Learning about perceived inflation target and stabilisation policy

Kosuke Aoki¹  Takeshi Kimura²

¹LSE
²Bank of Japan

22th August 2006
Objectives

- Analyse the interaction between:
  - private-sector uncertainty about inflation target ($\pi^*$)
  - central-bank uncertainty about private-sector belief about $\pi^*$ (perceived inflation target)

- Implications for
  - inflation persistence and volatility
  - time-varying inflation process
Motivation 1: Uncertainty about perceived inflation target

Measures of perceived inflation target are noisy when monetary policy loses nominal anchor
Example: US in late ’70s-’80s

- Survey measures of LR inflation expectations as proxies
  - Blue chip survey: 8%
  - Michigan survey: 10-11%

- Model-based measures of belief about inflation target
  - Kozicki-Tinsley (’01, ’05): 8 % (estimated target ≃ 3.5%)
  - Bekaert et. al. (’05): 14%
Survey measures of perceived inflation target

long run inflation expectations

Blue 10years
mich 5 years
Learning about perceived inflation target and stabilisation policy

Kosuke Aoki, Takeshi Kimura

INTRODUCTION

MOTIVATION

QUESTIONS AND ISSUES

OUTLINE

STRUCTURAL EQUATIONS

EQUILIBRIUM UNDER INCREDIBLE $\pi^*$

IMPLICATIONS FOR ‘GREAT INFLATION’

IMPLICATIONS FOR ‘GREAT MODERATION’

CONCLUSION

APPENDIX

Model-based measures of perceived inflation target

Bakaert, Cho, Moleno ’05

Kozicki and Tinsley ’05

Fig. 1. Time-varying inflation target, constant inflation target, perceived inflation target, and inflation.
Motivation 2: inflation and misinformation

Forecast errors larger in the 70s
Motivation 2: inflation and misinformation

Romer and Romer ’02
Motivation 2: inflation and misinformation

Estimate of natural rate biased. What caused this?
Related Literature: ‘Great Inflation’

- Time inconsistency (Kydland-Prescott)
- Sunspot fluctuations (Clarida-Gali-Gertler, ’99 QJE)
- Misspecified model (Sargent ’99, Romer-Romer ’02)
- Misinformation (Orphanides ’01 AER, ’02 AER, ’03 JME)
- Imperfect credibility (Erceg-Levin, ’03 JME)
- This paper is related to Orphanides and Erceg-Levin.
  - Weak nominal anchor disturbs stabilisation policy. How?
  - PS uncertainty about inflation target represents uncertainty facing Central Bank
Questions addressed

- Unobservable (or incredible) $\pi^*$
  - how does this affect private agents?
  - how does this affect central bank?
- What are the interaction between the two?
Issues

Feedback from private-sector (PS) uncertainty about $\pi^*$ to monetary policy

- PS belief about $\pi^*$
  - unobservable to CB
  - CB cannot distinguish from other shocks

- Expectations formation by PS affected by CB information problem
Negative feedback on stabilisation
an example

Suppose nominal interest rate $i_t$ increases

- Two possibilities ($i_t = r_t + E_{t|p} \pi_{t+1}$)
  - inflation expectations increased
  - natural rate increased

- When CB uncertain about PS belief about $\pi^*$, CB cannot distinguish those two.
Outline of the model

A simple model of inflation determination

- Flexible prices
- Exogenous output (exogenous natural interest rate = real rate)
- Monetary policy follows a simple rule (No optimisation. Focus on filtering and equilibrium)
  - $\pi^*$ unobservable to PS. Perceived target $\pi^*_{t|p}$
  - $\pi^*_{t|p}$ unobservable to CB (Information structure explained in detail later)
Results

- Inflation persistence caused by:
  - PS filtering about $\pi^*$
  - CB filtering about $\pi^*_{t|p}$
  (Recursive nature of filtering)

