log	-1.036	-7.000	1.17%	0.25%	0.10%
		log-log	-3.020	-0.500	1.76%	1.09%	0.85%
	1980:Q1-2013:Q4	semi-log	-1.778	-2.276	0.27%	0.04%	0.02%
		log-log	-2.089	-0.055	0.10%	0.04%	0.03%
Japan	1985:Q1-2017:Q4	semi-log	-0.220	-16.973	3.06%	0.99%	0.44%
	1985:Q1-2005:Q3	log-log	-1.623	-0.150	0.61%	0.27%	0.18%
	2005:Q4-2017:Q4	log-log	-0.404	-0.063	0.66%	0.27%	0.17%

Note: It is assumed that the real interest rate at the steady state is 3 percent, so that $r = 0.03$ corresponds to price stability, $r = 0.05$ to 2 percent inflation, and $r = 0.13$ to 10 percent inflation. The welfare cost of inflation, $w(r)$, is calculated as $w(r) = e^{\alpha}\left(-\beta/(1+\beta)\right) r^{1+\beta}$ for the log-log specification and $w(r) = -e^{\alpha}\left[1-(1-\beta r)e^{\beta r}\right]/\beta$ for the semi-log specification. The US parameter values for α and β are taken from Lucas (2000) for the period 1900-1994 and from Watanabe and Yabu (2018) for the period 1980:Q1-2013:Q4. The parameter values for Japan are from Table 1 for the semi-log specification and from eq. (6) for the log-log specification.

Figure 1. The demand for money in Japan, 1985-2017

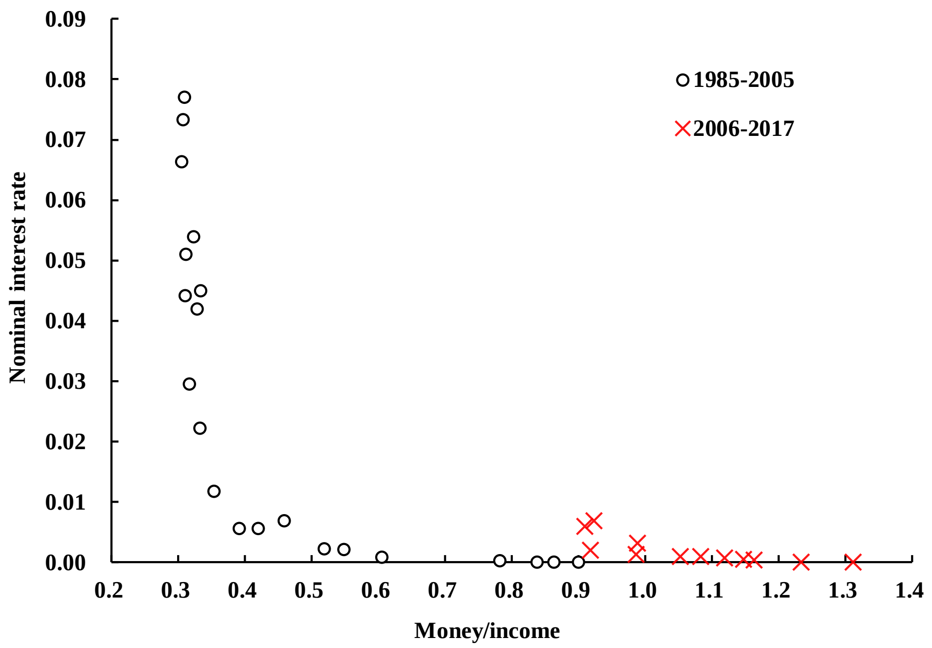
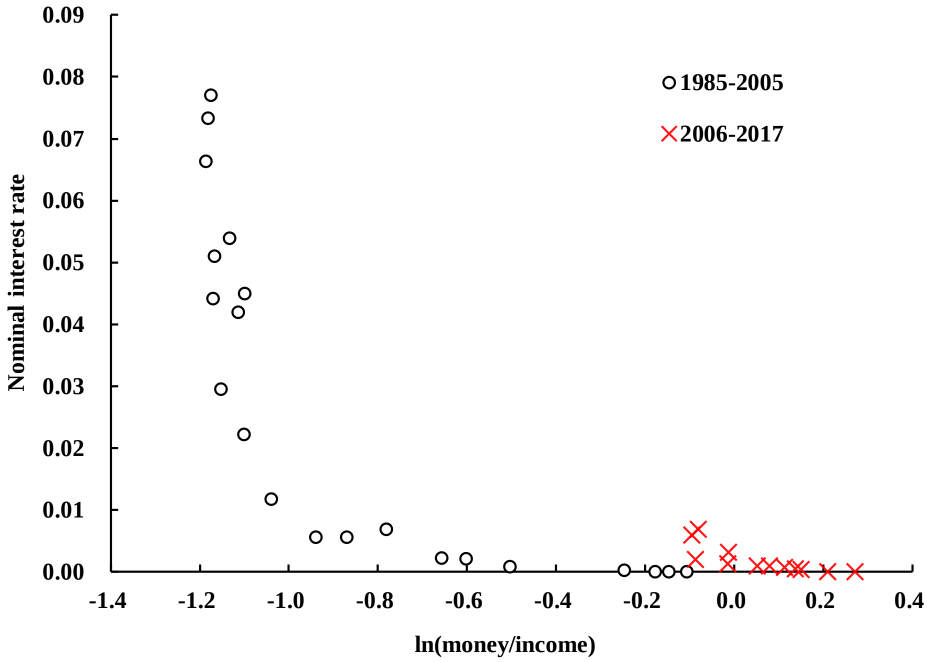


