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Endogenous Monetary Policy Shifts and the Term Structure: Evidence from Japanese Government Bond Yields

Junko Koeda*†

Abstract

I construct a no-arbitrage term structure model with endogenous regime shifts and apply it to Japanese government bond (JGB) yields. This application subjects the short-term interest rate to monetary regime shifts, such as a zero interest rate policy (ZIRP) and normal regimes, which depend on macroeconomic variables. The estimated results show that under a ZIRP, the deflationary effect on bond yields increases on the longer end of yield curves; on the other hand, the effect of output gaps on raising bond yields weakens for all maturities.

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1 Introduction

The short-term interest rate or the policy rate is subject to monetary regime shifts, such as a zero interest rate policy (ZIRP) and normal regimes, which depend on macroeconomic state variables. To examine how state-dependent policy shifts affect government bond yields, this paper constructs a no-arbitrage term structure model with discrete regime shifts. In the model (i) the transition probabilities depend on the state variables that appear in the monetary policy rule, for example, inflation and output gap, and (ii) the state vector depends on the current monetary policy regime rather than on the previous regime. This timing is not trivial because the data frequency of macroeconomic variables is usually not very high. Condition (ii) is suitable particularly when the state vector includes the policy rate to allow the dynamics of macroeconomic variables to depend on the lagged policy rate.

This paper adds to the existing literature in two ways. First, it extends no-arbitrage affine[1] term structure models with discrete regime shifts by incorporating both conditions (i) and (ii) above; to date, the literature has only adopted either (i) or (ii).[2] For example, Dai, Singleton, and Yang (2007), henceforth DSY, adopt condition (i) with state-dependent transition probabilities under $\mathbb{P}$, while their state vector depends on the previous regime.

[1] Alternatively, term structure models that lie outside of the affine family have been applied to the Japanese zero rate environments. See Ichiue and Ueno (2012) for an application of Black's (1995) model to JGB yields and Singleton and Kim (2012) for a comparison between Cox, Ingersoll, and Ross (1985) type affine model and non-affine models.

[2] Ang, Boivin, Dong, and Loo-Kung (2011) model monetary policy shifts with time-varying Taylor rule coefficients. These coefficients are treated as latent factors in their term structure model. This paper differs from their model as it focuses on discrete and observable monetary regime shifts.

On the other hand, Bansal and Zhou (2002), Ang, Bekaert, and Wei (2008), Hamilton and Wu (2011), and others adopt condition (ii) assuming a constant transition matrix. Via formal propositions and proofs and discussion on the link between the $\mathbb{P}$ and $\mathbb{Q}$ measures, this paper extends DSY's work with Hamilton's (1989) formulation, in which the current state variables depend on the current (not previous) regime.[3]

Second, as an application of the model, I estimate how Japanese government bond (JGB) yields respond to a ZIRP as well as macroeconomic conditions. The factor dynamics used in bond pricing are similar to that of Hayashi and Koeda (2012). These authors use a vector autoregression (VAR) model with endogenous monetary policy shifts that incorporate the key aspects under a ZIRP, such as (a) the zero lower bound and (b) the duration of the ZIRP committed to by the Bank of Japan, known as *Jikan Jiku*. This paper's model differs from Hayashi and Koeda's mainly by including fewer variables in the VAR system, and by not assuming any no a priori identification restrictions in the VAR system. As a result, uncertainty about the current macroeconomic variables affects the current regime determination.

I consider two regimes–the normal and ZIRP regimes–and two types of regime evolution, one that incorporates condition (a) above and one that incorporates both conditions (a) and (b). I first consider a simple evolution of the regime that depends solely on a Taylor-rule based policy rate (Type I evolution). If the Taylor-rule based policy rate hits the lower bound of interest rates, the policy rate is set at the bound under the ZIRP regime; otherwise the policy rate is set by the Taylor rule under the normal regime. Type I

[3]Another key difference from DSY's study is that this paper allows the factor coefficients in the short rate equation to depend on the regime, as explained in the next section.

evolution, however, does not take into account *Jikan Jiku,* a key feature of the ZIRP in Japan (for a discussion on *Jikan Jiku*, see e.g., Ueda, 2012a and 2012b and Ugai's survey, 2007). Thus I extend the Type I evolution by introducing a *Jikan Jiku* policy under which the ZIRP is maintained unless some inflation condition is satisfied (Type II evolution).

The main results are presented in two steps. First, I compare the empirical results of the two types of factor dynamics, that is, Type I (without policy duration) versus Type II (with policy duration). A notable difference is that the state-dependent transition probabilities are estimated to be more persistent under Type II evolution. Furthermore, empirical evidence indicates that Type II evolution has notably better fits to the data than Type I, whereas out-of-sample performance results are mixed. Second, I discuss the estimated yield curves and term premia using the term structure model with Type II factor dynamics as the benchmark model. The estimated yield curves indicate, under a ZIRP, the effect of output gaps on raising bond yields weakens for all maturities, whereas deflationary effect on JGB yields becomes stronger at the longer end of yield curves. Furthermore, the estimated term premia indicate that a large bond yield decline in the early 1990s was driven by expectation components, whereas that in the late 1990s was driven by both components. The term premia also declined after the introduction of the quantitative easing monetary policy (QEP) in March 2001.

This paper proceeds as follows. Section 2 describes a term structure model with endogenous regime shifts. Section 3 describes the specific regime evolutions considered. Section 4 and 5 discuss the estimation strategy and results. Section 6 concludes the paper.

2 The model

2.1 The $\mathbb{P}$ model

The state of the economy is assumed to follow a discrete time stationary Markov process $\{\mathbf{y}_t, s_t\}$ where $\mathbf{y}_t$ is a vector of continuous variables and s_t is a scalar discrete variable also called the regime. Both $\mathbf{y}_t$ and s_t are observable. For notational convenience, the current value is indicated with tilda, and the previous period's value with no time subscript.

The joint density-distribution function of $(\widetilde{\mathbf{y}}, \tilde{s})$ conditional on $(\mathbf{y}, s)$ is

$$p(\widetilde{\mathbf{y}}, \tilde{s}|\mathbf{y}, s) = f(\widetilde{\mathbf{y}}|\tilde{s}, \mathbf{y}, s)\, \rho(\tilde{s}|\mathbf{y}, s), \tag{1}$$

where $f(\widetilde{\mathbf{y}}|\tilde{s}, \mathbf{y}, s)$ is the conditional density of $\tilde{y}$ given $(\tilde{s}, \mathbf{y}, s)$ and $\rho(\tilde{s}|\mathbf{y}, s)$ is the transition probability of the regime.

N period bond prices are functions of the state, $P_n(\mathbf{y}, s)$. In particular, the short rate (r) is given by

$$r \equiv -\log(P_1(\mathbf{y}, s)) = \text{1st element of } \mathbf{y}.$$

The $\mathbb{P}$ model satisfies two assumptions. Assumption 1 follows Hamilton (1989) that the conditional density of $\tilde{y}$ given $(\tilde{s}, \mathbf{y}, s)$ depends on $\tilde{s}$ but not on s. This formulation differs from DSY, they instead assume that the conditional density of $\widetilde{\mathbf{y}}$ depends on s but not on $\tilde{s}$. Thus, the $\mathbb{P}$ model can be interpreted as an extension of DSY with the Hamilton (1989) formulation where $f(\widetilde{\mathbf{y}}|\tilde{s}, \mathbf{y}, s) = f(\widetilde{\mathbf{y}}|\tilde{s}, \mathbf{y})$. Hamilton's (1989) formulation is suitable particularly when the state vector includes the policy rate to allow the dynamics of macroeconomic variables to depend on the lagged policy rate, in a spirit similar to Ang, Piazzesi, and Wei (2006) and Hördahl, Tristani, and Vestin (2006).

Assumption 1: $f(\tilde{\mathbf{y}}|\tilde{s},\mathbf{y},s) = f(\tilde{\mathbf{y}}|\tilde{s},\mathbf{y})$**, and** $\tilde{\mathbf{y}}|\tilde{s},\mathbf{y} \sim N\left(\boldsymbol{\mu}(\tilde{s},\mathbf{y}), \boldsymbol{\Sigma}(\tilde{s})\boldsymbol{\Sigma}(\tilde{s})'\right)$ **under** $\mathbb{P}$**.**

Assumption 2 follows DSY's assumption on the Radon-Nikodym derivative or equivalently that on the pricing kernel ($\mathcal{M}$) that accommodates both regime shift and factor risks. $\boldsymbol{\lambda}(\tilde{s},\mathbf{y})$ and $\gamma(\tilde{s},\mathbf{y},s)$ are the prices-of-risk and regime-shift-risk coefficients, respectively.

Assumption 2: $\mathcal{M}(\tilde{\mathbf{y}},\tilde{s},\mathbf{y},s) = \exp\left[\begin{array}{c} -r(\mathbf{y}) - \gamma(\tilde{s},\mathbf{y},s) - \frac{1}{2}\boldsymbol{\lambda}(\tilde{s},\mathbf{y})'\boldsymbol{\lambda}(\tilde{s},\mathbf{y}) \\ -\boldsymbol{\lambda}(\tilde{s},\mathbf{y})'\boldsymbol{\Sigma}(\tilde{s})^{-1}(\tilde{\mathbf{y}} - \boldsymbol{\mu}(\tilde{s},\mathbf{y})) \end{array}\right]$**.**

No arbitrage requires that

$$P_{n+1}(\mathbf{y},s) = \sum_{\tilde{s}} \rho(\tilde{s}|\mathbf{y},s) E\left[\mathcal{M}(\tilde{\mathbf{y}},\tilde{s},\mathbf{y},s) P_n(\tilde{\mathbf{y}},\tilde{s}) |\tilde{s},\mathbf{y},s\right]. \tag{2}$$

For $n=0$, Since $\exp(-r(\mathbf{y})) = P_1(\mathbf{y},s)$ and $P_0 = 1$, equation (2) becomes

$$\begin{aligned} \exp[-r(\mathbf{y})] &= \sum_{\tilde{s}} \rho(\tilde{s}|\mathbf{y},s) E\left[\mathcal{M}(\tilde{\mathbf{y}},\tilde{s},\mathbf{y},s) |\tilde{s},\mathbf{y},s\right], \\ &= \sum_{\tilde{s}} \rho(\tilde{s}|\mathbf{y},s) \exp\left[-r(\mathbf{y}) - \gamma(\tilde{s},\mathbf{y},s)\right], \end{aligned}$$

where that the second equality holds since

$$E\left[\mathcal{M}(\tilde{\mathbf{y}},\tilde{s},\mathbf{y},s) |\tilde{s},\mathbf{y},s\right] = \exp\left[-r(\mathbf{y}) - \gamma(\tilde{s},\mathbf{y},s)\right]. \tag{3}$$

Thus,

$$\sum_{\tilde{s}} \rho(\tilde{s}|\mathbf{y},s) \exp\left[-\gamma(\tilde{s},\mathbf{y},s)\right] = 1. \tag{4}$$

Two interesting choices for $\gamma(\tilde{s},\mathbf{y},s)$ may be (i) $\gamma(\tilde{s},\mathbf{y},s) = 0$, i.e., the regime-shift risk is not priced and (ii) $\rho(\tilde{s}|\mathbf{y},s)\exp[-\gamma(\tilde{s},\mathbf{y},s)]$ does not depend on $\mathbf{y}$ and its sum over $\tilde{s}$ is 1, i.e., the regime-shift risk is "fully priced."

