Coexisting Exchange Platforms: Limit Order Books and Automated Market Makers

[draft available here]

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Decentralized Exchanges and Automated Market Makers

- Decentralized exchanges (DEXs): trading with smart contracts on Ethereum blockchain
  $\Leftrightarrow$ Centralized exchanges (CEXs), e.g., Binance, Bittrex, etc. DEX?
- Algorithm-based pricing by Automated Market Makers (AMM)
  $\Leftrightarrow$ Limit order books on CEXs DEX share LOB
- New market-making structure coexisting with traditional LOB

Centralized Exchanges

1. Custody of customer funds
2. KYC requirements
3. Control of transactions
4. Censorship
5. Settlement
6. Order matching
7. Liquidity
8. Infrastructure, Development

Permissioned DEX

1. Custody of customer funds
2. KYC requirements
3. Control of transactions
4. Censorship
5. Settlement
6. Order matching
7. Liquidity
8. Infrastructure, Development

Ability to reverse transactions, censor who has access to their services

Off-Chain DEX

1. Custody of customer funds
2. KYC requirements
3. Control of transactions
4. Censorship
5. Settlement
6. Order matching
7. Liquidity
8. Infrastructure, Development

Order matching and liquidity pools are managed off-chain (e.g., relayer mechanism)

On-Chain DEX

1. Custody of customer funds
2. KYC requirements
3. Control of transactions
4. Censorship
5. Settlement
6. Order matching
7. Liquidity
8. Infrastructure, Development

Still not fully decentralized: e.g., using Infura

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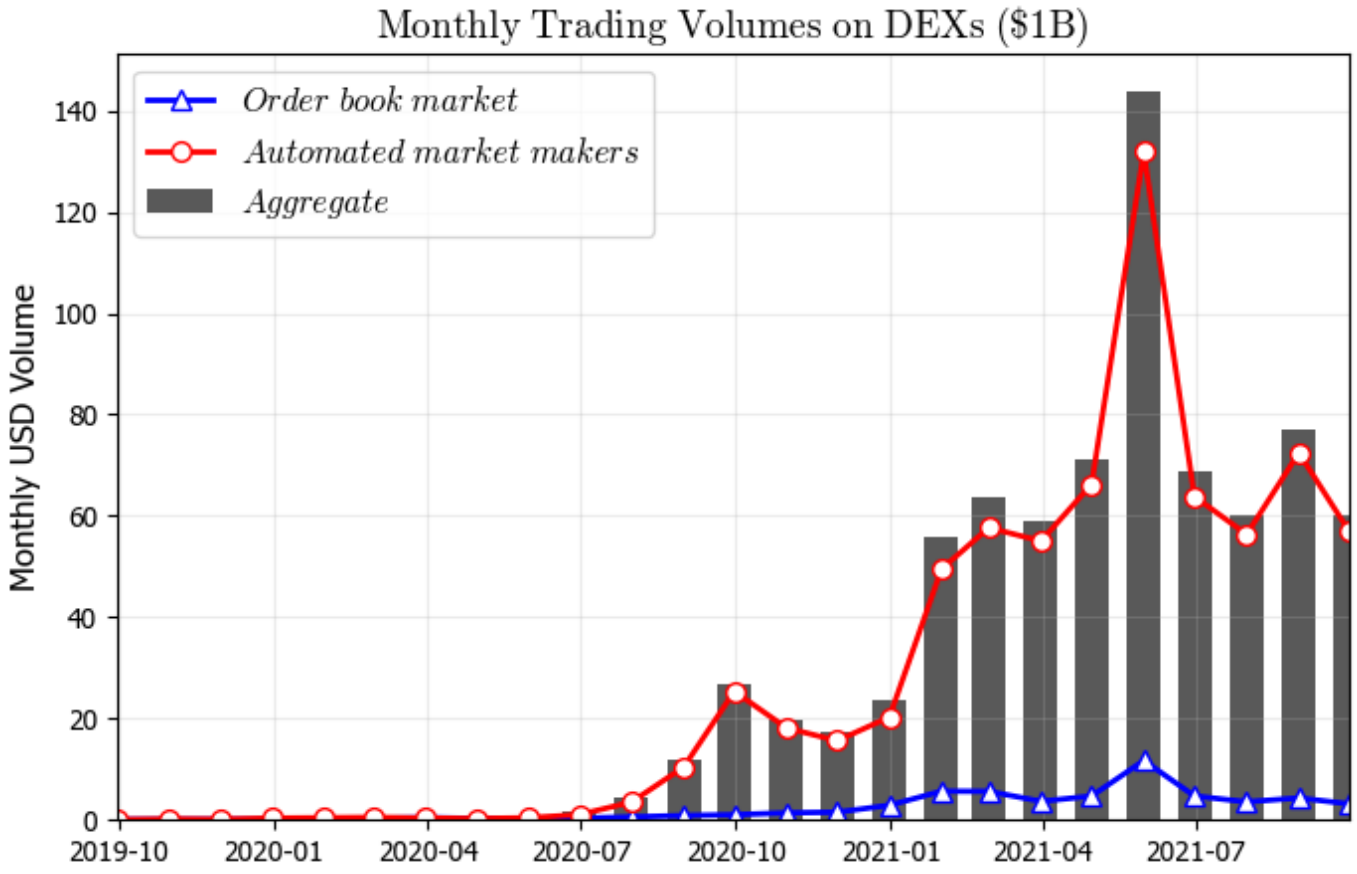
DEX and AMM