Figure 2. Semi-log vs. log-log plots

(a) Semi-log plot



(b) Log-log plot

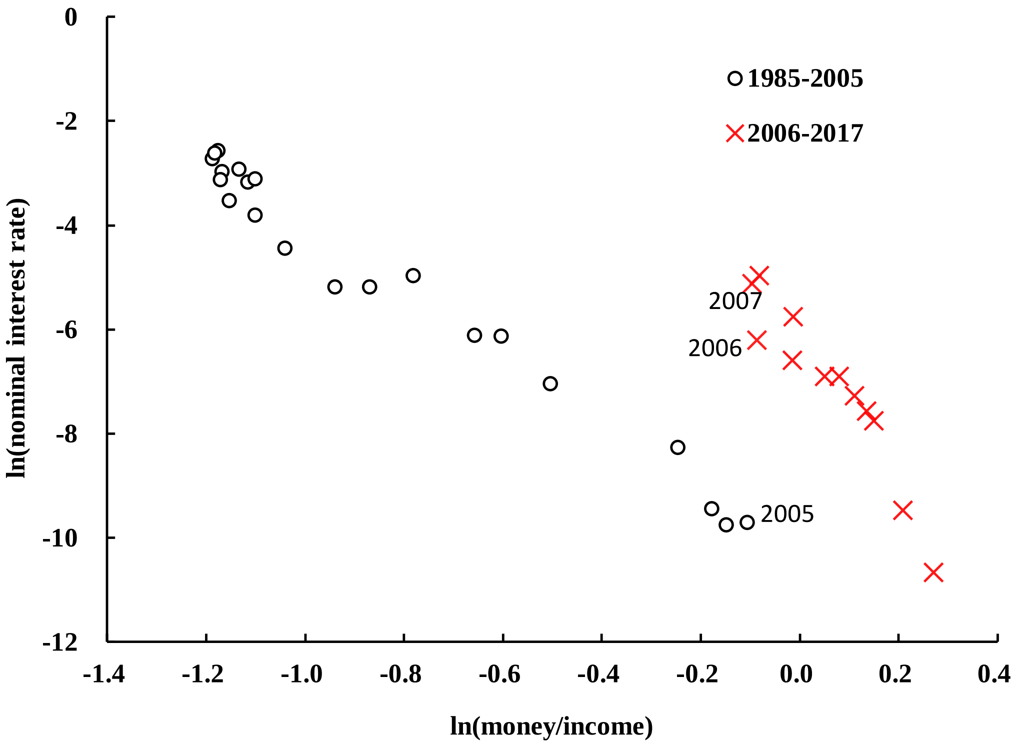


Figure 3. CD rate and the money-income ratio in 2004-2010

