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Framing Game Theory

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Framing Game Theory[1]

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Abstract

A real player sometimes fails to practice hypothetical thinking, which increases the occurrence of anomalies in various situations. This study incorporates psychology into game theory and demonstrates a cognitive method to encourage bounded-rational players to practice correct hypothetical thinking in strategic interactions with imperfect information. We introduce a concept termed "frame" as a description of a synchronized cognitive procedure through which each player decides multiple actions in a step-by-step manner, shaping his (or her) strategy selection. We could regard a frame as the supposedly irrelevant factors from the viewpoint of full rationality. However, this paper theoretically shows that in a multi-unit trading with private values, the ascending proxy auction has a significant advantage over the second-price auction in terms of the bounded-rational players' incentive to practice hypothetical thinking, because of the difference, not in physical rule, but in background frame. By designing a frame appropriately, we generally show that any static game that is solvable in iteratively undominated strategies is also solvable, even if players cannot practice hypothetical thinking without the help of a well-designed frame. We further investigate the possibility that even a detail-free frame design serves to overcome the difficulty of hypothetical thinking. We extend this investigation to the Bayesian environments.

Keywords: Hypothetical Thinking, Frame Design, Quasi-Obvious Dominance, Ascending Proxy Auction, Abreu-Matsushima Mechanism

JEL Classification Numbers: C72, D78, D82, D83, D91

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1. Introduction

A real player sometimes fails to practice hypothetical thinking (or conditional reasoning) in strategic situations[3]. Since each player cannot observe the other players' strategies, it is inevitable that he (or she) makes a hypothesis about the other players' strategies, provided he wants to carry out rational behavior. In this study, hypothetical thinking implies the "what-if" manner of strategic thought, such that a rational player first makes a hypothesis about the other players' strategies, and then reasons about his best strategy from this hypothesis, where he does not recognize whether the hypothesis is true.

A real player, however, sometimes avoids or incorrectly practices such hypothetical thinking. Instead of thinking hypothetically, he incorrectly thinks: "I expect the other players to select a strategy profile if I intend to select a strategy, while I expect them to select another strategy profile if I intend to select another strategy." In other words, he incorrectly expects the other players' strategies to depend on which strategy he intends to select, though he ought to recognize that they cannot observe his strategy selection.

In the prisoners' dilemma game, for instance, a bounded-rational player incorrectly thinks: "I expect the other player to select C (cooperation) if I intend to select C, while I expect him to select D (defection) if I intend to select D."

The failure of hypothetical thinking generally causes various anomalies in economics, such as the winner's curse, overbidding, non-pivotal voting, Ellsberg's paradox, and Allais paradox. Players may fail to think hypothetically even in simple situations that have dominant strategies, such as the prisoners' dilemma and a second price auction.

Hence, it is important in game theory to consider the possibility that players sometimes irrationally avoid hypothetical thinking. Furthermore, it is important to explore a psychological method that encourages such bounded-rational players to practice hypothetical thinking more appropriately.

This paper argues that *frame design* serves to promote correct hypothetical

[3] We have a literature in cognitive psychology that studied hypothetical thinking. For instance, see Evans (2007) and Nickerson (2015). This study attempts to incorporate cognitive psychology into game theory.

thinking. We define a frame as a description of the players' cognitive procedure, which is a re-formulation of the static game that players face, as an extensive form game with imperfect information. We regard a frame as the "supposedly irrelevant factors" from the viewpoint of full rationality. We assume that the frame is common knowledge among all players.

A frame divides each player's strategy selection into multiple cognitive steps of decision-making. According to the frame, we regard each player's strategy as a combination of multiple actions that he sequentially decides on through the cognitive procedure implied by the frame.

A frame synchronizes players' sequential decisions with each other. Importantly, at each step, a player assumes that the other players have already decided upon the actions that the frame requires them to before this step, whereas they have not yet done so at this and later steps. We show that this sequential and synchronized nature of a frame plays a significant role in helping players to practice correct hypothetical thinking.

This study categorizes hypothetical thinking into the following two types. The first type concerns the actions that the other players have already decided upon before the current step. The second type concerns the actions that the other players will decide upon at the current and future steps. We assume that players can correctly practice the first type of hypothetical thinking, whereas they fail to practice the second type of hypothetical thinking.

At each step, a player perceives the actions that the other players have already decided to take as *irreversible* ones. This perception leads the player to correctly perceive that his action decision at the current step has no relation with the actions that the other players have decided to take. On the other hand, the player does not perceive the actions that the other players will decide to take at the current and future steps as irreversible ones. This misperception prevents him from correctly perceiving that his action decision he takes at the current step has no relation with these actions. Hence, a player can practice the first type of hypothetical thinking, whereas he does not necessarily practice the second type.

To overcome the failure of applying the second type of hypothetical thinking, we propose methods of designing a frame, and then show various permissible results in the *frame design*. For instance, we can gain insights into the comparison between the second price auction and the ascending proxy auction, both of which are static games with imperfect information. The strategic interactions implied by these auctions are logically equivalent, but their background frames are different. The second price auction accompanies a degenerate frame, whereas the detailed description of the open-bid ascending auction protocol frames the ascending proxy auction. The latter frame positions the decisions that are more suspected of causing the failure on later steps. Because of this difference in background frame, players can practice hypothetical thinking more correctly in the ascending proxy auction than in the second price auction. This observation is consistent with the evidence that people have historically hesitated to apply the second price auction, whereas people are willing to use proxy bids in online open-bid ascending auctions.

The failure to think hypothetically also badly influences the practice of higher-order reasoning and iterative elimination in strategic situations. Without the help of appropriate frame design, a player cannot even expect the other players to play undominated strategies, because they do not necessarily practice correct hypothetical thinking. This significantly obstructs the practice of higher-order reasoning and iterative elimination of dominated strategies.

However, a well-designed frame can avoid such obstacles. We show that for any static game that is solvable in iteratively undominated strategies, there always exists a frame that motivates even bounded-rational players to practice both hypothetical thinking and higher-order reasoning, that is, to play the unique iteratively undominated strategy profile. To overcome the difficulties, we design a frame that positions the strategies that can be eliminated in earlier stages of iteration in later steps.

Unfortunately, this frame design is generally dependent on the finer details of payoff functions. However, this study also investigates the possibility that a detail-free frame design functions in a wide class of games with both complete and incomplete information.

2. Literature Review and Contributions

This study investigates a case of framing effects. In contrast with the previous works on framing effects and related topics such as prospect theory (Kahneman and Tversky, 1979) and focal points (Schelling, 1963), this study defines a frame as an extensive form game with imperfect information, and shows that it serves as a guidance for all players to behave rationally.

Game theory typically interprets an extensive form game as a physical rule of strategic interactions. However, this study should view an extensive form game as a frame, that is, a description of not a physical rule but a cognitive procedure. In this respect, this study is related to the seminal work by Glazer and Rubinstein (1996). Glazer and Rubinstein (1996) argued that an extensive form game provides useful information about how to carry out iterative elimination in a static game. In contrast, this study assumes that players are rational in iterative elimination, but not rational in hypothetical thinking. We then emphasize that an extensive form game provides useful information about how to carry out hypothetical thinking in a static game.

Because of the difference in role, the manner of designing extensive form games in this study is substantially different from Glazer and Rubinstein. While the physical rule is a static game with imperfect information, the extensive form game that Glazer and Rubinstein consider assumes perfect information. However, to make the cognitive procedure consistent with the physical rule, this study considers an extensive form game that assumes imperfect information. Moreover, Glazer and Rubinstein put the decisions in the order of elimination, while this study puts the decisions in the reverse order of elimination. For instance, in the study of Abreu-Matsushima mechanisms, Glazer and Rubinstein design the extensive form game that fines the first deviants from truthful revelation, while this study designs the extensive form game (with imperfect information) that fines the last deviants from truthful revelation.

The study of hypothetical thinking is a growing concern in experimental and theoretical economics. We can expect that the failure of hypothetical thinking contains a clue for discovering the origin of various anomalies in laboratory experiments such as the winner’s curse (Charness and Levin, 2009), non-pivotal voting (Esponda and Vespa, 2014), market failure caused by informational asymmetry (Ngangoue and Weizsacker, 2015), ambiguity, and loss aversion (Esponda and Vespa, 2016). For instance, Esponda and Vespa (2016) conducted laboratory experiments for testing the Sure-Thing Principle and showed that subjects tend to fail hypothetical thinking in various situations of single-person decision making. Esponda and Vesta compared a noncontingent treatment and a contingent treatment, where the contingent treatment added to the corresponding noncontingent treatment a device to guide a subject to hypothetical thinking.

This study generalizes the devices of guidance in single-person problems to apply to multi-person strategic problems. The problem with this generalization is that it is not always possible to provide all aspects of multi-person decision-making with the devices of guidance, because the order of decision-making across players matters in frame design. We need a careful frame design to divide each player's strategy selection into multiple steps of action decisions and then to specify the order of these action decisions. Hence, we need to formulate a frame as an extensive form game with imperfect information that is common to all players.

There are various equilibrium analyses in game theory that considered bounded rationality in hypothetical thinking, such as Jehiel (2005), Eyster and Rabin (2005), Esponda (2008), and Li (2017). Among them, Li (2017) is closely related to this study, but in a limited manner.

The difficulty of applying hypothetical thinking in daily life sometimes justifies the advantage of dynamic mechanism design with perfect information over static mechanism design. By assuming perfect information about the players' previous decisions, we can replace hypothetical thinking with information extraction from the observed data, which is easier to practice than hypothetical thinking. For this reason, Li (2017) emphasized that the open-bid ascending auction is better than the sealed-bid second-price auction.

In contrast with Li, this study does not consider such devices of mechanism design. Instead, we focus on fixed static games with imperfect information. We do not cover dynamic games with perfect information such as the open-bid ascending auction.