2.2 From $\mathbb{P}$ to $\mathbb{Q}$

This subsection provides two propositions to link measures $\mathbb{P}$ to $\mathbb{Q}$ by finding a conditional density $f^Q(\widetilde{\mathbf{y}}|\tilde{s},\mathbf{y},s)$ and a transition matrix $\rho^Q(\tilde{s}|\mathbf{y},s)$, such that

$$\exp[-r(\mathbf{y})]\underbrace{f^Q(\widetilde{\mathbf{y}}|\tilde{s},\mathbf{y},s)\,\rho^Q(\tilde{s}|\mathbf{y},s)}_{=p^Q(\widetilde{\mathbf{y}},\tilde{s}|\mathbf{y},s)} = \mathcal{M}(\widetilde{\mathbf{y}},\tilde{s},\mathbf{y},s)\underbrace{f(\widetilde{\mathbf{y}}|\tilde{s},\mathbf{y})\,\rho(\tilde{s}|\mathbf{y},s)}_{=p(\widetilde{\mathbf{y}},\tilde{s}|\mathbf{y},s)}, \tag{5}$$

where the Radon-Nikodym derivative is given by $1/[\exp[r(\mathbf{y})]\,\mathcal{M}(\widetilde{\mathbf{y}},\tilde{s},\mathbf{y},s)]$. For any random variable X, define $E^Q(X|\mathbf{y},s)$ by

$$E^Q(X|\mathbf{y},s) \equiv \sum_{\tilde{s}}\rho^Q(\tilde{s}|\mathbf{y},s)\,E^Q(X|\tilde{s},\mathbf{y},s)\,,\ \text{ where } E^Q(X|\tilde{s},\mathbf{y},s) \equiv \int_{\tilde{\mathbf{y}}} X f^Q(\tilde{\mathbf{y}}|\tilde{s},\mathbf{y},s)\,d\tilde{\mathbf{y}}.$$

Then $E^Q(\exp[-r(\mathbf{y})]\,X|\mathbf{y},s) = E(\mathcal{M}X|\mathbf{y},s)$ for f^Q and ρ^Q satisfying equation (5). In particular, the no-arbitrage condition (2) can be written as

$$P_{n+1}(\mathbf{y},s) = \sum_{\tilde{s}}\rho^Q(\tilde{s}|\mathbf{y},s)\,E^Q[\exp[-r(\mathbf{y})]\,P_n(\tilde{\mathbf{y}},\tilde{s})\,|\tilde{s},\mathbf{y},s]\,. \tag{6}$$

Proposition 1 *Under Assumptions 1 and 2, the transition probability $\rho^Q(\tilde{s}|\mathbf{y},s)$ is given by $\rho^Q(\tilde{s}|\mathbf{y},s) = \rho(\tilde{s}|\mathbf{y},s)\exp[-\gamma(\tilde{s},\mathbf{y},s)]$.*

Proof. Pin down ρ^Q. Integrate both sides of (5) over $\widetilde{\mathbf{y}}$.

$$\begin{aligned} LHS &= \int_{\tilde{\mathbf{y}}}\exp[-r(\mathbf{y})]\,f^Q(\widetilde{\mathbf{y}}|\tilde{s},\mathbf{y},s)\,\rho^Q(\tilde{s}|\mathbf{y},s)\,d\widetilde{\mathbf{y}}, \\ &= \exp[-r(\mathbf{y})]\,\rho^Q(\tilde{s}|\mathbf{y},s)\int_{\tilde{\mathbf{y}}} f^Q(\widetilde{\mathbf{y}}|\tilde{s},\mathbf{y},s)\,d\widetilde{\mathbf{y}}, \\ &= \exp[-r(\mathbf{y})]\,\rho^Q(\tilde{s}|\mathbf{y},s) \quad (\text{since } \int_{\tilde{\mathbf{y}}} f^Q(\widetilde{\mathbf{y}}|\tilde{s},\mathbf{y},s)\,d\widetilde{\mathbf{y}} = 1). \end{aligned}$$

$$
\begin{aligned}
RHS &= \int_{\widetilde{\mathbf{y}}} \mathcal{M}\left(\widetilde{\mathbf{y}}, \tilde{s}, \mathbf{y}, s\right) f\left(\widetilde{\mathbf{y}}|\tilde{s}, \mathbf{y}\right) \rho\left(\tilde{s}|\mathbf{y}, s\right) d\widetilde{\mathbf{y}}, \\
&= \rho\left(\tilde{s}|\mathbf{y}, s\right) \int_{\widetilde{\mathbf{y}}} \mathcal{M}\left(\widetilde{\mathbf{y}}, \tilde{s}, \mathbf{y}, s\right) f\left(\widetilde{\mathbf{y}}|\tilde{s}, \mathbf{y}\right) d\widetilde{\mathbf{y}}, \\
&= \rho\left(\tilde{s}|\mathbf{y}, s\right) E\left[\mathcal{M}\left(\widetilde{\mathbf{y}}, \tilde{s}, \mathbf{y}, s\right)|\tilde{s}, \mathbf{y}, s\right], \\
&= \rho\left(\tilde{s}|\mathbf{y}, s\right) \exp\left[-r\left(\mathbf{y}\right) - \gamma\left(\tilde{s}, \mathbf{y}, s\right)\right], \quad \text{(by equation (3))} \\
&= \exp\left[-r\left(\mathbf{y}\right)\right] \rho\left(\tilde{s}|\mathbf{y}, s\right) \exp\left[-\gamma\left(\tilde{s}, \mathbf{y}, s\right)\right].
\end{aligned}
$$

Q.E.D. ■

Proposition 2 *The conditional density* $f^Q\left(\widetilde{\mathbf{y}}|\tilde{s}, \mathbf{y}, s\right)$ *does not depend on* s *and is the density of* $N\left(\boldsymbol{\mu}^Q\left(\tilde{s}, \mathbf{y}\right), \boldsymbol{\Sigma}\left(\tilde{s}\right)\boldsymbol{\Sigma}\left(\tilde{s}\right)'\right)$ *with* $\boldsymbol{\mu}^Q\left(\tilde{s}, \mathbf{y}\right) \equiv \boldsymbol{\mu}\left(\tilde{s}, \mathbf{y}\right) - \boldsymbol{\Sigma}\left(\tilde{s}\right)\boldsymbol{\lambda}\left(\tilde{s}, \mathbf{y}\right)$.

Proof. Pin down f^Q. Divide both sides of equation (5) by the expression of ρ^Q in Proposition 1 and use Assumption 2 to obtain

$$
f^Q\left(\widetilde{\mathbf{y}}|\tilde{s}, \mathbf{y}, s\right) = \exp\left[-\frac{1}{2}\boldsymbol{\lambda}\left(\tilde{s}, \mathbf{y}\right)'\boldsymbol{\lambda}\left(\tilde{s}, \mathbf{y}\right) - \boldsymbol{\lambda}\left(\tilde{s}, \mathbf{y}\right)'\Sigma\left(\tilde{s}\right)^{-1}\left(\widetilde{\mathbf{y}} - \boldsymbol{\mu}\left(\tilde{s}, \mathbf{y}\right)\right)\right] \times f\left(\widetilde{\mathbf{y}}|\tilde{s}, \mathbf{y}\right).
$$

So the conditional moment-generating function of $\tilde{\mathbf{y}}$ under $\mathbb{Q}$ can be written as

$$
E^Q\left[\exp\left(\zeta'\widetilde{\mathbf{y}}\right)|\tilde{s}, \mathbf{y}, s\right] = \int_{\tilde{\mathbf{y}}} \exp\left(\zeta'\widetilde{\mathbf{y}}\right) f^Q\left(\widetilde{\mathbf{y}}|\tilde{s}, \mathbf{y}\right) d\tilde{\mathbf{y}} = \int_{\tilde{\mathbf{y}}} \exp\left(X\right) f\left(\widetilde{\mathbf{y}}|\tilde{s}, \mathbf{y}\right) d\widetilde{\mathbf{y}} = E\left[\exp\left(X\right)|\tilde{s}, \mathbf{y}\right],
$$

where $X \equiv \zeta'\widetilde{\mathbf{y}} - \frac{1}{2}\boldsymbol{\lambda}'\boldsymbol{\lambda} - \boldsymbol{\lambda}'\boldsymbol{\Sigma}^{-1}\left(\widetilde{\mathbf{y}} - \boldsymbol{\mu}\right)$ with $\boldsymbol{\mu}$ here being $\boldsymbol{\mu}\left(\tilde{s}, \mathbf{y}\right)$, $\boldsymbol{\lambda}$ being $\boldsymbol{\lambda}\left(\tilde{s}, \mathbf{y}\right)$, and $\boldsymbol{\Sigma}$ being $\boldsymbol{\Sigma}\left(\tilde{s}\right)$. Since $\widetilde{\mathbf{y}}|\tilde{s}, \mathbf{y} \sim N\left(\boldsymbol{\mu}, \boldsymbol{\Sigma}\boldsymbol{\Sigma}'\right)$ under $\mathbb{P}$ by Assumption 1, the conditional distribution of X is normal with

$$
E\left(X|\tilde{s}, \mathbf{y}\right) = \zeta'\boldsymbol{\mu} - \frac{1}{2}\boldsymbol{\lambda}'\boldsymbol{\lambda}, \; Var\left(X|\tilde{s}, \mathbf{y}\right) = \left(\zeta - \boldsymbol{\Sigma}^{-1\prime}\boldsymbol{\lambda}\right)\boldsymbol{\Sigma}\boldsymbol{\Sigma}'\left(\zeta - \boldsymbol{\Sigma}^{-1\prime}\boldsymbol{\lambda}\right).
$$

Thus

$$E\left[\exp\left(X\right)|\tilde{s},\mathbf{y}\right]=\exp\left[E\left(X|\tilde{s},\mathbf{y}\right)+\frac{1}{2}\left(X|\tilde{s},\mathbf{y}\right)\right]=\exp\left[\zeta'\left(\boldsymbol{\mu}-\boldsymbol{\Sigma\lambda}\right)+\frac{1}{2}\zeta\boldsymbol{\Sigma\Sigma}'\zeta\right],$$

which is the moment-generating function of a normal random variable with mean $\mu-\Sigma\lambda$ and variance $\Sigma\Sigma'$. Q.E.D. ■

2.3 Pricing under $\mathbb{Q}$

The no-arbitrage condition under $\mathbb{Q}$ can be rewritten as

$$1=\sum_{\tilde{s}}\rho^{Q}\left(\tilde{s}|\mathbf{y},s\right)E^{Q}\left\{\exp\left(\tilde{h}_{t+1}\right)|\tilde{s},\mathbf{y},s\right\}, \tag{7}$$

where

$$\begin{aligned}\tilde{h}_{t+1} &\equiv p_{n}\left(\tilde{\mathbf{y}},\tilde{s}\right)-p_{n+1}\left(\mathbf{y},s\right)-r\left(\mathbf{y}\right),\\ p_{n} &\equiv \log\left(P_{n}\right).\end{aligned}$$

The term $\tilde{h}_{t+1}$ is the log excess one-period return on $n+1$ period bonds.

