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Dark Sides of Patent Pools with Compulsory Independent Licensing

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Abstract

This paper examines roles of patent pools with compulsory independent licensing.A seminal work by Lerner and Tirole (2004) have shown that requiring independent licensing or compulsory independent licensing is a useful tool to select only desirable patent pools. In this paper, however, we are going to show that their argument is not always true, If there are users who demand only a part of the pooled technologies, the compulsory independent licensing gives a tool for price discrimination for the patent holders, and that is welfare decreasing under some conditions. Moreover, the compulsory independent licensing may promote entry deterrence when there are lower grade entrants. Even in this sense, compulsory independent licensing decreases social welfare. The welfare under the patent pool with independent licensing may become lower than that under the competitive licensing.

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1 Introduction

This paper examines roles of patent pools with compulsory independent licensing. It is now well known that patent pools have positive roles for economic welfare. If patent pools include substitute patents, however, they may decrease the total welfare, since they become a mechanism for promoting the collusive behaviors of patent holders. A seminal work by Lerner and Tirole (2004) have shown that requiring independent licensing or compulsory independent licensing is a useful tool to select only desirable patent pools. They have shown that by requiring independent licensing, only welfare improving patent pools are stable, and welfare decreasing patent pools, in which substitutable patents are included, become unstable.

Requiring independent licensing may have another positive role. By supplying independent licensing, those users who demand only a part of the pooled technology get benefit. For example, Commission Notice Guidelines on the application of Article 81 of the EC Treaty to technology transfer agreements (2004/C 101/02) states that "in cases where the pooled technologies have different applications some of which do not require use of all of the pooled technologies, the pool offers the technologies only as a single package or whether it offers separate packages for distinct applications. In the latter case it is avoided that technologies which are not essential to a particular product or process are tied to essential technologies". (222 (c)).

In this paper, however, we are going to show that this intuition is not correct. The above argument is implicitly assuming that patent pools ignore the users who demand a part of the pooled technologies. If the pools price by considering such users, their argument becomes quite different. Without independent licensing, the price of the pooled technology becomes low to promote the independent users. On the other hand, if the independent licensing is required, the price for the patent pool can be higher, since the profit from the independent market is derived by the independent licensing. In other words, the compulsory independent licensing gives a tool for price discrimination for the patent holders. It is well known that such price discrimination is welfare decreasing under some conditions. Hence, independent licensing has a negative impact for economic welfare.

2 Literature

Among a number of the papers which theoretically investigate patent pools,[1] one of the related paper to our research is Lerner and Tirole (2004) who show that formation of a patent pool is welfare-enhancing if and only if the included patents are sufficiently complementary and that forcing all the participants of a pool to also offer their patents individually (compulsory independent licensing) is always socially beneficial since it can work as a screening device to distinguish between welfare-enhancing pool and welfare-deteriorating pool.[2] We point out a drawback of the compulsory independent licensing. In particular, we argue that their clear result depends on homogeneity of patents in that consumers only cares about the number of the patents they purchase. We incorporate an idiosyncratic preference over the patents into the consumers and show that in this case, compulsory independent licensing can be a device for price discrimination which deteriorates the social welfare.

Quint (2009) shows results similar to our argument. He considers product differentiated oligopoly where for supplying the product, both the essential patents which are common

[1] Aoki and Nagaoka (2004, 2005, 2006) consider coalition formation games with externality and point out the difficulty of efficient patent pool formation through sequential negotiation among patent holders. Kim (2004) studies the effect of vertical integration on licensing behaviour with or without patent pools. Choi (2010) investigates the implication of patent pools and cross licensing for uncertain litigation. Schiff and Aoki (2007) investigate how competition of consortium standards among owners of essential patents influences formation of patent pools.

[2] Brenner (2009) considers an extended model in which formation of patent pool of fewer patents is available. He shows that a scheme of compulsory independent licensing with exclusive pool membership still works for screening welfare effect of pool formation.

in the market and the non-essential patents which are specific to the product are required. He shows that while the non-essential patents are assumed to be perfect complements in the sense that all the non-essential patents are necessary for supplying the product, forming patent pool including only the non-essential patents can be welfare decreasing and stable under compulsory independent licensing. Our setting is different from his model. We consider an oligopolistic patent market in which some consumers consider the patents perfect complements and the others purchase only one of the patents or no patent. We show that in segmented demand markets as we consider, compulsory independent licensing can be a price discrimination scheme profitable for the patent holders and it can reduce the welfare.

The present paper is also related to the economics of bundling. There is a bunch of papers studying price discrimination *via* mixed bundling of complementary goods in monopoly (Long, 1984; Lewbel, 1985; Bakos and Brynjolfsson, 1999; Venkatesh and Kamakura, 2003; Armstrong, 2010).[3] In our model, formation of patent pools is a kind of bundling scheme and depending on whether compulsory independent licensing is allowed or not, it can be interpreted as a kind of pure or mixed bundling. Nevertheless these schemes are not exactly the same in that since formation of a patent pool does not mean merger of the patent holders, independent licensing causes strategic interaction on pricing among the patent holders. In terms of oligopolistic competition of complementary goods with bundling sales,[4] Economides and Salop (1992) and Choi (2008) study an environment in which there are two perfect complementary components and each of them has two differentiated suppliers. and investigate the impact of merger between different component firms which allows the firm to attempt mixed or pure bundling. While the demand structure in their model is somewhat similar to

[3]There are also a number of papers studying bundling sales without complementarity between the goods. See Adams and Yellen (1976), Schmalensee (1982, 1984), McAfee et al. (1989), Salinger (1985), Fang and Norman (2006), Adachi et al. (2009), Adachi and Ebina (2010) and Chen and Riordan (2011).

[4]For a broad review of price discrimination and bundling in imperfect competitive markets, see Armstrong (2006, 2008) and Stole (2007).

ours, the difference is that in our model (i) there are consumers who purchase at most one component and (ii) pricing in patent pool with independent licensing is sequential and does not exactly coincides with mixed bundling in Choi (2008).

Furthermore our analysis has an aspect of application work of bundling sales in oligopolistic market.[5]

Empirical Studies on Patent Pool: Lerner et al. (2007), Layne-Farrar and Lerner (2011)

3 The Model

There are two patent (or license) holders, denoted by A and B respectively, and many potential users for their patents. Users are classified to the following three types: (i) those who demand only patent A, (ii) those who demand only patent B, (iii) those who treat patent A and B are perfect complements and demand both A and B. We call the market of type (i) and (ii) the “single market" and that of type (iii) the “bundled market". The demand function of each single market is given by

$$D(p_i) = \begin{cases} 1 - p_i & p_i \in [0, 1] \\ 0 & p_i > 1 \end{cases}$$

for $i = A, B$ where $p_i \geq 0$ is the license price of patent i.[6] We assume, for simplicity, that the cost of licensing of patent i is normalized to be zero. Hence, the profit from the single market of patent i is written as $\Pi(p_i) = p_i D(p_i)$. On the other hand, the demand of the

[5] In terms of perspective of applied works, Chae (1992) studies monopolistic bundling in subscription TV markets and Brueckner (2001) and Chen and Gayle (2007) investigate the economic effect of code-sharing in airline industries.

[6] Because we assume a symmetric situation for simplicity, the demand function itself does not depend on i.

bundled market is given by

$$\overline{D}(P) = \begin{cases} a - bP & P \in [0, a/b] \\ 0 & P > a/b \end{cases}$$

where $P \geq 0$ is the license price to obtain both patent A and B. We assume $a > 0$, and $b > 0$. Each of the patent holders maximizes the sum of the profits from these markets. The profit from the bundled market depends on the licensing process as will be explained below.

We assume here that patent holders do not have any information about the type of each consumer. Hence, it is impossible to set different prices to different types. It might be possible, however, to screen the user's type by offering menu of contracts. This is a key point of our setting, but even if patent holders have information about each user's type, qualitative properties of our results are not affected. We will explain this point more carefully in the later sections.

In this paper we examine the following three schemes and compare how the welfare under the pool with compulsory independent licensing is better or worse than that under the other schemes.