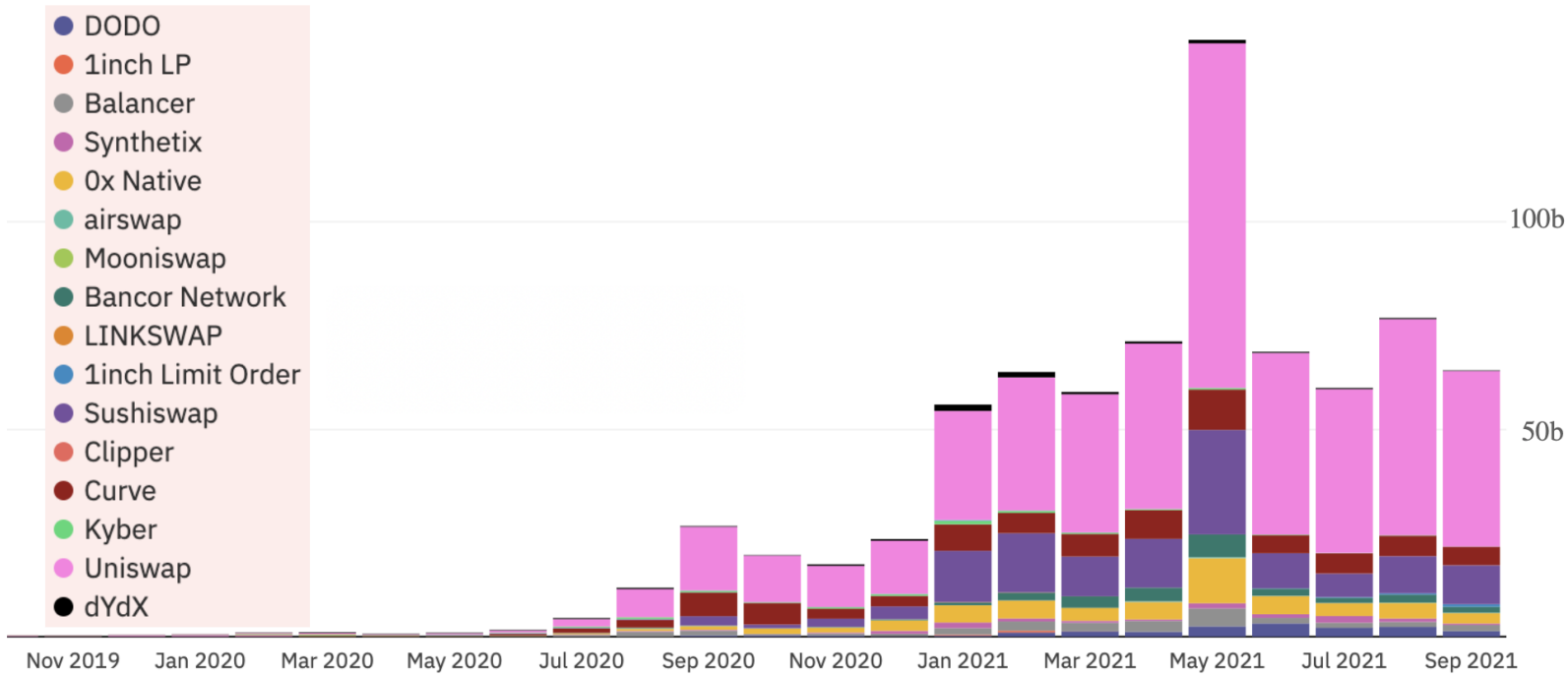
Figure: DEX traidng volume



Source: Dune Analytics

Automated Market Makers

- Constant Product MM: Uniswap, Sushiswap, PancakeSwap
- Constant Mean MM: Balancer
- Constant Function (hybrid) MM: Curve, DODO, Gnosis, etc.



Source: Dune Analytics

Compared to CEXs...

Spot | DEX | Derivatives

USD | All Countries

Top Cryptocurrency Exchanges Ranking by Trust Score - Spot

* Read the methodology

Overview | Cybersecurity New | Social

#	Exchange	Trust Score Beta	24h Volume (Normalized)	24h Volume	Visits (SimilarWeb)	# Coins	# Pairs	Last 7 Days
1	Binance Centralized	10	$25,870,516,427	$25,870,516,427	165,696,594.0	332	1230	
2	Huobi Global Centralized	10	$7,728,934,191	$7,728,934,191	5,854,617.0	353	934	
3	Crypto.com Exchange Centralized	10	$4,452,244,801	$4,452,244,801	9,764,011.0	120	219	
4	Coinbase Exchange Centralized	10	$3,830,167,342	$3,830,167,342	14,962,048.0	102	304	
5	FTX Centralized	10	$2,591,303,145	$2,591,303,145	21,690,867.0	273	437	
6	KuCoin Centralized	10	$1,624,103,572	$1,624,103,572	11,934,432.0	456	896	
7	Gate.io Centralized	10	$1,483,171,769	$1,483,171,769	12,469,972.0	922	1933	
8	Bitfinex Centralized	10	$1,127,968,565	$1,127,968,565	5,001,192.0	155	374	
9	Kraken Centralized	10	$909,598,391	$909,598,391	13,195,314.0	87	389	
10	Binance US Centralized	10	$619,719,350	$619,719,350	3,879,846.0	60	117	
11	Gemini Centralized	10	$184,959,401	$184,959,401	3,065,878.0	51	72	
12	Bittrex Centralized	10	$146,809,383	$146,809,383	3,424,540.0	420	969	

Source: CoinGecko

Decentralized Exchanges and Automated Market Makers

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Questions and Findings

Questions

- How DEX (AMM) liquidity is determined?
- How DEX (AMM) liquidity interacts with CEX (LOB) liquidity?
- How do traders choose their trading venue? Depends on trading motives (informed or uninformed)?
- Implications for asset prices and other market quality?

Questions and Findings

Result

- Information asymmetry matters to AMM liquidity
- Liquid DEX $\rightarrow$ positive spillover effect on CEX liquidity
- ("Buy" order flow tends to be more informative than "sell" on DEX)
- (Bid and ask prices tend to be asymmetrically distributed around true asset value)

Limit Order Market

Limit order market

- Centralized exchange with limit order book LOB
  - Liquidity providers place limit orders on limit order book (LOB)
  - Liquidity takers submit market orders
- Liquidity = bid-ask spread (trading cost for liquidity takers)
- Finance literature: asymmetric information matters
  - e.g., Glosten and Milgrom (1985), Kyle (1985).

Asymmetric information and liquidity

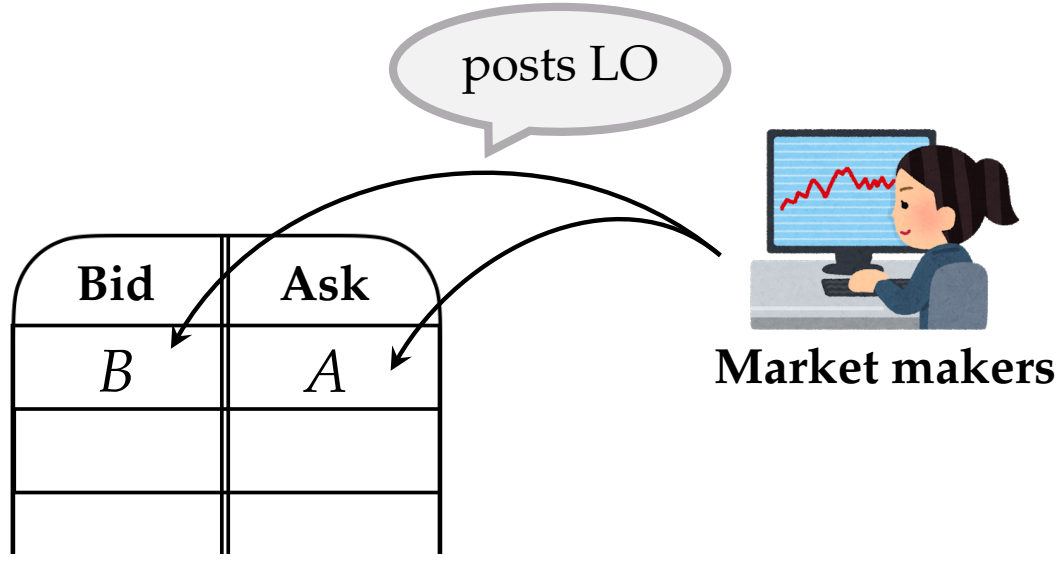
Environment:

- Asset value $\tilde{v} = \pm\sigma$ with prob. $1/2$
- β of takers know $\tilde{v} \rightarrow$ informed traders
- α of takers do not $\rightarrow$ uninformed (noise) traders
  - exogenous reasons: hedging, margin calls, needs for immediacy, etc.
- Market makers post limit orders (A, B) anticipating a trade (without knowing who takes LO)
- Assume that traders trade only one unit for simplicity

Market maker posts a limit order

Informed traders

Noise traders



Informed trading

Informed traders

learns news

$\tilde{v} = +\sigma$

market order

Bid	Ask
B	A

Limit order book

Noise traders

Market makers

Uninformed (noise) trading

Informed traders

need to buy/sell

Noise traders

prob. 1/2

Bid	**Ask**
B	A

Limit order book

Market makers

Market maker profit

- By posting LO with ask price A, a market maker expects to earn

$$\pi_{ask}(A) = \underbrace{\frac{\beta}{2}(A - \mathbb{E}[\tilde{v}|\text{informed buy}])}_{\text{informed trading}} + \underbrace{\frac{\alpha}{2}(A - \mathbb{E}[\tilde{v}|\text{noise buy}])}_{\text{noise trading}}$$

- Trading with informed trdr = losing money:

$$\mathbb{E}[\tilde{v}|\text{informed buy}] = \sigma > A$$

  - adverse selection cost

- Trading with noise trdr = earning money:

$$\mathbb{E}[\tilde{v}|\text{noise buy}] = 0 < A$$

- Adjust $A > 0$ so that the profit covers the cost in expectation

Bid-ask spread

- Competition $\rightarrow$ zero profit

$$\pi_{ask}(A) = \frac{\beta}{2}(A - \sigma) + \frac{\alpha}{2}A = 0$$

$$\therefore A = \frac{\beta}{\alpha + \beta}\sigma = \mathbb{E}[\tilde{v}|\text{trade at } A]$$

- $B = -A$ (everything is symmetric around 0)
- Bid-ask spread (= trading cost)

$$S \equiv A - B = 2\frac{\beta}{\alpha + \beta}\sigma$$

- $S \propto$ signal-to-noise ratio of trade = adverse selection cost for MM

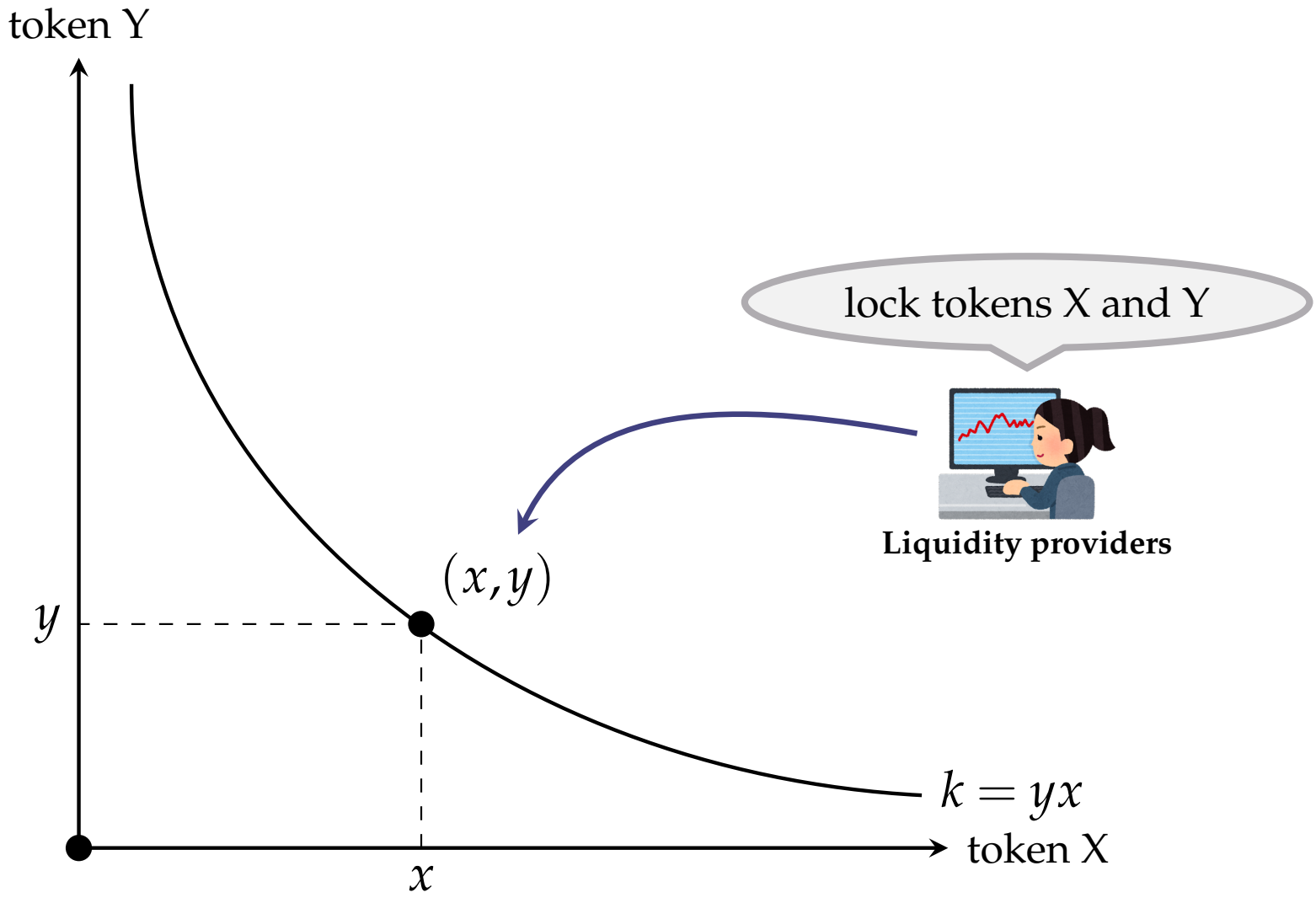
Automated Market Makers

AMMs: What are they and how they work?

