Empirical Analysis of Corporate Tax Reforms: What is the Null and Where Did It Come From?*

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Abstract

Absent theoretical guidance, empiricists have been forced to rely upon numerical comparative statics from constant tax rate models in formulating testable implications of tradeoff theory in the context of natural experiments. We fill the theoretical void by solving in closed-form a dynamic tradeoff theoretic model in which corporate taxes follow a multiple-state Markov process with exogenous rate changes. We simulate ideal difference-in-differences estimations, finding that constant tax rate models offer poor guidance regarding testable implications. While constant rate models predict large symmetric responses to rate changes, our model with stochastic tax rates predicts small, asymmetric, and often statistically insignificant responses. Even with very long regimes (one decade), under plausible parameterizations, the true underlying theory – that taxes matter – is incorrectly rejected in about *half* the simulated natural experiments. Moreover, tax response coefficients are actually smaller in simulated economies with larger tax-induced welfare losses.

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1 Introduction

In the field of corporate finance, understanding whether, and how much, taxes affect leverage is a first-order question. According to the canonical trade-off theory, firms weigh tax advantages of debt against bankruptcy and distress costs in optimizing leverage. Therefore, failure to reject the null hypothesis of zero tax effect would generally be viewed as constituting strong empirical evidence against trade-off theory. For example, in making his case against the trade-off theory, Myers (1984) writes, "I know of no study clearly demonstrating that a firm's tax status has predictable, material effects on its debt policy."

Gordon and MacKie-Mason (1990) offer what has become a consensus view regarding the empirical evidence on the effect of taxes on leverage, writing, "Past work has presumed that taxes play an important role in these decisions. If so, then the extensive changes in the tax law that were enacted in 1986 should have led to noticeable changes in these decisions... We find that the actual change in debt-to-value ratios has been substantially smaller than the models predict." The finding of a weak corporate leverage response to tax changes has been confirmed over much longer sample windows. For example, in their study of long-term changes in U.S. capital structures, Graham, Leary, and Roberts (2015) state, "Corporate taxes underwent 30 revisions over the past century and increased from 10% to 52% between 1920 and 1950. Yet we find no significant time-series relation between taxes and the margin between debt usage and common equity, in large part because of a near decade-long delay in the response of leverage to tax changes."[1] More generally, Slemrod (1992) states, "I do not mean to suggest that, for all aspects of behavior, all economists have lowered toward near zero their best guess of the tax elasticity... However, I do believe that this is an appropriate generalization about how views have changed in the past decade."

Reflecting the primacy currently placed upon avoiding endogeneity bias, many would argue that the chief barrier to correct statistical inference regarding the effect of taxes on corporate leverage is the fact that tax changes are endogenous governmental decisions. Although we do not dispute the importance of identifying sources of exogenous variation, in this paper we demonstrate quantitatively another fundamental constraint on our ability to use empirical tests in order to reach valid conclusions here: *We actually have no idea what kind of regression coefficients to expect from*

[1] In contrast, Graham (1999) does document a cross-sectional relationship between leverage and tax shield value.

trade-off theoretic firms even if we are given ideal natural experiments.

To see the point, suppose one were actually able to identify the type of ideal experiment that today's best empiricists seek out, a set of corporations hit with an exogenous tax rate change sitting alongside an ideal control group unaffected by the tax change. This still leaves open some basic questions: Do we actually have an informed prior regarding what coefficient magnitudes to expect? Should our interpretation of the evidence hinge upon the sign of the tax rate change? Finally, do we actually know the properties of the estimator here, such as the probability of Type II errors?

A broad range of sophisticated quantitative models have been used to motivate empirical tests for tax effects, e.g. Fisher, Heinkel and Zechner (1989), Leland (1994), and Goldstein, Ju and Leland (2001). The problem is not that empiricists are not sufficiently careful, or that they have failed to pay sufficient attention to the theoretical models. Rather, *the problem is that existing models do not actually speak to the empirical tests.*

In practice, it is common to proceed to the data using comparative statics analysis to formulate testable hypotheses regarding how firms should respond to tax rate changes. For example, Auerbach and Slemrod (1997) state, "we must specify why firms do not finance exclusively with debt... There are a variety of potential explantations in the literature, and they may give rise to different comparative static predictions regarding the impact of TRA86." Based on a simple stylized model, Welch (2010, 2013) writes, "The comparative statics are simple and obvious–The firm should have more debt if the interest rate is higher, if the corporate tax rate is higher, and if the cost of distress is lower." Leland (1994) reaches the same conclusion regarding comparative statics using his contingent-claims model. Goldstein, Ju and Leland (2001) present a similar numerical comparative statics result in their rich dynamic model. With such comparative statics results serving as a backdrop, the consensus interpretation of the empirical evidence looks uncontroversial: The theoretical canon predicts large symmetric leverage responses to tax rate changes, so apparently tax motives are a second-order effect, if they are an effect at all.

However, introspection suggests there is a wide gap between the empirical tests and models. After all, the econometrician is exploiting data in which she can observe the tax rate changing over time. In contrast, in existing theoretical models, the corporate tax rate is a parameter. In other words, the econometrician and the corporations inside the model occupy different worlds. The econometrician exploits real-world data with periodic changes in tax shield value. Yet the modeled firms expect tax shield value to remain constant forever, until the next tax rate innovation, after which they again expect the tax rate to remain constant forever.

The first contribution of our paper is to develop a theoretical model that remedies this internal inconsistency, narrowing the gap between model and data. Specifically, we extend the dynamic capital structure model of Goldstein, Ju and Leland (2001) to incorporate a stochastic corporate tax rate, taking the form of a multiple-state Markov chain with arbitrary expected regime durations. Despite the substantial increase in complexity, all components of firm value are derived in closed-form. Since there is no need for numerical solutions for value functions, the model represents a tractable and transparent framework for empiricists to contemplate various empirical thought-experiments, as we do below.

We use the simulated model as an ideal laboratory for conducting tests of empirical tests. To begin with, we focus our attention on the likely probability of Type II error in real-world empirical studies. Such studies generally focus on changes in corporate leverage ratios in response to small real-world tax rate changes. For example, even with the fundamental tax overhaul of the Tax Reform Act of 1986 (TRA86), Gordon and MacKie-Mason (1990) estimate a change in effective tax shield value of only 2.5 percentage points. To be conservative, we consider a larger tax rate change of 4 percentage points, and focus on a rate increase (from 36% to 40%), where statistical power is higher. Further, we consider long (10 year) expected tax regime lives. Here, if we first compare leverage across economies with constant tax rates of 36% and 40%, respectively, the average difference in leverage ratios is 141 basis points (0.0141). In contrast, the average DD regression coefficient on simulated data is only 45 basis points (0.0045). That is, the coefficient estimate obtained under a relatively long, yet realistic, regime life of one decade is only 32.2% of the average effect that would be expected by using the constant tax rate model. Further, the tax elasticity of leverage implied by a constant tax rate model is 1.20 whereas the implied elasticity in simulated data is only 0.35, a downward bias of 68 percent. Worse still, in about 45% of the simulated tax reforms studied, one fails to find a statistically significant coefficient at the 5% level of confidence. More generally, if the expected regime life falls just below one decade, the majority of simulated experiments fail to reject the null that taxes do not matter for leverage decisions– despite the fact that taxes are *the* cause of corporate leverage in our simulated economy. Importantly, we conduct a battery of sensitivity analyses, finding that this Type II error problem is robust to alternative parameterizations.

What explains these troubling findings? A number of mechanisms are at work. First, due to transactions costs, only a subset of firms react quickly to tax rate changes. Second, transient tax rates cause firms to be less aggressive conditional upon engaging in financial restructuring. Finally, although we impose common aggregate shocks on all firms, treated and control groups are not identical, as they necessarily differ in terms of idiosyncratic shocks. Here it is worth noting that we do not claim to be the first to point out these three challenges to inference in the context of capital structure research. Rather, we merely claim to be the first to develop an operational model incorporating the underlying frictions, allowing us to assess their quantitative importance in the context of state-of-the-art empirical tests featuring ideal exogenous policy changes. Our findings show that rather than being included in the boilerplate list of caveats, these concerns may be fatal, leading to frequent false falsifications of a correct theory or erroneous conclusions regarding whether, and how much, taxes matter for capital structure decisions.

We also consider the implications for the empirical analysis of social welfare. In our economy, the corporate income tax is only cause of all debt and therefore the welfare loss that could be measured as bankruptcy costs. As one would suspect, the present value of welfare loss is naturally increasing in the model's bankruptcy cost parameter. However, we find that the DD regression coefficient in simulated data is actually *decreases* with the bankruptcy cost parameter. Intuitively, higher bankruptcy costs diminish the firm's willingness to increase leverage in response to tax rate increases, lowering the regression coefficient. Ironically then, the regression output suggests that taxes are less important precisely when the tax system is responsible for extremely high welfare losses. Thus, aside from Type II errors, empirical tests, even those hard-wired to be clean, can be highly misleading as indicators of tax-induced welfare losses. In particular, these results stand in stark contrast to the conventional wisdom mapping elasticity estimates to welfare analysis. For example, de Mooij (2011) argues, "A key question underlying these discussions of debt bias is how much taxes actually matter for corporate debt policy. If behavioral responses were only weak, the welfare gains of corporate income tax reforms will be small." Our results show that the conventional wisdom should be revisited, using an operational model that directly speaks to the mapping between empirical evidence and welfare analysis.

We next analyze the question of whether responses to tax rate changes should be expected to be symmetric in real-world data. The workhorse approach in the literature, numerical comparative statics, mechanically predicts symmetric responses. In contrast, our model shows that financial policy should be more responsive to tax rate increases than to decreases. Intuitively, if the tax rate increases, there is a larger tax benefit associated with performing a leveraged recapitalization, one that is often sufficiently large to swamp the underwriting fees it will incur. In other words, when the tax rate is high, total firm value is more sensitive to the coupon level, and so there is more to be gained from rebalancing from a low to higher coupon. Conversely, if the tax rate decreases, the tax benefit to debt falls, so the firm is less willing to incur the underwriting fees associated with a new debt flotation.

In simulated data, we find that this asymmetry manifests itself in the differential response of firms to tax increases and tax decreases. This differential response is increasing in the size of the tax reform. For example, for very small tax reforms (a 1% change in the effective tax rate), on average firms respond to a tax increase by increasing their leverage on average by 4% more compared to their response to a tax decrease of the same size. However, for large tax reforms (a 15% change in the effective tax rate), the response to a tax increase is larger by a third.

We turn now to related literature. Danis, Rettl and Whited (2014) provide empirical evidence in favor of lumpy leverage adjustment models, such as we present here, documenting that market leverage decreases for several quarters for those firms that subsequently engage in large deliberate leverage increases. The most direct support in favor of our model is offered by Heider and Ljungqvist (2015) who use difference-in-differences estimation to document a positive response of corporate leverage to increases in state income tax rates, especially for more profitable and higher rated firms, but find zero response to decreases in tax rates. The difference-in-differences methodology is also applied by Panier, Perez-Gonzalez and Villanueva (2015) who document an increase in equity usage in conjunction with a novel Belgium tax reform allowing deductibility of notional interest on equity capital. Our model suggests that the causal effect of taxation on capital structure must be strong indeed in order to consistently generate statistically significant coefficients.

Hennessy and Strebulaev (2017) consider a stylized model with instantaneous investment. In their baseline setting, with quadratic adjustment costs, one would always find statistically significant responses to policy variable changes, since there is no region of optimal inaction. Further, in that paper, the focus is on the severe challenges to inference arising when the policy variable can take on more than two states. As they show, in such a setting, it is possible to observe attenuation, overshooting, and sign reversals relative to the underlying comparative statics. In the present paper's binary setting, the problem is always one of economic and statistical attenuation. The present paper considers a simpler policy generating process while tackling a much more complex optimization and pricing problem. While the general message of the papers is the same in that they call for a tighter nexus between models and empirical tests, with greater attention devoted to the policy generating process, there are two key differences between the present paper and that of Hennessy and Strebulaev. First, Hennessy and Strebulaev do not present a model of optimal capital structure in the presence of taxes, issuance costs and bankruptcy costs. Rather, their model treats the accumulation of a generic stock variable, such as real capital, under linear-quadratic adjustment costs. Second, Hennessy and Strebulaev focus on bias in estimates, and are silent on the issue of statistical significance. In stark contrast, the present paper contains a focused analysis of the potential for Type II errors in tests of the trade-off theory.

The rest of the paper is organized as follows. Section 2 develops the model. Section 3 discusses the parameterization and presents a standard numerical comparative statics exercise to set a benchmark. Section 4 describes the simulation procedure and empirical tests. Section 5 examines statistical power of empirical tests. Section 6 analyzes asymmetric tax responses. Section 7 analyzes the relationship between regression coefficients and welfare loss. Section 8 discusses implications for the design of empirical tests. Section 9 concludes. The Appendix lists the firm value components and derives the price of primitive claims under regime shifts.

2 Model

This section develops a model of capital structure decisions in an economy in which corporate income tax rates are stochastic. We build on the standard contingent-claims framework of Leland (1994) where firms face a corporate income tax, with bankruptcy being costly. Departing from Leland, firms can increase leverage dynamically, with each new bond flotation incurring proportional issuance costs. In these respects, our model follows a rich literature analyzing dynamic financing decisions, e.g. Fischer, Heinkel, and Zechner (1989), Goldstein, Ju, and Leland (2001), and Strebulaev (2007). Importantly, these models assume tax rates are constant.

2.1 EBIT and tax rate dynamics

Time is continuous and the horizon is infinite. There is a risk-free asset with interest rate $r > 0$. The economy is populated by N firms, each of which has monopolistic access to a project that generates an instantaneous flow of Earnings Before Interest and Taxes (EBIT). Each firm n has an EBIT process evolving as a geometric Brownian motion under the pricing measure, with:

$$\frac{dX_t^n}{X_t^n} = \mu dt + \sigma dZ_t^n.$$

In the preceding equation $X_0^n = 1$, while Z_t^n is a standard Brownian motion defined on a filtered probability space $(\Omega, \mathcal{F}, \mathbb{Q}, (\mathcal{F}_t)_{t\geq 0})$. The parameters μ and σ represent the instantaneous risk-neutral drift and volatility of EBIT, respectively. For notational simplicity, the superscript n is dropped when obvious.

There is a linear corporate income tax, with the tax rate evolving as a Markov chain. In the interest of tractability, we consider that there are two tax rate states, $i \in \{l, h\}$.[2] For example, one can think of the model as approximating an economy with a two-party system, e.g. the U.S., where one party favors a relatively high tax rate, and the other a relatively low tax rate. In reality, the tax advantage to debt varies with complex tax code provisions such as depreciation schedules, tax credits, loss limitations, and alternative minimum taxes. Since we abstract from such complications, one should think of the tax rate as capturing the effective debt tax shield. Of course, this represents only an approximation of reality since historically the effective debt tax shield has taken on multiple values. For example, Graham, Leary and Roberts (2015) document that the effective debt tax shield has taken on more than ten different values over the past century. The conclusion of our paper discusses the potential economic and statistical implications that would likely arise in a model featuring multiple tax states.

The process $\nu_t \in \{l, h\}$ denotes the current tax state. In state i, the corporate income tax rate is τ_i. It is assumed $0 \leq \tau_l < \tau_h \leq 1$. The tax rate switches from τ_i to τ_j with instantaneous probability $\lambda_i dt$. The parameter λ_i, $\lambda_i > 0$, therefore determines the expected time the tax rate will occupy regime i. Specifically, if the current tax regime is i, then the expected remaining occupation time

[2] We have also extended our model to generic N-state tax regimes, with the substantial loss of tractability. The details available upon request.

in that regime is equal to $1/\lambda_i$. The model therefore converges to a constant tax rate model if one were to let each transition rate tend to zero.

In reality, empiricists exert considerable energy trying to identify orthogonal tax rate changes, with the concern being that causal inference will be contaminated if the government changes the tax rate in response to changes in economic conditions. In our model, endogeneity is ruled out by construction, as we consider that the tax rate process is independent of each EBIT process.

2.2 Unlevered firm value

To understand how stochastic tax rates affect valuation, consider first the valuation of an unlevered firm. The value of unlevered assets at time t, denoted A_t, is equal to the present value of expected future after-tax earnings. This value depends upon current EBIT value, as well as the current tax regime τ_t. We have:

$$A(x,i) = \mathbb{E}\left[\int_0^\infty e^{-rs}(1-\tau_s)X_s ds \Big| X_t = x, \nu_t = i\right].$$

This value function must satisfy the following system of ordinary differential equations (ODEs) which demand that at each instant the expected holding return is just equal to the opportunity cost r:

$$\begin{aligned} rA(x,l) &= \mu x A_x(x,l) + \frac{1}{2}\sigma^2 x^2 A_{xx}(x,l) + \lambda_l[A(x,h) - A(x,l)] + (1-\tau_l)x; \qquad (1) \\ rA(x,h) &= \mu x A_x(x,h) + \frac{1}{2}\sigma^2 x^2 A_{xx}(x,h) + \lambda_h[A(x,l) - A(x,h)] + (1-\tau_h)x. \end{aligned}$$

The first two terms on the right hand side of both equations reflect expected capital gains due to the instantaneous evolution of EBIT, computed via Ito's lemma. The third term on the right hand side of both equations captures the discrete jump in unlevered asset value that will occur in the event of a tax rate transition. The final term in both equations captures the dividend flow. By solving the preceding system, one obtains an analytic expression for unlevered asset value, with value equal to a regime-contingent constant times x:

$$A(x,i) = \left[\frac{1-\tau_i}{r-\mu} + \frac{\lambda_i(\tau_i - \tau_j)}{(r-\mu)(r-\mu+\lambda_l+\lambda_h)}\right]x. \qquad (2)$$

The first term in the preceding equation captures unlevered asset value under a constant tax rate. The second term captures the value adjustment attributable to potential tax rate transitions.

2.3 Dynamic capital structure policy

This subsection considers dynamic capital structure policy. Following standard EBIT-based capital structure models (e.g., Goldstein, Ju, and Leland 2001), EBIT is split between equityholders, debtholders, and the government. More specifically, suppose the firm issues debt promising lenders an instantaneous coupon payment c. Then after paying corporate taxes and lenders, equityholders receive the residual amount $(1-\tau_i)(X_t-c)$ as a dividend. It follows that debt coupons increase the net flow to equity and debt by $\tau_i c$, coming at the expense of the tax collector.

Consider now capital structure dynamics. Suppose the tax state at date zero is i. Then at this point in time equityholders will choose a state-contingent coupon c_i to be written into the first bond issued. In order to ensure that re-levering is done optimally, maximizing total firm value, the debt contract will also specify a pair of mandatory refinancing thresholds (γ_i^l, γ_i^h).[3] Here the subscript indexes the tax state at the time the bond was issued, with the superscripts denoting the two potential tax states post-issuance. For example, if the tax regime at some future date, post-issuance, is equal to j, then the corporation is obligated to call the outstanding bond if EBIT is greater than or equal to γ_i^j. By writing the call thresholds into the bond at the time of issuance, value losses due to debt-equity agency conflicts regarding call timing are optimally avoided, as in Goldstein, Ju and Leland (2001). As in Goldstein, Ju and Leland, we also consider debt contracts, where $c < 1 < \min\left\{\gamma_i^j, \gamma_j^i\right\}$. The upper bound on the coupon ensures the firm will not default immediately, which is clearly suboptimal. The lower bound on the call threshold prevents the firm from circumventing bankruptcy costs by pre-committing with its existing lenders to do a distressed equity issuance to finance the retirement of outstanding debt replaced by debt with a lower coupon. The contractual call price is par value.[4]

Once it has called its outstanding bond, the firm is free to issue another bond. Refinancing is costly since new bond flotations force the firm to incur issuance costs equal to a fraction q of the proceeds raised. As is well known in the literature, in such a setting the firm will prefer to refinance only if EBIT is sufficiently high to justify incurring the issuance costs.

[3]We consider public debt and rule out renegotiation to lower the debt coupon. Renegotiation, which might be difficult to achieve given multiple debtholders, would be expected in response to a tax rate decrease, increasing firm responsiveness.

[4]Instead of contracting on the call threshold, state-contingent call prices could be set to achieve a similar outcome.

At all future restructuring dates, the so-called scaling property holds in that each element of the relevant tax-state-contingent debt contract will scale up in proportion to the increase in EBIT. To understand the mechanics here, for each possible date zero state $i \in \{l, h\}$, let the triple $\Omega_i \equiv (c_i, \gamma_i^l, \gamma_i^h)$ refer to the debt contract that would be optimally chosen by the firm at this point in time. Suppose then the date zero tax state was i, with the current tax state being j. And suppose further that the first upward restructuring threshold has just been hit, with $X_t = \gamma_i^j$. Then the firm will scale up the "baseline" state j debt contract in proportion to the increase in EBIT. That is, the new optimal debt contract will be:

$$\gamma_i^j \Omega_j = (\gamma_i^j c_j, \gamma_i^j \gamma_j^l, \gamma_i^j \gamma_j^h).$$

Of course, it is possible that instead of just reaching the restructuring threshold, through the path-continuous diffusion of the geometric Brownian motion EBIT, there will instead be a jump into restructuring due to a tax rate transition, in which case the debt contract would scale up by $x > \gamma_i^j$, a fact we will account for in the valuations.

Default occurs if EBIT falls below the promised coupon c_i on the current bond outstanding. That is, default results from illiquidity. Liquidity default captures the notion that a financially distressed firm may find it impossible to raise equity externally. An alternative assumption, not adopted here, would be to model default as a stopping problem assuming instead that the firm enjoys frictionless access to external equity injections. Solving the associated stopping time problem would complicate the analysis considerably with little relation to our main argument. In reality, defaults are driven by some combination of illiquidity and insolvency, with the dividing line between the two often difficult to distinguish.