Competitive Licensing (C) The patent holders do not form the patent pool and they simultaneously and noncooperatively choose their own patent price p_i. Given p_A and p_B, consumers in the bundled market face the sales price p_A+p_B to get both of the patents. Thus, the demand function of the bundled market is given by $\overline{D}(p_A+p_B)$. The patent holder i gets the profit $\Pi(p_i) = p_i D(p_i)$ from her own single market and $\overline{\Pi}_i^C(p_i, p_j) \equiv p_i \overline{D}(p_i + p_j)$ $(j \neq i)$ from the bundled market. Patent holder i maximizes the sum of them, $p_i D(p_i) + p_i \overline{D}(p_i + p_j)$ given p_j for $j \neq i$.

Patent Pool (P) The patent holders form a patent pool and jointly choose the price for the bundled patents to maximize their joint profit. Specifically, if the patent holders choose P as the price for the bundled patents, consumers in all the markets, including each of the single markets, face P as the sales price. We assume that the profit gained from the pool is equally divided into the patent holders as assumed in Lerner and Tirole (2004). Thus, the profit from the single market is $\Pi(P) = PD(P)$ and that of the bundled market is $\overline{\Pi}^P(P)/2 = P\overline{D}(P)/2$, and the profit of each patent holder is given as $\Pi(P) + \overline{\Pi}^P(P)/2 = PD(P) + P\overline{D}(P)/2$.

Patent Pool with Independent Licensing (I) This situation can be formulated as a two stage game. First, the patent holders form the a patent pool and choose the price of bundled patents P. Second, each of them simultaneously and noncooperatively chooses her own patent price p_i. In this case, consumers who only demand the patent i has two options, to purchase the independent patent i or to purchase the bundled patent. Hence, as long as p_i is lower than the price for the bundled patent, P, the each patent holder gets $\Pi(p_i)$. On the other hand, if $p_i \geq P$, the patent pool supplies the bundled patent, and each patent holder gets the profit $\Pi(P)/2$. Hence, the profit of the patent holder i from a single market i is[7]

$$\begin{cases} \Pi(p_i) = p_i D(p_i) & \text{if} \quad p_i < P \\ \frac{1}{2}\Pi(P) = PD(P)/2 & \text{if} \quad p_i \geq P. \end{cases}$$

Similarly, consumers in the bundled market need not purchase from the patent pool. They have an option to purchase two independet licensings. Hence, the profit of patent holder i

[7] In this paper we assume that purchasing patents from the individual and the pool is indifferent, a consumer goes to the pool. In what follows, we adopt this manner.

from the bundled market is

$$\begin{cases} \overline{\Pi}_i^C(p_i, p_j) = p_i \overline{D}(p_i + p_j) & \text{if} \quad p_i + p_j < P \\ \frac{1}{2}\overline{\Pi}^P(P) = P\overline{D}(P)/2 & \text{if} \quad p_i + p_j \geq P. \end{cases}$$

In the second stage, given the pool price P, the patent holder i chooses p_i so as to maximize the sum of them;

$$\Pi_i^I(p_i, p_j|P) := \begin{cases} \Pi(p_i) + \overline{\Pi}_i^C(p_i, p_j) & \text{if} \quad p_i < P - p_j \\ \Pi(p_i) + \frac{1}{2}\overline{\Pi}^P(P) & \text{if} \quad 0 < P - p_j \leq p_i < P \\ \Pi(p_i) + \frac{1}{2}\Pi(P) + \frac{1}{2}\overline{\Pi}^P(P) & \text{if} \quad P - p_j \leq 0 \leq p_i < P \\ \frac{1}{2}\Pi(P) + \frac{1}{2}\overline{\Pi}^P(P) & \text{if} \quad p_i \geq P > p_j \\ \Pi(P) + \frac{1}{2}\overline{\Pi}^P(P) & \text{if} \quad \min\{p_A, p_B\} \geq P. \end{cases}$$

Let $(p_A^*(P), p_B^*(P))$ be a Nash equilibrium price in the second stage given P. In the first stage, the patent holders decide P to achieve the Pareto optimum profits. Formally, we say that price of the pool P^* is supported by a subgame perfect equilibrium if there does not exist P such that

$$\begin{aligned} \Pi_A^I(p_A^*(P), p_B^*(P)|P) &\geq \Pi_A^I(p_A^*(P^*), p_B^*(P^*)|P^*) \\ \Pi_B^I(p_A^*(P), p_B^*(P)|P) &\geq \Pi_B^I(p_A^*(P^*), p_B^*(P^*)|P^*) \end{aligned}$$

where at least either of the inequalities strictly holds.

Recall that the monopoly price in the single market is $1/2$ and that in the bundle market is $a/2b$. Since it is natural to assume that the market size of the bundle market is larger than that of the single market, we assume that $1/2 < a/2b$. In other words, $b < a$. Moreover, we consider the situation in which the single market is sufficiently large and the patent pool does not exclude or ignore the single market. Thus it is natural to assume $a/2b < 1$ or $a < 2b$. Threfore, in the following analysis, we assume that $b < a < 2b$.

Assumption 1 $b < a < 2b$.

4 Equilibrium

4.1 Competitive Licensing

First we consider the equilibrium under the competitive licensing. For patent holder i, given the opponent price p_j, the profit is $\Pi(p_i) + \overline{\Pi}_i^C(p_i, p_j)$. In particular, for $p_i \in [0, \min\{1, a/b - p_j\}]$ the profit maximization becomes

$$\max_{p_i} p_i(1 - p_i) + p_i(a - b(p_i + p_j)).$$

The equilibrium pricing strategy is derived by the first-order condition, $1+a-bp_j-2(1+b)p_i = 0$. Rigorously, we have to care about the case where $p_i > \min\{1, a/b - p_j\}$, but the proof of the following proposition shows that the equilibrium solution can be derived directly from the first-order condition.

Proposition 1 *The equilibrium of licensing strategy (C) is* $p_1 = p_2 = p^C \equiv (1+a)/(2+3b)$.

Proof. See the appendix ■

Since each patent holder considers both the single market and the bundle market, the equilibrium price is different from the monopoly price $1/2$. We can easily show $p^C < 1/2$, that is the single market price becomes lower by the existence of the bundle market. On the other hand, the equilibrium price for the bundle market $2p^C$ is higher than the monopoly price $a/2b$.

4.2 Patent Pool

In the case of patent pool, the patent holders jointly maximize their aggregate profit $2PD(P) + P\overline{D}(P)$ by choosing P, the price for the bundled patent. In particular, for $P \in [0, 1]$, the aggregate profit is $2(1 - P)P + (a - bP)P$. It can be verified that its first order condition satisfies the optimal price while we have to carefully check the case where $P > 1$.

Proposition 2 *When the licensing strategy is (P), the patent holders choose the price as* $P^P = (2 + a)/2(2 + b)$.

Proof. See the appendix. ■

We obtain that $P^P = (2 + a)/2(2 + b) < a/2b$, which means that existence of the single markets pushes the price down from the monopoly price of the bundled market. If the patent holders maximize the profit only in the bundled market, they choose the monopoly price in the bundled market $a/2b$. Here, however, the patent holders account for not only the bundled market but also each of the single markets, and reduce the price to P^P.

Moreover, we can see tht $P^P < 2p^C$, that is the equilibrium price for the bundled market under the patent pool is lower than that under the competitive licensing. In other words, the total surplus from the bundled market becomes higher by formulating the patent pool. The reason is simple. Those two patents are complements for the bundled market, and thus we can aviod the double marginalization problem by formulating the patent pool. This logic is wellknown in the literature.

4.3 Patent Pool with Independent Licensing

In this game, the patent holders form the pool and choose the price of bundled patents P and given P, each of the patent holders noncooperatively and simultaneously choose her own patent price p_i.

Recall that the monopoly prices are $1/2$ in the single markets and $a/2b$ in the bundled market. Then, given the pool's price $P = a/2b$, if each of the patent holders chooses $1/2$ as her own patent price, consumers in the bundled market purchase the patents from the patent pool and those in the single market purchase the patent from the patent holder individually. It means that the patent holders can keep their monopoly profit in all of the markets.

More rigorously, In order to support a separating equilibrium, the following conditions should be satisfied.

