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Auction Platform Design and the Linkage Principle*

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Abstract

This paper examines an auction platform in which the platform provider maximizes profits by adjusting participation fees and by choosing an auction format. The seller has private information on the quality of the good, and each participating buyer receives a private signal about his valuation of the good. The choice of auction format determines the allocation of trading surplus among participating seller and buyers. This paper shows that when the seller's type is affiliated with buyers' signals, the platform provider can charge higher participation fees to both sides by choosing a first-price auction rather than a second-price or English auction. It also examines the effect of allowing participating buyers to acquire information on the seller's type and shows that the provider can charge higher participation fees under a non-transparency policy.

Keywords: multi-sided platforms, auctions, market transparency
JEL classification codes: L21, D44, D82, D83

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1 Introduction

Auctions are considered to be an effective tool for the exchange of goods. For example, eBay, the most successful online auction platform, reported that its gross fourth quarter merchandise volume in 2011 exceeded $16 billion.[1] Today, there are so many online auction sites, some of which specialize in goods as diverse as jewelry, musical instruments, and real estate. However, although economists have studied bidders' behavior in auctions and trading mechanisms that lead to an efficient allocation or that maximize revenue, few studies have incorporated the fact that sellers are typically mere users of auction platforms rather than designers of them.

Following the view of Hagiu and Wright (2011), this paper examines a simple model of a platform that enables direct interactions between participating agents. To focus on how the platform provider indirectly controls their interactions by charging fees and designing auctions, we assume that the platform provider chooses both the participation fees and the auction format. Implicit in this formulation is that it is infeasible for the platform provider to be involved in direct trading. Although real-world auction platforms such as eBay allow the use of price contingent fees, reserve prices, and buy-it-now prices, which have been the subject of both theoretical and empirical research, we rather focus on the informational aspect of auctions when designing platforms.[2]

Specifically, the auction model in the present paper is based on that proposed by Milgrom and Weber (1982). In this model, the seller has private information on the quality of the good and each participating buyer receives a private signal about his valuation of the seller's good. The buyer's valuation is then determined by his signal as well as by the seller's type. When the seller's type is affiliated with the buyer's signal, a first-price auction *on average* yields lower revenues to the seller than a second-price auction, which in turn yields lower revenues than an English (ascending) auction (this is the so-called linkage principle). Intuitively, the price in a first-price auction is less sensitive to bidders' information than it is in other auction formats. However, the question remains of whether the platform provider should choose an auction that has a stronger linkage.

The main conclusion of this paper is that the platform provider can charge higher participation fees to *both sides* and hence earn higher profits by choosing a first-price auction rather than a

[1] For further information, see http://www.investor.ebay.com.

[2] For surveys, see Bajari and Hortaçsu (2004), Ockenfels, Reiley and Sadrieh (2006), and Hasker and Sickles (2010).

second-price or English auction based on the linkage principle discussed herein.

Since the choice of auction format does not affect total surplus, the linkage principle implies that the buyer's surplus is higher in a first-price auction than it is in a second-price or English auction.[3] That is, the choice of auction format affects allocation of trading surplus between the two sides. Moreover, the provider can charge a higher fee to the seller since the expected payment to the *marginal type* is also higher in a first-price auction than it is in a second-price or English auction. The marginal type is a seller who is indifferent between participating in the platform or not. The seller's revenue reflects his type based on bidders' signals and their competitive bids. Expected revenue varies less with the seller's type in a first-price auction since it is less sensitive to bidders' information. Therefore, compared with a second-price or English auction, a first-price auction increases the expected revenue of lower types and reduces the information rents of higher types, and facilitates rent extraction by the platform provider through participation fees. In other words, the choice of auction format affects allocation of trading surplus within the seller side.

The linkage principle also allows us to conclude that the disclosure of the seller's information increases average prices. Put simply, market transparency benefits sellers *on average*. To examine whether the platform should be transparent, we consider an extension to the model in which the platform provider also determines whether participating buyers observe the seller's type (its so-called transparency policy). Figure 1 depicts the relationships between the four types of auctions (two auction formats, each with two policies) in the private value setting. The result in the baseline model is indicated as a dotted arrow on the right-hand side of the figure. The new result here is that opacity is preferred to transparency in a first-price auction (dotted arrow at the top). The intuition is analogous to that of the first result discussed above. In a first price auction, the equilibrium bidding strategy depends on the belief about the seller's type. Each bidder thus adjusts his bid to the seller's type when this is observable. In particular, when the seller has a lower type, each bidder is optimistic about the probability of winning and bids less aggressively. Therefore, the revelation of the seller's type to bidders decreases the expected revenue of the marginal type. Furthermore, the linkage principle implies that transparency decreases the buyer's surplus. Consequently, the platform provider can charge higher fees to both sides under opacity than under transparency. Other relationships follow from the strategic equivalence in a second-price auction under the private value

[3]This argument is true only when we consider the symmetric and monotone strategy equilibrium.

