Monetary and Fiscal Policy in a Cash-in-advance Economy with Quasi-geometric Discounting*

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Abstract

This paper develops a dynamic general equilibrium model in which households have a preference of quasi-geometric discounting, as in Krusell et al. (2002), and face a cash-in-advance constraint. Through this extension, we obtain the following three outcomes; (i) this economy engages in over-saving, although households have a present bias; (ii) when the government can control only money supply, the Friedman rule is optimal; (iii) when the government can control both money supply and income tax rates, the optimal inflation rate can be positive.

Keywords: Quasi-geometric discounting; Friedman rule

JEL classification: E21, E40, E70

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1 Introduction

Many experimental evidences suggest that discounting of future rewards is not geometric. Ainslie (1992) is one of the most famous study in experimental studies. Moreover, Salois and Moss (2011) directly estimate the hyperbolic discounting parameter from asset market data.

Krusell et al.(2002) (hereafter KKS) is an important theoretical study. They introduce quasi-geometric discounting into the standard discrete-time dynamic general equilibrium model. KKS solves the household's problem as a game between current self and future self. They obtain the result in which the saving rate in a competitive equilibrium is smaller than that for maximizing welfare. Krusell et al.(2000) study the case in which labor supply is determined endogenously.

In our paper, we extend KKS to a monetary economy. We assume that households face a cash-in-advance (CIA) constraint. Through this assumption, we obtain the following three outcomes. First, this economy engages in over-saving, although households have a present bias when a weak preference for leisure. Households who have a present bias want to delay saving more, and increase their consumption and leisure more than households with standard geometric discounting do. However, households do not succeed in increasing consumption because they face the CIA constraint. Households can make future themselves more save instead. Because the present households are also increased their saving by the past themselves, the present consumption is decreased. Although households cannot commit future labor supply, they do not decrease labor supply and income from it if they do not much prefer to leisure. Therefore, there exists the case in which over-saving occurs. This result is quite different from that of KKS in which the saving rate in a competitive equilibrium is too small relative to the optimal rate.

Second, when the government can control only money supply, the Friedman rule is optimal. The reason is that labor supply is decreased by the existence of the present bias. Generally, the government can increase labor supply by decreasing the inflation. In our model, when households have the preference of the present bias, their labor supply is smaller than the optimal one. Therefore, the government decreases the inflation, and the nominal interest rate is zero to increase the labor supply. This is the same as Chari and Kehoe (1999).

Finally, when the government controls both money supply and income tax rates, the optimal inflation rate can be positive. As mentioned before, there exists the case in which over-saving occurs. In this case, if the government taxes the capital income, it can improve the welfare by adjusting the saving. This relaxes the government's budget constraint, and thus the government decreases the labor income tax. By this government's policy, the saving is decreased, and the labor supply is increased. However, in some cases, households supply labor too much. This implies that by increasing the inflation to reduce the labor supply, the government can improve the welfare. Hence, there exists the case in which the Friedman rule is not optimal.

At the end of this subsection, we mention the organization of our paper. Section 2 provides our model. Section 3 studies the optimization of households and equilibrium. Section 4 analyzes welfare. Section 5 analyzes the government's policy. Section 6 concludes the paper.

1.1 Related Literature

Other representative works which study hyperbolic discounting are Laibson (1997) and Barro (1999). Laibson (1997) shows that hyperbolic discounting affects the resource allocation, and that if there exists the commitment device, the welfare is improved. Barro (1999) studies a Ramsey model which is continuous time model. Moreover, recent works are Hiraguchi (2016a) and Ojima (2017).

Gong and Zhu (2009), Boulware et al.(2013), and Graham and Snower (2008, 2013) incorporate money and hyperbolic discounting into a dynamic general equilibrium model. Gong and Zhu (2009) show that money is super-neutral. Boulware et al. (2013) study the model in which individuals accumulate only money, and show that the inflation is cost for the economy. Graham and Snower (2008, 2013) study the New Keynesian model with the stickiness of the wage. They show that the inflation has a long-run effect on a real variable, and that the Friedman rule is not optimal. Other works which study time-inconsistency are following. Hiraguchi (2016b) studies the monetary search model which introduces the temptation, and shows that the Friedman rule is not optimal. Futagami and Hori (2017), and Hori and Futagami (2018) study the Non-Unitary Discount model in which the household's discount rate is different between consumption and labor. They show that the Friedman rule is not optimal when the discount rate of the consumption is higher than that of the labor.

Finally, we mention the monetary model without hyperbolic discounting, which show that the Friedman rule is not optimal. Faig (1988) and Guidotti and Vegh (1993) study the shopping time model. Faig (1988) shows that the Friedman rule is not optimal by the magnitude of elasticity of money demand. Guidotti and Vegh (1993) show that the Friedman rule is not optimal if the function of shopping time cost is not homogeneous. Mulligan and Sala-I-Martin (1997) show that when they assume that the utility function which past studies (for example Chari et al. (1996)) do not assume that it occurs. Goodfriend and King (1997), and Schmitt-Grohe and Uribe (2011) are the New Keynesian model. Goodfriend and King (1997) show that the optimal inflation rate is zero. Schmitt-Grohe and Uribe (2011) study the case in which the quality of goods improve, and show that the optimal inflation rate is positive. The recent work which study the search model with money is Gomis-Porqueras and Peralta-Alva (2010).