(1) Incentive Condition of the Single User $\quad p_i^* < P^*$

(2) Incentive Condition of the Bundle User $\quad p_1^* + p_2^* \geq P^*$

(3)No undercut incentive of a patent holder $p_i^* D(p_i^*) + P^* D(P^*)/2 \geq p_i D(p_i) + p_i \overline{D}(p_i + p_j^*)$ for $\forall p_i < P^* - p_j^*,\ \ i \neq j$

Since $1/2 < a/2b < 1$, the condition (1) and (2) are satisfied. The point is the condition (3). Why does each patent holder have no incentive to undercut the price for the bundle? The key point is that the price of the other patent holder j is too high to undercut the price of pool. Second, each patent holder has its "loyality" users, sets its price is relatively high level, p_j^*.

We can show that such a pair of the prices is supported by a subgame perfect equilibrium.[8]

Proposition 3 *In licensing strategy (I), the price pair of* $p_A = p_B = p^I = 1/2,\ P = P^I = a/2b$ *are supported by a subgame perfect equilibrium. Furthermore, this is a unique symmetric equilibrium.*

Proof. See the appendix. ■

[8] We do not exclude the possibility of the asymmetric equilibrium. However, we believe that the fact that the monopoly prices are chosen in all the market and then the joint profits are maximized is enough for the focal point.

The intuitive reason of this result is quite simple. In the case of patent pool (without independent licensing), the price P must be lower than the monopoly price $a/2b$ to attract the consumers of the single market. In the case of the patent pool with independent licensing, each patent holder can attract the users of its single market by the independent licensing. Hence, the price for the bundled market can be equal to the monopoly price, $a/2b$, and the each independet licesing prices the monopoly price, $1/2$ for its loyal custmers.

Next, we examine the possibility that the patent pool offers menu of contracts. In this case, since there are tree types of users, the patent pool has an incentive to offer the price for the bundle market and the prices for each idividual market. However, the set of the prices, $a/2b$ and $1/2$ are the monopoly price of each market. Hence, even if the patent pool has a chance to offer menu of contracts, the optimal pricing strategy is just equal to the pricing under the patent pool with independent licensing as explained above. In other words, if the pool can ofer menu of contracts it offers the independent licensings. In this situation, the welfare under the patent pool and that under the patent pool with independent licensing must be the same since the equalibrium prices are the same. Even so, we can say that the independent licensing does not improve the social welfare. Moreover, if cumupulsoly independent licensing system is introduced and each patent holder (not patent pool) has to offer independent licesing, the patent pool will not offer the independent licensing and it only offer the price $a/2b$ to the bundled market. Even in this case, of course, the outcome is just same as that explained above.

4.4 Welfare Comparison

Now that we have derived the equilibrium price in each licensing strategy, we will demonstrate the welfare comparison among them.

From the equilibrium price, the supply quantity can be immediately computed as follows;

$$q^C := D\left(p^C\right) = \frac{1+3b-a}{2+3b}, \qquad Q^C := \overline{D}\left(2p^C\right) = \frac{ab+2a-2b}{2+3b},$$
$$q^P := D\left(P^P\right) = \frac{2-a+2b}{2(2+b)}, \qquad Q^P := \overline{D}\left(P^P\right) = \frac{4a-2b+ab}{2(2+b)},$$
$$q^I := D\left(p^I\right) = \frac{1}{2}, \qquad Q^I := \overline{D}\left(P^I\right) = \frac{a}{2}.$$

Let w and W (with the corresponding superscript) be the social welfare of (one of) the single markets and that of the bundled market, which can be computed as follows[9];

$$w^\ell := \int_0^{q^\ell} D^{-1}(q)dq = q^\ell - \frac{q^{\ell 2}}{2}, W^\ell := \int_0^{Q^\ell} \overline{D}^{-1}(Q)dQ = \frac{1}{b}\left(aQ^\ell - \frac{Q^{\ell 2}}{2}\right),$$

for $\ell = C, P, I$. Then the aggregate social welfare is the sum of the welfare of the single markets and of bundle markets, that is, $W^\ell + 2w^\ell$. By comparing this value among $\ell = C, P, I$, we obtain the order of the welfare. First, we can easily show that the welfare under the pool with independent licensing is lower than that under the patent pool. This is a simply application of the welknown result about price descrimination. Since we have assumed linear demand functions and the pool offers even to the individual markets (does not shut down the individual markets), the allowing price discrimination decreases the total welfare. Next we compare the welfare under the pool with independet licensing and that under the competitive licensing. As we have already mentioned $p^C < 1/2$ and $2p^C > a/2b$. Hence, by offering the licensings competitively, the welfare in the individual market is improved but that in the bundled market should be decreased. However, the market size of the bundled market is supposed to be relatively large, the total welfare must be decreased under the competitive licensing. We can summarize these results and show more regorous proof in the appendix.

[9] $D(\cdot)^{-1}$ and $\overline{D}(\cdot)^{-1}$ are the inverse functions with domain $[0,1]$ and $[0,a]$, respectively.

Proposition 4 $W^P + 2w^P > W^I + 2w^I > W^C + 2w^C$.

This result shows that the total welfare can be improved by only allowing the patent pool. This result is natural since the price for the bundled market can be lower by formulating the patent pool. On the other hand, by requiring the indipendent licensing, the total welfare must be decreased. Hence, from this result, we can say that formulating pool is a positive effect on economic welfare but requiring the independent licensing is not a good strategy since it reduces the total welfare.

5 Entry deterrece by Independent Licensing

In this section, we extend the argument in the previous section and show that there is a possibility that the patent pool with independent licensing may realize the “worst outcome"; the welfare under the patent pool with independent licensing is worse than both under competitive licensing and patent pool without independent licensing. In order to consider this possibility, we introduce vertical product differentiation; there are low grade patents, A' and B'. For the bundled market consumers, A' (B') can be a substitute for A (B) in the sense that the combination of A' and B (or A and B') becomes a perfect substitute for the combination of A and B by paying an additional cost C, which, for example, can be interpreted as an opportunity cost for installing the non-standard technology. However, the combination of A' and B' cannot satisfy demands in the bundled market. In this sense, A and B are essential patents in the bundled market. Furthermore, we assume that low grade patent A' (B') cannot be a substitute in the single market A (B). In this sense, the low grade patent holder does not have its own single market.

We maintain Assumption as in the previous section. Furthermore, throughout this section we make the following assumptions.

1. $C^{min} \leq C \leq C^{max}$ where

$$C^{min} \equiv \frac{(1+a) - \sqrt{(1+a)^2 - a - 2b}}{2(1+b)} \text{ and } C^{max} = \min\left\{\frac{1}{2}, \frac{1+a}{2+3b}\right\}.$$

2. Both low grade patent holders A' and B' always choose their price equal to zero.

3. If a consumer in the bundled market is indifferent between purchasing high grade patent $i(= A, B)$ and low grade patent i', then she purchases high grade one.

4. If a high grade patent holder is indifferent between attempting to sell the patent in the bundled market and not, then it chooses the former.

Part 1 restricts the range of the cost. The upper bound of C guarantees the low grade patent holders to be potential competitors in the bundled market. The lower bound of C is for the existence of pure strategy equilibria under competitive licensing and it is satisfied whenever a is sufficiently high (i.e., the market size in the bundled market is sufficiently large) and/or b is sufficiently low. Part 2 drastically simplifies our analysis since we only have to consider the game with two strategic players, patent holder A and B, instead of four players. Although this assumption seems to be restrictive, we can show that under each licensing scheme, there is a equilibrium which induces the same price even if the low grade patent holder strategically chooses its price.[10] Finally, Part 3 and 4 provide the tie-breaking rules.

[10] Roughly speaking, even if the low grade patent holder can be a competitor in the bundled demand market, there is a equilibrium on which none of the consumers in the bundled market purchase the low grade patent. In this case, as in the standard Bertrand competition, the loser chooses its price equal to the marginal cost, which is zero in our model. As a result, the low grade patent holder chooses its price equal to zero. The detail is available upon request. (or in the Appendix?)

5.1 Equilibrium under Competitive Licensing

First we examine the competitive licensing. In this situation, each high grade patent holder faces two markets, the single market and the bundled market. The demand in the bundled market is a little complicated. If patent holder of A (B) chooses its price higher than C, then it cannot get any share from the bundled market since a consumer who is willing to purchase in the bundled market actually purchases A' (B') instead of A (B) Then it must chooses the price equal to or lower than C as long as it tries to sell the patent in the bundled market. Thus, given $p_i \in [0, 1]$, the profit function for the patent holder $i = A, B$ is as follows.

$$\begin{cases} p_i(1 - p_i) + p_i(a - bp_i - b\min\{p_j, C\}) & \text{if } p_i \leq C \text{ and } p_i + \min\{p_j, C\} \leq a/b \\ p_i(1 - p_i) & \text{otherwise.} \end{cases}$$

Obviously, $\arg\max_{p_i} p_i(1-p_i) = 1/2$ and the profit is $1/4$. On the other hand, if there exists $p_i \leq \min\{C, a/b - \min\{p_j, C\}\}$ which attains the profit in the first line greater than $1/4$, then patent holder i chooses it rather than the monopoly price in the single market $1/2$. Actually we can show that both A and B attempt it by choosing the price equal to C.