- Inflation volatility caused by CB’s failure to keep track of $r_t$ (Feedback effects of PS uncertainty on stabilisation)

- Persistence and volatility decrease over time

- Weak nominal anchor and MP mistakes are related with each other
Learning about perceived inflation target and stabilisation policy

Kosuke Aoki, Takeshi Kimura

INTRODUCTION

STRUCTURAL EQUATIONS

EQUILIBRIUM UNDER INCREDIBLE $\pi^*$

IMPLICATIONS FOR ‘GREAT INFLATION’

IMPLICATIONS FOR ‘GREAT MODERATION’

CONCLUSION

APPENDIX

Structural Equations
Fisher equation

A version of ‘expectational’ IS curve

\[ i_t = r_t + E_t|_p \pi_{t+1} \]  \hspace{1cm} (IS)

- \( i_t \): nominal interest rate;
- \( r_t \): natural rate; \( \pi_t \): inflation

- Can be derived from Euler equation under flexible-price equilibrium (Woodford ’04, Ch2)

- \( E_t|_p \): expectation operator conditional on PS information
Monetary policy rule

CB follows a simple rule:

\[ i_t = \phi(\pi_t - \pi^*) + \pi^* + E_{t|c}r_t + u_t, \ \phi > 1. \]  \hspace{1cm} (MP)

*\pi^**: inflation target; *u_t*: monetary policy shock

▶ *E_{t|c}*: expectation conditional on CB information

▶ CB wants to keep track of natural rate *r_t*
Equilibrium

- Endogenous variables \( \{ i_t, \pi_t \}_{t=0}^{\infty} \) satisfy (IS) and (MP),
- taking exogenous variables \( \{ r^n_t, u_t \}_{t=0}^{\infty} \) as given,
- expectations are rational conditional on information set of PS and CB
Benchmark: When $\pi^*$ is credible

- $\pi^*$ is common knowledge
- From (IS) and (MP),
  \[ \pi_t = \pi^* + E_t \sum_{s=0}^{\infty} \phi^{-(s+1)} u_{t+s} \]
- When $E_t u_{t+s} = 0$ for $s > 1$,
  \[ \pi_t = \pi^* + \phi^{-1} u_t. \]
Benchmark: When $\pi^*$ is credible

Equilibrium is given by

$$\pi_t = \pi^* + \phi^{-1}u_t.$$ 

- CB fully offsets the effects of $r_t$ on $\pi_t$
- Inflation expectations anchored by $\pi^*$
- By looking at $i_t$, CB can identify $r_t$ even if $r_t$ not directly observable.

$$(i_t = r_t + E_t\pi_{t+1} = r_t + \pi^*)$$
Learning about perceived inflation target and stabilisation policy

Kosuke Aoki, Takeshi Kimura

INTRODUCTION

STRUCTURAL EQUATIONS

EQUILIBRIUM UNDER INCREDIBLE $\pi^*$

INFORMATION
EQUILIBRIUM GIVEN BELIEF
PS FILTERING
CB FILTERING
INFLATION DYNAMICS

IMPLICATIONS FOR ‘GREAT INFLATION’

IMPLICATIONS FOR ‘GREAT MODERATION’

CONCLUSION

APPENDIX

Equilibrium under incredible $\pi^*$
Assumptions on private-sector information

- $\pi^*$ and $u_t$: unobservable.
- PS belief about $\pi^*$: denoted by $\pi^*_{t|p}$
- $i_t$, $\pi_t$, $r_t$, $\phi$: observable
- CB belief ($E_t|c\pi^*_{t|p}$ and $E_t|c\pi^*_{t|p}$): observable (see next page)
  - Only need to analyse up to 3rd-order belief
Assumptions on Central-Bank information

- $r_t, E_t|p \pi_{t+1}, \pi^*_t$: unobservable
  - belief about belief: $E_t|c \pi^*_t$
  - belief about $r_t$: $E_t|c r_t$
  - CB announces its belief
    What we have in mind: CB publishes its economic outlook
- $i_t, \pi_t, u_t, \pi^*$: observable
Normality assumption