If $\tilde{h}_{t+1}$ is conditionally normally distributed given $(\tilde{s},\mathbf{y},s)$ under $\mathbb{Q}$, then equation (7) becomes

$$1=\sum_{\tilde{s}}\rho^{Q}\left(\tilde{s}|\mathbf{y},s\right)\exp\left[E^{Q}\left(\tilde{h}_{t+1}|\tilde{s},\mathbf{y},s\right)+\frac{1}{2}Var^{Q}\left(\tilde{h}_{t+1}|\tilde{s},\mathbf{y},s\right)\right]. \tag{8}$$

Furthermore, by applying the approximation used by Bansal and Zhou (2002) and Hamilton and Wu (2012) (i.e., $\exp(x)\approx 1+x$), equation (8) becomes

$$0\approx\sum_{\tilde{s}}\rho^{Q}\left(\tilde{s}|\mathbf{y},s\right)\left[E^{Q}\left(\tilde{h}_{t+1}|\tilde{s},\mathbf{y},s\right)+\frac{1}{2}Var^{Q}\left(\tilde{h}_{t+1}|\tilde{s},\mathbf{y},s\right)\right]. \tag{9}$$

In order to solve bond prices, I assume two additional assumptions that are commonly assumed in the existing literature. Assumption 3 assumes that the factors that explain the yield curves follow VAR(1) under $\mathbb{Q}$.

Assumption 3:

$$E^Q\left(\widetilde{\mathbf{y}}|\tilde{s},\mathbf{y},s\right)\equiv\boldsymbol{\mu}^Q\left(\tilde{s},\mathbf{y}\right)=\mathbf{c}^Q\left(\tilde{s}\right)+\mathbf{\Phi}^Q\left(\tilde{s}\right)\mathbf{y}.$$

Assumption 4 assumes that the transition probabilities under $\mathbb{Q}$ are constant, i.e., $\rho\left(\tilde{s}|\mathbf{y},s\right)\exp\left[-\gamma\left(\tilde{s},\mathbf{y},s\right)\right]$ in equation (4) does not depend on $\mathbf{y}$ and its sum over $\tilde{s}$ is 1, so that the regime-shift risk is "fully priced." I discuss the case without Assumption 4 in Appendix A.

Assumption 4:

$$\rho^Q\left(\tilde{s}|\mathbf{y},s\right)=\rho^Q\left(\tilde{s}|s\right).$$

To solve bond prices, I first conjecture that

$$p_n\left(\widetilde{\mathbf{y}},\tilde{s}\right)=-a_n\left(\tilde{s}\right)-\mathbf{b}_n\left(\tilde{s}\right)\tilde{y}.$$

Since $\widetilde{\mathbf{y}}|\tilde{s},\mathbf{y},s\sim N\left(\mu\left(\tilde{s},\mathbf{y}\right),\Sigma\left(\tilde{s}\right)\Sigma\left(\tilde{s}\right)'\right)$ under $\mathbb{Q}$, the log excess return $\tilde{h}_{t+1}$ is conditionally normal, as required above, with

$$\begin{aligned}
E^Q\left(\tilde{h}_{t+1}|\tilde{s},\mathbf{y},s\right) &= -a_n\left(\tilde{s}\right)-\mathbf{b}_n\left(\tilde{s}\right)\left[\mathbf{c}^Q\left(\tilde{s}\right)+\mathbf{\Phi}^Q\left(\tilde{s}\right)\mathbf{y}\right]+a_{n+1}\left(s\right)+\mathbf{b}_{n+1}\left(s\right)\mathbf{y}-r\left(\mathbf{y}\right),\\
Var^Q\left(\tilde{h}_{t+1}|\tilde{s},\mathbf{y},s\right) &= \mathbf{b}_n\left(\tilde{s}\right)\mathbf{\Sigma}\left(\tilde{s}\right)\mathbf{\Sigma}\left(\tilde{s}\right)'_n\mathbf{b}\left(\tilde{s}\right)'.
\end{aligned}$$

Substituting these equations into (9) yields

$$\begin{aligned}
0 &\approx -\left[\sum_{\tilde{s}}\rho^Q\left(\tilde{s}|s\right)\left(a_n\left(\tilde{s}\right)+\mathbf{b}_n\left(\tilde{s}\right)\mathbf{c}^Q\left(\tilde{s}\right)-\frac{1}{2}\mathbf{b}_n\left(\tilde{s}\right)\mathbf{\Sigma}\left(\tilde{s}\right)\mathbf{\Sigma}\left(\tilde{s}\right)'\mathbf{b}_n\left(\tilde{s}\right)'\right)-a_{n+1}\left(s\right)\right]\\
&\quad-\sum_{\tilde{s}}\rho^Q\left(\tilde{s}|s\right)\left[\mathbf{b}_n\left(\tilde{s}\right)\mathbf{\Phi}^Q\left(\tilde{s}\right)+\mathbf{e}_1'-\mathbf{b}_{n+1}\left(s\right)\right]\mathbf{y},
\end{aligned}$$

where $\mathbf{e}_1$ is a vector of zeros with 1st element being one (thus $r(\mathbf{y}) = \mathbf{e}_1'\mathbf{y}$). Since this has to hold for any $\mathbf{y}$, I obtain the recursion

$$\begin{aligned} a_{n+1}(s) &= \sum_{\tilde{s}} \rho^Q(\tilde{s}|s) \left(a_n(\tilde{s}) + \mathbf{b}_n(\tilde{s}) \mathbf{c}^Q(\tilde{s}) - \frac{1}{2} \mathbf{b}_n(\tilde{s}) \boldsymbol{\Sigma}(\tilde{s}) \boldsymbol{\Sigma}(\tilde{s})_n' \mathbf{b}(\tilde{s})' \right), \\ \mathbf{b}_{n+1}(s) &= \sum_{\tilde{s}} \rho^Q(\tilde{s}|s) \left(\mathbf{b}_n(\tilde{s}) \boldsymbol{\Phi}^Q(\tilde{s}) + \mathbf{e}_1' \right). \end{aligned}$$

The initial condition is $a_0(s) = 0$ and $\mathbf{b}_0(s) = 0$ for all s.

If I assume, as in DSY that $\boldsymbol{\Phi}^Q$ does not depend on the regime s, then it is not necessary to invoke the "$\exp(x) \approx 1 + x$" approximation. In this case, $\mathbf{b}_n$ does not depend on the regime and the recursion becomes

$$\begin{aligned} a_{n+1}(s) &= \log \left[\sum_{\tilde{s}} \rho^Q(\tilde{s}|s) \left(a_n(\tilde{s}) + \mathbf{b}_n(\tilde{s}) \mathbf{c}^Q(\tilde{s}) - \frac{1}{2} \mathbf{b}_n(\tilde{s}) \boldsymbol{\Sigma}(\tilde{s}) \boldsymbol{\Sigma}(\tilde{s})_n' \mathbf{b}(\tilde{s})' \right) \right], \\ \mathbf{b}_{n+1} &= \mathbf{b}_n \boldsymbol{\Phi}^Q + \mathbf{e}_1'. \end{aligned}$$

If I assume VAR(1) under $\mathbb{P}$, i.e., $\boldsymbol{\mu}(\tilde{s}, \mathbf{y}) = \mathbf{c}(\tilde{s}) + \boldsymbol{\Phi}(\tilde{s})\mathbf{y})$ and the prices of risk is affine in $\mathbf{y}$, i.e., $\boldsymbol{\lambda}(\tilde{s}, \mathbf{y}) = \boldsymbol{\lambda}_0(\tilde{s}) + \boldsymbol{\Lambda}_1(\tilde{s})\mathbf{y}$, by Proposition 2 and Assumption 3, $\mathbf{c}^Q(\tilde{s})$ and $\boldsymbol{\Phi}^Q(\tilde{s})$ can be expressed with the prices of risk coefficients

$$\mathbf{c}^Q(\tilde{s}) = \mathbf{c}(\tilde{s}) - \boldsymbol{\Sigma}(\tilde{s}) \boldsymbol{\lambda}_0(\tilde{s}), \ \boldsymbol{\Phi}^Q(\tilde{s}) = \boldsymbol{\Phi}(\tilde{s}) - \boldsymbol{\Sigma}(\tilde{s}) \boldsymbol{\Lambda}_1(\tilde{s}). \tag{10}$$

3 An application to JGB yields

In this section, I specify factor dynamics that correspond to $f(\tilde{\mathbf{y}}|\tilde{s}, \mathbf{y})$ in the $\mathbb{P}$ model and then endogenously derive the corresponding state-dependent transition probabilities $\rho(\tilde{s}|\mathbf{y}, s)$. The specified factor dynamics are similar to those in Hayashi and Koeda's (2012) VAR model. This study's VAR(1) model comprises three variables (i) the policy interest rate, (ii) inflation, and (iii) output gaps, with endogenous monetary policy regime shifts. Its main difference from Hayashi and Koeda's model is that it includes fewer variables in the VAR system, and it does not assume any a priori identification restrictions in the VAR system. As a result, uncertainty about current macroeconomic variables affects the current regime determination. The VAR explicitly models the key features of recent monetary policies in Japan, such as the zero lower bound of the policy interest rate and the duration of the zero interest rate policy (ZIRP).

3.1 Factor dynamics

The monetary policy regime (s) can be either normal (P) or a ZIRP (Z). I partition $\mathbf{y}_t$ as

$$\underset{3\times 1}{\mathbf{y}_t} = \begin{bmatrix} r_t \\ \underset{2\times 1}{\mathbf{y}_{2t}} \end{bmatrix},$$

where r is the policy interest rate (short rate) and $\mathbf{y}_2$ is a vector of macroeconomic variables (inflation and output gap).

