Bubble Cycles [#]

Very preliminary, welcome comments

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Abstract

We develop a simple model that explains episodes on bubbles that occurred in the past Japan and the recent United States. The basic idea is that the self-fulfilling change in the saving rate leads to bubble-induced business cycles with co-movement between bubbles and investment. We have a counter-intuitive implication of the stimulus package of fiscal expansion in the bubbly economy.

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1. Introduction

Business cycles are often accompanied by bubbles. The boost of bubbles create investment booms, but the crash of bubbles, sometimes triggered by financial crisis, gives rise to prolonged and severe recessions. The story of business fluctuations induced by bubbles explains the current economic slowdown of the U.S. economy that has been triggered by the sub-prime-loan syndrome as well as the prolonged recession in Japan in the 1990s.

The aim of this paper is to construct a simple model of business fluctuations induced by investments that go hand in hand with bubbles. The starting point of the argument is Tirole (1985) that develops the theory of rational bubbles in the growth model of overlapping generations. Although he demonstrates that bubbles can arise when the growth rate is above the interest rate, his argument fails in explaining two important observations on how bubbles impact on economic activities. First, Tirole (1985) demonstrates that the bubbles arise when capital over-accumulation arise in the bubbleless economy, and move the economy to the Golden Rule. But we have found rare evidence for capital over-accumulation (e.g., Abel et al, 1989). Second, in Tirole bubbles are assets that compete with investment in capital in the portfolio of investors so that *the portfolio effect* of bubbles crowds investment out, but the crowding-out view seems to contradict the recently observed comovement between bubbles and investment.

We consider two ingredients, the imperfections of capital markets, and the behavior of the aggregate saving that varies with respect to the interest rate, so as to explain the coexistence of bubbles and capital under-accumulation, and the comovement between bubbles and investment in a unified model.

The poor working of capital markets brought about by imperfect pledgeability gives rise to the wedge between the rate of return to capital and the real interest rate [e.g., Stiglitz and Weiss (1981), Gale and Hellwig (1985), Williamson (1986), and others], which in turn leads to an interesting situation when the rate of return to capital is greater but the interest rate is smaller than the growth rate so that bubbles can arise even when capital under-accumulation prevails. Imperfect pledgeability furthermore induces people to find bubbles as internal wealth valuable. The channel of *the collateral effect* enables investment to go hand in hand with bubbles (e.g., Kiyotaki and Moore (1997), Venture (2003), Caballero and Krishnamurthy (2006), Farhi and Tirole (2008)).

When the aggregate savings increase rapidly as the interest rate rise for some region, we find an interesting link between savings and bubbles. Bubbles increase the interest rate, and are followed by the rise in savings so that the self-fulfilling expectation on savings boosts bubbles. The link between bubbles and saving is not unconventional. A large part of the flows of income are consumed, but incomes from capital gains of assets are almost reinvested.

Whether bubbles crowd investment *in or out* depend on whether the collateral effect is stronger or weaker than the portfolio effect. Whether savings rise or not is essential to which effect is greater. When savings do not change whether bubbles arise, the collateral effect is always dominated by the portfolio effect, and then the bubbly steady state features less investment than the bubbleless steady state. By contrast, when savings change when bubbles arise, the collateral effect can dominate the portfolio effect, and the bubbly steady state will feature more investment than the bubbleless steady state.

We develop a story of business cycles induced by bubbles that are linked to the self-fulfilling behavior of savings. Appreciations in asset prices generate the economic boom, and the crash of asset prices leads to the recession, and importantly bubbles are induced by the self-fulfilling expectations on savings and/or the rate of return to bubbles. When people anticipate the high return of bubbles, they save and bubbles actually boost, whereas people anticipate the low return of bubbles, they dissave and bubbles burst.

This paper has also implications for dynamic efficiency/inefficiency. The greater rate of return to capital than the growth rate is neither a necessary nor sufficient condition for dynamic efficiency. Even when the rate of return to capital is less than the growth rate so that capital over-accumulation arises in the bubbleless economy, dynamic efficiency holds. Additionally, even when the rate of return to capital is greater than the growth rate holds, the bubbleless economy is dynamically inefficient when bubbles crowd investment in. When we consider an economy with imperfect pledgeability, the criterion proposed by Able et al (1989) gives a misleading implication for dynamic efficiency.

Caballero et al (2006) and Farhi and Tirole (2008), closely related to this paper, develop models so as to explain the comovement between investment and bubbles or asset prices. Caballero et al (2006) develop their argument for the growth-saving feedback so as to explain the high correlation between investment and the stock price, and bubbles, but in theirs the bubbly steady state does not feature more investment than the bubbleless steady state. Farhi and Tirole (2008) develop a model of imperfect pledgeability where the bubbly steady state features more investment than the bubbleless steady state, but relies on some non-standard assumption for the portfolio choice of entrepreneurs. Ventura (2003) provides a setting in which entrepreneurs issue debt and also equity to finance investment, where equity takes the form of bubble creation. Caballero and Krishnamurthy (2006) develop a small-open economy in which asset bubbles are used for collateral for financing productive investment so that bubbles can promote capital accumulation.

This paper is organized as follows. Section 2 looks at the overview of the advanced countries. Section 3 sets up the model and studies the benchmark economy. Section 4 analyzes the economy when there are frictions in financial markets. Section 5 develops the story of bubble cycles.

2. Overview of the Recent Observations in Advanced Economies

The coexistence of the historically low real interest rate, asset bubbles, and weak investment demand relative to great savings, in other words, capital under-accumulation is a dominant feature of the world economy over this decade. This phenomenon is seemingly casual, but unusual from economic theories. Investigating interest rates versus economic growth rates will give a hint to solve this puzzle.

Figure X illustrates several interest rates and the economic growth rate averaged over G7 countries (U.S., U.K., Japan, Germany, France, Canada, and Italy).[1] Until the end of 1990s, all the interest rates except for the deposit rate were greater than the economic growth rate, but this tendency has changed around 2000. Some of interest rates began to be smaller than the economic growth rate, and since 2004, all the interest rates are smaller than the economic growth rate.

The recent observation for the greater economic growth rates than interest rates is consistent with economic theories on bubbles. Diamond (1965), Ihori (1978), and Tirole (1985) explain the sustainability of asset bubbles in the model of finitely-lived agents, and show that asset bubbles exists when capital is overly accumulated in the bubbleless economy, put differently, when the rate of return to capital is smaller than economic growth rate. Since the late 1990s, we have witnessed several bubbles, including the dot-com bubbles, real estate booms, stock market booms, and recently appreciations in gold and oil.[2] Their story explains the coexistence of bubbles and low interest rates, but cannot explain why capital-under accumulation that coexists with bubbles.

We check capital under- or over-accumulation using the criterion proposed by Abel et al

[1] Each of interest rates and the economic growth rate is a simple average of G-7 countries (*source:* IFS). We exclude the 1991 data of German in calculating averages. Money market rate, treasury bills, treasury bills: 3years or longer, deposit rate, and max overdraft reflect call rate, short-term rate of the government bond, long-term rate of the government bond, short-term deposit interest rate, and loan interest, respectively.

[2] Additionally, behind asset bubbles in China that have been sustained for more than two decades is the far higher economic growth rate than the interest rates.

(1989) that compare capital income and the aggregate investment, not relying on rates of returns to capital that are often unobservable in actual economies. According to their criterion, capital is under- (over-) accumulated if and only if capital income is greater (smaller) than investment.

Table 1 describes gross capital income less gross investment divided by GDP over G7 countries for the sample of 1990-2004(source: *National Accounts of OECD Countries*). Every figure is positive in every country in every year so that we can safely conclude that all the listed countries have experienced capital under-accumulation.[3]

3. Basic Model

Let us consider an economy of overlapping generations that lasts for infinity. At each period $t = 0,1,2,\dots,\infty,$ the economy is populated by a continuum of agents that live for three periods. They are endowed with H_t units of the final good in the young age, and supply one unit of labor in the middle age. There is no population growth.