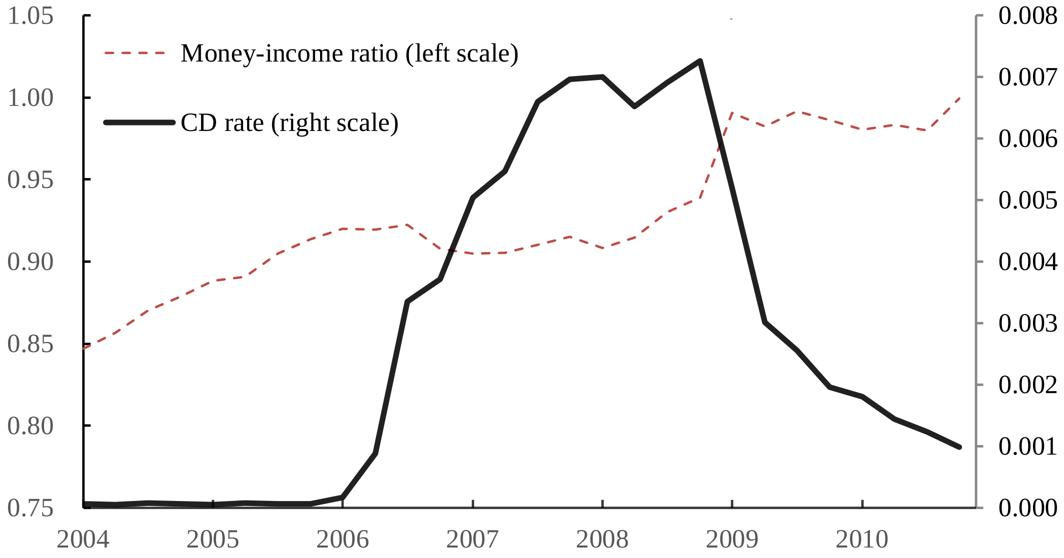


Figure 4: Interest semi-elasticity of money demand

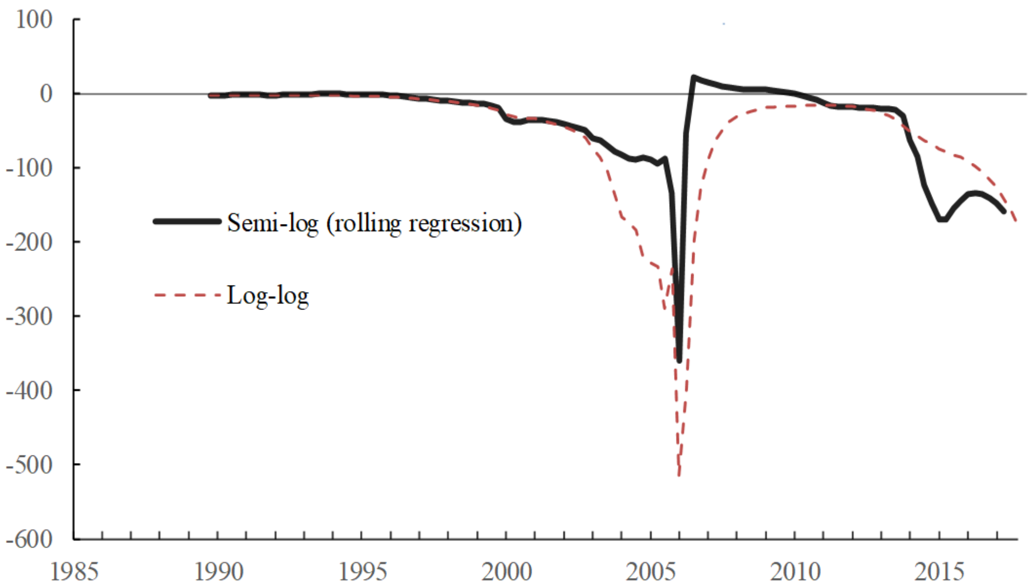


Figure 5: Quarterly data

