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Optimal Monetary Policy and Transparency under Informational Frictions*

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Abstract

This paper examines optimal monetary policy and central bank transparency in an economy where firms set prices under informational frictions. The economy modeled in this paper is subject to two types of shocks that determine the efficient level of output and firms' desired mark-ups. To minimize the welfare-reducing output gap and price dispersion among firms, the central bank controls firms' incentives and expectations by using a monetary instrument and by disclosing information on the fundamentals. This paper shows that the optimal policy comprises the partial disclosure of information and the adjustment of the monetary instrument contingent on the disclosed information. Under this optimal policy, public information is formed by the weighted difference of the two shocks in order to induce a negative correlation between their conditional expectations, while monetary policy should offset the detrimental effect of such a disclosure policy on price stabilization.

JEL classification codes: E31, E52, D83

Keywords: optimal monetary policy, central bank transparency, policy effectiveness, sticky information

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1 Introduction

Central banks can help stabilize an economy through two channels.[1] The first channel is by controlling market conditions such as money supply and interest rates through monetary instruments, thereby influencing private sector incentives (the monetary channel hereafter). The second is by controlling market expectations about macroeconomic fundamentals through public announcements and information disclosure (the expectations channel). The literature emphasizes the importance of managing market expectations in order to improve the effectiveness of monetary instruments (Woodford (2005)), reduce excess firm collaboration (Morris and Shin (2002)), and increase the credibility of monetary policy (Faust and Svensson (2001)). Despite recent contributions to the body of knowledge on this topic, however, little is known about the optimal policy for both channels.

To bridge this gap in the literature, this paper describes the optimal policy when the central bank uses both these channels. Specifically, the central bank chooses an instrument rule and a disclosure rule, which together determine nominal demand and public information for the realization of the fundamentals. By making its monetary instrument contingent on the realization of shocks, the central bank can thereby reduce welfare losses that occur because of fluctuations in these fundamentals. As pointed out by Baeriswyl and Cornand (2010), however, the central bank's monetary instrument may signal its private information to the private sector and, consequently, cause undesirable fluctuations in prices. Thus, market expectations depend not only on the explicitly disclosed information but also on the implicit signals that accompany the adjustment of the central bank's monetary instrument.

The model economy presented in this paper is based on the static general equilibrium model with flexible prices developed by Adam (2007) and followed by Baeriswyl and Cornand (2010) among others. The economy is subject to two types of shocks: the labor supply shock induces variations in the efficient level of output and the mark-up shock (a shock to product market competition) induces variations in firms' desired mark-ups. Hence, monopolistically competitive firms have incentives to accommodate these shocks as well as nominal demand, which is controlled by the central bank.

Although prices are flexible, monetary policy may be non-neutral due to informational frictions. In the present model, informational frictions stem from the existence of three types of firms that

[1] For recent discussions on central bank transparency in monetary policy, see Geraats (2002) and Blinder et al. (2008).

differ in their degree of information receipt. Fully informed firms observe both the fundamentals and the policy outcomes, partially informed firms observe policy outcomes but not the fundamentals, and uninformed firms observe neither. The idea behind this specification is based on the notion that some firms can quickly incorporate news or announcements by the central bank into their pricing structures, while others may take a longer time to analyze and process the macroeconomic impact of such news or monetary policy. In addition, this specification allows us to clarify the role of each channel. Specifically, the monetary channel affects the incentives of fully and partially informed firms, while the expectations channel affects the beliefs of partially informed firms about the fundamentals. A more detailed discussion on this division is provided in Subsection 2.3.

To understand how the monetary and expectations channels work in the present model, let us begin by discussing the role of fully informed firms. A positive mark-up shock increases the incentives of fully informed firms to raise prices and hence increases price dispersion and reduces the inflation gap.[2] Note that because the efficient allocation is independent of this mark-up shock, it generates inefficient fluctuations in prices (see Angeletos and Pavan (2007)). To drive prices down, the central bank should therefore tighten money supply and decrease nominal demand, whereas it should expand money supply and increase nominal demand in order to bridge the output gap. Hence, fluctuations in prices due to the mark-up shock cannot be fully neutralized through monetary policy, which is the well-known trade-off between price and output stabilization. Consequently, the monetary instrument should accommodate the mark-up shock.

Next, let us focus on the role of partially informed firms. A negative shock to labor supply increases the output gap when nominal demand remains constant, whereas a positive mark-up shock increases the desired mark-up. Therefore, firms raise prices when public information indicates a negative labor supply shock or a positive mark-up shock. One simple way to control prices is to withhold information on the mark-up shock and adjust nominal demand to the labor supply shock. Although such a policy perfectly stabilizes the prices set by partially informed firms, it cannot mitigate welfare losses that occur because of the mark-up shock discussed in the preceding paragraph. Thus, the signaling effects of the monetary instrument create a trade-off between expectations management for partially informed firms and effective monetary policy for fully informed firms. When the central bank reacts more sensitively to the mark-up shock, the monetary instrument becomes

[2] This can be interpreted as a negative shock to product market competition.

a more precise signal about the mark-up shock for partially informed firms, and hence the trade-off between price and output stabilization becomes more severe.

Based on the foregoing, the optimal policy involving both the monetary and the expectations channels is formulated as follows. First, the central bank creates an index based on the weighted difference of the two shocks. It then publicly discloses it and chooses the monetary instrument contingent only on that index. Crucially, the index should be constructed so that partially informed firms cannot distinguish a negative shock to labor supply from a positive mark-up shock. In other words, the optimal policy induces a negative correlation between the conditional expectations of the labor supply shock and those of the mark-up shock. Note that such a disclosure policy increases the price volatility of partially informed firms and decreases the additional price volatility of fully informed firms that respond to the forecast errors of estimates. The optimal policy therefore rests on the flexibility of the monetary instrument. By adjusting nominal demand contingent on the disclosed information, the central bank can offset the detrimental effect of partial disclosure. Thus, the expectations channel is directed at reducing volatility of those prices that cannot be neutralized through the monetary channel. Under the optimal policy, these two channels therefore complement each other.

To examine the role of the monetary instrument in more depth, this paper also describes the optimal disclosure policy when nominal demand is fixed (the inflexible instrument hereafter). Without the flexible monetary instrument, it is shown that the central bank needs to place more weight on controlling the beliefs of partially informed firms. If the share of uninformed firms is sufficiently high, which can be interpreted as a measure of inattentiveness, the optimal disclosure policy is represented by the weighted sum of the shocks, inducing a positive correlation between those conditional expectations.

Firstly, this paper contributes to the literature on central bank transparency by being the first study to present the optimal transparency policy in the general policy space.[3] In the growing debate on the social value of public information, as instigated by the paper of Morris and Shin (2002), most papers focus on the optimal precision of public information about the one dimensional state variable (e.g., Hellwig (2005), Morris and Shin (2007), Cornand and Heinemann (2008)). By contrast, the

[3]Recently, Kamenica and Gentzkow (2011) and Tamura (2012) analyze the optimal information structure in a sender-receiver game.

present paper examines how to control information on multidimensional state variables. To induce coordination among firms, the optimal disclosure rule proposed in the present paper controls the informational content of the public information by mixing the state variables rather than by mixing noises. Hahn (2012, 2013) also explores the interactions between the central bank and price-setting firms in models such as that proposed by Adam (2007) and focus on the optimal discretionary policy and transparency.

Secondly, this paper contributes to the literature on optimal monetary policy under informational frictions. Although informational frictions may stem from different sources, previous studies mostly focus on the case where each agent receives idiosyncratic information on the policy instrument or the fundamentals (e.g., Adam (2007), Baeriswyl and Cornand (2010), and Lorenzoni (2010)).[4] Angeletos and La'O (2012) investigate optimal monetary policy in a general model of informational frictions based on the primal approach. In a related paper, Baeriswyl and Cornand (2010) examine how the signaling effect of the choice of the monetary instrument distorts policy responses to shocks under three transparency regimes: transparent, opaque, and intermediate.[5] Unlike their study, however, the present paper provides an analytical solution to optimal monetary policy that fully internalizes its signaling effects.

The rest of the paper is organized as follows. Section 2 presents the model economy. Section 3 analyzes firms' incentives and describes the equilibrium pricing strategies. Section 4 reformulates the problem and presents the optimal policy. Section 5 examines the optimal transparency policy under the inflexible instrument. Section 6 concludes the paper with discussions about possible extensions.

