The Choice of Invoice Currency under Uncertainty: Theory and Evidence from Korea

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Abstract

The purpose of this paper is to investigate the choice of invoice currency under exchange rate uncertainty. The analysis is motivated by the fact that the U.S. dollar has been the dominant vehicle currency in developing countries. The theoretical analysis is based on an open economy model of monopolistic competition. The export prices are set before exchange rates are known. When the market is competitive enough, the exporting firms tend to set their prices not to deviate from those of the competitors. As a result, when the other exporters set their prices in the third currency, the exporting firm tends to choose the third currency as an equilibrium invoice currency. The tendency becomes conspicuous in the market where the shares of local firms are small. The latter part of the paper empirically investigates the relevancy of the theoretical results by using the export price data in Korea. We find that export prices in Korea are highly stable in terms of the US dollar even in the commodities for which Japan has had dominant shares. We also find that export prices in Korea are more stable against the US dollar in the commodities for which the shares of local firms are small in Japan. The empirical results are consistent with our theoretical model. The result may explain why the firm tends to set prices in the US dollar even if the United States is not a trade partner.

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1. Introduction

The purpose of this paper is to investigate the choice of invoice currency in international trade. There are several theoretical studies that investigated the choice of invoice currency in international trade. Baron (1976) and Giovannini (1988) are their early attempts. Most of the studies analyzed whether the exporting firm sets prices in its own currency or in the importer’s currency. It is, however, well known that some of international trades are invoiced in a third currency, that is, vehicle currency. In particular, the U.S. dollar tends to be the dominant vehicle currency in developing countries. By using an open economy model of monopolistic competition, this paper tries to explain the choice of invoice currency in developing countries.

Except for primary commodities, the role of vehicle currency is relatively limited in international trade among developed countries (see Magee and Rao [1980]). The U.S. dollar is, however, the dominant vehicle currency in many developing countries. For example, Table 1 reports the ratios of currencies used for payments in Thai international trade. It shows that payments in the US dollar have been dominant in Thai exports, although the ratios of the US dollar showed marginal declines in recent years (see Table 1a). The results hold true even if the export destinations are East Asian countries or European countries. In particular, the payment ratio of the Japanese yen is less than 10% even in Thai exports to Japan (see Table 1b). A similar result is observed for the currency ratios used for payments in Korean exports. In Korea, the dominant ratios of the US dollar declined during a past decade years (Table 2). However, even in recent years, the ratios of the US dollar still lied between 85% and 90% in Korean visible exports and around 75% in Korean invisible trades.

Table 3 summarizes the shares of each export destination from Korea and Thailand. We can see that Japan and Western Europe as well as other Asian countries have been the other important trade partners for Thai and Korean exports. The above evidence indicates that the U.S. dollar was chosen as the dominant vehicle currency even in the case where the United States is not a trade partner.

One may argue that these countries chose the U.S. dollar as the dominant invoice currency because their exchange rates were stable against the U.S. dollar. The argument may have been true before the Asian crisis when they effectively pegged their currencies to the U.S. dollar (see, for example, Frankel and Wei [1994]). However, after the crisis, these countries shifted the exchange rate regime from de facto dollar peg to float. As a result, there is no longer a natural reason for them to choose the U.S. dollar as the dominant invoice currency to stabilize their export prices in terms of domestic currencies.

The following theoretical analysis is based on an open economy model of monopolistic competition. Since the export prices are set before exchange rates are known, the exporting firms face uncertainty

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1 McKinnon (1979) is another seminal study that addressed this issue.
2 The authors such as Ito (1993), Fukuda (1995), and Kawai (1996) discussed why the Japanese yen has not been used in international trade.
of exchange rates. If necessary, the exporting firm set prices in its own currency or in the currency of the importing country. However, when the market is competitive enough, the exporting firms tend to set prices not to deviate from those of the competitors. As a result, when the other exporters set their prices in the third currency, the exporting firms tends to invoice in the third currency. The tendency becomes conspicuous in the market where the shares of local firms are small.

Our model follows a partial equilibrium model in Bacchetta and van Wincoop (2002). It, however, has two distinctive features that the previous study did not have. First, we allow the exporting firms to choose the third currency as an invoice currency. In developing countries, the exporting firms are under competition because of less differentiated products. It is thus a natural choice for the exporting firm to set prices in the third currency when the competitors set their prices in the third currency. Secondly, we show that coordination failures can lead the third currency to be an equilibrium invoice currency. Since multiple equilibria are Pareto ranked, it implies that the equilibrium choice of the invoice currency may lead to a less efficient equilibrium.

The latter part of the paper empirically investigates the relevancy of the theoretical results by using the export price data in Korea. The approach follows Fukuda and Ji (1994) that studied the pricing behavior of Japanese firms. We find that the export prices in Korea were highly stable in terms of the US dollar even in the commodities for which Japan had dominant shares. We also find that the export prices in Korea were more stable against the US dollar in the commodities for which the shares of local firms are small. The results are consistent with our theoretical model. They are, however, inconsistent with pricing-to-the-market models that have provided the dominant approaches in previous literature. Since the exporting products are less differentiated in developing countries, the exporting firms are under competition when they choose the invoice currency. The result thus explains why the firm tends to set prices in the US dollar even if the United States is not a trade partner in developing countries.