Proposition 5 *Under competitive licensing, there uniquely exists an equilibrium in which* $(p_A^*, p_B^*) = (C, C)$.

Proof. See the Appendix. ■

By comparing the case without low grade patent holders, we see that the equilibrium price is weakly lower since $C \leq (1 + a)/(2 + 3b)$. It is due to the effect of the competition with the low grade patent holders.

5.2 Equilibrium under Patent Pool with Independent Licensing

As in the case without low grade patent holders, this is a two stage game in which first high grade patent holders A and B form the pool and choose the price of bundled patents P and given P, each of A and B noncooperatively and simultaneously chooses the own patent price p_i. We demonstrate that even with low grade patent holders, patent pool with independent licensing can be a price discrimination scheme to gain a large profit for the high grade patent holders. Nevertheless the competition with the low grade patent holders makes the analysis a little bit complicated.

As in the case without low grade patent holders, it can be shown that independent licensing focuses on the single market and the bundling sales *via* the patent pool focuses on the bundled market. However, due to the existence of the low grade competitor, the bundling sale does not necessarily choose the monopoly price in the bundled market $a/2b$.

Suppose that consumers in the single market purchase the patent from patent holder A or B with price $p_i = 1/2$ and those of the bundled market purchase the patents from the patent pool with price P. Since the latter have no incentive to purchase the patents from the individual suppliers separately, that is

$$P \leq \min \{p_A + p_B, p_A + C, C + p_B\} = \min \left\{1, \frac{1}{2} + C\right\} = \frac{1}{2} + C. \tag{1}$$

Furthermore, if the patent holder $i = A, B$ lowers the independent price below $P - C$, the consumers in the bundled market purchases the patents from patent holder i and j' separately instead of the patent pool and then i's per-unit revenue from the bundled market is p_i instead of $P/2$. Thus the following No Undercut Incentive Condition (NUIC) must be satisfied in order that each patent holder does not compete with the patent pool by undercutting P^*,

$$\frac{P(a-bP)}{2} + \frac{1}{4} \geq (P-C)(a-bP) + (1-P+C)(P-C) \tag{NUIC}$$

The left hand side of is the profit for each patent holder when the independent pricing concentrates on its single market. The right hand side is the profit when the high grade patent holder deviates the equilibrium independent price from $1/2$ to $P-C$. As long as (NUIC) is satisfied, it is the best strategy for each high grade patent holder to concentrate to its own independent market. Furthermore, whenever (NUIC) is satisfied, the low grade patent holders have no chance to sell the patent in the bundled market. We can show that on the equilibrium, the patent pool chooses its price to satisfy (1) and (NUIC).

Note that (NUIC) is transformed to

$$F(P) \equiv (a-bP)\left(\frac{P}{2}-C\right) + (1-P+C)(P-C) \leq \frac{1}{4}. \tag{NUIC'}$$

If $F(a/2b) \leq 1/4$ and $a/2b \leq 1/2 + C$, then patent holder A and B have no incentive to undercut even if $P^* = a/2b$. However, if it is not satisfied, the price offered by the patent pool must be pushed down from the monopoly price $a/2b$. To see it explicitly, let $\tilde{P}(C) \in [0, 1/2 + C]$ satisfying that $F(\tilde{P}(C)) = 1/4$. We can check that such $\tilde{P}(C)$ uniquely exists and if (1) and (NUIC) are not satisfied at $P = a/2b$, then the patent pool reduces the price from $a/2b$ to $\tilde{P}(C) < a/2b$. In summary, the equilibrium under patent pool with independent licensing is as follows.

Proposition 6 *Under patent pool with independent licensing, the equilibrium pricing strategies are* $p_A^* = p_B^* = \frac{1}{2}$ *and* $P^* = \min\{a/2b, \tilde{P}(C)\}$.

The existence of low grade patent holders may lead the patent pool's price to be lower.

The intuitive reason of this result is simple. If the patent pool prices $P^* = a/2b$, there are two possibilities that the patent pool cannot get any profit. First, consumers in the bundled market may have an incentive to buy a low grade patent with the total cost $1/2 + C$. Hence, if $1/2 + C < a/2b$, the patent pool cannot win the competition in the bundled market given $p_A^* = p_B^* = 1/2$. Second, each patent holder may undercut the price of the patent pool. By pricing slightly lower than $P^* - C$, the patent holder can obtain more profit than the profit with the sales *via* patent pool if $P^* > \tilde{P}(C)$. These two effect may reduce the bundling price lower than $a/2b$.

Nevertheless, this transaction price in the bundled market, P^*, may be higher than under the competitive licensing, $2C$. This result is different from the case where there is no low grade competitors, and it has an important implication for the welfare comparison which will be explained below.

5.3 Equilibrium under Patent Pool

In the case of simple patent pool without independent licensing, it is efficient for the patent holders to form the combination of A and B. Hence, the pricing behavior of the patent pool is just same as that in the previous section.

Proposition 7 *The patent holders choose the price as $P^P = (2 + a)/2(2 + b)$, under the patent pool without independent licensing.*

5.4 Welfare Comparison

We now examine the welfare comparison. First, we examine the case of competitive licensing and the case of patent pool with independent licensing. As explained above, the patent price of independent licensing, $1/2$, is higher than the competitive price, C. Hence, we only check whether P^* is higher than $2C$ or not.

Proposition 8 *As long as* $C \leq \frac{a}{4b}$*, the welfare under the patent pool with independent licensing is lower than that under the competitive licensing.*

Proof. Since $F(2C) = (1-C)C < \frac{1}{4}$, $\tilde{P}(C) > 2C$.Thus, $P^* = \min\left[\tilde{P}(C), \frac{a}{2b}\right] > 2C$. Moreover, $p_A^* = p_B^* = \frac{1}{2} > C$. Hence all equilibrium prices are higher under the patent pool with independent licensing.The welfare under the patent pool with independent licensing is lower than that under the competitive licensing. ■

If and only if $C \leq \sqrt{\frac{2+a^2}{8(1+2b)}}$, the welfare under the patent pool with independent licensing is lower than that under the competitive equilibrium. As long as $b < 2$, the above condition is always satisfied. Even if $b \geq 2$, this condition is satisfied as long as $C \leq \frac{a}{4b}$.

A Proofs

A.1 Proof of Proposition 1

Suppose that an equilibrium satisfies $p_A + p_B \geq a/b$. Then there is no demand in the bundled market users and patent holder i's profit is $p_i(1-p_i)$ if $p_i \in [0, a]$ and 0 if $p_i > 1$. As long as $p_i \neq 1/2$, i's profit can be improved by choosing $p_i = 1/2$. Hence, $p_A = p_B = 1/2$ but it implies that $p_A + p_B = 1 < a/b$ by Assummption. Thus, an equilibrium must satisfy $p_A + p_B < a/b$ and we look for the optimal choices of patent prices within the range $p_A + p_B < a/b$. The profit function of patent holder i is

$$\begin{cases} (1-p_i)p_i + (a - b(p_i + p_j))p_i & \text{if} \quad p_i \in [0, 1] \\ (a - b(p_i + p_j))p_i & \text{if} \quad p_i \in [1, a/b - p_j]. \end{cases}$$

However, by the assumption $a < 2b$,

$$\frac{\partial}{\partial p_i}(a - b(p_i + p_j))p_i = a - 2bp_i - bp_j < 0 \quad \text{for } p_i \in [1, a/b - p_j].$$

Hence, we only have to check the following maximization problem for i.