2 The model

In this section, we explain the goods market, the capital and labor markets, the government, and households in this economy.

The goods market. In this economy, a good exists that is produced by inputting capital and labor. The production function is a Cobb-Douglas function. We assume that capital is fully depreciated. Therefore, the goods market clearing condition is as follows:

$$A\bar{k}_{t-1}^{\alpha}\bar{l}_t^{1-\alpha} = \bar{c}_t + \bar{k}_t, \tag{1}$$

where $\bar{k}_t$ is the capital accumulated by period t, $\bar{l}_t$ is the labor supply, $\bar{c}_t$ is consumption, $A > 0$ is the productivity parameter, and $\alpha \in (0,1)$ is the capital share[1]. $\bar{k}_t$, $\bar{l}_t$ and $\bar{c}_t$ are aggregate values in this economy. We also assume that the goods market is perfectly competitive.

The capital and labor markets. These markets are also perfectly competitive, implying marginal-product pricing of the capital and labor inputs:

$$r_t = \alpha A\bar{k}_{t-1}^{\alpha-1}\bar{l}_t^{1-\alpha} \tag{2}$$

$$w_t = (1-\alpha)A\bar{k}_{t-1}^{\alpha}\bar{l}_t^{-\alpha}, \tag{3}$$

where r_t is a real rental price for capital, and w_t is a real wage.

[1] We assume that the government transfer all of its income to households in the following paragraph. Therefore, (1) dose not include the government's spending.

Government. The government issues money at a constant growth rate, θ. Therefore, we obtain the dynamics of the real money stock as follows:

$$\bar{m}_t = \frac{1+\theta}{1+\pi_t}\bar{m}_{t-1}, \tag{4}$$

where $\bar{m}_t$ is the real money stock and π_t is the inflation rate. Notice that the government's real income from issuing money at period t is $\frac{\theta}{1+\pi_t}\bar{m}_{t-1}$. The government taxes households' income from capital and labor. Therefore, the government's real income from taxing households is:

$$\tau_{\mathrm{g},t} = \tau_r r_t \bar{k}_{t-1} + \tau_w w_t \bar{l}_t, \tag{5}$$

where $\tau_{\mathrm{g},t}$ is the government's real income from taxing, τ_r is the tax rate of the capital income, and τ_w is tax rate of the labor income. Moreover, we assume that tax rates do not change for all time. In this economy, the government transfers all of its income to households:

$$\tau_t = \tau_{\mathrm{g},t} + \frac{\theta}{1+\pi_t}\bar{m}_{t-1} = \tau_r r_t \bar{k}_{t-1} + \tau_w w_t \bar{l}_t + \frac{\theta}{1+\pi_t}\bar{m}_{t-1}, \tag{6}$$

where τ_t is the real transfer to households.

Households. There is one unit of a household and its size does not change. We assume that each household has one unit of time, and it is divided into leisure and labor. We also assume a utility function with consumption and leisure:

$$u(c_t, l_t) = \ln c_t + \mu \ln(1 - l_t), \tag{7}$$

where c_t is consumption, l_t is labor supply, and $\mu \geq 0$ is the parameter of the preference for leisure. In this economy, there are two assets: capital and money. Therefore, the household's real budget constraint is:

$$k_t + m_t = (1-\tau_r) r_t k_{t-1} + (1-\tau_w) w_t l_t + \frac{m_{t-1}}{1+\pi_t} + \tau_t - c_t, \tag{8}$$

where k_t is capital and m_t is money held by the household. We suppose that households face the CIA constraint as follows:

$$c_t \leq \frac{m_{t-1}}{1+\pi_t} + \frac{\theta}{1+\pi_t}\bar{m}_{t-1}. \tag{9}$$

Households have the preference of quasi-geometric discounting. Therefore, their time utility is:

$$U_t = u(c_t, l_t) + \beta \sum_{s=t+1}^{\infty} \delta^{s-t} u(c_s, l_s), \ (\beta > 0, 0 < \delta < 1) \tag{10}$$

where β is the parameter of the present bias and δ is the discount factor. Similar to KKS, we assume that households cannot commit their future actions, which is discussed in detail in the next section.

3 Optimization of households and Equilibrium

In this section, we explain the optimization problem of households and the market equilibrium. Before we solve the household's problem, we explain notations. Subscripts for all of the variables are omitted, such as l. " $'$ " has been added to the superscript of the stock variable, which is accumulated in the next period, such as k'.

