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Who buys what, where: Reconstruction of the  
international trade flows by commodity and industry

Yuichi Ikeda

Graduate School of Advanced Integrated Studies  
in Human Survivability, Kyoto University

Tsutomu Watanabe

Graduate School of Economics, University of Tokyo

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# Who buys what, where: Reconstruction of the international trade flows by commodity and industry

Yuichi Ikeda and Tsutomu Watanabe

**Abstract** We developed a model to reconstruct the international trade network by considering both commodities and industry sectors in order to study the effects of reduced trade costs. First, we estimated trade costs to reproduce WIOD and NBER-UN data. Using these costs, we estimated the trade costs of sector specific trade by types of commodities. We successfully reconstructed sector-specific trade for each types of commodities by maximizing the configuration entropy with the estimated costs. In WIOD, trade is actively conducted between the same industry sectors. On the other hand, in NBER-UN, trade is actively conducted between neighboring countries. This seems like a contradiction. We conducted community analysis for the reconstructed sector-specific trade network by type of commodities. The community analysis showed that products are actively traded among same industry sectors in neighboring countries. Therefore the observed features of the community structure for WIOD and NBER-UN are complementary.

## 1 Introduction

In the era of economic globalization, most national economies are linked by international trade, which in turn consequently forms a complex global economic network. It is believed that greater economic growth can be achieved through free trade based on the establishment of Free Trade Agreements (FTAs) and Economic Partnership Agreements (EPAs). However, there is limitation to the resolution of the currently available trade data. For instance, NBER-UN records trade amounts between bilateral countries without industry sector information for each type of commodities [1],

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Y. Ikeda

Graduate School of Advanced Integrated Studies in Human Survivability, Kyoto University, e-mail: ikeda.yuichi.2w@kyoto-u.ac.jp

T. Watanabe

Graduate School of Economics, University of Tokyo e-mail: watanabe@e.u-tokyo.ac.jp

and the World Input-Output Database (WIOD) records sector-specific trade amount without commodities information [2]. This limited resolution makes it difficult to analyze community structures in detail and systematically assess the effects of reduced trade tariffs and trade barriers.

In this paper, we reconstruct the sector-specific trade network for each type of commodities by maximizing the configuration entropy based on the local information about the inward and outward flow of trade. The reconstruction of interbank networks from local information has been studied intensively [3, 4, 5]. But these studies intend to reproduce the average nearest degree, the average nearest strength, and the expected weight for various weighted networks. Our goal is to reconstruct an international trade network by considering both commodities and industry sectors in order to systematically study the effects of reduced trade costs, such as trade tariffs and trade barriers.

This paper is organized as follows: Section 2 describes the model of network reconstruction, and Section 3 explains the existing trade data. Section 4 shows results of cost estimation, and finally Section 5 explains the identified community structure for the reconstructed international trade network. Section 6 provides a summary of the points presented in this paper.

## 2 Model of Network Reconstruction

We reconstruct the international trade network by considering both commodities and industry sectors in order to systematically study the effects of reduced trade costs. For this reason, the estimation of trade costs is indispensable. In this section, we describe our network reconstruction model.

### 2.1 Outline of Network Reconstruction Model

We reconstruct the sector-specific trade network for each type of commodities by maximizing the configuration entropy using existing international trade data: NBER-UN and WIOD. These two types of existing data will be explained in the following section.

The outline of our model of network reconstruction is as follows:

1. We estimate trade cost  $C_{AB}^{(G)}$  between country  $A$  and  $B$  for commodities ( $G$ ) to reproduce trade amount data NBER-UN  $T_{AB}^{(G)}$  by maximizing the configuration entropy with given strengths  $D_A^{(G)}$  and  $O_B^{(G)}$ .
2. We estimate trade cost  $C_{A\alpha B\beta}$  to reproduce trade amount data WIOD  $T_{A\alpha B\beta}$  between industry sector  $\alpha$  in country  $A$  and industry sector  $\beta$  in country  $B$  by maximizing the configuration entropy with given strengths  $D_{A\alpha}$  and  $O_{B\beta}$ .