$$\max_{p_i \in [0,1]} ((1 - p_i)p_i + (a - b(p_i + p_j))p_i.$$

From the two first order conditions, $p_A = (1+a-bp_B)/2(1+b)$, and $p_B = (1+a-bp_A)/2(1+b)$, we get $p_A = p_B = (1+a)/(2+3b)$. Note that $p_A = p_B < 1$ and

$$\frac{a}{b} - (p_A + p_B) = \frac{2(a-b) + ab}{(2+3b)b} > 0.$$

A.2 Proof of Proposition 2

The objective function can be described as following

$$\begin{cases} 2(1-P)P + (a - bP)P & \text{if} \quad P \in [0,1] \\ (a - bP)P & \text{if} \quad P \in [1, a/b] \\ 0 & \text{if} \quad P \in [a/b, \infty). \end{cases}$$

First consider

$$\max_{P} 2(1-P)P + (a - bP)P \text{s.t.} P \in [0,1].$$

The first order condition of the above problem gives us the solution $P^P = (2+a)/2(2+b) \in (0,1)$.Obviously $P \in [a/b, \infty)$ is never optimal and since $Max_P\ (a-bP)P = a/2b$, we obtain

$$\begin{aligned} 2(1-P^P)P^P + (a-bP^P)P^P &\geq 2\left(1-\frac{a}{2b}\right)\frac{a}{2b} + \left(a - b\left(\frac{a}{2b}\right)\right)\frac{a}{2b} \\ &> \left(a - b\left(\frac{a}{2b}\right)\right)\frac{a}{2b} \\ &= \max_P (a-bP)P. \end{aligned}$$

This means that $P \in [1, a/b]$ is not optimal and we get that P^P is the optimal price for the pool.

A.3 Proof of Proposition 3

Fix the pool's price $P = a/2b(< 1)$. Then given the opponent price $p_j = 1/2$, the profit of patent holder i is

$$\Pi_i^I\left(p_i, \frac{1}{2}\middle|\frac{a}{2b}\right) := \begin{cases} (1-p_i)p_i + \left(a - b\left(p_i + \dfrac{1}{2}\right)\right)p_i & \text{if} \quad p_i < \dfrac{a}{2b} - \dfrac{1}{2} \\ (1-p_i)p_i + \frac{1}{2}\left(a - b\frac{a}{2b}\right)\frac{a}{2b} & \text{if} \quad \dfrac{a}{2b} - \dfrac{1}{2} \leq p_i < \dfrac{a}{2b} \\ \dfrac{1}{2}\left(1-\dfrac{a}{2b}\right)\dfrac{a}{2b} + \dfrac{1}{2}\left(a - b\dfrac{a}{2b}\right)\dfrac{a}{2b} & \text{if} \quad p_i \geq \dfrac{a}{2b}. \end{cases}$$

If $p_i < \frac{a}{2b} - \frac{1}{2}$, it can get the monopoly profit from its own market $(1-p_i)p_i$ and the profit $(1-p_i)p_i + \left(a - b\left(p_i + \frac{1}{2}\right)\right)p_i$ from the bundled market since $p_i + \frac{1}{2}$ is lower than $P = a/2b$. On the other hand, if $\frac{a}{2b} - \frac{1}{2} \leq p_i < \frac{a}{2b}$, $p_i + \frac{1}{2}$ becomes higher than P and gets the profit of the bundled market from the patent pool, that is $\frac{1}{2}\left(a - b\frac{a}{2b}\right)\frac{a}{2b}$. Moreover, if $p_i \geq \frac{a}{2b}$, even the loyal users do not purchase from the independet license and purchase the patent pool. patent the price of the pool, $\frac{a}{2b}$.

Let define $\pi^1(p_i) = (1-p_i)p_i + \left(a - b\left(p_i + \frac{1}{2}\right)\right)p_i$, $\pi^2(p_i) = (1-p_i)p_i + \frac{1}{2}\left(a - b\frac{a}{2b}\right)\frac{a}{2b}$, and $\pi^3 = \frac{1}{2}\left(1-\frac{a}{2b}\right)\frac{a}{2b} + \frac{1}{2}\left(a - b\frac{a}{2b}\right)\frac{a}{2b}$.

As long as $p_i \leq \frac{a}{2b} - \frac{1}{2}$,

$$\begin{aligned}
\frac{\partial \pi^1}{\partial p_i} &= 1 + a - \frac{b}{2} - 2(1+b)p_i \\
&\geq 1 + a - \frac{b}{2} - 2(1+b)\left(\frac{a}{2b} - \frac{1}{2}\right) \\
&= 2 - \frac{a}{b} + \frac{b}{2} > 0.
\end{aligned}$$

Moreover,

$$\begin{aligned}
\pi^1\left(\frac{a}{2b} - \frac{1}{2}\right) &= \left(1 - \frac{a}{2b} + \frac{1}{2}\right)\left(\frac{a}{2b} - \frac{1}{2}\right) + \left(a - b\frac{a}{2b}\right)\left(\frac{a}{2b} - \frac{1}{2}\right) \\
&< \left(1 - \frac{a}{2b} + \frac{1}{2}\right)\left(\frac{a}{2b} - \frac{1}{2}\right) + \frac{1}{2}\left(a - b\frac{a}{2b}\right)\frac{a}{2b} \\
&= \pi^2\left(\frac{a}{2b} - \frac{1}{2}\right).
\end{aligned}$$

Hence $p_i < \frac{a}{2b} - \frac{1}{2}$ is not optimal. On the other hand, $\arg\max \pi^2(p_i) = \arg\max\,(1-p_i)p_i = 1/2$, and $\frac{a}{2b} - \frac{1}{2} \leq \frac{1}{2} < \frac{a}{2b}$. lastly

$$\begin{aligned}
\pi^2\left(\frac{1}{2}\right) &= \max_{p_i}\left[(1-p_i)p_i\right] + \frac{1}{2}\left(a - b\frac{a}{2b}\right)\frac{a}{2b} \\
&\geq \left(1 - \frac{a}{2b}\right)\frac{a}{2b} + \frac{1}{2}\left(a - b\frac{a}{2b}\right)\frac{a}{2b} \\
&> \frac{1}{2}\left(1 - \frac{a}{2b}\right)\frac{a}{2b} + \frac{1}{2}\left(a - b\frac{a}{2b}\right)\frac{a}{2b} \\
&= \pi^3.
\end{aligned}$$

Thus $p_i = 1/2$ is the best response for $P = a/2b$ and $p_j = 1/2$, and given $P = a/2b$, $p_A = p_B = 1/2$ are the equilibrium behaviors at the second stage. Since both of the prices are monopoly ones in each of the markets, the attained profit must be Pareto optimal. Then this pair of the prices is supported by a subgame perfect equilibrium.

Fix the pool's price $P = a/2b(< 1)$. Then given the opponent price p_j, the profit of patent holder i is

$$\Pi_i^I\left(p_i, \frac{1}{2}\Big|\frac{a}{2b}\right) := \begin{cases} (1-p_i)p_i + (a - b\,(p_i + p_j))\,p_i & \text{if } \; p_i < \dfrac{a}{2b} - p_j \\ (1-p_i)p_i + \frac{1}{2}\left(a - b\frac{a}{2b}\right)\frac{a}{2b} & \text{if } \; \dfrac{a}{2b} - p_j \le p_i < \dfrac{a}{2b} \\ \dfrac{1}{2}\left(1 - \dfrac{a}{2b}\right)\dfrac{a}{2b} + \dfrac{1}{2}\left(a - b\dfrac{a}{2b}\right)\dfrac{a}{2b} & \text{if } \; p_i \ge \dfrac{a}{2b}. \end{cases}$$

If $p_i < \frac{a}{2b} - p_j$, it can get the monopoly profit from its own market $(1-p_i)p_i$ and the profit $(1-p_i)p_i + \left(a - b\left(p_i + \frac{1}{2}\right)\right)p_i$ from the bundled market since $p_i + p_j$ is lower than $P = a/2b$. On the other hand, if $\frac{a}{2b} - p_j \le p_i < \frac{a}{2b}$, $p_i + p_j$ becomes higher than P and gets the profit of the bundled market from the patent pool, that is $\frac{1}{2}\left(a - b\frac{a}{2b}\right)\frac{a}{2b}$. Moreover, if $p_i \ge \frac{a}{2b}$, even the loyal users do not purchase from the independet license and purchase the patent pool. patent the price of the pool, $\frac{a}{2b}$.