$r_t$ and $u_t$ are iid normal

\[ r_t \sim N(0, \sigma_r^2), \quad \gamma_r = 1/\sigma_r^2 \ \text{(precision)} \]

\[ u_t \sim N(0, \sigma_u^2), \quad \gamma_u = 1/\sigma_u^2 \ \text{(precision)} \]

- Common knowledge
- Can obtain analytical results
- The main results survive if we allow shock-persistence
Equilibrium given belief

From (IS) and (MP),

\[ \pi_t = \phi^{-1} \left[ (\phi - 1)\pi^* - u_t + (r_t - E_t|c r_t) + E_t|p \pi_{t+1} \right] \]

- \( r_t - E_t|c r_t \): CB estimation error
- Inflation given CB belief

\[ \pi_t = \pi^* - \phi^{-1} u_t + \phi^{-1} (E_t|c \pi^*_t|p - \pi^*) \]

- Inflation given PA belief

\[ \pi_t = \pi^* - \phi^{-1} u_t + \phi^{-1} (E_t|p \pi^* - \pi^*) \]

\[ + E_t|p \sum_{j=0}^{\infty} \phi^{-(j+1)} \left[ r^n_{t+j} - E_{t+j}|c r^n_{t+j} \right] \]

- 2nd order belief matters
Private-sector filtering

PS observation equation (derived from MP rule)

\[ i_t - \phi \pi_t - E_{t|c} r_t = (1 - \phi) \pi^* + u_t. \]

- Observable: \( z_t \equiv i_t - \phi \pi_t - E_{t|c} r_t \)
- Sequential updating of \( \pi^*_{t|p} \)
Private-sector filtering

Perceived inflation target after $t$ observations:

$$\pi_{t|p} - \pi^* = b_t(\pi_{t-1|p} - \pi^*) + \frac{1 - b_t}{1 - \phi} u_t,$$

1. $b_t \to 1$ as $t \to \infty$
2. Private sector eventually learn $\pi^*$
Equilibrium and CB filtering

Simultaneity

- Equilibrium depends on CB policy
- CB policy depends on CB filtering
- CB filtering depends on statistical relation between observables and unobservables in equilibrium
- Solve by the method of undetermined coefficients (time-varying coefficients).
CB-filtering about \( \pi^*_t \)

- **Observation equation (Fisher equation)**
  \[
i_t = r_t + E_{t|p} \pi_{t+1}
\]

- \( E_{t|p} \pi_{t+1} \) is determined simultaneously with CB filtering

- **Estimated perceived inflation target**
  \[
  E_{t|c} \pi^*_t - \pi^*_t = f_t(E_{t-1|c} \pi^*_{t-1|p} - \pi^*_{t-1|p}) + g_t r_t \quad (2)
\]

- **Estimated natural rate**
  \[
  E_{t|c} r_t - r_t = h_t(E_{t-1|c} r_{t-1} - r_{t-1}) + k_t r_t \quad (3)
\]

- \( f_t, g_t, h_t, k_t: \) time-varying coefficients
Summary of Equilibrium

- Equilibrium is given by

\[
\pi_t = \pi^* - \phi^{-1} u_t + \phi^{-1} (E_t \mid \pi_t^* - \pi^*)
\]

- \( \tilde{\pi}_t = \phi^{-1} \left\{ (\pi_t^* - \pi^*) + (E_t \mid \pi_t^* - \pi_t^*) \right\} \)

- \( (\pi_t^* - \pi^*) \) is given by (1)

- \( (E_t \mid \pi_t^* - \pi_t^*) \) is given by (2)
Implications for great inflation: persistence and volatility
Inflation dynamics

Our model implies $\tilde{\pi}_t$ is persistence and volatile

$$(E_{t|c} \pi_{t|p}^* - \pi_{t|p}^*) \propto (r_t - E_{t|c} r_t):$$ represents estimation error of $r_t$.