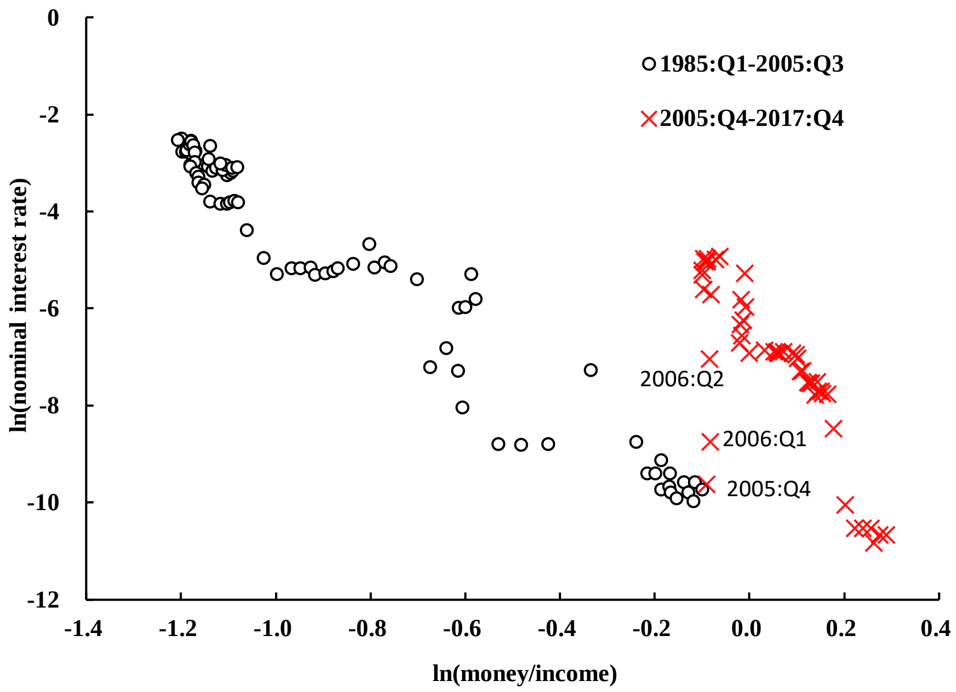


Figure 6: Estimated money demand functions

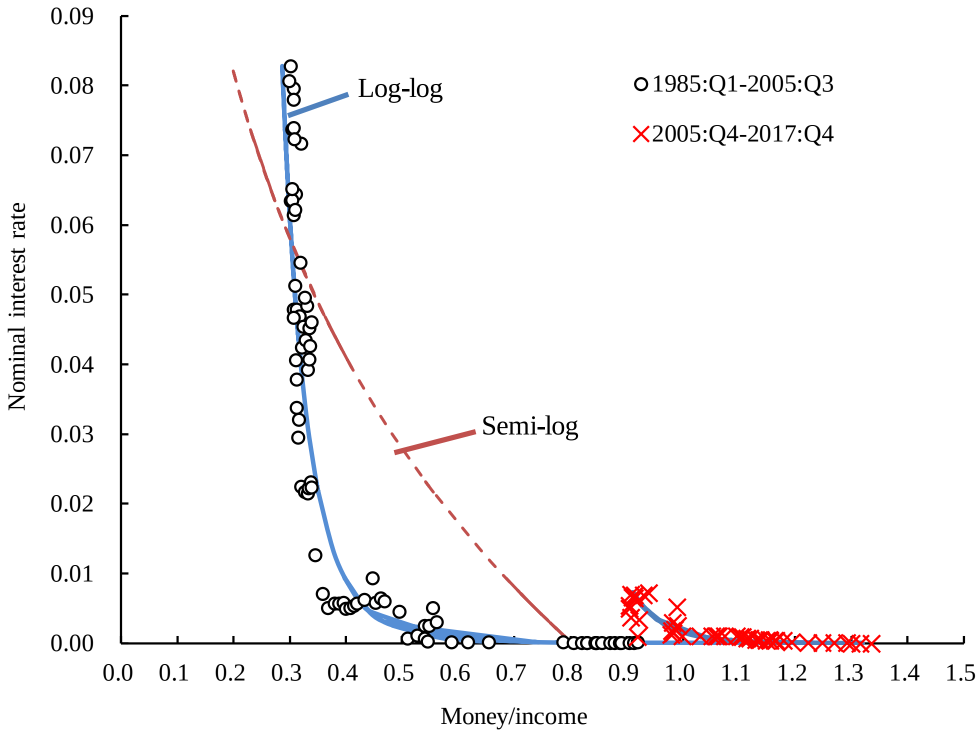
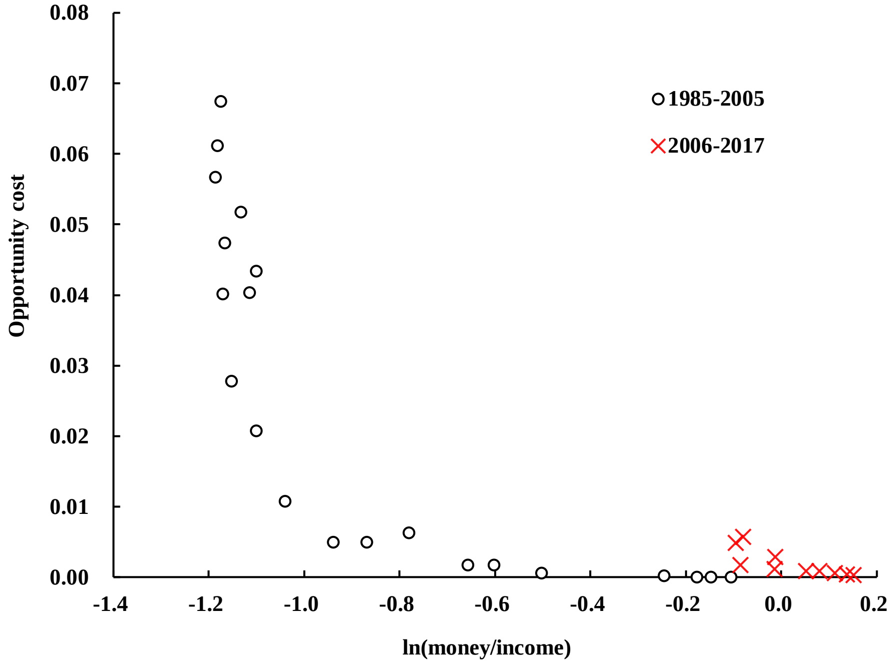