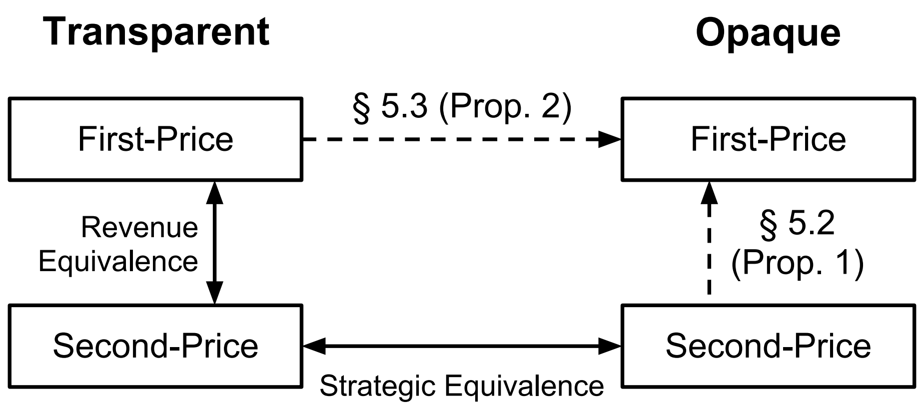

Figure 1: Relationships among four types of auctions in the private value model

setting (two-sided arrow at the bottom of the figure) and the standard revenue equivalence theorem under an assumption that the bidders' signals are conditionally independent given the seller's type (two-sided arrow on the left-hand side). Therefore, the revenue difference between a transparent and an opaque first-price auction (dotted arrow at the top) is equal to the difference between an opaque first-price auction and an opaque second-price auction (vertical dotted arrow).

Our results depend on several assumptions. First, we assume that potential bidders have no private information about valuation before participation. We consider situations where buyers have uncertain what types of items (design, color, specification, etc.) are present in the marketplace. So, his taste specific to the item can be determined only after he observes the item put up for auction. If each bidder observes a private signal that is correlated with his valuation before participation, the bidder's participation strategy may not be monotone in equilibrium (see Landsberger and Tsirelson (2000) for this issue). Our formulation avoids such complication. Second, we suppose that the platform provider charges constant participation fees. In practice, however, many Internet auction cites and art auction houses charges percentage fees. In Appendix A, we show that with a slight modification of the model, our results can be applied in a setting where the platform provider charges a percentage fee to the seller's revenue.

The rest of the paper is organized as follows. Section 2 reviews the related literature and Section 3 describes the model. Section 4 characterizes equilibrium in auctions and derives preliminary results. Section 5 characterizes equilibrium participation strategies. Section 6 presents the main results. Section 7 concludes the paper.

2 Related Literature

This paper contributes to the literature on two-sided markets and multi-sided platform design.[4] A number of papers including those by Armstrong (2006), Rochet and Tirole (2006), and Weyl (2010) have examined monopoly and/or competitive pricing by platform providers that internalize cross-side network externalities. Recently, some papers also consider non-price instruments that affect the interactions between participating agents. For example, Cañón (2011) explores a matching platform that controls information about the matching partner available to participating agents, while Veiga and Weyl (2012) investigate a general model of a multidimensional product monopoly (such as broadcast media) that controls price and non-price instruments in order to screen agents with multidimensional characteristics. As emphasized in the findings of these papers, the non-price instruments considered in the present paper (e.g., the choice of auction format and transparency policy) affect the interaction values of different types of users and thereby serve as a screening device. In the same vein, Lizzeri (1999) and Gaudeul and Jullien (2005) examine the informational role of intermediaries and demonstrate the suboptimality of full transparency about the seller's product quality. Unlike these papers, however, the present paper explores a different aspect of information mediation by addressing the effect of market transparency on competition among buyers.

The optimal mechanism for a mediator is studied by Myerson and Satterthwaite (1983), who consider the optimal intermediation mechanisms for bilateral trading in an independent private value setting. Likewise, Matros and Zapechelnyuk (2011) analyze a dynamic model of Internet auctions and show that the optimal mechanism involves charging no participation fee to the seller. Since these authors consider an independent private value setting, they do not focus on the informational aspect of auctions.

The present paper singles out a transparency policy as a non-price instrument for the platform provider and finds an interesting relationship between the effects of market transparency and that of auction choice. In the literature on auction theory, a number of papers have investigated the seller's incentives to reveal information both in independent private value settings (Bergemann and Pesendorfer (2007), Esö and Szentes (2007), Ganuza and Penalva (2010)) and in interdependent or common value settings (Gershkov (2009), Lauermann and Virág (2012)). In addition, Forand (2010)

[4] For surveys, see Rysman (2009) and Hagiu and Wright (2011).

and Troncoso-Valverde (2011) explore the incentives of competing sellers who post mechanisms and choose disclosure policies that reveal sellers' private information to (potential) buyers and examine whether full transparency arises in equilibrium. Finally, other authors assess the choice of auction formats (Kremer and Skrzypacz (2004)) and reserve prices (Jullien and Mariotti (2006)) by an informed seller.

3 The Model

Consider an auction platform for trading between a seller with one unit of a good for sale and N potential buyers. The platform provider chooses an auction format A and sets participation fees t^S and t^B for the seller and buyers, respectively. The format A is assumed to be one of the first-price ($A = I$), second-price ($A = II$) and English ($A = Eng$) auctions. In a later section, we include a transparency policy as an additional non-price instrument for the platform design. Given the provider's choice, each seller/buyer chooses whether or not to participate in the platform. When the seller and at least one buyer participate, an auction is held.

The seller has private information about the quality of the good. Let $\theta \in \Theta \equiv [0, 1]$ denote the seller's *type*. The distribution function of the seller's type is denoted by $\Phi_0(\theta)$ and is assumed to have density $\phi_0(\theta) > 0$ over θ. Let $\sigma^S : \Theta \to \{0, 1\}$ be the seller's participation strategy, where $\sigma^S(\theta) = 1$ denotes participation and $\sigma^S(\theta) = 0$ denotes non-participation. The conditional distribution $\Phi(\theta)$ of the seller's type given participation is determined by Φ_0 and σ^S.