Instead of designing mechanisms, we fix a static game and then design a frame as a description of a cognitive procedure, through which, under the imperfect information assumption, players sequentially decide multiple actions that shape their strategy selections. We should regard a frame as the supposedly irrelevant factors from the viewpoint of full rationality. However, a well-designed frame can nudge a bounded-rational player to think more rationally. In this respect, the sealed-bid ascending proxy auction has a much better frame design than the sealed-bid second-price auction, even though both auctions have the same physical rules of strategic interactions. This is in contrast with Li, because the open-bid ascending auction is different from the sealed-bid second-price auction even in terms of physical rule.

Players need hypothetical thinking even in dominant strategies or in eliminating dominated strategies. In games with imperfect information, Friedman and Shenker (1996) and Friedman (2002) introduced a stronger solution concept than dominance, which this study terms “obvious dominance.” This strategy totally excludes the practice of hypothetical thinking by regarding each player to be very pessimistic about other players’ strategy selection. Li (2017) extended this obviously dominant strategy to extensive form games without the imperfect information assumption. Friedman and Shenker (1996) and Friedman (2002) further introduced the solvability in iteratively undominated strategies. Note, however, that a static game with imperfect information generally has no obviously dominant strategy or does not satisfy the solvability in iteratively obviously undominated strategies, even if it does have a dominant strategy.

This study weakens the obviously dominant strategy and the solvability in iterative obvious dominance by permitting a player to practice the first type of hypothetical thinking, which this study terms “quasi-obviously dominant strategy,” and “solvability in iteratively quasi-obviously undominated strategies,” respectively. We then show that for every static game that is solvable in iteratively undominated strategies, there always exists a frame that makes this game solvable in iteratively quasi-obviously undominated strategies.

We further demonstrate a permissive result, indicating the class of games that are solvable in iteratively quasi-obviously undominated strategies with the help of only detail-free frames. We then apply this result to the robustness of the Abreu-Matsushima mechanisms in hypothetical thinking. Several works such as Abreu and Matsushima (1992a, 1992b, 1994) and Matsushima (2008a, 2008b, 2017) showed that every social choice function is uniquely implementable in iteratively undominated strategies, in the virtual or exact sense, provided it is incentive compatible. To prove this result, we designed so-called Abreu-Matsushima mechanisms, which require players to announce multiple messages about their types and use only small monetary fines. This study shows that there exists a detail-free frame, with the help of which, the game implied by the Abreu-Matsushima mechanism is solvable even in iteratively quasi-obviously undominated strategies. Importantly, this frame interprets the Abreu-Matsushima mechanism as punishing the last deviants from truthful revelation.

There is literature about level-k models in game theory, where a player has an exogenous limitation in the depth of cognitive hierarchy. See Nagel (1995) and Crawford and Iriberri (2007), for instance. In contrast, this study does not assume any exogenous limitation on the depth of cognitive hierarchy, and instead explains that each player's depth limitation is endogenously determined by the degree to which the other players fail to practice correct hypothetical thinking. This study explains that without the help of frame design, players fail to behave rationally, even if we assume unlimited depth of cognitive hierarchy.

This paper assumes that a player can practice correct hypothetical thinking regarding the actions that the other players have decided upon before. This assumption excludes the case that Shafir and Tversky (1992) investigated as a variant of the Newcomb paradox, where a decision maker behaves irrationally in front of a predictor with miraculous power irrespective of whether this predictor is the almighty or a high-quality artificial intelligence. In this sense, this study assumes players to be "slightly" bounded-rational in hypothetical thinking.

3. Outline

Section 4 demonstrates examples of prisoners' dilemma to give a background to this study. Section 5 defines basic notions such as a normal form game, dominant strategy, and dominated strategy. We then define obviously dominant strategy and obviously dominated strategy.

Section 6 introduces a frame as an extensive-form game with imperfect information. Section 7 introduces quasi-obviously dominant strategy and quasi-obviously dominated strategy. We show a necessary and sufficient condition for the existence of a frame such that the dominant strategy profile is also quasi-obviously dominant.

Section 8 introduces weak quasi-obvious dominance by replacing strict inequalities with weak inequalities. More importantly, we show that in a multi-unit ascending proxy auction with single-unit demands the sincere proxy bidding is the weakly quasi-obviously dominant strategy. This finding is in contrast with a second-price auction, which accompanies just a degenerate frame and fails to motivate bidders to play dominant strategies even if its physical rule is the same as that of the ascending proxy auction.

Section 9 introduces iterative quasi-obvious dominance. We show that whenever a game is solvable in iteratively undominated strategies, then we can design a frame, with the help of which, the game is solvable even in iteratively quasi-obviously undominated strategies.

Section 10 considers a case of frames that are detail-free, that is independent of the finer detail of payoff functions. We show a sufficient condition of games that are solvable in iteratively quasi-obviously undominated strategy by using only detail-free frames. Section 11 applies this detail-free frame design to the implementation problem where we permit ex-post verification of which allocation is desirable. We show that with the help of the detail-free frame investigated in Section 10, the game implied by the Abreu-Matsushima mechanism, which fines the last deviants, is solvable in iteratively quasi-obviously dominated strategies.

Section 12 investigates the frame design in the Bayesian environment. We can directly extend the argument of this study to the Bayesian environment by replacing the Bayesian game with its agent normal form game. In this case, however, we generally utilize a complicated frame design that is defined not for the set of players but for the set of type-contingent agents. To overcome this complexity, we demonstrate the possibility that a simple frame design, defined for the set of players, functions in the Bayesian environment.

Section 13 concludes this paper.

4. Intuition: Prisoners' Dilemma

Figure 1 describes a prisoners' dilemma. Strategy D (defection) is a dominant strategy for each player. It is obviously dominant for player 2, but it is not obviously dominant for player 1, because

$$\min[u_1(D,C),u_1(D,D)] = \min[2,0] = 0$$
$$< 1 = \max[1,-1] = \max[u_1(C,C),u_1(C,D)],$$

while

$$\min[u_2(C,D),u_2(D,D)] = \min[0,-2] = -2$$
$$> -3 = \max[-3,-3] = \max[u_2(C,C),u_2(D,C)].$$

Player 1's failure to play the dominant strategy D is caused by his (or her) failure to practice hypothetical thinking.

However, by designing a frame that assigns the first mover to player 2 and the second move to player 1, we can make the strategy profile (D,D) quasi-obviously dominant, because the second mover (player 1) can practice hypothetical thinking regarding the first mover's strategy selection correctly.

		player 2 C	player 2 D
Player 1	C	1 -3	-1 0
	D	2 -3	0 -2

Figure 1

Figure 2 describes another prisoners' dilemma. Strategy D is a dominant strategy, but not obviously dominant, for each player. In this case, irrespective of which frame we design, strategy D fails to be quasi-obviously dominant for the first mover.

However, if the first mover is rational in higher-order reasoning, then he can correctly anticipate that the frame nudges the second mover to practice correct hypothetical thinking, and then expects the second mover to play strategy D. Hence, with the help of the frame design, the first mover comes to play strategy D as the unique iteratively quasi-obviously undominated strategy, even if he is (slightly) bounded-rational in hypothetical thinking.

		player 2 C	player 2 D
Player	C	1 1	-1 2

1	D	2 -1	0 0

Figure 2

5. Normal Form Game as Physical Rule

We investigate a normal form game, a game in short, which is denoted by $G=(N,A,u)$, where $N\equiv\{1,...,n\}$ is the set of all players, $n\geq 2$; A is the set of all strategy profiles; $A\equiv\underset{i\in N}{\times}A_i$, where A_i is the set of all strategies for player $i\in N$; and $u\equiv(u_i)_{i\in N}$, where $u_i:A\to R$ is the payoff function for player i. Let $\hat{A}_i\subset A_i$ denote an arbitrary subset of strategies for player i. Let $\hat{A}\equiv\underset{i\in N}{\times}\hat{A}_i$ and $\hat{A}_{-i}\equiv\underset{j\in N\setminus\{i\}}{\times}\hat{A}_j$.

Definition 1: A strategy $a_i\in A_i$ for player i is said to be *dominated* for $\hat{A}$ in G if $a_i\in\hat{A}_i$, and there exists a mixed strategy $\alpha_i\in\Delta(\hat{A}_i)$ such that

$$u_i(a_i,\hat{a}_{-i})<u_i(\hat{a}_i,\hat{a}_{-i})\text{ for all }\hat{a}_{-i}\in\hat{A}_{-i}.$$[4]

It is said to be *dominant* for $\hat{A}$ in G if $a_i\in\hat{A}_i$, and

$$u_i(a_i,\hat{a}_{-i})>u_i(\hat{a}_i,\hat{a}_{-i})\text{ for all }\hat{a}_i\in\hat{A}_i\setminus\{a_i\}\text{ and }\hat{a}_{-i}\in\hat{A}_{-i}.$$

It is said to be *weakly dominated* for $\hat{A}$ in G if $a_i\in\hat{A}_i$, and there exists a mixed strategy $\alpha_i\in\Delta(\hat{A}_i)$ such that

$$u_i(a_i,\hat{a}_{-i})\leq u_i(\hat{a}_i,\hat{a}_{-i})\text{ for all }\hat{a}_{-i}\in\hat{A}_{-i},$$

and the strict inequality holds for some $a_{-i}\in A_{-i}$. It is said to be *weakly dominant* for $\hat{A}$ in G if $a_i\in\hat{A}_i$, and for every $\hat{a}_i\in\hat{A}_i\setminus\{a_i\}$,

$$u_i(a_i,\hat{a}_{-i})\geq u_i(\hat{a}_i,\hat{a}_{-i})\text{ for all }\hat{a}_{-i}\in\hat{A}_{-i},$$

and the strict inequality holds for some $\hat{a}_{-i}\in\hat{A}_{-i}$.