2 The model economy

An economy is populated by a representative household, a continuum of monopolistically competitive firms, and a central bank, and it is subject to two types of shocks: a labor supply shock y^*, which determines the efficient output level, and a mark-up shock (or real demand shock) u, which

[4] For example, the island model by Lucas (1972), the sticky information model by Mankiw and Reis (2002), and the rational inattention model by Sims (2003) provide the microfoundations of informational frictions.

[5] In their paper, transparency means full disclosure, opacity means no disclosure and a monetary instrument with no signaling effects, and intermediate transparency means no disclosure and a monetary instrument with signaling effects.

affects firms' desired mark-ups. Nominal demand q is determined by the monetary instrument.

2.1 Firms

The pricing rule of firm i under monopolistic competition is given by

$$p_i = \mathbb{E}_i \left[p + \xi (y - y^*) + u \right] \tag{1}$$

where the lower-case letters indicate a percentage deviation from the equilibrium under no uncertainty.[6]

The pricing rule in (1) depends on the expected values of (i) the aggregate price $p = \int p_j dj$, (ii) the real output gap $y - y^*$ where $y = \int y_j dj$, and (iii) the mark-up shock u. The parameter $\xi > 0$ determines the sensitivity of the optimal price to the output-gap. Since the nominal aggregate demand q is expressed as $q = y + p$, the firm's pricing rule is rewritten as

$$p_i = \mathbb{E}_i \left[(1 - \xi) p + \xi q - \xi y^* + u \right]. \tag{2}$$

Throughout this paper, prices are assumed to be strategic complements, i.e., $0 < \xi \leq 1$.

The *labor supply shock* $y^* \sim N(0, \sigma^2_{y^*})$ and the *mark-up shock* $u \sim N(0, \sigma^2_u)$ are assumed to be Gaussian. For simplicity, suppose that $\text{cov}(y^*, u) = 0$.

2.2 The central bank

The central bank maximizes the expected utility of the representative household by adjusting nominal demand and disclosing information. As shown in Adam (2007),[7] maximizing the second-order approximation of the welfare of the representative household is equivalent to minimizing the unconditional expectations of the following loss function:

$$L = (y - y^*)^2 + \frac{\bar{\theta}}{\xi} \int_0^1 (p_j - p)^2 dj$$

[6] Adam (2007) provides the micro-founded derivation.
[7] See Appendix A.2. Equation (66) in his paper.

where $\bar{\theta}$ is the average value of the price elasticity of demand. The welfare loss comes from the output gap and price dispersion.

A *policy* (f, g) consists of an *instrument rule* $f : \mathbb{R}^2 \to \mathbb{R}$ and a *disclosure rule* $g : \mathbb{R}^2 \to M$, which specify the monetary instrument $q = f(y^*, u) \in \mathbb{R}$ and the public information $m = g(y^*, u) \in M$ for each realization of (y^*, u). Note that one can extend the policy space to allow randomization, but this does not affect any results of the present paper.[8] A pair (q, m) represents the policy outcomes.

2.3 Information structure

Following the approach presented by Mankiw and Reis (2002), this paper assumes that information diffuses slowly through the economy. To express this, suppose that while the central bank perfectly observes (y^*, u), firms are divided into three types depending on their information receipt, as noted in Section 1.[9] A fully informed firm observes the fundamentals (y^*, u) and policy outcomes (q, m). A partially informed firm observes only (q, m). An uninformed firm observes neither (y^*, u) nor (q, m). Let $a_f \in [0, 1]$ denote the share of fully informed firms. Similarly, $\alpha_p \in [0, 1]$ is the share of partially informed firms and $\alpha_u \in [0, 1]$ the share of uninformed firms. Note that $\alpha_f + \alpha_p + \alpha_u = 1$.

The parameter α_f captures information stickiness in the spirit of the original work by Mankiw and Reis (2002), while α_u can be interpreted as inattentiveness to public information. One interpretation is that firms are ex ante heterogeneous and have different information-processing capacities. For example, large firms can allocate enough resources to market research and make decisions based on accurate information on the prevailing market conditions. Another example is organizational structure that determines the decision-making procedure. Although not explicitly modeled in the present paper, the central bank's information-processing capacity may matter. For example, a central bank that has a large research staff and takes a short time to process information can frequently change its policy course, suggesting that few firms receive information on the macroeconomic conditions before monetary policy responses have been revealed (i.e., small α_f).

[8] A (generalized) policy $\sigma : \mathbb{R}^2 \to \Delta(\mathbb{R} \times M)$ specifies a joint distribution $g(\cdot|y^*, u) \in \Delta(\mathbb{R} \times M)$ of $(q, m) \in \mathbb{R} \times M$ for each realization of (y^*, u).

[9] As discussed in Section 6, this can be extended to the case where the central bank observes imperfect signals about the fundamentals.

The specification of informational frictions presented in this paper has two main advantages. First, it allows us to obtain equilibrium pricing under any arbitrary policy. Second, it highlights the roles of the monetary and expectations channels; the former affects the incentives of fully informed and partially informed firms, while the latter affects the posterior beliefs of partially informed firms.[10]

Let us introduce some further notations: $\hat{y}^* \equiv \mathbb{E}[y^*|q, m]$ denotes the conditional expectations given policy outcomes (q, m) and $\Delta_{y^*} \equiv y^* - \hat{y}^*$ the residual. Other variables, $\hat{u}$ and Δ_u, are similarly defined.

2.4 Timing of events

The sequence of events is as follows. First, the central bank publicly chooses its policy (f, g). Second, the nature draws the labor supply shock y^* and mark-up shock u. Third, the central bank observes (y^*, u) and chooses the monetary instrument $q = f(y^*, u)$ and public information $m = g(y^*, u)$. Finally, firms simultaneously set their prices: fully informed firms condition their choices on (y^*, u) and (q, m), whereas partially informed firms condition on (q, m) and uninformed firms only on their prior information.

3 Equilibrium pricing

This section derives firms' pricing strategies in equilibrium. To understand the basic working of the model, Subsection 3.1 examines the incentives of each type of firm, while Subsection 3.2 describes the pricing strategies in equilibrium and the aggregate price level as a function of market expectations about the fundamentals. Throughout this section, the policy (f, g) is fixed.

3.1 Firms' incentives

3.1.1 Fully informed firms

First, the incentives of fully informed firms are analyzed. Because such firms know all publicly available information, their expectations about the aggregate price level must be consistent with

[10] As seen below, monetary neutrality holds if $\alpha_u = 0$ and information disclosure is ineffective if $\alpha_p = 0$.

the actual realization. Therefore, the pricing rule (2) is expressed as

$$p_f = (1-\xi)p + \xi q - \xi y^* + u. \tag{3}$$

As a benchmark, consider the case of $\alpha_f = 1$. In the symmetric equilibrium $p_f = p$, it follows from pricing rule (3) that

$$(y - y^*) = -\frac{u}{\xi}.$$

Note that the output gap and price dispersion are determined independent of the policy. In other words, the central bank has no influence over the real economy when all firms are fully informed. Hence, any welfare loss arises from firms' responses to the mark-up shock.

3.1.2 Partially informed firms

Each partially informed firm $i \in I_p$ observes only the realization of (q, m) and then forms conditional expectations of y^*, u, and p. Pricing rule (2) is expressed as

$$p_p = (1-\xi)\hat{p} + \xi q - \xi \hat{y}^* + \hat{u}. \tag{4}$$

As in the previous benchmark, consider the case of $\alpha_p = 1$. In the symmetric equilibrium $p_p = p$, the output gap is given by

$$(y - y^*) = -\frac{\hat{u}}{\xi} - (y^* - \hat{y}^*).$$

Again, the monetary channel is ineffective, whereas the expectations channel is effective. The output gap depends on the conditional expectations of the fundamentals, and hence the central bank can reduce losses by controlling the information available to partially informed firms. When the central bank chooses full disclosure $g(y^*, u) = (y^*, u)$, which induces $\hat{y}^* = y^*$ and $\hat{u} = u$ for every (y^*, u), the expected loss is given by $\mathrm{var}(u)/\xi^2$, which coincides with the first benchmark case. Under a disclosure rule $g(y^*, u) = y^*$, which induces $y^* = y^*$ and $\hat{u} = \mathbb{E}[u] = 0$ for every (y^*, u), the central bank can achieve the first-best outcome (i.e., zero welfare loss).