Each buyer $i \in \{1, \ldots, N\}$ has an opportunity cost $k_i \geq 0$ for participation. Let D be the distribution function of k_i over $\mathbb{R}_+$. Let $\sigma_i^B : \mathbb{R}_+ \to \{0, 1\}$ be the participation strategy of buyer i. Upon participation, each buyer i observes a private signal $X_i \in [0, 1]$ about the seller's good. The bidder i's valuation of the object is denoted by $u(\theta, x_i)$ when the seller's type is θ and his private signal is $X_i = x_i$. Assume that u is non-negative and non-decreasing in each of its arguments. We say the values are private if $u(\theta, x_i) = x_i$. Within an auction, the buyers' signals are independent and identically distributed conditional on the seller's type. The conditional distribution of X_i given θ is F_θ with a continuous density f_θ. We assume that F_θ satisfies the monotone likelihood ratio property: for $\theta > \theta'$, the likelihood $f_\theta(x)/f_{\theta'}(x)$ is increasing in x. This assumption means that in an auction with bidder $1, \ldots, n$, $(\theta, X_1, \ldots, X_n)$ are affiliated.

We assume that the number of participating buyers is publicly observable. We focus on the symmetric monotone Bayesian Nash equilibrium of the auction game. Let $\mu \in \Delta(\Theta)$ denote the posterior belief of the seller's type conditional on her participation. Let $U_\theta^A(n,\mu)$ be the expected revenue of a participating seller of type θ in auction A when she faces n bidders. Note that it depends on μ since the bidding strategy in equilibrium may depend on the bidders' beliefs about the seller's type.

Since the participation decisions are simultaneous, potential participants do not know how many bidders she/he will face. Let $\mathcal{U}_\theta^A(\sigma^S, \sigma_1^B, \ldots, \sigma_N^B)$ and $\mathcal{V}_i^A(\sigma^S, \sigma_1^B, \ldots, \sigma_N^B)$ denote the expected payoff from participation for the seller and the buyer, respectively. If the seller does not participate, she receives utility normalized to zero. Hence, the seller participates if $\mathcal{U}_\theta^A - t^S \geq 0$ and the buyer if $\mathcal{V}_i^A - t^B \geq k_i$.

The provider's profit is $\pi \equiv t^S \mathbb{E}\left[\sigma^S(\theta)\right] + t^B \sum_{i=1}^N \mathbb{E}\left[\sigma_i^B(k_i)\right]$ where $(\sigma^S, \sigma_1^B, \ldots, \sigma_N^B)$ are determined in equilibrium. We focus on a pure strategy symmetric equilibrium in which every buyer chooses the same participation strategy.

4 Equilibrium in Auctions

In this section, we present the equilibrium bidding strategy in an auction $A \in \{I, \mathit{II}, Eng\}$ with n bidders participate with the belief that the seller's type θ is distributed according to $\mu \in \Delta(\Theta)$.

4.1 Preliminary: Order Statistics

Fix $n \geq 2$. The density function of $(\theta, X_1, \ldots, X_n)$ is given by $\phi(\theta) f_\theta(x_1) \cdots f_\theta(x_n)$. Note that $\theta, X_1, \ldots, X_n$ are affiliated.

Now, fix bidder 1. Define $Y_1 \equiv \max\{X_2, \ldots, X_n\}$ to be the highest signal among the bidders other than bidder 1. Similarly, define Y_k to be the k-th highest signal among $X_2, \ldots, X_n$. Let $G_\theta(y_1) \equiv [F_\theta(y_1)]^{n-1}$ denote the conditional distribution of Y_1 given θ. Note that for any $\theta \geq \theta'$, $G_\theta(\cdot)$ dominates $G_{\theta'}(\cdot)$ in terms of the likelihood ratio.

Given θ, the (conditional) distribution function of (X, Y_1) is $F_\theta(x) G_\theta(y_1)$, so the joint distribution H of (X, Y_1) (not conditional on θ) is given by

$$H_\mu(x, y_1) = \int_{\theta \in \Theta} F_\theta(x) G_\theta(y_1) d\mu.$$

Thus, the distribution of Y_1 given $X_1 = x$ (i.e., posterior belief of bidder 1 when his signal is x) is

$$H_\mu(y_1|x) = \frac{\int_\theta f_\theta(x) G_\theta(y_1) d\mu}{\int_\theta f_\theta(x) d\mu}. \tag{1}$$

Let $h_\mu(y|x)$ denote the density of $H_\mu(y|x)$. Recall that each buyer is not informed of the type of the seller, so he may update his belief from the signal realization. Consequently, buyers may have different posterior beliefs about the seller's type and the rivals' valuation (see (1)). Note that X_1 and Y_1 are affiliated: i.e., for any μ and any $x > x'$, $H_\mu(\cdot|x)$ dominates $H_\mu(\cdot|x')$ in terms of the likelihood ratio.

Let $\hat{\theta}$ is the lowest type in the support of μ. Then for any x, $H_\mu(\cdot|x)$ dominates $G_{\hat{\theta}}(\cdot)$ in terms of the likelihood ratio. An immediate implication is that the reverse hazard rate of $H_\mu(\cdot|x)$ is greater than that of $G_{\hat{\theta}}(\cdot)$.

4.2 Second-Price Auctions

We now describe the equilibrium bidding strategy and the expected payment in the second-price auction. The results presented in the rest of this section draw on Milgrom and Weber (1982) except Lemma 3 below.

First, we define the function

$$v(x, y) = \mathbb{E}[u(\theta, x)|X_1 = x, Y_1 = y] \tag{2}$$

to be the expected valuation of bidder 1 when his signal is x and the highest signal among the competing bidders is y.