In the event of default, the firm will be liquidated by the bankruptcy courts, with debtholders receiving unlevered asset value net of liquidation costs. Liquidation costs absorb a fraction α of unlevered asset value.

2.4 Contingent claims

To express the components of levered firm value compactly, we introduce several contingent claims. For a given debt contract $\Omega = (c, \gamma^l, \gamma^h)$, these contingent claims pay off only under specific conditions. In each claim value expression, x denotes the current EBIT value and j denotes the current tax state. We have the following contingent claims.

1. *Contingent Down Claim* with price $d^i(x, j, \Omega)$: This claim pays one if and when default occurs in state i unless it has been knocked out by call or by default in the other tax state j.

2. *Adjusted Contingent Up Claim* with price $m^i(x, j, \Omega)$: This claim pays one if and when call occurs in state i unless it has been knocked out by default or by call in the other tax state j. This claim will be useful in our accounting for cases in which there is a gradual transition into restructuring through the path-continuous diffusion of EBIT.

3. *Contingent Up Claim* with price $u^i(x, j, \Omega)$: This claim pays $x(T)/\gamma^i$, where T is the time of the call, if and when call occurs in state i unless it has been knocked out by default or by call in the other tax state j. This claim will be useful in our accounting for jumps into restructuring resulting from tax rate transitions.

4. *Contingent Occupation Claim* with price $a^i(x, j, \Omega)$: This claim delivers an instantaneous unit flow of dt whenever the tax state is equal to i unless it has been knocked out by default or call.

In the Appendix we present closed-form expressions for each of these contingent claim prices by solving systems of two simple ODEs.

2.5 Levered firm value

From now on, we fix a pair of debt contracts (Ω_l, Ω_h) and compute levered firm value utilizing the contingent claims defined in the preceding subsection and priced in the Appendix. The levered firm value can be decomposed either into standard real-world financial claims (debt and equity) or into the following four components: (i) unlevered asset value plus (ii) tax benefits less (iii) bankruptcy costs less (iv) issuance costs. Each of the components is defined as the present value of corresponding future cash flows. Initially, we will utilize the latter decomposition of firm value, since equity value can then be readily computed as the residual of total firm value less debt value.

The unlevered asset value has a linear closed-form solution, as shown in equation (2). By definition, this value is independent of the choice of debt contracts (Ω_l, Ω_h). On the other hand, the other three components of firm value depend on (Ω_l, Ω_h) in a complex manner. Conveniently, it is possible to characterize these values analytically by exploiting simple recursive relationships. The next three subsections derive the value of these three components.

2.6 Bankruptcy costs

Let $BC(x, i, \Omega_j)$ denote the present value of bankruptcy costs when EBIT is x, the current tax state is i, and the outstanding debt contract is Ω_j. This valuation considers the costs of any default, not just default on the first bond issued, accounting for the fact that the firm can potentially issue an infinite sequence of bonds in the unlikely event of ever-growing EBIT. At each instant prior to the first default or first restructuring, we have the following analytic expressions for bankruptcy costs:

$$\begin{aligned} BC(x, l, \Omega_i) &= d^l(x, l, \Omega_i)\alpha A(c_i, l) + d^h(x, l, \Omega_i)\alpha A(c_i, h) \\ &\quad + u^l(x, l, \Omega_i)\gamma_i^l BC(1, l, \Omega_l) + u^h(x, l, \Omega_i)\gamma_i^h BC(1, h, \Omega_h); \end{aligned} \tag{3}$$

$$\begin{aligned} BC(x, h, \Omega_i) &= d^l(x, h, \Omega_i)\alpha A(c_i, l) + d^h(x, h, \Omega_i)\alpha A(c, h) \\ &\quad + u^l(x, h, \Omega_i)\gamma_i^l BC(1, l, \Omega_l) + u^h(x, h, \Omega_i)\gamma_i^h BC(1, h, \Omega_h). \end{aligned} \tag{4}$$

In each equation, the first two terms on the right hand side capture the present value of bankruptcy costs under the scenario that default occurs before the first upward levered recapitalization. Because unlevered asset value differs across the two tax states, these cases must be considered separately. The last two terms capture the scaling upward of the present value of bankruptcy costs if the first refinancing occurs prior to a default. The two terms account for the two potential tax states at the time of refinancing.

Conveniently, we can exploit recursive relationships to compute the present value of bankruptcy costs by evaluating the preceding equations at the two possible date-zero configurations of the exogenous state variables. Because the initial EBIT is equal to one and only two potential tax states can occur, date-zero bankruptcy costs must satisfy:

$$\begin{aligned} BC(1, l, \Omega_l) &= d^l(1, l, \Omega_l)\alpha A(c_l, l) + d^h(1, l, \Omega_l)\alpha A(c_l, h) \\ &\quad + u^l(1, l, \Omega_l)\gamma_l^l BC(1, l, \Omega_l) + u^h(1, l, \Omega_l)\gamma_l^h BC(1, h, \Omega_h); \end{aligned} \tag{5}$$

$$BC(1,h,\Omega_h) = d^l(1,h,\Omega_h)\alpha A(c_h,l) + d^h(1,h,\Omega_h)\alpha A(c_h,h) + u^l(1,h,\Omega_h)\gamma_h^l BC(1,l,\Omega_l) + u^h(1,h,\Omega_h)\gamma_h^h BC(1,h,\Omega_h). \quad (6)$$

As shown in the Appendix, the two preceding linear equations can be easily solved in order to obtain analytic expressions for the present value of bankruptcy costs evaluated at the initial date. Further, these expressions can then be substituted back into the generic bankruptcy cost equations (3–4) to evaluate bankruptcy costs at each instant prior to the first default or restructuring.

It is clear from the preceding equations that the present value of bankruptcy costs depend on the terms of each tax regime-contingent debt contract (Ω_l, Ω_h). To emphasize this dependence, we denote the date zero bankruptcy costs evaluated at each tax regime by $\{BC_0^l(\Omega_l,\Omega_h), BC_0^h(\Omega_l,\Omega_h)\}$.

2.7 Issuance costs

The present value of future issuance costs is denoted $IC(x,i,\Omega_j)$. This term considers the entire sequence of refinancing possibilities in the future. At each instant prior to the first default or restructuring, we have the following analytic expressions for issuance costs:

$$IC(x,l,\Omega_i) = u^l(x,l,\Omega_i)\gamma_i^l[qD(1,l,\Omega_l) + IC(1,l,\Omega_l)] + u^h(x,l,\Omega_i)\gamma_i^h[qD(1,h,\Omega_h) + IC(1,h,\Omega_h)]; \quad (7)$$

$$IC(x,h,\Omega_i) = u^l(x,h,\Omega_i)\gamma_i^l[qD(1,l,\Omega_l) + IC(1,l,\Omega_l)] + u^h(x,h,\Omega_i)\gamma_i^h[qD(1,h,\Omega_h) + IC(1,h,\Omega_h)]. \quad (8)$$

In the preceding equation, D denotes the market value of debt, a value function to be derived below. The equation reflects the issuance costs associated with the first potential upward restructuring, as well as the scaling up of the present value of issuance costs in the event of such a restructuring. Because the firm issues different amounts of debt depending on the tax state at the time of refinancing, there are two terms on the right hand sides of each equation. Notice, again we are utilizing the scaling property inherent in the model to evaluate the present value of issuance costs at the moment of refinancing. Finally, it is worth noting that $IC(1,i,\Omega_j)$ excludes the initial issuance costs paid at time zero in connection with the first bond flotation. This cost will be accounted for separately in our date zero firm valuation formulas.

As with bankruptcy costs, we can exploit recursive relationships to compute the present value of issuance costs at date zero by evaluating the two preceding equations at the two possible date-zero configurations of the exogenous state variables, EBIT and tax states. We have:

$$\begin{aligned} IC(1,l,\Omega_l) &= u^l(1,l,\Omega_l)\gamma_l^l[qD(1,l,\Omega_l)+IC(1,l,\Omega_l)] \\ &\quad +u^h(1,l,\Omega_l)\gamma_l^h[qD(1,h,\Omega_h)+IC(1,h,\Omega_h)]; \end{aligned} \tag{9}$$

$$\begin{aligned} IC(1,h,\Omega_h) &= u^l(1,h,\Omega_h)\gamma_h^l[qD(1,l,\Omega_l)+IC(1,l,\Omega_l)] \\ &\quad +u^h(1,h,\Omega_h)\gamma_h^h[qD(1,h,\Omega_h)+IC(1,h,\Omega_h)]. \end{aligned} \tag{10}$$

As shown in the Appendix, this pair of linear equations can be solved easily in order to obtain analytic expressions for issuance costs evaluated at the initial date. Further, these expressions can then be substituted back into the generic issuance cost equations (7–8) to compute the present value of issuance costs at each instant prior to the first default or restructuring. Of course, these present values will depend upon the chosen configuration of debt contracts. To emphasize this dependence, we denote the date-zero issuance costs under each tax regime as $\{IC_0^l(\Omega_l,\Omega_h), IC_0^h(\Omega_l,\Omega_h)\}$.

2.8 Tax benefits

The present value of tax benefits is denoted by $TB(x,i,\Omega_j)$. This value accounts for the tax savings generated by the entire sequence of potential debt flotations, not just that associated with the first bond issued. The tax benefits value function satisfies:

$$\begin{aligned} TB(x,l,\Omega_i) &= \tau_l c_i a^l(x,l,\Omega_i)+\tau_h c_i a^h(x,l,\Omega_i) \\ &\quad +u^l(x,l,\Omega_i)\gamma_i^l TB(1,l,\Omega_l)+u^h(x,l,\Omega_i)\gamma_i^h TB(1,h,\Omega_h); \end{aligned} \tag{11}$$

$$\begin{aligned} TB(x,h,\Omega_i) &= \tau_l c_i a^l(x,h,\Omega_i)+\tau_h c_i a^h(x,h,\Omega_i) \\ &\quad +u^l(x,h,\Omega_i)\gamma_i^l TB(1,l,\Omega_l)+u^h(x,h,\Omega_i)\gamma_i^h TB(1,h,\Omega_h). \end{aligned} \tag{12}$$

The first two terms on the right hand side of the preceding equations capture the present value of tax savings generated by the first bond flotation. The two last terms capture the present value of all future tax benefits after a first restructuring. Again, the scaling property inherent in the model is exploited here.

Once again, we can exploit recursive relationships to compute present values at date zero. We must simply evaluate the two preceding equations at the two possible date-zero configurations of the exogenous state variables, EBIT and tax states. We have:

$$\begin{aligned} TB(1,l,\Omega_l) &= \tau_l c_l a^l(1,l,\Omega_l) + \tau_h c_l a^h(1,l,\Omega_l) \\ &\quad + u^l(1,l,\Omega_l)\gamma_l^l TB(1,l,\Omega_l) + u^h(1,l,\Omega_i)\gamma_l^h TB(1,h,\Omega_h); \end{aligned} \tag{13}$$

$$\begin{aligned} TB(1,h,\Omega_h) &= \tau_l c_h a^l(1,h,\Omega_h) + \tau_h c_h a^h(1,h,\Omega_h) \\ &\quad + u^l(1,h,\Omega_h)\gamma_h^l TB(1,l,\Omega_l) + u^h(1,h,\Omega_h)\gamma_h^h TB(1,h,\Omega_h). \end{aligned} \tag{14}$$

As shown in the Appendix, this system of linear equations is easily solved to obtain analytic expressions for tax benefits evaluated at date zero. Further, these expressions can then be substituted back into the generic tax benefit equations (11–12) to evaluate tax benefits at each instant prior to the first default or restructuring. Of course, the present value of tax benefits will hinge upon the chosen configuration of debt contracts. To emphasize this dependence, we denote the date zero tax benefits under each tax regime by $\{TB_0^l(\Omega_l,\Omega_h), TB_0^h(\Omega_l,\Omega_h)\}$.

2.9 Optimal capital structure

At time zero, the firm chooses the optimal capital structure. Once the date zero capital structure is optimized, all future capital structures are also optimized since future debt contracts just scale up the date zero contracts in proportion to the EBIT increase. The optimal capital structure maximizes total firm value and thus solves the following program:

$$\begin{aligned} \Omega_l^* &\in \arg\max_{\Omega_l} \quad TB_0^l(\Omega_l,\Omega_h^*) - BC_0^l(\Omega_l,\Omega_h^*) - IC_0^l(\Omega_l,\Omega_h^*) - qD(1,l,\Omega_l); \\ \Omega_h^* &\in \arg\max_{\Omega_h} \quad TB_0^h(\Omega_l^*,\Omega_h) - BC_0^h(\Omega_l^*,\Omega_h) - IC_0^h(\Omega_l^*,\Omega_h) - qD(1,h,\Omega_h). \end{aligned} \tag{15}$$

It is worth recalling that analytic expressions for tax benefits, bankruptcy costs, and issuance costs were derived in the preceding subsections. Note, the final terms in the preceding equations account for issuance costs incurred on the first bond flotation. At any time after date zero, but prior to the first default or restructuring, total firm value F is computed as

$$F(x,i,\Omega_j) = A(x,i) + TB(x,i,\Omega_j) - BC(x,i,\Omega_j) - IC(x,i,\Omega_j).$$

We next derive debt and equity values. Let $D(x, i, \Omega_j)$ denote the value of a bond that was issued in regime j with the current regime being i. We have:

$$\begin{aligned} D(x, i, \Omega_j) &= [a^l(x, i, \Omega_j) + a^h(x, i, \Omega_j)]c_j + [m^l(x, i, \Omega_j) + m^h(x, i, \Omega_j)]D(1, j, \Omega_j) \\ &\quad +(1-\alpha)[d^l(x, i, \Omega_j)A(c_j, l) + d^h(x, i, \Omega_j)A(c_j, h)]. \end{aligned} \tag{16}$$

The first term in the preceding equation captures the present value of coupon payments. The second term captures the scenario under which the bond is called. The last term captures the liquidation value recovered by lenders if default occurs prior to call.

To pin down the date-zero debt pricing functions, we evaluate the above general debt pricing equation at date zero, so that $j = i$ and $x = 1$. We obtain:

$$D(1, i, \Omega_i) = \frac{[a^l(1, i, \Omega_i) + a^h(1, i, \Omega_i)]c_i + (1-\alpha)[d^l(1, i, \Omega_i)A(c_i, l) + d^h(1, i, \Omega_i)A(c_i, h)]}{1 - m^l(1, i, \Omega_i) - m^h(1, i, \Omega_i)}. \tag{17}$$

The initial debt price can be substituted back into the general debt pricing equation to evaluate debt value at any point in time after issuance. Levered equity value is denoted $E(x, i, \Omega_j)$, and is just equal to the residual of total firm value over and above debt value:

$$E(x, i, \Omega_j) = F(x, i, \Omega_j) - D(x, i, \Omega_j).$$

With all claims priced, we can compute the market leverage ratio L as follows:

$$L(x, i, \Omega_j) \equiv \frac{D(x, i, \Omega_j)}{F(x, i, \Omega_j)}.$$

3 Comparative statics analysis

In empirical corporate finance, a first-order question is whether leverage decisions are explained by the trade-off theory. Here too, in our thought-experiments, which treat the model economy as the true data generating process, the answer to the question is unambiguous. In the model economy, all corporate financing decisions are dictated by trade-off theoretic concerns. Tax advantages to increasing leverage are weighed against bankruptcy costs and issuance costs. The key question then is whether empirical tests will confirm what we know to be true in this economy: trade-off theory explains corporate financing decisions, and taxes matter.

Before going to the data, today's increasingly careful empiricists want to take the underlying theory seriously. And so they will look to the original published versions of models for guidance. Similarly, theorists are today strongly encouraged by editors to furnish empiricists with testable implications that follow from their models. The workhorse methodology for this purpose is comparative statics analysis. Model parameters are varied, with resulting changes in the endogenous variables analyzed. For example, Leland (1994) and Goldstein, Ju and Leland (2001) provide an exhaustive analysis of how optimal leverage changes with the various parameters. The objective of this section is to spell out, within the context of an economy populated by trade-off theoretic firms, the type of testable implications that one would derive by way of numerical comparative statics analysis.

With this in mind, let us now return to the model presented in Section 2. If we treat the tax rate as constant, by letting the transition rates (λ_l, λ_h) go to zero, the model will be nearly identical to that of the trade-off model of Goldstein, Ju, and Leland (2001).[5] This is by now a classical trade-off framework. Our objective is to examine testable implications that would be arrived at by way of comparative statics analysis, with our focus being on the predicted effect of tax rate changes. To this end, the following subsection discusses the parameter values we will use throughout our analysis.

3.1 Parameterization

This section describes the choice of parameters. In the interest of transparency, we choose parameter values that are standard in the literature. Below, we will discuss the model's ability to match some key moments.

Recall, in our model, the tax rate parameter captures the effective debt tax shield value. As discussed above, Graham, Leary and Roberts (2015) document that the effective debt tax shield has varied considerably over the past century. However, most of the empirical of analysis of tax effects on leverage has been confined to the post-war period. During this particular period, the effective debt tax shield has tended to fall within the band of 30 percent to 40 percent. Thus, our baseline analysis assumes $\tau_l = .36$ and $\tau_h = .40$. We assume a tax rate spread of 4 percentage points

[5] One difference is the modeling of default.

in the interest of conservatism. In particular, such a change in corporate tax rates is large relative to the typical tax rate shock exploited in empirical work. For example, Gordon and MacKie-Mason (1990) estimate that the landmark Tax Reform Act of 1986 led to an increase in the effective debt tax shield value of only 1.5 percentage points. To take another example, Heider and Ljundqvist (2015) estimate leverage responses to changes in state corporate income tax rates. In their sample, the average increase in tax rates is 74 basis points and the average decrease in tax rates is only 54 basis points. Thus, our assumed tax rate spread is relatively wide and thus conservative in the sense of biasing our simulated experiments towards finding significant tax effects.

In our baseline parameterization, the proportional debt issuance cost q is set at .001. By way of direct comparison, Fischer, Heinkel, and Zechner (1989) and Goldstein, Ju, and Leland (2001) define issuance costs in the same way as we have done here and utilize $q = .01$. In his calibrated simulation analysis, Strebulaev (2007) assumes $q = .002$. Thus, relative to existing parameterizations, our simulated model features small issuance costs. Of course, the dynamic capital structure literature's modeling of issuance costs represents a simplification. Firms may be able issue debt incrementally, with underwriting fees being levied only on the increment. If debt issuance costs were strictly incremental, firms would be more responsive to shocks. However, in reality debt issuance costs are not strictly proportional to incremental debt. For example, as documented by Altinkilic and Hansen (2000), there is an important fixed or lump-sum component charged by underwriters. Moreover, there are indirect costs of debt flotations. Fixed costs and indirect costs of debt issuance would tend to increase firm inertia. Finally, inertia might tend to be exacerbated due to general equilibrium effects. In particular, if an economy-wide tax rate change took place, direct and indirect issuance costs might increase as banks attempted to deal with the probable queue of customers seeking to refinance.

To better understand the magnitude of our assumed debt issuance costs, we note that a fee of $q = .001$ multiplied by total debt is approximately equal in magnitude to an underwriting fee of 1% applied to incremental debt, where incremental debt is 10% of total debt. With this in mind, we note that Kim, Palia, and Saunders (2009), Datta, Iskandar-Datta, and Patel (1997) report total expenses of new debt issuance from 1976 to 1992 equal to 2.96%. Kim, Palia, and Saunders (2003), in a study of underwriting spreads over a 30-year period, arrive at an estimate of 1.15%. Thus, total issuance costs in our simulated model are relatively low compared with direct empirical evidence. In this sense, our baseline model parameterization can be viewed as conservative since firm responsiveness to shocks tends to be inversely related to issuance costs. Nevertheless, we will examine and demonstrate the robustness of our main conclusions with respect to alternative assumptions about q.

The bankruptcy cost parameter α is set at 0.2. This value is consistent with the findings of Davydenko, Strebulaev and Zhao (2012), who estimate the change in the market value of firm assets in default to be 21.7%. The extant literature on dynamic trade-off models generally assumes α is between 0.05 and 0.30.

The risk-neutral drift of EBIT, μ, is set at 0.01. The risk premium, denoted as ψ, is set at 0.0645. These values are consistent with Strebulaev (2007). Thus, we assume the long-term nominal growth rate of firm payouts and correspondingly returns to be 0.0745. The instantaneous volatility of firm EBIT, σ, is set at 0.2, with the volatility of the idiosyncratic shock, σ_I, set at 0.14. This is consistent with a number of prior studies using volatility values between 0.15 and 0.30. Empirical estimates suggest that the asset volatility distribution can be quite diverse, ranging from 0.05 to more than 0.5 per annum. The constant risk-free rate, r, is assumed to be 0.05.

In Section 5, in which we analyze statistical power of empirical studies by modeling simulated economies, the number of units of observation (in our case, number of firms in the economy) is critical. In a classical study of the 1986 Tax Reform Act, that by Gordon and MacKie-Mason (1990), their number of firms, N, is 996 firms. We therefore adopt this N (rounded to 1000) as our benchmark number for the number of treated firms in the sample.