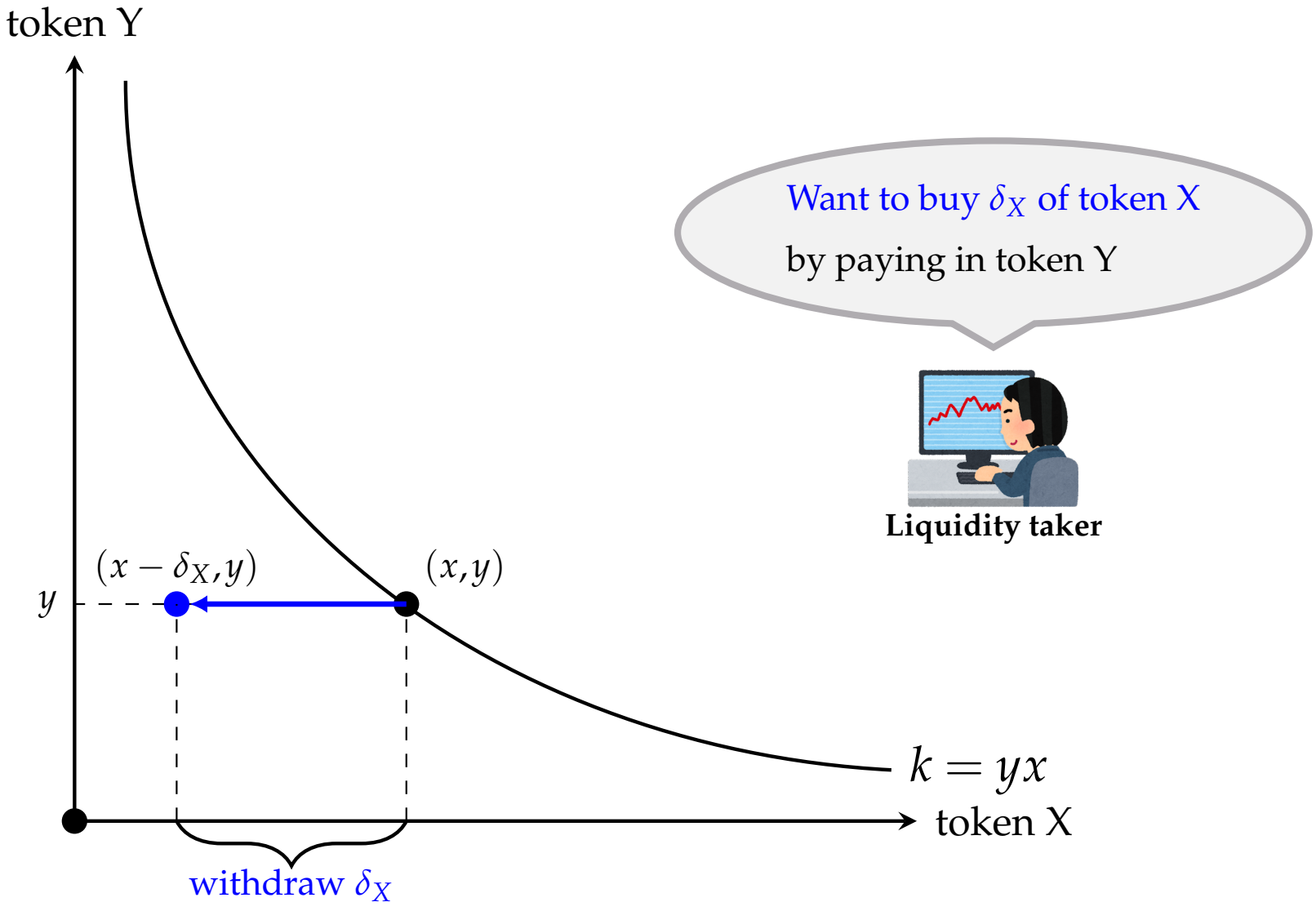
On DEX with AMMs:

- Liquidity providers inject assets into liquidity pools
- Liquidity takers trade against the pools (add and subtract tokens)
- Price is set by a pre-determined function (AMM)

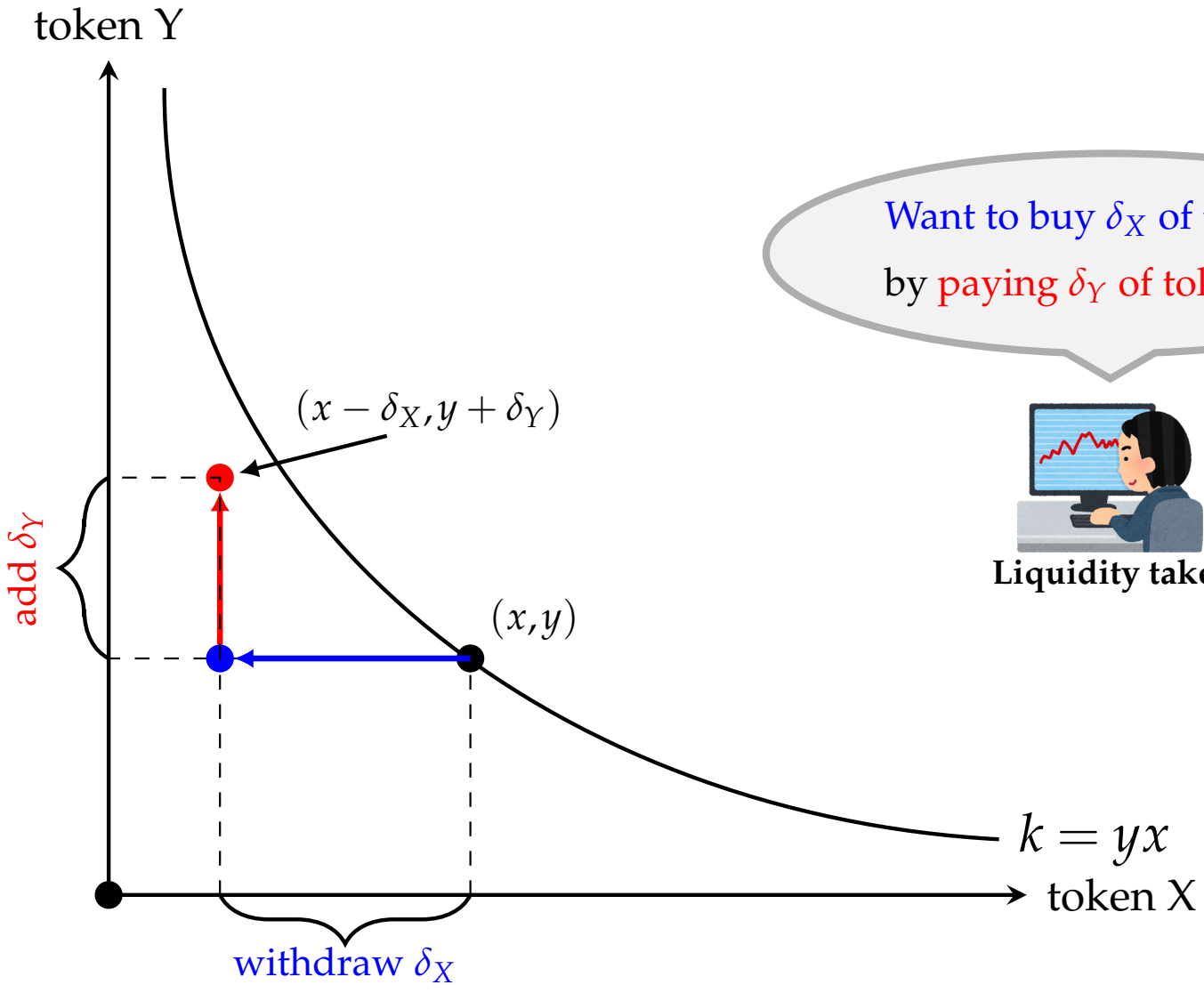
Constant Product Market Makers: $k = xy$