In previous literature, some exceptional studies explored the role of vehicle currency in international trade. Krugman (1980) and Rey (2001) show that transaction costs might make vehicle currency a dominant medium of exchange in international trade. These studies are, however, successful only in explaining the role of vehicle currency as a medium of exchange, through which transactions between currencies are made. In contrast, our approach tries to explain the role of vehicle currency as a unit account in terms of which prices of commodities are set. A unit account is another important function of vehicle currency. Friberg (1998) is an exceptional study that investigated the role of vehicle currency as a unit account. Assuming that the exporter commits to sell the demanded quantity at the ex post realized price, he explored under what conditions the monopolistic exporter chooses the third currency as vehicle currency. It was, however, demonstrated that setting price in the importer’s currency yields the highest expected profits for the exporters under reasonable demand and cost functions. Friberg thus cannot explain why the vehicle currency tends to be a dominant invoice
currency in international trade with developing countries. It has been widely observed that export prices are stable in terms of the US dollar in developing countries. We will show that the exporters’ pricing behavior is consistent with our model in developing countries.

2. The Model of Export Pricing Behavior under Uncertainty

The purpose of this section is to present the theoretical framework that discusses pricing behavior under uncertainty. The firms studied are exporters who produce only in their home country. For simplicity, we assume that all exporting firms are identical and sell all of their products in a single foreign market. There are three countries: the exporting country, the importing country, and the third country. The third country has no international trade with the other two countries. Each exporter, however, has the choice between setting the export price in its own, in the importers’, or in the third currencies. The exchange rates $s_0$ and $s$ are exogenous and assumed to be the only source of uncertainty. Selling $s_0$ units of the third currency leads to one unit of the exporter’s currency on the spot market and selling $s$ units of the importers’ currency leads to one unit of the exporter’s currency on the spot market. By definition, the exchange rate between the exporter’s and the third currency is given by the relation $s/s_0$. We denote their variances as $\sigma_0^2 = E (s_0 - E s_0)^2$ and $\sigma^2 = E (s - E s)^2$. For analytical simplicity, we assume that $s_0$ and $s$ are uncorrelated, so that $E (s_0 - E s_0)(s - E s) = 0$.

In the following analysis, each exporter is under monopolistic competition and firm $j$ faces the demand function $D(p_j, P^*)$, where $p_j$ is the price set by the firm $j$ measured in the importers’ currency. $P^*$ is the aggregate price index in the importers’ local market denominated in the importers’ currency. It is generally a function of prices set by local firms and prices set by exporting firms. The importers’ local firms always set their prices in the importers’ currency, so that the price set by domestic firms is independent of the exchange rate. The exporting firms, however, set their prices either in its own, in the importers’, or in the third currencies. The aggregate price index $P^*$ thus depends on the exchange rate unless all exporting firms set their prices in the importers’ currency. We assume that the total number of firms is large enough so that an individual firm does not affect the price index $P^*$.

The objective of each exporter is to maximize the expected profits in terms of his home currency. The central assumptions are that the exporter has to set price before the exchange rates are known and that demand is a function of the price that importers face after exchange rate uncertainty is resolved. Suppose that each exporter chooses $p^E$ when setting a price in its own currency, $p^I$ when setting a price in the importers’ currency, and $p^0$ when setting a price in the third currency. By definition, the unit price of imports in terms of the importers’ currency is $p^E/s$ when set in the exporters’ currency, $p^I$ when invoiced in the importers’ currency, and $p^0/s_0$ when invoiced in the third currency. Let $I^E$, $I^I$, $I^0$.
and \( \Pi \) respectively denote the exporter’s profit when the price \( p_j \) is set in its own, in the importers’, or in the third currencies. The profit is then respectively given by

\[
\begin{align*}
(1) \quad \Pi_E &= p_E D(p_E/s, P*) - C[D(p_E/s, P*)], \\
(2) \quad \Pi_I &= s p_I D(p_I, P*) - C[D(p_I, P*)], \\
(3) \quad \Pi_0 &= (s/s_0) p_0 D(p_0/s_0, P*) - C[D(p_0/s_0, P*)],
\end{align*}
\]

where \( C[\cdot] \) is cost function that is increasing and convex. We assume that the costs are incurred in terms of the exporter’s currency.

3. The Nash Equilibria

In equilibrium, each exporter sets a price in its own currency if \( \Pi_E \geq \Pi_I \) and \( \Pi_E \geq \Pi_0 \), in the importers’ currency if \( \Pi_I \geq \Pi_E \) and \( \Pi_I \geq \Pi_0 \), and in the third currency if \( \Pi_0 \geq \Pi_E \) and \( \Pi_0 \geq \Pi_I \). The optimal choice of the currency denomination thus generally depends on the forms of demand and cost functions. Let \( \Pi_j(s_0, s) \), \( \Pi_I(s_0, s) \), and \( \Pi_0(s_0, s) \) be profit functions in its own, in the importers’, or in the third currencies respectively. Then, a second order Taylor expansion near \( s_0 = E \) and \( s = E s \) leads to

\[
(4) \quad E \Pi_j(s_0, s) \approx \Pi_j(E s_0, E s) + (1/2) (\Pi'_1j \sigma_0^2 + \Pi'_2j \sigma^2), \quad \text{for } j = E, I, \text{ and } 0,
\]

where \( \Pi'_1j = d^2 \Pi_j(s_0, s)/ds^2_0 \) and \( \Pi'_2j = d^2 \Pi_j(s_0, s)/ds^2 \) at \( s_0 = E s_0 \) and \( s = E s \).