From this point, we discuss households' optimization problem. Households choose their behavior, c, l, k', and m', by assuming that the price functions—the rental price of capital $r(\bar{k}, \bar{l})$, wage $w(\bar{k}, \bar{l})$, and the inflation rate $\pi(\bar{k}, \bar{m}, \bar{l})$—, the dynamics of aggregate capital holdings $\bar{k}' = G(\bar{k}, \bar{m})$, the dynamics of money $\bar{m}_t = \frac{1+\theta}{1+\pi_t}\bar{m}_{t-1}$, and aggregate labor supply $\bar{l} = J(\bar{k}, \bar{m})$ are given, because these variables depend on the aggregate capital $\bar{k}$, labor $\bar{l}$, and the the government's policy $\bar{m}$ and θ which households cannot chose. We assume that the household behaves as given them future decision rules $k' = \mathrm{g}(k, m, \bar{k}, \bar{m}, \bar{l})$, $m' = h(k, m, \bar{k}, \bar{m}, \bar{l})$, and $l = j(k, m, \bar{k}, \bar{m}, \bar{l})$, and that the CIA constraint is binding. Here, we seek an equilibrium in which G, g, h, J, and j are time-invariant. The current self's problem is:

$$V_0(k, m, \bar{k}, \bar{m}) = \max_{l, k', m'} \left[\ln\left(\frac{1}{1+\pi}m + \frac{\theta}{1+\pi}\bar{m}\right) + \mu \ln(1-l) + \beta\delta V(k', m', \bar{k}', \bar{m}') \right] \tag{11}$$

$$s.t. \ \ (1-\tau_r)r(\bar{k}, \bar{l})k + (1-\tau_w)w(\bar{k}, \bar{l})l + \tau_{\mathrm{g},t} = k' + m', \tag{12}$$

where $V(k, m, \bar{k}, \bar{m})$ is the value function after a period. The current self believes that the future self commits to adopting the decision rules g, h, and j after a period. Therefore, V is defined by:

$$V(k, m, \bar{k}, \bar{m}) = \sum_{s=t}^{\infty} \delta^{s-t} u(c_s, l_s)$$
$$= \ln\left(\frac{1}{1+\pi}m + \frac{\theta}{1+\pi}\bar{m}\right) + \mu \ln(1 - j(k, m, \bar{k}, \bar{m}, \bar{l})) + \delta V(\mathrm{g}(k, m, \bar{k}, \bar{m}, \bar{l}), h(k, m, \bar{k}, \bar{m}, \bar{l}), \bar{k}', \bar{m}'). \tag{13}$$

The definition of equilibrium is as follows.

Definition 1. *A recursive competitive equilibrium for this economy consists of decision rules* $\mathrm{g}(k, m, \bar{k}, \bar{m}, \bar{l})$, $h(k, m, \bar{k}, \bar{m}, \bar{l})$, *and* $j(k, m, \bar{k}, \bar{m}, \bar{l})$, *a value function* $V(k, m, \bar{k}, \bar{m})$, *pricing functions* $r(\bar{k}, \bar{l}), w(\bar{k}, \bar{l})$ *and* $\pi(\bar{k}, \bar{m}, \bar{l})$, *law of motion for aggregate capital* $\bar{k}' = G(\bar{k}, \bar{m})$, *the government's policy* $\bar{m}' = \frac{1+\theta}{1+\pi}\bar{m}$, *and aggregate labor supply* $\bar{l} = J(\bar{k}, \bar{m})$ *such that:*

1. *given* $V(k, m, \bar{k}, \bar{m})$, $G(\bar{k}, \bar{m})$, $J(\bar{k}, \bar{m})$, $\bar{m}' = \frac{1+\theta}{1+\pi}\bar{m}$, *and the taxes* ($\tau_r$ *and* τ_w), *the rules* $\mathrm{g}(k, m, \bar{k}, \bar{m}, \bar{l})$, $h(k, m, \bar{k}, \bar{m}, \bar{l})$, *and* $j(k, m, \bar{k}, \bar{m}, \bar{l})$ *solve the optimization problem* (11);

2. *given* $\mathrm{g}(k, m, \bar{k}, \bar{m}, \bar{l})$, $h(k, m, \bar{k}, \bar{m}, \bar{l})$, $j(k, m, \bar{k}, \bar{m}, \bar{l})$, $G(\bar{k}, \bar{m})$, $\bar{m}' = \frac{1+\theta}{1+\pi}\bar{m}$, $J(\bar{k}, \bar{m})$, *and the taxes* (τ_r *and* τ_w), *the function* $V(k, m, \bar{k}, \bar{m})$ *satisfies* (13);

3. *the CIA constraint is binding;*

4. $r(\bar{k}, \bar{l}) = \alpha A \bar{k}^{\alpha-1} \bar{l}^{1-\alpha}, w(\bar{k}, \bar{l}) = (1-\alpha) A \bar{k}^{\alpha} \bar{l}^{-\alpha}$; *and*

5. $\mathrm{g}(\bar{k}, \bar{m}, \bar{k}, \bar{m}, \bar{l}) = G(\bar{k}, \bar{m})$, $h(\bar{k}, \bar{m}, \bar{k}, \bar{m}, \bar{l}) = \bar{m}'$ *and* $j(\bar{k}, \bar{m}, \bar{k}, \bar{m}, \bar{l}) = J(\bar{k}, \bar{m})$.

From this definition, we obtain the following proposition.