At each period the final good is produced by firms that use labor and capital as inputs according to the constant-returns-to-scale technology described as $Y_t = F(K_t, A_t N_t)$, where K_t and N_t are aggregate supplies of capital and labor, and A_t is the time-varying technology level that grows at an exogenous rate g. The endowment H_t also grows at rate g so that we denote $H_t = A_t h$ .The labor force is constant over time and normalized unity. Letting $k_t (\equiv K_t / A_t)$ denote capital per effective worker, the output per effective worker is described as $y_t \equiv Y_t / A_t = F(K_t / A_t, 1) \equiv f(k_t)$, where $f(.)$ is thrice continuously differentiable, increasing, concave, satisfying $f(0) = 0$, and $\lim_{k_t \to 0} f'(k_t) = +\infty$. Since the production technology is homogeneous of degree one, output of the final good can be described in terms of the action of a single, aggregate, price-taking firm. From the profit-maximizing behavior of that firm, output is exhausted by the payment to two inputs and each input is paid its marginal product. The rate of return to capital R_t and the wage rate w_t are determined to satisfy $R_t = f'(k_t)$ and $w_t = f(k_t) - k_t f'(k_t) \equiv W(k_t)$, respectively. The final good is numeraire.

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[3] This tendency for capital under-accumulation is not specific to advanced countries. The observation in the recent international capital mobility suggests the world-wide stagnant investment. Capital does not flow from OECD to non-OECD countries, but flows from non-OECD to OECD countries (e.g., Prasad et al(2007)), suggesting the evidence of stagnant investment in emerging economies.

Assume that capital depreciates fully after one period. The price of capital is then equal to R_t.

Within each generation, agents are divided into two types with respect to their preference. A fraction s_L $(0 < s_L < 1)$ of them are "patient", and maximize their old age consumption, while the remaining are "impatient" and maximize their linear combination of the middle age and old age consumptions.

A fraction α $(0 < \alpha < 1)$ of agents are "entrepreneurs" who have access in the middle age to one linear capital investment technology that transforms one unit of the final good into one unit of capital after one period, while the remaining fraction $1-\alpha$ of agents are "investors" who have not. Letting $i = E$ denote entrepreneurs and $i = I$ investors, impatient agents maximize their utility, $c_{t+1}^m + \beta_i c_{t+2}^o$, where $c_{t+1}^m (c_{t+2}^o)$ is consumption in the middle (old) age. and β_i is the discount factor. We allow for $\beta_E \neq \beta_I$.

Let s_t denote the aggregate saving rate. We focus on the case for $s_t = 1$ when all agents save in the middle age, and for $s_t = s_L$ when impatient agents do not save in the middle age. The essential feature is the aggregate saving function is that there is a region in which savings increase rapidly as the interest rate rises.

Assume that there is no enforcement mechanism to fulfill financial contracts between debtors and creditors and hence to enforce on borrowers to repay their debt. When debtors breach the contract and refuse to make their repayment, a portion $\lambda(0 < \lambda < 1)$ of their earnings are assumed to be forfeited by the creditor. The parameter λ is thus interpreted to capture the efficiency of the broadly defined financial system. A low λ is interpreted to capture weak bankruptcy procedure, poor bank monitoring, and low contract enforcement, and will be associated with poor financial development. Much literature on economic growth has argued the importance of institutional quality (e.g., North (1981), Hall and Jones(1999), Acemoglu et al(2001) and others). La Porta et al (1997, 1998), Beck et al(2000), and Levine et al(2000) have used indicators of investor rights and protection, and legal enforcement as instrumental variables for financial development so as to link economic and financial developments.

First of all, we investigate an economy with perfect pledgeability featured by perfect capital markets. Letting I_t denote the amount of investment, X_t denote the internal wealth of the middle age agents, and r_{t+1} denote the interest rate prevailing between t and t+1, any of entrepreneurs earns $f'(k_{t+1})I_t - (1+r_{t+1})(I_t - X_t)$ by implementing own project, while $(1+r_{t+1})X_t$ by supplying his wealth to others. Entrepreneurs are willing to start their own projects if

(2-1) $$f'(k_{t+1}) \geq 1 + r_{t+1} .$$

We call this inequality the *profitability constraint*. The "AK" structure of the investment project makes the inequality bind with equality. Eventually,

(2-2) $$f'(k_{t+1}) = 1 + r_{t+1}$$

is only sustainable in equilibrium. The aggregate capital is defined as the product of three terms, the population of entrepreneurs α, the aggregate saving rate s_t, and the individual investment size I_t so that $K_{t+1} = \alpha s_t I_t$, and hence we have

(2-3) $$(1+g)k_{t+1} = \alpha s_t \, I_t / A_t .$$

At the beginning of the middle age, the total income is composed of the wage income $A_t w_t$ and the interest income of the young age income $(1+r_t)H_{t-1}$. Given the saving rate s_t, the aggregate savings of the middle are $s_t\{A_t w_t + (1+r_t)H_{t-1}\}$. Let B_t denote units of aggregate bubbles, and p_t denote the price of bubbles. The aggregate savings are used for financing investment in capital $\alpha s_t I_t$ and purchasing bubbles $p_t B_t$, and its relation is described by

(2-4) $$s_t\{A_t w_t + (1+r_t)H_{t-1}\} + H_t = \alpha s_t I_t + p_t B_t .$$

Letting $b_t (\equiv p_t B_t / A_t)$ denote bubbles per effective worker at time t, and using (2-3), (2-4) is expressed by

(2-5) $$s_t\{W(k_t) + h\frac{1+r_t}{1+g}\} + h = (1+g)k_{t+1} + b_t .$$

Under perfect foresight, bubbles have to earn the same rate of return as that on capital to satisfy $p_{t+1}/p_t = 1 + r_{t+1}$. Given that the net supply of nominal bubbles is zero, the aggregate bubbles per effective worker thus grow to satisfy

(2-6) $$\frac{b_{t+1}}{b_t} = \frac{1+r_{t+1}}{1+g} .$$

Finally we exclude negative bubbles;

(2-7) $$b_t \geq 0 .$$

We define two kinds of equilibria. *A bubbleless economy* is defined as an equilibrium in which $b_t = 0$ for any t or b_t converges to zero if $b_t > 0$ for any t. *A bubbly economy* is defined as an equilibrium in which b_t does not converge to zero.

The steady state of a bubbleless economy is characterized by a triplet $\{\bar{k}, \bar{r}, \bar{s}\}$, satisfying $(1+g)\bar{k} = \bar{s}\{W(\bar{k}) + \frac{1+\bar{r}}{1+g}h\} + h$, $1+\bar{r} = f'(\bar{k})$, and $\bar{s} = s_L$ or 1. On the other hand, the steady state of a bubbly economy is characterized by four variables $\{k_{GR}, r_{GR}, s_{GR}, b_{GR}\}$, satisfying $1+r_{GR} = f'(k_{GR})$, $(1+g)k_{GR} + b_{GR} = s_{GR}\{W(k_{GR}) + \frac{1+r_{GR}}{1+g}h\} + h$, , and $r_{GR} = g$, with $b_{GR} > 0$.

We briefly summarize the properties of the economy under perfect financial markets. If $\bar{r} > g$, there exists a unique bubbleless economy and the interest rate converges to $\bar{r}$, while otherwise, there exists an asymptotically bubbly economy and the interest rate converges to g [Tirole (1985, Proposition 1)]. Ihori (1978) demonstrates that the government bond, which is intrinsically valueless, carries the long-run capital level to the Golden Rule level of capital when $\bar{r} < g$.