It is noteworthy that the choice of the currency denomination becomes irrelevant under certainty. It thus holds that

\[
(5) \quad \Pi_j(E s_0, E s) = \Pi_j(s_0, s), \\
(6) \quad p_E/E s = p_I = p_0/E s_0.
\]

After some tedious calculations shown in Appendix 1, we therefore obtain that when \( \partial P*/\partial s = 0 \),

\[
(7) \quad E \Pi_j - E \Pi_I = (1/2) [\partial^2 \Pi_j/\partial s^2_0 + 2 (\partial^2 \Pi_j/\partial P* \partial s_0)(\partial P*/\partial s_0)] \sigma_0^2,
\]

\[
(8) \quad E \Pi_j - E \Pi_E = (1/2) \left\{ [\partial^2 \Pi_j/\partial s^2_0 + 2 (\partial^2 \Pi_j/\partial P* \partial s_0)(\partial P*/\partial s_0)] \sigma_0^2 - (\partial^2 \Pi_j/\partial s^2) \sigma^2 \right\},
\]

\[
(9) \quad E \Pi_I - E \Pi_E = - (1/2) (\partial^2 \Pi_j/\partial s^2) \sigma^2,
\]

and that when \( \partial P*/\partial s_0 = 0 \),

4
Based on (7)-(12), we investigate which currency the exporters denominate their product in a Nash equilibrium. We first explore whether the denomination in the importers’ currency can be a Nash equilibrium. Assuming symmetry, we consider the case where \( \partial P^*/\partial s = \partial P^*/\partial s_0 = 0 \). This is the case where all of the other exporting firms set their prices in the importers’ currency. Equations (7)-(9) show that \( EI\bar{I} \geq EI\bar{E} \) and \( EI\bar{I} \geq EI\bar{O} \) if and only if

\[
\partial^2 \Pi_0 / \partial s_0^2 \leq 0 \quad \text{and} \quad \partial^2 \Pi_0 / \partial s^2 \leq 0.
\]

Since each exporter sets a price in the importers’ currency if \( EI\bar{I} \geq EI\bar{E} \) and \( EI\bar{I} \geq EI\bar{O} \), this implies that the denomination in the importers’ currency is a Nash equilibrium only if (13) holds. Each of two inequalities in (13) does not always hold under general demand and cost functions. We can, however, see that each inequality always holds when each profit function is concave in each exchange rate around its expected value.

We can similarly investigate whether the denomination in the third currency can be a Nash equilibrium. Assuming symmetry, we consider the case where \( \partial P^*/\partial s = \partial P^*/\partial s_0 \neq 0 \). This is the case where all of the other exporting firms set their prices in the third currency. Equations (10)-(12) show that \( EI\bar{I} \geq EI\bar{E} \) and \( EI\bar{I} \geq EI\bar{O} \) if and only if

\[
\partial^2 \Pi_0 / \partial s_0^2 + 2 (\partial^2 \Pi_0 / \partial P^* \partial s_0)(\partial P^* / \partial s_0) \geq 0,
\]

and

\[
\partial^2 \Pi_0 / \partial s_0^2 + 2 (\partial^2 \Pi_0 / \partial P^* \partial s_0)(\partial P^* / \partial s_0) \geq 0.
\]

This implies that the denomination in the third currency is a Nash equilibrium only if both (14) and (15) hold.

We finally investigate whether the denomination in the exporters’ currency can be a Nash equilibrium. Assuming symmetry, we consider the case where \( \partial P^*/\partial s = 0 \) but \( \partial P^*/\partial s_0 \neq 0 \). This is the case where some of the other exporting firms set their prices in the exporter’s currency. Equations (10)-(12) show that \( EI\bar{I} \geq EI\bar{E} \) and \( EI\bar{I} \geq EI\bar{O} \) if and only if

\[
\partial^2 \Pi_0 / \partial s_0^2 + 2 (\partial^2 \Pi_0 / \partial P^* \partial s_0)(\partial P^* / \partial s_0) \geq 0,
\]

and

\[
\partial^2 \Pi_0 / \partial s_0^2 + 2 (\partial^2 \Pi_0 / \partial P^* \partial s_0)(\partial P^* / \partial s_0) \geq 0.
\]
\[ (17) \quad (\partial^2 \bar{F} / \partial s^2) \sigma_0^2 \leq \left\{ (\partial^2 \bar{F} / \partial \sigma_0^2 s^2) + 2 \left[ (\partial^2 \bar{F} / \partial P^* \partial \sigma_0) - (\partial^2 \bar{F} / \partial P^* \partial s) \right] \right\} \sigma_0^2. \]

This implies that the denomination in the exporters’ currency is a Nash equilibrium only if both (16) and (17) hold.

It is interesting to note that both (14) and (15) can hold even if (13) holds and that both (16) and (17) can hold even if (13) holds. This indicates that the model can have multiple Nash equilibria for some demand and cost functions. Since multiple equilibria are Pareto ranked, coordination failures may thus make the equilibrium choice of invoice currency less efficient.

4. The Case of CES preferences

When we specify the demand and cost functions, our equilibrium conditions are solved explicitly. We consider the following set of constant elasticity demand and cost functions.

\[ (18) \quad D(p_j, P^*) = A \left( \frac{p_j}{P^*} \right)^\mu, \]
\[ (19) \quad C(D) = B D^\eta, \]

where \( \mu > 1 \) and \( \eta > 1 \).

If the importers have CES preferences with elasticity \( \mu > 1 \) among the different products, we can specify the demand for goods from firm \( j \) as (18). In this case, the aggregate price index in the importers’ local market \( P^* \) is given by

\[ (20) \quad P^* = \left( \frac{1}{N} \sum_{i=1}^{N} p_i^{1-\mu} \right)^{1/(1-\mu)} \]

where \( N \) is the number of firms in the importers’ local market and \( p_i \) is a price set by exporting firm \( i \) in the importers’ currency. In the local market, a fraction \( f \) of firms is identical exporting firms and a fraction \( 1-f \) is identical local firms. Let \( p_j \) denote a price set by exporting firm \( j \) in the importers’ currency and \( p^D \) a price set by local firms in the importers’ currency. The overall price index faced by foreign country consumers is then

\[ (21) \quad P^* = \left[ \frac{f}{H} \sum_{j=1}^{H} p_j^{1-\mu} + (1-f) (p^D)^{1-\mu} \right]^{1/(1-\mu)} \]
We assume that the local firms always set their prices in the local currency, that is, in the importers’ currency. Then, $p^D$ is always independent of the exchange rate. The exporting firms, however, set their prices either in its own, in the importers’, or in the third currencies. The price index thus depends on the exchange rate unless the exporting firms set their prices in the importers’ currency.