Figure 7: Alternative measure for the opportunity cost of holding money

(a) Semi-log plot



(b) Log-log plot

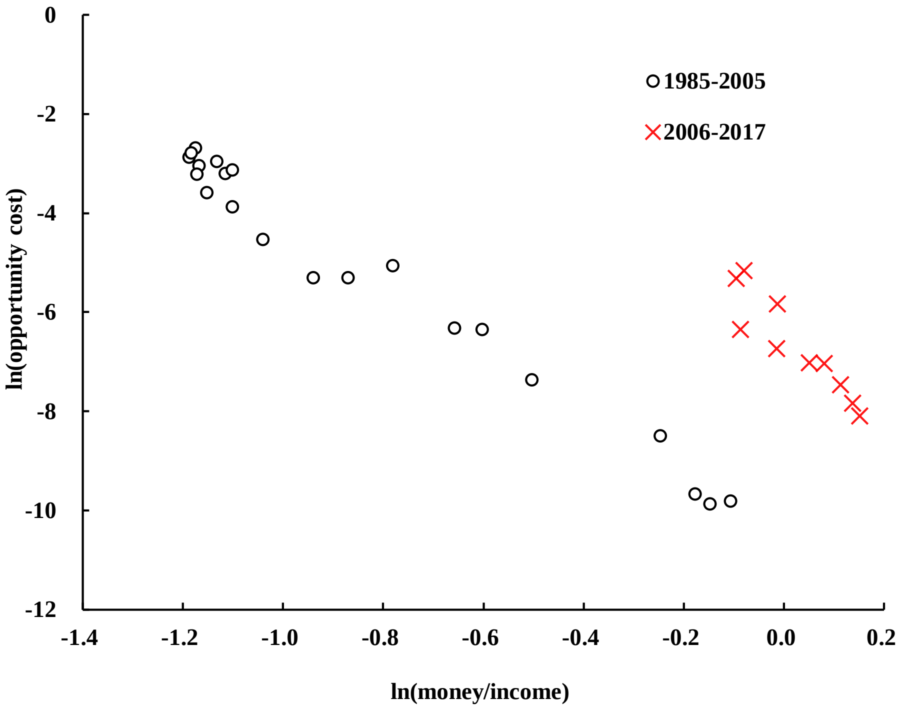