3. We obtain an analytic formulae to calculate trade cost  $C_{A\alpha B\beta}^{(G)}$  using costs estimated above:  $C_{AB}^{(G)}$  and  $C_{A\alpha B\beta}$ .
4. We calculate the trade cost  $C_{A\alpha B\beta}^{(G)}$  analytically and estimate the sector-specific trade for each type of commodities  $T_{A\alpha B\beta}^{(G)}$  by maximizing the configuration entropy with given strengths  $T_{AB}^{(G)}$  and  $T_{A\alpha B\beta}$ .

## 2.2 Maximization of the Configuration Entropy

A model that calculates the amount of traffic flow based on the local information for total outflow and inflow by maximizing the configuration entropy has been proposed [6]. We apply this model for our purpose. Suppose that the total amount of export  $O_i$  from country  $i$ , total amount of import  $D_j$  to country  $j$ , and trade cost  $c_{ij}$  from country  $i$  to country  $j$  are given:  $O_i = \sum_j t_{ij}$ ,  $D_j = \sum_i t_{ij}$ , and  $C = \sum_{ij} t_{ij} c_{ij}$ . The formulation of export  $t_{ij}$  from country  $i$  to  $j$  is obtained by maximizing the configuration entropy  $S = \log W = \log ((\sum_{ij} t_{ij})! / \prod_{ij} t_{ij}!)$  with the constraints using the Lagrange multiplier method. As a result, we obtain the closed relationship for export  $t_{ij}$  as follows:

$$t_{ij} = A_i B_j O_i D_j \exp(-\beta c_{ij}), \quad (1)$$

$$A_i = \left[ \sum_j B_j D_j \exp(-\beta c_{ij}) \right]^{-1}, \quad (2)$$

$$B_j = \left[ \sum_i A_i O_i \exp(-\beta c_{ij}) \right]^{-1}. \quad (3)$$

Here  $\beta$  is a multiplier that signifies the constraint for total trade cost  $C$ . Coefficients  $A_i$  and  $B_j$  are calculated iteratively from the appropriate initial values.

## 2.3 Algorithm of Cost Estimation

The trade cost is estimated using simulated annealing to reproduce the actual trade data. The simulated annealing takes a long time to compute, but shows a reasonably good convergence of the cost estimation. The cooling schedule of temperature  $T$  is given by  $T_n = (1 - 0.003)^n$ . Here  $n$  is the number of iteration step. At each temperature, the calculation is repeated using equilibrium samples. The root mean square error of calculated trade  $RMS_n = \sqrt{\sum_{ij} ((t_{ij}^{cal} - t_{ij}) / t_{ij})^2 / N}$  is calculated at each iteration step. Here  $N$  is the number of countries. If  $\Delta RMS_n = RMS_n - RMS_{n-1}$  is

negative, we accept cost  $c_{ij}$ , but if  $\Delta RMS_n$  is positive, the acceptance of cost is determined stochastically depending on the temperature.

## 2.4 Sector-Specific Cost by Commodities

The analytical formula of sector-specific cost  $c_{A\alpha B\beta}^{(G)}$  by type of commodities is obtained as a weighted average of the trade costs for WIOD and NBER-UN:  $c_{A\alpha B\beta}$  and  $c_{AB}^{(G)}$ . We have three identities:

$$\sum_{\alpha\beta} T_{A\alpha B\beta} C_{A\alpha B\beta} = \sum_G T_{AB}^{(G)} C_{AB}^{(G)} = \sum_{\alpha\beta G} T_{A\alpha B\beta}^{(G)} C_{A\alpha B\beta}^{(G)} = T_{AB} C_{AB}, \quad (4)$$

$$T_{A\alpha B\beta} C_{A\alpha B\beta} = \sum_G T_{A\alpha B\beta}^{(G)} C_{A\alpha B\beta}^{(G)}, \quad (5)$$

$$T_{AB}^{(G)} C_{AB}^{(G)} = \sum_{\alpha\beta} T_{A\alpha B\beta}^{(G)} C_{A\alpha B\beta}^{(G)}. \quad (6)$$

Using these identities, we write trade cost  $c_{A\alpha B\beta}^{(G)}$  as a weighted average of  $C_{A\alpha B\beta}$  and  $C_{AB}^{(G)}$ .