In a second-price auction, the symmetric equilibrium strategy is given by

$$b^{II}(x) = v(x, x).$$

The expected payment by a bidder with signal x to the seller of type θ is given by

$$e_\theta^{II}(x;n,\mu) = \int_0^x b^{II}(y) g_\theta(y) dy.$$

Note that b^{II} implicitly depends on n and μ while g_θ depends on n but not on μ.

4.3 English Auctions

In an English auction, the symmetric equilibrium strategy is recursively defined as follows:

$$b^{Eng(k)}(x, p_{k+1}, \ldots, p_n) = \mathbb{E}[u(\theta, x) | X_1 = Y_j = x \ \forall j < k, \ Y_k = y_k, \ldots, Y_{n-1} = y_{n-1}]$$

where y_k is defined by $b^{Eng(k+1)}(y_k, p_{k+2}, \ldots, p_n) = p_{k+1}$.

The expected payment function in the English auction is

$$e_\theta^{Eng}(x;n,\mu) = \mathbb{E}\left[\mathbb{E}\left[u(\theta,x)|X_1 = Y_1 = x, Y_j = y_j \forall j > 1\right]|Y_1 \leq x, \theta\right] \cdot \Pr(Y_1 \leq x|\theta).$$

4.4 First-Price Auctions

The symmetric equilibrium in a first-price auction under (n, μ) is

$$b^I(x) = \int_0^x v(y,y) dL(y|x)$$

where

$$L(y|x) = \exp\left(-\int_y^x \frac{h_\mu(t|t)}{H_\mu(t|t)} dt\right).$$

The expected payment function in the first-price auction is

$$e_\theta^I(x;n,\mu) = G_\theta(x) b^I(x).$$

4.5 Preliminary Results

This subsection provides payoff comparisons among the three auction formats. Note that for $A \in \{I, II, Eng\}$, the expected revenue of type θ is

$$U_\theta^A(n,\mu) = n \int_0^1 e_\theta^A(x;n,\mu) f_\theta(x)dx.$$

The first result is that the expected revenue in auction is increasing in the seller's type. Intuitively, higher types generate higher revenues.

Lemma 1 *For every $A \in \{I, II, Eng\}$ and (n,μ), $U_\theta^A(n,\mu)$ is increasing in θ.*

The second result is an application of the *linkage principle* (Milgrom and Weber (1982)).

Lemma 2 *For any $n \geq 2$ and μ, the expected payment by a bidder with signal x is ordered as follows*

$$\int_{\theta\in\Theta} e_\theta^{Eng}(x;n,\mu)d\mu(\theta|x) \geq \int_{\theta\in\Theta} e_\theta^{II}(x;n,\mu)d\mu(\theta|x) \geq \int_{\theta\in\Theta} e_\theta^{I}(x;n,\mu)d\mu(\theta|x).$$

Proof. See Krishna (2002). ■

It immediately follows from Lemma 2 that

$$\int U_\theta^{Eng}(n,\mu)d\mu \geq \int U_\theta^{II}(n,\mu)d\mu \geq \int U_\theta^{I}(n,\mu)d\mu.$$

It is worth noting that Lemma 2 does not mean that every type θ prefers the English auction to the second-price and first-price auctions. Indeed, for the lowest type $\hat{\theta}$ in the support of μ, the revenue ranking of the three auction formats is reversed.

Lemma 3 *For any $n \geq 2$ and μ with the lowest type $\hat{\theta}$ in the support, the expected payments from a bidder with signal x to a seller of type $\hat{\theta}$ are ordered as follows:*

$$e_{\hat{\theta}}^{I}(x;n,\mu) \geq e_{\hat{\theta}}^{II}(x;n,\mu) \geq e_{\hat{\theta}}^{Eng}(x;n,\mu).$$

Proof. See Appendix B. ■

The first-price auction is less sensitive to the bidders' signals compared with the other auction formats. Although this property decreases average prices in auctions, it increases the expected revenue of lower types. Intuitively, higher types gain and lower type lose when the bidders' information is well reflected to the price through competitive bidding.

5 Equilibrium Participation Strategy

5.1 Expected Payoffs and Welfare

Pick an auction with a seller of type θ and n bidders. In the symmetric equilibrium, a bidder with the highest signal wins the object. Let $Z = \max\{X_1, X_2, \ldots, X_n\}$. The distribution of Z is given by $F_\theta(\cdot)^n$ and the expected surplus is $W_\theta(n) \equiv \mathbb{E}[u(\theta, Z)|\theta] = \int u(\theta, z) dF_\theta(z)^n$, which is independent of the auction format. The seller's expected revenue is $U_\theta^A(n, \mu)$ and the total surplus to n bidders as a whole is given by $W_\theta(n) - U_\theta^A(n, \mu)$. Thus, the expected payoff for each bidder in an auction with n bidders (i.e., $n - 1$ competitors) is

$$V^A(n, \mu) = \frac{1}{n} \int_{\theta \in \Theta} (W_\theta(n) - U_\theta^A(n, \mu)) d\mu. \tag{3}$$

5.2 Cut-off Strategy

A participation strategy profile $(\sigma^S, \sigma_1^B, \ldots, \sigma_N^B)$ induces a distribution (θ, n). In equilibrium, the buyer's belief μ must be consistent with the conditional distribution Φ induced by the seller's participation strategy σ^S and the prior Φ_0. Since the seller's expected revenue $U_\theta^A(n, \mu)$ is increasing in θ for any (A, n, μ) (Lemma 1), so is its expected value. This implies that the seller's participating strategy in equilibrium has the following cut-off property: there is a unique *marginal type* $\hat{\theta} \in [0, 1]$ such that any seller of type $\theta > \hat{\theta}$ chooses to participate and receives a positive (expected) payoff and no type $\theta < \hat{\theta}$ participate. That is, the equilibrium strategy is given by

$$\sigma^S(\theta) = \begin{cases} 1 & \text{if } \theta \geq \hat{\theta} \\ 0 & \text{otherwise.} \end{cases}$$

There is a unique distribution of the seller's type that is consistent with the above cut-off strategy. Define $\Phi_{\hat{\theta}}$ by

$$\Phi_{\hat{\theta}}(\theta) = \frac{\Phi_0(\theta) - \Phi_0(\hat{\theta})}{1 - \Phi_0(\hat{\theta})}$$

with support $\theta \in [\hat{\theta}, 1]$.