Under (18) and (19), each profit function is respectively written as

$$
\Pi_E = \frac{A}{s} (p_E / s) - A B (p_E / s)^{\eta B} - \mu \eta
$$

$$
\Pi_I = \frac{A}{s} (p_I / s) - A B (p_I / s)^{\eta B} - \mu \eta
$$

$$
\Pi_0 = \frac{A}{s} (p_0 / s_0) - A B (p_0 / s_0)^{\eta B} - \mu \eta
$$

Assuming that all domestic and exporting firms are identical under certainty, it holds that $P^* = p^D = p^E / s = p^I = p_0 / E s_0$. The first-order conditions thus lead to

$$
p^E = p^I E s = p_0 (E s / s_0) = A^{-1} B \mu \eta / (\mu - 1)
$$

at $s_0 = E s_0$ and $s = E s$. In addition, after some tedious calculations shown in Appendix 2, we can derive that at $s_0 = E s_0$ and $s = E s$,

$$
\frac{\partial^2 \Pi}{\partial s^2} = (1/s_0)^2 (p^E - (\mu - 1)(\mu(\eta - 1))) < 0,
$$

$$
\frac{\partial^2 \Pi}{\partial s^2} = (1/s)^2 (p^E - (\mu(\eta - 1) - 1)) < 0,
$$

$$
\frac{\partial^2 \Pi}{\partial s^2} = (p^E / p_0)(\mu - 1)(\mu(\eta - 1) - 1) < 0,
$$

$$
\frac{\partial^2 \Pi}{\partial s} = -\mu(\mu(\eta - 1) - 1),
$$

$$
\frac{\partial^2 \Pi}{\partial p} = \mu,
$$

Equations (26) and (27) imply that the condition (13) holds if and only if

$$
\mu(\eta - 1) \geq 1.
$$

The denomination in the importers’ currency is thus a Nash equilibrium if and only if (32) holds. On the other hand, from equations (28)-(31), we can show that the condition (14) holds if and if

$$
\mu(\eta - 1)(2f - 1) \geq 1,
$$

and that the condition (16) holds if and if
\((34) \mu(\eta-1)(2fh-1) + 1 \geq 0,\)

where parameter \(h\) is a fraction of exporting firms that set their prices in the exporter’s currency. Since \(\partial^2 I\Phi/\partial s^2 < 0\) when \((33)\) holds, the condition \((15)\) holds if \((33)\) holds. This implies that the denomination in the third currency is a Nash equilibrium if and only if \((33)\) holds. Similarly, since \(\partial^2 I\Phi/\partial s_0^2 < 0\), the condition \((17)\) holds if \((16)\) holds. The denomination in the exporters’ currency is therefore a Nash equilibrium if and only if \((34)\) holds.

Among the above three inequalities, the condition \((34)\) is the only inequality that holds when \(\mu(\eta-1) < 1\). Since the parameter \(\mu\) becomes small when the importers’ local market is less competitive, the denomination in the exporters’ currency is thus a Nash equilibrium when the local market is less competitive. This implies that the exports of differentiated products, which prevail in developing countries, tend to be denominated the exporters’ currency.

In contrast, when \(\mu(\eta-1) \geq 1\), the denomination in the importers’ currency is always a Nash equilibrium. Since the parameter \(\mu\) becomes large enough when the local market is competitive, this indicates that the denomination in the importers’ currency is a Nash equilibrium in the competitive local market. However, when \(\mu(\eta-1) \geq 1\), Nash equilibrium may not be unique in general. The denomination in the third currency is another Nash equilibrium when \(2f-1 \geq 1/[\mu(\eta-1)]\). The denomination in the exporters’ currency is another Nash equilibrium when \(2fh-1 \geq -1/[\mu(\eta-1)]\). When the market is competitive, the model therefore has multiple Nash equilibria for some parameter set, particular when a fraction of exporting firms in the importers’ local market \(f\) is large.

When the local market is competitive enough, the exporting firms tend to keep their prices not to deviate from those of the competitors. As a result, when the other exporters are expected to set their prices in some currency, the exporting firm tends to set its price in the same currency. The denomination in the arbitrary currency can therefore be a Nash equilibrium depending on the expectations on the choice of invoice currency of the other exporters.

Because of the less differentiated exporting products, the exporters in developing countries tend to face serious competition in the importers’ local markets. The above result thus indicates that when a fraction of exporting firms is large in the local market, the choice of invoice currency can be arbitrary in the exports from developing countries. In particular, since the US dollar has historically been the dominant invoice currency in most developing countries, the exporters in the developing countries may not have an incentive to change their invoice currency from the US dollar to the other currency even if the United States is not a trade partner. It is noteworthy that multiple equilibria are Pareto ranked. This implies that the equilibrium choice of invoice currency may be a less efficient equilibrium.
5. Empirical Evidence

(i) Framework

It is well known that the U.S. dollar has been the dominant vehicle currency in many developing countries. In particular, as we showed in introduction, payments by the US dollar have been dominant in most East Asian countries even though other East Asian countries, particularly Japan, are important trade partners. There is, however, no direct evidence that shows how dominant the use of US dollar was as “contract currency” of export prices in the East Asian countries. The contract currency is usually the same as the payment currency in international trade. The role of medium of exchange is, however, theoretically different from that of a unit account in international trade. We thus need some formal tests to examine to what extent export prices are stable in terms of the US dollar in most of East Asian international trade.