Figure 8: Adjustment for the effect of the 2002 deposit insurance reform

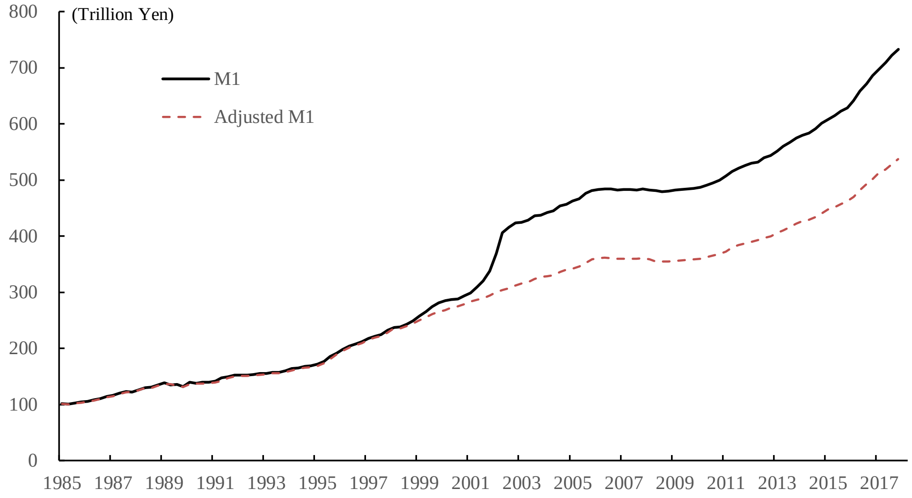
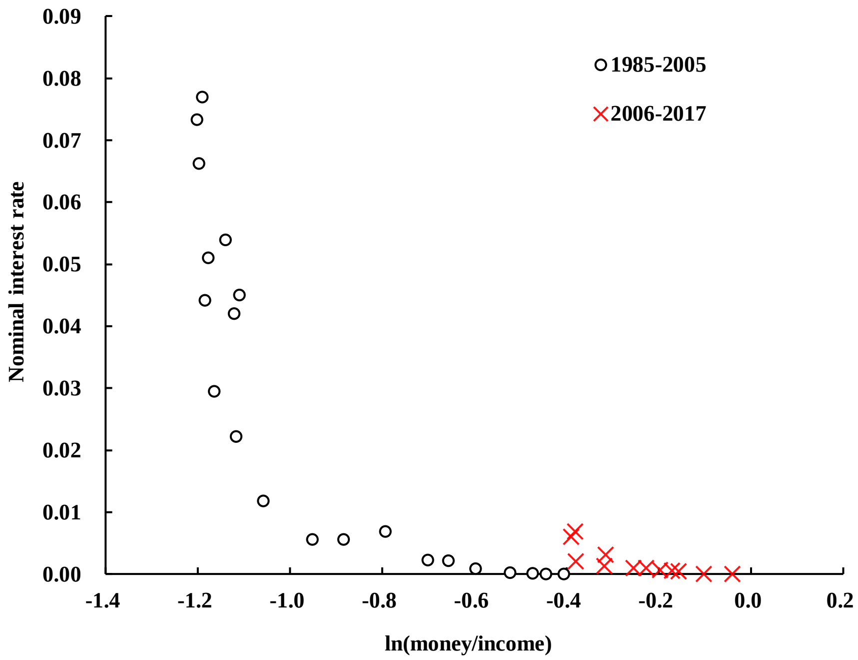