3.1.3 Uninformed firms

Each uninformed firm $i \in I_u$ chooses its price so that

$$p_u = (1-\xi)\mathbb{E}[p] + \xi\mathbb{E}[q] - \xi\mathbb{E}[y^*] + \mathbb{E}[u]. \tag{5}$$

When $\alpha_u = 1$, the symmetric equilibrium $p_u = p$ implies

$$y = q - \mathbb{E}[q].$$

Hence, by choosing an instrument rule $f(y^*, u) = y^*$, the central bank achieves the first-best outcome. Note that monetary policy is non-neutral as Lucas (1972) pointed out, while the disclosure rule has (trivially) no effects.

3.2 Equilibrium

From (3) - (5) with $p = \alpha_f p_f + \alpha_p p_p + \alpha_u p_u$, a Bayesian Nash equilibrium is characterized as follows.

Lemma 1 *Pricing strategies and the aggregate price level in equilibrium are as follows*

$$\begin{aligned}
p_u &= \bar{q} && (6)\\
p_p &= p_u + \frac{\lambda}{1-\alpha_u}\left((q-\bar{q}) - \hat{y}^* + \frac{1}{\xi}\hat{u}\right) && (7)\\
p_f &= p_p + \frac{\kappa}{\alpha_f}\left(-\Delta_{y^*} + \frac{1}{\xi}\Delta_u\right) && (8)\\
p &= \bar{q} + \lambda\left((q-\bar{q}) - \hat{y}^* + \frac{1}{\xi}\hat{u}\right) + \kappa\left(-\Delta_{y^*} + \frac{1}{\xi}\Delta_u\right) && (9)
\end{aligned}$$

where $\lambda = \frac{(1-\alpha_u)\xi}{(1-\alpha_u)\xi+\alpha_u}$ *and* $\kappa = \frac{\alpha_f\xi}{\alpha_f\xi+(1-\alpha_f)}$.

Proof. See Appendix A.1. ■

The aggregate price level depends on nominal demand q and market expectations $\hat{y}^*$ and $\hat{u}$, which are observed by both fully and partially informed firms, as well as on the residuals Δ_{y^*} and Δ_u, which are observed only by fully informed firms. The two parameters λ and κ, which are determined by the distribution of types $(\alpha_u : \alpha_p : \alpha_f)$ and the degree of strategic complementarity

($0 \leq 1-\xi < 1$), measure the sensitivities of the aggregate price level to these factors.[11] The price of partially informed firms becomes more volatile as α_u decreases.[12] Similarly, the volatility of $(p_f - p_p)$, which is induced by the variability of the residuals, increases with α_f. Intuitively, as more firms respond to the monetary instrument and to the fundamentals, firms have strong incentives to respond to them due to strategic complementarities.

Several remarks should be made at this point about monetary (non-)neutrality. First, an increase in $\bar{q}$ affects neither the expected output gap nor the degree of price dispersion since it is exactly canceled out by an increase in prices Therefore, it can be assumed that $\bar{q} = 0$ without loss of generality. Second, monetary neutrality holds if there are no uninformed firms ($\alpha_u = 0$). In this case, $\lambda = 1$, meaning that an increase in q by one unit results in an increase in p_f and p_p (and hence p when $\alpha_u = 0$) by one unit. Conversely, the monetary instrument is non-neutral and affects the real economy whenever there are uninformed firms.

3.3 Welfare loss in equilibrium

Under equilibrium pricing (6)–(9), the output gap volatility is written as

$$\mathbb{E}(y-y^*)^2 = \mathbb{E}\left((1-\lambda)(q-\hat{y}^*) - \lambda\frac{\hat{u}}{\xi}\right)^2 + \mathbb{E}\left((1-\kappa)\Delta_{y^*} + \kappa\frac{\Delta_u}{\xi}\right)^2. \tag{10}$$

Note that no disclosure induces $\hat{y}^* = \hat{u} = 0$, while full disclosure induces $\Delta_{y^*} = \Delta_u = 0$.

Similarly, the unconditional expectation of price dispersion is written as

$$\begin{aligned}\frac{\bar{\theta}}{\xi}\mathbb{E}\left[\int_0^1 (p_j - p)dj\right] =& \frac{\bar{\theta}}{\xi}\alpha_u(1-\alpha_u)\mathbb{E}(p_p - p_u)^2 + \frac{\bar{\theta}}{\xi}\alpha_f(1-\alpha_f)\mathbb{E}(p_f - p_p)^2 \\ =& \bar{\theta}\lambda(1-\lambda)\mathbb{E}\left(q - \hat{y}^* + \frac{\hat{u}}{\xi}\right)^2 + \bar{\theta}\kappa(1-\kappa)\mathbb{E}\left(-\Delta_{y^*} + \frac{\Delta_u}{\xi}\right)^2. \end{aligned} \tag{11}$$

Thus, the central bank should reduce the volatility of $(p_p - p_u)$, which is induced by fluctuations in conditional expectations, as well as that of $(p_f - p_p)$, which is induced by fluctuations in the residuals. Note that $\mathbb{E}(p_p - p_u)^2$ is minimized under no disclosure, while $\mathbb{E}(p_f - p_p)$ is minimized under full disclosure.

[11]Indeed, λ and κ are increasing with ξ. Note that $\lambda, \kappa \in [0,1]$ and that $\lambda \geq \kappa$.

[12]Note that the coefficient in (7), $\lambda/(1-\alpha_u) = \xi/((1-\alpha_u)\xi + \alpha_u)$, is decreasing in α_u if $\xi \in (0,1)$.

4 Optimal policy

The central bank's problem is formulated as follows:

$$\min_{(f,g)} \ \mathbb{E}\left[(y-y^*)^2 + \frac{\bar{\theta}}{\xi}\int_0^1 (p_j - p)^2 dj\right]$$
$$\text{s.t.} \ \ (6)-(9) \text{ and } y = q - p.$$

In general, the first-order approach in the standard optimal control problem cannot be applied since the optimal instrument rule must internalize its signaling effects. In other words, the information revealed through the monetary instrument is determined by the entire policy and not by its local conditions. By virtue of the generality of disclosure rules, however, the choice of an instrument rule can be restricted to the class of functions that depend only on public information m.

Lemma 2 *In the set of optimal policies, if nonempty, there exists a policy (f, g) such that the instrument rule f can be written as a function of public information $m = g(y^*, u)$ (i.e., $f(y^*, u) = h(g(y^*, u))$ for some $h : M \to \mathbb{R}$).*

Intuitively, the optimal instrument rule can be designed in order to influence the economy only through the monetary channel. Formally, for any policy (f, g), there is a policy $(\tilde{f}, \tilde{g})$ such that $\tilde{g}$ reveals the same information $\tilde{m} = (q, m)$ revealed under (f, g), while $\tilde{f}$ specifies the same level of nominal demand q as f chooses.[13]

Obviously, $(\tilde{f}, \tilde{g})$ induces the same outcome for the realization of each fundamental but $\tilde{f}$ has no signaling effects in the sense that it reveals no additional information on the fundamentals. Hence, the optimal policy can be represented by the pair (g, h) where $g : \mathbb{R}^2 \to M$ is a disclosure rule and $h : M \to \mathbb{R}$ is an instrument rule.

In Subsection 4.1, the optimal instrument rule $h : M \to \mathbb{R}$ given an arbitrary disclosure rule g is described. In Subsection 4.2, the "indirect" loss function, which is a function of the disclosure rule, is derived and the optimal disclosure rule obtained.

[13] Fix a policy (f, g). Now consider a policy $\tilde{f} : \mathbb{R}^2 \to \mathbb{R}$ and $\tilde{g} : \mathbb{R}^2 \to \tilde{M}$ with $\tilde{M} = \mathbb{R} \times M$ such that $\tilde{f}(y^*, u) = f(y^*, u)$ and $\tilde{g}(y^*, u) = (f(y^*, u), g(y^*, u)) \in \tilde{M}$. Then, the instrument rule $\tilde{f}$ can be written as a function of $\tilde{m} = \tilde{g}(y^*, u)$. That is, $\tilde{f} = h \circ \tilde{g}$ where $h(q, m) = q$.