This section empirically investigates the stability of export prices against the US dollar in Korea. We used the export prices in Korea because the data are available for varieties of commodities. We examine how the export prices of various commodities are correlated with the US dollar, the Japanese yen, and the Euro. In particular, we explore whether export prices in Korea can be highly stable in terms of the US dollar even in the commodities which were exported to Japan.

All data are monthly. Define the relative export price of commodity $i$ at time $t$ by $REP_{i,t} = \frac{EPI_{i,t}}{PPI_{i,t}}$, where $EPI_{i,t}$ is the export price index of commodity $i$ at time $t$ and $PPI_{i,t}$ is the producer price index of commodity $i$ at time $t$. We regressed its growth rate on the growth rates of USD (= the exchange rate of the US dollar), JPY (= the exchange rates of the Japanese Yen), and EUR (= the exchange rate of the Euro [the German Mark before December 2002]).

For each relative export price of commodity $i$, we estimated the following equation

$$d \ REP_{i,t} = \text{constant} + \sum_{k=0}^{K} a_k \ d \ \ln \ USD_{t-k} + \sum_{k=0}^{K} b_k \ d \ \ln \ JPY_{t-k} + \sum_{k=0}^{K} c_k \ d \ \ln \ EUR_{t-k},$$

where $d \ REP_{i,t} = \ln \ REP_{i,t} - \ln \ REP_{i,t-1}$ and $d \ \ln \ S_{t-k} = \ln \ S_{t-k} - \ln \ S_{t-k-1}$ for $S = \text{USD, JPY, and EUR}$. All exchange rates are monthly average rates in terms of the Korean won. To allow the lag structure, we used the Almon lag, where $K$ is the number of lags. Some preliminary estimations could not reject the hypothesis that the end point constraint that $a_3 = b_3 = c_3 = 0$ when $K = 2$. We thus estimated equation (35) assuming that $K = 2$ and imposing the end point constraint that $a_3 = b_3 = c_3 = 0$.

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3 When converting the German Mark to the Euro, we applied the euro conversion rate: 1 Euro = 1.95583 Mark.
4 We first estimated (35) with seasonal dummies. None of them was, however, significant. The following analysis reports the estimation results without seasonal dummies.
The sum of the coefficients respectively reflects the impact of the exchange rate change on the export price in Korea. That is, $\sum_{k=0}^{K} a_k$ reflects the impact of the change of the US dollar, $\sum_{k=0}^{K} b_k$ that of the change of the Japanese Yen, and $\sum_{k=0}^{K} c_k$ that of the change of the Euro. If the export price in Korea is denominated by the currency of the export destination, all of $\sum_{k=0}^{K} a_k$, $\sum_{k=0}^{K} b_k$, and $\sum_{k=0}^{K} c_k$ would lie between zero and one. In contrast, if the US dollar is the dominant invoice currency, $\sum_{k=0}^{K} a_k$ would be close to one and $\sum_{k=0}^{K} b_k$ but $\sum_{k=0}^{K} c_k$ would be small.

(ii) The data

The sample period of estimations is from March 1998 to December 2002. In order to exclude the turbulent period after the currency crisis, we start the sample period from March 1998. We ended the sample period in December 2002 because the commodity classification of the export price index was revised after January 2003.

The data of the export price index (EPI) and the producer price index (PPI) are downloaded from the website of the Bank of Korea. The base year of each index is 1995. The commodity classifications are based on “won basis Basic Groups” of each index. Unfortunately, the classifications do not have one-to-one correspondences between the two indexes. We therefore reclassified each classification and sorted out 21 commodities. Among these 21 commodities, we excluded agricultural products and marine products from our samples because the export prices were highly volatile over time. We consequently obtained 19 commodities: (1) mining products, (2) processed marine products, (3) plastic products, (4) non-metallic mineral products, (5) iron & steel, (6) basic nonferrous metals & related primary, (7) furniture, (8) footwear, (9) hand tools & general hardware, (10) electric machinery & apparatus, (11) precision instrument, (12) sports & leisure goods, (13): musical instruments, (14) rubber products, (15) general purpose machinery, (16) special purpose machinery, (17) radio, TV, & communication equipment, (18) transportation equipment, and (19) apparel.

The column (A) in table 4 displays the list of 19 commodities we use in the following analysis. The columns (B) and (C) in table 4 summarize how each classified commodity corresponds to that in the export price index (EPI) and that in the producer price index (PPI) in the following analysis.

(iii) The Estimation Results

Table 5 summarizes the results of regressions for the 19 commodities. The sum of the coefficient $\sum_{k=0}^{K} a_k$ was significantly positive in eighteen among the 19 commodities. The exception was non-
metallic mineral products in which none of \( \sum_{k=0}^{K} a_k \), \( \sum_{k=0}^{K} b_k \), and \( \sum_{k=0}^{K} c_k \) was significantly positive (No.6-1 in the table). However, even in non-metallic mineral products, \( \sum_{k=0}^{K} a_k \) turned out significantly positive when we estimate (35) with the restriction that \( \sum_{k=0}^{K} b_k = \sum_{k=0}^{K} c_k = 0 \) (No.6-2 in the table). Excluding No.6-1, the average value of \( \sum_{k=0}^{K} a_k \) was 0.837, which implies that export prices are highly stable in terms of the US dollar in Korean exports. In particular, \( \sum_{k=0}^{K} a_k \) was greater than 0.9 in eight commodities: mining products, plastic products, nonmetallic mineral products, footwear, sports & leisure goods, musical instruments, rubber products, and general purpose machinery.