Similarly, the buyer's participation strategy in equilibrium is

$$\sigma^B(k) = \begin{cases} 1 & \text{if } k \leq \hat{k} \\ 0 & \text{otherwise} \end{cases}$$

for some $\hat{k}$. Then the probability of participation is given by $D(\hat{k})$.

Suppose that a pair $(\hat{\theta}, \hat{k})$ of cut-off strategies constitutes an equilibrium. Then, the expected payoff from participation for a seller of type θ is

$$\mathcal{U}^A_\theta(\hat{\theta}, \hat{k}) = \sum_{n=0}^{N} \binom{N}{n} D(\hat{k})^n (1 - D(\hat{k}))^{N-n} U^A_\theta(n, \Phi_{\hat{\theta}}).$$

The value of participation for a buyer is

$$\begin{aligned}\mathcal{V}^A(\hat{\theta}, \hat{k}) &= \left(1 - \Phi_0(\hat{\theta})\right) \sum_{l=0}^{N-1} \binom{N-1}{l} D(\hat{k})^l (1 - D(\hat{k}))^{N-1-l} V^A(l+1, \Phi_{\hat{\theta}}) \\ &= \frac{1}{ND(\hat{k})} \left(1 - \Phi_0(\hat{\theta})\right) \int \left(\mathcal{W}_\theta(\hat{k}) - \mathcal{U}^A_\theta(\hat{\theta}, \hat{k})\right) d\Phi_{\hat{\theta}}(\theta)\end{aligned}$$

where $\mathcal{W}_\theta$ is the expected surplus when the seller participates as defined by

$$\mathcal{W}_\theta(\hat{k}) = \sum_{n=0}^{N} \binom{N}{n} D(\hat{k})^n (1 - D(\hat{k}))^{N-n} W_\theta(n).$$

6 Platform Design

Given any $(\hat{\theta}, \hat{k})$, there is a unique pair of participation fees that sustains these $(\hat{\theta}, \hat{k})$ as an equilibrium. Indeed, under $t^S = \mathcal{U}^A_{\hat{\theta}}(\hat{\theta}, \hat{k})$ and $t^B = \mathcal{V}^A(\hat{\theta}, \hat{k}) - \hat{k}$, the strategy profile $(\hat{\theta}, \hat{k})$ constitutes an equilibrium. Following Weyl (2010), we investigate $(A, \hat{\theta}, \hat{k})$ that is optimal for the platform provider.

6.1 Optimality of First-price Auctions

The provider's problem is written as

$$\max_{A,\hat{\theta},\hat{k}} \left(1-\Phi_0(\hat{\theta})\right) t^S + ND(\hat{k})t^B$$

$$\text{subject to } \Phi_{\hat{\theta}}(\theta) = (\Phi_0(\theta)-\Phi_0(\hat{\theta}))/(1-\Phi_0(\hat{\theta})) \text{ for } \theta \in [\hat{\theta}, 1]$$

$$t^S = \mathcal{U}^A_{\hat{\theta}}(\hat{\theta}, \hat{k})$$

$$t^B = \mathcal{V}^A(\hat{\theta}, \hat{k}) - \hat{k}.$$

Notice that given any $(\hat{\theta}, \hat{k})$, the choice of an auction format affects the profit only through t^S and t^B. From Lemma 2 and equation (3), we have

$$\mathcal{V}^I(\hat{\theta}, \hat{k}) \geq \mathcal{V}^{II}(\hat{\theta}, \hat{k}) \geq \mathcal{V}^{Eng}(\hat{\theta}, \hat{k}).$$

On the other hand, Lemma 3 implies that

$$\mathcal{U}^I_{\hat{\theta}}(\hat{\theta}, \hat{k}) \geq \mathcal{U}^{II}_{\hat{\theta}}(\hat{\theta}, \hat{k}) \geq \mathcal{U}^{Eng}_{\hat{\theta}}(\hat{\theta}, \hat{k}).$$

From these facts, we find that for *any* $(\hat{\theta}, \hat{k})$, the provider can charge higher fees to *both sides* in the first-price auction than in the second-price or English auction.

Proposition 1 *The first-price auction yields higher profits to the platform provider than does the second-price or English auction.*

Proposition 1 provides a sharp contrast to the revenue ranking principle by Milgrom and Weber (1982). Compared with the other auction formats, the first-price auction yields higher revenues to the lowest type and higher payoffs to the participating buyers. Such a change in the distribution of expected payoffs among participating users allows the provider to charge higher participation fees.