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[4] $\Delta(X)$ denotes the set of all lotteries over X.

A strategy for player i is dominant for $\hat{A}$ in G if and only if it is the unique undominated strategy for $\hat{A}$ in G. It is weakly dominant for $\hat{A}$ in G if and only if it is the unique weakly dominated strategy for $\hat{A}$ in G. If it is dominant for $\hat{A}$ in G, it is also weakly dominant for $\hat{A}$ in G. If it is dominated for $\hat{A}$ in G, it is also weakly dominated for $\hat{A}$ in G. If $\hat{A} = A$, we will simply say that a_i is dominated in G. We will say similarly for the other definitions.

According to Friedman and Shenker (1996), Friedman (2002), and Li (2017), we introduce *obvious dominance* as follows. Li (2017) defined obviously dominant strategy for extensive form games, while this paper defines it for normal form games.

Definition 2: A strategy $a_i \in A_i$ for player i is said to be *obviously dominated* for $\hat{A}$ in G if $a_i \in \hat{A}_i$, and there exists $\hat{a}_i \in \hat{A}_i$ such that

$$\max_{\hat{a}_{-i} \in \hat{A}_{-i}} u_i(a_i, \hat{a}_{-i}) < \min_{\hat{a}_{-i} \in \hat{A}_{-i}} u_i(\hat{a}_i, \hat{a}_{-i}).$$

It is said to be *obviously dominant* for $\hat{A}$ in G if $a_i \in \hat{A}_i$, and

$$\min_{\hat{a}_{-i} \in \hat{A}_{-i}} u_i(a_i, \hat{a}_{-i}) > \max_{\hat{a}_{-i} \in \hat{A}_{-i}} u_i(\hat{a}_i, \hat{a}_{-i}) \quad \text{for all } \hat{a}_i \in \hat{A}_i \setminus \{a_i\}.$$

Definition 2 permits each player's expectation regarding the other players' strategies to depend on his (or her) strategy selection. This permission implies that each player $i \in N$ is not rational in hypothetical thinking, that is, he (or she) fails to practice correct hypothetical thinking in the "what-if" manner, such that he (or she) selects a strategy a_i if the other players select a profile of strategies a_{-i}, whereas he selects another strategy a_i' if the other players select another profile of strategies a_{-i}'. Instead of practicing such hypothetical thinking, a player i incorrectly thinks about the other players' strategies in a "strategy-dependent" manner, such that he selects a strategy a_i, and the other players then select a profile of strategies a_{-i}, while he selects another strategy $a_i' \neq a_i$, and the other players then select another profile of strategies $a_{-i}' \neq a_{-i}$.

Obviously dominated strategy implies that a player hesitates to select a strategy even if he is the most optimistic in his strategy-dependent expectation about the other players' strategy selections. Obviously dominant strategy implies that a player prefers selecting a strategy even if he is the most pessimistic in his strategy-dependent expectation about the other players' strategy selections. If a strategy for player i is obviously dominant for $\hat{A}$ in G, then it is also dominant for $\hat{A}$ in G. If it is obviously dominated for $\hat{A}$ in G, then it is also dominated for $\hat{A}$ in G. It is obviously dominant for $\hat{A}$ in G if and only if it is the unique obviously undominated strategy for $\hat{A}$ in G.

We regard a normal form game as a *physical rule* of strategic interactions, implying a full description of the supposedly relevant factors to ideally rational players. To nudge bounded-rational players to play rationally, we will introduce a *cognitive procedure* that is a description of the factors that are supposedly irrelevant to ideally rational players but nudge bounded-rational players to play rationally.

6. Frame

This section defines a *frame* as an extensive game form with imperfect information that is consistent with a given normal form game. We regard a frame as a cognitive procedure that is common to all players.

6.1. Definition

Associated with a game G, we introduce a concept that we term a *frame*, which is denoted by $\Gamma = (T, (A_{i,t}, \tilde{A}_{i,t}(\cdot))_{t\in T}, \delta_i)_{i\in N})$. Each player makes multiple action decisions from step 1 to step T. At each step $t \in \{1,...,T\}$, each player i selects an action $a_{i,t}$ from a finite set $A_{i,t}$. Let $a_i^t \equiv (a_{i,1},...,a_{i,t})$ denote a sequence of player $i's$ action decisions up to the step t. For every $t \in \{1,...,T\}$, we define the set of possible sequences of player $i's$ action decisions up to the step t by $A_i^t \subset \underset{\tau=1}{\overset{t}{\times}} A_{i,\tau}$. We also define the sequence-dependent set of actions at step t, which is described as a function

$\tilde{A}_{i,t}: A_i^{t-1} \to 2^{A_{i,t}}$, where

$$\tilde{A}_{i,1}(a_i^0) = A_{i,1} = A_i^1,$$

and for every $t \in \{2,...,T\}$,

$$[a_i^t \in A_i^t] \Leftrightarrow [a_{i,\tau} \in \tilde{A}_{i,\tau}(a_i^{\tau-1}) \ \text{for all}\ \tau \in \{1,...,t\}].$$

At each step $t \in \{1,...,T\}$, where player i has determined $a_i^{t-1} = (a_{i,1},...,a_{i,t-1})$ up to the step $t-1$, he selects an action $a_{i,t}$ from the sequence-dependent set $\tilde{A}_{i,t}(a_i^{t-1}) \subset A_{i,t}$.

Let $\delta_i : A_i \to A_i^T$ denote a one-to-one correspondence, where we regard a strategy $a_i \in A_i$ for player i in the game G as the complete sequence of player $i's$ action decisions $\delta_i(a_i) \in A_i^T$ in the frame Γ. We will write $a_i = \delta_i(a_i) = (a_{i,1},...,a_{i,T})$.

We interpret a frame as a description of players' *cognitive procedures* regarding how to determine their strategy selections. That is, each player i determines his strategy selection in the game G according to the cognitive procedure implied by the frame Γ. We assume that not only the game G, but also the frame Γ, is common knowledge among all players. Importantly, at each step $t \in \{1,...,T\}$, each player i perceives that any other player $j \neq i$ has already decided the sequence of actions $a_j^{t-1} = (a_{j,1},...,a_{j,t-1})$ up to the step $t-1$, but has not decided on $(a_{j,t},...,a_{j,T})$ yet.

6.2. Specifications

We introduce two specifications of frame as follows.

Specification (1): Fix an arbitrary strategy profile $\overline{a} \in A$ as the default. Let

$$\rho : \bigcup_{i \in N} A_i \setminus \{\overline{a}_i\} \to \{1,..., \sum_{i \in N} |A_i| - n\}$$

denote a one-to-one correspondence that describes an *order of all players' strategies* except for $\{\overline{a}_1,...,\overline{a}_n\}$. We define $\mu(\cdot,\rho) : \{1,..., \sum_{i \in N} |A_i| - n\} \to N$ by

$$[\rho(a_i)=t] \Rightarrow [\mu(t,\rho)=i] \qquad \text{for all } i \in N,\ a_i \in A_i \setminus \{\bar{a}_i\}, \text{ and}$$
$$t \in \{1,\dots,\sum_{i\in N}|A_i|-n\}.$$

According to ρ, player $i's$ strategy $a_i \in A_i \setminus \{\bar{a}_i\}$ is placed in the position $\rho(a_i) \in \{1,\dots,\sum_{i\in N}|A_i|-n\}$, and $\mu(\rho(a_i),\rho)=i$ identifies the player who occupies the position $\rho(a_i)$.

We specify a frame, denoted by Γ^{ρ}, such that

$$T=\sum_{i\in N}|A_i|-n,$$

$$A_{i,t}=\{0,1\},$$

$$\tilde{A}_{i,t}(a_i^{t-1})=\{0\} \qquad \text{if either } \mu(t,\rho)\neq i \text{ or}$$
$$a_{i,\tau}=1 \text{ for some } \tau \in \{1,\dots,t-1\},$$

$$\tilde{A}_{i,t}(a_i^{t-1})=\{0,1\} \qquad \text{otherwise,}$$

and $\delta_i(a_i)=(\delta_{i,t}(a_i))_{t=1}^{\sum_{i\in N}|A_i|-n}$, where

$$\delta_{i,t}(\bar{a}_i)=0 \text{ for all } t\in\{1,\dots,T\},$$

and for every $a_i \in A_i \setminus \{\bar{a}_i\}$,

$$\delta_{i,\rho(a_i)}(a_i)=1, \text{ and } \delta_{i,t}(a_i)=0 \text{ for all } t\neq\rho(a_i).$$

At each step $t\in\{1,\dots,T\}$, only player $\mu(t,\rho)$ is active and decides whether to select strategy $a_i=\rho^{-1}(t)\in A_i\setminus\{\bar{a}_i\}$, that is, decide “action 1,” or not, that is, decide "action 0." By deciding action 0 at all steps, player i selects the default strategy $\bar{a}_i$. By deciding action 1 at the step $t=\rho(a_i)$, player i selects strategy $a_i\in A_i\setminus\{\bar{a}_i\}$. Each player can choose action 1 only once during the cognitive procedure implied by Γ^{ρ}.

Specification (2): Let $\mu: N\to N$ denote a permutation on N. We specify a frame, denoted by Γ^{μ}, such that

$$T=n,$$

$$A_{i,\mu(i)}=A_{i,\mu(i)}(a_i^{\mu(i)-1})=A_i \text{ for all } i\in N \text{ and } a_i^{\mu(i)-1}\in A_i^{\mu(i)-1},$$

$$A_{i,t} \text{ is a singleton for all } i \in N \text{ and } t \neq \mu(i),$$

and $\delta_i(a_i) = (\delta_{i,t}(a_i))_{t=1}^{\sum_{i\in N}|A_i|-n}$, where for every $a_i \in A_i$,

$$\delta_{i,\mu(i)}(a_i) = a_i, \text{ and } \delta_{i,t}(a_i) = 0 \text{ for all } t \neq \mu(i).$$

Each player selects a strategy in the order of the permutation μ. At each step $t \in \{1,...,n\}$, only player $\mu^{-1}(t)$ is active and selects his strategy.