In contrast, the sum of the coefficient \( \sum_{k=0}^{K} b_k \) was significantly positive in eight commodities but not in eleven commodities. Even in the eight commodities in which \( \sum_{k=0}^{K} b_k \) was significantly positive, \( \sum_{k=0}^{K} b_k \) was less than 0.4 except for processed marine products. The sum of the coefficient \( \sum_{k=0}^{K} c_k \) was significantly positive only in one commodity and not in the other eighteen commodities. The results indicate that export prices in Korea had only modest correlation with the Japanese yen and little correlation with the Euro.

(iv) Trade Destinations and Export Prices

In the last subsection, we showed that the US dollar was the dominant invoice currency in Korean exports. However, interpreting the results in Table 5, we need to note that the United States is the largest export destination from Korea and that China and Hong Kong peg their exchange rates to the US dollar. To the extent that the US dollar is dominant in invoicing the Korean exports to the United States, China, and Hong Kong, the results do not necessarily mean that the US dollar is the dominant invoice currency in Korean exports to the other countries. The following analysis investigates whether export prices in Korea are highly stable in terms of the US dollar even in the commodities for which the United States is not a dominant export partner.

Table 6 summarizes the shares of the United States (US/W) and the shares of Japan (JP/W) in the Korean exports for each of the 19 commodities over 1998 - 2001. It also reports the summed shares of China and Hong Kong \([(C+HK)/W]\) and those of the United States, China, and Hong Kong \([(US+C+HK)/W]\). The table shows that the United States has large shares in several commodities. In particular, the summed shares of the United States, China, and Hong Kong exceed 50% in eight commodities. Japan, however, has large shares in several commodities. In particular, the shares of Japan exceed 20% in seven commodities.
If the US dollar is the dominant invoice currency only in the Korean exports to the United States, export prices in Korea would be stable in terms of the US dollar only in the commodities for which the United States has large shares. Because China and Hong Kong peg their exchange rates to the US dollar, it is also likely that the US dollar is dominant in invoicing the Korean exports to China and Hong Kong. In contrast, if the US dollar is the dominant invoice currency in all of the Korean exports, export prices in Korea would be stable in terms of the US dollar even in the commodities for which Japan has large shares.

By using the estimated coefficients in equation (35), we test these alternative hypotheses. Denoting commodity $i$ by subscript $i$, we estimate the following equations:

\[(36a) \left( \sum_{k=0}^{K} a_k \right)_{ij} = \text{constant} + d_1 \text{US}_i/W_i + d_2 (\text{C+HK})_i/W_i + d_3 \text{JP}_i/W_i,\]

\[(36b) \left( \sum_{k=0}^{K} b_k \right)_{ij} = \text{constant} + e_1 \text{US}_i/W_i + e_2 (\text{C+HK})_i/W_i + e_3 \text{JP}_i/W_i,\]

\[(36c) \left( \sum_{k=0}^{K} a_k \right)_{ij} - \left( \sum_{k=0}^{K} b_k \right)_{ij} = \text{constant} + g_1 \text{US}_i/W_i + g_2 (\text{C+HK})_i/W_i + g_3 \text{JP}_i/W_i,\]

where \(\text{US}_i/W_i\) denotes the shares of the United States, \((\text{C+HK})_i/W_i\) the shares of China and Hong Kong, and \(\text{JP}_i/W_i\) the shares of Japan. If the invoice currency is determined by the currency of the export destination, we can expect that the parameters \(d_1, d_2, e_3, g_1,\) and \(g_2\) would be significantly positive and that the parameter \(g_3\) would be significantly negative.

Table 7 reports the regression results. In the table, all of the parameters had expected signs. This implies that invoicing in the US dollar was more dominant in the Korean exports to the United States, China, and Hong Kong but less in the Korean exports to Japan. However, except for \(g_1\) and \(g_2\), the estimated parameters were not significantly positive. Even the parameters \(g_1\) and \(g_2\) took small positive values. The results suggest that export prices in Korea were highly stable in terms of the US dollar even in the commodities for which Japan has had dominant shares.

6. Consistency of our Empirical Results with our Theoretical Results

One of the most prominent features in our theoretical model is that the third currency can be an equilibrium invoice currency when the exporters are under competition and when local firms have small shares in the market. Because of less differentiated products, the first condition tends to hold in the exports from developing countries. Our theoretical implication will thus be supported if the third currency is used as an invoice currency in the exports from developing countries when the second condition holds, that is, when a fraction of local firms is small in the market. In this section, we examine this hypothesis by using the regression results in the last section. Specifically, we explore
whether the US dollar is the dominant invoice currency in the products of which local firms have small shares in the competitive market.

In the analysis, we investigate how the difference in the import shares in Japan affects the choice of invoice currency of each commodity. We chose the Japanese market as a representative local market because Japan had been the second biggest export destination for Korea for a long period. If the theoretical hypothesis is true, we expect that invoicing in the U.S. dollar tends to be large in commodities for which the import share is large in the Japanese market.

Define the import share in Japan \( f_i \) as

\[
(37) \quad f_i \equiv \frac{\text{[the amount of imports in commodity } i\text{]}}{\text{[the amount of sales in commodity } i\text{]}},
\]

where domestic sales \( \equiv \text{total domestic production} – \text{exports} + \text{imports}. \) We calculate \( f_i \) by using the 2000 Input-Output Tables reported by the Ministry of Economy, Trade, and Industry. To avoid the aggregation biases, we first calculated \( f_i \)’s for basic 71 commodities and then used those for which imports from Korea are more relevant in Japan (see Appendix 3).