Taking liquidity



Taking liquidity

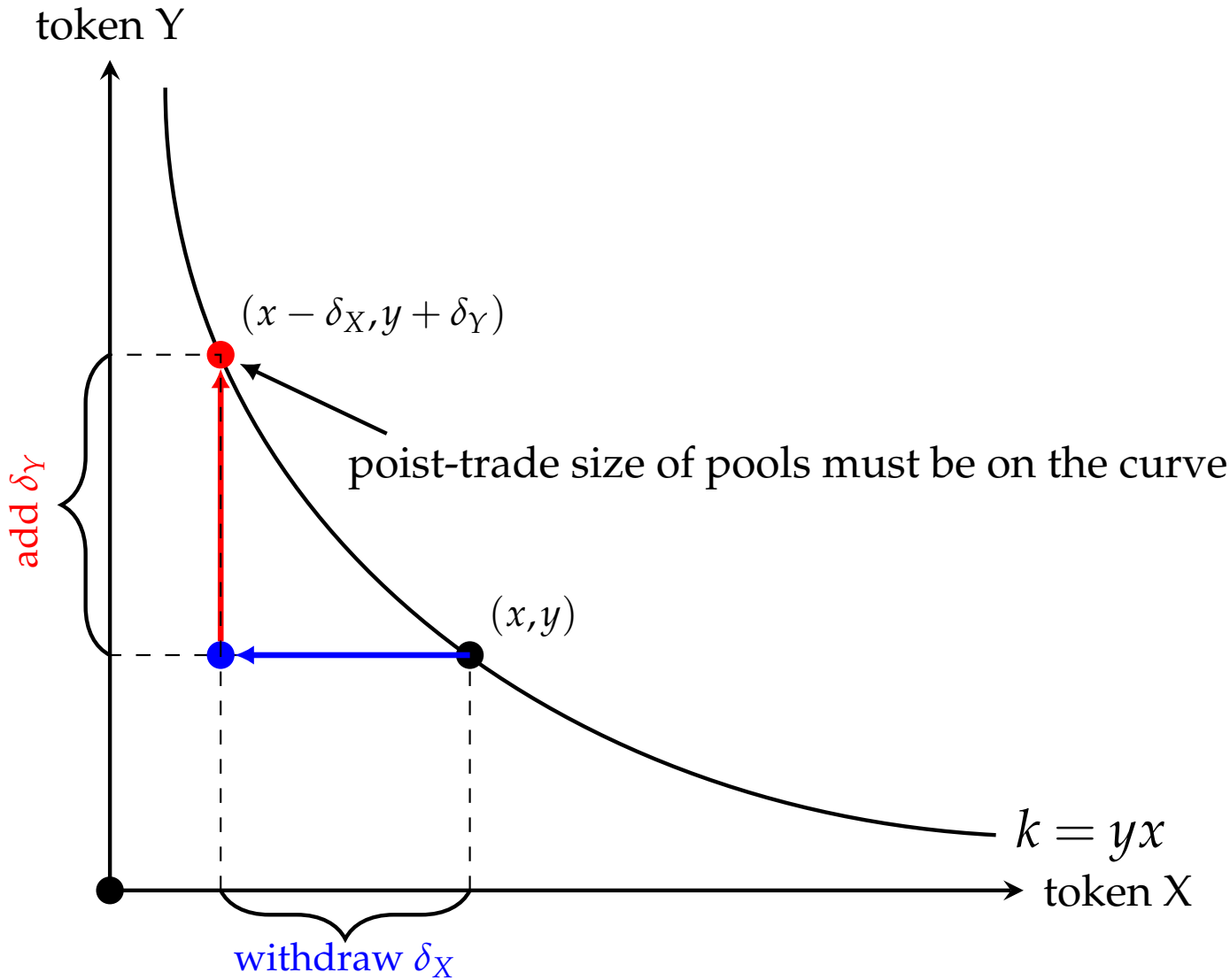


Want to buy δ_X of token X

by paying δ_Y of token Y

Liquidity taker

Execution price by CPMM



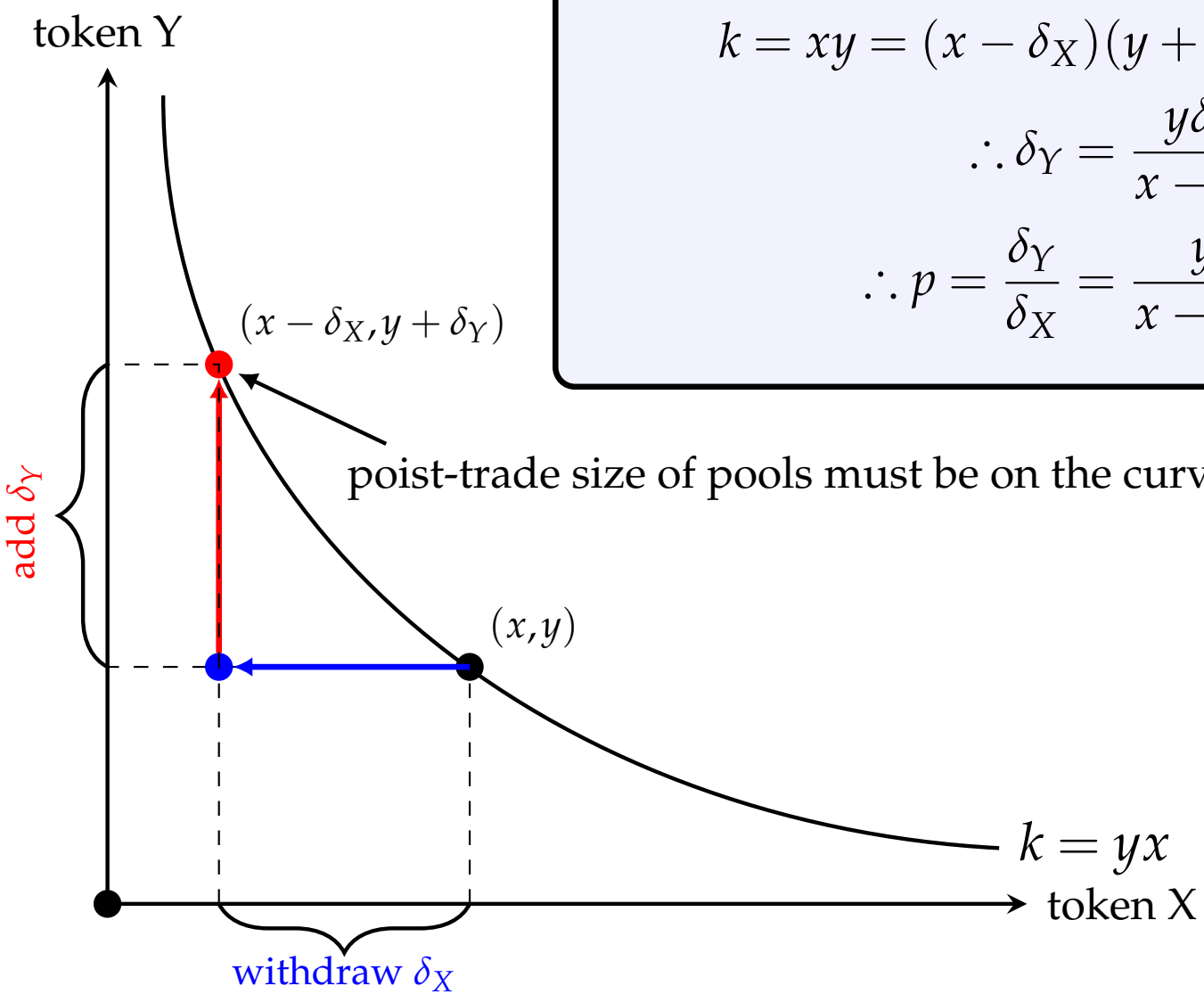
Execution price by CPMM

CPMM pricing

$$k = xy = (x - \delta_X)(y + \delta_Y)$$

$$\therefore \delta_Y = \frac{y\delta_X}{x - \delta_X}$$

$$\therefore p = \frac{\delta_Y}{\delta_X} = \frac{y}{x - \delta_X}$$



Profit for liquidity providers

- Why they provide liquidity?
- Other rewards: liquidity reward (fees), staking reward, governance right

Generality: Constant Function Market Makers

CFMM sets the price so that

$$f(x,y) = f(x - \delta_X, y + p\delta_X)$$

- CPMM $f(x,y) = xy$; CMMM $f(x,y;\omega) = x^{\omega}y^{1-\omega}$
- With some regularity conditions, trade with size $\delta \neq 0$ is executed at