Two parameters of particular interest are (λ_l, λ_h). Recall, the expected duration of tax regime i is simply λ_i^{-1}. To reduce the number of cases to consider, we assume transition rates are equal across the two states, setting $\lambda_l = \lambda_h = \lambda$. To shed light on the importance of tax regime duration, different values of λ will be considered. For the base case, we will assume $\lambda = .10$, implying an expected regime duration of one decade. By post-war standards this is a conservative assumption given that the effective tax shield value has changed much more frequently due to frequent tax reforms and the effects of inflation. For example, we recall that Graham, Leary, and Roberts (2015) find that corporate taxes underwent 30 revisions over the past century, an average regime life of only 3.3 years. However, in the last 50 years, the corporate tax rate only changed six times (reported in Graham, Leary, and Roberts (2015), Figure 10, Panel A), making our assumption more in line with the recent data. In addition, we report many results for an alternative specification of λ equal to 0.5, which is a very frequent transiency with an expected regime duration of only 2 years, and report comparative statics as λ varies.

3.2 Optimal leverage and tax elasticity

Suppose first that we are interested in formulating testable predictions regarding how firms will respond to an increase in the corporate income tax rate from, say, 36% to 40%. Table 1 reports the results of our numerical comparative statics exercise. The table reports the optimal date-zero capital structure of a firm. We focus on three variables: the market leverage ratio L, debt value D, and interest coverage ratio X_0/c. Of course, given the scaling property inherent in the model, the leverage ratio and coverage ratio shown will also be optimal at each future recapitalization date.

The top row of Table 1 shows the optimal capital structure of a company facing a tax rate equal to 36%. This is the initial capital structure of the firm in the present thought-experiment. Facing this tax rate, the firm optimally chose a leverage ratio of 28.59%, debt value equal to 5.243, and an interest coverage ratio of 3.428. The second row of the table shows the optimal capital structure of a company facing a tax rate of 40%. A firm facing the higher tax rate would adopt a more aggressive financial policy, optimally choosing a higher leverage ratio of 30.00%, a higher debt value of 5.307, and a lower interest coverage ratio of 3.324. Of course, if we were to instead contemplate a tax rate decrease from 40% to 36%, the numerical comparative statics analysis necessarily predicts a symmetric leverage reduction. Starting from an initial leverage ratio of 30% at the higher tax rate, optimal leverage falls to 28.59% at the lower tax rate.

In summary, the numerical comparative statics analysis implies a large symmetric tax-sensitivity of leverage. In particular, here the leverage ratio changes by about 1.4 percentage points given a 4 percentage point change in the tax rate. Following de Mooij (2011), one can estimate the tax elasticity of leverage as:

$$\frac{(.30 - .2859)/\frac{(.30+.2859)}{2}}{(.40 - .36)} = 1.20.$$

4 Empirical tests using simulated economy data

This section investigates the strengths and weaknesses of state-of-the-art empirical methods for testing whether taxes matter, and whether trade-off theory is a first-order driver of capital structure decisions. The comparative statics analysis in the preceding section suggested that corporate leverage ratios should be responsive to changes in the effective tax shield value, here the corporate income tax rate, with an implied tax-elasticity of leverage above one.

We now ask the following question: What would an empiricist, armed with the best empirical methods used for natural experiments, find when the true data generating process is the model itself? To achieve this goal, we first simulate the model to generate panel data on firm leverage ratios and tax rates. Next, we use this data to mimic real-world studies estimating the responsiveness of capital structure to changes in corporate tax rates.

A first-order issue in any real-world empirical study of this type is avoiding the possibility of endogeneity bias. As an example, perhaps the government has decided to lower the corporate tax rate in response to a bad systematic shock? If this is the case, then the observed leverage ratio change is partly attributable to the negative macroeconomic shock, as opposed to tax rate change. Conveniently, the econometrician in our economy enjoys immunity from endogeneity bias since the tax rate process is independent of the EBIT processes. Importantly, this is not to claim that endogeneity bias does not exist or that it is unimportant – it obviously is – but to show that there are other quantitatively significant barriers to correct inference that must be cleared.

As in other similar studies of this kind, the complexity of dynamic effects makes it impossible to derive closed-form expressions for regression coefficients, despite the fact that we have closed-form expressions for all components of firm value. Therefore, we use simulations to generate artificial data from the model, choosing parameter values and simulated experiments to mimic the real-world data as closely as possible.

4.1 Simulation procedure

This subsection describes the simulation procedure. The goal is to generate simulated data that mimics key features of real-world empirical data. Recall, we have already solved for the optimal financial policies of our firms. In particular, we know the firms in our economy will scale up their debt contracts in proportion to the increase in EBIT each time one of the contractually specified call thresholds is crossed, and we know the values of the call thresholds. Thus, simulation simply requires implementing these decision rules for each firm given its respective EBIT path, as well as the path of the corporate income tax rate. There is no need for any optimization at this stage, as optimal policies have already been pinned down.

Of course, in real-world data firms face both idiosyncratic and systematic shocks. We therefore specify an EBIT process for each firm that captures both types of shocks. The EBIT process for an arbitrary firm n evolves as follows:

$$\frac{dX_t^n}{X_t^n} = \mu dt + \sigma_I dZ_t^n + \theta \sigma_S dZ_t^S. \tag{18}$$

In the preceding equation, Z_t^n and Z_t^S are idiosyncratic and systematic shocks, respectively. The parameter θ determines the sensitivity of EBIT to systematic shocks. Recall, the sum of two Brownian motions is itself a Brownian motion. We can apply the results from Section 2 by letting

$$\sigma \equiv \sqrt{\sigma_I^2 + \theta^2 \sigma_S^2}. \tag{19}$$

The model determines the optimal capital structure decisions of firms at any point in time, taking into account all the information available up to that point. The optimal decision includes the amount of debt to issue, as well as the timing of refinancing. In particular, if the tax rate changes at any date t, the firm's optimization problem immediately adjusts to take into account the new tax state.

At date zero of the simulation, all firms in the economy are born and choose their optimal capital structure given the tax rate at that time. The initial EBIT of each firm is set to 1. The cross-sectional properties of the economy at date zero are unrealistic because in reality firms refinance at different times and their decisions depend on past realizations of firm-specific shocks. In addition, the capital structure distribution at any point in time is the outcome of past changes in tax rates. Thus, the first step in our simulation is the construction of a stationary cross-sectional distribution of capital structures.

Conveniently, in the model, the leverage ratio can be uniquely derived from EBIT. A firm actively adjusts its capital structure if and only if its EBIT crosses one of the optimally-set tax-specific refinancing thresholds. Therefore, to generate a stationary distribution of leverage ratios, it is sufficient to generate a stationary distribution of EBITs.

To this end, we discretize the continuous-time model and set each period length equal to one quarter. We then simulate quarterly data for $N = 1000$ treated firms (and $N = 1000$ control firms when performing difference-in-differences estimation). Each simulation consists of two steps. First, to achieve the stationary distribution, we simulate 200 quarters and then drop these initial observations.[6] Next, we continue the simulation until the arrival of a tax rate change that fits the relevant event inclusion criteria, as defined in detail below. We refer to the resulting panel data set as one "simulated economy" or just "economy." It is worth stressing that each economy simply represents one draw of a tax change meeting the stated inclusion criteria. We repeat this procedure 1000 times to study the sampling distribution of test statistics for 1000 stationary economies experiencing the relevant tax rate change.[7]

In any period, each firm observes the random shock hitting its respective EBIT over the last quarter as well as any changes in the tax rate. To determine whether a tax rate change occurs in the quarter, we draw an independent binomial random variable with transition probability λ_i, where i is the tax state at the beginning of the quarter. If the tax rate switches, the firm's refinancing threshold switches accordingly. And here we recall that the refinancing threshold at each point in time depends on the tax rate at that point in time as well as on the tax rate at the time the outstanding bond was issued.

To simulate the EBIT, we discretize its dynamics as follows:

$$dX^n_{t+\Delta t} = X^n_t \left\{ (\mu + \psi)\Delta t + \sigma_I \Delta Z^n_t + \theta \sigma_S \Delta Z^S_t \right\},$$

and draw $(\{\Delta Z^n_t\}^N_{n=1}, \Delta Z^S_t)$ from independent normal distributions with mean zero and variance Δt, where Δt is set to one quarter in the simulation analysis. Because the actual stochastic process occurs under the natural distribution, we adjust the instantaneous drift term μ by adding a risk premium ψ. If EBIT does not cross any boundary (the default threshold or the state-specific refinancing threshold), the firm takes no action. If EBIT crosses a refinancing threshold, we reset EBIT to 1 at the beginning of next quarter, with the debt contract reset to the state-contingent optimum. Here it is worth noting that we are not interested in the size distribution of firms. And

[6] Our analysis suggests that dropping the initial 200 quarters is more than sufficient for achieving the stationary distribution. The EBIT distribution is invariant after about 50 quarters.

[7] Unreported, we find that sampling distribution 1000 times is sufficient to estimate the extent of statistical power.

the scalability property inherent in the model ensures that at each refinancing we can simply re-set EBIT to 1 with no contamination of financial ratios. If EBIT reaches the outstanding debt coupon, the firm is liquidated, EBIT is re-set, and the new shareholders re-optimize its capital structure. However, in the analysis of simulated tax reforms, simulated firms that default during the sample window are removed, in order to mimic the real-world data.

An important concern for empiricists is the possibility of closely overlapping events. For example, if a second tax rate change occurs soon after the first change, firm responses to the first reform are difficult to gauge. Empirical studies therefore routinely exclude such contaminated events. We mimic such restrictions in the course of our simulations. In particular, in order for a tax reform to meet our inclusion criteria, there can be no other tax reform within $\pm$ 12 quarters. Therefore, we continue a given simulation run until the occurrence of the first tax reform (of the desired sign) meeting these criteria.

4.2 Optimal policies at refinancing points

Before analyzing in detail our simulated natural experiments, this subsection discusses the key mechanisms that drive firm financial decisions. Table 2 compares the optimal values of control variables, those written into the date zero debt contract, across alternative environments. In particular, the table contrasts the optimal debt contract in the case of constant tax rates with optimal debt contract terms when tax rates vary stochastically, considering optimal terms for a ten year expected tax regime duration ($\lambda = 0.1$), as well as a two year expected tax regime duration ($\lambda = 0.5$).

In all cases, one sees that the optimal coupon for debt issued in the high tax state, c_h, is greater than the optimal coupon for debt issued in the low tax state, c_l. Intuitively, the firm is willing to incur greater bankruptcy risk when the flow of debt tax shields is higher. Notice also that the difference between optimal coupons across the low and the high tax states grows larger as the likelihood of a tax rate change falls (lower λ). Intuitively, if the current tax regime is expected to last longer, the firm issues more debt in the high tax regime and less debt in the low tax regime. In fact, in the limit as λ tends to zero, the firm acts as if it will face a constant tax rate forever. Conversely, in the limit, as tax reforms become extremely frequent, the gap between the optimal coupons across the low and high tax rate states vanishes, with the firm acting as if it faces the average of the two tax rates.

To understand firm behavior, it is also critical to understand the optimal refinancing thresholds. To begin, suppose the outstanding debt was issued in the low tax state, and consider the effect of a change in the tax rate from low to high. As shown in Table 2, the optimal refinancing threshold will jump down from γ_l^l to $\gamma_l^h < \gamma_l^l$ implying either an immediate jump into restructuring or a hastening of restructuring. Intuitively, if the tax rate increases, the firm becomes increasingly willing to incur issuance costs in order to capture the higher flow of tax shields. Continuing with the present scenario, comparing across λ values we see that as the expected duration of the tax regime increases, the refinancing threshold for the high rate state (γ_l^h) decreases. The firm becomes more willing to incur new issuance costs if the higher flow of tax savings is more stable. Conversely, as the expected duration of the tax regime increases, the refinancing threshold for the low rate state (γ_l^l) increases. The firm becomes less willing to incur new issuance costs given the lower likelihood of a switch to a higher tax rate, where tax shields are particularly valuable.

Suppose next that the outstanding debt was issued in the high tax state, and consider the effect of a change in the tax rate from high to low. As shown in Table 2, the optimal refinancing threshold will jump up from γ_h^h to $\gamma_h^l > \gamma_h^h$ implying a delay in restructuring. Intuitively, if the tax rate decreases, the firm becomes less willing to incur issuance costs because the expected present value of the additional tax shield is lower. Continuing with the present scenario, comparing across λ values we see that as the expected duration of the tax regime increases, the refinancing threshold for the high rate state (γ_h^h) decreases. Again, the firm becomes more willing to incur new issuance costs if the high flow of tax savings is more stable. Conversely, as the expected duration of the tax regime increases, the refinancing threshold for the low rate state (γ_h^l) increases. Again, the firm is less willing to incur new issuance costs if there is a lower probability of a switch to a higher tax rate.

In summary, in terms of financial control variables, the optimal interest coverage ratio at refinancing points is lower if the tax rate at that point is high. The wedge between tax rate-contingent coverage ratios widens as the tax rate process becomes more persistent. Refinancing thresholds jump downward (upward) if there is the tax rate jumps from low to high (high to low), implying hastened (delayed) recapitalizations. As tax rates become more persistent, the wedge between tax rate-contingent refinancing thresholds widens further, with high persistence encouraging greatly hastened (greatly delayed) recapitalizations after a jump to high (low) tax rates.

Table 3 reports financial metrics at refinancing points under optimal capital structure policies. The left (right) panel considers financial metrics the expected duration of the tax rate regime is one decade (two years). These financial metrics should be compared to the same ones reported in Table 1 under constant tax rates. Consider first leverage ratios. Under constant tax rates, the optimal leverage ratio is 1.41 percentage points higher at the high tax rate than at the low tax rate, a 4.9% difference. However, with stochastic tax rates for the case of λ equal to 0.1, the leverage gap is nearly cut in half, to only 0.81 percentage points, representing a wedge of only 2.8%. And note, this halving of the leverage gap occurs here despite the fact that we have considered a conservative decade-long tax regime duration. Consider next the gap between optimal interest coverage ratios across the two tax rates. Under constant tax rates, the optimal coverage ratio is 3.1% higher at the low tax rate than at the high tax rate. However, under the ten year duration case, the coverage ratio is only 2.6% higher at the low tax rate. As Panel B shows, if the tax regime is even shorter with an average duration of two years, the wedge is even smaller. For example, the leverage wedge falls from 2.8% to 1.1%.

Of course, as pointed out forcefully by Strebulaev (2007), these findings are not directly informative about empirical tests since, when working with real-world panel data, empiricists do not observe snapshots of firms at their refinancing points. Rather, a subset of firms will be at their refinancing points and the remainder will be in the region of optimal inaction. Our simulated empirical tests will certainly take account of this fact. However, the analysis of this subsection does suggest that even a modest degree of policy transience, e.g. decade-long regimes, may considerably weaken the power of empirical tests, as the leverage ratio wedge narrows under stochastic tax rates.

Although not definitive, the analysis also suggests that the power of empirical tests may well hinge upon the direction of the tax rate change, with corporations being more responsive to tax rate increases. After all, as shown in Table 3, the model predicts that an increase in the corporate tax rate will hasten a leverage increase as the call threshold falls. Conversely, the model predicts that a decrease in the corporate tax rate will lead to a delayed leverage decrease, as the call threshold increases. Intuitively, the firm stands to capture a larger tax shield value if it recapitalizes in response to a tax rate increase, but not for a tax rate decrease. This causes total firm value to be more responsive to the leverage choice when the tax rate is high. Formally, the net gain from leverage, accounting for bankruptcy and issuance costs, is higher under a higher tax rate. This effect is present in the models of Leland (1994), Goldstein, Ju, and Leland (2001) and in our model. This conjecture is explored further in Section 6.

To explore how well the calibrated model fits the broad stylized facts on leverage, Table 4 reports the values of capital structure variables in the stationary environment in the two-state model for values of λ equal to 0.1 and 0.5. The first row shows that the stationary leverage ratios are about 0.355. These values are substantially higher than the optimal leverage ratios at date zero as reported in Table 1. This property of our simulated model data is standard in the literature (see Strebulaev (2007)). These stationary leverage ratios are also in the ballpark of those reported in the empirical literature. For example, Rajan and Zingales (1995) find that the mean US debt-to-capital ratio during the 1987–1991 period is 0.32. Using more recent data, Graham, Leary and Roberts (2015) report that the mean market leverage ratio during the 2001–2010 period is 0.28. Heider and Ljungqvist (2015) find that the average long-term market leverage ratio from 1989 to 2011 is 0.172. While substantially lower, this leverage ratio uses only long-term debt.

The second row of Table 4 reports annualized refinancing frequencies defined as the fraction of firms in the economy that refinance in one year. Our benchmark simulations produce refinancing frequencies around 0.26, meaning that a quarter of firms on average refinance every year. Leary and Roberts (2005) find that firms refinance by issuing debt at annualized frequency of 0.50. This relatively lower refinancing activity is to be expected in a simulated model that abstracts from important non-tax sources of refinancing such as real investments opportunities or shocks to other causal variables such as volatility, bankruptcy costs, risk premia, or risk-free rates. Despite the absence of these factors, the magnitude of refinancing frequency is similar in the data and the model.

Finally, the third row of Table 4 reports the stationary values of the interest coverage ratio, defined as the ratio of EBIT over debt coupon. The stationary values are about 2.91, which are almost the same as the interest coverage ratios at date zero reported in Table 1. These values are similar to the value of 2.41 reported for a similarly defined measure by Rajan and Zingales (1995).

4.3 Difference-in-differences

The difference-in-differences method (DD) has recently attracted widespread usage. This is unsurprising given that, in their influential textbook on empirical methods, Angrist and Pischke (2009) treat DD as one of the key tools for achieving identification. Our objective here is not to take exception with the DD method itself, but rather to consider its performance in the specific, and important, context of empirical research on the effect/non-effect of taxation on firm financing policies.

In DD, the behavior of treated agents (e.g. firms) is compared with that of non-treated agents. Of course, a key concern is selection bias, so that in practice great care is taken to verify to the extent possible that the treated and non-treated agents are otherwise comparable, aside from the (hopefully) orthogonal treatment shock. For example, in their study of Belgian tax reforms, Panier, Perez-Gonzalez, and Villanueva (2015) compare the capital structure change of treated Belgian firms with the capital structure change of firms in neighboring countries, such as France and the Netherlands. The firms in those countries were not directly affected by the Belgian reform. The main assumption underlying the DD analysis is that the treated and the non-treated samples do not differ along other important dimensions that could affect the empirical outcome variable, the change in capital structure. For example, if the firms in France and the Netherlands happened to experience changes in their investment opportunity set, while the firms in Belgium did not, the DD analysis would be undermined.

Our simulated model represents an ideal laboratory for assessing the performance of the DD methodology in the context of capital structure research since, as we describe, we can design the treated and non-treated firms so that they are identical in terms of all key risk factors, aggregate shocks, and deep technological parameters. Specifically, for each simulated economy of 1000 firms, we will generate another parallel economy that we call the non-treated economy. The two economies have exactly the same realized path of systematic shocks. There are only two differences between the treated and non-treated economies. First, as in the real-world data, realized firm-specific idiosyncratic EBIT shocks will differ across all firms. Again, coming back to our example of the study by Panier, Perez-Gonzalez, and Villanueva (2015), while ideally the aggregate shocks affecting France and Belgium would be the same, ensuring common average leverage trends, idiosyncratic shocks affecting individual French and Belgian firms will necessarily differ in reality. Second, in the non-treated economy, there is no tax reform within the simulated event observation window.

It is important to stress that, although the non-treated economy does not experience a tax rate change within the studied event window, the tax rate process in this economy has the exactly the same parameterization as the tax rate process for the treated economy. In other words, the two economies face the same probability and magnitude of tax rate changes, but the realized path of tax rate changes differs. Although we have not seen this assumption discussed in empirical work, commonality of the driving process for the policy shocks is likely to be essential for ensuring common trends, and that is certainly the case in our simulated DD analysis.

After constructing the simulated panels, we run standard DD regressions, following, for example, the empirical methodology of Wooldridge (2010) or Angrist and Pischke (2009) (see for example Panier, Perez-Gonzalez, and Villanueva (2015) for the application to corporate leverage response to tax changes). In order to study the impact of tax rate changes on leverage ratios, we run the following panel regression:

$$\begin{aligned} L_{i,j,t} &= \beta_0 + \beta_1 \mathbf{1}(j = \textit{treatment group}) \times \mathbf{1}(t = \textit{post treatment}) \\ &\quad + \beta_2 \mathbf{1}(j = \textit{treatment group}) + \beta_3 \mathbf{1}(t = \textit{post treatment}) + \sum_{t'} \beta_4^{t'} \mathbf{1}(t = t') + \epsilon_{i,j,t}, \end{aligned} \tag{20}$$

where i, j and t index firm, group and time, respectively. We also run the same regressions for refinancing frequency and the interest coverage ratio. In the benchmark analysis, one simulated economy consists of 1000 treated and 1000 control firms and the panel regression is run over 8 quarters of data, centered around the tax rate change event. Here, $\mathbf{1}(j = \textit{treatment group})$ and $\mathbf{1}(t = \textit{post treatment})$ are dummy variables that equal one for treated firms and post-treatment quarters, respectively, with β_1 being the coefficient of interest. The summation term captures the time-fixed effects that soak up systematic shocks. We omit two time periods to ensure the full-rank condition. Unlike in Panier, Perez-Gonzalez, and Villanueva (2015), we do not need to include group-fixed effects because the treated and control groups are ex-ante homogeneous and face the same systematic shocks. For the same reason, we do not include other fixed effects, such as those controlling for industry and country.

5 An examination of statistical power

In this section, we evaluate the power of statistical tests regarding corporate tax effects using simulated tax reform data. To begin, it is useful to consider historical tax reforms in which the tax change was generally accepted as being large. An important tax reform in this regard was the Tax Reform Act of 1986 (TRA86). TRA86 attempted to reduce distortions by way of a combination of lower tax rates and base broadening, moving the code closer to a pure Haig-Simons income tax. The statutory corporate tax rate fell sharply from 46% to 34%, prompting some superficial observers to expect a large corporate leverage response. However, the change in the effective debt tax shield value was substantially lower in magnitude. In fact, although the statutory corporate tax rate fell by 12 percentage points, the effective tax advantage to debt was judged by economists to have increased. For example, Gordon and MacKie-Mason (1990) argue that, due to all the other provisions in TRA86, the effective tax advantage to debt rose by 2.5 cents per dollar of interest expense. The point to note here is that empiricists have not generally enjoyed the luxury of observing very large changes in the effective tax shield value. Therefore, any examination of statistical power must consider realistic changes in tax shield value.