The estimated values of \( \sum_{k=0}^{K} a_k \) reflect not only the impacts of the dollar-won exchange rate on Korean export prices to non-US dollar areas such as Japan but also those to the US dollar areas (i.e., the U.S., China, and Hong Kong). To identify the invoice ratios in the exports to Japan, we thus need to subtract those to the US dollar areas from \( \sum_{k=0}^{K} a_k \). Define \( A(\text{USD})_i \) as the invoice ratio in the exports to the US dollar areas and \( A(\text{Non-USD})_i \) as that to non-US dollar areas. For commodity \( i \), the value of \( \sum_{k=0}^{K} a_k \) is then represented as the weighted average of \( A(\text{USD})_i \) and \( A(\text{Non-USD})_i \) as follows

\[
(38) \quad [\sum_{k=0}^{K} a_k]_i = w_i \cdot A(\text{USD})_i + (1 - w_i) \cdot A(\text{Non-USD})_i,
\]

where \( w_i \) is the ratio of Korean exports to the US dollar areas divided by Korean exports to the World.

We use the value of \( (\text{US+C+HK})/W \) in Table 6 for \( w_i \), and the estimates in Table 5 for \( [\sum_{k=0}^{K} a_k]_i \). Assuming that all of the exports to the US dollar areas are invoiced in the US dollar, that is, \( A(\text{USD})_i \equiv 1 \) for all \( i \), equation (38) then leads to the estimates of \( A(\text{Non-USD})_i \). For Korea, a primary part of \( 1 - w_i \) is the weight of the exports to Japan. The estimates of \( A(\text{Non-USD})_i \) would therefore reflect the extent to which the US dollar is chosen as an invoice currency in Korean exports to Japan.

\[ \text{In recent years, China became the second biggest export destination for Korea.} \]
Table 8 reports the value of $f_i$ as well as the estimates of $A(\text{Non-USD})_i$ for 19 commodities. Because the aggregation biases still remain, the calculated values of $f_i$ were less than 50% except for mining products. We can, however, observe a tendency that $A(\text{Non-USD})_i$ is large in commodities for which $f_i$ is large. For example, $f_i$ exceeds 40% in three commodities: mining products, footwear, and apparel. In these commodities, the average of $A(\text{Non-USD})_i$ is 1.103. In contrast, in the other sixteen commodities, the average of $A(\text{Non-USD})_i$ is 0.761. Welch’s test reveals that the former average is statistically greater than the latter one at the 1% significance level. The results support our theoretical hypothesis that the US dollar tends to be the dominant invoice currency in the products for which local firms have small shares.

When the market is competitive enough, the exporting firms tend to keep their prices not to deviate from those of the competitors. As a result, when the competitors are expected to set their prices in the US dollar, the exporting firm tends to set its price in the same currency. The local firms usually set their prices in the local currency. To the extent that the shares of local firms are large, it is thus unlikely that the competitors are expected to set their prices in the US dollar outside the United States. However, when the shares of local firms are small, it is possible that the competitors are expected to set their prices in the US dollar outside the US market. Our empirical result supports this view.

7. Concluding Remarks

This paper investigated the choice of invoice currency under exchange rate uncertainty. The analysis was motivated by the fact that the U.S. dollar has been the dominant vehicle currency in developing countries. Our theoretical analysis was based on an open economy model of monopolistic competition. When the market is competitive enough, the exporting firms tend to set their prices not to deviate from those of the competitors. As a result, when the other exporters set their prices in the third currency, the exporting firm tends to choose the third currency as the invoice currency. The tendency becomes conspicuous in the market where the shares of local firms are small. The latter part of the paper empirically investigated the relevancy of the theoretical results by using the export price data in Korea. We found that export prices in Korea were highly stable in terms of the US dollar even in the commodities for which Japan had dominant shares. We also found that export prices in Korea were more stable against the US dollar in the commodities for which the shares of local firms were small.

The empirical results are consistent with our theoretical model. They are, however, inconsistent with pricing-to-the-market models that have analyzed whether the exporting firm sets prices in its own currency or in the importer’s currency. It is well known that some of international trades are invoiced in the U.S. dollar in developing countries. Since the exporting products are less differentiated in developing countries, our results may provide one plausible explanation on why the exporting firms in developing countries tend to set prices in the US dollar even if the United States is not a trade partner.
References


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Source: Bank of Thailand.
### Table 1b. Structure of export receipts from major trading partners classified by currency in Thailand (Percent share)

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Source: Bank of Thailand.
Table 2. The Shares of Payment Currencies in Korean Exports

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**注記：** 以上の内容は、印刷された文書の自然な読解を目的として、テキストベースの理解と分析に基づいて作成されました。
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※詳細は前記の申請書をご参照ください。
Appendix 1: Derivations of (7)-(10) and (11)-(14).

Equations (1)-(3) and (6) imply that \( \partial I \partial P^* = \partial I \partial P^* = \partial I \partial P^* \) and \( \partial I \partial P^* s = \partial I \partial P^* s \), and \( \partial^2 I \partial P^* = \partial^2 I \partial P^* = \partial^2 I \partial P^* at s_0 = E s_0 and s = E s \). It thus holds that when \( \partial P^*/\partial s = 0 \),

(A1) \( \Pi_{11}^E = \Pi_{11}^l = \partial^2 I \partial P^* (\partial P^*/\partial s s_0^2 + (\partial I \partial P^*) (\partial^2 P^*/\partial s s_0^2) s_0) \),

(A2) \( \Pi_{11}^0 = \Pi_{11}^E + \partial^2 I \partial P^* s_0^2 + 2 (\partial^2 I \partial P^* s_0^2) s_0 (\partial P^*/\partial s s_0) \),

(A3) \( \Pi_{22}^l = \Pi_{22}^0 = 0 \),

(A4) \( \Pi_{22}^E = \partial^2 I \partial P^* s_0^2 \),

at \( s_0 = E s_0 and s = E s \). Similarly, when \( \partial P^*/\partial s_0 = 0 \),

(A5) \( \Pi_{11}^E = \Pi_{11}^l = 0 \),

(A6) \( \Pi_{11}^0 = \partial^2 I \partial P^* s_0^2 \),

(A7) \( \Pi_{22}^l = \Pi_{22}^0 = \partial^2 I \partial P^* s^2 + (\partial I \partial P^*) (\partial^2 P^*/\partial s^2 s) + 2 (\partial^2 I \partial P^* s_0^2) (\partial P^*/\partial s s_0) \),

(A8) \( \Pi_{22}^E = \Pi_{22}^l + \partial^2 I \partial P^* s^2 + 2 ([\partial^2 I \partial P^* s^2] - (\partial^2 I \partial P^* s_0^2)) (\partial P^*/\partial s) \),

at \( s_0 = E s_0 and s = E s \).