4.1 Optimal instrument rule

Fix a disclosure rule. From Lemma 2, this section addresses the optimal instrument rule $h : M \to \mathbb{R}$. The optimal instrument rule solves the following problem for each realization of m:

$$\begin{aligned} \min_q \quad & \mathbb{E}\left[(y-y^*)^2 + \frac{\bar{\theta}}{\xi}\int_0^1 (p_j - p)^2 dj | m\right] \\ \text{s.t.} \quad & (6) - (9) \text{ and } y = q - p. \end{aligned}$$

From (10) and (11), the above problem is equivalent to

$$\min_q \quad \left((1-\lambda)(q-\hat{y}^*) - \lambda\frac{\hat{u}}{\xi}\right)^2 + \bar{\theta}\lambda(1-\lambda)\left(q - \hat{y}^* + \frac{\hat{u}}{\xi}\right)^2.$$

Since the problem is concave, the first-order condition fully describes the optimal instrument rule.[14]

Proposition 1 *Under the optimal instrument rule, nominal demand is determined to satisfy*

$$q = \hat{y}^* - \gamma\frac{\hat{u}}{\xi} \tag{12}$$

where $\gamma \equiv \frac{(\bar{\theta}-1)\lambda}{(\bar{\theta}-1)\lambda+1}$.

Proof. See Appendix A.2. ∎

Given a disclosure rule, welfare losses come from the volatility in conditional expectations $(\hat{y}^*, \hat{u})$ and residuals (Δ_{y^*}, Δ_u). The optimal instrument rule should minimize welfare losses due to $(\hat{y}^*, \hat{u})$. When nominal demand satisfies (12), the "indirect" loss function is given by

$$\begin{aligned} L = & \frac{\bar{\theta}\lambda}{\bar{\theta}\lambda + (1-\lambda)}\left(\frac{\hat{u}}{\xi}\right)^2 + \frac{\bar{\theta}\kappa}{\bar{\theta}\kappa + (1-\kappa)}\left(\frac{\Delta_u}{\xi}\right)^2 \\ & + \frac{1-\kappa}{\bar{\theta}\kappa + 1 - \kappa}\left(\left(\bar{\theta}\kappa + 1 - \kappa\right)\Delta_{y^*} - (\bar{\theta}-1)\kappa\frac{\Delta_u}{\xi}\right)^2 + t.i.p. \end{aligned} \tag{13}$$

where $t.i.p.$ is such that $\mathbb{E}(t.i.p.) = 0$ for any disclosure rule. Note that by choosing a disclosure rule, the central bank controls the joint distribution of $(\hat{u}, \Delta_{y^*}, \Delta_u)$.

[14] If $\lambda = 1$, or equivalently if $\alpha_u = 0$, then any level of q leads to the same welfare loss. In other words, monetary neutrality holds when there are no uninformed firms.

It may be useful to consider benchmark cases before presenting the optimal disclosure rule. When $\alpha_f = 0$, the indirect loss function is

$$L = \frac{\bar{\theta}\xi(1-\alpha_u)}{\bar{\theta}\xi(1-\alpha_u)+\alpha_u}\left(\frac{\hat{u}}{\xi}\right)^2 + \Delta_{y^*}^2 + t.i.p.$$

By choosing $g(y^*, u) = y^*$, the central bank can achieve the first-best as in Benchmarks 2 and 3. By contrast, when $\alpha_f > 0$ (and hence $\lambda, \kappa > 0$), the first-best tends to require $\hat{u}^2 = 0$ and $\Delta_u^2 = 0$, which is clearly impossible unless $\sigma_u^2 = 0$.

When $\alpha_p = 0$ (or equivalently $\alpha_f = 1 - \alpha_u$), the central bank does not need to take account of the signaling effects of the monetary instrument. Therefore, the optimal monetary policy can be conducted under full disclosure. Thus, the unconditional expected loss is $\mathbb{E}L = \frac{\bar{\theta}\xi\alpha_f}{\bar{\theta}\xi\alpha_f+1-\alpha_f}\left(\frac{\mathrm{var}(u)}{\xi^2}\right)$, which concurs with the findings in Baeriswyl and Cornand's (2010) benchmark case.

4.2 Optimal disclosure rule

Let us now return to the original problem. Note that according to the law of iterated expectations, the variances and the covariances of the residuals are written as follows: $\mathbb{E}[\Delta_{y^*}^2] = \mathbb{E}[(y^* - \hat{y}^*)^2] = \mathbb{E}[y^*] - \mathbb{E}[(\hat{y}^*)^2]$ and $\mathbb{E}[\Delta_u^2] = \mathbb{E}[u^2] - \mathbb{E}[\hat{u}^2]$, and $\mathbb{E}[\Delta_{y^*}\Delta_u] = \mathbb{E}[y^* u] - \mathbb{E}[\hat{y}^*\hat{u}]$. Then, the unconditional expectations of the indirect loss function (13) are expressed as

$$\mathbb{E}L = -\mathbb{E}\Phi(\hat{y}^*, \hat{u}/\xi) + \widehat{t.i.p.}$$

where $\Phi(\hat{y}^*, \hat{u})$ is the quadratic function given below and $\widehat{t.i.p.}$ denotes terms that are independent of policy:[15]

$$\begin{aligned}\Phi(\hat{y}^*, \hat{u}/\xi) =& (1-\kappa)(\bar{\theta}\kappa + 1 - \kappa)(\hat{y}^*)^2 - 2(\bar{\theta}-1)\kappa(1-\kappa)(\hat{y}^*\hat{u}/\xi) \\ &+ \left[\kappa(\kappa + \bar{\theta}(1-\kappa)) - \frac{\bar{\theta}\lambda}{\bar{\theta}\lambda + 1 - \lambda}\right](\hat{u}/\xi)^2 \end{aligned} \tag{14}$$

Hence, the optimal disclosure rule is the solution to $\max_g \mathbb{E}\Phi(\hat{y}^*, \hat{u}/\xi)$.

[15]Explicitly, this is given by $\widehat{t.i.p.} = \frac{\bar{\theta}\kappa}{\bar{\theta}\kappa+1-\kappa}(u/\xi)^2 + \frac{1-\kappa}{\bar{\theta}\kappa+1-\kappa}\left[(\bar{\theta}\kappa + 1 - \kappa)y^* - (\bar{\theta}-1)\kappa(u/\xi)\right]^2$.

Let $H = \begin{pmatrix} H_{11} & H_{12} \\ H_{12} & H_{22} \end{pmatrix}$ be the Hessian matrix of Φ. It can then be shown that H has a unique positive eigenvalue. From Tamura (2012) follows the next proposition.[16]

Proposition 2 *If y^* and u are normally distributed, then the optimal disclosure rule reveals a weighted difference in the shocks given by*

$$g(y^*, u) = b_{y^*}\sigma_{y^*}^{-1}y^* - b_u\sigma_u^{-1}u \tag{15}$$

where $(b_{y^*}, -b_u)$ is the eigenvector associated with the unique positive eigenvalue of $\Sigma^{\frac{1}{2}}H\Sigma^{\frac{1}{2}}$, and $\Sigma = \mathrm{var}(y^*, (u/\xi))$. Specifically, the weights (b_{y^*}, b_u) satisfy $b_{y^*}, b_u \geq 0$, $b_{y^*}^2 + b_u^2 = 1$ and $b_{y^*}/b_u = B + \sqrt{B^2+1}$ where

$$B = \frac{\sigma_{y^*}^2 H_{11} - (\sigma_u/\xi)^2 H_{22}}{-2\sigma_{y^*}(\sigma_u/\xi)H_{12}}. \tag{16}$$

Proof. See Appendix A.3. ■

The most important property of the optimal disclosure rule is that it induces a *negative correlation between the estimates*, i.e., $\mathrm{cov}(\hat{y}^*, \hat{u}) < 0$. This approach may seem to be detrimental to price stabilization since the volatilities of p_p and p_f in equilibrium are both decreasing in the covariance between the estimates. However, the monetary instrument can perfectly offset the effects of $\hat{y}$. Then, it becomes more important to reduce the fluctuations in $(p_f - p_p)$ generated by the variability of the residuals. Creating a positive correlation between Δ_{y^*} and Δ_u results in a negative correlation between $\hat{y}^*$ and $\hat{u}$. Intuitively, the disclosure rule should therefore be designed to reduce the variability of $(p_f - p_p)$ that cannot be offset by the monetary instrument.