The policy interest rate follows a regime-dependent Taylor rule

$$r_t = \underset{(1\times 1)}{\alpha^{s_t}} + \underset{(1\times 2)}{\boldsymbol{\beta}^{s_t\prime}} \mathbf{y}_{2t} + \underset{(1\times 1)}{\delta^{s_t}} r_{t-1} + \underset{(1\times 1)}{\sigma_r^{s_t}} u_{r,t}, \quad u_{r,t} \sim N(0,1), \tag{11}$$

where the ZIRP regime can be represented as

$$\alpha^Z \approx 0,\ \boldsymbol{\beta}^Z = \mathbf{0},\ \delta^Z = 0,\ \sigma_r^Z \approx 0.$$

The rest of system is also regime dependent

$$\mathbf{y}_{2,t} = \underset{(2\times 1)}{\mathbf{c}_2^{s_t}} + \underset{(2\times 3)}{\boldsymbol{\Phi}_2^{s_t}} \mathbf{y}_{t-1} + \underset{(2\times 2)}{\boldsymbol{\Sigma}_{22}^{s_t}} \mathbf{u}_{2t}, \quad \mathbf{u}_{2t} \sim N(\mathbf{0}, \mathbf{I}). \tag{12}$$

All shocks are jointly standard normal and independent to each other and over time. Substituting (12) in the short rate equation yields

$$r_t = \alpha^{s_t} + \boldsymbol{\beta}^{s_t\prime}\mathbf{c}_2^{s_t} + \left[\boldsymbol{\beta}^{s_t\prime}\boldsymbol{\Phi}_2^{s_t} + [\delta^{s_t}, 0, 0, 0]\right]\mathbf{y}_{t-1} + \boldsymbol{\beta}^{s_t\prime}\boldsymbol{\Sigma}_{22}^{s_t}\mathbf{u}_{2t} + \sigma_r^{s_t}u_{r,t}.$$

Stacking the above equation over (12) results in the VAR

$$\mathbf{y}_t = \underset{(3\times1)}{\mathbf{c}^{s_t}} + \underset{(3\times3)}{\boldsymbol{\Phi}^{s_t}}\mathbf{y}_{t-1} + \underset{(3\times3)}{\boldsymbol{\Sigma}^{s_t}}\mathbf{u}_t, \quad \mathbf{u}_t \sim N(\mathbf{0}, \mathbf{I}), \tag{13}$$

where

$$\underset{(3\times1)}{\mathbf{c}^{s_t}} = \begin{bmatrix} \alpha^{s_t} + \boldsymbol{\beta}^{s_t\prime}\mathbf{c}_2^{s_t} \\ \underset{(2\times1)}{\mathbf{c}_2^{s_t}} \end{bmatrix}, \quad \underset{(3\times3)}{\boldsymbol{\Phi}^{s_t}} = \begin{bmatrix} \boldsymbol{\beta}^{s_t\prime}\boldsymbol{\Phi}_2^{s_t} + [\delta^{s_t}, 0, 0, 0] \\ \underset{(2\times3)}{\boldsymbol{\Phi}_2^{s_t}} \end{bmatrix}, \quad \underset{(3\times3)}{\boldsymbol{\Sigma}^{s_t}} = \begin{bmatrix} \sigma_r^{s_t} & \boldsymbol{\beta}^{s_t\prime}\boldsymbol{\Sigma}_{22}^{s_t} \\ \mathbf{0} & \boldsymbol{\Sigma}_{22}^{s_t} \end{bmatrix}.$$

Equation (13) is a restricted VAR because the first row of $\boldsymbol{\Phi}^Z$ is a vector of zeros and the six elements of $\boldsymbol{\Sigma}^Z$ are a function of four parameters.

3.2 Regime determination

I first consider a simple regime evolution that depends solely on the level of the Taylor-rule-based (TRB) policy rate (Type I evolution). If the Taylor-rule based policy hits the lower bound of interest rate (ZLB), the policy rate is set at the bound under the ZIRP regime ($s = Z$), otherwise the policy rate is set by the Taylor rule under the normal regime ($s = P$).

Type I evolution, however, does not take into account a key feature of the ZIRP in Japan: *Jikan Jiku,* the duration of the ZIRP, committed to by the Bank of Japan. Thus I extend the Type I evolution by introducing a *Jikan Jiku* policy under which the ZIRP regime is maintained unless the expected year-on-year core inflation exceeds a certain level

(Type II evolution). Such a level can be interpreted as the exit condition based on core inflation rate ($\bar{\pi}$), a parameter that must be estimated since the Bank of Japan did not commit a specific rate during the investigated period.

For notational convenience,

$$\begin{aligned} r^e\left(\mathbf{y}_{t-1}\right) &\equiv \alpha^P + \boldsymbol{\beta}^{P\prime}\left[\mathbf{c}_2^P + \boldsymbol{\Phi}_2^P \mathbf{y}_{t-1}\right] + \delta^P r_{t-1}, \\ \pi^e\left(\mathbf{y}_{t-1}\right) &\equiv [1,0]\left[\mathbf{c}_2^P + \boldsymbol{\Phi}_2^P \mathbf{y}_{t-1}\right]. \end{aligned}$$

3.2.1 Evolution with the zero rate bound: Type I evolution

Under Type I evolution, the regime is a ZIRP if and only if the TRB rate hits the ZLB; otherwise it is normal. The corresponding transition probabilities are

$$\begin{aligned} \Pr\left(s_t = P|\mathbf{y}_{t-1}, s_{t-1}\right) &= \Pr\left(\underbrace{\alpha^P + \boldsymbol{\beta}^{P\prime}\mathbf{y}_{2,t} + \delta^P r_{t-1} + \sigma_r^P u_{r,t}}_{\text{TRB rate}_t} > \underbrace{\alpha^Z + \sigma_r^Z u_{r,t}}_{\text{ZLB}_t}\middle| \mathbf{y}_{t-1}, s_{t-1}\right), \\ &= \Pr\left(\left\{\begin{array}{c} \alpha^P + \boldsymbol{\beta}^{P\prime}\left(\mathbf{c}_2^P + \boldsymbol{\Phi}_2^P \mathbf{y}_{t-1} + \boldsymbol{\Sigma}_{22}^P \mathbf{u}_{2t}\right) \\ +\delta^P r_{t-1} + \sigma_r^P u_{r,t} \end{array}\right\} > \alpha^Z + \sigma_r^Z u_{r,t}\right), \\ &= \Pr\left(r^e\left(\mathbf{y}_{t-1}\right) > \alpha^Z - \xi_t\right), \\ \Pr\left(s_t = Z|\mathbf{y}_{t-1}, s_{t-1}\right) &= \Pr\left(r^e\left(\mathbf{y}_{t-1}\right) \leqq \alpha^Z - \xi_t\right), \end{aligned}$$

where $\xi_t \equiv \boldsymbol{\beta}^{P\prime}\boldsymbol{\Sigma}_{22}^P \mathbf{u}_{2t} + \left[\sigma_r^P - \sigma_r^Z\right] u_{r,t}$. Since u_t and $\mathbf{u}_{2t}$ are normal and independent

$$\xi_t|\mathbf{y}_{t-1}, s_{t-1} \sim N\left(0, \sigma_\xi^2\right),\ \sigma_\xi^2 \equiv \left[\sigma_r^P - \sigma_r^Z\right]^2 + \boldsymbol{\beta}^{P\prime}\boldsymbol{\Sigma}_{22}^P \boldsymbol{\Sigma}_{22}^{P\prime}\boldsymbol{\beta}^P.$$

The Type I transition probabilities can be rewritten as

$$\Pr\left(s_t = P|\mathbf{y}_{t-1}, s_{t-1}\right) = \Pr\left(s_t = P|\mathbf{y}_{t-1}\right) = F\left(\frac{r^e\left(\mathbf{y}_{t-1}\right) - \alpha^Z}{\sigma_\xi}\right), \tag{14}$$

$$\Pr\left(s_t = Z|\mathbf{y}_{t-1}, s_{t-1}\right) = \Pr\left(s_t = Z|\mathbf{y}_{t-1}\right) = 1 - F\left(\frac{r^e\left(\mathbf{y}_{t-1}\right) - \alpha^Z}{\sigma_\xi}\right), \tag{15}$$

where $F(.)$ is the cumulative distribution function of $N(0,1)$. These transition probabilities do not depend on s_{t-1}.

3.2.2 Policy duration: Type II evolution

How does the regime evolution change when the *Jikan Jiku* policy is introduced? Now, the determination of the regime depends on the inflation rate relative to $\bar{\pi}$, in addition to the TRB policy rate.

If the previous regime is normal, then the transition probabilities are unchanged (i.e., equations (14) and (15)). On the other hand, if the previous regime is a ZIRP, the probability of it returning to normal is

$$\begin{aligned}
& \Pr\left(s_t = P|\mathbf{y}_{t-1}, s_{t-1} = Z\right) \\
= & \Pr\left(\left.\underbrace{\alpha^P + \boldsymbol{\beta}^{P\prime}\mathbf{y}_{2,t} + \delta^P r_{t-1} + \sigma_r^P u_{r,t}}_{\text{TRB rate}_t} > \underbrace{\alpha^Z + \sigma_r^Z u_{r,t}}_{\text{ZLB}_t}, \pi_t > \bar{\pi}\right| \mathbf{y}_{t-1}, s_{t-1} = Z\right), \\
= & \Pr\left(r^e\left(\mathbf{y}_{t-1}\right) > \alpha^Z - \xi_t, \pi^e\left(\mathbf{y}_{t-1}\right) > \bar{\pi} - \sigma_\pi u_{\pi,t}\right). \qquad (16)
\end{aligned}$$

where $u_{\pi,t}$ and σ_π are the shock and volatility parameters, respectively, calculated in the inflation equation as $u_{\pi,t} = [1,0]\,\mathbf{u}_{2t}$ and $\sigma_\pi = [1,0]\,\boldsymbol{\Sigma}_{22}^P\boldsymbol{\Sigma}_{22}^{P\prime}\,[1,0]'$.