Whether TRA86 led to an increase or decrease in the effective tax shield value is important given the potential for response asymmetry results as in the previous subsection. After all, if tax reform leads to an increase in the effective tax shield value, we are more likely to detect it empirically. To be on the conservative side, in other words, to increase the probability of finding significant responses to our simulated tax reforms, this section considers an increase in the corporate income tax rate. Further, we consider a large increase in tax shield value of 4 percentage points (relative to 2.5 percentage points under TRA86), from $\tau_l = 0.36$ to $\tau_h = 0.4$. We continue to assume tax regimes are expected to last a decade on average, setting $\lambda = 0.1$ as our baseline. This too may be considered a conservative assumption given that, at the time of TRA86, greater tax rate transiency might well have been expected given the frequency of tax code changes in the run-up to the legislation. For this reason, we also continue to consider the case of λ equal 0.5, where tax regimes are expected to last for two years.

Table 5 provides detail on the distribution of DD estimates of β_1 obtained from our 1000 simulated tax reforms, with the leverage ratio serving as the dependent variable here. To set a benchmark, it is useful to first recall the difference in the optimal date-zero leverage in the one-state model. This difference is equal to 0.0141 as shown in Table 1. With this in mind, consider the results from the simulated DD instead using a ten year ($\lambda = 0.1$) expected regime duration reported in the first column of Panel A. Here the mean value of β_1 is only 0.0045, which is only 32.2% of the difference in the optimal date-zero leverage in the one-state model. The response to the same level of a tax increase is even smaller when the tax reform is expected to be shorter. With $\lambda = 0.5$, the mean β_1 is 0.0019, which is only 13.8% of the difference in the one-state model. Figure 1 plots the distribution of estimated β_1 over 1000 simulated economies for $\lambda = 0.1$ as a histogram. We observe that the estimated β_1 is actually *negative* in a non-negligible fraction of economies.

Recall, that the tax elasticity of leverage implied by a constant tax rate model is 1.20. Estimating the implied elasticity in the simulated data and recalling that the average leverage in simulation equals 0.3553, we get the implied elasticity equal to:

$$\frac{0.0045/.3553}{0.40 - 0.36} = 0.32.$$

The implied elasticity in simulated data is only 0.32, a downward bias of 68 percent compared to the constant tax rate model. de Mooij (2011) surveys an extensive literature on the tax-elasticity of leverage, reporting a median estimate of 0.51 and a mean estimate of 0.65. Our simulation results suggest that tax rate transience may well explain the low estimated tax elasticities relative to what would be expected based upon constant tax rate models.

Returning to Table 5, we see that if attention is turned to statistical significance, the picture is even gloomier. In 44.4% of the simulated tax reforms studied, one fails to find a statistically significant coefficient at the 5% level of confidence. That is, in almost half the studies, we fail to reject the null that taxes do not matter for leverage decisions despite the fact that taxes are the cause of corporate leverage in the simulated economy. With an expected regime life of two years, one fails to find a significant coefficient in 72.8% of the simulated tax reforms analyzed. Panel B shows that similar, and even worse, lack of statistical significance is observed in tax decreasing reforms.

The next two columns in Panel A of Table 5 report the results from analogous DD estimations in which the percentage of firms undertaking refinancing activities in a quarter, the Refinancing

Frequency, is used as the dependent variable. Here the results are also discouraging in terms of statistical power. Even under a decade-long expected tax regime life, one fails to obtain a statistically significant coefficient in 42.4% of the simulated tax reform experiments. The final two columns report results from analogous DD estimations in which the interest coverage ratio is treated as the dependent variable. The results are similarly discouraging in terms of statistical power. Even under a decade-long expected tax regime life, one fails to obtain a statistically significant coefficient in 47.9% of the simulated tax reform experiments.

What explains these troubling findings? First, given that firms face debt issuance costs, one should only expect a minority of firms to react immediately to changes in the tax advantage of debt. The majority of firms will delay debt market activity until there is a sufficient value gain to justify incurring transactions costs. Second, we have seen that transient tax rates cause firms to be less aggressive at restructuring dates. Finally, in reality one works with finite samples, so that the treated and control groups are not identical. Instead they differ in terms of the realization of idiosyncratic shocks. For some sample paths, the non-treated group will experience relatively large increases in EBIT, causing their leverage to increase at a greater rate, despite the lack of a tax increase.

Obviously, the expected duration of the tax regime will influence the magnitude of regression coefficients and statistical power. In particular, as tax regimes become more transient (λ increases) one expects the DD slope coefficient to fall, along with the probability of rejecting the null of zero tax effect. Consistent with this intuition, Figure 2 plots how the results of the present simulation experiment will change as we vary the λ parameter, focusing on the change in market leverage as the dependent variable. We see that soon after λ exceeds .10, so that expected regime duration falls a bit below one decade, one fails to reject the faulty null of zero tax effect in the majority of simulated tax reform experiments.

Our benchmark choice of the number of firms in the economy, $N = 1000$, follows a classical study of TRA86, Gordon and MacKie-Mason (1990). In any empirical study of statistical power, the number of units of observation is critical. To explore the effect of the number of firms in the economy on the (lack of) statistical power, we replicate our main result of Table 5 as N varies. The first plot in Figure 3 shows that statistical significance increases as the sample size becomes larger, though at a relatively slow pace. For example, even as the sample size doubles to 2000 firms, statistical significance is still low and occurs in only 67% of empirical tests. However, as the sample size becomes really large, statistical significance power improves further. For example, for the value of N equal to 5000, statistical significance is expected in about 84% of replicated empirical tests, when tax reforms are expected to last a decade. Thus, for relatively long-lasting tax reforms, increasing the size sample substantially improves the power of empirical tests. Recent empirical studies, such as Heider and Ljunqvist (2015), use a substantial sample size, which increases their power. However, the plot also shows that for short-term lasting tax reforms statistical power is still low. For example, for the same sample size of 5000, when reforms last on average only 2 years, statistical significance is achieved only in about 50% of replicated empirical tests. It is also worth noting that statistical power attenuates very quickly as the sample size falls below 1000.

The second plot of Figure 3 shows a similar comparative statistics analysis of statistical power with respect to the size of the tax reform. For this analysis, the mean of the average tax rate in the economy is kept constant, at 0.38. The plot shows that statistical power rises rapidly for larger tax reforms. For example, when the reform increases the effective tax rate by 10%, statistical significance in leverage panel regressions is found in 87% (54%) of replicated empirical tests when tax reforms are expected to last ten (two) years. For tax reforms that do not last long, however, statistical significance is still below 80% even for very large tax reforms. When tax reforms are smaller than the benchmark case of 0.04, statistical significance attenuates quickly.

The third plot of Figure 3 shows that statistical power is insensitive to the debt issuance costs for the value of q between 0.0005 and 0.01. The fact that statistical power is largely unaffected by the debt issuance cost might at first appear surprising. However, even with extremely small issuance costs, the firm will still find it optimal to build in a non-trivial degree of financial inertia. That is, it will optimally refrain from setting a call threshold very close to the initial EBIT. After all, it is well known that once a Brownian motion crosses a threshold, it will cross the same threshold infinitely many times over any finite time period afterwards.[8] Thus, a call threshold too low will generate extremely high issuance costs, while providing a minor increase in expected firm value given that leverage is already in the neighborhood of the optimum.

The last plot explores to what extent statistical power is sensitive to changing the sample window. We use 8 quarters as our benchmark case. Increasing sample window improves statistical

[8] See, for example, Bielecki et al. (2003, p. 238)

power in our simulations because it increases the size of the panel and allows more time for firms to respond. Note, however, that even for a very sizable sample window of 24 quarters, statistical significance is still not achieved in many cases, especially in relatively short lasting tax reforms. In addition, any effect of higher statistical power is likely counterbalanced in reality by increased noise. For example, over a longer window macroeconomic shocks may cloud inference. Unreported, if we allow for future tax reforms to take place after the tax reform of interest (i.e., future tax reforms could be within the sample window), statistical power is indeed lower. Therefore, choosing a relatively small sample window makes sense. What is most worrying, therefore, is that for smaller sample window (for example, of 4 quarters), the lack of statistical power is really dramatic: even for tax reforms that last ten years, the effect will not be statistically significant in 47% of cases.

6 Asymmetric responses to tax increases and decreases

In this section, we use the simulated data to further examine our conjecture that there will be an asymmetry in empirical tests regarding tax effects on capital structure. In particular, we conjectured above that firms will respond more aggressively to tax rate increases than to tax rate decreases. Formally, such an effect might well be expected in the simulated model data because tax rate increases (decreases) are associated with a jump downwards (upwards) in optimal refinancing thresholds (Table 2), so that refinancing occurs sooner (later). Intuitively, when the tax rate is high, total firm value is more sensitive to the coupon level, and so there is more to be gained from rebalancing from a low to higher coupon. Conversely, when the tax rate is low, total firm value is relatively insensitive to the coupon level, so there is less to be gained from rebalancing from the high to low coupon. In the former case the firm is more willing to pay the issuance costs associated with restructuring. In the latter case, the firm is less willing to do so.

For this analysis, we use exactly the same economy and simulation parameters we use throughout the paper, with the only difference being the size of the tax reform. Our goal is to explore the asymmetry of the tax response as the size of the tax reform changes. Given that our benchmark tax parameters are $\tau_l = 0.36$ and $\tau_h = 0.40$, we assume that the average tax rate between the two states stays the same at 0.38 and vary the mean-preserving spread of the tax reform $\Delta\tau$. For example, for the value of $\Delta\tau = 0.10$, the state-specific tax parameters are $\tau_l = 0.33$ and $\tau_h = 0.43$.

The variable of interest is the relative response of firms to tax increases versus tax decreases. Recall that the response of firms to tax reform is measured as β_1 in Equation (20). If we define the average response of firms over simulated economies to tax increases as β_1^{INC} and the corresponding average response of firms to tax decreases as β_1^{DEC}, a variable that measures asymmetric response of firms is

$$z = \frac{\beta_1^{INC}}{\beta_1^{DEC}}. \tag{21}$$

Figure 4 shows z, the relative response of firms to tax increases and increases, as the size of the tax reform, $\Delta\tau$, varies. The figure shows that firms are more responsive to tax increases than decreases for any size of the tax reform. Even when the tax reform is very small, $\Delta\tau = 0.01$, the average response to tax increase is about 4% larger than the average response to a tax decrease. The asymmetry, however, becomes much more pronounced for larger tax reforms. For example, for a value of $\Delta\tau$ of 15%, the average response to a tax increase is larger by a third compared to the average response to a tax decrease. Not shown, similar or more pronounced asymmetry can be seen if we compare relative responses of firms using other measures, such as the frequency of refinancing. The magnitude of the asymmetry for a large tax reform is far from a theoretical exercise. The tax reform of 2018, still in its infancy at the time of writing, resulted in a very large decrease in the corporate tax rate, reducing the top rate by 14%. Our analysis shows that the response of firms to such a large change will exhibit lower statistical significance and economic magnitude than might be otherwise expected by many observers.

7 Are the regressions informative about welfare costs?

In this section we examine the relationship between social welfare and the regression coefficients that would be obtained by an empiricist working with our ideal simulated tax reform data. Casual intuition might lead one to assume that there should be a positive monotonic relationship between regression coefficients and welfare loss. After all, welfare losses arise from distortions in behavior, so greater tax-sensitivity, as conventionally gauged by regression coefficients, suggests greater welfare loss. Or, to put it another way, if the regression coefficient is closer to zero one might conclude that taxes have only a negligible effect on decisions, and so the welfare costs resulting from tax-induced behavioral distortions might well be expected to be small.

Recall that the simulated DD estimation is hard-wired with ideal exogenous tax changes, and so easily clears the "clean identification" hurdle, as it is conventionally construed. Our model is also useful in terms of thinking about welfare losses attributable to the tax system. To begin, we note that in the absence of a corporate income tax, the firms in our economy would never tap external financing of any sort. To see this, note that the firm always has positive cash flow and so has no need to incur underwriting fees associated with raising external debt or external equity financing. Thus, the only reason the firms in our simulated economy issue debt, faced with issuance and bankruptcy costs, is for the debt tax shield they provide. With this in mind, we treat bankruptcy costs as our measure of the welfare loss attributable to tax distortions. Of course, one might here view debt flotation costs as a welfare loss. However, to the extent that flotation costs represent a payment from the firm to their investment bankers, these fees simply represent a transfer, not a loss of output.

With this in mind, we conduct the following thought-experiment. Consider economies on parallel planets identical in all respects with the exception of being endowed with different bankruptcy cost parameters (α). And suppose that on each of these planets we were to repeat the ideal simulated DD exercises described in the preceding sections. The question is whether there will, in fact, be a positive relationship between welfare loss and the econometrically estimated tax-responsiveness of firms.

Figure 5 reports the results of the analysis. On the horizontal axis of each panel is the assumed value of the bankruptcy cost parameter. The top panel plots the average level of welfare loss (BC) in the simulated economies, conditional upon the current tax rate. As intuition would suggest, welfare loss does indeed increase with α. And this occurs despite the fact that the simulated firms find it optimal to adopt less aggressive leverage policies when faced with higher bankruptcy costs. The middle panel plots the difference in welfare loss across the high and low tax rate states. Again, as intuition would suggest, the average welfare loss is higher in the high tax state since the simulated firms adopt more aggressive leverage policies when confronted with higher present-day tax rates.

The bottom panel of Figure 5 contains the key result. Strikingly, the DD regression coefficient is actually decreasing in α. That is, although welfare losses rise with the bankruptcy cost parameter α, the DD regression coefficient decreases with α. Intuitively, with higher bankruptcy costs, firms find it optimal to increase leverage less aggressively in response to a tax rate increase. Thus, the simulated DD coefficient is decreasing in α.

It follows from this analysis that, for an economy endowed with high bankruptcy costs, due to say an inefficient bankruptcy procedure, the econometrician would here find that taxes are "less important" despite the fact that the tax system is here uniquely responsible for very high welfare losses. Apparently, the regression coefficients would constitute a poor gauge of welfare losses arising from the tax system.

8 Implications for empirical research

In this section, we summarize practical implications that could help empirical researchers in developing and evaluating empirical analysis of corporate responses to tax reforms. The implications are particularly geared towards improving reduced-form empirical analysis.

Sample size. Our results suggest that for sample sizes that are often used in corporate empirical research, statistical power could be on the lower side. For example, as shown above, with a sample size of 1000, the probability of Type II error in simulated data is approximately 50 percent under plausible parameterizations. Moreover, there is a concave relationship between sample size and statistical power. Specifically, for relatively small samples, lack of statistical power is so dramatic that absence of statistical significance should not be interpreted as absence of corporate response. However, even for large samples (the size of 5000 firms roughly approximates the upper ceiling of public non-financial US firms that could be studied in any single tax reform episode), statistical power is surprisingly low. As sample size increases further, statistical power increases slowly. Thus, while finding a significant corporate response indicates the importance of taxation, even for large sample sizes lack of empirical response should be interpreted cautiously and ideally supplemented with additional evidence, e.g. by utilizing a number of tax reforms. Of course, this raises some important methodological questions. In particular, it will be hard for any particular study to "falsify" a given theory, such as tradeoff theory. Thus, in contrast to, say, physics, financial economists must rely on an accumulation of evidence, interpreted in light of models.

Expected tax regime life. In studies of leverage ratios, the corporate response to a tax change that is expected to be reversed relatively soon is substantially smaller and less significant than to a long-lasting tax change. For example, in our simulated model data for tax changes that are expected to revert on average within two years (say, because of the frequency of elections and diametrically opposed tax views of the two main political parties), even for a very large sample size of 5000, statistical significance is not achieved in more than 50 percent of cases. This finding in our simulated model data is very general.

A practical implication is that any evidence of expectations of the tax reform duration should be incorporated into the empirical analysis. For example, empirical researchers might want to peruse media reports surrounding the tax reform debates or take into account expected political changes that might bring to power those who would be more likely to change the tax reform. Historical evidence on the tax regime duration in a specific strata (such as a country) also should be taken into account. In addition, stock market behavior should be informative about the expected duration of tax reforms, since prices capitalize expectations. Finally, it is worth emphasizing that this is a prediction regarding the behavior of corporate leverage ratios in response to changes in effective tax shield value. Other variables might well be expected to be more responsive to short-lived changes, e.g. capital gains realizations or reductions in taxation on repatriated dividends.

Size of the tax reform. Our findings suggest that it could be difficult to find significant evidence of corporate responses to relatively small tax reforms. In the absence of some offsetting force, such as a very large sample size, our findings suggest that it could be difficult to find significant evidence of corporate leverage responses to all but extremely large changes in effective tax shield values. For example, even in a simulated tax reform more than two times the size of TRA86, we fail to reject the null of zero tax response in roughly have the simulated experiments. The practical implication is that empirical researchers should estimate the change in the effective corporate tax rate and focus on tax reforms with appropriate size.

Tax increases vs. tax decreases. Our results suggest that corporate responses to tax increases should be more easily discernible in the data than corporate response to tax decreases. In our context, leverage is more responsive to increases in debt tax shield value than to decreases. As we show, if the tax rate increases, there is a larger tax benefit associated with performing a leveraged recapitalization than a corresponding tax loss associated with a tax rate decrease. This is especially true for sizable tax reforms. For example, for a 15% change in the effective tax rate, the response to a tax increase is larger by a third compared to a similar response to a tax decrease.

An empirical study dealing with a tax decreasing reform should take this finding into account.

Sample window. Increasing the sample window after the tax reform increases statistical power, because it increases the number of firms that over time decided to respond to the reform. Our simulated model data suggests that short sample windows, such as 2 quarters to a year, are more likely to result in lack of statistical significance. Empirical researchers should therefore test for statistical significance over more extensive sample windows than currently practiced. Note, however, that increasing the sample window inevitably increases noise and noise would contribute to lower statistical power. Therefore, finding no effect does not necessarily provide strong evidence in favor of the absence of corporate responses. Finding statistical significance at longer, but not at shorter, horizons would suggest that corporate response is present, but obscured by using a small sample window. An additional contaminating effect that should be considered is the presence of other large shocks as the sample window increases. For example, if we do not select tax reforms in the simulated data by requiring that no other tax reforms took place in the sample window, then if any additional tax reform does take place within such a window, statistical power drops precipitously.

Refinancing sample. Another possibility is to concentrate only on the sample of firms that decided to refinance in the wake of the tax reform to explore the direction of their refinancing. Unreported, our results suggest that this conditional exercise substantially increases statistical power.

To summarize, empirical researchers may use the above as a check list in their analysis of a specific empirical exercise. We realize that no empirical test could satisfy all the conditions. However, if a researcher has identified a natural experiment in which the size of the tax reform is deemed sizable, the sample window is relatively long, the tax reform is sizable and results in the tax increase, the experiment could be quite successful even if the sample is relatively small. Conversely, if the natural experiment consists of a tax-decreasing reform of a very small size and the analysis suggests that investors expect the tax reform to be of short duration, this experiment is likely not worth running.

9 Conclusions and Future Directions

An important question for corporate finance and public finance researchers is whether and how taxation influences financing decisions. Although this question has received considerable attention from empiricists, the ability to interpret this evidence has been severely hampered by the absence of theoretical models speaking to the empirical tests. In particular, empiricists exploit (hopefully) exogenous variation in tax shield value, while null hypotheses are being informed by comparative statics performed on theoretical models in which the tax rate is constant. In this paper, we develop a theoretical model intended to narrow the gap between model and data. Specifically, we develop a dynamic trade-off theoretic model of capital structure in which firms face stochastic tax rates, with leverage adjustments weighing tax benefits of debt against bankruptcy and adjustment costs. Conveniently, all components of firm value are derived in closed-form, which should facilitate future use of the model in considering various empirical thought-experiments.

To this end, we used the model as a basis for generating null hypotheses and assessing statistical power regarding tax effects in simulated laboratory-type controlled experiments. Problematically, we showed that even in ideal settings, the probability of Type II error regarding tax effects is unacceptably high. In particular, in reasonably parameterized economies, with decade-long expected policy regimes, one fails to reject the incorrect null of zero tax effect in roughly one-half of the simulated difference-in-differences studies. Further, in terms of magnitudes, simulated tax elasticities are about 70% lower than those implied by the constant tax rate model and broadly consistent with existing empirical evidence.

This lack of statistical power for tax effects stems from: a degree of optimal inaction in the face of transactions costs; firms reacting less to current tax rates given that tax rates are transient; and noise due to idiosyncratic shocks hitting firms. While some empiricists have listed these factors as caveats, our quantitative analysis shows that these concerns may well be fatal. Moreover, the commonality of false-falsifications in our simulated experiments suggests that these concerns perhaps deserve more attention than inclusion in a list of disclaimers after conclusions have been reached, and policy prescriptions offered, nevertheless. Our quantitative results suggest that it is premature to reach any strong consensus regarding tax effects on capital structure. Rather, empirical tests, including those predicated upon ideal randomizations, must be placed upon firmer theoretical foundations. The present paper represents an attempt to make progress on this front.