In the next step, we assess how informational frictions affect the optimal communication policy. In the present model, the degree of informational frictions is characterized by parameters $(\alpha_u, \alpha_p, \alpha_f)$. As discussed in Section 2, α_u is interpreted as the degree of inattentiveness to policy outcomes and $1-\alpha_f$ as the stickiness of information on the prevailing macroeconomic conditions. It can thus be shown that B in (16) is increasing in $\lambda = (1-\alpha_u)\xi/((1-\alpha_u)\xi + \alpha_u)$ and decreasing in $\kappa = \alpha_f\xi/(\alpha_f\xi + 1 - \alpha_f)$. Hence, an increase in $1-\alpha_u$ increases b_{y^*}/b_u, while an increase in α_f

[16] Note that $H_{12} = -(\bar{\theta}-1)\kappa(1-\kappa)$ is negative when $\bar{\theta} > 1$ and $1 > \alpha_f > 0$. Since the optimal policy for $\alpha_f = 1$ or $\alpha_f = 0$ has already been checked, I focus here on the case of $\alpha_f \in (0, 1)$.

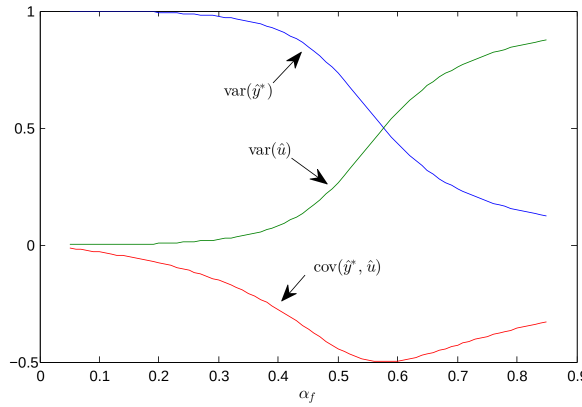


Figure 1: Second moments of the estimates ($\alpha_u = 0.1$)

decreases b_{y^*}/b_u. The next proposition summarizes these comparative statics.

Proposition 3 *Under the optimal policy, the public information reveals more about the labor supply shock and less about the mark-up shock ($b_{y^*}/b_u \uparrow$) if the degree of inattentiveness decreases ($\alpha_u \downarrow$) or information stickiness increases ($\alpha_\kappa \downarrow$).*

The intuition behind this finding is rather straightforward. A rise in α_p increases the costs of information revelation on the mark-up shock. Therefore, the central bank should decrease the relative weight placed on the mark-up shock. One implication of Proposition 3 is that it is important to distinguish two types of informational frictions, namely inattentiveness and information stickiness.

Figure 1 illustrates how the share of fully informed firms affects the second moments of the estimates induced by the optimal disclosure rule. Since m is a linear combination of (y^*, u), the variances of $\hat{y}^*$ and $\hat{u}$ satisfy the following property:

$$\frac{\mathrm{var}(\hat{y}^*)}{\mathrm{var}(y^*)} + \frac{\mathrm{var}(\hat{u})}{\mathrm{var}(u)} = 1.$$

As α_p decreases and α_f increases, it becomes less important to withhold information on u. These effects lead to the monotonic change in $\mathrm{var}(\hat{y}^*)$ and $\mathrm{var}(\hat{u})$ described in Proposition 3.

4.3 Discussion

From Propositions 1 and 2, the optimal policy pair (g, h) can expressed as the following two-step policy:

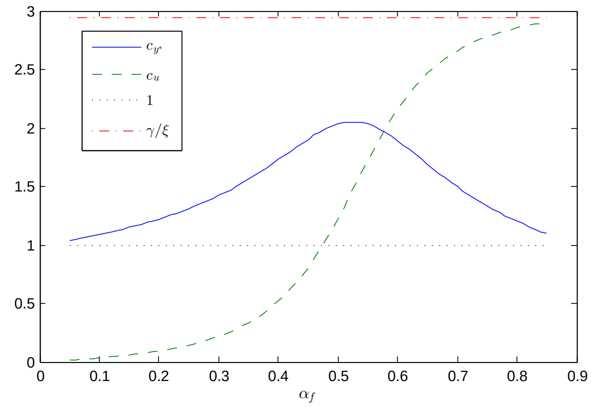


Figure 2: Effective policy responsiveness ($\alpha_u = 0.1$)

Step 1. Disclose a policy index of the shocks $m = g(y^*, u) = b_{y^*}\sigma_{y^*}^{-1}y^* - b_u\sigma_u^{-1}u$.

Step 2. Adjust the monetary instrument contingent on that index so that $q = h(m) = \beta m$ where $\beta = b_{y^*}\sigma_{y^*} + \gamma b_u(\sigma_u/\xi)$.[17]

The optimal instrument rule is essentially described as a linear function of the fundamentals,

$$f(y^*, u) = h(g(y^*, u)) = c_{y^*}y^* - c_u u \tag{17}$$

where $c_{y^*} = \beta b_{y^*}\sigma_{y^*}^{-1}$ and $c_u = \beta b_u\sigma_u^{-1}$. By construction, this contains the same information as that revealed by the optimal policy pair (g, h). Thus, an alternative form of the optimal policy pair (f, g) is obtained.

Corollary 1 *An instrument rule* $f(y^*, u) = c_{y^*}y^* - c_u u$ *with no disclosure* $g(y^*, u) = 0$ *is also optimal.*

The coefficients (c_{y^*}, c_u) represent the effective responsiveness of the monetary instrument to the fundamentals and are illustrated in Figure 2. As pointed out by Baeriswyl and Cornand (2010), the central bank distorts its policy response in order to prevent information revelation about the

[17] Since m is a linear combination of the two shocks, (y^*, u, m) are normally distributed. Therefore, the conditional expectation is given by

$$\mathbb{E}\left[\begin{pmatrix} y^* \\ u \end{pmatrix} | m\right] = \begin{pmatrix} b_{y^*}\sigma_{y^*} \\ b_u\sigma_u \end{pmatrix} m.$$

By plugging this into (12), the optimal instrument rule $q = \beta m$ is obtained.

mark-up shock. As α_f increases, the monetary instrument should offset the responses of fully informed firms to u. However, information revelation about u to partially informed firms reduces the effectiveness of the monetary channel when α_p is high (or α_f is low). To reduce detrimental signaling effects, the optimal instrument rule thus becomes more responsive to y^*, thereby changing the informational content of the monetary instrument.

5 Optimal disclosure policy when under the inflexible instrument

The choice of a disclosure rule changes the distribution of market expectations about the fundamentals. As discussed above, the optimality of a negative correlation between $\hat{y}^*$ and $\hat{u}$ relies on the fact that the central bank optimally controls the monetary instrument. In other words, it may be optimal to induce a positive correlation between $\hat{y}^*$ and $\hat{u}$ when the monetary instrument cannot offset fluctuations in p_p. To examine this possibility, this section considers the case where the monetary instrument is *inflexible* and the only means available to the central bank is to control information revelation to the private sector.

Suppose the monetary instrument is *inflexible* in the sense that nominal demand is fixed as a constant value $q = 0$. Then, the loss function (denoted by L_0) is given by

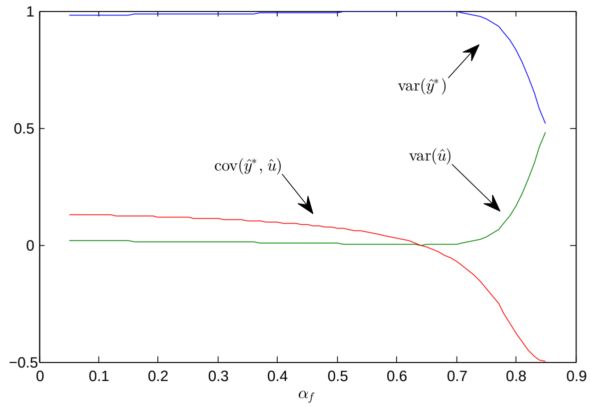
$$\begin{aligned} L_0 = & \frac{1-\lambda}{\bar{\theta}\lambda + 1 - \lambda}\left(\left(\bar{\theta}\lambda + 1 - \lambda\right)\hat{y}^* - (\bar{\theta}-1)\lambda\frac{\hat{u}}{\xi}\right)^2 + \frac{\bar{\theta}\lambda}{\bar{\theta}\lambda + (1-\lambda)}\left(\frac{\hat{u}}{\xi}\right)^2 \\ & + \frac{\bar{\theta}\kappa}{\bar{\theta}\kappa + (1-\kappa)}\left(\frac{\Delta_u}{\xi}\right)^2 + \frac{1-\kappa}{\bar{\theta}\kappa + 1 - \kappa}\left(\left(\bar{\theta}\kappa + 1 - \kappa\right)\Delta_{y^*} - (\bar{\theta}-1)\kappa\frac{\Delta_u}{\xi}\right)^2 + t.i.p. \end{aligned} \tag{18}$$

where $t.i.p.$ is again the collection of terms whose expected values equal zero under any disclosure rule. The first term in (18) can be interpreted as the *value of the monetary channel* since that term disappears in (13). As before, the unconditional expected loss is