Let define $\pi^1(p_i; p_j) = (1-p_i)p_i + (a - b\,(p_i + p_j))\,p_i$, $\pi^2(p_i) = (1-p_i)p_i + \frac{1}{2}\left(a - b\frac{a}{2b}\right)\frac{a}{2b}$, and $\pi^3 = \frac{1}{2}\left(1 - \frac{a}{2b}\right)\frac{a}{2b} + \frac{1}{2}\left(a - b\frac{a}{2b}\right)\frac{a}{2b}$.

As long as $p_i \le \frac{a}{2b} - p_j$,

$$\begin{aligned} \frac{\partial \pi^1}{\partial p_i} &= 1 + a - bp_j - 2(1+b)p_i = 0 \\ &\Leftrightarrow \; p_i = \frac{1 + a - bp_j}{2(1+b)}. \end{aligned}$$

$$\frac{a}{2b} - p_j - \frac{1 + a - bp_j}{2(1+b)} \ge 0 \tag{2}$$

$$\Leftrightarrow \; p_j < \frac{a - b}{b(2+b)}. \tag{3}$$

On the other hand, $\arg\max\ \pi^2(p_i) = \arg\max\ (1-p_i)p_i = 1/2$, and $\frac{a}{2b} - \frac{1}{2} \leq \frac{1}{2} < \frac{a}{2b}$. lastly

$$\begin{aligned}
\pi^2(\frac{1}{2}) &= \max_{p_i}[(1-p_i)p_i] + \frac{1}{2}\left(a - b\frac{a}{2b}\right)\frac{a}{2b} \\
&\geq \left(1 - \frac{a}{2b}\right)\frac{a}{2b} + \frac{1}{2}\left(a - b\frac{a}{2b}\right)\frac{a}{2b} \\
&> \frac{1}{2}\left(1 - \frac{a}{2b}\right)\frac{a}{2b} + \frac{1}{2}\left(a - b\frac{a}{2b}\right)\frac{a}{2b} \\
&= \pi^3.
\end{aligned}$$

$$p_i^*(p_j) = \begin{cases} \frac{1+a-bp_j}{2(1+b)} & \text{if} \quad p_j < \frac{a}{2b} - \frac{1}{2} \\ \frac{1+a-bp_j}{2(1+b)} \text{ or } \frac{1}{2} & \text{if} \quad \frac{a}{2b} - \frac{1}{2} \leq p_j < \frac{a-b}{b(2+b)} \\ \frac{1}{2} & \text{if} \quad p_j \geq \frac{a-b}{b(2+b)} \end{cases}$$

If $p_j < \frac{a-b}{b(2+b)}$, $p_i^*(p_j) \geq \frac{a}{2b} - \frac{a-b}{b(2+b)}$. However, $p_i^*(p_j) \geq \frac{a}{2b} - \frac{a-b}{b(2+b)} > \frac{a-b}{b(2+b)}$ since $\frac{a}{2b} - 2\frac{a-b}{b(2+b)} = \frac{1}{2b(2+b)}\{(a(2+b) - 4(a-b)\} = \frac{1}{2b(2+b)}\{(ab + 2(2b-a)\} > 0$. This means $p_j^*(p_i^*(p_j))$ must be equal to $\frac{1}{2} < \frac{a-b}{b(2+b)}$. Hence $p_j < \frac{a-b}{b(2+b)}$ cannot be an equilibrium price and only $p_i = p_j = \frac{1}{2}$ is the unique equilibrum.

Finally, in the class of symmetric prices, any other pair of the price attains the payoff less than the pair of p^I and P^I. No other pair of the price is chosen on equilibrium.

A.4 Proof of Proposition 4

$$\begin{aligned}
q^C &:= D\left(p^C\right) = \frac{1+3b-a}{2+3b}, \qquad Q^C := \overline{D}\left(2p^C\right) = \frac{ab+2a-2b}{2+3b},\\
q^P &:= D\left(P^P\right) = \frac{2-a+2b}{2(2+b)}, \qquad Q^P := \overline{D}\left(P^P\right) = \frac{4a-2b+ab}{2(2+b)},\\
q^I &:= D\left(p^I\right) = \frac{1}{2}, \qquad Q^I := \overline{D}\left(P^I\right) = \frac{a}{2}.
\end{aligned}$$

$$w^\ell := \int_0^{q^\ell} D^{-1}(q)dq = q^\ell - \frac{{q^\ell}^2}{2}, W^\ell := \int_0^{Q^\ell} \overline{D}^{-1}(Q)dQ = \frac{1}{b}\left(aQ^\ell - \frac{{Q^\ell}^2}{2}\right),$$

The proof consists of two claims.

Step 1: $W^P + 2w^P > W^I + 2w^I$. Since $Q^P > Q^I$ and $q^P < q^I$,

$$\begin{aligned}
W^P - W^I &= \frac{1}{b}\left[a(Q^P - Q^I) - \left(\frac{{Q^P}^2}{2} - \frac{{Q^I}^2}{2}\right)\right]\\
&= \frac{1}{b}(Q^P - Q^I)\left[a - \frac{1}{2}(Q^P + Q^I)\right]\\
&> \frac{1}{b}(Q^P - Q^I)\left[a - \frac{1}{2}(Q^P + Q^P)\right]\\
&= \frac{1}{b}(Q^P - Q^I)(a - Q^P)\\
&= P^P(Q^P - Q^I)
\end{aligned}$$

and

$$\begin{aligned}
w^P - w^I &= \left[q^P - q^I - \frac{q^{P2}}{2} + \frac{{q^I}^2}{2}\right] \\
&= (q^P - q^I)\left[1 - \frac{1}{2}(q^P + q^I)\right] \\
&= -(q^I - q^P)\left[1 - \frac{1}{2}(q^P + q^I)\right] \\
&> -(q^I - q^P)\left[1 - \frac{1}{2}(q^P + q^P)\right] \\
&= -(q^I - q^P)(1 - q^P) \\
&= -P^P(q^I - q^P).
\end{aligned}$$

Then

$$\begin{aligned}
W^P + 2w^P - W^I - 2w^I &> P^P(Q^P - Q^I) - 2P^P(q^I - q^P) \\
&= P^P(q^I - q^P)\left[\frac{Q^P - Q^I}{q^I - q^P} - 2\right]. \qquad (4)
\end{aligned}$$

Finally,

$$\frac{Q^P - Q^I}{q^I - q^P} = \frac{\frac{4a - 2b + ab}{2(2+b)} - \frac{a}{2}}{\frac{1}{2} - \frac{2 - a + 2b}{2(2+b)}} = 2$$

which implies that equation (4) is 0.

Step 2: $W^I + 2w^I > W^C + 2w^C$ Since $Q^I > Q^C$, obviosuly $W^I + 2w^I > W^C + 2w^C$ if $q^I \geq q^C$. Thus, we examine the case in which $q^I < q^C$. First,

$$\begin{aligned}
W^I - W^C &= \frac{1}{b}\left[a(Q^I - Q^C) - \left(\frac{Q^{I2}}{2} - \frac{Q^{C2}}{2}\right)\right] \\
&= \frac{1}{b}(Q^I - Q^C)\left[a - \frac{1}{2}(Q^I + Q^C)\right] \\
&> \frac{1}{b}(Q^I - Q^C)\left[a - \frac{1}{2}(Q^I + Q^I)\right] \\
&= \frac{1}{b}(Q^I - Q^C)(a - Q^I) \\
&= P^I(Q^I - Q^C) \\
&= \frac{a}{2b}(Q^I - Q^C) > 0,
\end{aligned}$$

and

$$\begin{aligned}
w^I - w^C &= \left[q^I - q^C - \frac{q^{I2}}{2} + \frac{q^{C2}}{2}\right] \\
&= (q^I - q^C)\left[1 - \frac{1}{2}(q^I + q^C)\right] \\
&= -(q^C - q^I)\left[1 - \frac{1}{2}(q^I + q^C)\right] \\
&> -(q^C - q^I)\left[1 - \frac{1}{2}(q^I + q^I)\right] \\
&= -(q^C - q^I)(1 - q^I) \\
&= -p^I(q^C - q^I) \\
&= -\frac{1}{2}(q^C - q^I).
\end{aligned}$$

Then

$$\begin{aligned}
W^I + 2w^I - W^C - 2w^C &> \frac{a}{2b}(Q^I - Q^C) - (q^C - q^I) \\
&= \frac{a}{2b}(q^C - q^I)\left[\frac{Q^I - Q^C}{q^C - q^I} - \frac{2b}{a}\right]. \qquad (5)
\end{aligned}$$

Finally,

$$\begin{aligned}
\frac{Q^I - Q^C}{q^C - q^I} - \frac{2b}{a} &= \frac{\dfrac{a}{2} - \dfrac{ab + 2a - 2b}{2 + 3b}}{\dfrac{1 + 3b - a}{2 + 3b} - \dfrac{1}{2}} - \frac{2b}{a} \\
&= \frac{ab - 2a + 4b}{3b - 2a} - \frac{2b}{a} \\
&= \frac{a^2 b + 2(3b - a)(a - b)}{a(3b - 2a)}.
\end{aligned}$$

However, $q^I < q^C \iff 3b > 2a$. Hence, $W^I + 2w^I - W^C - 2w^C > 0$.