- persistence: recursive nature of learning.

- volatility: Negative feedback of uncertainty about $\pi^*$ on stabilisation policy
Great inflation

- High and persistent inflation in the late ’70s-early ’80s
- Orphanides (’01 AER, ’02 AER, ’03: JME): Mis-measurement in the output gap/natural interest rate
  - Misinformation is exogenously given.
- Erceg-Levin (’03 JME): weak nominal anchor (imperfect credibility) causes inflation persistence
  - Mainly focuses on persistence but not volatility.
Great inflation

- This paper connects Orphanides and Erceg-Levin
- Imperfect credibility creates uncertainty about perceived inflation target
  - → identification of shocks difficult. → source of natural rate mis-measurement
- this causes policy mistakes, generating inflation volatility and persistence.
Policy implications

- Orphanides
  - Misinformation causes inflation
  - Policy recommendation: avoid responding to noisy estimates of output gap and natural rates

- Our paper
  - Weak nominal anchor creates misinformation
  - Policy recommendation: make nominal anchor strong. If MP becomes credible, misinformation becomes smaller.
Implications for great moderation: time-varying stochastic properties of inflation
Time-varying stochastic process of inflation

Our model implies

- \( \pi_t \to \bar{\pi}_t \) as \( t \to \infty \).

- Contribution of \( \tilde{\pi}_t \) becomes smaller over time
  - \( \pi_t \) becomes less persistent over time
  - \( \pi_t \) becomes less volatile over time
Great moderation

- UK inflation: less volatile and less persistent after ’92 (Benati ’04)
- time-varying stochastic process of $\pi$ (Cogley-Sargent (’02,’04), Stock-Watson (’02), Ahmed-Levin-Wilson (’04))
- good policy or good luck?
  Existing literature: likely to be good luck.
Bernanke’s conjecture
(Bernanke ’04 speech)

Econometric methods confuse good policy with good luck
▶ don’t take into account of impact of systematic component of monetary policy on inflation expectations
▶ fluctuations caused by de-anchored expectations get confused with genuine non-policy shocks
Reduced-form regression of model-generated data

- Estimation of

\[ \pi_t = c + \alpha \pi_{t-1} + \varepsilon_t \]
Reduced-form regression

- We are interested in:
  - change in $\alpha$
  - change in $SD(\varepsilon_t)$

- Literature on ‘great moderation’ interprets
  - change in $\alpha$ as change in propagation
  - change in $SD(\varepsilon_t)$ as change in innovation
Learning about perceived inflation target and stabilisation policy

Kosuke Aoki, Takeshi Kimura

INTRODUCTION

STRUCTURAL EQUATIONS

EQUILIBRIUM UNDER INCREDIBLE $\pi^*$

IMPLICATIONS FOR ‘GREAT INFLATION’

IMPLICATIONS FOR ‘GREAT MODERATION’

CONCLUSION

APPENDIX

Numerical example

- $\pi^* = 2, \pi_0|_p = 10, 0 \leq E_0|_c \pi_0^* - \pi_0|_p \leq 5$
  - $\pi_0|_p$ in line with US estimates in ’80-’81
  - $E_0|_c \pi_0^* - \pi_0|_p$ in line with differences among US estimates of perceived target in ’80s.