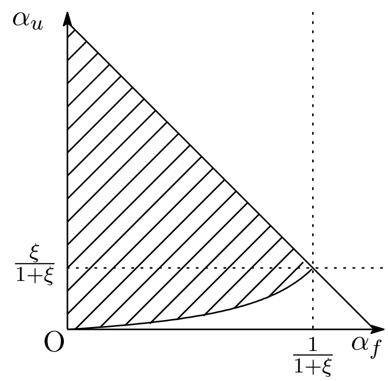
$$\mathbb{E}L_0 = -\mathbb{E}\Phi_0(\hat{y}^*, \hat{u}/\xi) + \widehat{t.i.p.}$$

where Φ_0 is a quadratic function defined by

$$\Phi_0(\hat{y}^*, \hat{u}/\xi) = \left(\hat{y}^* - (\hat{u}/\xi)\right)\left(d_{y^*}\hat{y}^* + d_u(\hat{u}/\xi)\right) \tag{19}$$



(a) Second moments of the estimates ($\alpha_u = 0.1$)



(b) Condition for a positive correlation

Figure 3: Properties of optimal disclosure rule with the inflexible instrument

with coefficients $d_{y^*} = (\lambda-\kappa)-(\bar{\theta}-1)[(1-\lambda)\lambda-(1-\kappa)\kappa]$ and $d_u = (\lambda-\kappa)+(\bar{\theta}-1)[(1-\lambda)\lambda-(1-\kappa)\kappa]$. It can then be shown that if $\lambda = \kappa$ (i.e., $\alpha_p = 0$), then $\Phi_0(\hat{y}^*, \hat{u}/\xi) = 0$ for all $(\hat{y}^*, \hat{u})$. Otherwise, Φ_0 is neither concave nor convex (or equivalently, its Hessian matrix $H_0 \equiv \partial^2\Phi_0/\partial\hat{y}^*\partial(\hat{u}/\xi)$ is indefinite). Hence, the optimal disclosure rule is characterized as follows.

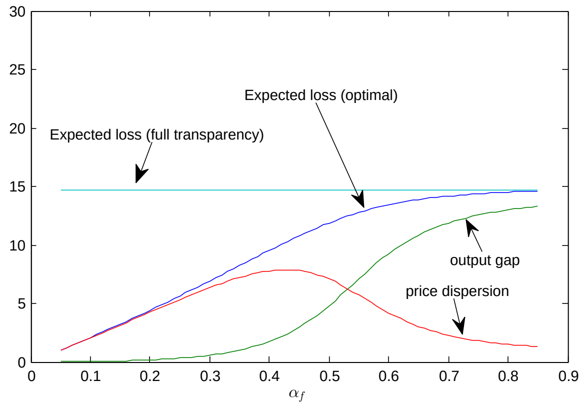
Proposition 4 *Suppose that the monetary instrument is inflexible (i.e., $q = 0$). If y^* and u are normally distributed, then the optimal disclosure rule is given by*

$$g_0(y^*, u) = \tilde{b}_{y^*}\sigma_{y^*}^{-1}y^* + \tilde{b}_u\sigma_u^{-1}u \tag{20}$$

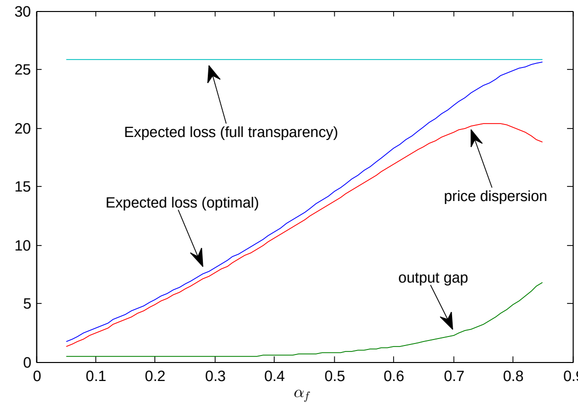
where $(\tilde{b}_{y^*}, \tilde{b}_u)$ is the eigenvector associated with the unique positive eigenvalue of $\Sigma^{\frac{1}{2}}H_0\Sigma^{\frac{1}{2}}$.

Figure 3a depicts the second moments of the market expectations induced by the optimal disclosure rule under the inflexible policy. Interestingly, $\mathrm{var}(\hat{u})$, which is interpreted as a measure of information revelation about u, is U-shaped. In other words, as α_f increases, the weight on u in (20) decreases from a positive to a negative value.

The condition under which the optimal disclosure rule induces a positive covariance between $\hat{y}^*$ and $\hat{u}$ is given by $d_{y^*} < d_u$, or equivalently $\lambda + \kappa < 1$. In Figure 3b, the shaded area represents the set of (α_f, α_u) for which this condition holds. Note that $2\lambda < 1$, or equivalently $\alpha_u > \xi/(1+\xi)$, is sufficient for this condition. Moreover, it can be shown that $\alpha_u \geq \alpha_f$ is a sufficient condition for a positive correlation. Conversely, $2\kappa > 1$, or equivalently $\alpha_f > 1/(1+\xi)$, is sufficient for a negative



(a) Flexible instrument



(b) Inflexible instrument

Figure 4: Expected losses under the flexible and inflexible instruments

correlation. When the share of uninformed firms is sufficiently high, the central bank therefore benefits from reducing the volatility of $(p_p - p_u)$ rather than that of $(p_f - p_p)$.

Figure 4 compares the expected losses from the output gap volatility with those from price dispersion under flexible and inflexible instruments. As α_f increases, the output gap volatility monotonically increases, while price dispersion has a single peak. Compared with the expected losses under full transparency, the optimal policy reduces a greater amount of welfare losses when the share of partially informed firms is high. Interestingly, compared with the flexible instrument case (the left-hand panel), the optimal disclosure rule with the inflexible instrument may allow a lower output gap volatility.

6 Concluding remarks

This paper described optimal monetary policy and discussed central bank transparency when the economy is subject to two types of shocks and when monopolistically competitive firms set their prices under informational frictions. Under the optimal policy presented herein, the instrument rule is used to reduce the price variability of partially informed firms, while the disclosure rule is directed at reducing the price variability of fully informed firms. An important finding of the present paper is therefore that market expectations about the labor market and product market should exhibit a negative correlation. Below are some discussions.

– Output stabilization vs. price stabilization:

The central bank has two objectives: to stabilize aggregate output around the efficient level and to reduce price dispersion. To examine this stabilization trade-off under optimal monetary policy, two extreme cases are examined below where the central bank's objective is either output stabilization or price stabilization.

When the central bank needs to minimize only the volatility in the output gap, nominal demand should be chosen to satisfy $q = \hat{y} + \frac{\lambda}{1+\lambda}\frac{\hat{u}}{\xi}$. Given this instrument rule, it is easily shown that full transparency achieves a perfect stabilization of the output gap regardless of the informational heterogeneity among firms. By contrast, when the central bank minimizes only price dispersion, it is optimal to choose $q = \hat{y}^* - \frac{\hat{u}}{\xi}$. It can also be shown that full disclosure achieves no price dispersion.

In both cases, the monetary instrument can perfectly eliminate the effects of $\hat{y}^*$ and $\hat{u}$ simultaneously. Then, it makes no sense to withhold information and increase the variability of the residuals. In other words, the optimality of partial disclosure comes from the need to accommodate these possibly conflicting objectives.