$$C_{A\alpha B\beta}^{(G)} = \frac{1}{2} \left( u_G \frac{T_{A\alpha B\beta}}{T_{A\alpha B\beta}^{(G)}} C_{A\alpha B\beta} + v_{\alpha\beta} \frac{T_{AB}^{(G)}}{T_{A\alpha B\beta}^{(G)}} C_{AB}^{(G)} \right), \quad (7)$$

$$u_G = \frac{C_{AB}^{(G)}}{C_{AB}}, \quad (8)$$

$$v_{\alpha\beta} = \frac{C_{A\alpha B\beta}}{C_{AB}}. \quad (9)$$

We obtain the following analytical formula of the sector-specific cost by type of commodities as an approximation:

$$C_{A\alpha B\beta}^{(G)} \cong \frac{1}{2} \left( u_G G C_{A\alpha B\beta} + v_{\alpha\beta} S^2 C_{AB}^{(G)} \right). \quad (10)$$

## 2.5 Sector-Specific Trade by Type of Commodities

Once  $C_{A\alpha B\beta}^{(G)}$  is obtained, we estimate  $T_{A\alpha B\beta}^{(G)}$  based on the given local information  $T_{AB}^{(G)}$  and  $T_{A\alpha B\beta}$  by maximizing entropy iteratively, in the same manner as before.

$$T_{A\alpha B\beta}^{(G)} = \tilde{A}_{AB}^{(G)} \tilde{B}_{A\alpha B\beta} T_{AB}^{(G)} T_{A\alpha B\beta} \exp \left( -\beta C_{A\alpha B\beta}^{(G)} \right), \quad (11)$$

$$\tilde{A}_{AB}^{(G)} = \left[ \sum_{\alpha\beta} \tilde{B}_{A\alpha B\beta} T_{A\alpha B\beta} \exp\left(-\beta C_{A\alpha B\beta}^{(G)}\right) \right]^{-1}, \quad (12)$$

$$\tilde{B}_{A\alpha B\beta} = \left[ \sum_G \tilde{A}_{AB}^{(G)} T_{AB}^{(G)} \exp\left(-\beta C_{A\alpha B\beta}^{(G)}\right) \right]^{-1}. \quad (13)$$

### 3 Trade Data

We used bilateral trade data between countries for each type of commodities NBER-UN and sector-specific trade data WIOD at year 2000. Table 1 shows the list of commodities for NBER-UN. Table 2 shows the list of countries for NBER-UN and WIOD. Table 3 shows the list of industry sectors for WIOD. Here the number of commodities  $G$  is 10, the number of countries  $N$  is 31, and the number of industry sectors  $S$  is 35.

Figure 1 shows that the relationship between NBER-UN and WIOD for the total amount of exports and imports of 31 countries. We note that WIOD is about 50% to 60% of NBER-UN for both exports and imports. We assume that the difference between the two databases comes from the lack of a consumer sector in WIOD.

**Table 1** Commodities for NBER-UN

Symbol	Description
g0	<i>FOOD AND LIVE ANIMALS CHIEFLY FOR FOOD</i>
g1	<i>BEVERAGES AND TOBACCO</i>
g2	<i>CRUDE MATERIALS, INEDIBLE, EXCEPT FUELS</i>
g3	<i>MINERAL FUELS, LUBRICANTS AND RELATED MATERIALS</i>
g4	<i>ANIMAL AND VEGETABLE OILS, FATS AND WAXES</i>
g5	<i>CHEMICALS AND RELATED PRODUCTS, N.E.S.</i>
g6	<i>MANUFACTURED GOODS CLASSIFIED CHIEFLY BY MATERIAL</i>
g7	<i>MACHINERY AND TRANSPORT EQUIPMENT</i>
g8	<i>MISCELLANEOUS MANUFACTURED ARTICLES</i>
g9	<i>COMMODITIES &amp; TRANS. NOT CLASSIFIED ELSEWHERE</i>

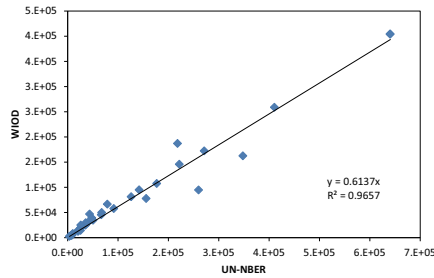
**Table 2** Countries for NBER-UN and WIOD