Figure 9: Money demand function with adjusted M1

(a) Semi-log plot



(b) Log-log plot

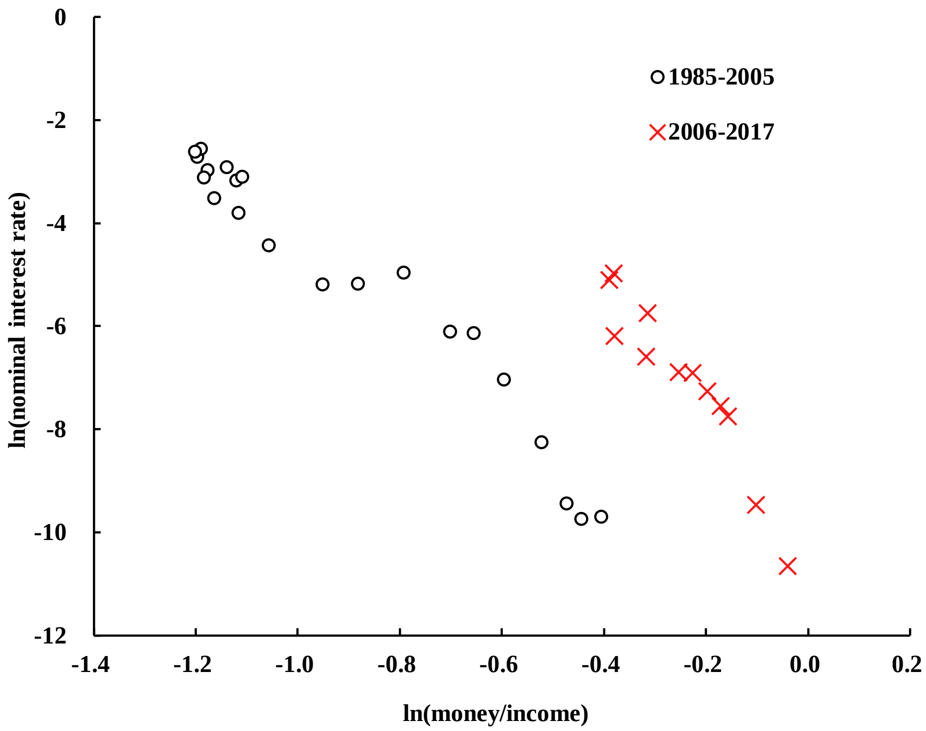
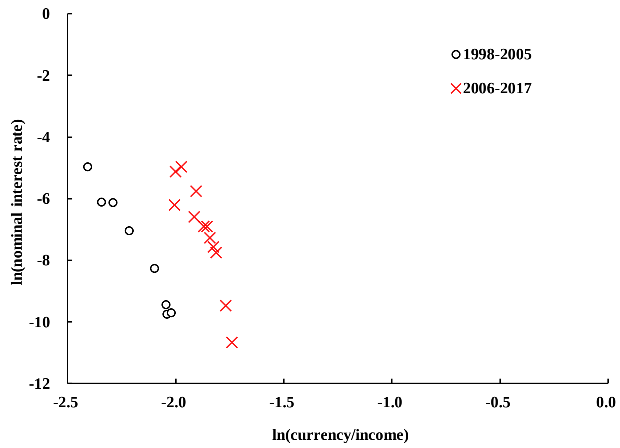
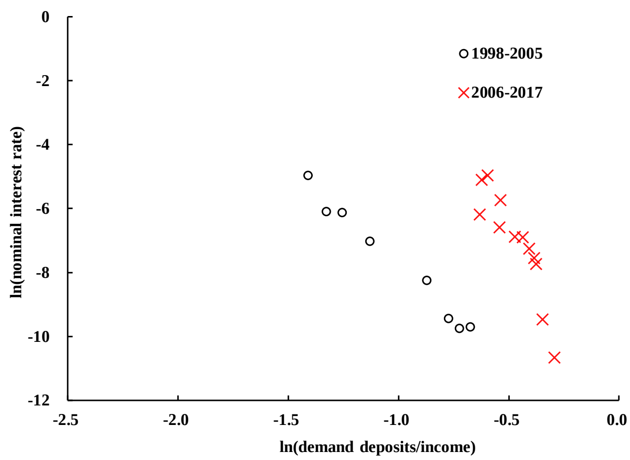