- $\gamma_r = 0.44, \gamma_u = 1, \phi = 1.5$

- Simulation for 40 periods, 1000 replications

- Estimate for two sub-samples (1-20, 21-40)

- Sensitivity analysis
Simulation results

- $\alpha$ and $SD(\varepsilon_t)$ become smaller in the second half
- $\alpha$ and $SD(\varepsilon_t)$ become larger as $E_0|c\pi^*_0|p - \pi^*_0|p$ becomes larger
Reduced-form regression of inflation

- Both ‘innovation’ and ‘persistence’ decline in reduced-form regression
- But, in our model, policy and structural shocks are constant over time
- In our model, change in stochastic process of $\pi_t$ is generated by change in expectations (beliefs) — consistent with Bernanke’s conjecture
Summary

- Analysis of uncertainty about perceived inflation target
- Mis-measurement of natural rates endogenously determined
- A unified analysis of weak nominal anchor and misinformation
- Change in stochastic process of inflation driven by changes in expectations — existing literature on Great Moderation has not fully explored yet
Future work

- Implication for yield curve
  - Excess sensitivity of long rates due to lack of nominal anchor
How is private-sector inflation expectation affected by CB-uncertainty about perceived target?

\[
E_{t|p} \pi_{t+1} = (1 - \phi^{-1})\pi^*_{t|p} + \phi^{-1}E_{t|p}E_{t+1|c} \pi^*_{t+1|p}.
\]

3rd-order belief

- In general, \(E_{t|p} \pi_{t+1} \neq \pi^*_{t|p}\)
- PS expectation about how CB will learn about future \(\pi^*_{t+1|p}\) matters
Learning about perceived inflation target and stabilisation policy

Kosuke Aoki, Takeshi Kimura

INTRODUCTION

STRUCTURAL EQUATIONS

EQUILIBRIUM UNDER INCREDIBLE $\pi^*$

IMPLICATIONS FOR ‘GREAT INFLATION’

IMPLICATIONS FOR ‘GREAT MODERATION’

CONCLUSION

APPENDIX

EQUILIBRIUM AND CB FILTERING

EQUILIBRIUM PROPERTIES

SENSITIVITY ANALYSIS

CB-filtering about $\pi^*_{t|p}$

- Observation equation (Fisher equation)

$$i_t = r_t + E_{t|p} \pi_{t+1}$$

$$E_{t|p} \pi_{t+1} = (1 - \phi^{-1}) \pi^*_{t|p} + \phi^{-1} E_{t|p} E_{t+1|c} \pi^*_{t+1|p}$$

- CB knows $\pi^*_{t|p}$ evolves by:

$$\pi^*_{t|p} = \alpha_t \pi^*_{0|p} + (1 - \alpha_t) \pi^* + \frac{1 - \alpha_t}{1 - \phi} \bar{u}_t \quad \text{(PSB)}$$

$\pi^*_{0|p}$: only uncertainty to CB

- $E_{t|p} \pi_{t+1}$ is determined simultaneously with CB filtering about $\pi^*_{0|p}$

- Solve by the method of undetermined coefficients (time-varying coefficients).
Equilibrium and CB filtering (1)

Solve by the method of undetermined coefficients.

- Define observables by
  \[ X_t \equiv i_t - (1 - a_t)\pi^* - \frac{1-a_t}{1-\phi} \bar{u}_t. \]

- Guess:
  \[ A_tX_t = r_t + B_t\pi^*_0|p + C_tE_{t-1}|c\pi^*_0|p \] (G)

  \( A_t, B_t, C_t \) to be determined jointly with Kalman filtering about \( r_t \).

- \( B_t \) represents the effects of initial perceived target \( (\pi^*_0|p) \) on current equilibrium.
Equilibrium and CB filtering (2)

Derive Kalman filter based on (G), and substitute it back to (G). Then solve for $A_t$, $B_t$, $C_t$. Then $B_t$ satisfies

$$B_t = a_t - \phi^{-1}a_{t+1} \frac{\frac{B_{t+1}^2}{B_t^2} \tau_{t|c}}{\frac{B_{t+1}^2}{B_t^2} \tau_{t|c} + \gamma r},$$

$$\tau_{t|c} = \frac{B_{t-1}^2}{B_t^2} \tau_{t-1|c} + \gamma r.$$ 

Once $B_t$ is determined, $A_t$ and $C_t$ are determined.
Define new observation equation by

\[ Y_t \equiv A_t X_t - C_t E_{t-1} \mid c \pi_0^* = r_t + B_t \pi_0^* \mid p \text{ unobservable} \]