Symbol	Description	Symbol	Description	Symbol	Description	Symbol	Description
c1	<i>Australia</i>	c2	<i>Austria</i>	c3	<i>Bulgaria</i>	c4	<i>Brazil</i>
c5	<i>Canada</i>	c6	<i>China</i>	c7	<i>CzechRep</i>	c8	<i>Germany</i>
c9	<i>Denmark</i>	c10	<i>Spain</i>	c11	<i>Finland</i>	c12	<i>France</i>
c13	<i>UK</i>	c14	<i>Greece</i>	c15	<i>Hungary</i>	c16	<i>Indonesia</i>
c17	<i>Ireland</i>	c18	<i>Italy</i>	c19	<i>Japan</i>	c20	<i>KoreaRep</i>
c21	<i>Mexico</i>	c22	<i>Netherlands</i>	c23	<i>Poland</i>	c24	<i>Portugal</i>
c25	<i>Romania</i>	c26	<i>RussianFed</i>	c27	<i>Slovakia</i>	c28	<i>Slovenia</i>
c29	<i>Sweden</i>	c30	<i>Turkey</i>	c31	<i>USA</i>		

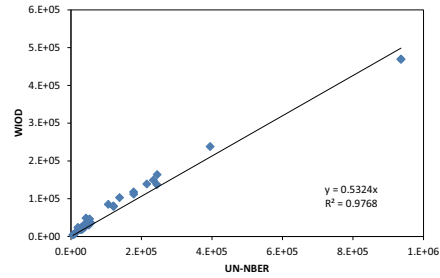
**Table 3** Industry Sectors for WIOD

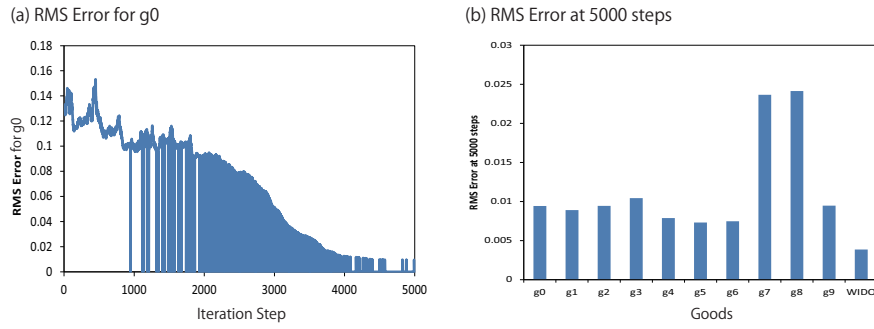
Symbol	Description
s1	<i>Agriculture, Hunting, Forestry and Fishing</i>
s2	<i>Mining and Quarrying</i>
s3	<i>Food, Beverages and Tobacco</i>
s4	<i>Textiles and Textile Products</i>
s5	<i>Leather, Leather and Footwear</i>
s6	<i>Wood and Products of Wood and Cork</i>
s7	<i>Pulp, Paper, Paper, Printing and Publishing</i>
s8	<i>Coke, Refined Petroleum and Nuclear Fuel</i>
s9	<i>Chemicals and Chemical Products</i>
s10	<i>Rubber and Plastics</i>
s11	<i>Other Non-Metallic Mineral</i>
s12	<i>Basic Metals and Fabricated Metal</i>
s13	<i>Machinery, Nec</i>
s14	<i>Electrical and Optical Equipment</i>
s15	<i>Transport Equipment</i>
s16	<i>Manufacturing, Nec; Recycling</i>
s17	<i>Electricity, Gas and Water Supply</i>
s18	<i>Construction</i>
s19	<i>Sale, Maintenance and Repair of Motor Vehicles and Motorcycles; Retail Sale of Fuel</i>
s20	<i>Wholesale Trade and Commission Trade, Except of Motor Vehicles and Motorcycles</i>
s21	<i>Retail Trade, Except of Motor Vehicles and Motorcycles; Repair of Household Goods</i>
s22	<i>Hotels and Restaurants</i>
s23	<i>Inland Transport</i>
s24	<i>Water Transport</i>
s25	<i>Air Transport</i>
s26	<i>Other Supporting and Auxiliary Transport Activities; Activities of Travel Agencies</i>
s27	<i>Post and Telecommunications</i>
s28	<i>Financial Intermediation</i>
s29	<i>Real Estate Activities</i>
s30	<i>Renting of M&amp;Eq and Other Business Activities</i>
s31	<i>Public Admin and Defence; Compulsory Social Security</i>
s32	<i>Education</i>
s33	<i>Health and Social Work</i>
s34	<i>Other Community, Social and Personal Services</i>
s35	<i>Private Households with Employed Persons</i>

(a) Export (M\$)



(b) Import (M\$)

**Fig. 1** The relationship between NBER-UN and WIOD for the total amount of exports and imports of 31 countries.



**Fig. 2** RMS error for g0: food and live animals chiefly, and RMS errors at 5000 steps for various commodities and WIOD.