$$p(\delta,x) = \frac{1}{\delta}\int_0^{\delta} \frac{f_2(h(x-\tilde{\delta}),x-\tilde{\delta})}{f_1(h(x-\tilde{\delta}),x-\tilde{\delta})}d\tilde{\delta}$$

- p is increasing and convex in δ
- $\frac{dp}{d\delta}$ is decreasing in x

AMM and asymmetric information

Informed trading: "impermanent loss"

- Buy (resp. sell) when $v = +\sigma$ (resp. $-\sigma$) $\Rightarrow$ always withdraw more valuable asset
  $\Rightarrow$ pools' value deteriorates
  $\Rightarrow$ LPs lose money

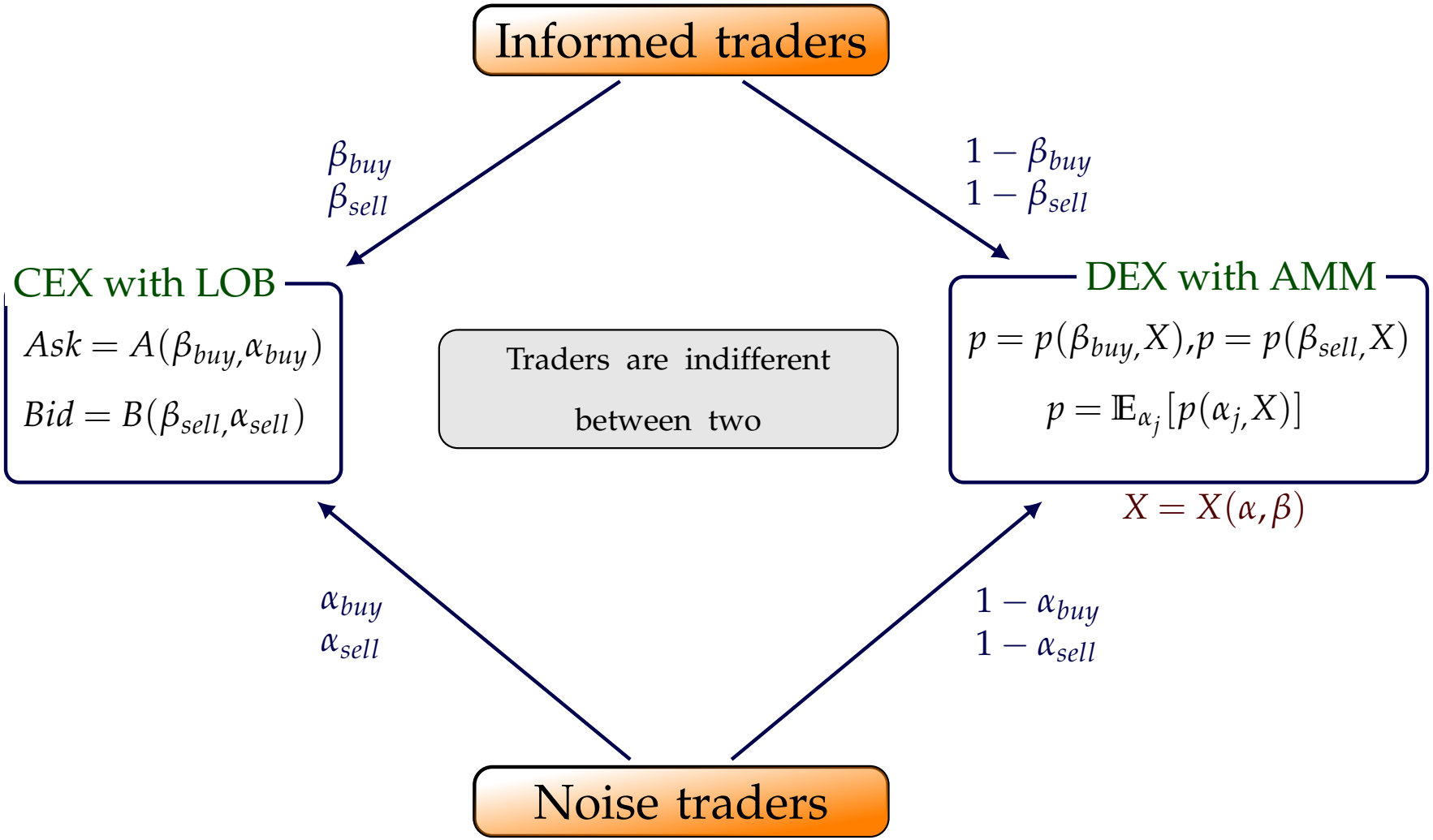
Noise trading:

- Random trading with convex pricing p
  $\Rightarrow$ always improves pools' value (Jensen's ineq.) noise
  $\Rightarrow$ LPs earn money

Result:

- Signal-to-noise ratio negatively affects the size of liquidity pools

Equilibrium with AMM and LOB



How traders are differentiated?