Note that the provider's profit is written as

$$\begin{aligned}\pi =&(1-\Phi_0(\hat{\theta}))\mathcal{U}^A_{\hat{\theta}}(\hat{\theta},\hat{k}) + ND(\hat{k})\left(\mathcal{V}^A(\hat{\theta},\hat{k})-\hat{k}\right)\\ =&\underbrace{\int_{\hat{\theta}}^1 \mathcal{W}_\theta(\hat{k})d\Phi_0(\theta)}_{\text{trading surplus}} - \underbrace{\int_{\hat{\theta}}^1 \left(\mathcal{U}^A_\theta(\hat{\theta},\hat{k})-\mathcal{U}^A_{\hat{\theta}}(\hat{\theta},\hat{k})\right)d\Phi_0(\theta)}_{\text{seller's rents}} - \underbrace{\hat{k}ND(\hat{k})}_{\text{opportunity cost + buyers' rents}}\end{aligned} \tag{4}$$

Intuitively, the provider should increase the total surplus from trade and decrease the sellers' information rents. By choosing an auction format that is less sensitive to the bidders' information (e.g., FPA), the provider can reduce the seller's information rent.

6.2 Optimality of Opaque Auctions

One main finding of Milgrom and Weber (1982) is that disclosure of the seller's information increases the *ex ante* expected revenues. Intuitively, when the seller's information is available to the bidders, the payments become more sensitive to the signals. Our previous result suggests that the provider prefers an auction with a weaker linkage. In this subsection, we investigate the optimality of opaque auctions for the provider. Specifically, suppose that the provider can choose either a *transparent auction* indexed by A_{Tr} where the seller's type is observable to the bidders or an *opaque auction* A_{Op} where it is unobservable. To make the point clear, we focus on a private value setting (that is, $u(\theta,x)=x$ for all θ and x). In the private value, any information revelation about the seller's type does not affect his valuation of the object, and hence it allows us to focus on the effect on competition among bidders. Note that in the private value setting, the English auction is equivalent to the second-price auction, so this section considers only $A\in\{I,\mathit{II}\}$.

In the second-price auction, it is a weakly dominant strategy to bid the true valuation:

$$b^{\mathit{II}_{Tr}}_\theta(x)=b^{\mathit{II}_{Op}}(x)=x.$$

This strategy does not depend on the belief about the seller's type, so the two policies induce the same expected revenue to every type. Hence, $U^{\mathit{II}_{Tr}}_\theta(n,\mu=\{\theta\})=U^{\mathit{II}_{Op}}_\theta(n,\mu)$ for all θ where $\mu=\{\theta\}$ denotes the degenerate distribution at θ.

On the other hand, in the first-price auction, the buyers' belief about the seller's type matters.

Under transparency, the bidders face an auction in which their signals are independently distributed. In the independent private value model, the well-known revenue equivalence theorem holds. Namely, in the class of standard auctions including the first- and second-price auctions, the expected revenue in equilibrium is independent of the auction format. Hence, for every θ, $U_{\theta}^{I_{Tr}}(n, \mu = \{\theta\}) = U_{\theta}^{II_{Tr}}(n, \mu = \{\theta\})$.

Under transparency, the equilibrium strategy in the first-price auction is expressed as the expectation of the highest signal among the rival bidders conditional on winning. Formally, the equilibrium strategy is given by

$$\begin{aligned} b_{\theta}^{I_{Tr}}(x) =& \mathbb{E}[Y_1 | Y_1 < x, \theta] \\ =& \int_0^x y dG_{\theta}(y | y < x). \end{aligned}$$

On the other hand, under opaqueness, the equilibrium strategy is given by

$$b^{I_{Op}}(x) = \int_0^x y dL(y|x)$$

where $L(y|x) = \exp\left(- \int_y^x \frac{h(t|t)}{H(t|t)} dt\right)$. Since for any Φ with the lowest type $\hat{\theta}$ in its support, $L(y|x)$ is first-order stochastically dominates $G_{\hat{\theta}}(y|y < x)$ for all x, we obtain $b_{\hat{\theta}}^{I_{Tr}}(x) \leq b^{I_{Op}}(x)$. This implies that $U_{\hat{\theta}}^{I_{Tr}}(n, \mu = \{\hat{\theta}\}) \leq U_{\hat{\theta}}^{I_{Op}}(n, \mu)$. Intuitively, for the marginal type $\hat{\theta}$, information revelation about her type makes bidders less aggressive, thereby decreasing her revenues.

Note that the standard linkage principle implies that full transparency is the best policy for the seller on average, that is,

$$\int U_{\theta}^{I_{Tr}}(n, \mu) d\mu \geq \int U_{\theta}^{I_{Op}}(n, \mu) d\mu.$$

Hence, the expected payoffs from participation in the four types of auctions are ordered as

$$\begin{aligned} \mathcal{U}_{\hat{\theta}}^{I_{Op}}(\hat{\theta}, \hat{k}) \geq \mathcal{U}_{\hat{\theta}}^{II_{Op}}(\hat{\theta}, \hat{k}) = \mathcal{U}_{\hat{\theta}}^{II_{Tr}}(\hat{\theta}, \hat{k}) = \mathcal{U}_{\hat{\theta}}^{I_{Tr}}(\hat{\theta}, \hat{k}) \\ \mathcal{V}^{I_{Op}}(\hat{\theta}, \hat{k}) \geq \mathcal{V}^{II_{Op}}(\hat{\theta}, \hat{k}) = \mathcal{V}^{II_{Tr}}(\hat{\theta}, \hat{k}) = \mathcal{V}^{I_{Tr}}(\hat{\theta}, \hat{k}). \end{aligned}$$

Applying the same argument as in Proposition 1, we obtain the following result.