The Type II transition probabilities can be rewritten as

$$\Pr\left(s_t = P|\mathbf{y}_{t-1}, s_{t-1} = Z\right) = B\left(r^e\left(\mathbf{y}_{t-1}\right) - \alpha^Z, \pi^e\left(\mathbf{y}_{t-1}\right) - \bar{\pi}; \mathbf{W}\right), \qquad (17)$$

$$\Pr\left(s_t = Z|\mathbf{y}_{t-1}, s_{t-1} = Z\right) = 1 - B\left(r^e\left(\mathbf{y}_{t-1}\right) - \alpha^Z, \pi^e\left(\mathbf{y}_{t-1}\right) - \bar{\pi}; \mathbf{W}\right), \qquad (18)$$

$$\Pr\left(s_t = P|\mathbf{y}_{t-1}, s_{t-1} = P\right) = F\left(\frac{r^e\left(\mathbf{y}_{t-1}\right) - \alpha^Z}{\sigma_\xi}\right), \qquad (19)$$

$$\Pr\left(s_t = Z|\mathbf{y}_{t-1}, s_{t-1} = P\right) = 1 - F\left(\frac{r^e\left(\mathbf{y}_{t-1}\right) - \alpha^Z}{\sigma_\xi}\right), \qquad (20)$$

where $B(a, b; \mathbf{W})$ is the cumulative distribution function of the bivariate normal distribution with mean zero and the variance covariance matrix of $(\xi_t, u_{\pi,t})$. The variance covariance matrix, which is denoted as $\mathbf{W}$, is a known function of $\left(\sigma_r^P, \sigma_r^Z, \boldsymbol{\beta}^P, \boldsymbol{\Sigma}_{22}^P\right)$

$$\mathbf{W} = \begin{bmatrix} \sigma_\xi^2 & \boldsymbol{\beta}^{P\prime}\boldsymbol{\Sigma}_{22}^P\boldsymbol{\Sigma}_{22}^{P\prime}\begin{bmatrix} 1 \\ 0 \end{bmatrix} \\ \boldsymbol{\beta}^{P\prime}\boldsymbol{\Sigma}_{22}^P\boldsymbol{\Sigma}_{22}^{P\prime}\begin{bmatrix} 1 \\ 0 \end{bmatrix} & \begin{bmatrix} 1 & 0 \end{bmatrix}\boldsymbol{\Sigma}_{22}^P\boldsymbol{\Sigma}_{22}^{P\prime}\begin{bmatrix} 1 \\ 0 \end{bmatrix} \end{bmatrix}.$$

4 Estimating JGB yield curves

4.1 Data

I use quarterly data on interest rates and the macro variables of inflation and output gap from 1985Q1 to 2008Q2. I use quarterly data because it may reflect Japan's overall economic activity more precisely than readily available monthly real activity measures, such as, industrial production, unemployment, and machinery orders. The sample period starts in 1985Q1 because reliable zero coupon bond yield data are available from that quarter; it ends in 2008Q2, the period prior to the Lehman shock.

The uncollaterized overnight call rate[4] is used for the short-term interest rate. Zero coupon bond yields of 4, 12, 20, and 40 quarter maturities are used for longer maturities. These bond yields are obtained from Wright's (2011) dataset and are the end of period rates expressed at annualized rates in percent.

Regarding the macro variables, inflation is measured by the percentage change in the Consumer Price Index, excluding fresh food, from the same quarter in the previous, ob-

[4] I use the average rate of the last month in each quarter to remove end-of-month fluctuations in the call rate.

tained from the Ministry of Internal Affairs and Communications[5]; real activity is measured by output gaps estimated by applying the Hodrick-Prescott filter to the logs of the seasonally adjusted GDP at 2000 prices, obtained from the Japanese Cabinet Office. Output gaps are expressed in percentage points.

The regime series is constructed based on public statements by the end of each quarter. One issue is how to differentiate the normal from the ZIRP regime; the regime can be normal even if the short rate is almost zero if the Taylor-rule-based interest rate indicates it. I thus identify the ZIRP regime with the *Jikan Jiku* policy within Bank of Japan's public statements. This implies that the period from March to June 2006, when the targeted rate was zero in the absence of a *Jikan Jiku* policy, was under the "normal" regime.

4.2 Estimation strategy

The model consists of macro dynamics and static yield equations. The macro dynamics are summarized by equation (13) and the static yield equations are

$$\underset{4\times1}{\mathbf{z}_t} = \underset{4\times1}{A^{s_t}} + \underset{4\times3}{\mathbf{B}^{s_t}}\mathbf{y}_t + \underset{4\times1}{\mathbf{v}_t},$$

where $\mathbf{z}_t = [r_t^4, r_t^{12}, r_t^{20}, r_t^{40}]'$ is a 4×1 vector of bond yields with maturities corresponding to the superscript numbers (in quarters). The yield equations are an affine function of the state variables with 4×1 coefficient vectors A and a 4×3 coefficient matrix $\mathbf{B}$ corresponding to (i) a constant, (ii) the short-term interest rate, and (iii) the macro variables, respectively. The subscript numbers in A and $\mathbf{B}$ correspond to maturities, that is,

[5] I use the 2000-base CPI up to mid-2006 since policy decisions were not made based on the 2005-base CPI up to that point.

$A^{s_t} = \left[\frac{a_4(s_t)}{4}, \frac{a_{12}(s_t)}{12}, \frac{a_{20}(s_t)}{20}, \frac{a_{40}(s_t)}{40}\right]'$, $\mathbf{B}^{s_t} = \left[\frac{\mathbf{b}_4(s_t)}{4}, \frac{\mathbf{b}_{12}(s_t)}{12}, \frac{\mathbf{b}_{20}(s_t)}{20}, \frac{\mathbf{b}_{40}(s_t)}{40}\right]'$.[6] The elements in A and $\mathbf{B}$ are derived from the recursive equations with the subscript numbers corresponding to maturities. Measurement errors $\mathbf{v}$ are assumed to have constant variance.

The system of equations to be estimated can be summarized as

$$\begin{aligned}
\mathbf{y}_t &= \mathbf{c}^{s_t} + \mathbf{\Phi}^{s_t}\mathbf{y}_{t-1} + \mathbf{u}_t, && (21)\\
s_t &= h\left(s_{t-1}, \mathbf{y}_{t-1}, \mathbf{u}_{2t}, u_{r,t}\right), && (22)\\
\mathbf{z}_t &= A^{s_t} + \mathbf{B}^{s_t}\mathbf{y}_t + \mathbf{v}_t, &&\\
\mathbf{u}_t &\sim N\left(\mathbf{0}, \mathbf{\Sigma}^{s_t}\mathbf{\Sigma}^{s_t\prime}\right),\ \mathbf{v}_t \sim N\left(\mathbf{0}, \mathbf{V}\right). &&
\end{aligned}$$

where (22) is defined by (14) and (15) for Type I factor dynamics and by (17) to (20) for Type II factor dynamics. All the shocks are iid and $\mathbf{u}_t$ and $\mathbf{v}_s$ are independent for all (t, s). Thus, the observation equation linking $\mathbf{z}_t$ to the state $(\mathbf{y}_t)$ is appended to the VAR equations describing the state dynamics. I apply a two step procedure to estimate the model (e.g., Ang, Piazzesi, and Wei, 2006). Appendix B shows the derivation of the likelihood functions.

4.3 Estimated results

I present the estimated results in two steps. First, I compare the estimated factor dynamics with (Type II) and without (Type I) the *Jikan Jiku* effect. Second, I discuss the estimated yield curves and term premium dynamics using the term structure model with Type II factor dynamics as the benchmark model. The estimated results using the term structure

[6] According to the basic relation between bond price and yield, the n-period bond yield is given by $\frac{a_n}{n} + \frac{\mathbf{b}_n}{b}\mathbf{y}$.

model with Type I factor dynamics are available upon request.

4.3.1 Factor dynamics: Type II versus Type I

How do the Type I and Type II factor dynamics differ from each other? Figure 1 shows the estimated state-dependent transition probabilities under each type of evolution. Under both types, $\Pr(s_t = P|\mathbf{y}_{t-1}, s_{t-1} = P)$, that is, the estimated probability that the normal regime continues into the next period, is one until mid-1995. This is a reasonable result, since nobody would have imagined a ZIRP up to that point. A notable difference between the Type I and Type II transition probabilities is that the latter are more persistent. For example, Figure 1 shows that during the quantitative easing monetary policy (QEP) of March 2001 to February 2006, $\Pr(s_t = Z|\mathbf{y}_{t-1}, s_{t-1} = Z)$ is estimated to be much higher under the Type II regime than under the Type I regime. This may reflect market pessimism over the recovery from deflation with the Bank of Japan's commitment that it would maintain the zero rate until some inflationary condition was satisfied.

The Taylor rule coefficients under the two types of factor dynamics are reported in Table 1. Under both types of regime evolution, the estimated coefficients have the right signs in terms of economic interpretation, and the long-run response of the short-term interest rate to a unit increase in inflation well exceeds one (3.6 and 2.2 under Types I and II, respectively). The Taylor rule coefficient with respect to inflation (i.e., the first element of $\boldsymbol{\beta}^P$) under Type I is higher than that under Type II (0.34 versus 0.22 respectively), possibly reflecting the omission of the inflation variable in the regime evolution under Type I (see equations (23) and (24)). The estimated $\mathbf{c}_2^Z$, $\mathbf{\Phi}_2^Z$,and $\mathbf{\Sigma}_{22}^Z$ are the same under Type I and

II since they can be estimated separately (see Appendix B for details).

Figure 1 and Table 1 here

Which type of factor dynamics is more appropriate? I compare the empirical performances of the two types of models, with Type I and Type II factor dynamics, using two different approaches, and find that Type II has notably better fits to the data, though out-of-sample performance results are mixed. The first approach is the likelihood-ratio test. Setting Type 1 as the null and Type 2 as the alternative, a large positive test statistic (14.1) rejects the null. The second approach is one-period-ahead out-of-sample forecasting on the state variables (i.e., the short rate, inflation, and output gaps) to check these models' predictive accuracy. This exercise involves a rolling forecast covering the last three years of the QEP period. I evaluate the predictive accuracy by the following two measures: (i) the root mean square error ratios (Type I relative to Type II) and (ii) the modified Diebold-Mariano test statistics[7] proposed by Harvey et al (1998) with differential loss based on the mean-squared errors. The results are summarized in Table 2. Type 2 weakly outperforms Type 1 for the policy rate and output gap forecasts, whereas it underperforms Type I for the inflation forecasts. Type 2's poor forecasting performance for inflation forecasts may be due to the imprecise estimate of $\bar{\pi}$.

Table 2 here

[7] Type II does not nest Type I factor dynamics. One may wonder whether Type II reduces to Type I when $\bar{\pi}$ is sufficiently negative (so that the inflation condition is always satisfied); however, this is not true since $u_\pi \sim N(0,1)$ and u_π can be $-\infty$.

4.3.2 The yield curves and term premia

I now discuss the estimated yield curves and term premia under the benchmark model. The estimated prices-of-risk coefficients and transition probabilities under $\mathbb{Q}$ are reported in Table 3. The estimated prices-of-risk level coefficients $(\lambda_0(P),\ \lambda_0(Z))$ differ for the two regimes, particularly those corresponding to inflation (the second element of each λ_0) which increase notably under the ZIRP, from a negative value under the normal regime to a positive value under the the ZIRP regime. In the benchmark estimation, given large standard errors with limited sample size, the remaining prices of risk coefficients are set equal to zero except for the (1,1) element of $\Lambda_1(P)$, that is, under the normal regime the prices of risk are allowed to change with short rate fluctuations.