Informed traders on DEX:

- Each informed trader anticipates trading direction of other inf. traders
  $\Rightarrow$ tend to cluster on the same side of the market
  $\Rightarrow$ cause a large shift in liquidity pools (= large price cost)

Noise traders on DEX:

- Random trading volume and direction
  $\Rightarrow$ buy and sell orders tend to be netted out
  $\Rightarrow$ price impact is limited

Result:

- Informed tends to be more sensitive to a change in DEX liquidity
- DEX liquidity $\uparrow \Rightarrow$ informed enjoys it more than noise trdr $\Rightarrow$ CEX liquidity improves (bid-ask shrinks)
- DEX and CEX complement each other in liquidity

Asymmetric price impact

AMM pricing is convex

- Buy and sell market orders bear asymmetric price impact
  - buying 1 unit triggers a larger shift than selling 1 unit

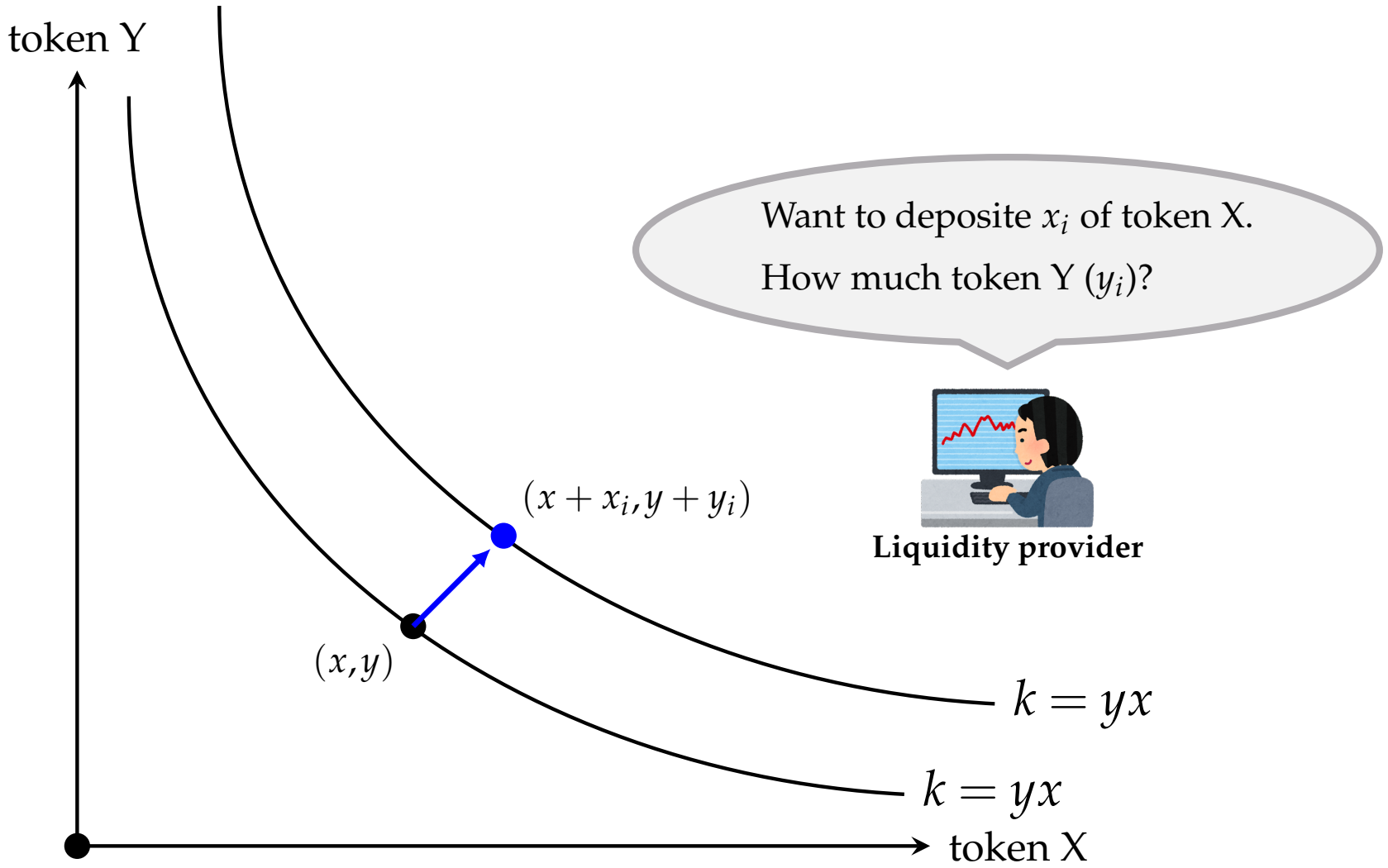
1. Informed $\times$ buy is most reactive to exogenous variations
2. By non-arb., trading cost on CEX becomes asymmetric for buy and sell
   - Bid and ask are not symmetric around $\mathbb{E}[\tilde{v}]$
   - Midpoint of bid and ask does not work as a proxy of efficient price

$$\frac{\text{Ask} + \text{Bid}}{2} \neq \mathbb{E}[\tilde{v}]$$

Takeaway

- DEXs have proposed/implemented new market-making structure called AMMs
- A couple of papers have studied how AMMs behave
  - perfect information: Angeris, Kao, Chiang, Noyes and Chitra (2019), Park (2021),
  - asymmetric information: Aoyagi (2020), Capponi and Jia (2021)
- In reality, CEXs operate in parallel with DEXs
  - Lehar and Parlour (2021) compare LOB and AMM separately
  - This paper considers the coexisting LOB and AMM, trader differentiation, and liquidity spillover effect