4. The Economy with Financial Market Frictions

We now consider an economy with imperfect pledgeability featured by financial market frictions. The financial market is competitive in the sense that both entrepreneurs (borrowers) and investors (lenders) take the equilibrium rate r_{t+1} as given. Importantly, now entrepreneurs have an incentive to cheat creditors. If any of entrepreneurs borrows the amount $(I_t - X_t)$ and repays $(1+r_{t+1})$ $(I_t - X_t)$ honestly, his earnings will be $f'(k_{t+1})I_t - (1+r_{t+1})(I_t - X_t)$, whereas if he breaches the promise for repayment, a portion λ of his earning is forfeited, and his earning would be $(1-\lambda) f'(k_{t+1})I_t$. The incentive compatibility constraint that induces entrepreneurs to commit the truthful behavior is described as

(3-1) $$(1+r_{t+1})(I_t - X_t) \leq \lambda f'(k_{t+1})I_t .$$ [5]

Equation (3-1) implies that entrepreneurs can borrow up to some fraction of the project revenue

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[5] Implicit in (3-1) is that entrepreneurs do not use the borrowed fund to buy bubbles. Entrepreneurs will borrow only for capital investment because the rate of return from capital is greater than the one from holding bubbles when (3-1) binds with equality, as argued below.

so that we will call this inequality the "*borrowing constraint*".[6] The market clearing in the capital market is given by $\alpha s_t(I_t - X_t) = (1-\alpha)s_t X_t + H_t - p_t B_t$, where the L.H.S. is the aggregate demand for funds by entrepreneurs, and the R.H.S. is the aggregate supply of fund by investors, except for holding bubbles. Incorporating the agent's wealth $X_t = A_t w_t + H_{t-1}(1+r_t)$ into the above equality, rearranging terms, we have

(3-2) $$s_t\{W(k_t) + h\frac{1+r_t}{1+g}\} + h = (1+g)k_{t+1} + b_t,$$

which is the same as (2-5).

The determination of the real interest rate requires careful analysis. Either the profitability constraint or the borrowing constraint should bind with equality. If the borrowing constraint is not binding, the profitability constraint should bind with equality, whereas if the borrowing constraint is binding with equality, the profitability constraint may not be binding. With the market clearing in the financial market, we summarize the above argument by;

(3-3) $$1 + r_{t+1} = \min\{f'(k_{t+1}), \lambda f'(k_{t+1})\frac{I_t}{I_t - X_t}\}.$$

Without loss of generality, we confine attention on symmetric equilibria in which all entrepreneurs choose the same amount of investment.

When the borrowing constraint binds with equality, (3-3) is replaced by

(3-4) $$1 + r_{t+1} = \frac{\lambda f'(k_{t+1})(1+g)k_{t+1}}{(1+g)k_{t+1} - \alpha s_t(w_t + h\frac{1+r_t}{1+g})}.$$

When the borrowing constraint is binding with equality, three equations (2-6), (3-2), (3-4), and $s_t = s_L$ or 1 define a bubbly economy with $b_t > 0$ for $t \to \infty$. The steady state is expressed as four variables $\{\tilde{k}_B, \tilde{b}, \tilde{s}, \tilde{r}_B\}$, satisfying

(3-5) $$\tilde{s}\{W(\tilde{k}_B) + h\} + h = (1+g)\tilde{k}_B + \tilde{b}$$

(3-6) $$1 + \tilde{r}_B = \lambda f'(\tilde{k}_B)\frac{(1+g)\tilde{k}_B}{(1+g)\tilde{k}_B - \alpha\tilde{s}\{W(\tilde{k}_B) + h\}},$$

(3-7) $$\tilde{r}_B = g,$$

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[6] A number of other incentive considerations allow one to derive the similar borrowing constraint. For example, the literature on credit rationing (e.g, Stiglitz and Weiss (1981), Williamson (1986), Aghion and Bolton (1997), and others) leads eventually to the same specification.

and $\tilde{s} = s_L$ or 1.

Before proceeding to the analysis of the bubbly economy, it is useful to study the bubbleless economy. Equation (3-3), using (3-2), reduces to

(3-8) $$1+r_{t+1} = \min\{f'(k_{t+1}), \lambda f'(k_{t+1})\frac{k_{t+1}}{(1-\alpha)k_{t+1}+\alpha h/(1+g)}\}.$$

We see that $\frac{k_{t+1}}{(1-\alpha)k_{t+1}+\alpha h/(1+g)}$ is increasing in k_{t+1} but below $\frac{1}{1-\alpha}$. It is obvious to verify that $\alpha+\lambda<1$ is a sufficient condition under which the borrowing constraint is binding, to have

(3-9) $$1+r_{t+1} = \lambda f'(k_{t+1})\frac{k_{t+1}}{(1-\alpha)k_{t+1}+\alpha h/(1+g)}.$$

We focus on the case when the borrowing constraint is binding, and so impose the following:

Assumption 1 $\alpha+\lambda<1$

Assumption 1 is intended to describe an economy when financial market imperfections are serious.[7] Assumption 1 guarantees the wedge between the rate of return to capital and the real interest rate, that is, $1+r_{t+1} < f'(k_{t+1})$.

Incorporating (3-9) into the market clearing (3-2) leads to the following dynamics of capital;

(3-10) $$(1+g)k_{t+1} = s_t\{W(k_t)+h\frac{\lambda f'(k_t)k_t}{(1-\alpha)k_t+\alpha h/(1+g)}\}+h.$$

The steady state is characterized by the triplet $\{\tilde{k},\tilde{r},\tilde{s}\}$, satisfying

(3-11) $$(1+g)\tilde{k} = \tilde{s}\{W(\tilde{k})+h\frac{\lambda f'(\tilde{k})\tilde{k}}{(1-\alpha)\tilde{k}+\alpha h/(1+g)}\}+h,$$

7 The fraction of entrepreneurs α is a measure of separation between creditors and debtors, and matters when the borrowing constraint is crucial. As $\alpha\to1$, outside funds are negligible, and all investment is carried out directly by entrepreneurs, while as $\alpha\to0$, outside funds are more important, and each of entrepreneurs has to borrow the greater amount from investors. Another parameter λ capture the development of the contract enforcement mechanism as argued above. As $\lambda\to1$, the incentive compatibility constraint is always satisfied, and entrepreneurs would be able to borrow as much as possible, taking r_{t+1} as given. As $\lambda\to0$, entrepreneurs would be able to borrow nothing and hence have to self-finance their investment entirely.

(3-12) $$1+\tilde{r} = \frac{\lambda f'(\tilde{k})\tilde{k}}{(1-\alpha)\tilde{k}+\alpha h/(1+g)},$$

and $\tilde{s} = s_L$ or 1. Given k_0, the economy converges to $\tilde{k}$ (need detailed explanation)

We turn to the analysis of the bubbly economy. Our primary interest is whether bubbles are complements or substitutes with investment. The central argument is which is stronger between *the collateral effect* and *the portfolio effect.* The portfolio effect implies that bubbles compete with investment in capital in the asset portfolio of investors, and works to crowd out investment via the route of rising interest rate. The collateral effect implies that bubbles accumulated by entrepreneurs as internal wealth for investment projects promote investment when bubbles boost.

Tirole (1985) demonstrates that bubbles arise when there is capital over-accumulation in the bubbleless economy, and move the capital stock to the Golden Rule in accordance with improved efficiency. However, when there financial market imperfections, we derive an interesting feature that is distinguishable from Tirole and useful for our analysis.

Proposition 1

If the borrowing constraint is binding with equality, the bubbly steady state features less stock of capital than the Golden Rule.