7. Quasi-Obvious Dominance

Because of the imperfect information assumption, it is a precondition that each player i cannot observe the other players' action decisions during the cognitive procedure implied by the frame Γ. However, at each step $t \in \{1,...,T\}$, each player i perceives that the other players have already decided upon actions a_{-i}^{t-1} as irreversible ones up to the step t. With the help of this perception, he can correctly recognize that his action decision $a_{i,t}$ at the step t has no relation to a_{-i}^{t-1}, and can therefore practice correct hypothetical thinking regarding the other players' past action decisions a_{-i}^{t-1}.

On the other hand, he perceives that the other players have not decided $(a_{-i}^{t},...,a_{-i}^{T})$ yet. This perception motivates him to incorrectly expect that his action decision may influence the other players' future and current action decisions, thus, leading to his failure to enforce hypothetical thinking regarding the other players' future and current action decisions.

This section introduces a new concept that we term *quasi-obvious dominance*. For each sequence of player $i's$ action decisions up to step t, $a_i^t \in A_i^t$, we define the set of all strategies for player i that are consistent with a_i^t by

$$A_i(a_i^t) \equiv \{\hat{a}_i \in A_i \mid \hat{a}_i^t = a_i^t\}.$$

Definition 3: A strategy $a_i \in A_i$ for player i is said to be *quasi-obviously dominated for* $\hat{A}$ *in a game with a frame* (G,Γ) if $a_i \in \hat{A}_i$, and there exist $t \in \{1,...,T\}$ and

$\hat{a}_i \in \hat{A}_i$ such that

$$\hat{a}_i \in A_i(a_i^{t-1}),$$

$$\hat{a}_{i,t} \neq a_{i,t},$$

and

(1) $$\max_{\hat{a}_{-i} \in \times_{j \neq i} \hat{A}_j \cap A_j(a_j^{t-1})} u_i(a_i, \hat{a}_{-i}) < \min_{\hat{a}_{-i} \in \times_{j \neq i} \hat{A}_j \cap A_j(a_j^{t-1})} u_i(\hat{a}_i, \hat{a}_{-i}) \text{ for all } a_{-i}^{t-1} \in \hat{A}_{-i}^{t-1}.$$

A strategy $a_i \in A_i$ for player i is said to be *quasi-obviously dominant for* $\hat{A}$ *in* (G, Γ) if $a_i \in \hat{A}_i$, and for every $t \in \{1,...,T\}$ and $\hat{a}_i \in \hat{A}_i$, whenever $\hat{a}_i \in A_i(a_i^{t-1})$ and $\hat{a}_{i,t} \neq a_{i,t}$,

(2) $$\min_{\hat{a}_{-i} \in \times_{j \neq i} \hat{A}_j \cap A_j(a_j^{t-1})} u_i(a_i, \hat{a}_{-i}) > \max_{\hat{a}_{-i} \in \times_{j \neq i} \hat{A}_j \cap A_j(a_j^{t-1})} u_i(\hat{a}_i, \hat{a}_{-i}) \text{ for all } a_{-i}^{t-1} \in \hat{A}_{-i}^{t-1}.$$

According to a frame Γ, each player i determines whether or not he selects a strategy a_i, and also determines whether or not he selects another strategy $\hat{a}_i$, at the first step that distinguishes these strategies, that is, at the step t where $a_i^{t-1} = \hat{a}_i^t$ and $a_{i,t} \neq \hat{a}_{i,t}$. More importantly, he perceives that the other players have already made the action decisions that the frame requires them to make up to the step $t-1$, whereas they do not have decided their future and current actions yet.

Quasi-obviously dominated strategy implies that a player hesitates to select one strategy over another even if, at the first step that distinguishes these strategies, he is the most optimistic in terms of his strategy-dependent expectation about the other players' future and current action decisions. Quasi-obviously dominant strategy implies that a player prefers selecting a strategy over another strategy even if, at the first step that distinguishes these strategies, he is the most pessimistic in terms of his strategy-dependent expectation about the other players' future and current action decisions.

The main difference between quasi-obvious dominance and obvious dominance is that at each step during the cognitive procedure implied by the frame, any player can practice hypothetical thinking regarding the other players' past action decisions correctly.

Li (2017) defined obviously dominant strategy for the general class of extensive form games that includes both imperfect information and perfect information. In contrast with quasi-obviously dominant strategy in this study, Li's concept does not require players to practice hypothetical thinking at all. Hence, for extensive form games with imperfect information games, Li's concept is the same as obviously dominant strategy in this study.

If a strategy for player i is quasi-obviously dominant for $\hat{A}$ in G, then it is dominant for $\hat{A}$ in G. If it is obviously dominant for $\hat{A}$ in G, then it is quasi-obviously dominant for $\hat{A}$ in G. If it is quasi-obviously dominated for $\hat{A}$ in G, then it is dominated for $\hat{A}$ in G. If it is obviously dominated for $\hat{A}$ in G, then it is quasi-obviously dominated for $\hat{A}$ in G. It is quasi-obviously dominant for $\hat{A}$ in G, if and only if it is the unique quasi-obviously undominated strategy for $\hat{A}$ in G.

Consider an arbitrary strategy profile $a^* \in A$ as the targeted strategy profile. We will demonstrate a necessary and sufficient condition for the existence of a frame Γ such that a^* is quasi-obviously dominant in (G,Γ).

Consider the specification of frame (1), Γ^ρ, where we regard a^* as the default, that is,

$$\bar{a} = a^* .$$

For each $i \in N$, $a_{-i} \in A_{-i}$, and $t \in \{1,...,T\}$, we define the set of all players $j \neq i$ other than player i who select strategy a_j after the step t by

$$C(a_{-i}, i, t, \rho) \equiv \{ j \in N \setminus \{i\} \mid \rho(a_j) > t \} .$$

We define the set of all strategies that player i can select after the step t by

$$A_i(t,\rho) \equiv \{a_i \in A_i \mid \rho(a_i) > t\} .$$

From the specification of Γ^ρ and the definitions of $C(a_{-i}, i, t, \rho)$ and $A_i(t,\rho)$, it follows that the inequalities (2) for $a_i = a_i^*$ in Definition 3 are equivalent to the following inequalities: for every $a_i \neq a_i^*$ and $a_{-i} \in A_{-i}$,

(3)
$$\min_{\tilde{a}_{C(a_{-i},i,t,\rho)} \in A_{C(a_{-i},i,t,\rho)}(t,\rho)} u_i(a_i^*, a_{-i-C(a_{-i},i,t,\rho)}, \tilde{a}_{C(a_{-i},i,t,\rho)})$$
$$> \max_{\tilde{a}_{C(a_{-i},i,t,\rho)} \in A_{C(a_{-i},i,t,\rho)}(t,\rho)} u_i(a_i, a_{-i-C(a_{-i},i,t,\rho)}, \tilde{a}_{C(a_{-i},i,t,\rho)}),$$

where we denote $t = \min[\rho(a_i^*), \rho(a_i)]$. Hence, the inequality (3) is necessary and sufficient for a^* to be quasi-obviously dominant in (G, Γ^ρ).

The implication of (3) is as follows. Suppose that a player i considers selecting $a_i \neq a_i^*$ instead of a_i^* and expects that the other players select a_{-i}. In this case, player i irrationally expects that if he selects a_i^* instead of $a_i \neq a_i^*$, any other player $j \neq i$, who can select a_j after the step $t = \min[\rho(a_i^*), \rho(a_i)]$, will select the worst strategy among strategies that player j can select after this step, instead of a_j. However, the inequality (3) implies that player i is still willing to select a_i^* even if he is pessimistic about the other players' strategy selection in such a manner.

Since

$$C(a_{-i}, i, t, \rho) \subset C(a_{-i}, i, t', \rho) \text{ for all } t \text{ and all } t' < t,$$

it follows from (3) that to make a^* quasi-obviously dominant, we should design the order ρ that positions the action decisions that are more likely the cause of failure during later steps.

We show that if there exists no ρ such that a^* is quasi-obviously dominant in (G, Γ^ρ), then there generally exists no frame Γ such that it is quasi-obviously dominant in (G, Γ). Hence, we only have to consider the specification of frame (1) for quasi-obvious dominance.

Theorem 1: *There exists a frame Γ such that a strategy profile a^* is quasi-obviously dominant in (G, Γ) if and only if there exists ρ such that a^* is quasi-obviously dominant in (G, Γ^ρ), that is, the inequality (3) holds.*

Proof: Suppose that a^* is quasi-obviously dominant in (G, Γ). For each $i \in N$ and $t \in \{1, ..., T\}$, we define

$$\bar{A}_i(t) \equiv \{a_i \in A_i \mid a_i \in A_i(a_i^{*t-1}) \quad and \quad a_i^t \neq a_i^{*t}\},$$

which is the set of all strategies a_i such that the step t distinguishes a_i and a_i^* at the first time. Let $\bar{a} = a^*$. We specify ρ in the manner that for every $(i, j) \in N^2$, $(t, t') \in \{1, ..., T\}^2$, and $(a_i, a'_j) \in \bar{A}_i(t) \times \bar{A}_j(t')$,

$$[t > t'] \Rightarrow [\rho(a_i) > \rho(a'_j)].$$

Note that ρ positions action decisions that are eliminated earlier in iterative dominance on later steps.