Proposition 1. *For our economy, the recursive competitive equilibrium is given by:*

1. $V(k, m, \bar{k}, \bar{m}) = B + \ln(m + \theta\bar{m}) - \ln \bar{m} + D \ln \bar{k} + E \ln(k + F\bar{k})$
   where $D = \frac{\alpha(1-\delta)-(1-\alpha\delta)(\mu+\delta)}{(1-\alpha\delta)(1-\delta)}$, $E = \frac{\mu+\delta}{1-\delta}$,
   $F = \frac{\beta\delta[1+\tau_i-\alpha\delta(1-\tau_r)]+\mu\{[1-\delta(1-\beta)]-\alpha\beta\delta(1-\tau_r)\}}{\alpha\beta\delta(1-\delta)(1-\tau_r)}(1+\theta) - 1$

2. $G(\bar{k}, \bar{m}) = s\alpha A \bar{k}^{\alpha} J(\bar{k}, \bar{m})^{1-\alpha}$ *where* $s\alpha = \frac{\alpha\beta\delta(\delta+\mu)}{\beta\delta+\mu[1-\delta(1-\beta)]}(1-\tau_r)$ *(this is the saving rate),*
   $J(\bar{k}, \bar{m}) = \bar{l}^* = \frac{1-\alpha}{1-\alpha+\frac{1+\theta}{\beta\delta(1-\tau_w)}\mu(1-s\alpha)}$, *and*
   $\pi(\bar{k}, \bar{m}, \bar{l}) = \frac{(1+\theta)\bar{m}}{(1-s\alpha)A\bar{k}^{\alpha}J(\bar{k},\bar{m})^{1-\alpha}} - 1$.

Proof. See Appendix A. □

4 Welfare Property

In this section, we discuss welfare property. First, we seek the saving rate which maximizes the welfare function. Because households maximize this function in a competitive equilibrium, we define the value function $V_0(k, m, \bar{k}, \bar{m})$ as the welfare function. This point is the same to KKS. By maximizing the value function as in Appendix B, we obtain the saving rate, s^{op}, which maximizes the value function as follows:

$$s^{op} = \frac{\alpha\beta\delta}{(1-\alpha\delta)[1-\delta(1-\beta)]+\alpha\beta\delta} \tag{14}$$

If the government dose not tax, then we compare the saving rate of the competitive equilibrium and the optimal saving rate, and obtain the following proposition:

Proposition 2. *The saving rate is excessive when* $\beta < (>)1$ *if* $\mu < (>)\frac{1-\alpha\delta}{\alpha}$.

Proof. Because $s^{op} - s\alpha = \frac{\alpha\beta\delta^2(1-\delta)(1-\beta)[\alpha\mu-(1-\alpha\delta)]}{\{(1-\alpha\delta)[1-\delta(1-\beta)]+\alpha\beta\delta\}\{\beta\delta+\mu[1-\delta(1-\beta)]\}}$, we find $s\alpha > s^{op}$ when $\beta < (>)1$ if $\mu < (>)\frac{1-\alpha\delta}{\alpha}$. □

From this proposition, we find that over-saving occurs when households have a present bias ($\beta < 1$) and a weak preference for leisure. This is cased by the CIA constraint becomes a commitment device in this economy. To understand it, we think about the case in which households supply their labor force inelastically, $\mu = 0$. In this case, all of households at any time cannot choose their present consumption and can choose only their investment in capital and money. This implies that households choose the amount of consumption in the next period and in subsequent periods. This decision making is equal to maximizing only the second term of household's life time utility (10). Hence, in this case, the saving rate is $s\alpha = \alpha\delta$. This is equal to the case in which $\beta = 1$ and is larger than the case without the CIA constraint. Although households at any time succeed to make future themselves more save, they are forced not to consume more, which is why over-saving occurs. Over-saving occurs when $0 < \mu < \frac{1-\alpha\delta}{\alpha}$ for almost the same reason. In this case, a household's decision making depends on the present bias because the household cannot commit future labor supply. However, the effect of the present bias is limited when μ is small. Therefore, labor supply and income do not decrease as much, and over-saving occurs.

5 Policy Analysis

In this section, we discuss the government's policy. In the first subsection, we consider only the monetary policy. In the second subsection, we consider both policies, the monetary and fiscal policy.

5.1 Monetary policy

In this subsection, to consider only the monetary policy, $\tau_r = \tau_w = 0$. We define the value function $V_0(k, m, \bar{k}, \bar{m})$ as the welfare function as in section 4. Therefore, the government maximizes following equation as given households' response, the accumulated capital, $\bar{k}$, and the accumulated money, $\bar{m}$, in past time:

$$V_0(\bar{k}, \bar{m}) = \ln(1 - s\alpha)A\bar{k}^{\alpha}(\bar{l}^*)^{1-\alpha} + \mu \ln(1 - \bar{l}^*) + \beta\delta V(\bar{k}', \bar{m}'). \tag{15}$$

From Proposition 1, we find that θ affects only labor supply. Therefore, the optimal monetary policy is the same to maximizing following function:

$$\begin{aligned} v(\theta) = [1-&\delta(1-\beta)]\mu \ln(1+\theta) \\ &- \left\{[1-\delta(1-\beta)]\mu + \frac{1-\alpha}{1-\alpha\delta}[(1-\delta(1-\beta)) + \alpha\beta\delta]\right\} \ln\left[1 - \alpha + \frac{1+\theta}{\beta\delta}\mu(1 - s\alpha)\right]. \end{aligned} \tag{16}$$

From this equation, we obtain the optimal monetary policy as follows:

$$\bar{\theta} = \beta\delta\frac{1 - s^{op}}{1 - s\alpha} - 1. \tag{17}$$

Because the second term of (16), $[1 - \delta(1 - \beta)]\mu + \frac{1-\alpha}{1-\alpha\delta}[(1 - \delta(1 - \beta)) + \alpha\beta\delta]$, is positive, we find that this function is concave. Therefore, $\bar{\theta}$ which satisfies $v'(\theta) = 0$ is optimal.