Proof. We derive the steady-state relation between k and r as $1+r = \min\{f'(k), \Lambda(k)\}$, where $\Lambda(k)$ satisfies $\Lambda(k) = \dfrac{\lambda f'(k)(1+g)k}{(1+g)k - \alpha s(W(k)+h\dfrac{\Lambda(k)}{1+g})}$. In the (k,r) plane, the real interest rate should be lower between the two curves. In Figure 1, we illustrate the case when $\Lambda(k)$ is always below $f'(k)$. When the borrowing constraint is binding, for any given k, capital should be smaller than otherwise. We have $\tilde{k}_B <\ k_{GR}$ for $r = g$. Q.E.D.

Bubbles move the capital stock to the smaller level than the Golden Rule, and are coexistent with capital under-accumulation. This finding suggests that capital over-accumulation is not necessary in the bubbleless economy for bubbles to emerge.

We investigate dynamic properties. The term capturing the collateral effect $(1+r_t)/(1+g)$ in (3-2) and (3-4) makes the analysis a little complicated. Using (2-6), (3-2), (3-4), rearranging terms, we have

(3-13) $$\frac{1+r_t}{1+g}=\frac{\lambda f'(k_t)k_t+\alpha b_t}{(1-\alpha)(1+g)k_t+\alpha h}\equiv\phi(k_t,b_t)$$ (need detailed explanation)

Using (3-13), we rewrite the asset market clearing (3-2) as

(3-14)

$$(1+g)k_{t+1}+b_t=s_t w_t+s_t h\frac{\lambda f'(k_t)k_t}{(1-\alpha)(1+g)k_t+\alpha h}+\frac{\alpha s_t h}{(1-\alpha)(1+g)k_t+\alpha h}b_t+h\,.$$

Importantly, we find b_t in two terms. First, the second term in the LHS captures the portfolio effect, and secondly the third term in the RHS the collateral effect that emerges when entrepreneurs receive endowment in the first period. To the extent that the coefficient of the latter is great, the collateral effect weakens the portfolio effect, but we see $\frac{\alpha s_t h}{(1-\alpha)(1+g)k_t+\alpha h}$ <1 so that the collateral effect does not dominate the portfolio effect, given the saving rate s_t, and as seen below, the negative correlation will be found between bubbles and investment.

We roughly characterize the economy with a two-dimensional dynamic system by the two loci, $k_{t+1}=k_t$ and $b_{t+1}=b_t$, given $s_t=s_L$ or 1 although this economy is in principle a three-dimensional dynamic system. Demonstrating the dynamic properties is conventional with a phase diagram. The derivation of the curve $k_{t+1}=k_t$ that follows from (3-2) and (3-13) is given by

(3-15) $$[1-\frac{\alpha s_t h}{(1-\alpha)(1+g)k_t+\alpha h}]b_t=s_t W(k_t)-(1+g)k_t+hs_t\frac{\lambda f'(k_t)k_t}{(1-\alpha)(1+g)k_t+\alpha h}+h\,.$$

The curve $b_{t+1}=b_t$ follows from (3-8) and (3-11), and k_t and b_t satisfy

(3-16) $$1+g=\lambda f'(\frac{s_t W(k_t)-b_t+\phi(k_t.b_t)+h}{1+g})\frac{s_t W(k_t)-b_t+\phi(k_t,b_t)+h}{(1-\alpha)s_t W(k_t)-b_t+\phi(k_t,b_t)+h}\,.$$

Figure 2 illustrates a phase diagram representing the dynamics of the economy with the unchanging saving rate $s_t = s_L$. The curve $b_{t+1} = b_t$ is typically upward sloping, and crosses the curve $k_{t+1} = k_t$ from below (need detailed explanation). The bubbly steady state is attained at W. The bubbly steady state features less investment than the bubbleless steady state.

Dynamic properties are qualitatively the same as those developed by Tirole (1985) and Weil (1987). There exists an upwardly sloping saddle path to the bubbly steady state. Given k_0 , all dynamic paths originating from below b_0 converge toward the bubbleless steady state. Trajectories starting above b_0 are infeasible as they all lead to the resource constraint being violated in finite periods.

We are now in a position to describe the dynamics when bubbles emerge. Suppose that the economy is in the bubbleless steady state. As bubbles emerge, the economy jumps upwardly to reach the saddle path of the bubbly steady state. The interest rate rises, investment declines, and bubbles also decline as the economy converges to the bubbly steady state. Bubbles necessarily crowd investment out.

Let $z(r,k) \equiv \frac{k}{W(k)} f'(k) + \{1 - \frac{k}{W(k)}\}(1+r)$ denote the rate of return to capital for entrepreneurs that are strictly greater than investors. Sufficient conditions under which the aggregate saving rate does not change when bubbles arise are $(1+g)\beta_I < 1$ and $z(g,\tilde{k}_b)\beta_E < 1$ (need detailed explanation). Impatient agents consume in both the bubbleless and bubbly economies so that bubbles do not change the saving rate. We summarize properties of equilibria in the following.

Proposition 2

Suppose that both $(1+g)\beta_I < 1$ and $z(g,\tilde{k}_b)\beta < 1$ hold so that the saving rate remains unchanged when bubbles arise.

(a) If $\tilde{r} > g$, the economy is bubbleless and the interest rate converges to $\tilde{r}$.

(b) If $f'(\tilde{k}) > 1+g > 1+\tilde{r}$, there exists a unique bubbly economy with initial bubble b_0. The per-effective-worker bubbles converge to $\tilde{b}$ and the interest rate converges to g . In the bubbly economy, the steady-state per-effective-worker capital, denoted $\tilde{k}_B$, satisfies

$f'(\tilde{k}_B) > 1+g = f'(k_{GR})$, with $\tilde{k}_B < \tilde{k} < k_{GR}$.

(c) If $1+g > f'(\tilde{k}) > 1+\tilde{r}$, there exists a unique bubbly economy with initial bubble b_0. The per-effective-worker bubbles converge to $\tilde{b}$ and the interest rate converges to g. In the bubbly economy, the steady-state per-effective-worker capital satisfies $f'(\tilde{k}_B) > 1+g = f'(k_{GR})$, with $\tilde{k}_B < k_{GR} < \tilde{k}$.

A heuristic proof of Proposition 2 is as follows. Agents require that, at the stationary state, the rate of return on bubbles, $1+g$, be at least equal to the rate of return on lending, $1+\tilde{r}_B$, so that it must be the case that $g \geq \tilde{r}_B$ if bubbles should happen. On the other hand, at the steady state, the presence of bubbles decreases the capital stock relative to the bubbleless economy, so that we must have $\tilde{k}_B < \tilde{k}$ and hence $\tilde{r}_B > \tilde{r}$ if $\tilde{b} > 0$. Therefore, the necessary condition for bubbles to be sustainable is $g > \tilde{r}$. Conversely, if $g > \tilde{r}$, bubbles absorb the aggregate savings and reduces the capital stock until the interest rate $1+\tilde{r}$ is pushed up to $1+g$. Finally, if $\tilde{r} > g$, it must be the case that $\tilde{r}_B > \tilde{r}$, but then it follows that the aggregate bubbles per-effective-worker should grow indefinitely, which is infeasible.

Bubbles can arise even if the rate of return to capital is greater than the growth rate (Proposition 2(b)). The sustainability of bubbles depends on the relation between the growth rate and the real interest rate, not the rate of return to capital.

The case of Proposition 2(b) is interesting and will be relevant for the current observation of the world economy. The borrowing constraint induced by the weak financial system gives rise to the weak demand for investment and the low real interest rate that calls for asset bubbles, which in turn shrinks further investment. We succeed in explaining the low real interest rate, asset bubbles, and capital under-accumulation in a unified model.

5. Increase in Savings and Bubble Cycles

Having so far studied the case when the saving rate does not change, we turn to the case when the saving rate changes when bubbles emerge.