## 4 Cost Estimation

In this section, first we estimated  $C_{AB}^{(G)}$  to reproduce the trade amount data for NBER-UN. Then, we estimated  $C_{A\alpha B\beta}$  to reproduce the trade amount data for WIOD. Finally, we calculated  $C_{A\alpha B\beta}^{(G)}$  using the analytic formula in Eq. (10) as a weighted average of  $C_{A\alpha B\beta}$  and  $C_{AB}^{(G)}$ .

### 4.1 Trade Cost of WIOD

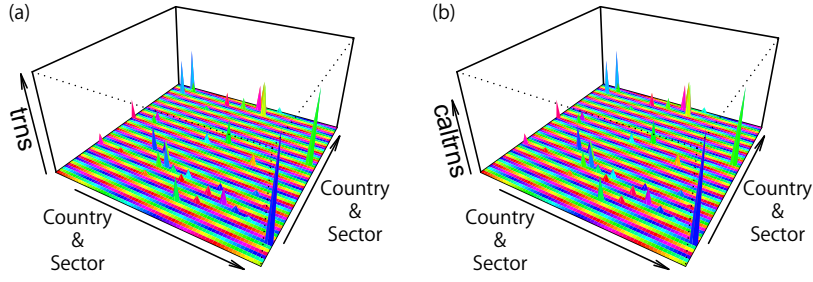
The left panel of Fig. 2 shows that the convergence of the RMS errors for g0: *FOOD AND LIVE ANIMALS CHIEFLY FOR FOOD* and the right panel shows the RMS errors at 5000 steps for various type of commodities and WIOD. The error for each trade cost is 0.5% to 2% for all commodities and WIOD.

Figure 3 shows the comparison of (a) actual trade  $T_{A\alpha B\beta}$  and (b) calculated trade  $T_{A\alpha B\beta}$  using estimated cost  $C_{A\alpha B\beta}$ . The agreement between two types of trade is quite good.

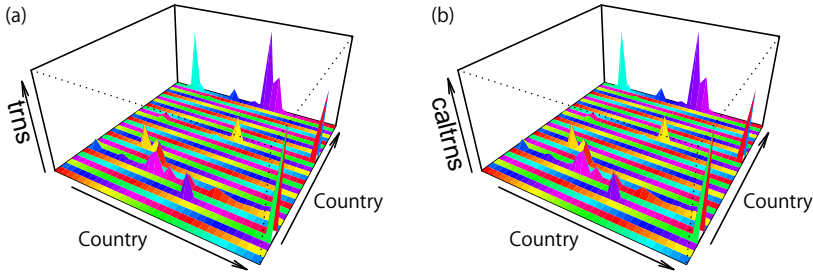
### 4.2 Trade Cost of NBER-UN

Figure 4 shows the comparison of (a) actual trade  $T_{AB}^{(G)}$  and (b) calculated trade  $T_{AB}^{(G)}$  using estimated cost  $C_{AB}^{(G)}$  for commodity g7: *MACHINERY AND TRANSPORT EQUIPMENT*. The agreement between these two types of trade is once again quite good.





**Fig. 3** Comparison of actual trade  $T_{\alpha\beta\beta}$  and calculated trade  $T_{\alpha\beta\beta}$  using estimated cost  $C_{\alpha\beta\beta}$ .