While the present model represents an advance of the state of the art in terms of thinking about the mapping between capital structure models and data, a number of extensions seem worthwhile. For example, in the interest of tractability we considered that there are two potential tax rates while in reality tax rates can take on multiple values. Based on the results in Hennessy and Strebulaev (2017) we speculate that the econometric challenges here would only become more severe with multiple tax rates. In particular, they show that with multiple tax rates, observed investment responses to shocks can overshoot, undershoot and even have signs opposite to the comparative static effect. However, on average the typical effect is attenuation of measured responses relative to comparative statics. Similar effects would likely arise with respect to leverage decisions. That is, we conjecture that the average effect of tax rate transience is to attenuate shock responses consistent with our findings here regarding transience creating Type II errors.

A second simplification adopted in our analysis is to assume, following the existing dynamic capital structure literature, that debt issuance costs are proportional to total debt. In reality, firms may be able to issue incremental debt, increasing their level of activity. Conversely, there may be fixed or lump-sum costs of debt issuance that would tend to exacerbate inertia. Therefore, adding truly lump-sum costs to our setup would likely strengthen our main results, including the results on the lack of statistical power. We have also set the model in partial equilibrium to the extent that we have ignored the possibility of debt flotations costs varying with the overall level of debt market activity. In reality, one might expect issuance costs to rise in response to higher activity levels brought on by fundamental tax reforms. Incorporating more realistic specifications of debt issuance costs, with micro-foundations, is likely to be a fruitful avenue for future research.

10 Appendix

In this Appendix, we show how to derive contingent claim prices and exploit the recursive relationships derived in the body of the paper to pin down the components of firm value.

10.1 Canonical ODE

As a preparation, we solve the following ODE:

$$\rho v = \mu x v' + \frac{1}{2}\sigma^2 x^2 v'' + zx + Z \tag{22}$$

We know the solution is of the form:

$$v = x^{B_1} K_1 + x^{B_2} K_2 + \frac{zx}{\rho - \mu} + \frac{Z}{\rho} \tag{23}$$

where the exponents are the negative and positive roots of

$$\frac{1}{2}\sigma^2 B^2 + \left(\mu - \frac{1}{2}\sigma^2\right) B - \rho = 0. \tag{24}$$

It will be useful to keep in mind this solution as we seek to price the various contingent claims.

10.2 Canonical ODE System

Moreover, the following system of ODEs will also appear frequently in our valuations:

$$\begin{aligned} rv(x,l) &= \mu x v_x(x,l) + \frac{1}{2}\sigma^2 x^2 v_{xx}(x,l) + \lambda_l[v(x,h) - v(x,l)] \\ rv(x,h) &= \mu x v_x(x,h) + \frac{1}{2}\sigma^2 x^2 v_{xx}(x,h) + \lambda_h[v(x,l) - v(x,h)]. \end{aligned} \tag{25}$$

We conjecture solutions of the form:

$$\begin{aligned} v(x,l) &= Lx^\beta \\ v(x,h) &= Hx^\beta \end{aligned} \tag{26}$$

Substituting these derivatives back into the ODEs one obtains:

$$\begin{aligned} (r+\lambda_l)Lx^\beta &= \mu x \beta L x^{\beta-1} + \frac{1}{2}\sigma^2 x^2(\beta^2 - \beta)Lx^{\beta-2} + \lambda_l H x^\beta \\ (r+\lambda_h)Hx^\beta &= \mu x \beta H x^{\beta-1} + \frac{1}{2}\sigma^2 x^2(\beta^2 - \beta)Hx^{\beta-2} + \lambda_h L x^\beta, \end{aligned} \tag{27}$$

which is equivalent to

$$\begin{aligned}\left[(r+\lambda_l)-\left(\mu-\frac{1}{2}\sigma^2\right)\beta-\frac{1}{2}\sigma^2\beta^2\right]L &= \lambda_l H \\ \left[(r+\lambda_h)-\left(\mu-\frac{1}{2}\sigma^2\right)\beta-\frac{1}{2}\sigma^2\beta^2\right]H &= \lambda_h L.\end{aligned} \tag{28}$$

Thus we demand:

$$\left[(r+\lambda_l)-\left(\mu-\frac{1}{2}\sigma^2\right)\beta-\frac{1}{2}\sigma^2\beta^2\right]\left[(r+\lambda_h)-\left(\mu-\frac{1}{2}\sigma^2\right)\beta-\frac{1}{2}\sigma^2\beta^2\right]=\lambda_l\lambda_h. \tag{29}$$

Letting

$$\begin{aligned}g_l(\beta) &\equiv (r+\lambda_l)-\left(\mu-\frac{1}{2}\sigma^2\right)\beta-\frac{1}{2}\sigma^2\beta^2 \\ g_h(\beta) &\equiv (r+\lambda_h)-\left(\mu-\frac{1}{2}\sigma^2\right)\beta-\frac{1}{2}\sigma^2\beta^2,\end{aligned} \tag{30}$$

we then demand that any candidate exponent β must satisfy the following characteristic equation:

$$g_l(\beta)g_h(\beta)=\lambda_l\lambda_h. \tag{31}$$

Thus, the general form of the solution is:

$$\begin{aligned}v(x,l) &= L_1x^{\beta_1}+L_2x^{\beta_2}+L_3x^{\beta_3}+L_4x^{\beta_4} \\ v(x,h) &= H_1x^{\beta_1}+H_2x^{\beta_2}+H_3x^{\beta_3}+H_4x^{\beta_4},\end{aligned} \tag{32}$$

where the β_n are the roots of the characteristic equation, with:

$$\beta_1<\beta_2<0<\beta_3<\beta_4. \tag{33}$$

We know the respective constants are linked via:

$$H_n=\left[\frac{g_l(\beta_n)}{\lambda_l}\right]L_n. \tag{34}$$

So the Canonical ODE System has solutions of the form:

$$\begin{aligned}v(x,l) &= L_1x^{\beta_1}+L_2x^{\beta_2}+L_3x^{\beta_3}+L_4x^{\beta_4} \\ v(x,h) &= H_1x^{\beta_1}+H_2x^{\beta_2}+H_3x^{\beta_3}+H_4x^{\beta_4} \\ &= L_1\left[\frac{g_l(\beta_1)}{\lambda_l}\right]x^{\beta_1}+L_2\left[\frac{g_l(\beta_2)}{\lambda_l}\right]x^{\beta_2}+L_3\left[\frac{g_l(\beta_3)}{\lambda_l}\right]x^{\beta_3}+L_4\left[\frac{g_l(\beta_4)}{\lambda_l}\right]x^{\beta_4}.\end{aligned} \tag{35}$$

Or for brevity we can write:

$$v(x,l) = \sum_{n=1}^{4} x^{\beta_n} L_n \tag{36}$$

$$v(x,h) = \sum_{n=1}^{4} \left[\frac{g_l(\beta_n)}{\lambda_l}\right] x^{\beta_n} L_n. \tag{37}$$

With these solutions in mind, we can now price each of the contingent claims.

10.3 Contingent Down Claim

This claim pays one if and when default occurs in state i unless it has been knocked out by call or default in the other tax state. Let $d^i(x,j,\Omega)$ denote the price of this claim when EBIT is x and the current tax state is j. By the Feynman-Kac formula, this function must satisfy the following system of ODEs:

$$\begin{aligned}
rd^i(x,l,\Omega) &= \mu x d^i_x(x,l,\Omega) + \frac{1}{2}\sigma^2 x^2 d^i_{xx}(x,l,\Omega) + \lambda_l[d^i(x,h,\Omega) - d^i(x,l,\Omega)] \qquad (38)\\
rd^i(x,h,\Omega) &= \mu x d^i_x(x,h,\Omega) + \frac{1}{2}\sigma^2 x^2 d^i_{xx}(x,h,\Omega) + \lambda_h[d^i(x,l,\Omega) - d^i(x,h,\Omega)]\\
d^i(x,i,\Omega) &= 1 \quad \text{if } x \in (0,c]\\
d^i(x,j,\Omega) &= 0 \quad \text{if } x \in (0,c]\\
d^i(x,l,\Omega) &= 0 \quad \text{if } x \in [\gamma^l,\infty)\\
d^i(x,h,\Omega) &= 0 \quad \text{if } x \in [\gamma^h,\infty).
\end{aligned}$$

To solve this system, we need to consider three cases separately depending on the ranking between γ^l and γ^h.

- Case 1: $\gamma^l > \gamma^h$.

  First, we assume that the refinancing threshold is higher in the low tax state. On the region of $[c,\gamma^h]$, equation (38) reduces to a Canonical ODE System with a solution

$$\begin{aligned}
d^i(x,l) &= \sum_{n=1}^{4} x^{\beta_n} L_n \qquad (39)\\
d^i(x,h) &= \sum_{n=1}^{4} \left[\frac{g_l(\beta_n)}{\lambda_l}\right] x^{\beta_n} L_n.
\end{aligned}$$

On the other hand, when $x \in [\gamma^h, \gamma^l]$, equation (38) reduces to a Canonical ODE:

$$\begin{aligned} d^i(x,h) &= 0 \\ (r+\lambda^l)\widetilde{d^i}(x,l) &= \mu x \widetilde{d^i_x}(x,l) + \frac{1}{2}\sigma^2 x^2 \widetilde{d^i_{xx}}(x,l) \\ &\Rightarrow \widetilde{d^i}(x,l) = x^{B_1}K_1 + x^{B_2}K_2. \end{aligned} \tag{40}$$

The boundary conditions give us six linear equations in the six constants, $(L_1, L_2, L_3, L_4, K_1, K_2)$:

$$\begin{aligned} d^i(c,l) &= \sum_{n=1}^{4} c^{\beta_n} L_n = \Phi(i=l) \\ d^i(c,h) &= \sum_{n=1}^{4} \left[\frac{g_l(\beta_n)}{\lambda_l}\right] c^{\beta_n} L_n = \Phi(i=h) \\ d^i(\gamma^h,h) &= \sum_{n=1}^{4} \left[\frac{g_l(\beta_n)}{\lambda_l}\right] (\gamma^h)^{\beta_n} L_n = 0 \\ \widetilde{d^i}(\gamma^l,l) &= (\gamma^l)^{B_1}K_1 + (\gamma^l)^{B_2}K_2 = 0 \\ d^i(\gamma^h,l) &= \widetilde{d^i}(\gamma^h,l) \Rightarrow \sum_{n=1}^{4} (\gamma^h)^{\beta_n} L_n = (\gamma^h)^{B_1}K_1 + (\gamma^h)^{B_2}K_2 \\ d^i_x(\gamma^h,l) &= \widetilde{d^i_x}(\gamma^h,l) \Rightarrow \sum_{n=1}^{4} \beta_n (\gamma^h)^{\beta_n - 1} L_n = B_1(\gamma^h)^{B_1-1}K_1 + B_2(\gamma^h)^{B_2-1}K_2. \end{aligned}$$

Here $\Phi(i=l)$ and $\Phi(i=h)$ are indicator variables. By solving these linear equations, we can pin down the constants and then compute a contingent down claim price.

- Case 2: $\gamma^h > \gamma^l$.

Next, we consider the case where the refinancing threshold is higher in the high tax state. On the region of $[c, \gamma^l]$, equation (38) again reduces to the Canonical ODE System with the solution of form

$$\begin{aligned} d^i(x,l) &= \sum_{n=1}^{4} x^{\beta_n} L_n \\ d^i(x,h) &= \sum_{n=1}^{4} \left[\frac{g_l(\beta_n)}{\lambda_l}\right] x^{\beta_n} L_n. \end{aligned} \tag{41}$$

On the other hand, when $x \in [\gamma^l, \gamma^h]$, equation (38) reduces to the Canonical ODE:

$$\begin{aligned} d^i(x,l) &= 0 \\ (r+\lambda^h)\widetilde{d}^i(x,h) &= \mu x \widetilde{d}^i_x(x,h) + \frac{1}{2}\sigma^2 x^2 \widetilde{d}^i_{xx}(x,h) \\ &\Rightarrow \widetilde{d}^i(x,h) = x^{B_1}K_1 + x^{B_2}K_2. \end{aligned} \quad (42)$$

The boundary conditions give us six linear equations in the six constants, $(L_1, L_2, L_3, L_4, K_1, K_2)$:

$$\begin{aligned} d^i(c,l) &= \sum_{n=1}^{4} c^{\beta_n} L_n = \Phi(i=l) \\ d^i(c,h) &= \sum_{n=1}^{4} \left[\frac{g_l(\beta_n)}{\lambda_l}\right] c^{\beta_n} L_n = \Phi(i=h) \\ d^i(\gamma^l,l) &= \sum_{n=1}^{4} (\gamma^l)^{\beta_n} L_n = 0 \\ \widetilde{d}^i(\gamma^h,h) &= (\gamma^h)^{B_1}K_1 + (\gamma^h)^{B_2}K_2 = 0 \\ d^i(\gamma^l,h) &= \widetilde{d}^i(\gamma^l,h) \Rightarrow \sum_{n=1}^{4} \left[\frac{g_l(\beta_n)}{\lambda_l}\right] (\gamma^l)^{\beta_n} L_n = (\gamma^l)^{B_1}K_1 + (\gamma^l)^{B_2}K_2 \\ d^i_x(\gamma^l,h) &= \widetilde{d}^i_x(\gamma^l,h) \Rightarrow \sum_{n=1}^{4} \left[\frac{g_l(\beta_n)}{\lambda_l}\right] \beta_n (\gamma^l)^{\beta_n - 1} L_n = B_1(\gamma^l)^{B_1-1}K_1 + B_2(\gamma^l)^{B_2-1}K_2. \end{aligned}$$

By solving these linear equations, we can pin down the constants and then compute contingent down claim prices.

- Case 3: $\gamma^h = \gamma^l = \gamma$.

In this case, equation (38) is simply the Canonical ODE System has a solution of the form:

$$\begin{aligned} d^i(x,l) &= \sum_{n=1}^{4} x^{\beta_n} L_n \\ d^i(x,h) &= \sum_{n=1}^{4} \left[\frac{g_l(\beta_n)}{\lambda_l}\right] x^{\beta_n} L_n. \end{aligned} \quad (43)$$

The boundary conditions give us four linear equations in the four constants, (L_1, L_2, L_3, L_4):

$$\begin{aligned}
d^i(c,l) &= \sum_{n=1}^{4} c^{\beta_n} L_n = \Phi(i=l) \\
d^i(c,h) &= \sum_{n=1}^{4} \left[\frac{g_l(\beta_n)}{\lambda_l}\right] c^{\beta_n} L_n = \Phi(i=h) \\
d^i(\gamma,h) &= \sum_{n=1}^{4} \left[\frac{g_l(\beta_n)}{\lambda_l}\right] (\gamma)^{\beta_n} L_n = 0 \\
d^i(\gamma,l) &= \sum_{n=1}^{4} (\gamma)^{\beta_n} L_n = 0.
\end{aligned}$$

By solving these linear equations, we can pin down the constants and then compute contingent down claim prices.

10.4 Contingent Up Claim

This claim pays $x(T)/\gamma^i$, where T is the time of the call, if and when call occurs under tax regime i unless it has been knocked out by default or call in the other tax state j. Let $u^i(x, j, \Omega)$ denote the price of this claim when EBIT is x and the current tax state is j. By the Feynman-Kac formula, this function must satisfy the following system of ODEs:

$$\begin{aligned}
ru^i(x,l,\Omega) &= \mu x u_x^i(x,l,\Omega) + \frac{1}{2}\sigma^2 x^2 u_{xx}^i(x,l,\Omega) + \lambda_l[u^i(x,h,\Omega) - u^i(x,l,\Omega)] \qquad (44) \\
ru^i(x,h,\Omega) &= \mu x u_x^i(x,h,\Omega) + \frac{1}{2}\sigma^2 x^2 u_{xx}^i(x,h,\Omega) + \lambda_h[u^i(x,l,\Omega) - u^i(x,h,\Omega)] \\
u^i(x,i,\Omega) &= 0 \quad \text{if } x \in (0, c^i] \\
u^i(x,j,\Omega) &= 0 \quad \text{if } x \in (0, c^j] \\
u^i(x,i,\Omega) &= \frac{x}{\gamma^i} \quad \text{if } x \in [\gamma^i, \infty) \\
u^i(x,j,\Omega) &= 0 \quad \text{if } x \in [\gamma^j, \infty).
\end{aligned}$$

Note that this formulation ensures that the call (and thus restructuring) might occur due to a jump in a tax regime. Similarly, we consider two cases separately with different payoff states.

10.4.1 Contingent Up Claim: l-State Payoff

- Case 1: $\gamma^l > \gamma^h$.

On the region of $[c, \gamma^h]$, equation (44) reduces to a Canonical ODE System with a solution

$$\begin{aligned} u^l(x,l) &= \sum_{n=1}^{4} x^{\beta_n} L_n \qquad (45)\\ u^l(x,h) &= \sum_{n=1}^{4} \left[\frac{g_l(\beta_n)}{\lambda_l}\right] x^{\beta_n} L_n. \end{aligned}$$

When $x \in [\gamma^h, \gamma^l]$, the claim is worth 0 in the high tax rate state and so the Canonical ODE System Reduces to a Canonical ODE

$$\begin{aligned} u^l(x,h) &= 0\\ (r+\lambda_l)\widetilde{u}^l(x,l) &= \mu x \widetilde{u}^l_x(x,l) + \frac{1}{2}\sigma^2 x^2 \widetilde{u}^l_{xx}(x,l)\\ &\Rightarrow \widetilde{u}^l(x,l) = x^{B_1}K_1 + x^{B_2}K_2. \end{aligned}$$

The boundary conditions give us six linear equations in the six constants, $(L_1, L_2, L_3, L_4, K_1, K_2)$:

$$\begin{aligned} u^l(c,l) &= \sum_{n=1}^{4} c^{\beta_n} L_n = 0\\ u^l(c,h) &= \sum_{n=1}^{4} \left[\frac{g_l(\beta_n)}{\lambda_l}\right] c^{\beta_n} L_n = 0\\ u^l(\gamma^h,h) &= \sum_{n=1}^{4} \left[\frac{g_l(\beta_n)}{\lambda_l}\right] (\gamma^h)^{\beta_n} L_n = 0\\ \widetilde{u}^l(\gamma^l,l) &= (\gamma^l)^{B_1}K_1 + (\gamma^l)^{B_2}K_2 = 1\\ u^l(\gamma^h,l) &= \widetilde{u}^l(\gamma^h,l) \Rightarrow \sum_{n=1}^{4} (\gamma^h)^{\beta_n} L_n = (\gamma^h)^{B_1}K_1 + (\gamma^h)^{B_2}K_2\\ u^l_x(\gamma^h,l) &= \widetilde{u}^l_x(\gamma^h,l) \Rightarrow \sum_{n=1}^{4} \beta_n (\gamma^h)^{\beta_n - 1} L_n = B_1(\gamma^h)^{B_1-1}K_1 + B_2(\gamma^h)^{B_2-1}K_2. \end{aligned}$$

By solving these linear equations, we can pin down the constants and compute the contingent up claim price.

- Case 2: $\gamma^h > \gamma^l$.

On the region of $[c, \gamma^h]$, equation (44) reduces to a Canonical ODE System with a solution of the form

$$
\begin{aligned}
u^l(x,l) &= \sum_{n=1}^{4} x^{\beta_n} L_n \qquad (46)\\
u^l(x,h) &= \sum_{n=1}^{4} \left[\frac{g_l(\beta_n)}{\lambda_l}\right] x^{\beta_n} L_n.
\end{aligned}
$$

When $x \in [\gamma^l, \gamma^h]$, the claim is worth x/γ^l in the low tax rate state and so the Canonical ODE System Reduces to a Canonical ODE

$$
\begin{aligned}
u^l(x,l) &= \frac{x}{\gamma^l}\\
(r+\lambda_h)\widetilde{u}^l(x,h) &= \mu x \widetilde{u}^l_x(x,h) + \frac{1}{2}\sigma^2 x^2 \widetilde{u}^l_{xx}(x,h) + \frac{\lambda_h x}{\gamma^l}\\
&\Rightarrow \ \widetilde{u}^l(x,h) = x^{B_1}K_1 + x^{B_2}K_2 + \frac{\lambda_h x}{\gamma^l(r+\lambda_h-\mu)}.
\end{aligned}
$$

The boundary conditions give us six linear equations in the six constants $(L_1, L_2, L_3, L_4, K_1, K_2)$:

$$
\begin{aligned}
u^l(c,l) &= \sum_{n=1}^{4} c^{\beta_n} L_n = 0\\
u^l(c,h) &= \sum_{n=1}^{4} \left[\frac{g_l(\beta_n)}{\lambda_l}\right] c^{\beta_n} L_n = 0\\
u^l(\gamma^l,l) &= \sum_{n=1}^{4} (\gamma^l)^{\beta_n} L_n = 1\\
\widetilde{u}^l(\gamma^h,h) &= (\gamma^h)^{B_1}K_1 + (\gamma^h)^{B_2}K_2 + \frac{\lambda_h \gamma^h}{\gamma^l(r+\lambda_h-\mu)} = 0\\
u^l(\gamma^l,h) &= \widetilde{u}^l(\gamma^l,h) \Rightarrow \sum_{n=1}^{4} \left[\frac{g_l(\beta_n)}{\lambda_l}\right] (\gamma^l)^{\beta_n} L_n = (\gamma^l)^{B_1}K_1 + (\gamma^l)^{B_2}K_2 + \frac{\lambda_h}{r+\lambda_h-\mu}\\
u^h_x(\gamma^l,h) &= \widetilde{u}^h_x(\gamma^l,h) \Rightarrow \sum_{n=1}^{4} \beta_n \left[\frac{g_l(\beta_n)}{\lambda_l}\right] (\gamma^l)^{\beta_n-1} L_n = B_1(\gamma^l)^{B_1-1}K_1 + B_2(\gamma^l)^{B_2-1}K_2 + \frac{\lambda_h}{\gamma^l(r+\lambda_h-\mu)}.
\end{aligned}
$$

By solving these linear equations, we can pin down the constants and compute the contingent up claim price.