B Proofs

The proof consists of several steps.

B.1 Proof of Proposition 5

Step 1: There is a positive amount of transaction in the bundled market. Suppose that no users in the bundled market purchase the patents. Since patent holder A and B gains the profit only from its own single market respectively, A and B should choose $p_A = p_B = 1/2$ to gain the monopoly profit in the single market. Then the user can obtain both patents at most with the price $p_A + p_B = 1$. Since $1 < a/b$, there exist a number of users in the bundled market being willing to purchase both patents with price 1, a contradiction.

Step 2: If high grade patent holder $i = A, B$ does not sell the patent in the bundled market, then $p_i^* = 1/2$. It is straightforward form the discussion in Step 1.

Step 3: If high grade patent holder $i = A, B$ sells the patent in the bundled market on equilibria, then $p_i^* \leq C$. Suppose i sells the patent in the bundled market and $p_i^* > C$.

Note that in this case since patent holder i' does not sell the product, $C+p_j^* \geq p_i^*+\min\{p_j^*, C\}$ for $j = A, B$ and $j \neq i$. Furthermore Step 1 implies that $a/b > p_i^* + \min\{p_j^*, C\}$. If $p_j^* \leq C$, then since $C + p^{j*} < \min\{a/b, p_i^* + \min\{p^{j*}, C\}\}$, there are consumers in the bundled market who purchases i' and j. Then $p_j^* > C$, which implies that consumers in the bundled market purchase not patent j but patent j'. Furthermore the above constraints become $a/b > p_i^* + C$ and $p_j^* \geq p_i^*$. Since patent holder j sells its patent only in the single market, it should choose the monopoly price in the single market, i.e., $p_j^* = 1/2$ and the profit is $1/4$. If patent holder j chooses the price less than p_i^*, the demand in the bundled market purchases patent i' and j instead of i and j' and the profit would be $p_j[1-p_j]+p_j[a-b(p_i^*+p_j)]$. In order to guarantee $p_j^* = 1/2$ on the equilibrium, $1/2 \geq p_i^* > C$ and

$$\frac{1}{4} > \max_{p_j \leq p_i^*} \left[p_j[1-p_j] + p_j[a - b(p_i^* + p_j)]\right].$$

The first order condition implies that

$$\begin{aligned} & argmax_{p_j \leq p_i^*} \left[p_j[1-p_j] + p_j[a - b(p_i^* + p_j)]\right] \\ = \; & \min\left\{\frac{1+a-bp_i^*}{2(1+b)}, p_i^*\right\} = \begin{cases} \dfrac{1+a-bp_i^*}{2(1+b)} & \text{if } p_i^* \geq \dfrac{1+a}{2+3b} \\ p_i^* & \text{if } p_i^* \leq \dfrac{1+a}{2+3b}. \end{cases} \end{aligned}$$

If $p_i^* \geq (1+a)/(2+3b)$, the constraint is equivalent to

$$\frac{1}{4} > \frac{(1+a-bp_i^*)^2}{4(1+b)} \iff \frac{a}{b} + \frac{\sqrt{1+b}+1}{b} > p_i^* > \frac{a}{b} - \frac{\sqrt{1+b}-1}{b}.$$

Since $p_i^* \leq 1/2$,

$$\frac{1}{2} > \frac{a}{b} - \frac{\sqrt{1+b}-1}{b} \iff b - \frac{1}{2}\left(\sqrt{1+b}-1\right)^2 > a$$

implying that $b > a$, which contradicts Assumption **??**. If $p_i^* < (1+a)/(2+3b)$, the constraint is equivalent to

$$\begin{aligned} & \frac{1}{4} > p_i^*[1+a-(1+2b)p_i^*] \\ \iff \quad & p_i^* > \frac{1+a+\sqrt{(1+a)^2-1-2b}}{2(1+2b)} \text{ or } p_i^* < C^{min}, \end{aligned}$$

where it should be noted that $p_i^* > C \geq C^{min}$. However since $(1+a)/(2+3b) > p_i^* > [1+a+\sqrt{(1+a)^2-1-2b}]/[2(1+2b)]$,

$$\begin{aligned} & \left[\frac{1+a}{2+3b} - \frac{1+a}{2(1+2b)}\right]^2 > \left[\frac{\sqrt{(1+a)^2-1-2b}}{2(1+2b)}\right]^2 \\ \iff & 0 > 2(a+b)+(a+b)(a-b)+b\left(2a-\frac{5b}{4}\right)+a^2b, \end{aligned}$$

which is obviously a contradiction.

Step 4: If high grade patent holder $i = A, B$ sells the patent in the bundled market on equilibria, then $p_i^* = C$. Step 3 implies that if patent holder i sells the patent in the bundled market, then $p_i^* \leq C$. Given p_j^* and $p_i \in [0,1]$, the profit for patent holder i is

$$\begin{cases} p_i[1-p_i+a-b(p_i+\min\{p_j^*, C\})] & \text{if } p_i+\min\{p_j^*, C\} \leq a/b \text{ and } p_i \leq C \\ p_i(1-p_i) & \text{otherwise.} \end{cases}$$

Note that $b/a - \min\{p_j^*, C\} - C \geq b/a - 2C \geq b/a - 1 > 0$. Furthermore, $p_i[1-p_i+a-b(p_i+\min\{p_j^*, C\})]$ is concave in p_i and the first order condition implies that $\arg\max_{p_i} p_i[1-$

$p_i + a - b(p_i + \min\{p_j^*, C\})] = (1 + a - b\min\{p_j^*, C\})/[2(1+b)]$. Since

$$\frac{1 + a - b\min\{p_j^*, C\}}{2(1+b)} - C \geq \frac{1 + a - bC}{2(1+b)} - C = \frac{1 + a - (2+3b)C}{2(1+b)} \geq 0$$

due to Assumption 5,

$$\arg\max_{p_i \leq \min\{C, a/b - \min\{p_j^*, C\}\}} p_i[1 - p_i + a - b(p_i + \min\{p_j^*, C\})] = C$$

for any p_j^*.

Step 5: On the equilibrium, $p_A^* = p_B^* = C$. Step 2 and 4 imply that either $(p_i^*, p_j^*) = (1/2, 1/2), (1/2, C), (C, C)$. Suppose that $C \neq 1/2$ and $p_i^* = 1/2$. Note that by Assumption 5, $C < 1/2$ and patent holder i does not sell the patent in the bundled market implying that i's profit from the single market is $1/4$. If i chooses the price equal to C instead of $1/2$, then since the consumers in the bundled market purchase patent i instead of patent i', the profit for patent holder i would be

$$C[1 - C + a - b(C + \min\{p_j^*, C\})] \geq C[1 - C + a - 2bC] \geq 1/4$$

where the last inequality is due to that $C \geq C^{min}$. It means that patent holder i prefers pricing $p_i^* = C$ to $p_i^* = 1/2$ no matter what p_j^* is. Therefore the unique equilibrium is such that $(p_A^*, p_B^*) = (C, C)$ and since the consumers in the bundled market purchase patent A and B, the profit for patent holder A and B is given by $C[1 - C + a - 2bC]$.

B.2 Proof of Proposition 6

Let P^* be the price offered by the patent pool and $p_i^*(P)$ be the equilibrium price of patent holder i given the pool's price P (if exists).

The proof consists of several steps.

Step 1: $p_i^*(P) = C$ when $P > a/b$. If $P > a/b$, consumers in the bundled market never purchase the patents from the patent pool. Thus the independent pricing behaviour is exactly the same as the competitive licensing, implying the result.