In the steady state. Here, we focus the steady state because we analyze whether the Friedman rule is satisfied. We derive the rental price of capital in steady state. From Proposition 1, the rental price of capital in steady state is:

$$r^* = \frac{1}{s}. \tag{18}$$

The inflation rate in steady state is equal to θ. From (18) and $\pi = \theta$, the nominal interest rate in steady state is:

$$i = \frac{1+\theta}{s} - 1 \tag{19}$$

Substituting (17) into (19), we obtain the following proposition.

Proposition 3. *If $\beta \leq (>)1$ in the steady state, the Friedman rule is (not) optimal.*

Proof. Substituting (17) into (19), we obtain the nominal interest rate:

$$\bar{i} = \frac{(\beta-1)\{(1-\alpha\delta)(1+\mu)[\mu(\delta-1)^2+\beta^2\delta^2(1+\mu)]+\beta(1-\delta)\delta[(1-\alpha\delta)((1-\alpha\delta)+\mu(3-\alpha)+\mu^2)+\alpha^2(\delta+\mu)\mu]\}}{\beta(1-\alpha\delta)[1-\delta(1-\beta)](\delta+\mu)[\mu(1-\delta)\beta\delta(1+\mu)]}. \tag{20}$$

If $\beta \leq (>)1$, (20) is negative or 0 (positive). Because the nominal return of money is 0, the nominal interest rate dose not negative. Therefore, if $\beta \leq (>)1$, the Friedman rule is (not) optimal. $\square$

This proposition implies that if households are impatient (β is small), the optimal nominal interest rate is small. Because $\frac{\partial s}{\partial \beta} = \frac{\alpha\delta(\delta+\mu)\mu(1-\delta)}{\{\beta\delta+\mu[1-\delta(1-\beta)]\}^2} > 0$, we find that:

$$\frac{\partial \bar{l}^*}{\partial \beta} = \frac{(1-\alpha)\left[\delta + \mu(1+\theta)\frac{\partial s}{\partial \beta}\right]}{[1-\alpha+\frac{1+\theta}{\mu}(1-s)]^2} > 0. \tag{21}$$

This implies that if β is small, the labor supply may be too small. Therefore, the government deceases the money to increase labor supply. This is the intuition of the Friedman rule being optimal.

5.2 Monetary and Fiscal policy

In Section 4, we found that over-saving occurs if households have a weak preference for leisure. However, the monetary policy dose not affect the saving rate. Therefore, the fiscal policy might improve the welfare. In this subsection, we introduce taxes, and seek the optimal policy.

In this economy, the government's transfer dose not affect the saving rate, and labor supply. Therefore, we give the amount of transfer exogenously as follows:

$$\tau_{\mathrm{g},t} = 0, \tag{22}$$

$$\tau_t = \frac{\theta}{1+\pi}\bar{m} \quad for\ all\ t. \tag{23}$$

We define $V_0(k, m, \bar{k}, \bar{m})$ as the welfare function as in section 4. In section 4, we have obtained the optimal saving rate (14). However, the optimal labor supply has not been obtained yet. By calculating the value function as in Appendix B, we obtain:

$$\bar{l}^{op} = \frac{1-\alpha}{1-\alpha+\mu(1-s^{op})}. \tag{24}$$

Therefore, we can maximize the welfare by adjusting the saving rate and labor supply in competitive equilibrium as follows: $s\alpha = s^{op}$, $\bar{l}^* = \bar{l}$. Therefore, we obtain the optimal policy as follows:

$$\tau_r^* = \frac{(1-\beta)\delta(1-\delta)(1-\alpha\delta-\alpha\mu)}{\{(1-\alpha\delta)[1-\delta(1-\beta)]+\alpha\beta\delta\}(\delta+\mu)} \tag{25}$$

$$\tau_w^* = -\frac{\alpha}{1-\alpha}\frac{(1-\beta)\delta(1-\delta)(1-\alpha\delta-\alpha\mu)}{\{(1-\alpha\delta)[1-\delta(1-\beta)]+\alpha\beta\delta\}(\delta+\mu)} \tag{26}$$

$$\theta^* = \beta\delta\left\{1+\frac{\alpha}{1-\alpha}\frac{(1-\beta)\delta(1-\delta)(1-\alpha\delta-\alpha\mu)}{\{(1-\alpha\delta)[1-\delta(1-\beta)]+\alpha\beta\delta\}(\delta+\mu)}\right\}-1. \tag{27}$$

From (25), (26), and (27), we obtain following proposition.

Proposition 4. *In the steady state, when the government uses the monetary and fiscal policy to improve the welfare, the Friedman rule is not optimal if* $\beta < 1$ *and* $\mu < \frac{\alpha(1+\delta^2)-\delta(1+\alpha)}{1-\alpha\delta}$.