The rise in the interest rate in the bubbly economy may increase savings, which in turn will generate the positive feedback between bubbles and investment. When $(1+g)\beta_E > 1$ and $(1+g)\beta_I > 1$ are both met, all agents save all the middle-aged wealth at least at the steady state of the bubbly economy. If $(1+\tilde{r})\beta_I < 1 < (1+g)\beta_I$ and $z(\tilde{r},\tilde{k})\beta_E < 1 < z(g,\tilde{k}_b)\beta_E$ jointly hold, then impatient agents consume in the bubbleless economy, but save in the bubbly economy. When bubbles emerge, the boost of savings weakens the portfolio effect, and reinforces the collateral/investment feedback so as to generate the comovement between bubbles and investment. The rise in savings enables the collateral effect to be stronger than the portfolio effect, and the positive feedback between bubbles and investment ledas to the comovement.

Figure 3 illustrates a phase diagram representing the dynamics of the economy when the saving rate changes when bubbles arise. The curve $k_t = k_{t+1}$ shifts outwardly when bubbles arise so that the bubbly steady state features more investment than the bubbleless steady state.

Suppose that the economy is in the bubbleless steady state. As bubbles emerge, the economy jumps upwardly to reach the saddle path of the bubbly steady state, V. In response to the rise in the interest rate, savings increase, investment booms, and bubbles also appreciate as the economy converges to the bubbly steady state. Bubbles crowd investment in. We summarize properties of equilibria in the following.

Proposition 3

Suppose that $(1+\tilde{r})\beta_I < 1 < (1+g)\beta_I$ and $z(\tilde{r},\tilde{k})\beta_E < 1 < z(g,\tilde{k}_b)\beta_E$ so that the saving rate rises when bubbles arise.

(a) If $\tilde{r} > g$, the economy is bubbleless and the interest rate converges to $\tilde{r}$.

(b) If $f'(\tilde{k}) > 1+g > 1+\tilde{r}$, there exists a unique bubbly economy with initial bubble b_0. Bubbles per effective worker converge to $\tilde{b}$ and the interest rate converges to g. In the bubbly economy, the steady-state capital per effective worker, $\tilde{k}_B$, satisfies $f'(\tilde{k}_B) > 1+g = f'(k_{GR})$, with $\tilde{k}_B < k_{GR}$.

(c) If $1+g > f'(\tilde{k}) > 1+\tilde{r}$, there exists a unique bubbly economy with initial bubble b_0. The per-effective-worker bubbles converge to $\tilde{b}$ and the interest rate converges to g. In the bubbly economy, the steady-state per-effective-worker capital satisfies $f'(\tilde{k}_B) > 1 + g = f'(k_{GR})$, with $\tilde{k}_B < k_{GR} < \tilde{k}$.

As have been demonstrated by Proposition 1, bubbly economy attains capital under-accumulation so that only in case (b), crowding-in happens. Importantly, the increasing saving function with respect to the interest rate is not sufficient for the bubbly economy to feature more investment than the bubbleless economy at the steady state. Imperfect pledgeability is also necessary to have this feature. More investment can be compatible with the rise in the interest rate in the economy of imperfect pledgeability. In the economy of perfect pledgeability, the bubbly economy features the greater interest rate and thus less investment than the bubbly economy.

The emergence of bubbles is closely linked to the saving behavior of agents. When people anticipate bubbles, all people save, and bubbles are self-fulfilling. We provide a story of business cycles that are driven by the boost and burst of bubbles.

Suppose that the sunspot variable is a Markov chain with two states, $\{b, c\}$. When the state is b, all agents coordinate their expectations on the bubbly path. When the state is c, they do on the path toward the low capital steady state D. Let the transition probability in which the state b (or c) occurs next period given that the current state is b (or c) is δ, with $\delta \to 1$. The assumption $\delta \to 1$ allows us to approximate the sunspot economy with the deterministic model.

Figure 4 illustrates a sample path. Suppose first that the state b continues, and the economy goes on the saddle path from V to the bubbly steady state W. Bubbles and investment are increasing. Suppose that the state c occurs when the economy almost reaches W. Bubbles crash and the economy goes down to Z, for example, and then begins to approaches the bubbleless steady state D along one exploding path. So long as the state c continues, bubbles are shrinking and investment declines. Suppose that the state b occurs when the economy almost reaches D. Bubbles boost and the economy goes up to V. The business cycle of $D \to V \to W \to Z \to D$ is featured by the investment boom and bust induced by the boost and burst of bubbles, and are reminiscent of business fluctuations of the classical boom and bust of investment (e.g., Kalecki 1937, and Kardor 1940) .

We interpret the change in savings in a number of ways. Basically, the link between bubbles and saving is not unconventional. A large part of the flows of income are consumed, but incomes from capital gains of assets are almost reinvested. When the boom is accompanied by assets appreciations, the saving rate should have risen, whereas when the bust is accompanied by assets depreciations, the saving rate should have fallen.

Our theory should predict the Granger causality both from growth to saving and from saving to growth. However, Carroll and Weil (1994) find that growth Granger-causes saving with a positive sign, but that saving does not Granger-cause growth. The discrepancy may arise from the fact that the data does not typically account for capital gains as income so that the calculated saving rate may be undervalued (overvalued) in the period of boom (bust).

Although we have constructed the closed economy, the increased savings may be interpreted to include capital inflows from foreign countries. Foreign saving can flow into the home country, and generate bubbles accompanied by the rise in the domestic interest rate, which in turn gives rise to the investment boom. The major source of funding for the investment boom in the United States was the current account.

6. Crash of Bubbles and Issuance of New Bubbles

In response to the crash of bubbles, many industrialized countries take the stimulus package of government expansion. The one important aspect is that this stimulus package will possibly be accompanied by the large scale of issuance of government bonds. The media report suggests that the total size of government expenditure amounts to about 5 trillion dollars in the world.

In our model, the issuance of government bonds will be identified as the creation of new bubbles that may coexist and/or compete with other existing bubbles, bubbles on real estate, stocks, and commodities. In the following we make an experiment to investigate what will happen if, when some bubbles have crashed, bubbles are newly created.

We assume that bubbles are composed of two bubbles, bubbles on real estate Q_t, and those on the government bond, $p_t^G B_t^G$, to satisfy $p_t B_t = p_t^G B_t^G + Q_t$, where B_t^G and p_t^G denote units and the price of the government bond, and the aggregate amount of real estate is normalized unity.

We identify the crash of bubbles as the unexpected depreciation of real estate bubbles from Q_t^+ to Q_t^-. We assume that in the vent of crash, the government bond does not depreciate possibly due to the implicit government guarantee. At the instant of the crash, the aggregate bubbles drop to $p_t^- B_t = p_t^G B_t^G + Q_t^-$ and $b_t^- (\equiv p_t^- B_t / A_t)$ in per efficient worker term. The new issuance of the government bond ΔB_t^G can add bubbles so as to offset the lost values of bubbles. Looking again at Figure 4, the economy may temporarily drop down to Z, but can promptly go back near the bubbly steady state W.

This is an optimistic scenario of the government expansion that will be made possible when the creation of bubbles is temporary. We provide another story when the government issues the bond for long time, maybe because the stimulating effect of the government expansion is weak. Suppose that the government keeps to issue the bond at the rate $B_{t+1}^G / B_t^G = 1+\mu$. Letting $q_t \, (\equiv Q_t / A_t)$ denote the real estate price in per efficient worker term, no-arbitrage conditions require

(5-1) $$\frac{b_{t+1}-q_{t+1}}{b_t - q_t} = \frac{(1+\mu)(1+r_{t+1})}{1+g}$$, and

(5-2) $$\frac{q_{t+1}}{q_t} = \frac{1+r_{t+1}}{1+g}$$,

where (5-1) represents the no-arbitrage condition between the government bond and the private lending and (5-2) the one between the real estate and the private lending.