Fix $i \in N$ and $a_i \in A_i \setminus \{a_i^*\}$ arbitrarily. Let $t \in \{1, ..., T\}$ denote the step in the frame Γ such that $a_i^{t-1} = a_i^{*t-1}$ and $a_i^t \neq a_i^{*t}$, that is, $a_i \in \bar{A}_i(t)$. From (2), it follows that for every $a_{-i} \in A_{-i}$,

(4) $$\min_{\hat{a}_{-i} \in A_{-i}(a_{-i}^{t-1})} u_i(a_i^*, \hat{a}_{-i}) > \max_{\hat{a}_{-i} \in A_{-i}(a_{-i}^{t-1})} u_i(a_i, \hat{a}_{-i}).$$

From the specification of ρ, it follows that for every $j \in C(a_{-i}, i, t, \rho)$,

$$A_j(t, \rho) \subset A_j(a_j^{t-1}),$$

where we denote $t = \min[\rho(a_i^*), \rho(a_i)]$. From this inclusion, $a_j \in A_j(a_j^{t-1})$, and the inequality (4), it follows that for every $a_i \neq a_i^*$ and $a_{-i} \in A_{-i}$,

$$\min_{\tilde{a}_{C(a_{-i},i,t,\rho)} \in A_{C(a_{-i},i,t,\rho)}(t,\rho)} u_i(a_i^*, a_{-i-C(a_{-i},i,t,\rho)}, \tilde{a}_{C(a_{-i},i,t,\rho)})$$

$$> \max_{\tilde{a}_{C(a_{-i},i,t,\rho)} \in A_{C(a_{-i},i,t,\rho)}(t,\rho)} u_i(a_i, a_{-i-C(a_{-i},i,t,\rho)}, \tilde{a}_{C(a_{-i},i,t,\rho)}),$$

which implies the inequality (3). Hence, we have proved Theorem 1.

Q.E.D.

Example 1 (Two-Strategy Game): Consider a two-strategy game G, where

$$A_i = \{0,1\} \quad \text{for all} \quad i \in N.$$

In this game, the specification of frame (1) is essentially the same as the specification of frame (2). Let $a^* = (1, ..., 1)$ and $\bar{a} = (0, ..., 0)$. Fix an arbitrary μ, and consider the associated frame Γ^μ. At each step $t \in \{1, ..., n\}$, player $i = \mu^{-1}(t) \in N$ selects a strategy between $a_i^* = 1$ and $\overline{a}_i = 0$.

Proposition 1: *In a two-strategy game G, there exists a frame Γ such that a^* is the quasi-obviously dominant strategy profile in (G, Γ) if and only if there exists μ such that a^* is the quasi-obviously dominant strategy profile in (G, Γ^μ), that is, for every $i \in N$ and $\tilde{a}_{-i} \in A_{-i}$,*

$$u_i(a_i^*, \tilde{a}_{-i}) > \max_{a_{C(\mu(i),\mu)} \in A_{C(\mu(i),\mu)}} u_i(\overline{a}_i, \tilde{a}_{-i-C(\mu(i),\mu)}, a_{C(\mu(i),\mu)}),$$

where $C(t,\mu) \equiv \{i \in N \mid \mu(i) > t\}$ denotes the set of all players who select strategies after the step t.

Proof: At each step $t \in \{1,...,n\}$, player $i = \mu^{-1}(t)$ fails to practice the hypothetical thinking regarding all players who move after this step, that is, all players who belong to $C(t,\mu)$. This implies that a^* is the quasi-obviously dominant strategy profile in (G, Γ^μ) if and only if for every $i \in N$ and $\tilde{a}_{-i} \in A_{-i}$,

$$u_i(a_i^*, \tilde{a}_{-i}) > \max_{a_{C(\mu(i),\mu)} \in A_{C(\mu(i),\mu)}} u_i(\overline{a}_i, \tilde{a}_{-i-C(\mu(i),\mu)}, a_{C(\mu(i),\mu)}).$$

It is clear from Theorem 1 that in this example, if there exists a frame Γ such that a^* is the quasi-obviously dominant strategy profile in (G, Γ), then there exists μ such that a^* is the quasi-obviously dominant strategy profile in (G, Γ^μ).

Q.E.D.

Since $C(t,\mu) \subset C(t',\mu)$ for all t and $t' < t$, it follows from Proposition 1 that to make a^* quasi-obviously dominant, we should design μ that *positions players who are more likely to be the cause of failure on later steps*.

8. Weak Quasi-Obvious Dominance

The following is a weaker version of quasi-obvious dominance, where we replace strict inequalities with weak inequalities.

Definition 4: A strategy $a_i \in A_i$ for player i is said to be *weakly quasi-obviously dominant for* $\hat{A}$ *in* (G,Γ) if it is weakly dominant for $\hat{A}$ in G, and for every $t \in \{1,...,T\}$ and $\hat{a}_i \in \hat{A}_i$, whenever $\hat{a}_i \in A_i(a_i^{t-1})$ and $\hat{a}_{i,t} \neq a_{i,t}$, then

$$\min_{\hat{a}_{-i} \in \times_{j \neq i} \hat{A}_j \cap A_j(a_j^{t-1})} u_i(a_i, \hat{a}_{-i}) \geq \max_{\hat{a}_{-i} \in \times_{j \neq i} \hat{A}_j \cap A_j(a_j^{t-1})} u_i(\hat{a}_i, \hat{a}_{-i}) \text{ for all } a_j^{t-1} \in \hat{A}_{-i}^{t-1}.$$

The following theorem parallels Theorem 1, which we can prove in the same way as Theorem 1.

Theorem 2: Suppose that *a strategy profile* a^* *is weakly dominant in* G. *There exists a frame* Γ *such that* a^* *is weakly quasi-obviously dominant in* (G,Γ) *if and only if there exists* ρ *such that* a^* *is weakly quasi-obviously dominant in* (G,Γ^ρ)*, that is,* a^* *is weakly dominant in* G*, and for every* $a_i \neq a_i^*$ *and* $a_{-i} \in A_{-i}$,

(5) $$\min_{\tilde{a}_{C(a_{-i},i,t,\rho)} \in A_{C(a_{-i},i,t,\rho)}(t,\rho)} u_i(a_i^*, a_{-i-C(a_{-i},i,t,\rho)}, \tilde{a}_{C(a_{-i},i,t,\rho)})$$

$$\geq \max_{\tilde{a}_{C(a_{-i},i,t,\rho)} \in A_{C(a_{-i},i,t,\rho)}(t,\rho)} u_i(a_i, a_{-i-C(a_{-i},i,t,\rho)}, \tilde{a}_{C(a_{-i},i,t,\rho)}),$$

where we denote $t = \min[\rho(a_i^*), \rho(a_i)]$.

Q.E.D.

Example 2 (Ascending Order): Consider a game G where A_i is a nonempty finite set of real numbers, and players select different numbers each as their strategies, that is,

$$A_i \cap A_j = \phi \text{ for all } i \neq j.$$

Let $\bar{a} = (\max a_1, ..., \max a_n)$. We specify the order $\rho = \rho^*$ that arranges all strategies except for $\{\bar{a}_i\}_{i \in N}$ in ascending order, that is, for every $(i,j) \in N^2$ and $(a_i, a'_j) \in A_i \times A_j$,

$$[\rho^*(a_i) > \rho^*(a'_j)] \Leftrightarrow [a_i > a'_j].$$

Proposition 2: *Consider Example 2. Suppose that* a^* *is a weakly dominant strategy profile. Then, it is weakly quasi-obviously dominant in* (G,Γ^{ρ^*}) *if and only if for every* $i \in N$, $a_i \in A_i \setminus \{a_i^*\}$, $a_{-i} \in A_{-i}$, *and* $a'_{-i} \in A_{-i}$, *whenever for every* $j \in N \setminus \{i\}$,

$$\text{either } a_j = a'_j \text{ or } \min[a_j, a'_j] > \min[a_i, a_i^*],$$

then

$$u_i(a_i^*, a'_{-i}) \geq u_i(a).$$

Proof: From the specification of ρ^*, it follows that

$$C(a_{-i}, i, t, \rho^*) = \{ j \in N \setminus \{i\} \mid a_j > \min[a_i^*, a_i]\},$$

where we denote $t = \min[\rho^*(a_i^*), \rho^*(a_i)]$. That is, a player $j \neq i$ selects a_j after the step $t = \min[\rho(a_i^*), \rho(a_i)]$ if and only if $a_j > \min[a_i^*, a_i]$. This implies that the inequality (5) is equivalent to the condition of this proposition.

Q.E.D.

A special case of Example 2 is a multi-unit ascending proxy auction with single-unit demands. Let $m \in \{1, 2, ..., n-1\}$ denote the total number of units of homogeneous commodities to be sold, where $1 \leq m < n$. We regard a strategy as a proxy bid. Each player i simultaneously makes his proxy bid. He obtains a single unit of the commodity if and only if his proxy bid is greater than the $(m+1)th$ highest proxy bid $p(a, m+1)$. Let

$$A_i = \{i, n+i, 2n+i, ..., (L-1)n+i\} \quad \text{for all } i \in N,$$

where L is a positive integer. We specify

$$u_i(a) = v_i - p(a, m+1) \qquad \text{if } a_i > p(a, m+1),$$

$$u_i(a) = 0 \qquad \text{otherwise,}$$

where $p(a, m+1)$ denotes the $(m+1)-th$ highest proxy bid, and v_i denotes the valuation of player i for the commodity. We assume

$$v_i \in A_i \quad \text{for all } i \in N.$$

Let a_i^* be the sincere strategy for player (bidder) i, that is,

$$a_i^* = v_i .$$

Note that for every bidder, the sincere strategy is weakly dominant, but not obviously dominant.

The frame Γ^{ρ^*} associated with the above-mentioned game describes the cognitive procedure through which bidders select their respective proxy bids. Note that the game along with this frame satisfies the conditions in Proposition 2. With the help of frame Γ^{ρ^*}, the sincere strategy becomes weakly quasi-obviously dominant.