Distribution of \( Y_t \) is

\[ Y_t \sim N \left( B_t \pi_0^* \mid p, \sigma_r^2 \right). \]
Equilibrium and CB filtering (4)

Posterior mean of $B_t \pi^*_0 \mid p$ at time $t$:

$$B_t E_t \mid c \pi^*_0 \mid p = d_t B_{t-1} E_{t-1} \mid c \pi^*_0 \mid p + (1 - d_t) Y_t,$$

(4)

where

$$d_t \equiv \frac{B_{t-1}^2 \tau_{t-1} \mid c}{B_t^2 \tau_{t-1} \mid c + \gamma_r}$$

(5)

and $\gamma_r \equiv 1/\sigma_r^2$. 
Equilibrium properties (1)

- Simultaneity of equilibrium and CB filtering
- PS expectations about future CB filtering matters to $\pi_t$
- Current CB filtering depends on PS expectations about future CB filtering
- Intuition:
  - Forward-looking nature of inflation
  - Inflation determined by expectations about future MP
  - Future MP depends on future CB filtering
Equilibrium property (2)

\( B_t \) depends on:

- \( B_{t-1} \): recursive nature of filtering
- \( B_{t+1} \): forward-looking nature of inflation
  - \( \pi_t \) depends on PS expectations about future MP
  - future MP depends on filtering \( d_{t+1} \)
  - current filtering depends on PS expectations
Sensitivity

Basic results robust against different $\pi^*_0|_p$, $\phi$, $\gamma_u$, $\tau_0|PA$

- High perceived target ($\pi^*_0|_p$) results in high inflation persistence
- Aggressive MP ($\phi$) results in smaller SD
- Smaller MP shock (larger $\gamma_u$) results in smaller SD and less persistence (because PS learning is quicker)
- More stubborn belief (larger $\tau_0|PA$) results in larger SD and more persistence (because PS learning is slower)
Sensitivity analysis (1)

Benchmark ($\pi^*_{0PA} = 10$, Black line) vs. Higher perceived target ($\pi^*_{0PA} = 20$, Gray line)

\[ E_0^{CB} \pi^*_{0PA} - \pi^*_{0PA} \]

\[ E_0^{CB} \pi^*_{0PA} - \pi^*_{0PA} \]

\[ V_\text{VAR}[\pi_t] \]
Learning about perceived inflation target and stabilisation policy

Kosuke Aoki, Takeshi Kimura

INTRODUCTION

STRUCTURAL EQUATIONS

EQUILIBRIUM UNDER INCREDIBLE $\pi^*$

IMPLICATIONS FOR ‘GREAT INFLATION’

IMPLICATIONS FOR ‘GREAT MODERATION’

CONCLUSION

APPENDIX

EQUILIBRIUM AND CB FILTERING

EQUILIBRIUM PROPERTIES

SENSITIVITY ANALYSIS

Sensitivity analysis (2)

Benchmark ($\phi = 1.5$, Black line) vs. Less aggressive monetary policy ($\phi = 1.1$, Gray line)
Sensitivity analysis (3)

Benchmark (γ_e = 1, Black line) vs. Smaller monetary policy shock (γ_e = 4, Gray line)

\[ E_0^{CB} \hat{\pi}_{0,PA}^* - \pi_{0,PA}^* \]

\[ E_0^{CB} \hat{\pi}_{0,PA}^* - \pi_{0,PA}^* \]

\[ Var[\pi_t] \]
Sensitivity analysis (4)

Benchmark ($\tau_{APA} = 1$, Black line) vs. More stubborn belief ($\tau_{APA} = 10$, Gray line)