- Case 3: $\gamma^h = \gamma^l = \gamma$.

In this case, equation (44) is simply a Canonical ODE System with a solution of the form

$$\begin{aligned} u^l(x,l) &= \sum_{n=1}^{4} x^{\beta_n} L_n \\ u^l(x,h) &= \sum_{n=1}^{4} \left[\frac{g_l(\beta_n)}{\lambda_l}\right] x^{\beta_n} L_n. \end{aligned} \tag{47}$$

The boundary conditions give us four linear equations in the four constants, (L_1, L_2, L_3, L_4):

$$\begin{aligned} u^l(c,l) &= \sum_{n=1}^{4} c^{\beta_n} L_n = 0 \\ u^l(c,h) &= \sum_{n=1}^{4} \left[\frac{g_l(\beta_n)}{\lambda_l}\right] c^{\beta_n} L_n = 0 \\ u^l(\gamma,l) &= \sum_{n=1}^{4} (\gamma^l)^{\beta_n} L_n = 1 \\ u^l(\gamma,h) &= \sum_{n=1}^{4} \left[\frac{g_l(\beta_n)}{\lambda_l}\right] c^{\beta_n} L_n = 0. \end{aligned}$$

By solving these linear equations, we can pin down the constants and compute the contingent up claim price.

10.4.2 Contingent Up Claim: h-State Payoff

- Case 1: $\gamma^l > \gamma^h$.

  On the region of $[c, \gamma^h]$, equation (44) reduces to a Canonical ODE System with a solution of the form

$$\begin{aligned} u^h(x,l) &= \sum_{n=1}^{4} x^{\beta_n} L_n \\ u^h(x,h) &= \sum_{n=1}^{4} \left[\frac{g_l(\beta_n)}{\lambda_l}\right] x^{\beta_n} L_n. \end{aligned} \tag{48}$$

  When $x \in [\gamma^h, \gamma^l]$, the claim is worth x/γ^h in the high tax rate state and so the Canonical

ODE System Reduces to a Canonical ODE

$$\begin{aligned}
u^h(x,h) &= \frac{x}{\gamma^h} \\
(r+\lambda_l)\widetilde{u}^h(x,l) &= \mu x \widetilde{u}^h_x(x,l) + \frac{1}{2}\sigma^2 x^2 \widetilde{u}^h_{xx}(x,l) + \frac{\lambda_l x}{\gamma^h} \\
&\Rightarrow \widetilde{u}^h(x,l) = x^{B_1}K_1 + x^{B_2}K_2 + \frac{\lambda_l x}{\gamma^h(r+\lambda_l-\mu)}.
\end{aligned}$$

The boundary conditions give us six linear equations in the six constants, $(L_1, L_2, L_3, L_4, K_1, K_2)$:

$$\begin{aligned}
u^h(c,l) &= \sum_{n=1}^{4} c^{\beta_n} L_n = 0 \\
u^h(c,h) &= \sum_{n=1}^{4} \left[\frac{g_l(\beta_n)}{\lambda_l}\right] c^{\beta_n} L_n = 0 \\
u^h(\gamma^h,h) &= \sum_{n=1}^{4} \left[\frac{g_l(\beta_n)}{\lambda_l}\right] (\gamma^h)^{\beta_n} L_n = 1 \\
\widetilde{u}^h(\gamma^l,l) &= (\gamma^l)^{B_1}K_1 + (\gamma^l)^{B_2}K_2 + \frac{\lambda_l \gamma^l}{\gamma^h(r+\lambda_l-\mu)} = 0 \\
u^h(\gamma^h,l) &= \widetilde{u}^h(\gamma^h,l) \Rightarrow \sum_{n=1}^{4}(\gamma^h)^{\beta_n} L_n = (\gamma^h)^{B_1}K_1 + (\gamma^h)^{B_2}K_2 + \frac{\lambda_l}{r+\lambda_l-\mu} \\
u^h_x(\gamma^h,l) &= \widetilde{u}^h_x(\gamma^h,l) \Rightarrow \sum_{n=1}^{4}\beta_n(\gamma^h)^{\beta_n-1} L_n = B_1(\gamma^h)^{B_1-1}K_1 + B_2(\gamma^h)^{B_2-1}K_2 + \frac{\lambda_l}{\gamma^h(r+\lambda_l-\mu)}.
\end{aligned}$$

By solving these linear equations, we can pin down the constants and compute the contingent up claim price.

- Case 2: $\gamma^h > \gamma^l$.

On the region of $[c, \gamma^l]$, equation (44) reduces to a Canonical ODE System with a solution of the form

$$\begin{aligned}
u^h(x,l) &= \sum_{n=1}^{4} x^{\beta_n} L_n \qquad (49)\\
u^h(x,h) &= \sum_{n=1}^{4} \left[\frac{g_l(\beta_n)}{\lambda_l}\right] x^{\beta_n} L_n.
\end{aligned}$$

When $x \in [\gamma^l, \gamma^h]$, the claim is worth 0 in the low tax rate state and so the Canonical ODE

System Reduces to a Canonical ODE

$$\begin{aligned}
u^h(x,l) &= 0 \\
(r+\lambda_h)\widetilde{u}^h(x,h) &= \mu x \widetilde{u}^h_x(x,h) + \frac{1}{2}\sigma^2 x^2 \widetilde{u}^h_{xx}(x,h) \\
&\Rightarrow \widetilde{u}^h(x,h) = x^{B_1}K_1 + x^{B_2}K_2.
\end{aligned}$$

The boundary conditions give us six linear equations in the six constants, $(L_1, L_2, L_3, L_4, K_1, K_2)$:

$$\begin{aligned}
u^h(c,l) &= \sum_{n=1}^{4} c^{\beta_n} L_n = 0 \\
u^h(c,h) &= \sum_{n=1}^{4} \left[\frac{g_l(\beta_n)}{\lambda_l}\right] c^{\beta_n} L_n = 0 \\
u^h(\gamma^l,l) &= \sum_{n=1}^{4} (\gamma^l)^{\beta_n} L_n = 0 \\
\widetilde{u}^h(\gamma^h,h) &= (\gamma^h)^{B_1}K_1 + (\gamma^h)^{B_2}K_2 = 1 \\
u^h(\gamma^l,h) &= \widetilde{u}^h(\gamma^l,h) \Rightarrow \sum_{n=1}^{4} \left[\frac{g_l(\beta_n)}{\lambda_l}\right] (\gamma^l)^{\beta_n} L_n = (\gamma^l)^{B_1}K_1 + (\gamma^l)^{B_2}K_2 \\
u^h_x(\gamma^l,h) &= \widetilde{u}^h_x(\gamma^l,h) \Rightarrow \sum_{n=1}^{4} \beta_n \left[\frac{g_l(\beta_n)}{\lambda_l}\right] (\gamma^l)^{\beta_n - 1} L_n = B_1(\gamma^l)^{B_1-1}K_1 + B_2(\gamma^l)^{B_2-1}K_2.
\end{aligned}$$

By solving these linear equations, we can pin down the constants and then compute contingent up claim prices.

- Case 3: $\gamma^h = \gamma^l = \gamma$.

In this case, equation (44) is simply a Canonical ODE System with a solution of the form

$$\begin{aligned}
u^h(x,l) &= \sum_{n=1}^{4} x^{\beta_n} L_n \\
u^h(x,h) &= \sum_{n=1}^{4} \left[\frac{g_l(\beta_n)}{\lambda_l}\right] x^{\beta_n} L_n.
\end{aligned} \tag{50}$$

The boundary conditions give us four linear equations in the four constants, (L_1, L_2, L_3, L_4):

$$
\begin{aligned}
u^h(c,l) &= \sum_{n=1}^{4} c^{\beta_n} L_n = 0 \\
u^h(c,h) &= \sum_{n=1}^{4} \left[\frac{g_l(\beta_n)}{\lambda_l}\right] c^{\beta_n} L_n = 0 \\
u^h(\gamma,l) &= \sum_{n=1}^{4} (\gamma^l)^{\beta_n} L_n = 0 \\
u^h(\gamma,h) &= \sum_{n=1}^{4} \left[\frac{g_l(\beta_n)}{\lambda_l}\right] c^{\beta_n} L_n = 1.
\end{aligned}
$$

By solving these linear equations, we can pin down the constants and compute the contingent up claim price.

10.5 Adjusted Contingent Up Claim

This claim pays one if and when call occurs under tax regime i unless it has been knocked out by default or upward restructuring in the other tax state j. Let $m^i(x,j,\Omega)$ denote the price of this claim when EBIT is x and the current tax state is j. By the Feynman-Kac formula, this function must satisfy the following system of ODEs:

$$
\begin{aligned}
rm^i(x,l,\Omega) &= \mu x m^i_x(x,l,\Omega) + \frac{1}{2}\sigma^2 x^2 m^i_{xx}(x,l,\Omega) + \lambda_l[m^i(x,h,\Omega) - m^i(x,l,\Omega)] \qquad (51) \\
rm^i(x,h,\Omega) &= \mu x m^i_x(x,h,\Omega) + \frac{1}{2}\sigma^2 x^2 m^i_{xx}(x,h,\Omega) + \lambda_h[m^i(x,l,\Omega) - m^i(x,h,\Omega)] \\
m^i(x,i,\Omega) &= 0 \qquad \text{if } x \in (0,c] \\
m^i(x,j,\Omega) &= 0 \qquad \text{if } x \in (0,c] \\
m^i(x,i,\Omega) &= 1 \qquad \text{if } x \in [\gamma^i,\infty) \\
m^i(x,j,\Omega) &= 0 \qquad \text{if } x \in [\gamma^j,\infty).
\end{aligned}
$$

In the following, we derive a price expression for each payoff state ($i = l, h$).

10.5.1 Adjusted Contingent Up Claim: l-State Payoff

- Case 1: $\gamma^l > \gamma^h$.

On the region of $[c, \gamma^h]$: equation (51) reduces to a Canonical ODE System with a solution

$$\begin{aligned} m^l(x,l) &= \sum_{n=1}^{4} x^{\beta_n} L_n \\ m^l(x,h) &= \sum_{n=1}^{4} \left[\frac{g_l(\beta_n)}{\lambda_l}\right] x^{\beta_n} L_n. \end{aligned} \tag{52}$$

When $x \in [\gamma^h, \gamma^l]$, the claim is worth 0 in the high tax rate state and so the Canonical ODE System Reduces to a Canonical ODE

$$\begin{aligned} m^l(x,h) &= 0 \\ (r+\lambda_l)\widetilde{m}^l(x,l) &= \mu x \widetilde{m}^l_x(x,l) + \frac{1}{2}\sigma^2 x^2 \widetilde{m}^l_{xx}(x,l) \\ &\Rightarrow \widetilde{m}^l(x,l) = x^{B_1} K_1 + x^{B_2} K_2. \end{aligned}$$

The boundary conditions give us six linear equations in the six constants, $(L_1, L_2, L_3, L_4, K_1, K_2)$:

$$\begin{aligned} m^l(c,l) &= \sum_{n=1}^{4} c^{\beta_n} L_n = 0 \\ m^l(c,h) &= \sum_{n=1}^{4} \left[\frac{g_l(\beta_n)}{\lambda_l}\right] c^{\beta_n} L_n = 0 \\ m^l(\gamma^h,h) &= \sum_{n=1}^{4} \left[\frac{g_l(\beta_n)}{\lambda_l}\right] (\gamma^h)^{\beta_n} L_n = 0 \\ \widetilde{m}^l(\gamma^l,l) &= (\gamma^l)^{B_1} K_1 + (\gamma^l)^{B_2} K_2 = 1 \\ m^l(\gamma^h,l) &= \widetilde{m}^l(\gamma^h,l) \Rightarrow \sum_{n=1}^{4} (\gamma^h)^{\beta_n} L_n = (\gamma^h)^{B_1} K_1 + (\gamma^h)^{B_2} K_2 \\ m^l_x(\gamma^h,l) &= \widetilde{m}^l_x(\gamma^h,l) \Rightarrow \sum_{n=1}^{4} \beta_n (\gamma^h)^{\beta_n - 1} L_n = B_1 (\gamma^h)^{B_1 - 1} K_1 + B_2 (\gamma^h)^{B_2 - 1} K_2. \end{aligned}$$

By solving these linear equations, we can pin down the constants and compute the adjusted contingent up claim price.

- Case 2: $\gamma^h > \gamma^l$.

On the region of $[c, \gamma^l]$, equation (51) reduces to a Canonical ODE System with a solution

$$\begin{aligned} m^l(x,l) &= \sum_{n=1}^{4} x^{\beta_n} L_n \qquad (53) \\ m^l(x,h) &= \sum_{n=1}^{4} \left[\frac{g_l(\beta_n)}{\lambda_l}\right] x^{\beta_n} L_n. \end{aligned}$$

When $x \in [\gamma^l, \gamma^h]$, the claim is worth one in the low tax rate state and so the Canonical ODE System Reduces to a Canonical ODE

$$\begin{aligned} \widetilde{m}^l(x,l) &= 1 \\ (r+\lambda_h)\widetilde{m}^l(x,h) &= \mu x \widetilde{m}^l_x(x,h) + \frac{1}{2}\sigma^2 x^2 \widetilde{m}^l_{xx}(x,h) + \lambda_h \\ &\Rightarrow \widetilde{m}^l(x,h) = x^{B_1}K_1 + x^{B_2}K_2 + \frac{\lambda_h}{r+\lambda_h}. \end{aligned}$$

The boundary conditions give us six linear equations in the six constants, $(L_1, L_2, L_3, L_4, K_1, K_2)$:

$$\begin{aligned} m^l(c,l) &= \sum_{n=1}^{4} c^{\beta_n} L_n = 0 \\ m^l(c,h) &= \sum_{n=1}^{4} \left[\frac{g_l(\beta_n)}{\lambda_l}\right] c^{\beta_n} L_n = 0 \\ \widetilde{m}^l(\gamma^h,h) &= (\gamma^h)^{B_1}K_1 + (\gamma^h)^{B_2}K_2 + \frac{\lambda_h}{r+\lambda_h} = 0 \\ m^l(\gamma^l,l) &= \sum_{n=1}^{4} (\gamma^l)^{\beta_n} L_n = 1 \\ m^l(\gamma^l,h) &= \widetilde{m}^h(\gamma^l,h) \Rightarrow \sum_{n=1}^{4} \left[\frac{g_l(\beta_n)}{\lambda_l}\right] (\gamma^l)^{\beta_n} L_n = (\gamma^l)^{B_1}K_1 + (\gamma^l)^{B_2}K_2 + \frac{\lambda_h}{r+\lambda_h} \\ m^l_x(\gamma^l,h) &= \widetilde{m}^l_x(\gamma^l,h) \Rightarrow \sum_{n=1}^{4} \beta_n \left[\frac{g_l(\beta_n)}{\lambda_l}\right] (\gamma^l)^{\beta_n - 1} L_n = B_1(\gamma^l)^{B_1-1}K_1 + B_2(\gamma^l)^{B_2-1}K_2. \end{aligned}$$

By solving these linear equations, we can pin down the constants and compute the adjusted contingent up claim price.

- Case 3: $\gamma^h = \gamma^l = \gamma$.

In this case, equation (51) is simply a Canonical ODE System with a solution of the form:

$$\begin{aligned} m^l(x,l) &= \sum_{n=1}^{4} x^{\beta_n} L_n \\ m^l(x,h) &= \sum_{n=1}^{4} \left[\frac{g_l(\beta_n)}{\lambda_l}\right] x^{\beta_n} L_n. \end{aligned} \tag{54}$$

The boundary conditions give us four linear equations in the four constants, (L_1, L_2, L_3, L_4):

$$\begin{aligned} m^l(c,l) &= \sum_{n=1}^{4} c^{\beta_n} L_n = 0 \\ m^l(c,h) &= \sum_{n=1}^{4} \left[\frac{g_l(\beta_n)}{\lambda_l}\right] c^{\beta_n} L_n = 0 \\ m^l(\gamma,l) &= \sum_{n=1}^{4} \gamma^{\beta_n} L_n = 1 \\ m^l(\gamma,h) &= \sum_{n=1}^{4} \left[\frac{g_l(\beta_n)}{\lambda_l}\right] \gamma^{\beta_n} L_n = 0. \end{aligned}$$

By solving these linear equations, we can pin down the constants and compute the adjusted contingent up claim price.

10.5.2 Adjusted Contingent Up Claim: h-State Payoff

We next consider the pricing of an adjusted contingent up claim with payoff state is h.

- Case 1: $\gamma^l > \gamma^h$.

  On the region of $[c, \gamma^h]$, equation (51) reduces to the Canonical ODE System with the solution form

$$\begin{aligned} m^h(x,l) &= \sum_{n=1}^{4} x^{\beta_n} L_n \\ m^h(x,h) &= \sum_{n=1}^{4} \left[\frac{g_l(\beta_n)}{\lambda_l}\right] x^{\beta_n} L_n. \end{aligned} \tag{55}$$

  When $x \in [\gamma^h, \gamma^l]$, equation (51) instead reduces to the Canonical ODE

$$\begin{aligned} m^h(x,h) &= 1 \\ (r+\lambda_l)\widetilde{m}^h(x,l) &= \mu x \widetilde{m}^h_x(x,l) + \frac{1}{2}\sigma^2 x^2 \widetilde{m}^h_{xx}(x,l) + \lambda_l \\ &\Rightarrow \widetilde{m}^h(x,l) = x^{B_1} K_1 + x^{B_2} K_2 + \frac{\lambda_l}{r+\lambda_l}. \end{aligned}$$

The boundary conditions give us six linear equations in the six constants, $(L_1, L_2, L_3, L_4, K_1, K_2)$:

$$
\begin{aligned}
m^h(c,l) &= \sum_{n=1}^{4} c^{\beta_n} L_n = 0 \\
m^h(c,h) &= \sum_{n=1}^{4} \left[\frac{g_l(\beta_n)}{\lambda_l}\right] c^{\beta_n} L_n = 0 \\
m^h(\gamma^h,h) &= \sum_{n=1}^{4} \left[\frac{g_l(\beta_n)}{\lambda_l}\right] (\gamma^h)^{\beta_n} L_n = 1 \\
\widetilde{m}^h(\gamma^l,l) &= (\gamma^l)^{B_1} K_1 + (\gamma^l)^{B_2} K_2 + \frac{\lambda_l}{r+\lambda_l} = 0 \\
m^h(\gamma^h,l) &= \widetilde{m}^h(\gamma^h,l) \Rightarrow \sum_{n=1}^{4} (\gamma^h)^{\beta_n} L_n = (\gamma^h)^{B_1} K_1 + (\gamma^h)^{B_2} K_2 + \frac{\lambda_l}{r+\lambda_l} \\
m^h_x(\gamma^h,l) &= \widetilde{m}^h_x(\gamma^h,l) \Rightarrow \sum_{n=1}^{4} \beta_n (\gamma^h)^{\beta_n - 1} L_n = B_1 (\gamma^h)^{B_1 - 1} K_1 + B_2 (\gamma^h)^{B_2 - 1} K_2.
\end{aligned}
$$

By solving these linear equations, we can pin down the constants and compute the adjusted contingent up claim price.

- Case 2: $\gamma^h > \gamma^l$.

On the region of $[c, \gamma^l]$, equation (51) reduces to a Canonical ODE System with a solution

$$
\begin{aligned}
m^h(x,l) &= \sum_{n=1}^{4} x^{\beta_n} L_n \\
m^h(x,h) &= \sum_{n=1}^{4} \left[\frac{g_l(\beta_n)}{\lambda_l}\right] x^{\beta_n} L_n.
\end{aligned} \tag{56}
$$

When $x \in [\gamma^l, \gamma^h]$, the claim is worth 0 in the low tax rate state and so the Canonical ODE System Reduces to a Canonical ODE:

$$
\begin{aligned}
\widetilde{m}^h(x,l) &= 0 \\
(r+\lambda_h)\widetilde{m}^h(x,h) &= \mu x \widetilde{m}^h_x(x,h) + \frac{1}{2}\sigma^2 x^2 \widetilde{m}^h_{xx}(x,h) \\
&\Rightarrow \widetilde{m}^h(x,h) = x^{B_1} K_1 + x^{B_2} K_2.
\end{aligned}
$$

The boundary conditions give us six linear equations in the six constants, $(L_1, L_2, L_3, L_4, K_1, K_2)$:

$$\begin{aligned}
m^h(c,l) &= \sum_{n=1}^{4} c^{\beta_n} L_n = 0 \\
m^h(c,h) &= \sum_{n=1}^{4} \left[\frac{g_l(\beta_n)}{\lambda_l}\right] c^{\beta_n} L_n = 0 \\
\widetilde{m}^h(\gamma^h,h) &= (\gamma^h)^{B_1} K_1 + (\gamma^h)^{B_2} K_2 = 1 \\
m^h(\gamma^l,l) &= \sum_{n=1}^{4} (\gamma^l)^{\beta_n} L_n = 0 \\
m^h(\gamma^l,h) &= \widetilde{m}^h(\gamma^l,h) \Rightarrow \sum_{n=1}^{4} \left[\frac{g_l(\beta_n)}{\lambda_l}\right] (\gamma^l)^{\beta_n} L_n = (\gamma^l)^{B_1} K_1 + (\gamma^l)^{B_2} K_2 \\
m^h_x(\gamma^l,h) &= \widetilde{m}^h_x(\gamma^l,h) \Rightarrow \sum_{n=1}^{4} \beta_n \left[\frac{g_l(\beta_n)}{\lambda_l}\right] (\gamma^l)^{\beta_n - 1} L_n = B_1 (\gamma^l)^{B_1 - 1} K_1 + B_2 (\gamma^l)^{B_2 - 1} K_2.
\end{aligned}$$

By solving these linear equations, we can pin down the constants and compute the adjusted contingent up claim price.