Figure 10: Cash and demand deposits

(a) Currency



(b) Demand deposits held by households



(c) Demand deposits held by non-financial firms

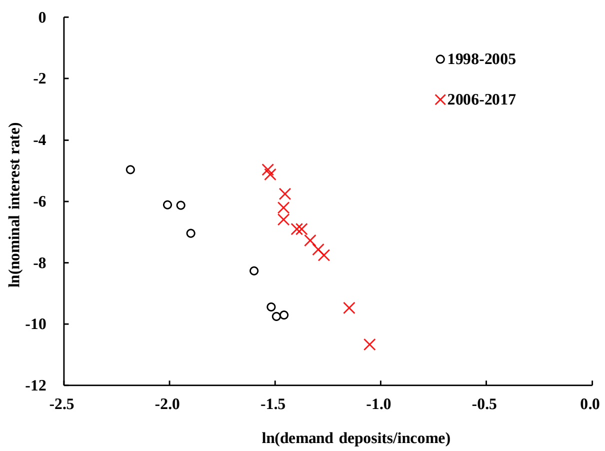


Figure 11: Financial assets held by households

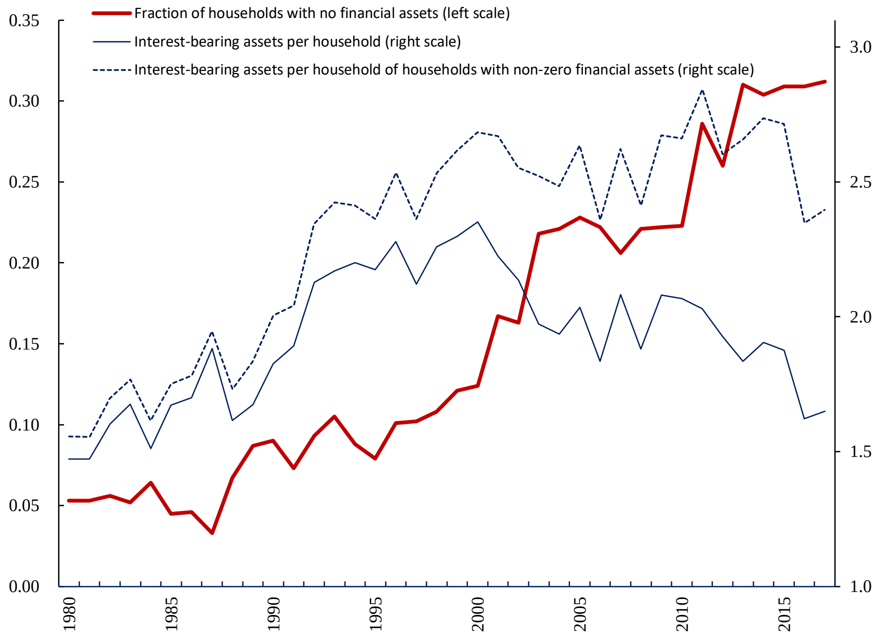


Figure 12: Welfare cost of inflation