Step 2: If $a/b \geq P > \min\{p_A^*(P) + p_B^*(P), p_A^*(P) + C, C + p_B^*(P)\}$, then $\min\{p_A^*(P) + p_B^*(P), p_A^*(P) + C, C + p_B^*(P)\} = p_A^*(P) + p_B^*(P)$. Suppose that $p_i^*(P) + C < \min\{p_A^*(P) + p_B^*(P), p_j^*(P) + C, P\}$ for some $i = A, B$ and $j \neq i$. Then patent holder j sells the patent only in the single market, which implies that $p_j^*(P) = 1/2$ and the profit is $1/4$. Note that since $p_i^*(P) + C < p_A^*(P) + p_B^*(P)$, $C < 1/2$. If j chooses the price p_j less than or equal to C, then since $p_j + p_i^*(P) \leq p_i^*(P) + C < \min\{p_j^*(P) + C, P\}$, the tie-breaking rule implies that consumers in the bundled market purchase patent i and j. Then the profit at $p_j = C$ would be

$$C[1 - C + a - b(C + \min\{p_i^*, C\})] \geq C[1 + a - (1 + 2b)C] \geq \frac{1}{4}$$

where the last inequality is due to that $C \geq C^{min}$. Thus the tie-breaking rule implies that patent holder j has no incentive to keep the price $1/2$, a contradiction.

Step 3: If $a/b \geq P > \min\{p_A^*(P) + p_B^*(P), p_A^*(P) + C, C + p_B^*(P)\}$, then $p_i^*(P) = C$ for $i = A, B$. Step 2 implies that if $a/b \geq P > \min\{p_A^*(P) + p_B^*(P), p_A^*(P) + C, C + p_B^*(P)\}$, then consumers in the bundled market purchase the patents from patent holder A and B

separately and $p_i^*(P) \leq C$ for $i = A, B$, implying that $P > 2C$. Since $p_j^* + C \leq 2C < P$, given any $p_j^*(P) \leq C$, when patent holder i chooses price p_i, the profit is

$$\begin{cases} p_i[1 - p_i + a - b(p_i + p_j^*(P))] & \text{if } p_i \in [0, C] \\ p_i[1 - p_i] & \text{if } p_i \in (C, 1] \\ 0 & \text{if } p_i > 1. \end{cases}$$

Note that the concavity and the fact that $p_j^*(P) \leq C$ imply that $\arg\max_{p_i \in [0,C]} p_i[1 - p_i + a - b(p_i + p_j^*(P))] = \min\{(1 + a - bp_j^*(P))/[2(1 + b)], C\} = C$. Since $C \geq C^{min}$, we see that patent holder i optimally chooses $p_i^*(P) = C$ for any $p_j^*(P) \leq C$. Thus $p_i^*(P) = C$ for $i = A, B$.

Step 4: If $P \leq \min\{p_A^*(P) + p_B^*(P), p_A^*(P) + C, C + p_B^*(P), a/b\}$, then $p_i^*(P) = 1/2$ for $i = A, B$. Since consumers in the bundled market purchase the patents from the patent pool, the independent pricing must focus on the single market, implying that $p_i^*(P) = 1/2$ for $i = A, B$.

Step 5: Given $P \leq \min\{p_A^*(P) + p_B^*(P), p_A^*(P) + C, C + p_B^*(P), a/b\}$, the joint profit between patent holder A and B is maximized at $P = \min\{\tilde{P}(C), a/2b\}$ and $p_i^*(P) = 1/2$ for $i = A, B$. Given $P \leq \min\{p_A^*(P) + p_B^*(P), p_A^*(P) + C, C + p_B^*(P), a/b\}$, Step 4 implies that $p_i^*(P) = 1/2$ for $i = A, B$ and since $C \leq 1/2$, $P \leq 1/2 + C$ implying that $P - C < 1$. When patent holder i independently chooses price p_i, the profit is

$$\begin{cases} \dfrac{P(a - bP)}{2} + p_i[1 - p_i] & \text{if } p_i \in [P - C, 1] \\ p_i[1 - p_i + a - b(p_i + C)] & \text{if } p_i \in [0, P - C) \\ \dfrac{P(a - bP)}{2} & \text{if } p_i > 1. \end{cases}$$

Obviously i prefers $p_i \in [P-C, 1]$ to $p_i > 1$ and since $1/2 \geq P - C$, $\max_{p_i \in [P-C,1]}[P(a - bP)/2 + p_i[1-p_i]] = P(a-bP)/2 + 1/4$ at $p_i = 1/2$. Note that $\arg\max_{p_i} p_i[1 - p_i + a - b(p_i + C)] = [1+a-bC]/(1+b)$ and since $P \leq 1/2 + C \leq 1$, $P - C - (1+a-bC)/(1+b) = P - (1+a+C)/(1+b) < 1 - 1 < 0$. Then the concavity implies that i's profit is maximized at $p_i = 1/2$ if and only if

$$\frac{P(a-bP)}{2} + \frac{1}{4} \geq (P-C)[1-(P-C)+a-b(P-C+C)] = (P-C)[1+a+C-(1+b)P]$$

or P satisfies (NUIC). By solving the quadratic equation, we see that the roots of $F(P) = 1/4$ are

$$\frac{2+a}{2(2+b)} + C \pm \frac{\sqrt{G}}{2+b} \text{ where } G \equiv \left(a - \frac{b}{2} - bC\right)(1-2C) + \left(\frac{a}{2} - bC\right)^2.$$

Note that G is decreasing in $C \in [0, 1/2]$ and when $C = 1/2$, $G > 0$, implying that there are two real number roots of $F(P) = 1/4$. Since $a > b$, the larger root is greater than $1/2 + C$. Thus, given $P \leq 1/2 + C$, (NUIC) is satisfied if and only if $P \leq \tilde{P}(C)$ where $\tilde{P}(C)$ is the lower root.

Suppose that $a/2b \leq 1/2 + C$. Then, since $P(a-bP)/2$ is concave in P, given $P \leq 1/2+C$ and (NUIC), the joint profit between patent holder A and B is obviously maximized at $P = \min\{a/2b, \tilde{P}(C)\}$ and $p_i^*(P) = 1/2$. Suppose by contrast that $a/2b > 1/2 + C$. Then it is maximized at $P = \min\{1/2 + C, \tilde{P}(C)\}$ and $p_i^*(P) = 1/2$. Note that since $a > b + 2bC$,

$$F\left(\frac{1}{2} + C\right) = \frac{1}{8}(2a - b - 2bC)(1-2C) + \frac{1}{4} \geq \frac{1}{4},$$

implying that due to the concavity of $F(P)$, $a/2b > 1/2 + C \geq \tilde{P}(C)$. Then $\min\{1/2 + C, \tilde{P}(C)\} = \min\{a/2b, \tilde{P}(C)\}$. Finally it is easily verified that $\tilde{P}(C) \leq \min\{1, 1/2 + C, a/b\}$.

Step 6: The patent pool chooses the price $P^* = \min\{a/2b, \tilde{P}(C)\}$ and patent holder i chooses $p_i^*(P^*) = 1/2$. When $a/2b \leq \tilde{P}(C)$, the high grade patent holders can achieve the monopoly price in each markets meaning that they can achieve jointly maximum profit.

When $a/2b > \tilde{P}(C)$, Step 1, 3, and 5 imply that the equilibrium pair of the transaction prices in the single market and the bundled market is either $(p, P) = (C, 2C)$ or $(1/2, \tilde{P}(C))$. Note that $\tilde{P}(C) > 0$ and

$$\begin{aligned}
\tilde{P}(C) - 2C &= \frac{2+a}{2(2+b)} + C - \frac{\sqrt{G}}{2+b} - 2C \\
&= \frac{1}{(2+b)(1+a/2-(2+b)C+\sqrt{G})}\left[\left(1+\frac{a}{2}-(2+b)C\right)^2 - G\right] \\
&= \frac{2(2+b)(1/2-C)^2}{(2+b)(1+a/2-(2+b)C+\sqrt{G})} \geq 0,
\end{aligned}$$

which implies that $a/2b > \tilde{P}(C) > 2C$. Since $\tilde{P}(C)$ is closer to the monopoly price in the bundled market than $2C$, price pair $(1/2, \tilde{P}(C))$ generates larger joint profit than $(C, 2C)$.

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