- Case 3: $\gamma^h = \gamma^l = \gamma$.

  In this case, equation (51) is simply the Canonical ODE System with a solution of form:

$$\begin{aligned}
m^h(x,l) &= \sum_{n=1}^{4} x^{\beta_n} L_n \\
m^h(x,h) &= \sum_{n=1}^{4} \left[\frac{g_l(\beta_n)}{\lambda_l}\right] x^{\beta_n} L_n.
\end{aligned} \tag{57}$$

  The boundary conditions give us four linear equations in the four constants, (L_1, L_2, L_3, L_4):

$$\begin{aligned}
m^h(c,l) &= \sum_{n=1}^{4} c^{\beta_n} L_n = 0 \\
m^h(c,h) &= \sum_{n=1}^{4} \left[\frac{g_l(\beta_n)}{\lambda_l}\right] c^{\beta_n} L_n = 0 \\
m^h(\gamma,l) &= \sum_{n=1}^{4} \gamma^{\beta_n} L_n = 0 \\
m^h(\gamma,h) &= \sum_{n=1}^{4} \left[\frac{g_l(\beta_n)}{\lambda_l}\right] \gamma^{\beta_n} L_n = 1.
\end{aligned}$$

  By solving these linear equations, we can pin down the constants and compute the adjusted contingent up claim price.

10.6 Contingent Occupation Claims

This claim delivers an instantaneous unit flow of dt whenever the tax regime is equal to i. The claim is knocked by default or call. Let $a^i(x,j,\Omega)$ denote the price of this claim when EBIT is x and the current tax state is j. By the Feynman-Kac formula, this function must satisfy the following system of ODEs:

$$
\begin{aligned}
ra^i(x,i,\Omega) &= 1+\mu x a^i_x(x,i,\Omega)+\frac{1}{2}\sigma^2x^2a^i_{xx}(x,i,\Omega)+\lambda_i[a^i(x,j,\Omega)-a^i(x,i,\Omega)] \\
ra^i(x,j,\Omega) &= \mu x a^i_x(x,j,\Omega)+\frac{1}{2}\sigma^2x^2a^i_{xx}(x,j,\Omega)+\lambda_j[a^i(x,i,\Omega)-a^i(x,j,\Omega)] \\
a^i(x,i,\Omega) &= 0 \quad \text{if } x\in(0,c] \\
a^i(x,j,\Omega) &= 0 \quad \text{if } x\in(0,c] \\
a^i(x,i,\Omega) &= 0 \quad \text{if } x\in[\gamma^i,\infty) \\
a^i(x,j,\Omega) &= 0 \quad \text{if } x\in[\gamma^j,\infty).
\end{aligned} \tag{58}
$$

In the following, we derive a price expression for each payoff state ($i=l,h$).

10.6.1 Contingent Occupation Claims: l-State Payoff

- Case 1: $\gamma^l>\gamma^h$.

  On the region of $[c,\gamma^h]$, equation (58) reduces to a Canonical ODE System with additional constant terms:

$$
\begin{aligned}
ra^l(x,l) &= 1+\mu x a^l_x(x,l)+\frac{1}{2}\sigma^2x^2a^l_{xx}(x,l)+\lambda_l[a^l(x,h)-a^l(x,l)] \\
ra^l(x,h) &= \mu x a^l_x(x,h)+\frac{1}{2}\sigma^2x^2a^l_{xx}(x,h)+\lambda_h[a^l(x,l)-a^l(x,h)].
\end{aligned} \tag{59}
$$

  We can also find a closed-form solution:

$$
\begin{aligned}
a^l(x,l) &= \sum_{n=1}^{4}x^{\beta_n}L_n+\frac{1}{r}-\frac{\lambda_l}{r(r+\lambda_l+\lambda_h)} \\
a^l(x,h) &= \sum_{n=1}^{4}\left[\frac{g_l(\beta_n)}{\lambda_l}\right]x^{\beta_n}L_n+\frac{\lambda_h}{r(r+\lambda_l+\lambda_h)}.
\end{aligned} \tag{60}
$$

When $x \in [\gamma^h, \gamma^l]$, equation (58) reduces to a Canonical ODE:

$$\begin{aligned} a^l(x,h) &= 0 \\ (r+\lambda_l)\widetilde{a}^l(x,l) &= 1 + \mu x \widetilde{a}^l_x(x,l) + \frac{1}{2}\sigma^2 x^2 \widetilde{a}^l_{xx}(x,l) \\ \widetilde{a}^l(x,l) &= x^{B_1}K_1 + x^{B_2}K_2 + \frac{1}{r+\lambda_l}. \end{aligned}$$

The boundary conditions give us six linear equations in the six constants $(L_1, L_2, L_3, L_4, K_1, K_2)$:

$$\begin{aligned} a^l(c,l) &= \sum_{n=1}^{4} c^{\beta_n} L_n + \frac{1}{r} - \frac{\lambda_l}{r(r+\lambda_l+\lambda_h)} = 0 \qquad (61) \\ a^l(c,h) &= \sum_{n=1}^{4} \left[\frac{g_l(\beta_n)}{\lambda_l}\right] c^{\beta_n} L_n + \frac{\lambda_h}{r(r+\lambda_l+\lambda_h)} = 0 \\ a^l(\gamma^h,h) &= \sum_{n=1}^{4} \left[\frac{g_l(\beta_n)}{\lambda_l}\right] (\gamma^h)^{\beta_n} L_n + \frac{\lambda_h}{r(r+\lambda_l+\lambda_h)} = 0 \\ \widetilde{a}^l(\gamma^l,l) &= (\gamma^l)^{B_1}K_1 + (\gamma^l)^{B_2}K_2 + \frac{1}{r+\lambda_l} = 0 \\ a^l(\gamma^h,l) &= \widetilde{a}^l(\gamma^h,l) \Rightarrow \sum_{n=1}^{4} (\gamma^h)^{\beta_n} L_n + \frac{1}{r} - \frac{\lambda_l}{r(r+\lambda_l+\lambda_h)} = (\gamma^h)^{B_1}K_1 + (\gamma^h)^{B_2}K_2 + \frac{1}{r+\lambda_l} \\ a^l_x(\gamma^h,l) &= \widetilde{a}^l_x(\gamma^h,l) \Rightarrow \sum_{n=1}^{4} \beta_n (\gamma^h)^{\beta_n - 1} L_n = B_1(\gamma^h)^{B_1-1}K_1 + B_2(\gamma^h)^{B_2-1}K_2. \end{aligned}$$

By solving these linear equations, we can pin down the constants and then compute contingent occupation claim prices.

- Case 2: $\gamma^h > \gamma^l$

We have the following differential equations that must be satisfied in the interval (c, γ^l):

$$\begin{aligned} ra^l(x,l) &= 1 + \mu x a^l_x(x,l) + \frac{1}{2}\sigma^2 x^2 a^l_{xx}(x,l) + \lambda_l[a^l(x,h) - a^l(x,l)] \qquad (62) \\ ra^l(x,h) &= \mu x a^l_x(x,h) + \frac{1}{2}\sigma^2 x^2 a^l_{xx}(x,h) + \lambda_h[a^l(x,l) - a^l(x,h)]. \end{aligned}$$

The solution is:

$$\begin{aligned} a^l(x,l) &= \sum_{n=1}^{4} x^{\beta_n} L_n + \frac{1}{r} - \frac{\lambda_l}{r(r+\lambda_l+\lambda_h)} \qquad (63) \\ a^l(x,h) &= \sum_{n=1}^{4} \left[\frac{g_l(\beta_n)}{\lambda_l}\right] x^{\beta_n} L_n + \frac{\lambda_h}{r(r+\lambda_l+\lambda_h)}. \end{aligned}$$

And on the interval $[\gamma^l, \gamma^h]$ we have:

$$\begin{aligned} a^l(x,l) &= 0 \\ (r+\lambda_h)\widetilde{a}^l(x,h) &= \mu x \widetilde{a}^l_x(x,h) + \frac{1}{2}\sigma^2 x^2 \widetilde{a}^l_{xx}(x,h) \\ \widetilde{a}^l(x,h) &= x^{B_1}K_1 + x^{B_2}K_2. \end{aligned}$$

The boundary conditions give us six linear equations in the six constants $(L_1, L_2, L_3, L_4, K_1, K_2)$:

$$\begin{aligned} a^l(c,l) &= \sum_{n=1}^{4} c^{\beta_n} L_n + \frac{1}{r} - \frac{\lambda_l}{r(r+\lambda_l+\lambda_h)} = 0 \qquad (64) \\ a^l(c,h) &= \sum_{n=1}^{4} \left[\frac{g_l(\beta_n)}{\lambda_l}\right] c^{\beta_n} L_n + \frac{\lambda_h}{r(r+\lambda_l+\lambda_h)} = 0 \\ a^l(\gamma^l,l) &= \sum_{n=1}^{4} (\gamma^l)^{\beta_n} L_n + \frac{1}{r} - \frac{\lambda_l}{r(r+\lambda_l+\lambda_h)} = 0 \\ \widetilde{a}^l(\gamma^h,h) &= (\gamma^h)^{B_1}K_1 + (\gamma^h)^{B_2}K_2 = 0 \\ a^l(\gamma^l,h) &= \widetilde{a}^l(\gamma^l,h) \Rightarrow \sum_{n=1}^{4} \left[\frac{g_l(\beta_n)}{\lambda_l}\right] (\gamma^l)^{\beta_n} L_n + \frac{\lambda_h}{r(r+\lambda_l+\lambda_h)} = (\gamma^l)^{B_1}K_1 + (\gamma^l)^{B_2}K_2 \\ a^l_x(\gamma^l,h) &= \widetilde{a}^l_x(\gamma^l,h) \Rightarrow \sum_{n=1}^{4} \beta_n \left[\frac{g_l(\beta_n)}{\lambda_l}\right] (\gamma^l)^{\beta_n-1} L_n = B_1(\gamma^l)^{B_1-1}K_1 + B_2(\gamma^l)^{B_2-1}K_2. \end{aligned}$$

By solving these linear equations, we can pin down the constants and then compute contingent occupation claim prices.

- Case 3: $\gamma^h = \gamma^l = \gamma$

We have the following differential equations that must be satisfied in the interval (c, γ):

$$\begin{aligned} ra^l(x,l) &= 1 + \mu x a^l_x(x,l) + \frac{1}{2}\sigma^2 x^2 a^l_{xx}(x,l) + \lambda_l[a^l(x,h) - a^l(x,l)] \qquad (65) \\ ra^l(x,h) &= \mu x a^l_x(x,h) + \frac{1}{2}\sigma^2 x^2 a^l_{xx}(x,h) + \lambda_h[a^l(x,l) - a^l(x,h)]. \end{aligned}$$

The solution is:

$$\begin{aligned} a^l(x,l) &= \sum_{n=1}^{4} x^{\beta_n} L_n + \frac{1}{r} - \frac{\lambda_l}{r(r+\lambda_l+\lambda_h)} \qquad (66) \\ a^l(x,h) &= \sum_{n=1}^{4} \left[\frac{g_l(\beta_n)}{\lambda_l}\right] x^{\beta_n} L_n + \frac{\lambda_h}{r(r+\lambda_l+\lambda_h)} \end{aligned}$$

The boundary conditions give us four linear equations in the four constants, (L_1, L_2, L_3, L_4):

$$\begin{aligned} a^l(c,l) &= \sum_{n=1}^{4} c^{\beta_n} L_n + \frac{1}{r} - \frac{\lambda_l}{r(r+\lambda_l+\lambda_h)} = 0 \\ a^l(c,h) &= \sum_{n=1}^{4} \left[\frac{g_l(\beta_n)}{\lambda_l}\right] c^{\beta_n} L_n + \frac{\lambda_h}{r(r+\lambda_l+\lambda_h)} = 0 \\ a^l(\gamma,l) &= \sum_{n=1}^{4} \gamma^{\beta_n} L_n + \frac{1}{r} - \frac{\lambda_l}{r(r+\lambda_l+\lambda_h)} = 0 \\ a^l(\gamma,h) &= \sum_{n=1}^{4} \left[\frac{g_l(\beta_n)}{\lambda_l}\right] \gamma^{\beta_n} L_n + \frac{\lambda_h}{r(r+\lambda_l+\lambda_h)} = 0. \end{aligned} \tag{67}$$

By solving these linear equations, we can pin down the constants and then compute contingent occupation claim prices.

10.6.2 Contingent Occupation Claims: h-State Payoff

- Case 1: $\gamma^l > \gamma^h$

  We have the following differential equations that must be satisfied in the interval (c, γ^h):

$$\begin{aligned} ra^h(x,l) &= \mu x a^h_x(x,l) + \frac{1}{2}\sigma^2 x^2 a^h_{xx}(x,l) + \lambda_l[a^h(x,h) - a^h(x,l)] \\ ra^h(x,h) &= 1 + \mu x a^h_x(x,h) + \frac{1}{2}\sigma^2 x^2 a^h_{xx}(x,h) + \lambda_h[a^h(x,l) - a^h(x,h)]. \end{aligned} \tag{68}$$

  The solution is:

$$\begin{aligned} a^h(x,l) &= \sum_{n=1}^{4} x^{\beta_n} L_n + \frac{\lambda_l}{r(r+\lambda_l+\lambda_h)} \\ a^h(x,h) &= \sum_{n=1}^{4} \left[\frac{g_l(\beta_n)}{\lambda_l}\right] x^{\beta_n} L_n + \frac{1}{r} - \frac{\lambda_h}{r(r+\lambda_l+\lambda_h)}. \end{aligned} \tag{69}$$

  And on the interval $[\gamma^h, \gamma^l]$ we have:

$$\begin{aligned} a^h(x,h) &= 0 \\ (r+\lambda_l)\widetilde{a}^h(x,l) &= \mu x \widetilde{a}^h_x(x,l) + \frac{1}{2}\sigma^2 x^2 \widetilde{a}^h_{xx}(x,l) \\ \widetilde{a}^h(x,l) &= x^{B_1} K_1 + x^{B_2} K_2 \end{aligned}$$

Boundary conditions:

$$\begin{aligned} a^h(c,l) &= \sum_{n=1}^{4} c^{\beta_n} L_n + \frac{\lambda_l}{r(r+\lambda_l+\lambda_h)} = 0 \\ a^h(c,h) &= \sum_{n=1}^{4} \left[\frac{g_l(\beta_n)}{\lambda_l}\right] c^{\beta_n} L_n + \frac{1}{r} - \frac{\lambda_h}{r(r+\lambda_l+\lambda_h)} = 0 \\ a^h(\gamma^h,h) &= \sum_{n=1}^{4} \left[\frac{g_l(\beta_n)}{\lambda_l}\right] (\gamma^h)^{\beta_n} L_n + \frac{1}{r} - \frac{\lambda_h}{r(r+\lambda_l+\lambda_h)} = 0 \\ \widetilde{a}^h(\gamma^l,l) &= (\gamma^l)^{B_1} K_1 + (\gamma^l)^{B_2} K_2 = 0 \\ a^h(\gamma^h,l) &= \widetilde{a}^h(\gamma^h,l) \Rightarrow \sum_{n=1}^{4} (\gamma^h)^{\beta_n} L_n + \frac{\lambda_l}{r(r+\lambda_l+\lambda_h)} = (\gamma^h)^{B_1} K_1 + (\gamma^h)^{B_2} K_2 \\ a^h_x(\gamma^h,l) &= \widetilde{a}^h_x(\gamma^h,l) \Rightarrow \sum_{n=1}^{4} \beta_n (\gamma^h)^{\beta_n - 1} L_n = B_1 (\gamma^h)^{B_1 - 1} K_1 + B_2 (\gamma^h)^{B_2 - 1} K_2. \end{aligned} \tag{70}$$

- Case 2: $\gamma^h > \gamma^l$

  We have the following differential equations that must be satisfied in the interval (c, γ^l):

$$\begin{aligned} ra^h(x,l) &= \mu x a^h_x(x,l) + \frac{1}{2}\sigma^2 x^2 a^h_{xx}(x,l) + \lambda_l [a^h(x,h) - a^h(x,l)] \\ ra^h(x,h) &= 1 + \mu x a^h_x(x,h) + \frac{1}{2}\sigma^2 x^2 a^h_{xx}(x,h) + \lambda_h [a^h(x,l) - a^h(x,h)]. \end{aligned} \tag{71}$$

  The solution is:

$$\begin{aligned} a^h(x,l) &= \sum_{n=1}^{4} x^{\beta_n} L_n + \frac{\lambda_l}{r(r+\lambda_l+\lambda_h)} \\ a^h(x,h) &= \sum_{n=1}^{4} \left[\frac{g_l(\beta_n)}{\lambda_l}\right] x^{\beta_n} L_n + \frac{1}{r} - \frac{\lambda_h}{r(r+\lambda_l+\lambda_h)}. \end{aligned} \tag{72}$$

  And on the interval $[\gamma^l, \gamma^h]$ we have:

$$\begin{aligned} a^h(x,l) &= 0 \\ (r+\lambda_h)\widetilde{a}^h(x,h) &= 1 + \mu x \widetilde{a}^h_x(x,h) + \frac{1}{2}\sigma^2 x^2 \widetilde{a}^h_{xx}(x,h) \\ \widetilde{a}^h(x,h) &= x^{B_1} K_1 + x^{B_2} K_2 + \frac{1}{r+\lambda_h} \end{aligned}$$

Boundary conditions:

$$
\begin{aligned}
a^h(c,l) &= \sum_{n=1}^{4} c^{\beta_n} L_n + \frac{\lambda_l}{r(r+\lambda_l+\lambda_h)} = 0 \qquad (73)\\
a^h(c,h) &= \sum_{n=1}^{4} \left[\frac{g_l(\beta_n)}{\lambda_l}\right] c^{\beta_n} L_n + \frac{1}{r} - \frac{\lambda_h}{r(r+\lambda_l+\lambda_h)} = 0\\
a^h(\gamma^l,l) &= \sum_{n=1}^{4} (\gamma^l)^{\beta_n} L_n + \frac{\lambda_l}{r(r+\lambda_l+\lambda_h)} = 0\\
\widetilde{a}^h(\gamma^h,h) &= (\gamma^h)^{B_1} K_1 + (\gamma^h)^{B_2} K_2 + \frac{1}{r+\lambda_h} = 0\\
a^h(\gamma^l,h) &= \widetilde{a}^h(\gamma^l,h) \Rightarrow \sum_{n=1}^{4} \left[\frac{g_l(\beta_n)}{\lambda_l}\right] (\gamma^l)^{\beta_n} L_n + \frac{1}{r} - \frac{\lambda_h}{r(r+\lambda_l+\lambda_h)} = (\gamma^l)^{B_1} K_1 + (\gamma^l)^{B_2} K_2 + \frac{1}{r+\lambda_h}\\
a^h_x(\gamma^l,h) &= \widetilde{a}^h_x(\gamma^l,h) \Rightarrow \sum_{n=1}^{4} \beta_n \left[\frac{g_l(\beta_n)}{\lambda_l}\right] (\gamma^l)^{\beta_n-1} L_n = B_1(\gamma^l)^{B_1-1} K_1 + B_2(\gamma^l)^{B_2-1} K_2.
\end{aligned}
$$

- Case 3: $\gamma^h = \gamma^l = \gamma$

We have the following differential equations that must be satisfied in the interval (c,γ):

$$
\begin{aligned}
ra^h(x,l) &= \mu x a^h_x(x,l) + \frac{1}{2}\sigma^2 x^2 a^h_{xx}(x,l) + \lambda_l[a^h(x,h) - a^h(x,l)] \qquad (74)\\
ra^h(x,h) &= 1 + \mu x a^h_x(x,h) + \frac{1}{2}\sigma^2 x^2 a^h_{xx}(x,h) + \lambda_h[a^h(x,l) - a^h(x,h)].
\end{aligned}
$$

The solution is:

$$
\begin{aligned}
a^h(x,l) &= \sum_{n=1}^{4} x^{\beta_n} L_n + \frac{\lambda_l}{r(r+\lambda_l+\lambda_h)} \qquad (75)\\
a^h(x,h) &= \sum_{n=1}^{4} \left[\frac{g_l(\beta_n)}{\lambda_l}\right] x^{\beta_n} L_n + \frac{1}{r} - \frac{\lambda_h}{r(r+\lambda_l+\lambda_h)}.
\end{aligned}
$$

Boundary conditions:

$$
\begin{aligned}
a^h(c,l) &= \sum_{n=1}^{4} c^{\beta_n} L_n + \frac{\lambda_l}{r(r+\lambda_l+\lambda_h)} = 0 \qquad (76)\\
a^h(c,h) &= \sum_{n=1}^{4} \left[\frac{g_l(\beta_n)}{\lambda_l}\right] c^{\beta_n} L_n + \frac{1}{r} - \frac{\lambda_h}{r(r+\lambda_l+\lambda_h)} = 0\\
a^h(\gamma,l) &= \sum_{n=1}^{4} \gamma^{\beta_n} L_n + \frac{\lambda_l}{r(r+\lambda_l+\lambda_h)} = 0\\
a^h(\gamma,h) &= \sum_{n=1}^{4} \left[\frac{g_l(\beta_n)}{\lambda_l}\right] \gamma^{\beta_n} L_n + \frac{1}{r} - \frac{\lambda_h}{r(r+\lambda_l+\lambda_h)} = 0.
\end{aligned}
$$