The frame Γ^{ρ^*} describes the format of ascending proxy auction. The auctioneer asks player 1 whether he wants to purchase one unit of the commodity at the price $n+1$ (cents). Player 1 gives an answer to the auctioneer by saying either “yes” or “no.” The auctioneer ascends the price by one cent and asks the next player, that is, player 2, whether he wants to purchase the commodity at the price $n+2$. The auctioneer continues to ask players in rotation. The auctioneer stops ascending prices in the end after the auctioneer has asked player n whether he wants to purchase the commodity at the price Ln. We assume that any player can answer “no” only once. The auctioneer sells the commodities to the last m players who answered “yes” at the highest price among the prices to which the remaining $n-m$ players answered “yes.” Here, we implicitly assume that any player i answered “yes” to the price i.

We regard the highest price to which a player i answers “yes” as his proxy bid. Hence, the auctioneer sells the commodities to all players whose proxy bids are greater than the $(m+1)th$ highest proxy bid at this $(m+1)th$ highest proxy bid price.

From these observations, we can regard the frame Γ^{ρ^*} as the cognitive procedure implied by the multi-unit ascending proxy auction with single-unit demands. With single-unit demands, bounded-rational players fail to play the strategy of sincere bidding in the uniform-price auction (i.e., the Vickrey multi-unit auction) even if it is the dominant strategy, while they can successfully play the strategy of sincere bidding with the help of the frame that implies the ascending proxy auction format even if the physical aspects of strategic interactions are logically unchanged.

9. Iterative Quasi-Obvious Dominance

We define *iterative dominance* in G as the following iterative elimination of dominated strategies: Let

$$A_i(0) = A_i .$$

For every $k \geq 1$, we define $A_i(k) \subset A_i$ by

$$[\, a_i \in A_i(k) \,]$$

$$\Leftrightarrow [\, a_i \in A_i(k-1), \text{ and } a_i \text{ is undominated for } A(k-1) \text{ in } G \,],$$

where we denote $A(k-1) \equiv \underset{i \in N}{\times} A_i(k-1)$. Let $A_i(\infty) \equiv \bigcap_{k=0}^{\infty} A_i(k)$.

Definition 5: A strategy $a_i \in A_i$ for player i is said to be *iteratively undominated* in G if $a_i \in A_i(\infty)$.

We define *iterative obvious dominance* in G by replacing "undominated" in Definition 5 with "obviously undominated."

Definition 6: A strategy $a_i \in A_i$ for player i is said to be *iteratively obviously undominated* in G if

$$a_i \in A_i^*(\infty) \equiv \bigcap_{k=0}^{\infty} A_i^*(k),$$

where we define $A_i^*(k)$ similarly to $A_i(k)$ by replacing "undominated" with "obviously undominated."

Iterative obvious dominance assumes that a player is bounded-rational in hypothetical thinking, but is rational in higher-order reasoning. If a strategy for player i is the unique iteratively obviously undominated strategy, then it is the unique iteratively undominated strategy. However, even if it is the unique iteratively undominated strategy, it is not necessarily the unique iteratively obviously undominated strategy. In other words, if a game is solvable in iterative obvious dominance, then it is solvable in iterative dominance. Even if a game is solvable in iterative dominance, it is not necessarily solvable in iterative obvious dominance.[5]

We define *iterative quasi-obvious dominance* in (G,Γ) by replacing "dominated in G" with "quasi-obviously dominated in (G,Γ)."

Definition 7: A strategy $a_i \in A_i$ for player i is said to be *iteratively quasi-obviously undominated* in (G,Γ) if

$$a_i \in A_i^*(\infty,\Gamma) \equiv \bigcap_{k=0}^{\infty} A_i^*(k,\Gamma),$$

where we define $A_i^*(k,\Gamma)$ similarly to $A_i(k)$ by replacing "dominated in G" with "quasi-obviously dominated in (G,Γ)."

Iterative quasi-obvious dominance assumes that a player is "slightly" bounded-rational in hypothetical thinking, but is rational in higher-order reasoning. Such a slightly bounded-rational player can practice hypothetical thinking regarding the other players' previous action decisions correctly, but fails hypothetical thinking regarding their current and future action decisions.

The following proposition shows that *the dominant strategy profile is always the unique iteratively quasi-obviously undominated strategy profile*. To prove the proposition, we utilize the specification of frame (2), Γ^μ, in a detail-free manner.

Proposition 3: *If a^* is the dominant strategy profile in G, then, irrespective of the specification of μ, it is the unique iteratively quasi-obviously dominant strategy profile in (G,Γ^μ).*

Proof: At the last step $T = n$ in the frame Γ^μ, player $\mu(n)$, who is the last person to move, is willing to select the strategy $a^*_{\mu(n)}$, because it is the dominant strategy in G

[5] The solvability in iterative obvious dominance corresponds to the O-solvability in Friedman and Shenker (1996) and Friedman (2002).

and the last mover can practice hypothetical thinking correctly. Consider an arbitrary step $t \in \{1,...,n-1\}$. Suppose that at every subsequent step $t' \in \{t+1,...,n\}$, player $\mu(t')$ selects $a^*_{\mu(t')}$ as the unique iteratively quasi-obviously undominated strategy in (G,Γ^μ). Then, player $\mu(t)$ can also select $a^*_{\mu(t)}$ as the unique quasi-obviously dominant strategy, because he perceives that any subsequent movers will play according to a^*, and player $\mu(t)$ can practice hypothetical thinking regarding the previous movers' decisions correctly. These observations imply that a^* is the unique iteratively quasi-obviously undominated strategy profile in (G,Γ^μ). The above argument holds irrespective of the specification of μ.

Q.E.D.

We show that *whenever a strategy profile is the unique iteratively undominated strategy profile in* G*, then there always exists a frame* Γ *such that it is the unique iteratively quasi-obviously undominated strategy profile in* (G,Γ). Hence, with the help of the appropriate frame design, we can make the solvability in iterative dominance equivalent to the solvability in iterative quasi-obvious dominance.

Theorem 3: *There exists* ρ *such that a strategy profile* a^* *is the unique iteratively quasi-obviously undominated strategy profile in* (G,Γ^ρ) *if and only if it is the unique iteratively undominated strategy profile in* G.

Proof: Let $A_i(t)$ be the subset of strategies for player i that survives through the $t-time$ iterative eliminations of dominated strategies. Let

$$\bar{a} = a^*.$$

We specify ρ so that

$$T = \sum_{i \in N} |A_i| - n,$$

and for every $(t,t') \in T^2$, $(i,j) \in N^2$, $a_i \in A_i(t) \setminus A_i(t-1)$, and $a'_j \in A_j(t') \setminus A_j(t'-1)$,

$$\rho(a_i) < \rho(a'_j) \text{ whenever } t > t'.$$

The order ρ positions strategies that can be eliminated in earlier stages of iteration on later steps.

It is clear that at the last step, player $\mu(T,\rho)$ decides on action 0, that is, eliminates the strategy $\rho^{-1}(T)\in A_{\mu(T,\rho)}$, because $\rho^{-1}(T)$ is quasi-obviously dominated for A.

Consider an arbitrary $t\in\{1,...,T-1\}$. Note that there exists an integer k such that $\rho^{-1}(t)\in A_{\mu(t,\rho)}(k)\setminus A_{\mu(t,\rho)}(k-1)$. Suppose that at any later step $t'\in\{t+1,...,T\}$, player $\mu(t',\rho)$ decides on action 0, that is, eliminates $\rho^{-1}(t')\in A_{\mu(t',\rho)}$. Then, from the specification of ρ, every strategy in $A_i\setminus A_i(k-1)$ will be eliminated at the later steps as iteratively quasi-obviously dominated strategies. This implies that player $\mu(t,\rho)$ is willing to select action 0, that is, eliminate $\rho^{-1}(t)\in A_{\mu(t,\rho)}$, because $\rho^{-1}(t)$ is quasi-obviously dominated for $A(k-1)$.

From these observations, all players decide action 0 at all times during the cognitive procedure implied by the frame Γ^{ρ}. Hence, $\bar{a}=a^*$ is the unique iteratively quasi-obviously undominated strategy profile in (G,Γ^{ρ}).

Q.E.D.

Example 3 (Proxy Centipede Game): Consider a two-player game that we term a *proxy centipede game*. Let $n=\{1,2\}$,

$$A_1=\{1,3,5,...,2L-1\}\text{, and } A_2=\{2,4,6,...,2L\}.$$

Player 1 selects a positive odd number less than $2L$, while player 2 selects a positive even number less than or equal to $2L$. Each player prefers the strategy that is smaller than the other player's strategy for one point: For every $a\in A$ and $a_i'\in A_i$,

$$u_i(a)>u_i(a_i',a_j) \qquad \text{if } a_i=a_j-1 \text{ and } a_i'=a_j+1.$$

$$u_i(a)>u_i(a_i',a_j) \qquad \text{if } a_i<a_i'\le a_j-1,$$

and

$$u_i(a)>u_i(a_i',a_j) \qquad \text{if } a_i'>a_i\ge a_j-1.$$

Note that the strategy profile $a^* = (1,2) \in A$ is the unique iteratively undominated strategy profile. However, it is not the unique iteratively obviously undominated strategy profile.

To nudge slightly bounded-rational players to select the targeted strategy profile a^*, we consider the ascending order $\rho = \rho^*$, where

$$T = 2L,$$

and

$$\rho^*(t) = t \quad \text{for all} \quad t \in \{1,...,2L\}.$$

In the frame Γ^{ρ^*}, we can iteratively eliminate strategies in descending order. Hence, with the help of the frame Γ^{ρ^*}, we can see that $a^* = (1,2)$ becomes the unique iteratively quasi-obviously undominated strategy profile.

To prove Theorem 3, we utilized the order ρ that *positions strategies that can be eliminated in earlier stages of iteration on later steps*. This implies that the frame design Γ^{ρ} in the proof of Theorem 3 generally depends on the finer details of the payoff functions, that is, it is not detail-free.