Proposition 2 *The opaque auction yields higher profits to the platform provider than does the transparent auction.*

Note that in the private value setting, the transparency policy affects the bidder's belief of the competitors' bids (i.e., his winning probability) but not his willingness to pay for the good. In this sense, we focus on the effect of market transparency on competition among the bidders. Proposition 2 remains true even when some assumptions are relaxed. Under the interdependent value setting, the strategic equivalence between $b_\theta^{II_{Tr}}$ and $b^{II_{Op}}$ collapses. Thus, the revenue ranking of the marginal type becomes $\mathcal{U}_{\hat{\theta}}^{I_{Op}} \geq \mathcal{U}_{\hat{\theta}}^{II_{Op}} \geq \mathcal{U}_{\hat{\theta}}^{II_{Tr}} = \mathcal{U}_{\hat{\theta}}^{I_{Tr}}$. The revenue equivalence between I_{Tr} and II_{Tr} depends on the conditional independence of the bidders' signals. If the signals are affiliated conditional on θ but the value is private, the linkage principle applies and hence $\mathcal{U}_{\hat{\theta}}^{I_{Op}} \geq \mathcal{U}_{\hat{\theta}}^{I_{Tr}} \geq \mathcal{U}_{\hat{\theta}}^{II_{Tr}} = \mathcal{U}_{\hat{\theta}}^{II_{Op}}$.

The transparency policy can be interpreted as a communication environment on the platform. For example, suppose that the seller can credibly transmit her private information to the bidders when communication is allowed. In such a situation, the unraveling argument works and full revelation occurs. Intuitively, the highest type among participating types has an incentive to reveal all information she possesses, and then the second highest type also reveals her information in order to escape from the pool of lower types, and so on. Our result suggests that communication among users enhances competition for higher types and reduces for lower types, thereby increasing information rents of the seller.

7 Conclusion

This paper considered a model of an auction platform in order to analyze the problem of a monopoly platform provider that maximizes profits by charging participation fees and choosing an auction format. In a setting that displays affiliated signals, we refocused on the informational role of competitive bidding in auctions from the perspective of the platform provider and showed that it is optimal for the platform provider to choose a first-price auction and an opacity policy in preference to other formats and a transparency policy.

Future work should aim to investigate a multidimensional screening problem in which both the quality of the object for buyers and the reservation value for sellers constitute the seller's type. In particular, when the platform provider has limited price instruments available, the choice of auction format becomes a useful screening device. For example, a second-price auction yields higher expected revenues to higher types of sellers and lower expected revenues to lower types, which may improve the quality of participating seller types in the platform. In contrast to our main result, the platform provider may also face a trade-off between increasing total surplus and reducing information rents.

Appendix

A Extension

A.1 Variable fees

We now examine the case where the platform provider makes fees contingent on the seller's revenue. Specifically, we suppose that the provider charges a fixed percentage $\tau \in [0,1]$ from the seller's revenue in stead of a participation fee t^S. In the baseline model, we assume that the seller's reservation utility is zero, so the provider can extract almost all surplus by setting $\tau = 1 - \epsilon$. To avoid such a trivial solution, we assume that the seller incurs a fixed opportunity cost $c > 0$ when she participates.

The provider's problem is now written as

$$\begin{aligned}
&\max_{A,\hat{\theta},\hat{k}} \tau \int_{\hat{\theta}}^{1} U_{\theta}^{A}(n,\Phi)d\Phi_0(\theta) + ND(\hat{k})t^B \\
&\text{subject to } \Phi_{\hat{\theta}}(\theta) = (\Phi_0(\theta) - \Phi_0(\hat{\theta}))/(1-\Phi_0(\hat{\theta})) \text{ for } \theta \in [\hat{\theta},1] \\
&\qquad (1-\tau)\mathcal{U}_{\hat{\theta}}^{A}(\hat{\theta},\hat{k}) = c \\
&\qquad t^B = \mathcal{V}^A(\hat{\theta},\hat{k}) - \hat{k}.
\end{aligned}$$

The second constraint is the break-even condition for the marginal type, who gets $1-\tau$ fraction of the revenue and pays opportunity costs c. As in (4), the platform provider's profit is essentially expressed as total surplus minus information rents minus opportunity costs.

$$
\begin{aligned}
\pi =& \tau^A(\hat{\theta},\hat{k}) \int_{\hat{\theta}}^1 \mathcal{U}^A_\theta(n,\Phi) d\Phi_0(\theta) + ND(\hat{k}) t^B \\
=& \int_{\hat{\theta}}^1 \mathcal{W}_\theta(\hat{k}) d\Phi_0(\theta) - (1-\tau^A(\hat{\theta},\hat{k})) \int_{\hat{\theta}}^1 \left(\mathcal{U}^A_\theta(\hat{\theta},\hat{k}) - \mathcal{U}^A_{\hat{\theta}}(\hat{\theta},\hat{k}) \right) d\Phi_0(\theta) \\
& - \underbrace{(1-\tau^A(\hat{\theta},\hat{k}))\mathcal{U}^A_{\hat{\theta}}(\hat{\theta},\hat{k})}_{c}(1-\Phi_0(\hat{\theta})) - \hat{k} ND(\hat{k}).
\end{aligned}
$$

Note that $\mathcal{U}^{I}_{\hat{\theta}}(\hat{\theta},\hat{k}) \geq \mathcal{U}^{II}_{\hat{\theta}}(\hat{\theta},\hat{k})$ implies $1-\tau^{II}(\hat{\theta},\hat{k}) \geq 1-\tau^{I}(\hat{\theta},\hat{k})$. Furthermore, as explained in equation 4, we know that $\int_{\hat{\theta}}^1 \left(\mathcal{U}^{II}_\theta(\hat{\theta},\hat{k}) - \mathcal{U}^{II}_{\hat{\theta}}(\hat{\theta},\hat{k}) \right) d\Phi_0(\theta)$ is greater than $\int_{\hat{\theta}}^1 \left(\mathcal{U}^{I}_\theta(\hat{\theta},\hat{k}) - \mathcal{U}^{I}_{\hat{\theta}}(\hat{\theta},\hat{k}) \right) d\Phi_0(\theta)$. Hence, the seller's information rents is smaller when the provider chooses a first-price auction then a second-price or English auction.