Proof. When the government dose the policy like (25), (26) and (27), in steady state, the nominal interest rate is: $i^* = \alpha\frac{1+\theta^*}{s^{op}} - 1$. From this equation, we find that if $1+\theta^* - \frac{s^{op}}{\alpha} > 0$, the Friedman rule is not optimal. When we calculate it, we obtain as follows:

$$1+\theta^* - \frac{s^{op}}{\alpha} = \frac{(\beta-1)\beta\delta^2[\delta(1+\alpha)-\alpha(1+\delta^2)+\mu(1-\alpha\delta)]}{\{(1-\alpha\delta)[1-\delta(1-\beta)]+\alpha\beta\delta\}(\delta+\mu)}. \tag{28}$$

This equation implies that if $\beta < 1$ and $\mu < \frac{\alpha(1+\delta^2)-\delta(1+\alpha)}{1-\alpha\delta}$, $1+\theta^* - \frac{s^{op}}{\alpha} > 0$. Therefore, in this case, we find that the Friedman rule is not optimal. $\square$

This proposition implies that the Friedman rule is not optimal when over-saving occurs because $\frac{\alpha(1+\delta^2)-\delta(1+\alpha)}{1-\alpha\delta} < \frac{1-\alpha\delta}{\alpha}$. The intuition of this proposition is very simple. When over-saving occurs, the government taxes the capital income to decrease the saving rate. In such case, the labor income tax is negative because we assume the government's budget constraint is (22). Then, households more supply their labor force. This effect is too strong when $\mu < \frac{\alpha(1+\delta^2)-\delta(1+\alpha)}{1-\alpha\delta}$. Therefore, the government induces the nominal interest to be positive to reduce labor supply.

6 Conclusion

In this paper, we studied a general equilibrium model which households have a preference of quasi-geometric discounting and face a cash-in-advance constraint.

There are three contributions in this paper. First, we showed that over-saving occurs although households have the preference of present bias. In our model, households cannot decide on present consumption but can decide on future consumption because the CIA constraint is assumed. Households with quasi-geometric discounting want to make future themselves save more. Therefore, households hold money too little and capital too much to reduce the consumption of future themselves. Second, we showed that when the government can control only money supply, the Friedman rule is optimal. The reason is that households do not supply labor sufficiently when they have the preference of quasi-geometric discounting. Households who have the preference of the present bias more enjoy leisure, and decrease labor supply. Therefore, the government decreases the nominal interest rate to increase the labor supply. Finally, we showed that when the government can control both money supply and income tax rates, there exists the case in which the Friedman rule is not optimal. In our model, over-saving occurs when households have the preference of quasi-geometric discounting and the weak preference for leisure. In such case, the optimal fiscal policy is reducing the investment in capital by taxing the capital income. The government decreases the labor income tax because they have to satisfy their budget constraint. Therefore, households' labor supply increase too much. The government increases the nominal interest rate to a positive level to suppress this households' labor supply.

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Appendix

A Proof of Proposition 1

We solve this problem by the Guess and Verify method. The value function $V(k, m, \bar{k}, \bar{m})$ is guessed as follows:

$$V(k, m, \bar{k}, \bar{m}) = B + \ln(m + \theta\bar{m}) - \ln\bar{m} + D\ln\bar{k} + E\ln(k + F\bar{k}), \tag{A1}$$

where B, D, E, and F are constant. Because we assume that (9) is binding, we obtain the first-order conditions with respect to k', m', and l as follows:

$$\frac{\beta\delta E}{k' + F\bar{k}'} - \lambda = 0 \tag{A2}$$

$$\frac{\beta\delta}{m' + \theta\bar{m}'} - \lambda = 0 \tag{A3}$$

$$-\frac{\mu}{1-l} + \lambda w(1-\tau_w) = 0, \tag{A4}$$

where λ is the Lagrangian multiplier for the budget constraint (12). From (A2), (A3), and (A4), we obtain:

$$k' = \frac{1}{1+\tau_i}[E(m' + \theta\bar{m}') - F\bar{k}']. \tag{A5}$$

This equation substitutes the budget constraint (12), and we obtain:

$$m' = \frac{1}{1+E}\left\{(1-\tau_r)rk + (1-\tau_w)wl + \tau_{\mathrm{g}} + F\bar{k}' - E\theta\bar{m}\right\} \tag{A6}$$

In the equilibrium, $\bar{c} = \frac{1+\theta}{1+\pi}\bar{m}$ because the CIA constraint (9) is binding. The right-hand side of this equation equals the right-hand side of equation (4). Therefore, $\bar{c} = \bar{m}'$. By the assumption, $m = \bar{m}$ and $k = \bar{k}$. From these equation, goods market clearing condition (1), rental price of capital (2), wage (3), and the government's income from the taxes (5) we obtain:

$$\bar{m}' = \frac{(1+F)}{(1+F) + E(1+\theta)}A\bar{k}^{\alpha}\bar{l}^{1-\alpha}. \tag{A7}$$

Moreover, from (A7) and , we obtain:

$$\frac{1}{1+\pi} = \frac{(1+F)}{[(1+F)+E(1+\theta)](1+\theta)} \frac{A\bar{k}^{\alpha}\bar{l}^{1-\alpha}}{\bar{m}}. \tag{A8}$$