Table 3 and Figure 2 here

Figure 2 shows how the yield-equation coefficients, that is the constant, short-rate, inflation, and output-gap coefficients in the yield equation, change against maturity (in quarters) under the normal regime (dashed black lines) and the ZIRP regime (red solid lines). The model-implied yields are expressed as the annualized rate in percent. The upward slopes of the constant coefficients represent the shapes of the average yield curves under the normal and ZIRP regimes. They imply that yield curves flatten on average under the ZIRP regime. The downward slopes of the short-rate coefficients imply that an increase in the short rate has a more positive impact on the shorter end of yield curves.

The bottom two charts in Figure 2 demonstrate how differently deflation and low growth contribute to lowering longer-term JGB yields between the normal and ZIRP regimes; the shapes of the inflation coefficients imply that the inflationary effect on the longer end of yield curves increases under the ZIRP; the shapes of the output-gap coefficients imply that growth effects on JGB yields weaken under the ZIRP. Quantitatively, the estimated results indicate that under the normal regime, 1-percent deflation lowers 10-year JGB yields by 14 basis points, and 1-percent output gap increase raises 10-year JGB yields by 7 basis points. On the other hand, under the ZIRP regime, 1-percent deflation lowers 10-year JGB yields by 23 basis points, and 1-percent output gap increase raises 10-year JGB yields by 2 basis points. A closer look at the recursive equations for $\mathbf{b}_n^{s_t}$ indicates that the shapes of inflation and output-gap coefficients are generated largely by the differences between macroeconomic factor coefficients (i.e., Φ^{s_t} and $\Sigma_{22}^{s_t}$) across regimes with persistent transition probabilities under $\mathbb{Q}$.

Lastly, I decompose the long term bond yields into the expectations and term premium components. Following the typical definition in the literature, the term premium of an n-period bond yield is defined as the actual n-period bond yield minus the average expected future short-term interest rates (i.e., $\frac{1}{n}E_t\left\{\sum_{j=0}^{n-1} r_{1,t+j}\right\}$). To calculate the expectations components, I first obtain $1, 2, ..., n$ period forecasts of the future short-term interest rates at each quarter via two-regime three-variable VAR forecasting, I then use these forecasts to calculate the average expected future short-term interest rates. Figure 4 reports the model implied term premia of 10-year bonds, the corresponding averages of expected future short-term interest rates, and the actual yields. It indicates that term premia declined after the second ZIRP introduction (i.e., the QEP started in March 2001). It also indicates that the large bond yield decline in the early 1990s was driven by the expectations components, whereas that in the late 1990s was driven by both expectations and term premium components.

Figure 3 here

5 Conclusion

This paper constructs a no-arbitrage affine term structure model with state-dependent regime shifts in which the state vector depends on the current policy regime. As an application of the model, it examines how the JGB yields fluctuate with macroeconomic variables with endogenous monetary policy shifts that incorporate the key ZIRP features, such as the zero lower bound of the policy rate and the *Jikan Jiku* policy. The estimated results indicate that under the ZIRP, deflation plays a growing role in lowering JGB yields, especially at the longer end of yield curves. On the other hand, the effect of output gap fluctuations on bond yields weakens. Furthermore, term premium components contributed to bond yield declined in the late 1990s and after the introduction of the quantitative easing policy.

Looking forward, when Japan finally emerges from a zero rate environment, it is important to understand not only "normal" bond yield responses to moderate inflation and economic growth, but also the channels that can steeply raise macroeconomic variables and thus jeopardizing the JGB markets.

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A A non-ATSM without Assumption 4

One may want to do away with Assumption 4 of fully priced regime risk. Under Assumptions 1-3, $p_1(\tilde{\mathbf{y}}, \tilde{s}) = -r(\tilde{\mathbf{y}})$ is conditionally normally distributed under $\mathbb{Q}$. So, (8) holds for $n = 1$. It can be rewritten as

$$\exp\left[p_2(\tilde{\mathbf{y}}, \tilde{s})\right] = \sum_{\tilde{s}} \rho^Q(\tilde{s}|\mathbf{y}, s) \exp\left[E^Q(p_1(\tilde{\mathbf{y}})|\tilde{s}, \mathbf{y}, s) + \frac{1}{2} Var^Q(p_1(\tilde{\mathbf{y}})|\tilde{s}, \mathbf{y}, s) - r(\mathbf{y})\right].$$

The term in the brackets can be rewritten as

$$\begin{aligned} & -E^Q(r(\tilde{\mathbf{y}})|\tilde{s}, \mathbf{y}, s) + \frac{1}{2} Var^Q(p_1(\tilde{\mathbf{y}})|\tilde{s}, \mathbf{y}, s) - r(\mathbf{y}), \\ = \; & -\mathbf{e}_1' E^Q(\tilde{\mathbf{y}}|\tilde{s}, \mathbf{y}, s) + \frac{1}{2} \mathbf{e}_1' Var^Q(p_1(\tilde{\mathbf{y}})|\tilde{s}, \mathbf{y}, s)\, \mathbf{e}_1 - \mathbf{e}_1'\mathbf{y}, \\ = \; & -\mathbf{e}_1' \left(\mathbf{c}^Q(\tilde{s}) + \mathbf{\Phi}^Q(\tilde{s})\,\mathbf{y}\right) + \frac{1}{2} \mathbf{e}_1' \Sigma(\tilde{s})\, \Sigma(\tilde{s})' \,\mathbf{e}_1 - \mathbf{e}_1'\mathbf{y}, \end{aligned}$$

that is,

$$p_2(\tilde{\mathbf{y}}, \tilde{s}) = \log\left(\sum_{\tilde{s}} \rho^Q(\tilde{s}|\mathbf{y}, s) \exp\left[-\mathbf{e}_1' \left(\mathbf{c}^Q(\tilde{s}) + \mathbf{\Phi}^Q(\tilde{s})\,\mathbf{y}\right) + \frac{1}{2} \mathbf{e}_1' \Sigma(\tilde{s})\, \Sigma(\tilde{s})'\, \mathbf{e}_1 - \mathbf{e}_1'\mathbf{y}\right]\right).$$

The whole term structure can be generated by the no-arbitrage condition for $n = 2, 3, ...,$ once the model parameters are estimated by using only p_1 and p_2.

B The likelihood function

B.1 Separating yield information from factor dynamics

The goal is to derive the likelihood of the data, i.e.,

$$\mathcal{L} \equiv p\left(\mathbf{z}_0, ..., \mathbf{z}_T, \mathbf{y}_1, ..., \mathbf{y}_T, s_1, ..., s_T | \mathbf{y}_0, s_0\right),$$

which can be decomposed as

$$\mathcal{L} \equiv \underbrace{f\left(\mathbf{z}_0, ..., \mathbf{z}_T | \mathbf{y}_0, ..., \mathbf{y}_T, s_0, ..., s_T\right)}_{\mathcal{L}_1} \underbrace{p\left(\mathbf{y}_1, ..., \mathbf{y}_T, s_0, ..., s_T | \mathbf{y}_0, s_0\right)}_{\mathcal{L}_2}.$$

B.2 Likelihood for yields ($\mathcal{L}_1$)

The model-implied static yield equation

$$\mathbf{z}_t = A^{s_t} + \mathbf{B}^{s_t}\mathbf{y}_t + \mathbf{v}_t,$$

where

$$A^{s_t} \equiv \begin{bmatrix} a_1^{s_t} \\ \vdots \\ a_{40}^{s_t}/40 \end{bmatrix}, \ \mathbf{B}^{s_t} \equiv \begin{bmatrix} \mathbf{b}_1\left(s_t\right) \\ \vdots \\ \mathbf{b}_{40}\left(s_t\right)/40 \end{bmatrix}.$$

The usual assumption that the error is iid normal can be stated precisely as

$$\left.\begin{bmatrix} \mathbf{v}_1 \\ \vdots \\ \mathbf{v}_T \end{bmatrix}\right| \left(\mathbf{y}_0, ..., \mathbf{y}_T, s_0, ..., s_T\right) \sim N\left(\mathbf{0}, \mathbf{I}_T \otimes \mathbf{V}\right), \ \ \mathbf{V} \equiv Var\left(\mathbf{v}_t\right).$$

So

$$\mathcal{L}_1 \equiv f(\mathbf{y}_0, ..., \mathbf{y}_T, s_0, ..., s_T) = \prod g_1(\mathbf{z}_t - A^{s_t} - \mathbf{B}^{s_t}\mathbf{y}_t; \mathbf{V}),$$

$$\text{Or } L_1 \equiv \log(\mathcal{L}_1) = \sum \log[g_1(\mathbf{z}_t - A^{s_t} - \mathbf{B}^{s_t}\mathbf{y}_t; \mathbf{V})],$$

where g_1 is the density of $N(0, \mathbf{V})$. Furthermore, the log likelihood can be concnetreated (Hayashi (2000), eq. (8.5.23)) as follows

$$L_1 = const. - \frac{1}{2}\log\left|\frac{1}{T+1}\sum_{t=1}^{T}(\mathbf{z}_t - A^{s_t} - \mathbf{B}^{s_t}\mathbf{y}_t)(\mathbf{z}_t - A^{s_t} - \mathbf{B}^{s_t}\mathbf{y}_t)'\right|.$$

B.3 Likelihood for factor dynamics ($\mathcal{L}_2$)

Since $\{\mathbf{y}, s\}$ is Markov, the usual sequantial factorization argument yields

$$\mathcal{L}_2 \equiv p(\mathbf{y}_1, ..., \mathbf{y}_T, s_1, ...s_T|\mathbf{y}_0, s_0) = \prod_{t=1}^{T} p(\mathbf{y}_t, s_t|\mathbf{y}_{t-1}, s_{t-1}).$$