B Proof of Lemma 3

First-price vs. Second-price auctions. Fix n and Φ with the lowest type $\hat{\theta}$. We will show that $e^{I}_{\hat{\theta}}(x;n,\Phi) \geq e^{II}_{\hat{\theta}}(x;n,\Phi)$ for all x. That is, $G_{\hat{\theta}}(x) \int_0^x v(y,y) dL(y|x) \geq \int_0^x v(y,y) g_{\hat{\theta}}(y) dy$ for all x. To show this, it suffices to show that $L(\cdot|x)$ first-order stochastically dominates $G_{\hat{\theta}}(\cdot|Y^{(1)}_{\hat{\theta}} \leq x, \hat{\theta})$. It is equivalent to show that for all $y < x$, $L(y|x) \leq \frac{G_{\hat{\theta}}(y)}{G_{\hat{\theta}}(x)}$.

Note that

$$
\begin{aligned}
-\int_y^x \frac{h(t|t)}{H(t|t)} dt \leq & -\int_y^x \frac{g_{\hat{\theta}}(t)}{G_{\hat{\theta}}(t)} dt \\
=& \ln G_{\hat{\theta}}(y) - \ln G_{\hat{\theta}}(x) \\
=& \ln \left(\frac{G_{\hat{\theta}}(y)}{G_{\hat{\theta}}(x)} \right).
\end{aligned}
$$

Hence we have $L(y|x) \leq \frac{G_{\hat{\theta}}(y)}{G_{\hat{\theta}}(x)}$ for all $y < x$. ■

Second-price vs. English auctions. The symmetric bidding strategy in the second-price auction is written as

$$\begin{aligned}
b^{II}(y) =& \mathbb{E}[u(\theta, y)|X_1 = y, Y_1 = y] \\
=& \mathbb{E}[\mathbb{E}[u(\theta, y)|X_1 = y, Y_1 = y, Y_2 = y_2, \ldots, Y_{n-1} = y_{n-1}]|X_1 = y, Y_1 = y] \\
=& \mathbb{E}[b^{Eng(2)}(y, y_2, y_3, \ldots, y_{n-1})|X_1 = y, Y_1 = y].
\end{aligned}$$

When his signal is x and the seller's type is θ, the expected payment by bidder 1

$$\begin{aligned}
e^{II}_{\theta}(x) =& \mathbb{E}[b^{II}(Y_1)|Y_1 < x, \theta] \cdot \Pr(Y_1 < x|\theta) \\
=& \mathbb{E}[\mathbb{E}[b^{Eng(2)}(Y_1, Y_2, \ldots, Y_{n-1})|X_1 = y, Y_1 = y]|Y_1 < x, \theta] \cdot \Pr(Y_1 < x|\theta).
\end{aligned}$$

On the other hand, in English auction, bidder 1's expected payment is

$$\begin{aligned}
e^{Eng}_{\theta}(x) =& \mathbb{E}[b^{Eng(2)}(Y_1, Y_2, \ldots, Y_{n-1})|Y_1 < x, \theta] \cdot \Pr(Y_1 < x|\theta) \\
=& \mathbb{E}[\mathbb{E}[b^{Eng(2)}(Y_1, Y_2, \ldots, Y_{n-1})|X_1 = x, Y_1 = y, \theta]|Y_1 < x, \theta] \cdot \Pr(Y_1 < x|\theta).
\end{aligned}$$

Since $b^{Eng(2)}(y_1, y_2, \ldots, y_{n-1})$ is nondecreasing in each argument, it suffices to show that the distribution $H(\mathbf{y}_{-1}|x, y_1)$ of $\mathbf{y}_{-1} \equiv (y_2, \ldots, y_{n-1})$ given $X_1 = x$ and $Y_1 = y_1$ first-order stochastically dominates the distribution $G_{\hat{\theta}}(\mathbf{y}_{-1}|x, y)$ of $\mathbf{y}_{-1}$ given $X_1 = x$, $Y_1 = y_1$ and $\theta = \hat{\theta}$. The density function of $H(\mathbf{y}_{-1}|x, y_1)$ is

$$h(\mathbf{y}_{-1}|x, y_1) = \frac{\int (n-1)! f_\theta(x) f_\theta(y_1) f_\theta(y_2) \cdots f_\theta(x_{n-1}) d\Phi(\theta)}{\int f_\theta(x) g_\theta(y_1) d\Phi(\theta)}$$

and the density of $G_{\hat{\theta}}(\mathbf{y}_{-1}|x, y_1)$ does not depend on x and is written as

$$g_{\hat{\theta}}(\mathbf{y}_{-1}|x, y_1) = \frac{(n-1)! f_{\hat{\theta}}(y_1) f_{\hat{\theta}}(y_2) \cdots f_{\hat{\theta}}(y_{n-1})}{g_{\hat{\theta}}(y_1)}.$$

Since $f_\theta(y)/f_{\hat{\theta}}(y)$ is increasing in y when θ belongs to the support of Φ, so is $\frac{h(\mathbf{y}_{-1}|x, y_1)}{g_{\hat{\theta}}(\mathbf{y}_{-1}|x, y_1)}$. That is, $H(\mathbf{y}_{-1}|x, y_1)$ dominates $G_{\hat{\theta}}(\mathbf{y}_{-1}|x, y)$ in terms of the likelihood ratio. It suffices to obtain the result. ■

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