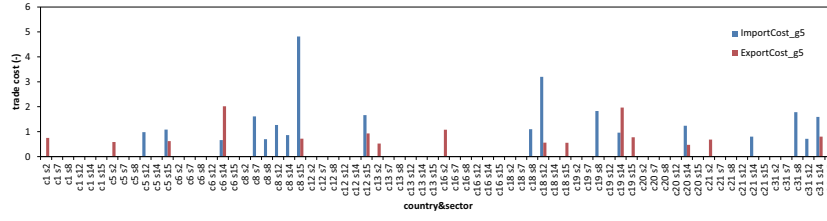
**Fig. 4** Comparison of actual trade  $T_{AB}^{(G)}$  and calculated trade  $T_{AB}^{(G)}$  using estimated cost  $C_{AB}^{(G)}$  for commodity g7.

### 4.3 Estimated Sector-Specific Cost by Type of Commodities

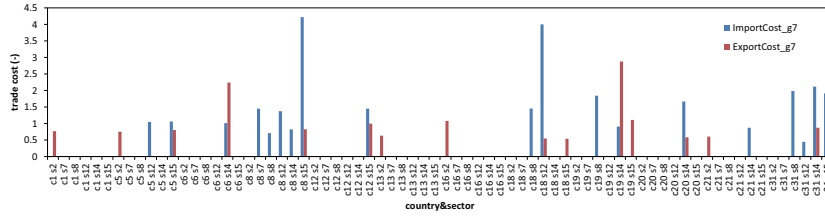
Sector-specific cost by type of commodities was estimated using Eq. (10). The estimated cost  $C_{\alpha\beta\beta}^{(G)}$  for commodity g5: *CHEMICALS AND RELATED PRODUCTS, N.E.S.* and g7: *MACHINERY AND TRANSPORT EQUIPMENT* are shown in Figs. 5 and 6, respectively. Note that we have common characteristics for both g5 and g7. For example, import costs in the german transport equipment industry are very high compared with other industry sectors of various countries. In the US, import costs are higher than export costs for many industries. On the other hand, in Japan, export costs are higher than import costs for some industries.

## 5 Reconstructed Sector-Specific Trade Network by Type of Commodities

$T_{\alpha\beta\beta}^{(G)}$  provides a weight for links of the sector-specific trade network by each type of commodities. For the reconstructed international trade network, we identify a community structure that corresponds to economic clusters linked by the trade of



**Fig. 5** Estimated trade cost  $C_{\alpha\beta\beta}^{(G)}$  for commodity g5: *CHEMICALS AND RELATED PRODUCTS, N.E.S.*



**Fig. 6** Estimated trade cost  $C_{\alpha\beta\beta}^{(G)}$  for commodity g7: *MACHINERY AND TRANSPORT EQUIPMENT.*

various type of commodities. In past analysis of the sector-specific trade network (WIOD), we obtained communities consisting of the same industry sector across countries [7, 8, 9]. In this section, we describe the characteristics of the community structure identified for the reconstructed sector-specific trade network by type of commodities.

### 5.1 Community Structure in WIOD

The community structure was identified by maximizing the modularity for WIOD. The identified community shows that the international trade is actively conducted between the same or similar industry sectors [9], but it is not know which commodities are traded. We note that a defect has been pointed out for the null model used in the definition of the modularity for weighted networks [3]. We conducted community analysis using map equation [10] for WIOD to confirm the community structure identified by modularity maximization. We confirmed that international trade is actively conducted between the same or similar industry sectors. The largest community consists of industry sector: *Renting of M&Eq, Financial Intermediation*, the second is industry sector: *Chemical Products*, the third is industry sector: *Basic Metals and Fabricated Metal*, the fourth is industry sector: *Mining and Quarrying*,

the fifth is industry sector: *Electrical and Optical Equipment*, and the sixth is industry sector: *Transport Equipment*.

### 5.2 Community Structure in NBER-UN

Community analysis for NBER-UN shows that international trade is actively conducted between neighboring countries, but industry sectors in which trade is conducted are not known. For example, we found five communities for g5: *CHEMICALS AND RELATED PRODUCTS, N.E.S.*. The largest community is Europe, consisting of Austria, Bulgaria, the Czech Republic, Germany, Spain, Finland, France, the UK, Hungary, Italy, the Netherlands, Poland, Portugal, the Russian Federation, Slovakia, and Slovenia. The second is South & North America, consisting of Brazil, Canada, Ireland, Mexico, and the USA. The third is Asia, consisting of Australia, China, Indonesia, Japan, and Korea Republic. The fourth is West Asia & East Europe, consisting of Greece, Romania, and Turkey. The fifth is North Europe, consisting of Denmark and Sweden.