– Accuracy of the central bank's information:

Second, in the baseline model, the central bank has perfect information on the fundamentals. This can be extended to the case where it receives an imperfect signal about the fundamentals. For instance, suppose that the central bank observes a set of signals (s1, ..., sn). When the fundamentals and signals have a joint normal distribution, the optimal policy pair is described in a manner similar to in the perfect information case. Put simply, it is optimal to disclose a one-dimensional policy index (i.e., a linear combination of the signals) and adjust nominal demand contingent only on this. Note that a central bank that receives perfect information can always replicate the optimal policy under the imperfect information case. Generally, it can be shown that the welfare loss under the optimal policy pair decreases with the rising accuracy of the signals. Formally, a set of signals s is said to be

In the baseline model, the central bank has perfect information about the fundamentals. It can be extended to the case where it receives an imperfect signal about the fundamentals. For instance, suppose that the central bank observes a set of signals $(s_1, \ldots, s_n)$. When the fundamentals and the signals have a joint normal distribution, the optimal policy pair is described in a manner similar to in the perfect information case. Put simply, it is optimal to disclose a one-dimensional policy index (i.e., a linear combination of the signals) and adjust nominal demand contingent only on this. Note that the central bank that receives perfect information can always replicate the optimal policy under the imperfect information case. Generally, it can be shown that the welfare loss under the optimal policy pair decreases with the rising accuracy of the signals. Formally, a set of signals $\mathbf{s}$ is said to be more accurate than $\mathbf{s}'$ if the variance-covariance matrix of the conditional expectations given $\mathbf{s}$ is greater than that given $\mathbf{s}'$ with respect to the Löwner partial ordering.[18]

– Noisy public information with the inflexible instrument:

An important feature of the optimal transparency policy is to induce a correlation, whether positive or negative, between the conditional expectations $\hat{y}^*$ and $\hat{u}$. Now consider a disclosure rule that reveals noisy information on each of the following two shocks: $m = (m_{y^*}, m_u)$ such that $m_{y^*} = y^* + \epsilon_{y^*}$ and $m_u = u + \epsilon_u$. The variance of each noise component, $\text{var}(\epsilon_{y^*}) = \varphi_{y^*}$ and $\text{var}(u) = \varphi_u$, is controlled by the central bank. Intuitively, the central bank controls how precise *each* message is. The optimal noisy disclosure rule under the inflexible instrument is thus given as follows:[19] if $d_u \leq 0$ (or equivalently, $\bar{\theta}/(\bar{\theta}-1) \leq \lambda + \kappa$), then full disclosure ($\varphi_{y^*} = \varphi_u = 0$) is optimal; if $d_{y^*} > 0 > -d_u$ (or equivalently, $(\bar{\theta}-2)/(\bar{\theta}-1) < \lambda + \kappa < \bar{\theta}/(\bar{\theta}-1)$), then partial disclosure ($\varphi_{y^*} = 0$ and $\varphi_u = \infty$) is optimal; if $d_{y^*} \leq 0$ (or equivalently, $\lambda + \kappa \leq (\bar{\theta}-2)/(\bar{\theta}-1)$), then opacity ($\varphi_{y^*} = \varphi_u = \infty$) is optimal. In this way, weak strategic complementarities (higher ξ), weak information stickiness (higher α_f), and low inattentiveness (lower α_u) all increase the social value of public information on the mark-up shock.

– Unobservable monetary instrument

This paper presents a novel approach to resolve the difficulty caused by the signaling effects of policy actions. As shown in Appendix B, if the choice of the monetary instrument is not publicly observed, then optimal monetary policy is chosen as though partially informed firms are uninformed. Specifically, optimal policy pair is given by an instrument rule $q = y^* - \tilde{\gamma}(u/\xi)$ with no disclosure

[18]A matrix A is said to be greater than B if $A - B$ is positive semidefinite.

[19]Recall that the objective function can be written as shown in (19).

where $\tilde{\gamma} = \frac{(\bar{\theta}-1)\kappa}{(\bar{\theta}-1)\kappa+1} \leq \frac{(\bar{\theta}-1)\lambda}{(\bar{\theta}-1)\lambda+1} = \gamma$. This instrument rule concurs with the optimal instrument rule in Corollary 1 for $\alpha_p = 0$.

Appendix

A Proofs

A.1 Proof of Lemma 1

The best-response of each type is given by (3), (4) and (5). First, the equilibrium strategy of uninformed firms (6) is derived. Taking unconditional expectations on both sides of (3) and (4), the expected aggregate price level is obtained as follows.

$$\begin{aligned}
\mathbb{E}[p] =& \alpha_f \mathbb{E}[p_f] + \alpha_p \mathbb{E}[p_p] + \alpha_u \mathbb{E}[p_u] \\
=& (1-\xi)\mathbb{E}[p] + \xi \mathbb{E}[q] \\
\implies \mathbb{E}[p] =& \mathbb{E}[q].
\end{aligned}$$

From (5), $p_u = \mathbb{E}[q]$ is obtained.

Next, the equilibrium strategy of partially informed firms (7) is derived. Taking conditional expectations on both sides of (3), the conditional expectation of the aggregate price level is obtained as follows.

$$\begin{aligned}
\mathbb{E}[p|q,m] =& \alpha_f \mathbb{E}[p_f|q,m] + \alpha_p \mathbb{E}[p_p|q,m] + \alpha_u \mathbb{E}[p_u|q,m] \\
=& \alpha_f \mathbb{E}[p_f|q,m] + \alpha_p \mathbb{E}[p_p|q,m] + \alpha_u \mathbb{E}[p_u|q,m] \\
=& (\alpha_f + \alpha_p)\left[(1-\xi)\hat{p} + \xi q - \xi \hat{y}^* + \hat{u}\right] + \alpha_u \mathbb{E}[q] \\
\implies \hat{p} =& \mathbb{E}[q] + \frac{(1-\alpha_u)\xi}{(1-\alpha_u)\xi + \alpha_u}\left(q - \hat{y}^* + \frac{\hat{u}}{\xi}\right)
\end{aligned}$$

From (4), $p_p = \mathbb{E}[q] + \frac{\xi}{(1-\alpha_u)\xi+\alpha_u}(q - \hat{y}^* + \hat{u}/\xi)$ is obtained.

Finally, the equilibrium strategy of fully informed firms and the aggregate price level are derived.

From (3),

$$\begin{aligned} p - \hat{p} =& \alpha_f (p_f - \hat{p}_f) \\ =& \alpha_f \left[(1-\xi)(p-\hat{p}) - \xi \Delta_{y^*} + \Delta_u \right] \end{aligned}$$

where $\Delta_{y^*} \equiv y - \hat{y}^*$ and $\Delta_u \equiv u - \hat{u}$. It follows that $p = \hat{p} + \frac{\alpha_f \xi}{\alpha_f \xi + 1 - \alpha_f} \left(-\Delta_{y^*} + \Delta_u / \xi\right)$. Note that taking conditional expectations on (3), it is shown that $\hat{p}_f = p_p$, and hence $p_f = p_p + \frac{p - \hat{p}}{\alpha_f}$, which can be written as (8). ■

A.2 Proof of Proposition 1

Suppose that a policy pair (f, g) is optimal. Lemma 2 implies that there exists a policy pair $(\tilde{f}, \tilde{g})$ and an associated function $h : M \to \mathbb{R}$ such that for every (y^*, u), $(f(y^*, u), g(y^*, u)) = \tilde{g}(y^*, u)$ and $f(y^*, u) = \tilde{f}(y^*, u) = h(\tilde{g}(y^*, u))$. Intuitively, under $(\tilde{f}, \tilde{g})$, q has no signaling effect since q is determined as a function of the public information $\tilde{m} = \tilde{g}(y^*, u)$. Then, h must determine q for each $\tilde{m}$ so as to minimize the conditional expected loss. Specifically, the optimal instrument rule solves the following problem.

$$\min_q \quad \left((1-\lambda)(q - \hat{y}^*) - \lambda \frac{\hat{u}}{\xi} \right)^2 + \bar{\theta} \lambda (1-\lambda) \left(q - \hat{y}^* + \frac{\hat{u}}{\xi} \right)^2 .$$

If $\lambda = 1$ (i.e., $\alpha_u = 0$), then the objective function does not depend on q. Hence the optimal instrument rule is indeterminate. When $\lambda < 1$, the first-order condition identifies the optimal nominal demand level that solves $\min_q \mathbb{E}[L|m]$. The first-order condition is

$$\begin{aligned} 2(1-\lambda) \left((1-\lambda)(q - \hat{y}^*) - \lambda \frac{\hat{u}}{\xi} \right) + 2\bar{\theta}(1-\lambda)\lambda \left(q - \hat{y}^* + \frac{\hat{u}}{\xi} \right) = 0 \\ (1 - \lambda + \bar{\theta}\lambda)(q - \hat{y}^*) =& (\lambda - \bar{\theta}\lambda) \frac{\hat{u}}{\xi} \\ q =& \hat{y}^* - \frac{\lambda(\bar{\theta} - 1)}{\lambda(\bar{\theta} - 1) + 1} \frac{\hat{u}}{\xi}. \quad ■ \end{aligned}$$