Equations (4) and (5) lead to

(A9) \( E I \partial^0 - E I \partial = (1/2) [(\Pi_{11}^0 - \Pi_{11}^l) \sigma_0^2 + (\Pi_{22}^0 - \Pi_{22}^l) \sigma^2] \),

(A10) \( E I \partial^0 - E I \partial = (1/2) [(\Pi_{11}^0 - \Pi_{11}^l) \sigma_0^2 + (\Pi_{22}^0 - \Pi_{22}^l) \sigma^2] \),

(A11) \( E I \partial^0 - E I \partial = (1/2) [(\Pi_{11}^0 - \Pi_{11}^l) \sigma_0^2 + (\Pi_{22}^0 - \Pi_{22}^l) \sigma^2] \).

We can therefore derive that when \( \partial P^*/\partial s = 0 \),

(A12) \( E I \partial^0 - E I \partial = (1/2) [\partial^2 I \partial P^* s_0^2 + 2 (\partial^2 I \partial P^* \partial s_0^2) (\partial P^*/\partial s_0)] \sigma_0^2 \),

(A13) \( E I \partial^0 - E I \partial = (1/2) [\partial^2 I \partial P^* s_0^2 + 2 (\partial^2 I \partial P^* \partial s_0^2) (\partial P^*/\partial s_0)] \sigma_0^2 - (\partial^2 I \partial P^* s_0^2) \sigma^2 \),

(A14) \( E I \partial^0 - E I \partial = - (1/2) (\partial^2 I \partial P^* s) \sigma^2 \),

and that when \( \partial P^*/\partial s_0 = 0 \),

(A15) \( E I \partial^0 - E I \partial = (1/2) (\partial^2 I \partial P^* s_0^3) \sigma_0^2 \),

(A16) \( E I \partial^0 - E I \partial = (1/2) ([\partial^2 I \partial P^* s_0^3]) \sigma_0^2 \).
Equations (22), (23), and (24) lead to

Appendix 2: Derivations of (29)-(34)

Equations (22), (23), and (24) lead to

\[(A18)\] \( \partial I^f / \partial s = A \mu p^E \left[ s^{\mu-1} P^* \mu - \mu \eta A^n B s^{\mu n-1} (P^*/s^E) \mu \eta \right], \]
\[(A19)\] \( \partial I^f / \partial s = A p^1 \left[ \mu \mu - P^* \mu \right], \]
\[(A20)\] \( \partial I^f / \partial s = A (\mu-1) s p^0 \left[ s_0^{\mu-2} P^* \mu - \mu \eta A^n B s_0^{\mu n-1} (P^*/s^0) \mu \eta \right]. \]

Since \( P^* E s = p^E = p^1 E s = p^0 (E s / E s_0) = A p^{-1} B \mu \eta / (\mu-1) \) at \( s_0 = E s_0 \) and \( s = E s \), it holds that

\[(A21)\] \( \partial^2 I^f / \partial s^2 = (\mu-1) A p^E s^2 \left[ P^* s/p^E \right] \mu - \mu \eta A^n B (\mu \eta-1) \]
\[= (1/s_0)^2 \left[ (\mu-1) A p^0 \left( s/s_0 \right) - \mu \eta A^n B (\mu \eta-1) \right], \]
\[= - (1/s_0)^2 p^E (\mu-1) \left[ \mu (\eta-1) + 1 \right] < 0 \]

\[(A22)\] \( \partial^2 I^f / \partial s^2 = (\mu-1) A p^E s^2 \left[ P^* s/p^E \right] \mu - \mu \eta A^n B (\mu \eta-1) s^2 \left( P^* s/p^E \right) \mu \]
\[= (1/s)^2 \left[ (\mu-1) A p^E s^2 - \mu \eta A^n B (\mu \eta-1) \right], \]
\[= - (1/s)^2 p^E (\mu-1) \left[ \mu (\eta-1) - 1 \right]. \]

\[(A23)\] \( \partial^2 I^f / \partial s s = A \mu (\mu-1) \left( s/s_0 \right) \left( P^* s_0/p^0 \right)^{\mu-1} - (\mu \eta)^2 A^n B \left( 1/p^0 \right) \left( P^* s_0/p^0 \right)^{\mu n-1}, \]
\[= (1/p^0) \left[ A (\mu-1) \left( p^0 / s_0 \right) - (\mu \eta)^2 A^n B \right], \]
\[= - (p^E/p^0) (\mu-1) \left[ \mu (\eta-1) - 1 \right]. \]

\[(A24)\] \( \partial^2 I^f / \partial P^* s = A \mu^2 \left( P^* s/p^E \right)^{\mu-1} - (\mu \eta)^2 A^n B \left( 1/p^E \right) \left( P^* / s^E \right)^{\mu n-1}, \]
\[= A \mu^2 - (\mu \eta)^2 A^n B \left( 1/p^E \right), \]
\[= - \mu (\mu \eta-1) \eta. \]

\[(A25)\] \( \partial^2 I^f / \partial P^* s = A \mu (P^*/s^E)^{\mu-1} = \mu. \)