As mentioned in the text, $p(\mathbf{y}_t, s_t|\mathbf{y}_{t-1}, s_{t-1}) = f(\mathbf{y}_t|s_t, \mathbf{y}_{t-1}, s_{t-1})\,\rho(s_t|\mathbf{y}_{t-1}, s_{t-1})$. The component $f(\mathbf{y}_t|s_t, \mathbf{y}_{t-1}, s_{t-1})$ can be decomposed as

$$\begin{aligned} f(\mathbf{y}_t|s_t, \mathbf{y}_{t-1}, s_{t-1}) &= f(r_t|\mathbf{y}_{2,t}, s_t, \mathbf{y}_{t-1}, s_{t-1}) \times f(\mathbf{y}_{2,t}|s_t, \mathbf{y}_{t-1}, s_{t-1}), \\ f(r_t|\mathbf{y}_{2,t}, s_t, \mathbf{y}_{t-1}, s_{t-1}) &= g_2\left(r_t - \alpha^{s_t} - \delta^{s_t} r_{t-1} - \boldsymbol{\beta}^{s_t\prime}\mathbf{y}_{2,t}; (\sigma_r^{s_t})^2\right), \\ f(\mathbf{y}_{2,t}|s_t, \mathbf{y}_t, s_t) &= g_3(\mathbf{y}_{2,t} - \mathbf{c}_2^{s_t} - \mathbf{\Phi}_2^{s_t}\mathbf{y}_{t-1}; \mathbf{\Sigma}_{22}^{s_t}\mathbf{\Sigma}_{22}^{s_t\prime}), \end{aligned}$$

where g_2 is the density of $N\left(0, (\sigma_r^{s_t})^2\right)$ and g_3 is the density of $N(0, \mathbf{\Sigma}^{s_t}\mathbf{\Sigma}^{s_t\prime})$. The other component in (1) $\rho(s_t|\mathbf{y}_{t-1}, s_{t-1})$ was derived in the text. Putting all pieces about $\mathcal{L}_2$

together, $L_2 \equiv \log(\mathcal{L}_2)$ can be written as

$$\text{Type I:} \qquad L_2 = \sum_{t=1}^{T} \log\left[g_2\left(r_t - \alpha^{s_t} - \delta^{s_t} r_{t-1} - \boldsymbol{\beta}^{s_t\prime}\mathbf{y}_{2,t}; (\sigma_r^{s_t})^2\right)\right] \tag{23}$$
$$+ \sum_{t=1}^{T} \log\left[g_3\left(\mathbf{y}_{2,t} - \mathbf{c}_2^{s_t} - \boldsymbol{\Phi}_2^{s_t}\mathbf{y}_{t-1}; \boldsymbol{\Sigma}_{22}^{s_t}\boldsymbol{\Sigma}_{22}^{s_t\prime}\right)\right]$$
$$+ \sum_{t=1}^{T} \left\{ s_t \log\left[F\left(\frac{r^e(\mathbf{y}_{t-1}) - \alpha^Z}{\sigma_\xi}\right)\right] + (1 - s_t)\log\left[1 - F\left(\frac{r^e(\mathbf{y}_{t-1}) - \alpha^Z}{\sigma_\xi}\right)\right]\right\}.$$

$$\text{Type II:} \qquad L_2 = \sum_{t=1}^{T} \log\left[g_2\left(r_t - \alpha^{s_t} - \delta^{s_t} r_{t-1} - \boldsymbol{\beta}^{s_t\prime}\mathbf{y}_{2,t}; (\sigma_r^{s_t})^2\right)\right] \tag{24}$$
$$+ \sum_{t=1}^{T} \log\left[g_3\left(\mathbf{y}_{2,t} - \mathbf{c}_2^{s_t} - \boldsymbol{\Phi}_2^{s_t}\mathbf{y}_{t-1}; \boldsymbol{\Sigma}_{22}^{s_t}\boldsymbol{\Sigma}_{22}^{s_t\prime}\right)\right]$$
$$+ \sum_{t=1}^{T} \left\{ \begin{array}{c} s_{t-1}s_t \log\left[F\left(\frac{r^e(\mathbf{y}_{t-1}) - \alpha^Z}{\sigma_\xi}\right)\right] + s_{t-1}(1 - s_t)\log\left[1 - F\left(\frac{r^e(\mathbf{y}_{t-1}) - \alpha^Z}{\sigma_\xi}\right)\right] \\ + (1 - s_{t-1}) s_t \log\left[B\left(r^e(\mathbf{y}_{t-1}) - \alpha^Z, \pi^e(\mathbf{y}_{t-1}) - \bar{\pi}; \mathbf{W}\right)\right] \\ + (1 - s_{t-1})(1 - s_t)\log\left[1 - B\left(r^e(\mathbf{y}_{t-1}) - \alpha^Z, \pi^e(\mathbf{y}_{t-1}) - \bar{\pi}; \mathbf{W}\right)\right] \end{array} \right\}.$$

Maximization of L_2 can be simplified because $\mathbf{c}_2^Z, \boldsymbol{\Phi}_2^Z$,and $\boldsymbol{\Sigma}_{22}^Z$ appear only in the g_3 component of the second summation in L_2.

B.4 Choices of parameters

The model parameters are

$$\boldsymbol{\theta} \equiv (\boldsymbol{\theta}_1, \boldsymbol{\theta}_2),$$
$$\boldsymbol{\theta}_1 \equiv \left(\rho^Q(P|P), \rho^Q(Z|Z), \boldsymbol{\lambda}_0(P), \boldsymbol{\lambda}_0(Z), \boldsymbol{\Lambda}_1(P), \boldsymbol{\Lambda}_1(Z)\right),$$
$$\text{Type I:} \qquad \boldsymbol{\theta}_2 \equiv \left(\begin{array}{c} \alpha^P, \alpha^Z, \boldsymbol{\beta}^P, \delta^P, \sigma_r^P, \sigma_r^Z, \\ \mathbf{c}_2^P, \mathbf{c}_2^Z, \boldsymbol{\Phi}_2^P, \boldsymbol{\Phi}_2^Z, \boldsymbol{\Sigma}_{22}^P, \boldsymbol{\Sigma}_{22}^Z \end{array} \right),$$
$$\text{Type II:} \qquad \boldsymbol{\theta}_2 \equiv \left(\begin{array}{c} \alpha^P, \alpha^Z, \boldsymbol{\beta}^P, \delta^P, \sigma_r^P, \sigma_r^Z, \\ \mathbf{c}_2^P, \mathbf{c}_2^Z, \boldsymbol{\Phi}_2^P, \boldsymbol{\Phi}_2^Z, \boldsymbol{\Sigma}_{22}^P, \boldsymbol{\Sigma}_{22}^Z, \bar{\pi} \end{array} \right).$$

Type I factor dynamics

Taylor rule

α^{P}	δ^{P}	$\boldsymbol{\beta}^{P'}$		σ_r^{P}
0.06	0.91	0.34	0.11	0.49
(0.098)	(0.029)	(0.168)	(0.041)	(0.040)

α^{Z}	σ_r^{Z}
0.01	0.02
(0.004)	(0.003)

Dynamics of macroeconomic variables

$\mathbf{c}_2^{P}$	$\boldsymbol{\Phi}_2^{P}$			$\boldsymbol{\Sigma}_{22}^{P}$	
0.17	-0.01	0.73	0.09	0.18	---
(0.044)	(0.012)	(0.070)	(0.017)	(0.016)	---
-0.22	0.06	-0.10	0.83	-0.05	0.93
(0.223)	(0.063)	(0.359)	(0.086)	(0.112)	(0.079)

$\mathbf{c}_2^{Z}$	$\boldsymbol{\Phi}_2^{Z}$			$\boldsymbol{\Sigma}_{22}^{Z}$	
0.01	-0.11	0.92	3.8E-03	0.06	---
(0.019)	(0.091)	(0.043)	(0.007)	(0.004)	---
-0.27	-1.36	1.39	0.77	-0.09	0.57
(0.191)	(0.903)	(0.422)	(0.069)	(0.059)	(0.042)

Type II factor dynamics

Taylor rule

α^{P}	δ^{P}	$\boldsymbol{\beta}^{P'}$		σ_r^{P}
0.20	0.90	0.22	0.10	0.50
(0.110)	(0.030)	(0.178)	(0.043)	(0.041)

α^{Z}	σ_r^{Z}
0.01	0.02
(0.004)	(0.003)

Dynamics of macroeconomic variables

$\mathbf{c}_2^{P}$	$\boldsymbol{\Phi}_2^{P}$			$\boldsymbol{\Sigma}_{22}^{P}$	
0.17	-0.01	0.72	0.09	0.18	---
(0.043)	(0.012)	(0.070)	(0.016)	(0.015)	---
-0.19	0.06	-0.15	0.82	-0.05	0.93
(0.224)	(0.063)	(0.360)	(0.086)	(0.113)	(0.080)

$\mathbf{c}_2^{Z}$	$\boldsymbol{\Phi}_2^{Z}$			$\boldsymbol{\Sigma}_{22}^{Z}$	
0.01	-0.11	0.92	3.8E-03	0.06	---
(0.019)	(0.091)	(0.043)	(0.007)	(0.004)	---
-0.27	-1.36	1.39	0.77	-0.09	0.57
(0.191)	(0.903)	(0.422)	(0.069)	(0.059)	(0.042)

$\bar{\pi}$
0.11
(0.104)

Table 1. The factor dynamics coefficients. This table reports the estimated coefficients of Type I (left panel) and Type II (right panel) factor dynamics.

	RMSE ratios	DM test
r	1.30	-1.33
π	0.77	2.89**
g	1.43	-1.52

Table 2. (Pseudo) out of sample performance. The second column reports the root mean square error (RMSE) ratios of the Type I factor dynamics relative to the Type II factor dynamics. The third column reports modified Diebold-Mariano test statistics. Significantly negative statistics indicate that the Type II specification outperforms the Type I specification. The out-of-sample period consists of the last three years of the QEP period. The superscript ** indicate significance at the 1% level.

	The prices of risk			Transition probabilities under **Q**	
	$\lambda_0(Z)$	$\lambda_0(P)$	$\Lambda_1(P)$	$\rho^Q(Z \mid Z)$	$\rho^Q(P \mid P)$
r	-15.50	2.93	-0.50	0.94	0.77
	(0.10)	(50.44)	(0.001)	(2.7e-6)	(4.9e-6)
π	10.81	-22.20			
	(0.56)	(0.25)			
g	-9.25	-4.78			
	(0.24)	(0.79)			

Table 3. Yield curve coefficients. This table reports the coefficients of the prices of risk and transition probabilities under **Q** estimated for the benchmark model.