10. Detail-Free Frame Design

To prove Theorem 3, we have utilized the frame design Γ^{ρ} that we tailored to the finer details of payoff structure, that is, the order of iterative eliminations. This section investigates the possibility that even a detail-free frame design promotes the solvability in iterative quasi-obvious dominance.

Fix an arbitrary positive integer T. We consider games where each player's strategy is decomposed into T elements as follows. For every $t \in \{1,...,T\}$, the $t-th$ element of a strategy a_i is denoted by $a_{i,t} \in A_{i,t}$, where $A_{i,t}$ is the set of possible $t-th$ elements. We define A_i as a nonempty subset of $\underset{t=1}{\overset{T}{\times}} A_{i,t}$, that is, $A_i \subset \underset{t=1}{\overset{T}{\times}} A_{i,t}$. Let $a_i = (a_{i,1},...,a_{i,T})$ and $a_i^t = (a_{i,1},...,a_{i,t})$.

We introduce another specification of frame, that is, Γ^*, which does not depend on the payoff functions, that is, is detail-free.

Specification (3): We specify a frame Γ^* as follows; for every $(i,t) \in N \times \{1,...,T\}$, $A_{i,t}(a_i^{t-1}) \subset A_{i,t}$ is defined as

$$[a_{i,t} \in A_{i,t}(a_i^{t-1})] \Leftrightarrow [(a_i^{t-1}, a_{i,t}) = \tilde{a}_i^t \text{ for some } \tilde{a}_i \in A_i].$$

According to Γ^*, at each step t, each player i decides his $t-th$ element $a_{i,t}$.

Fix an arbitrary $a^* \in A$ as the targeted strategy profile. For every $t \in \{0,...,T\}$, let

$$\bar{A}_i(t, a_i^*) \equiv \{a_i \in A_i \mid a_{i,t'} = a_{i,t'}^* \textit{ for all } t' \in \{t+1,...,T\}\}$$

denote the set of all strategies for player i whose elements after the $(t+1)-th$ element is the same as a_i^*. Note $\bar{A}_i(0, a_i^*) = \{a_i^*\}$. Let $\bar{A}(t, a^*) \equiv \underset{i \in N}{\times} \bar{A}_i(t, a_i^*)$.

The following theorem shows a sufficient condition for the solvability in iterative quasi-obvious dominance through detail-free frame design. The sufficient condition implies that at each round of iteration t, each player eliminates all strategies whose $(T-t)-th$ element is different from a_i^*.

Theorem 4: *A strategy profile $a^* \in A$ is the unique iteratively quasi-obviously undominated strategy rule profile in (G, Γ^*) if for every $i \in N$ and $t \in \{0,...,T\}$,*

$$A_i(t) \subset \bar{A}_i(T-t, a_i^*),$$

where $A_i(t)$ is the set of all strategies for player i that can survive through the $t-time$ iterative elimination of dominated strategies.

Proof: It is clear that at the last step T, each player i hesitates to select any $a_{i,T} \neq a_{i,T}^*$ as being dominated, that is, quasi-obviously dominated, for $A^*(0, \Gamma^*) = A$ in (G, Γ^*). Hence,

$$A_i^*(1, \Gamma^*) \subset \bar{A}_i(T-1, \bar{a}_i^*).$$

Fix an arbitrary $t \in \{2,...,T\}$ and $i \in N$. Suppose that

$$A_j^*(t-1,\Gamma^*) \subset \overline{A}_j(T-t+1,\overline{a}_j^*) \text{ for all } j \in N.$$

From the condition of this theorem, it is clear that at the step $T-t$, each player i hesitates to select any $a_{i,T-t} \neq a_{i,T-t}^*$ as being dominated, that is, quasi-obviously dominated, for $\overline{A}(T-t+1,\overline{a}^*)$. This along with the supposition implies that each player i hesitates to select any $a_{i,T-t} \neq a_{i,T-t}^*$ as being dominated, that is, quasi-obviously dominated, for $A^*(t-1,\Gamma^*)$. Hence,

$$A_i^*(t,\Gamma^*) \subset \overline{A}_i(T-t,\overline{a}_i^*).$$

The backward induction implies that for every $i \in N$,

$$A_i^*(T,\Gamma^*) \subset \overline{A}_i(0) = \{a_i^*\}, \text{ that is, } A_i^*(\infty,\Gamma^*) = \{a_i^*\}.$$

Q.E.D.

11. Framing Abreu-Matsushima Mechanisms

As an application of the detail-free frame design, this section considers the following allocation problem. Let C denote the set of possible allocations. The central planner attempts to determine the desirable allocation, but does not know which allocation is desirable at the time of his determination. We assume *ex-post verification* in that the desirable allocation becomes verifiable and contractible not before but after the central planner determines the allocation.[6]

Assume $n=3$. These three players know which allocation is desirable even before the central planner's allocation determination. Hence, the central planner requires each player to make some announcement regarding the desirable allocation, and then make his allocation determination contingent on their announcements.

We define a mechanism as $\Lambda = (A, g, x)$, where $A = \underset{i \in N}{\times} A_i$, A_i is the set of possible messages for player i, $\Delta(C)$ denotes the set of all lotteries over allocations, $g: A \to \Delta(C)$ denotes an allocation rule, and $x = (x_i)_{i \in N} : A \times C \to R^n$ denotes a

[6] See Matsushima (2017) for implementation with ex-post verification.

payment rule. Based on the announced message profile $a \in A$, the central planner determines an allocation according to the lottery $g(a) \in \Delta(C)$. After this determination, the central planner recognizes which allocation was actually desirable and then makes the side payment $x_i(a,c)$ to each player $i \in \{1,2,3\}$, where c denotes the desirable allocation.

We permit only small fines. Fix $\varepsilon \geq 0$ and $\eta > 0$ arbitrarily, both of which are close to zero. The side payments are less than or equal to $\varepsilon + \eta$, that is,

$$\left| x_i(a,c) \right| \leq \varepsilon + \eta \quad \text{for all } i \in N,\ a \in A, \text{ and } c \in C.$$

Fix a permutation $\xi : \{1,...,T\} \to \{1,...,T\}$ arbitrarily. We specify the Abreu-Matsushima mechanism $\Lambda = \Lambda(\xi)$ as follows. Let

$$A_i = A_{i,1} \times \cdots \times A_{i,T},$$

where

$$A_{i,t} = C \quad \text{for all } t \in \{1,...,T\}.$$

Each player announces T sub-messages about which allocation is desirable. Let $a^{(t)} = (a_{i,t})_{i \in N}$. We specify the allocation rule g by

$$g(a) = \frac{\sum_{t=1}^{T} g_t(a^{(t)})}{T},$$

where $g_t : C^3 \to C$ denotes the majority rule with respect to their $(t)th$ announcement. That is,

$$g_t(a^{(t)}) = c \qquad \text{if } a_{i,t} = c \text{ for two or more players,}$$

and

$$g_t(a^{(t)}) = \overline{c} \qquad \text{if there exists no such } c \in C,$$

where $\overline{c}$ is an arbitrarily fixed allocation. The central planner randomly selects an integer $t \in \{1,...,T\}$ and determines the allocation $g_t(a^{(t)}) \in C$.

We specify the payment rule x by

$$x_i(a,c) = -\varepsilon - \frac{k_i(a_i,c)}{T}\eta \qquad \text{if there exists } t \in \{1,...,T\} \text{ such that}$$

$$a_{i,t} \neq c, \text{ and } a_{j,t'} = c \quad \text{for all } j \neq i$$

and $t' \in \{1,...,T\}$ such that

$\xi(t') > \xi(t)$,

and

$$x_i(a,c) = -\frac{k_i(a_i,c)}{T}\eta \qquad \text{otherwise,}$$

where $k_i(a_i,c)$ denotes the number of elements of player $i's$ sub-message that are different from c, that is,

$$k_i(a_i,c) = \left|\{t \in \{1,...,T\} \mid a_{i,t} \neq c\}\right|.$$

Clearly, $\left|x_i(a,c)\right| \leq \varepsilon + \eta$. More importantly, the central planner fines the *last deviant* from the truthful revelation in the order of the permutation ξ. Moreover, for every $t \in \{1,...,T\}$, each player additionally pays the monetary fine $\frac{\eta}{T}$ whenever his $t-th$ announcement is different from c.

Each player $i's$ valuation function is given by $v_i : \Delta(C) \to R$. We assume expected utility and quasi-linearity. Fix an arbitrary allocation $c^* \in C$ as the desirable allocation. The game implied by the Abreu-Matsushima mechanism is denoted by $G = G(\xi, c^*)$, where the payoff function $u_i : A \to R$ for each player i is given by

$$u_i(a) = v_i(g(a)) + x_i(a,c^*) \text{ for all } a \in A.$$

Let $a^* = (a_i^*)_{i \in N}$ denote the sincere strategy profile, where

$$a_{i,t}^* = c^* \text{ for all } i \in N \text{ and } t \in \{1,...,T\}.$$

Note that the central planner can achieve the desirable allocation c^* without monetary fines, provided that the players select the sincere strategy profile a^*.

We assume that

$$(6) \qquad T\varepsilon + \eta > \max_{(i,c) \in N \times C} \{v_i(c) - v_i(c^*)\},$$

which is not very restrictive even if ε and η are as close to zero as possible, because we can set the positive integer T large enough to satisfy the inequality (6). With this assumption, we can prove in the same way as Matsushima (2017) that a^* *is the unique iteratively undominated strategy profile in the game* $G(\xi, c^*)$ *implied by the*

Abreu-Matsushima mechanism $\Lambda(\xi)$.

To exclude trivial cases, we assume that there exists $\tilde{c} \neq c^*$ such that

$$u_i(\tilde{c}) - u_i(c^*) > \eta \quad \text{for two or more players.}$$

(Otherwise, a^* is the unique iteratively undominated strategy profile even if $T = 1$ and $\varepsilon = 0$.)