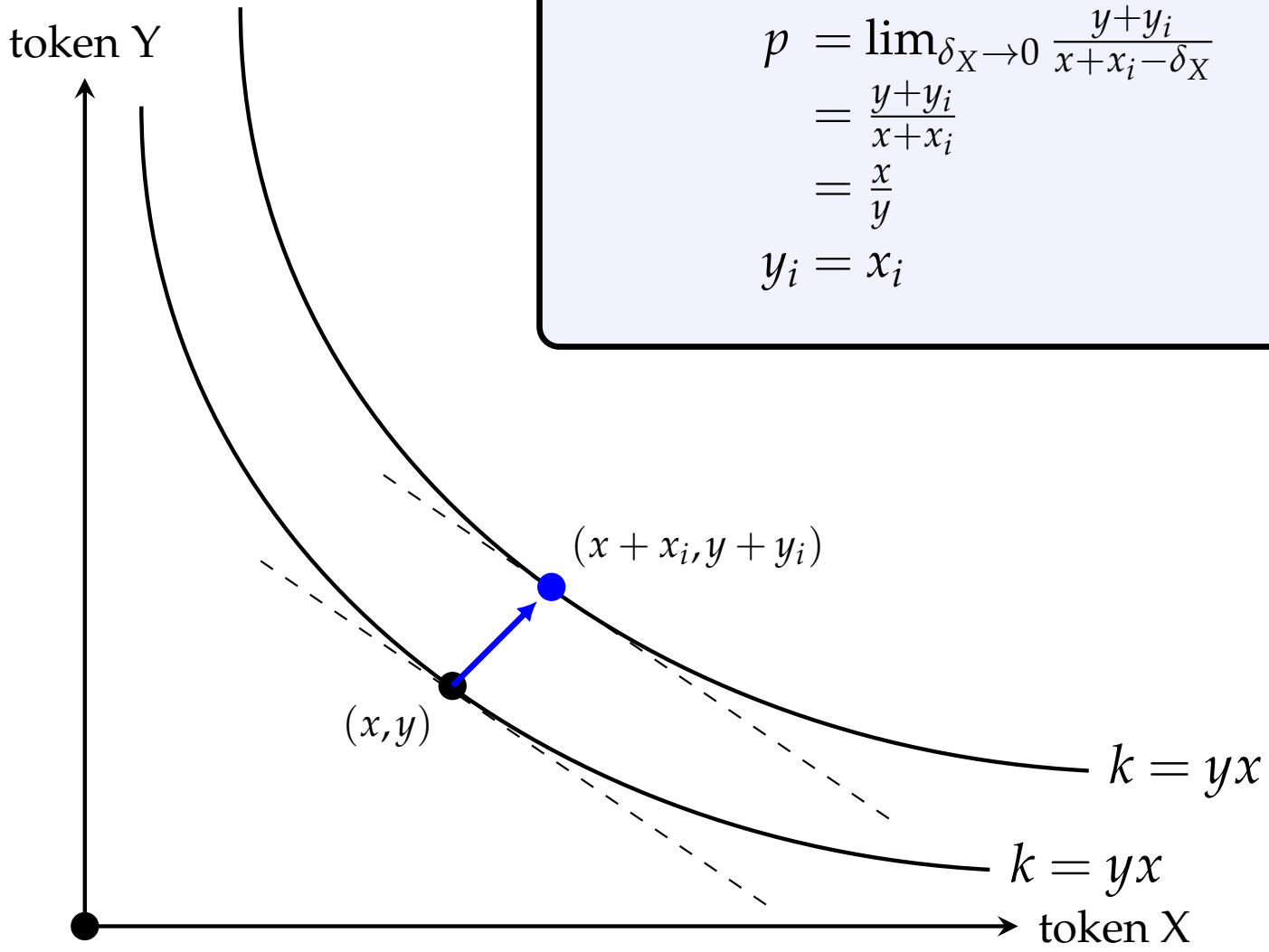
Liquidity provider



Marginal price should not change

Marginal price

$$
\begin{aligned}
p &= \lim_{\delta_X \to 0} \frac{y+y_i}{x+x_i-\delta_X} \\
&= \frac{y+y_i}{x+x_i} \\
&= \frac{x}{y} \\
y_i &= x_i
\end{aligned}
\tag{1}
$$



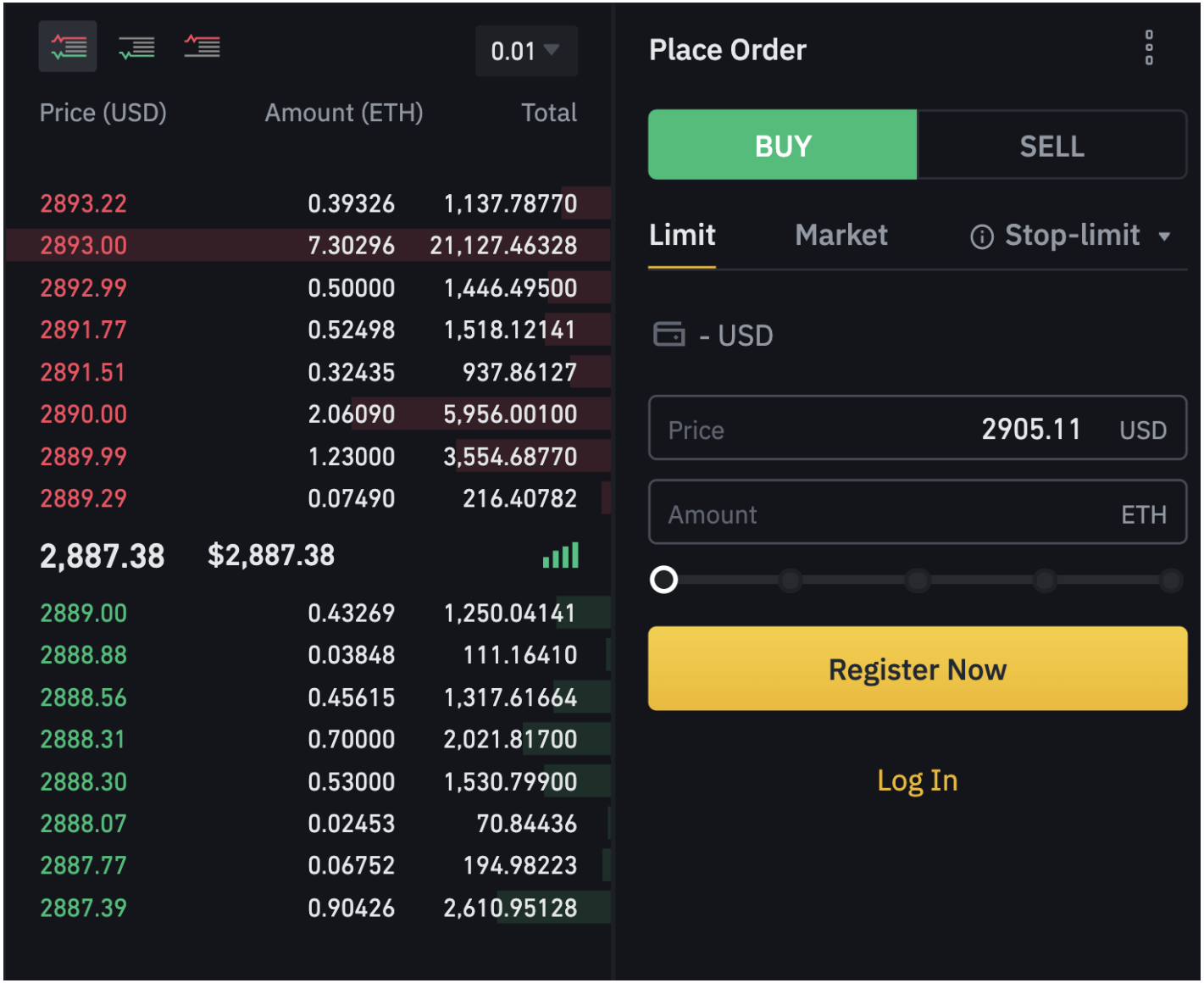
Some jargons

- Market makers: those who provide liquidity by submitting limit orders
- Limit order: quote (price and quantity) at which a market maker is willing to trade
  - bid (ask) = price to buy (sell)
  - limit orders are stored on a book (limit order book) and wait to be matched
- Market order (marketable limit order): limit order with infinite bid and ask prices ("want to trade whatsoever")

Limit order book

"Ask"

"Bid"



If noise trading is $\tilde{\Delta}x = \pm\Delta x$, pools value improves