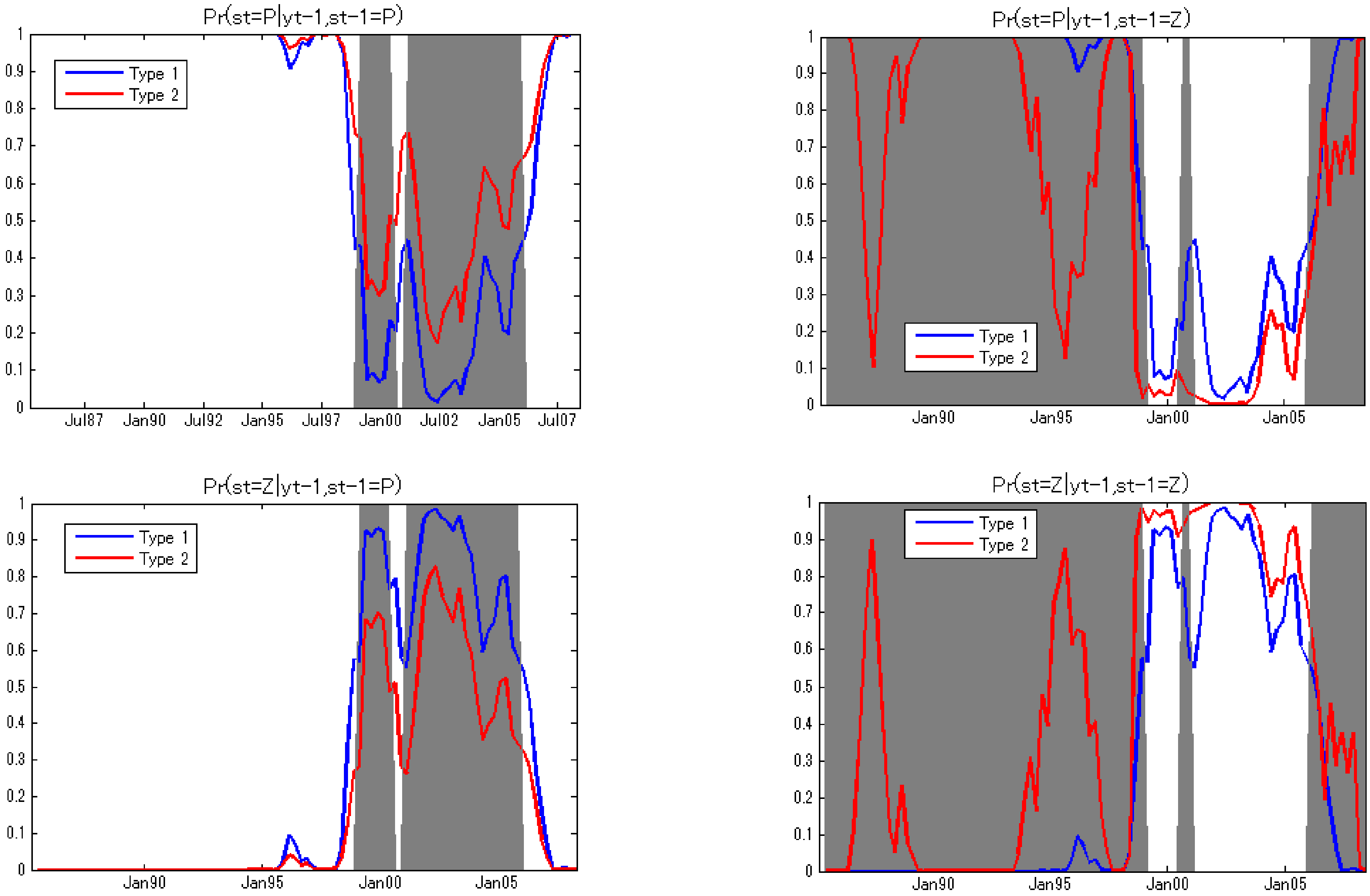

Figure 1. State-dependent transition probabilities. The left column reports the probability that the regime is normal (top) or a ZIRP (bottom) given that the previous regime is normal. The right column reports the probability that the regime is normal (top) or a ZIRP (bottom) given that the previous regime is ZIRP. The blue lines are transition probabilities under Type I evolution and red lines are those under Type II evolution. The periods that the current regime is not normal (left column) or a ZIRP (right column) are shaded in gray.

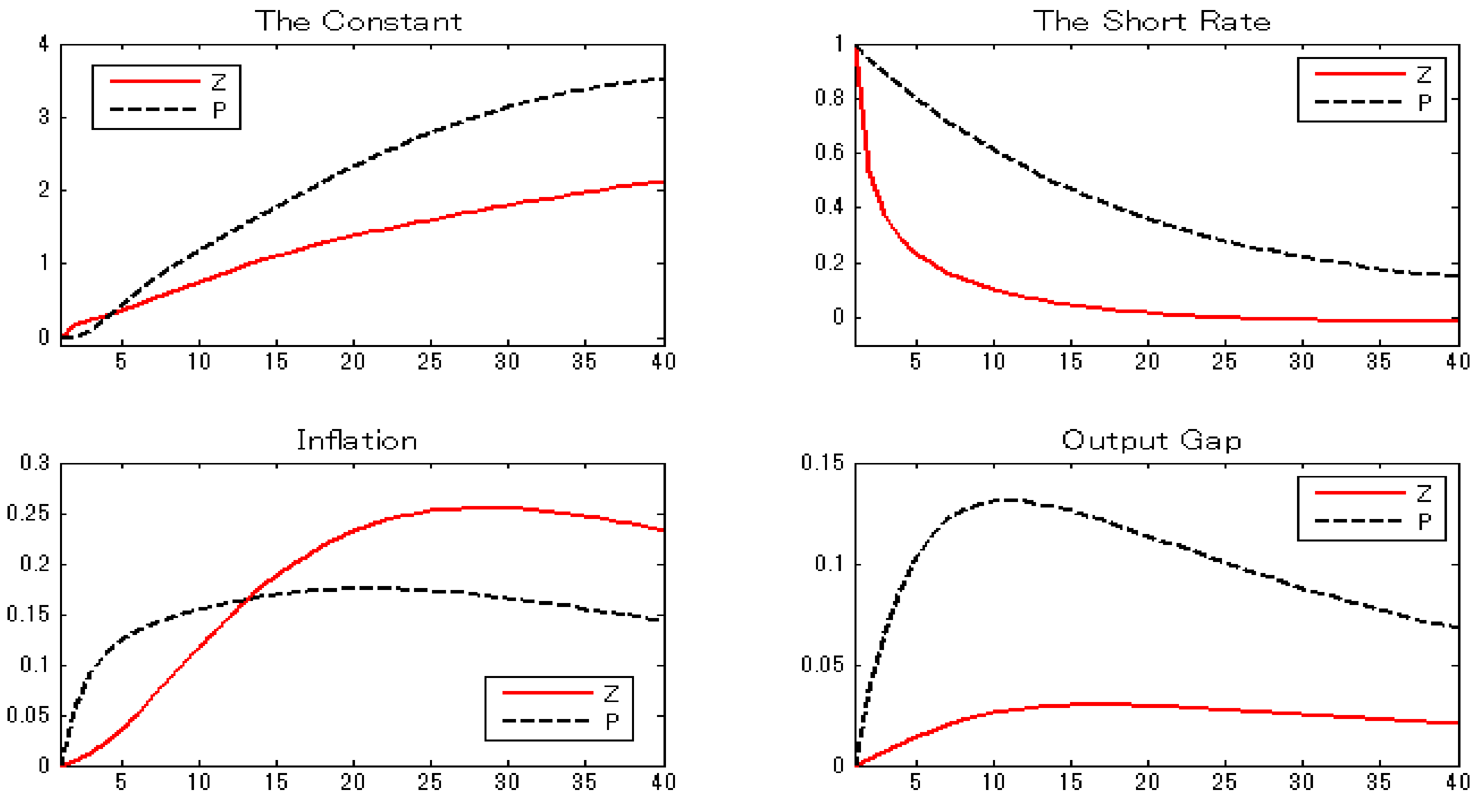

Figure 2. Factor weights against maturity. This figure plots the coefficients of the yield equation against maturity (in quarters) estimated for the benchmark model. The coefficients correspond to the constant, short-rate, inflation, and output-gap terms in the yield equation under the normal regime (dashed black lines) and the ZIRP regime (red solid lines). The model-implied yields are expressed as the annualized rate in percent.

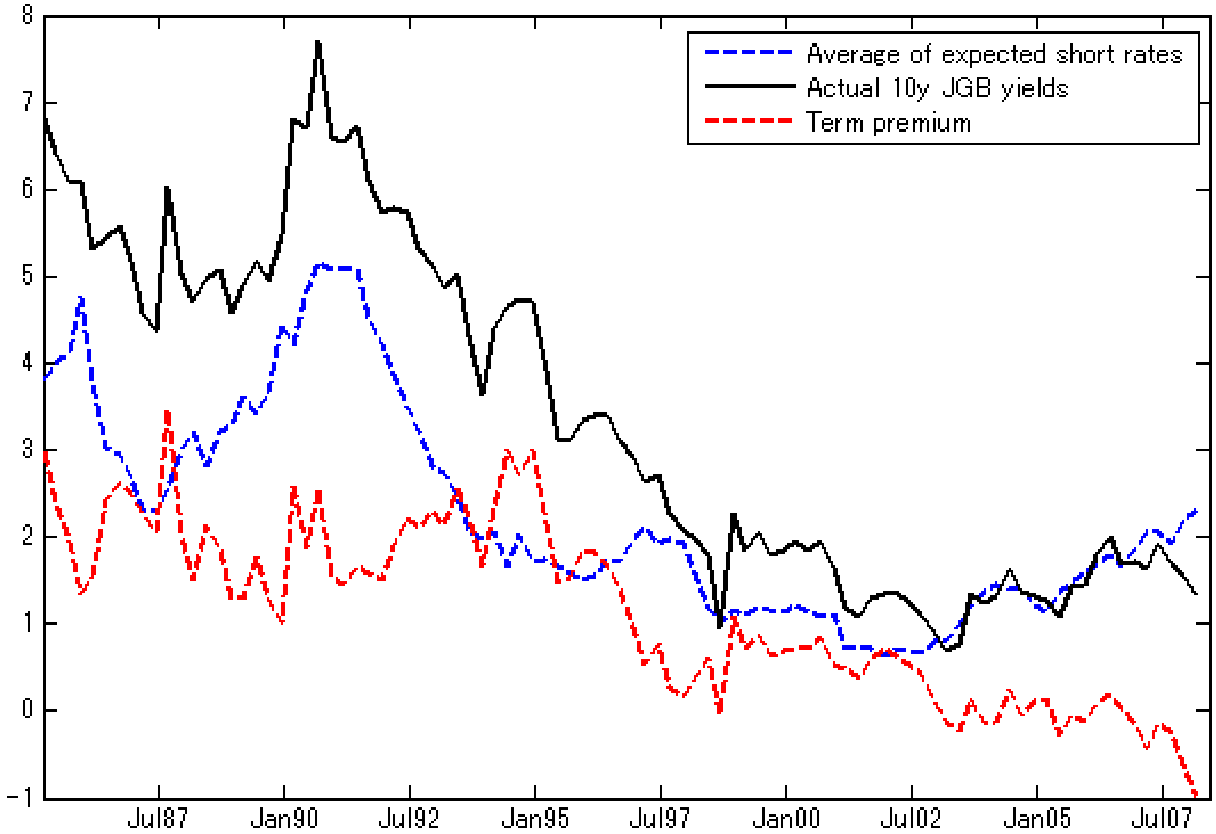

Figure 3. Estimation of expectations and term premium components of 10-year bond yields (annualized rates in percent). This figure plots the actual 10-year bond yields, the average expected future short-term interest rates over the life of the bond, and its difference from the actual yields (i.e., term premium) obtained via two-regime three-variable VAR forecasting for the benchmark model.