Suppose that q_t takes some positive value at the steady state, $\tilde{q}$ >0, for example. We have $r_{t+1} = g$ from (5-2), and then b_t should explode from (5-1), a contradiction. Therefore, we should have $q_t \to 0$. Real estate bubbles asymptotically disappear in terms of the aggregate bubbles, and are finally replaced by the government bond.

We have a direct evidence of "bubble substitution" in Japan. At the beginning of the 1990s, bubbles on stocks and land crashed, and the abundant savings were directed toward the finance of increasing government bonds that were issued at large scale for fiscal expansion. Figure Y illustrates how bubbles have been substituted. Much of the decline in the aggregate land values has been offset by the increasing outstanding government debt. The sum of land values and government debt is roughly constant over the period 1994-2006.

Another important feature is the "dilution" of bubbles leads to the decline in the long-run interest rate, which may influence the saving behavior of agents. Indeed, since $q_t \to 0$, dynamic features are preserved except for that the steady state interest rate decline down to $(1+g)/(1+\mu)$.

Figure 5 illustrates the case in which the bubble dilution gives rise to the shrink of saving. Suppose that people anticipate that the stimulus package goes long. Anticipating the decline in the interest rate, impatient agents begin to stop savings. Once the economy has temporarily drop down to Z, it partially recovers to V', but the economy converges to the bubbly steady state

W', with less investment and output than the bubbleless economy. The consequence of the long-run stimulus package is the severe stagnation.

7. Implications for Dynamic Efficiency

We now turn to the question of dynamic efficiency. We use the dynamic efficiency criterion by Cass (1972) who defines that the economy is *dynamically efficient* if there does not exists another feasible sequence of capital which provides at least as much *aggregate* consumption at all dates and strictly higher aggregate consumption in at least one date. We should note that our economy is Pareto sub-optimal in the sense that some intra-generational transfer of income can make all agents better off. If an intra-generational transfer of income would be permitted between investors and entrepreneurs through government intervention at the first period of their lives, an appropriate tax-subsidy scheme will move the economy substantially to the Diamond-Tirole model.[10] It turns out that we investigate dynamic efficiency of the economy that is not Pareto optimal.

First, if $\tilde{r} > g$, the efficiency result is straightforward because the sustainable bubbles are ruled out under perfect foresight. Secondly, we consider the case for $f'(\tilde{k}) > 1+g > 1+\tilde{r}$, where the per-effective-worker aggregate consumption is less than the Golden Rule at the initial equilibrium without bubbles. Bubbles move the capital stock down to an even smaller level than the Golden Rule, and decrease the consumption. Finally, we consider the case of capital over-accumulation in the bubbleless equilibrium, with $1+g > f'(\tilde{k}) > 1+\tilde{r}$. As shown in Figure 5, there exists a $\underline{k}$, under which the per-effective-worker aggregate consumption is the same as the one at $\tilde{k}$, satisfying $f(\underline{k}) - (1+g)\underline{k} = f(\tilde{k}) - (1+g)\tilde{k}$ and less than k_{GR}. The welfare implications differ according to whether the steady state of the asymptotically bubbly equilibrium $\tilde{k}_B$ lies greater than $\underline{k}$ or not. We summarize the following.

Proposition 4

Suppose that the saving rate remains unchanged when bubbles arise.

(a) If $\tilde{r} > g$, the bubbleless economy is dynamically constrained efficient.

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[10] In this economy, the efficiency result is then standard. The bubbleless equilibrium is dynamically efficient if $\tilde{r} > g$, while otherwise, it is dynamically inefficient and the asymptotically bubbly equilibrium is dynamically efficient [Tirole (1985, Proposition 2)].

(b) If $f'(\tilde{k}) > 1 + g > 1 + \tilde{r}$, the asymptotically bubbly economy does not improve efficiency. The bubbleless economy is dynamically constrained efficient.

(c) If $1 + g > f'(\tilde{k}) > 1 + \tilde{r}$, the asymptotically bubbly economy may or may not improve efficiency. If $\tilde{k}_B < \underline{k}$, the asymptotically bubbly economy does not improve efficiency, and the bubbleless economy is dynamically constrained efficient, while if $\tilde{k}_B > \underline{k}$, it improves efficiency so that the bubbleless economy is dynamically constrained inefficient.

Proposition 3(a) and 3(b) say that $f'(\tilde{k}) > 1 + g$ is a sufficient condition under which the bubbleless economy is dynamic efficient. Combined with Proposition 2(b), it turns out that bubbles can arise even when the bubbleless economy is dynamic efficient. Proposition 3(c) says that $1 + g > f'(\tilde{k})$ is not a sufficient condition under which the bubbleless economy is dynamic inefficient.

The mechanism under which asset bubbles asset bubbles arise even when the bubbleless economy is dynamically efficient is closely related to the fact that the bubbleless economy is Pareto sub-optimal. Several other papers have demonstrated that, in the presence of wedge between social and private returns to capital, bubbles can arise even when the bubbleless economy is dynamic efficient (e.g., Saint-Paul (1992), Grossman and Yanagawa (1993) and Femminis (2002)).[12]

Abel et al (1989), Zilcha (1992), and Bohn (1995) investigate dynamic efficiency in stochastic models with infinitely-lived agents. Their implication is that dynamic efficiency depends on the relation between the growth rate and the rate of return on “risky” capital, not the “safe” interest rate, and that the Non-Ponzi condition is satisfied and so sustainable bubbles are excluded.

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[12] Saint-Paul (1992), Grossman and Yanagawa (1993) provide endogenous growth models in which bubbles arise even when the private return to capital falls short of the growth rate, and bubbles are not Pareto-improving bur rather lowers the growth rate by crowding out capital accumulation.

[15] Kraay and Ventura (2005) provide a counterexample in which the Abel’s condition is satisfied but the economy is dynamically inefficient.

Proposition 5

Suppose that the saving rate rises when bubbles arise.

(a) If $\tilde{r} > g$, the bubbleless economy is dynamically constrained efficient.

(b) If $f'(\tilde{k}) > 1 + g > 1 + \tilde{r}$ and $\tilde{k}_B < \tilde{k}$ hold, the bubbly economy does not improve efficiency. The bubbleless economy is dynamically constrained efficient. If $f'(\tilde{k}) > 1 + g > 1 + \tilde{r}$ and $\tilde{k}_B > \tilde{k}$ hold, the bubbly economy improves efficiency. The bubbleless economy is dynamically inefficient.

(c) If $1 + g > f'(\tilde{k}) > 1 + \tilde{r}$, the bubbly economy may or may not improve efficiency. If $\tilde{k}_B < \underline{k}$, the bubbly economy does not improve efficiency, and the bubbleless economy is dynamically constrained efficient, while if $\tilde{k}_B > \underline{k}$, it improves efficiency so that the bubbleless economy is dynamically constrained inefficient.

When the saving rate rises, (b) will change. When crowding-in arises, bubbles move the capital stock closer to the Golden Rule so that the bubbleless economy is dynamically inefficient. The qualitative properties do not change in case (c).

The inequality for $f'(\tilde{k}) > 1 + g$ is neither a necessary nor sufficient condition for dynamic efficiency. Even if $f'(\tilde{k}) > 1 + g$ holds, the bubbleless economy is dynamically inefficient when bubbles crowd investment in.[15] Even if $f'(\tilde{k}) < 1 + g$ holds so that capital over-accumulation arises in the bubbleless economy, dynamic efficiency holds.