10.7 Components of firm value

We here exploit the recursive relationships derived in the body of the paper to pin down the components of firm value. Beginning first with Bankruptcy Costs, we have:

$$BC(1,l,\Omega_l) = \frac{d^l(1,l,\Omega_l)\alpha A(c_l,l) + d^h(1,l,\Omega_l)\alpha A(c_l,h) + u^h(1,l,\Omega_l)\gamma_l^h BC(1,h,\Omega_h)}{1 - u^l(1,l,\Omega_l)\gamma_l^l} \tag{77}$$

$$BC(1,h,\Omega_h) = \frac{d^l(1,h,\Omega_h)\alpha A(c_h,l) + d^h(1,h,\Omega_h)\alpha A(c_h,h) + u^l(1,h,\Omega_h)\gamma_h^l BC(1,l,\Omega_l)}{1 - u^h(1,h,\Omega_h)\gamma_h^h}. \tag{78}$$

Consider next Issuance Costs. We have:

$$IC(1,l,\Omega_l) = \frac{u^l(1,l,\Omega_l)\gamma_l^l qD(1,l,\Omega_l) + u^h(1,l,\Omega_l)\gamma_l^h[qD(1,h,\Omega_h) + IC(1,h,\Omega_h)]}{1 - u^l(1,l,\Omega_l)\gamma_l^l} \tag{79}$$

$$IC(1,h,\Omega_h) = \frac{u^h(1,h,\Omega_h)\gamma_h^h qD(1,h,\Omega_h) + u^l(1,h,\Omega_h)\gamma_h^l[qD(1,l,\Omega_l) + IC(1,l,\Omega_l)]}{1 - u^h(1,h,\Omega_h)\gamma_h^h} \tag{80}$$

Consider finally Tax Benefits. We have:

$$TB(1,l,\Omega_l) = \frac{\tau_l c_l a^l(1,l,\Omega_l) + \tau_h c_l a^h(1,l,\Omega_l) + u^h(1,l,\Omega_i)\gamma_l^h TB(1,h,\Omega_h)}{1 - u^l(1,l,\Omega_l)\gamma_l^l} \tag{81}$$

$$TB(1,h,\Omega_h) = \frac{\tau_l c_h a^l(1,h,\Omega_h) + \tau_h c_h a^h(1,h,\Omega_h) + u^l(1,h,\Omega_h)\gamma_h^l TB(1,l,\Omega_l)}{1 - u^h(1,h,\Omega_h)\gamma_h^h} \tag{82}$$

10.8 Bankruptcy Costs, Issuance Costs and Tax Benefits

Beginning first with Bankruptcy Costs, we have:

$$BC(1,l,\Omega_l) = \frac{d^l(1,l,\Omega_l)\alpha A(c_l,l) + d^h(1,l,\Omega_l)\alpha A(c_l,h) + u^h(1,l,\Omega_l)\gamma_l^h BC(1,h,\Omega_h)}{1 - u^l(1,l,\Omega_l)\gamma_l^l} \tag{83}$$

$$BC(1,h,\Omega_h) = \frac{d^l(1,h,\Omega_h)\alpha A(c_h,l) + d^h(1,h,\Omega_h)\alpha A(c_h,h) + u^l(1,h,\Omega_h)\gamma_h^l BC(1,l,\Omega_l)}{1 - u^h(1,h,\Omega_h)\gamma_h^h} \tag{84}$$

Consider next Issuance Costs. We have:

$$IC(1,l,\Omega_l) = \frac{u^l(1,l,\Omega_l)\gamma_l^l qD(1,l,\Omega_l) + u^h(1,l,\Omega_l)\gamma_l^h[qD(1,h,\Omega_h) + IC(1,h,\Omega_h)]}{1 - u^l(1,l,\Omega_l)\gamma_l^l} \tag{85}$$

$$IC(1,h,\Omega_h) = \frac{u^h(1,h,\Omega_h)\gamma_h^h qD(1,h,\Omega_h) + u^l(1,h,\Omega_h)\gamma_h^l[qD(1,l,\Omega_l) + IC(1,l,\Omega_l)]}{1 - u^h(1,h,\Omega_h)\gamma_h^h} \tag{86}$$

Consider finally Tax Benefits. We have:

$$TB(1,l,\Omega_l) = \frac{\tau_l c_l a^l(1,l,\Omega_l) + \tau_h c_l a^h(1,l,\Omega_l) + u^h(1,l,\Omega_i)\gamma_l^h TB(1,h,\Omega_h)}{1 - u^l(1,l,\Omega_l)\gamma_l^l} \tag{87}$$

$$TB(1,h,\Omega_h) = \frac{\tau_l c_h a^l(1,h,\Omega_h) + \tau_h c_h a^h(1,h,\Omega_h) + u^l(1,h,\Omega_h)\gamma_h^l TB(1,l,\Omega_l)}{1 - u^h(1,h,\Omega_h)\gamma_h^h} \tag{88}$$

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Table 1: Optimal Capital Structure: The Constant Tax Rate

This table reports the optimal date zero capital structure for different assumed values of the tax rate parameter τ in a constant tax rate model. Here L is the market leverage ratio, D is the market value of debt, X_0 is EBIT at date zero, c is the debt coupon, and l and h are tax states at date 0.

	L	D	X_0/c
$i = l$	0.2859	5.243	3.428
$i = h$	0.3000	5.307	3.324

Table 2: Control Variables

This tables reports the optimal capital structure policies at date zero for three value of λ, the tax regime frequency change parameter. Here c_i denotes coupon rate when the refinancing state is i, and γ_i^j is the optimal state j refinancing threshold for debt issued in tax state i. Tax states at date 0 are denoted by l and h.

Panel A: $\lambda = 0$

	c_i	γ_i
$i = l$	0.292	1.259
$i = h$	0.301	1.247

Panel B: $\lambda = 0.1$

	c_i	γ_i^l	γ_i^h
$i = l$	0.293	1.2580	1.216
$i = h$	0.300	1.291	1.2476

Panel C: $\lambda = 0.5$

	c_i	γ_i^l	γ_i^h
$i = l$	0.295	1.2579	1.234
$i = h$	0.298	1.272	1.2479

Table 3: Optimal Capital Structure at Date Zero

This table reports optimal capital structure variables at date zero for two values of λ, the tax regime frequency change parameter. Here L is the market leverage ratio, D is the market value of debt, X_0 is EBIT at date 0, c is the debt coupon, and l and h are tax states at date 0.

Panel A: $\lambda = 0.1$

	L	D	X_0/c
$i = l$	0.2889	5.220	3.419
$i = h$	0.2970	5.334	3.331

Panel B: $\lambda = 0.5$

	L	D	X_0/c
$i = l$	0.2914	5.252	3.393
$i = h$	0.2947	5.304	3.354

Table 4: Stationary Capital Structure

This table reports the values of capital structure variables in the stationary environment in the two-state tax regime model for two values of λ, the tax regime frequency change parameter. The two tax states are $\tau_l = 0.36$ and $\tau_h = 0.4$. Leverage is the market leverage ratio, Refinancing Frequency is the fraction of firms that refinance in one year, and Interest Coverage Ratio is the ratio of EBIT over debt coupon.

	$\lambda = 0.1$	$\lambda = 0.5$
Leverage	0.3553	0.3558
Refinancing Frequency	0.2550	0.2548
Interest Coverage Ratio	2.913	2.909

Table 5: Difference-in-Differences Regressions and Statistical Power

This table reports the analysis of difference-in-differences panel regressions in a two-state tax regime model, with $\tau_l = 0.36$ and $\tau_h = 0.4$. The results are reported for 1000 simulation economies, with each economy featuring 1000 treated and 1000 control firms. The sample window is 8 quarters, centered around the tax rate change event. Mean Beta is the average value of β_1 over all the simulated economies, where β_1 is estimated from a panel regression given in Equation (20). Leverage is the market leverage ratio, Refinancing Frequency (RF) is the fraction of firms that refinance in one quarter, and Interest Coverage Ratio (ICR) is the ratio of EBIT over debt coupon. Numbers in brackets show Mean Beta scaled by differences in the optimal leverage ratio (optimal ICR) between the two tax-rate states in the one-state model at date zero for the Leverage (ICR) columns, and scaled by the average refinancing frequency in the one-state model in the old tax regime for the RF columns. 95% Statistical Significance is the fraction of the simulated economies, in which the β_1 coefficient is statistically significant from zero at the 95% level.

Panel A: Tax Increase

	Leverage		Refinancing Frequency		Interest Coverage Ratio	
	$\lambda = 0.1$	$\lambda = 0.5$	$\lambda = 0.1$	$\lambda = 0.5$	$\lambda = 0.1$	$\lambda = 0.5$
Mean Beta	0.0045	0.0019	0.0125	0.0071	-0.0383	-0.0200
	(32.2%)	(13.8%)	(25.9%)	(14.8%)	(36.9%)	(19.3%)
Median Beta	0.0045	0.0020	0.0104	0.0053	-0.0334	-0.0177
Standard Deviation of Beta	0.0028	0.0021	0.0108	0.0087	0.0329	0.0212
95% Statistical Significance	55.6%	27.2%	57.6%	27.6%	52.1%	32.8%

Panel B: Tax Decrease

	Leverage		Refinancing Frequency		Interest Coverage Ratio	
	$\lambda = 0.1$	$\lambda = 0.5$	$\lambda = 0.1$	$\lambda = 0.5$	$\lambda = 0.1$	$\lambda = 0.5$
Mean Beta	-0.0041	-0.0017	-0.0114	-0.0069	0.0329	0.0178
	(28.8%)	(11.8%)	(-22.9%)	(-13.8%)	(31.7%)	(17.2%)
Median Beta	-0.0039	-0.0018	-0.0093	-0.0050	0.0268	0.0165
Standard Deviation of Beta	0.0028	0.0022	0.0104	0.0082	0.0327	0.0214
95% Statistical Significance	50.2%	25.7%	54.1%	27.8%	46.5%	31.0%

Figure 1: Difference-in-Differences Coefficient for Leverage Ratio

The histogram reports the distribution of coefficient β_1 in the difference-in-differences panel regression on leverage ratio for 1000 simulated economies in the case of the tax increasing reform with $\lambda = 0.1$.

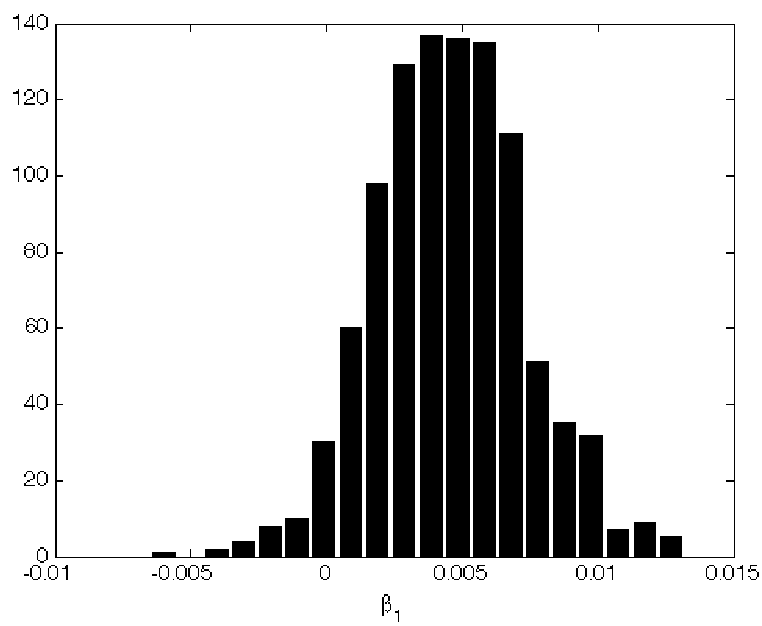

Figure 2: Leverage Response to Tax Reforms and λ

This figure shows the comparative statistics of leverage response to a tax increasing reform, β_1, with respect to the tax reform frequency parameter, λ. The first panel plots average DD estimates of β_1. The second panel shows mean β_1 scaled by differences in the optimal leverage ratios for the one-state model at date zero for $\tau_l = 0.36$ and $\tau_h = 0.40$. The third panel reports the fraction of the simulated economies, for which β_1 is statistically significant from zero at the 95% level.

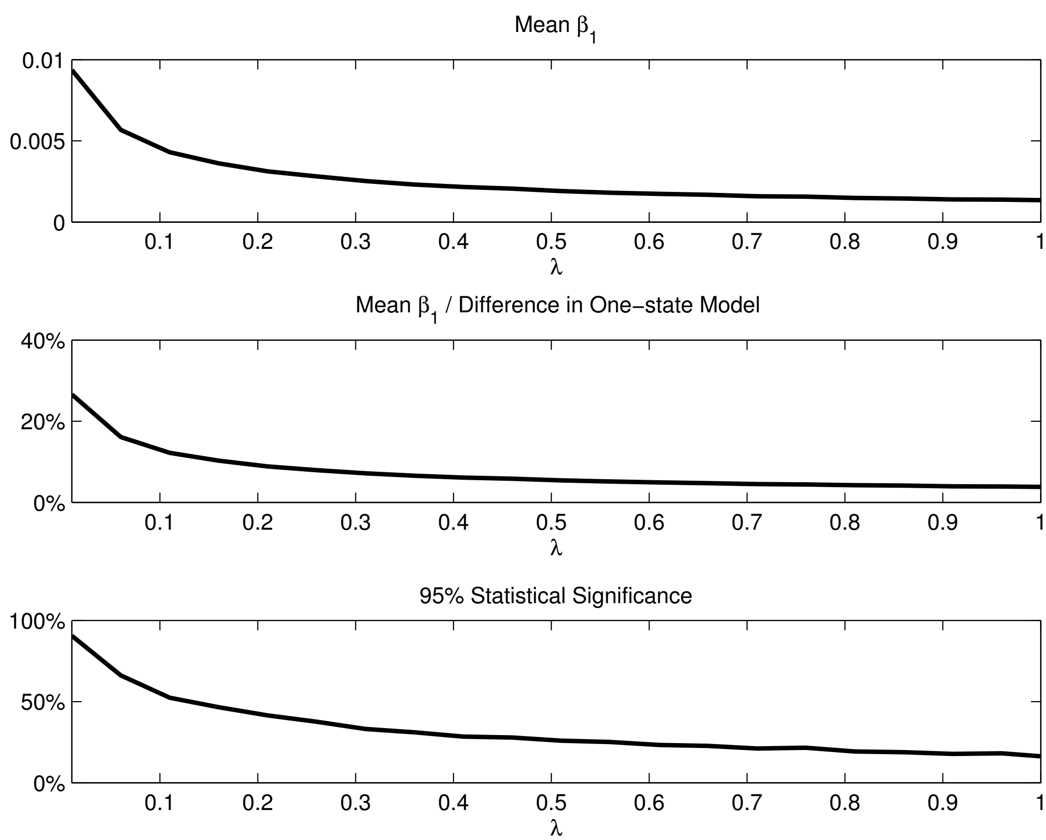

Figure 3: Statistical Power and Model Parameters

This figure plots the comparative statics of statistical significance with respect to the model parameters. Statistical significance is measured as the fraction of simulated economies in which the DD regression on leverage ratio in a tax increasing reform scenario, β_1, is statistically significant from zero at the 95% level. N is the number of treated firms in each simulated economy, $\Delta\tau$ is the difference between τ_h and τ_l, q is the size of debt issuance costs, and Quarter is the length of the sample window in quarters.

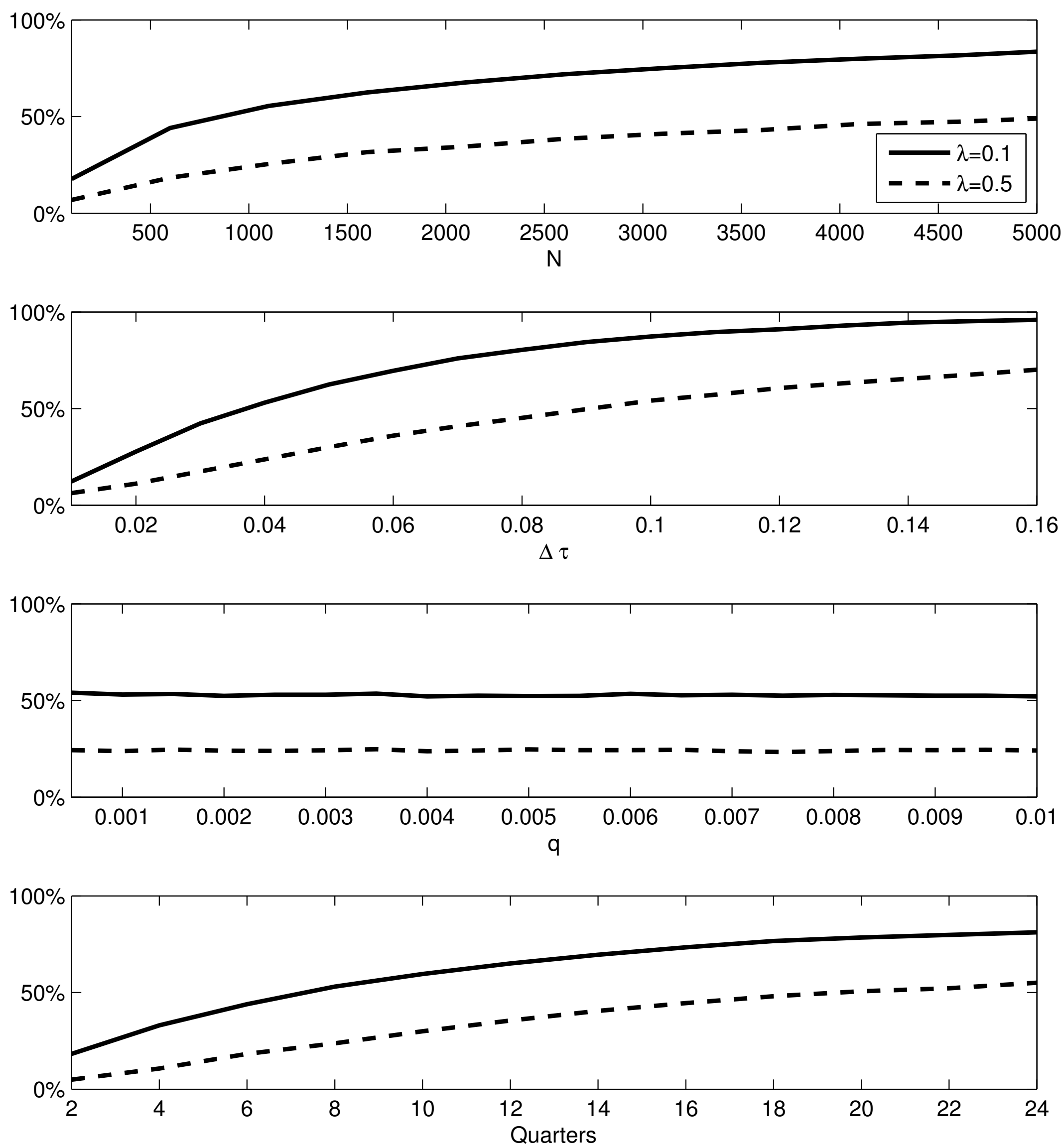

Figure 4: Asymmetric Response to Tax Increase and Decrease

This figure plots the relative response of leverage ratios to a tax increase versus a tax decrease, measured as z given in Equation (21), with respect to $\Delta\tau$, the difference between τ_h and τ_l.

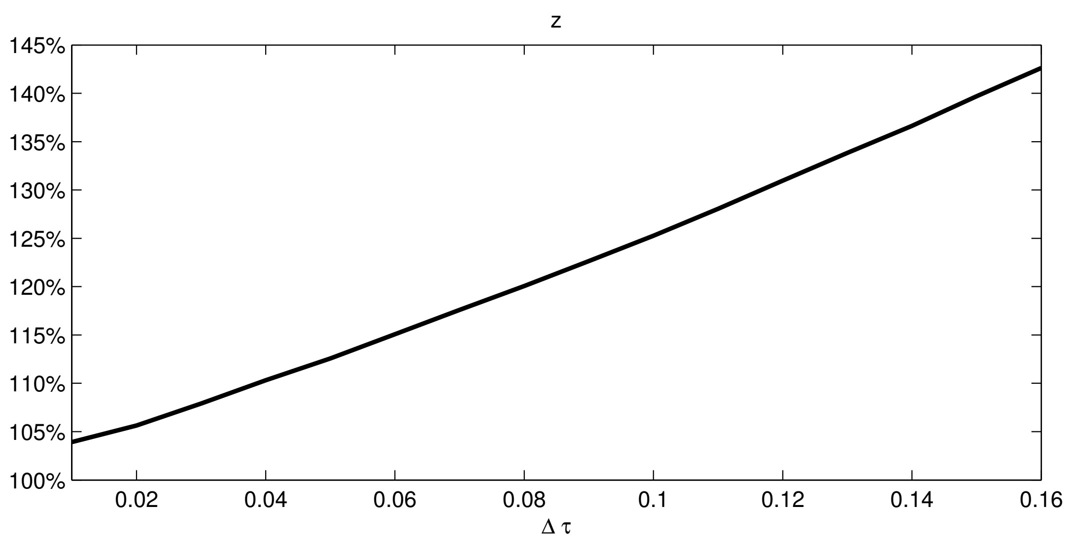

Figure 5: Welfare Losses, Tax Reforms, and Bankruptcy Costs

This figure plots deadweight losses with respect to the bankruptcy cost parameter, α. Deadweight loss is defined as the present value of future bankruptcy costs. The first panel plots the average deadweight loss in low and high tax states. The second panel shows the difference in deadweight losses between the high and the low tax states. The third panel shows the average value of leverage response to a tax increasing reform, β_1, over all the simulated economies.