### 5.3 Community Structure in Reconstructed Sector-Specific Trade Network by Commodities

Community analysis of the sector-specific trade network (WIOD) shows that international trade is actively conducted between the same or similar industry sectors. On the other hand, community analysis of the trade network for a specific type of commodities (NBER-UN) shows that international trade is actively conducted between neighboring countries. At first glance, these results seem to be contradictory. What do these results really mean?

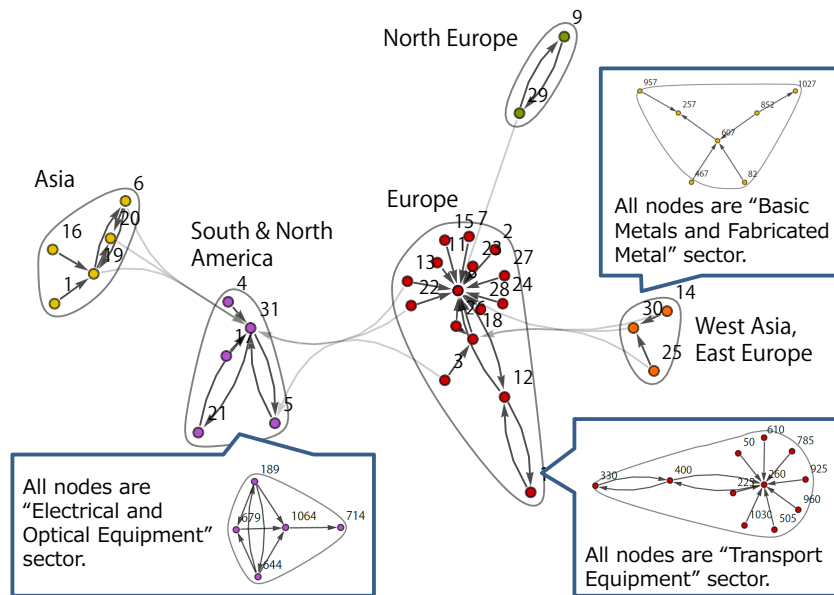
We conducted community analysis for the reconstructed sector-specific trade network by type of commodity g5: *CHEMICALS AND RELATED PRODUCTS, N.E.S.*. The identified community structure is shown in Fig. 7. The largest community corresponds to Europe, and all nodes in this community are in the "Transport Equipment" industry sector. The second largest community corresponds to South & North America, and all nodes are in the "Electrical and Optical Equipment" industry sector. In a similar way, the third largest community corresponds to West Asia & East Europe, and all nodes are in the "Basic Metals and Fabricated Metal" industry sector. Analysis showed that products are actively traded between the same industry sectors in neighboring countries. Therefore, we can say that the observed features of the community structure for WIOD and NBER-UN are not contradictory but rather that they are complementary.

## 6 Summary

We developed a model to reconstruct the international trade network by considering both commodities and industry sectors in order to study the effects of reduction of various trade costs. First, we estimated the trade cost to reproduce WIOD and NBER-UN data. Using these costs, we estimated the trade cost of sector specific trade by type of commodities. We successfully reconstructed sector-specific trade for each type of commodities by maximizing the configuration entropy with the estimated cost.

In WIOD, trade is actively conducted between the same industry sectors. On the other hand, in NBER-UN, trade is actively conducted between neighboring countries. This seems like a contradiction. We conducted community analysis for the reconstructed sector-specific trade network by type of commodity g5. The community analysis showed that products are actively traded between the same industry sectors in neighboring countries. The observed features of the community structure for WIOD and NBER-UN are complementary.

In future studies, we intend to analyze the effect of reduced trade tariffs and trade barriers. For instance, the Trans-Pacific Partnership (TPP) is expected to achieve a high-level of free trade in the Asia-Pacific region, which accounts for more than 40% of the world's GDP. Trade costs are estimated at 170% of the price of commodities. The breakdown in transportation costs is 21%, and the rest is trade tariffs and trade barriers [11]. We will discuss the effect of reduced trade tariffs and trade barriers



**Fig. 7** Community Structure in Reconstructed Sector-Specific Trade Network for commodity g5.

on the change in the community structure of the international trade network. This will enable us to arrive at better understanding of international trade after the TPP agreement goes into effect.

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