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Figure 1

Capital under-Accumulation in the Bubbly Economy

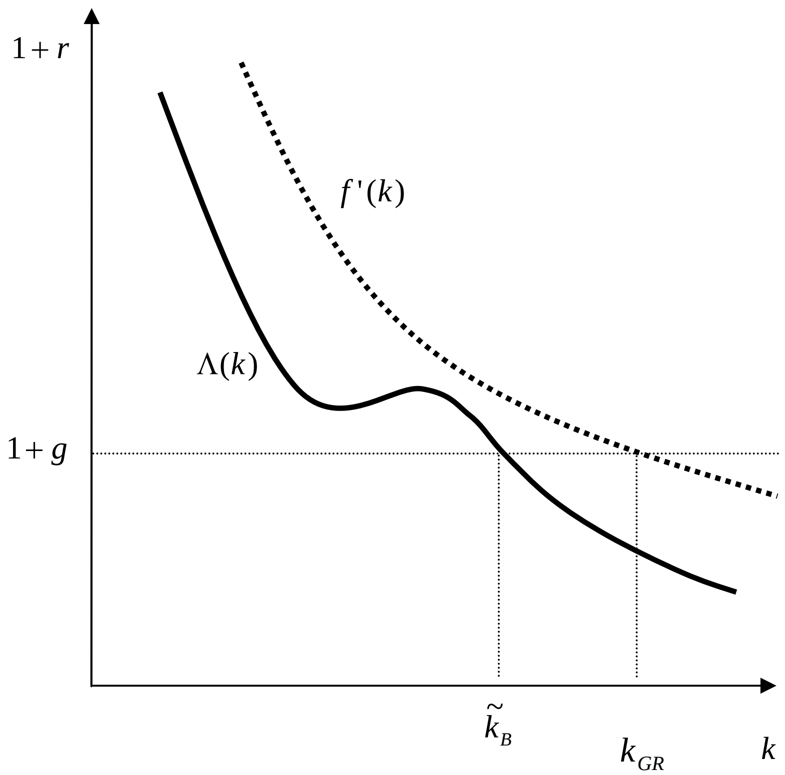


Figure 2

Bubbly Economy with Unchanging Saving Rate

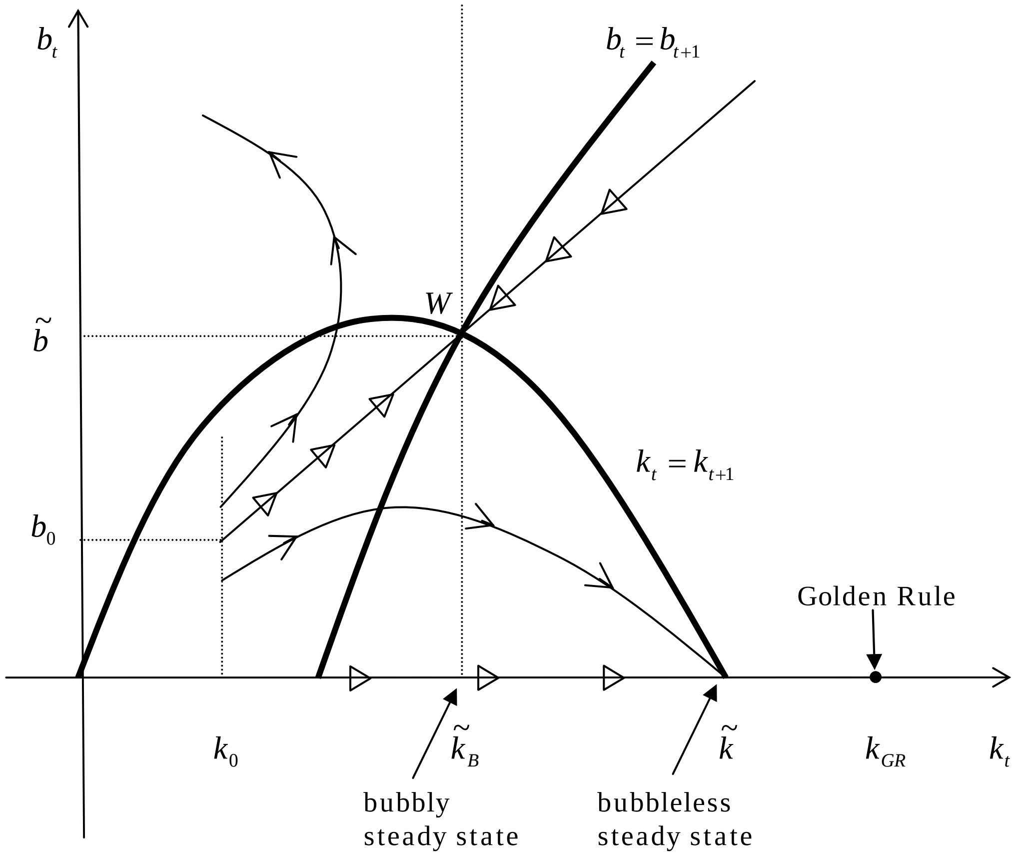


Figure 3
Bubbly Economy with Increased Saving Rate

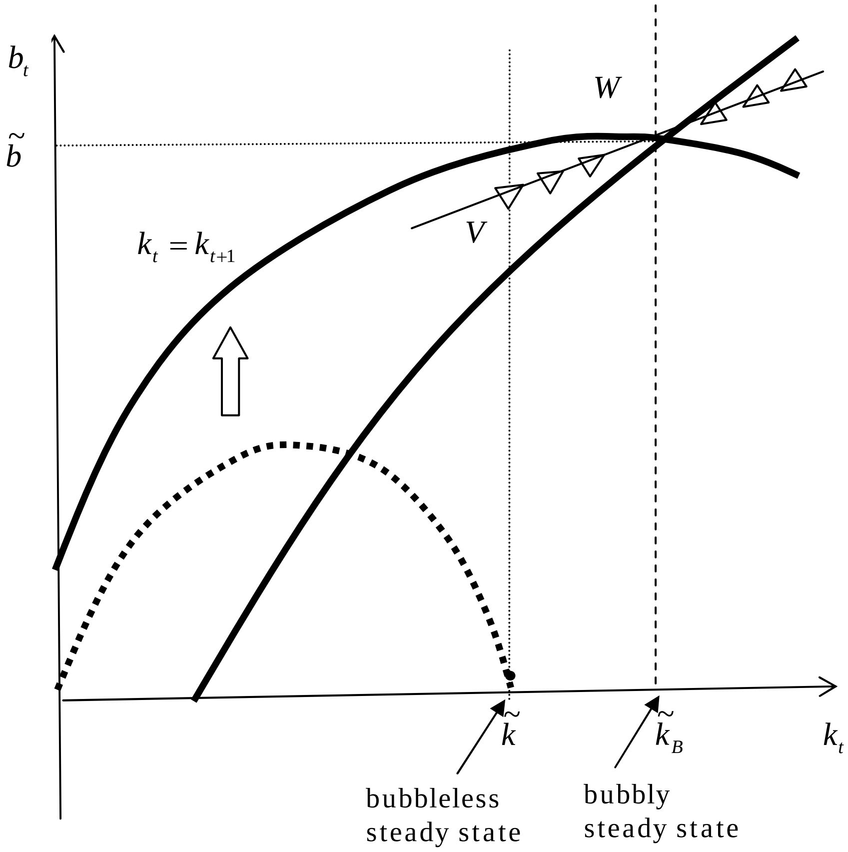
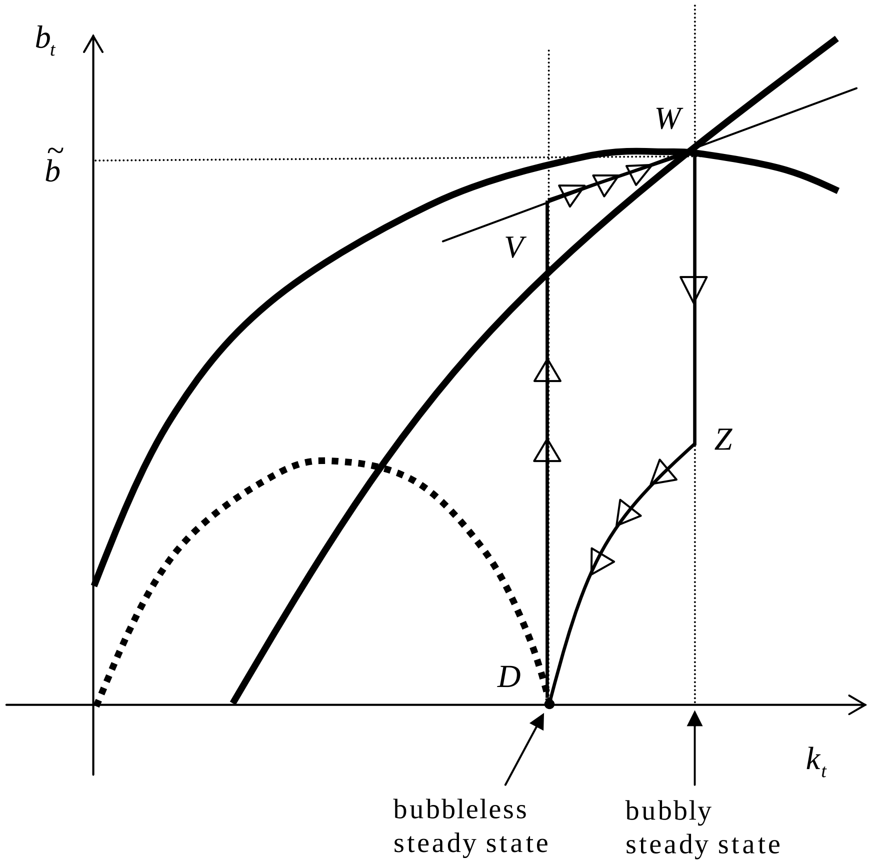