Let ξ^* denote the identity mapping, where

$$\xi^*(t) = t \quad \text{for all } t \in \{1,...,T\}.$$

In the game $G(\xi^*, c^*)$, we can iteratively eliminate strategies whose later elements are different from c^*. We therefore have

$$A_i(t) \subset \bar{A}_i(T - t, a_i^*) \quad \text{for all } i \in N \text{ and } t \in \{0,...,T\}.$$

This inclusion along with Theorem 4 implies that a^* is the unique iteratively quasi-obviously undominated strategy profile in $G(\xi^*, c^*)$.

Note that whenever $\xi \neq \xi^*$, then a^* is not the unique iteratively quasi-obviously undominated strategy profile in $G(\xi, c^*)$. In this case, there exist t and $t' > t$ such that $\xi(t) > \xi(t')$. Note that players cannot eliminate strategies whose $t-th$ elements are different from c^* before eliminating strategies whose $(t')th$ elements are different from c^*. This implies that a^* is not the unique strategy profile that survives through the iterative eliminations of quasi-obviously dominated strategies. Hence, we have proved the following theorem.

Theorem 5: *The strategy profile* a^* *is the unique iteratively quasi-obviously undominated strategy profile in* $(G(\xi, c^*), \Gamma^*)$ *if and only if*

$$\xi = \xi^*.$$

Theorem 5 implies that the Abreu-Matsushima mechanism is robust in hypothetical thinking, but the associated frame must interpret this mechanism to fine the last deviants from truthful revelation.[7]

12. Incomplete Information

Throughout this study, we have assumed complete information. This section considers a game with incomplete information as a Bayesian game, which is denoted by $\Psi \equiv (N, A, u, \Omega, p)$, where Ω is the set of all states, $\Omega = \underset{i\in N}{\times} \Omega_i$, Ω_i is the set of all types for player i, $u = (u_i)_{i\in N}$, $u_i : A \times \Omega \to R$ is a state-contingent payoff function for player i, $p \equiv ((p_i(\cdot \mid \omega_i))_{\omega_i \in \Omega_i})_{i\in N}$ is a belief system, $p_i(\cdot \mid \omega_i) : \Omega_{-i} \to [0,1]$, and $p_i(\omega_{-i} \mid \omega_i)$ is the probability of the occurrence of the other players' type profile ω_{-i} conditional on ω_i.

Let $s_i : \Omega_i \to \Delta(A_i)$ denote a *strategy rule* for player i. Player i whose type is given by ω_i selects a strategy according to the lottery $s_i(\omega_i) \in \Delta(A_i)$. Let S_i denote the set of all strategy rules for player i. Let $s \equiv (s_i)_{i\in N}$ and $S \equiv \underset{i\in N}{\times} S_i$.

It is implicit to assume that the state ω is determined before the Bayesian game starts. Each player i therefore has no trouble with hypothetical thinking regarding ω_{-i}.

By treating each type as an individual agent, we can regard the Bayesian game Ψ as being equivalent to the agent-normal form game, denoted by $G = G(\Psi) \equiv (M, B, w)$, where M is the set of all agents, that is,

$$M = \{1, 2, ..., \sum_{i\in N} |\Omega_i|\},$$

$B \equiv \underset{m\in M}{\times} B_m$, B_m is the set of all strategies for agent $m \in M$, $w \equiv (w_m)_{m\in M}$, and $w_m : B \to R$ is the payoff function for agent m. We assume that there exists a

[7] Glazer and Rubinstein (1996) argued that an extensive form game with perfect information helps players to practice iterative elimination of dominated strategies, which interprets the Abreu-Matsushima mechanism to fine, not the last deviants, but the first deviants.

one-to-one correspondence $\lambda : \bigcup_{i\in N} \Omega_i \to M$ such that for every $i \in N$ and $\omega_i \in \Omega_i$,

$$B_{\lambda(\omega_i)} = A_i ,$$

and

$$w_{\lambda(\omega_i)}(b) \equiv E[u_i((b_{\lambda(\omega_j)})_{j\in N}, \omega) \mid \omega_i] \text{ for all } b \in B .$$

We treat each type ω_i in the Bayesian game Ψ as the agent $\lambda(\omega_i) \in M$ in the agent-normal form game $G(\Psi)$. By replacing a Bayesian game Ψ with the agent-normal form game $G(\Psi)$, we can directly apply the arguments in the previous sections to the Bayesian environments.

We denote by $\Pi = (T, (B_{m,t}, \tilde{B}_{m,t}(\cdot))_{t\in T}, \iota_m)_{m\in M})$ a frame associated with the agent-normal form game $G(\Psi)$, which we define in the same way as Γ by replacing N, A, and δ with M, B, and ι, respectively.

Definition 8: A strategy rule profile $s \in S$ is said to be *iteratively quasi-obviously undominated in a Bayesian game with a frame* (Ψ, Π) if the associated strategy profile $b \in B$ is iteratively quasi-obviously undominated in the agent normal form game with the frame $(G(\Psi), \Pi)$, where

$$b_{\lambda(\omega_i)} = s_i(\omega_i) \text{ for all } i \in N \text{ and } \omega_i \in \Omega_i .$$

Note that a frame Π generally has a non-negligible complexity in that it is a cognitive procedure not for the set of all (real) players N but for the set of all type-contingent (hypothetical) agents M. To address such cognitive complexity, we should investigate the solvability in iterative quasi-obvious dominance in Bayesian environments by using only a simple frame that is defined not for M but for N. We regard the frame Γ for complete information defined in Section 6 as a simple case of the frame Ψ for incomplete information defined in this section. The frame Γ for complete information is equivalent to the frame Ψ for incomplete information whenever for every $i \in N$,

$$(B_{m,t}, \tilde{B}_{m,t}(\cdot))_{t\in T} = (B_{m',t}, \tilde{B}_{m',t}(\cdot))_{t\in T} \text{ if } m \in \Omega_i \text{ and } m' \in \Omega_i ,$$

and

$$(B_{m,t},\tilde{B}_{m,t}(\cdot))_{t\in T}=(A_{i,t},\tilde{A}_{i,t}(\cdot))_{t\in T}\,.$$

In this case, we will simply write Γ instead of Ψ.

Let us consider a Bayesian game Λ where

$$A_i=A_{i,1}\times\cdots\times A_{i,T}\quad\text{for all }\; i\in N\,.$$

We denote by $s_i=(s_{i,t})_{t=1}^{T}$ a strategy rule for player i, where we denote $s_{i,t}:\Omega_i\to A_{i,t}$. Fix an arbitrary strategy rule profile $s^*=(s_i^*)_{i\in N}$ as the targeted strategy rule profile. For every $t\in\{1,...,T\}$, $i\in N$, and $\omega_i\in\Omega_i$, let

$$\bar{A}_i(t,s_i^*(\omega_i))\equiv\{a_i\in A_i\,|\,a_{i,t'}=s_{i,t'}^*(\omega_i)\;\textit{for all}\;t'\in\{t+1,...,T\}\}$$

denote the set of all strategies for player i with type ω_i whose elements after the $(t+1)th$ element are the same as $s_i^*(\omega_i)$.

We specify a frame Γ^* for the set of all players N so that at each step $t\in\{1,...,T\}$, each player i selects an action $a_{i,t}$ from $A_{i,t}$ under the imperfect information assumption. The following theorem shows a sufficient condition for the solvability in iterative quasi-obvious dominance in the Bayesian environments. This theorem is an extension of Theorem 4, which we can prove in the same way as Theorem 4.

Theorem 6: *In the above-mentioned Bayesian game* Ψ*,* s^* *is the unique iteratively quasi-obviously undominated strategy rule profile in* (Λ,Γ^*) *if for every* $i\in N$*,* $\omega_i\in\Omega_i$*, and* $t\in\{0,...,T\}$*,*

$$B_{\lambda(\omega_i)}(t)\subset\bar{A}_i(T-t,s_i^*(\omega_i))\,,$$

where $B_{\lambda(\omega_i)}(t)$ *is the set of all strategies for agent* $\lambda(\omega_i)\in M$ *that can survive through the* $t-time$ *iterative elimination of dominated strategies in* $(G(\Psi),\Gamma^*)$.

Based on this theorem, we can extend the argument in Section 10 to the Bayesian environment. We can show in the same manner as in Theorem 5 that in the Bayesian environment, any social choice function that is uniquely implementable in iterative dominance through the Abreu-Matsushima mechanism is also uniquely implementable in iterative quasi-obvious dominance through the Abreu-Matsushima mechanism that fines the last deviants.

13. Conclusion

This study investigated the possibility that even bounded-rational players employ rational behavior. We assumed that each player fails to practice hypothetical thinking regarding the present and future actions of other players, but not the previous actions of the other players. We proposed the method of frame design that induces such players to practice correct hypothetical thinking as much as possible.

With the help of appropriate frame design, we showed that the solvability in iterative undominated strategies is equivalent to the solvability in iteratively quasi-obviously undominated strategies. Hence, a well-designed frame successfully motivates bounded-rational players to employ rational behavior.

This study provided a cogent explanation about why the ascending proxy auction has more popularity than the second price auction even if both have the same physical rule. This study also showed that the Abreu-Matsushima mechanism is robust in the practice of hypothetical thinking, even if we only consider detail-free frames. We further extended the method of detail-free frame design to the Bayesian environments.

Frame design bridges the gap between rationality and bounded rationality. It might be anticipated that frame design avoids the obstruction caused by various aspects of bounded rationality besides hypothetical thinking. Our main concern is that different frame designs are needed to overcome different aspects of bounded rationality such as higher-order reasoning. It would be an important future research topic to consider how to design frame systems that can solve multiple issues on bounded rationality altogether.

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