Because $\bar{k}' = A\bar{k}^{\alpha}\bar{l}^{1-\alpha} - \bar{c} = A\bar{k}^{\alpha}\bar{l}^{1-\alpha} - \bar{m}'$ from (1) and (9), we obtain:

$$\bar{k}' = G(\bar{k}, \bar{m}) = sA\bar{k}^{\alpha}\bar{l}^{1-\alpha} \tag{A9}$$

$$where \quad s\alpha \equiv \frac{E(1+\theta)}{1+F+E(1+\theta)}. \tag{A10}$$

From (A3) and (A4), we have:

$$1 - l = \frac{\mu}{\beta\delta(1-\tau_w)w}(m' + \theta\bar{m}') \tag{A11}$$

From (A6), (A7), and (A9), we obtain:

$$\begin{aligned} &m' + \theta\bar{m}' \\ &= \frac{1}{1+E}\left\{(1-\tau_r)rk + (1-\tau_w)wl + \left[\alpha\tau_r + (1-\alpha)\tau_w + \frac{FE(1+\theta)+\theta(1+F)}{(1+F)+E(1+\theta)}\right]A\bar{k}^{\alpha}\bar{l}^{1-\alpha}\right\} \end{aligned} \tag{A12}$$

From this equation and (A11), we obtain:

$$\begin{aligned} &(1-\tau_w)wl \\ &= \frac{1}{\beta\delta(1+E)+\mu}\left\{\beta\delta(1+E)(1-\tau_w)w - \mu\left[(1-\tau_r)rk + \frac{FE(1+\theta)+\theta(1+F)}{(1+F)+E(1+\theta)}\right]A\bar{k}^{\alpha}\bar{l}^{1-\alpha}\right\}. \end{aligned} \tag{A13}$$

Substituting (2) and (3) into this equation, we obtain:

$$\bar{l}^* = \frac{\beta\delta(1-\alpha)(1-\tau_w)[(1+F)+E(1+\theta)]}{\beta\delta(1-\alpha)(1-\tau_w)[(1+F)+E(1+\theta)] + \mu(1+\theta)(1+F)}. \tag{A14}$$

Moreover, substituting (A13) and (A14) into (A12), we obtain:

$$\begin{aligned} &m' + \theta\bar{m}' \\ &= \frac{\beta\delta(1-\tau_r)r}{\beta\delta(1+E)+\mu}\left\{k + \left[\frac{[(1+E)\beta\delta+\mu](1+F)(1+\theta)}{\alpha\beta\delta(1-\tau_r)[(1+F)+E(1+\theta)]} - 1\right]\bar{k}\right\} \end{aligned} \tag{A15}$$

By the definition 1, we must satisfy (13). Therefore, the following equation is satisfied:

$$
\begin{aligned}
&B + \ln(m + \theta\bar{m}) - \ln\bar{m} + D\ln\bar{k} + E\ln(k + F\bar{k}) \\
&= \delta[B + D\ln s\alpha + E\ln E - D\ln] + (1-\delta)\ln(1-s\alpha) + [1 + \delta(D+E)]\ln A \\
&+ \mu[\ln\mu - \ln(1-\tau_w) - \ln(1-\alpha)] + \delta(1+E)\ln\beta\delta + (\mu + \delta + \delta E)[\ln\alpha + \ln(1-\tau_r)] \\
&- \ln(1+\theta) + \{\alpha\mu + (1-\alpha)[1 + \mu + \delta(D+E)]\}\ln\bar{l} \\
&+ \ln(m + \theta\bar{m}) - \ln\bar{m} + [\alpha - \delta - (1-\alpha)(\mu + \delta E) + \alpha\delta D - \alpha\mu]\ln\bar{k} \\
&+ (\mu + \delta + \delta E)\ln\left\{k + \left[\frac{[(1+E)\beta\delta + \mu](1+F)(1+\theta)}{\alpha\beta\delta(1-\tau_r)[(1+F) + E(1+\theta)]} - 1\right]\bar{k}\right\}. \qquad \text{(A16)}
\end{aligned}
$$

From this equation, we obtain:

$$E = \mu + \delta + \delta E \ \rightarrow\ E = \frac{\mu + \delta}{1 - \delta} \qquad \text{(A17)}$$

$$D = \alpha - \delta - (1-\alpha)(\mu + \delta E) + \alpha\delta D - \alpha\mu \ \rightarrow\ D = \frac{\alpha(1-\delta) - (1-\alpha\delta)(\mu+\delta)}{(1-\alpha\delta)(1-\delta)} \qquad \text{(A18)}$$

$$
\begin{aligned}
F &= \frac{[(1+E)\beta\delta + \mu](1+F)(1+\theta)}{\alpha\beta\delta(1-\tau_r)[(1+F) + E(1+\theta)]} - 1 \\
&\rightarrow F = \frac{\beta\delta[1 - \alpha\delta(1-\tau_r)] + \mu\{[1 - \delta(1-\beta)] - \alpha\beta\delta(1-\tau_r)\}}{\alpha\beta\delta(1-\delta)(1-\tau_r)}(1+\theta) - 1 \qquad \text{(A19)}
\end{aligned}
$$

Finally, these equations substitute (A10) and (A14), and we obtain:

$$s = \frac{\beta\delta(\mu+\delta)}{\beta\delta + \mu[1 - \delta(1-\beta)]}(1-\tau_r) \qquad \text{(A20)}$$

$$\bar{l}^* = J(\bar{k}, \bar{m}) = \frac{1-\alpha}{1 - \alpha + \frac{1+\theta}{\beta\delta(1-\tau_w)}\mu(1-s\alpha)}. \qquad \text{(A21)}$$

$$\pi(\bar{k}, \bar{m}, \bar{l}) = \frac{(1+\theta)\bar{m}}{(1-s\alpha)A\bar{k}^{\alpha}J(\bar{k},\bar{m})^{1-\alpha}} - 1 \qquad \text{(A22)}$$