Figure 4 Bubble Cycle



v **Figure 5 Bubble Dilution**

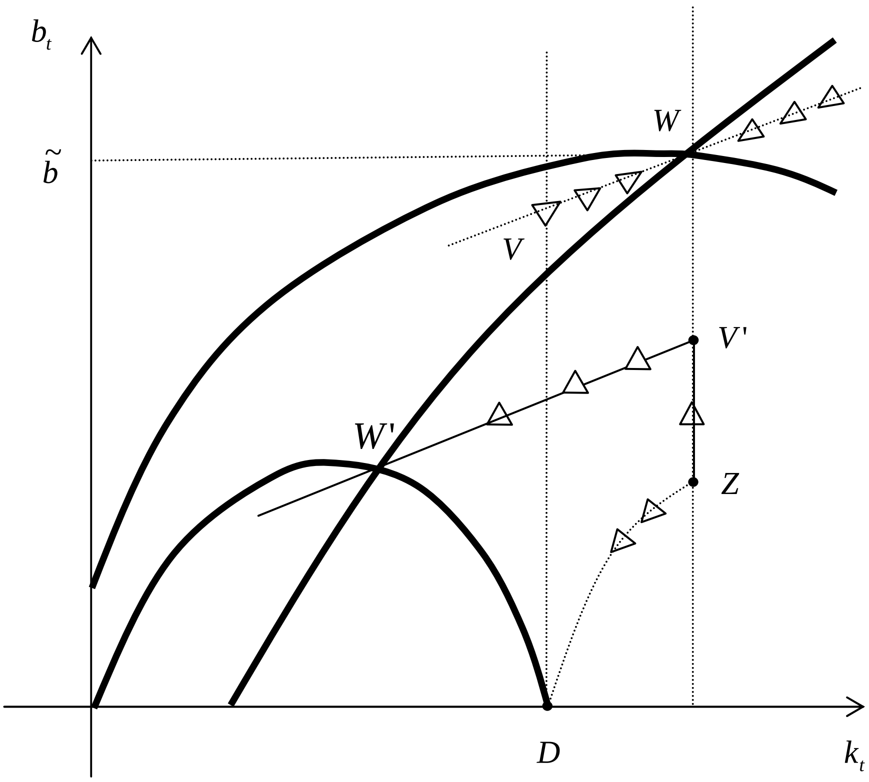


Figure X　Interest Rates and Growth Rate

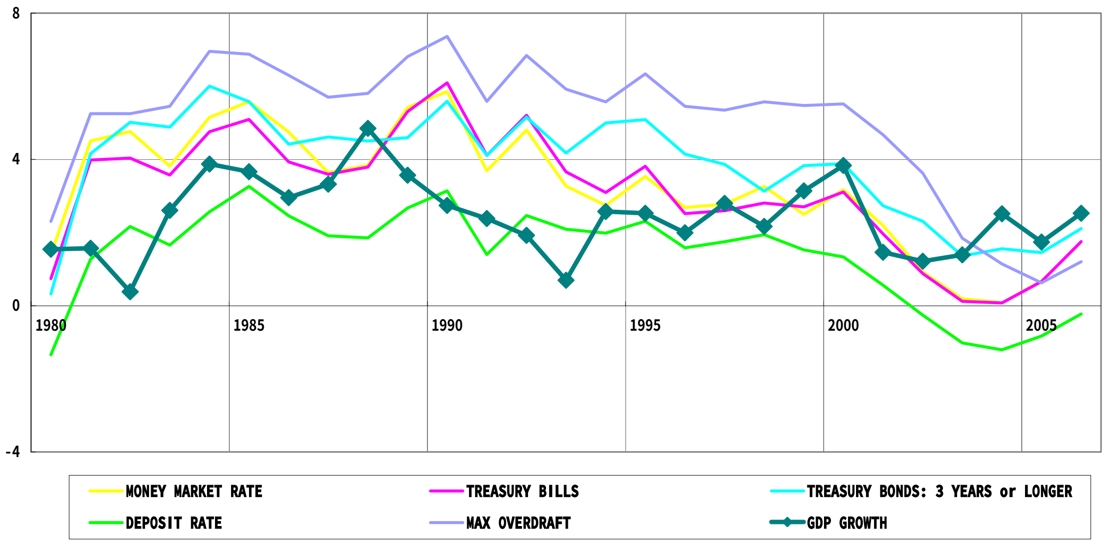


Table 1 Test for Capital Over (Under) -Accumulation

	Japan	Canada	France	German	Italia	UK	USA
1990	0.054	0.199	0.197		0.149	0.271	0.216
1991	0.052	0.196	0.202	0.153	0.149	0.281	0.224
1992	0.066	0.201	0.208	0.147	0.157	0.281	0.220
1993	0.074	0.208	0.237	0.169	0.217	0.296	0.215
1994	0.067	0.220	0.240	0.178	0.231	0.304	0.211
1995	0.063	0.230	0.238	0.179	0.241	0.308	0.209
1996	0.062	0.234	0.249	0.192	0.243	0.318	0.209
1997	0.069	0.209	0.254	0.201	0.236	0.308	0.199
1998	0.084	0.206	0.247	0.197	0.256	0.283	0.178
1999	0.101	0.215	0.233	0.194	0.249	0.276	0.170
2000	0.104	0.221	0.224	0.189	0.256	0.267	0.155
2001	0.113	0.227	0.226	0.211	0.252	0.258	0.173
2002	0.140	0.227	0.231	0.233	0.246	0.264	0.185
2003	0.154	0.217	0.233	0.231	0.246	0.270	0.188
2004	0.165	0.222	0.229	0.238	0.246	0.267	0.185

Figure Y　　Bubble Sbstitutioon in Japan