A.3 Proof of Proposition 2

For notational simplicity, let $e \equiv u/\xi$ and $\sigma_e = \sigma_u/\xi$. The problem is $\mathbb{E}\Phi(\hat{y}^*, \hat{e})$. Tamura (2012) shows that the optimal disclosure rule is a linear combination of (y^*, e) if the Hessian matrix of Φ is neither positive nor negative semidefinite. Note that (14) is written as

$$\Phi(\hat{y}^*, \hat{e}) = -\left[\frac{\bar{\theta}\lambda}{\bar{\theta}\lambda + (1-\lambda)} - \frac{\bar{\theta}\kappa}{\bar{\theta}\kappa + (1-\kappa)}\right]\hat{e}^2 + \frac{(1-\kappa)\left((\bar{\theta}\kappa + 1 - \kappa)\hat{y}^* - (\bar{\theta}-1)\kappa\hat{e}\right)^2}{\bar{\theta}\kappa + 1 - \kappa}$$

Suppose that $\alpha_p > 0$. Then $\lambda > \kappa$ and $1 - \kappa > 0$. In this case, Φ is neither concave nor convex, or equivalently, the Hessian matrix of Φ is neither positive nor negative semidefinite. Define

$$\begin{aligned} H_{11} =& (1-\kappa)(\bar{\theta}\kappa + 1 - \kappa) \\ H_{12} =& -(\bar{\theta}-1)\kappa(1-\kappa) \\ H_{22} =& \kappa(\kappa + \bar{\theta}(1-\kappa)) - \frac{\bar{\theta}\lambda}{\bar{\theta}\lambda + 1 - \lambda} \end{aligned}$$

According to Tamura (2012), the optimal disclosure rule is such that

$$g(y^*, u) = b_{y^*}\sigma_{y^*}^{-1}y^* - b_u\sigma_e^{-1}e$$

where

$$\frac{b_{y^*}}{b_u} = \frac{\sigma_{y^*}^2 H_{11} - \sigma_e^2 H_{22} + \sqrt{(\sigma_{y^*}^2 H_{11} - \sigma_e^2 H_{22})^2 + 4\sigma_{y^*}^2\sigma_e^2 H_{12}}}{-2\sigma_{y^*}\sigma_e H_{12}}.$$

Since any monotone transformation leads the same outcomes, the optimal weights (b_{y^*}, b_e) in Proposition 2 are normalized as $b_{y^*}^2 + b_u^2 = 1$. ■

B Optimal policy with an unobservable instrument

This section considers the optimal policy when the choice of the monetary instrument is not publicly observed. To keep exposition as simple as possible, suppose that the central bank chooses a deterministic policy pair (f, g) where $q = f(y^*, u)$ and $m = g(y^*, u)$. Then, fully informed firms can correctly predict the choice of the monetary instrument from their observation of (y^*, u). On the other hand, partially informed firms have to set prices conditional only on m. Hence, their pricing rules (4) are replaced by

$$p_p = (1-\xi)\hat{p} + \xi\hat{q} - \xi\hat{y}^* + \hat{u}.$$

Note that the second term in the right hand side is now the conditional expectation $\hat{q} = \mathbb{E}[q|m]$ of nominal demand. Consequently, the equilibrium pricing rules are given as

$$\begin{aligned}
p_p &= p_u + \frac{\lambda}{1-\alpha_u}\left(\hat{q} - \hat{y}^* + \frac{1}{\xi}\hat{u}\right) \\
p_f &= p_p + \frac{\kappa}{\alpha_f}\left(\Delta_q - \Delta_{y^*} + \frac{1}{\xi}\Delta_u\right) \\
p &= \lambda\left(\hat{q} - \hat{y}^* + \frac{1}{\xi}\hat{u}\right) + \kappa\left(\Delta_q - \Delta_{y^*} + \frac{1}{\xi}\Delta_u\right).
\end{aligned}$$

Let $a = q - y^*$. With some notations $\hat{a} = \hat{q} - \hat{y^*}$ and $\Delta_a = \Delta_q - \Delta_{y^*}$, the expected welfare loss in equilibrium is expressed as follows.

$$\begin{aligned}
\mathbb{E}L =& \mathbb{E}[(1-\lambda)\hat{a} - \lambda\hat{u}/\xi]^2 + \mathbb{E}[(1-\kappa)\hat{a} - \kappa\hat{u}/\xi]^2 \\
&+ \bar{\theta}\lambda(1-\lambda)\mathbb{E}(\hat{a} + \hat{u}/\xi)^2 + \bar{\theta}\kappa(1-\kappa)\mathbb{E}(\Delta_a + \Delta_u/\xi)^2
\end{aligned} \tag{21}$$

The central bank controls the distribution of $(\hat{a}, \hat{u}, \Delta_a, \Delta_u)$ through the choice of (f, g). I will solve the problem according to the following steps. First, given any $(\hat{u}, \Delta_u)$, choose $(\hat{a}, \Delta_a)$ as direct control variables to minimize the expected loss. Second, find the optimal disclosure policy when $(\hat{a}, \Delta_a)$ are determined as in step 1. Third, find a as a function of (y^*, u) that is consistent with $(\hat{a}, \Delta_a)$ in step 1 and the optimal policy in step 2. In the original problem, the central bank *indirectly* controls $(\hat{a}, \Delta_a)$ through the choice of a policy that determines a and m. So, the expected loss when $(\hat{a}, \hat{u}, \Delta_a, \Delta_u)$ are determined by step 1 and 2 should be less than or equal to that under the optimal policy. Therefore, from step 1 and 2, we will obtain relationships among $(\hat{a}, \hat{u}, \Delta_a, \Delta_u)$ that achieve an upper bound utility.

Step 1. $\hat{a}$ solves the following problem.

$$\min_{\hat{a}} [(1-\lambda)\hat{a} - \lambda\hat{u}/\xi]^2 + \bar{\theta}\lambda(1-\lambda)\mathbb{E}(\hat{a} + \hat{u}/\xi)^2$$

From the first-order condition, the solution is given by

$$\hat{a} = -\frac{(\bar{\theta}-1)\lambda}{(\bar{\theta}-1)\lambda+1}\frac{\hat{u}}{\xi} \tag{22}$$

Similarly, Δ_a is given by

$$\Delta_a = -\frac{(\bar{\theta}-1)\kappa}{(\bar{\theta}-1)\kappa+1}\frac{\Delta_u}{\xi} \tag{23}$$

Step 2. Substitute (22) and (23) into (21) and obtain the indirect loss function

$$\begin{aligned}\mathbb{E}L =& \frac{\bar{\theta}\lambda}{\bar{\theta}\lambda+1-\lambda}\mathbb{E}\left(\frac{\hat{u}}{\xi}\right)^2 + \frac{\bar{\theta}\kappa}{\bar{\theta}\kappa+1-\kappa}\mathbb{E}\left(\frac{\Delta_u}{\xi}\right)^2 \\ =& \left[\frac{\bar{\theta}\lambda}{\bar{\theta}\lambda+1-\lambda} - \frac{\bar{\theta}\kappa}{\bar{\theta}\kappa+1-\kappa}\right]\mathbb{E}\left(\frac{\hat{u}}{\xi}\right)^2 + \frac{\bar{\theta}\kappa}{\bar{\theta}\kappa+1-\kappa}\mathbb{E}\left(\frac{u}{\xi}\right)^2.\end{aligned}$$

The first term is minimized under no disclosure and the second term is independent of the disclosure rule. Hence, no disclosure is optimal.

Step 3. Combine (22) and (23) and obtain

$$\begin{aligned}a =& -\frac{(\bar{\theta}-1)\kappa}{(\bar{\theta}-1)\kappa+1}\frac{u}{\xi} - \left[\frac{(\bar{\theta}-1)\lambda}{(\bar{\theta}-1)\lambda+1} - -\frac{(\bar{\theta}-1)\kappa}{(\bar{\theta}-1)\kappa+1}\right]\frac{\hat{u}}{\xi} \\ =& -\frac{(\bar{\theta}-1)\kappa}{(\bar{\theta}-1)\kappa+1}\frac{u}{\xi}.\end{aligned}$$

Note that the second equality holds since $\hat{u} = 0$ under no disclosure. Hence, the optimal policy under the unobserved instrument is characterized by an instrument rule $f(y^*, u) = y^* - -\frac{(\bar{\theta}-1)\kappa}{(\bar{\theta}-1)\kappa+1}\frac{u}{\xi}$ with no disclosure.

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