B Derivation of (14) and (24)

By definition, the future value function $V(k, m, \bar{k}, \bar{m})$ must satisfy the following equation:

$$V(k, m, \bar{k}, \bar{m}) = \sum_{s=t}^{\infty}\delta^{s-t}u(c_s, l_s) = \sum_{s=t}^{\infty}\delta^{s-t}\ln c_s + \mu\sum_{s=t}^{\infty}\delta^{s-t}\ln(1-l_s). \qquad \text{(B1)}$$

In this paper, the saving rate and labor supply are constant at any time. Therefore, the second term of this equation can be written as follows:

$$\mu \sum_{s=t}^{\infty} \delta^{s-t} \ln(1-l_s) = \frac{\mu}{1-\delta} \ln(1-l). \tag{B2}$$

Because $c_t = (1-s)Ak_t^{\alpha} l_t^{1-\alpha}$, the first term is written as follows:

$$\begin{aligned}\sum_{s=t}^{\infty} \delta^{s-t} \ln c_s &= \ln(1-s)Ak^{\alpha}l^{1-\alpha} + \delta \ln(1-s)Ak'^{\alpha}l^{1-\alpha} + \delta^2 \ln(1-s)Ak''^{\alpha}l^{1-\alpha} + \cdots \\ &= \frac{1}{1-\delta}\ln(1-s) + \frac{1-\alpha}{1-\delta}\ln l + \frac{1}{1-\delta}\ln A + \alpha(\underbrace{\ln k + \delta \ln k' + \delta^2 \ln k'' + \cdots}_{\equiv K}).\end{aligned} \tag{B3}$$

I define the final term as K, which can be expressed as:

$$\begin{aligned}K &= \ln k + \delta \ln sAk^{\alpha}l^{1-\alpha} + \delta^2 \ln sAk'^{\alpha}l^{1-\alpha} + \delta^3 \ln sAk''^{\alpha}l^{1-\alpha} + \cdots \\ &= \frac{\delta}{1-\delta}\ln s + \delta\frac{1-\alpha}{1-\delta}\ln l + \frac{\delta}{1-\delta}\ln A + \ln k + \alpha\delta(\underbrace{\ln k + \delta \ln k' + \delta^2 \ln k'' + \cdots}_{=K}).\end{aligned} \tag{B4}$$

Therefore,

$$K = \frac{1}{1-\alpha\delta}\left\{\frac{\delta}{1-\delta}[\ln s + (1-\alpha)\ln l + \ln A] + \ln k\right\}. \tag{B5}$$

This equation substitutes (B3), and we obtain:

$$\sum_{s=t}^{\infty} \delta^{s-t} \ln c_s = \frac{1}{1-\delta}\ln(1-s) + \frac{1-\alpha}{(1-\delta)(1-\alpha\delta)}\ln l + \frac{1}{(1-\delta)(1-\alpha\delta)}\ln A + \frac{\alpha\delta}{(1-\delta)(1-\alpha\delta)}\ln l + \frac{\alpha}{1-\alpha\delta}\ln k. \tag{B6}$$

From (B2) and this,

$$\begin{aligned}&V(k, m, \bar{k}, \bar{m}) \\ &= \underbrace{\frac{1}{1-\delta}\ln(1-s) + \frac{1}{(1-\delta)(1-\alpha\delta)}\ln A + \frac{\alpha\delta}{(1-\delta)(1-\alpha\delta)}\ln s + \frac{\alpha}{1-\alpha\delta}\ln k + \frac{1-\alpha}{(1-\delta)(1-\alpha\delta)}\ln l}_{\sum_{s=t}^{\infty}\delta^{s-t}\ln c_s} \\ &+ \underbrace{\frac{\mu}{1-\delta}\ln(1-l)}_{\mu\sum_{s=t}^{\infty}\delta^{s-t}\ln(1-l_s)}.\end{aligned} \tag{B7}$$

Because $V_0(k, m, \bar{k}, \bar{m}) = \ln c + \mu \ln(1 - l) + \beta\delta V(k', m', \bar{k}', \bar{m}')$, we substitute (B7) for this equation and obtain:

$$
\begin{aligned}
&V_0(k, m, \bar{k}, \bar{m}) \\
&= \frac{1-\delta(1-\beta)}{1-\delta}\ln(1-s) + \frac{1-\delta(1-\beta)}{(1-\delta)(1-\alpha\delta)}\ln A + \frac{\alpha\beta\delta}{(1-\delta)(1-\alpha\delta)}\ln s + \frac{\alpha\beta\delta}{1-\alpha\delta}\ln k \\
&+ \frac{(1-\alpha)\{(1-\alpha\delta)[1-\delta(1-\beta)]+\alpha\beta\delta\}}{(1-\delta)(1-\alpha\delta)}\ln l + \frac{\mu[1-\delta(1-\beta)]}{1-\delta}\ln(1-l) \qquad \text{(B8)}
\end{aligned}
$$

s and l, which maximize this equation, are (14) and (24).