Maturity Structure of Haircut of Sovereign Bonds

Kenji Wada[†]

[†]Graduate School of Economics, Hitotsubashi University

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Abstract

Why does haircuts of sovereign bonds differ, depending on the maturity length? The empirical literature shows that the short-term bonds suffer from more severe haircuts than the long-term debt. This variation across maturity length should be a part of term premium of sovereign bonds, which is a determinant of maturity structure of sovereign debt in emerging markets. In order to answer that question posed above, I extend the Eaton-Gersovitz type of quantitative models. Unlike the models previously developed, I allow the sovereign to exchange the set of old short and long-term debt prior to default with the newly issued corresponding debt in debt restructurings. My model can capture the negative association between haircut and maturity length for the following reasons. On the one hand, the sovereign prefers to issue long-term debt since its repayment for long-term debt is negatively correlated with intertemporal MRS. On the other hand, the external creditor tries to avoid the large bias toward long-term debt, which decreases the price of long-term debt by increasing the default incentive. Quantitatively, the former dominates the latter so that the model can explain the negative correlation of haircut with maturity length.

[†]*Contacting address:* Graduate School of Economics, Hitotsubashi University, Naka 2-1, Kunitachi-city, Tokyo, 186-8601, Japan. *Emails:* em162005@g.hit-u.ac.jp

I. Introduction

Several sovereign debt restructurings have experienced huge variations in haircut across debt instruments within exchanges. Haircut means the ratio of debt reduction in debt restructuring with the external creditors after sovereign default to the amount of debt prior to default. Surprisingly, the empirical literature observes the negative relationship between haircut and maturity length. This is described in Table 1 below. Sturzenegger and Zettelmeyer (2008) collect the data on debt restructurings from 1998 to 2005. I cited their result only about the external debt[1]. The first column describes the name of countries in default. You can see the correlation of haircut with maturity length of sovereign debt on second column. Most of the default episodes show that the investors in short-term debt are more likely to suffer from larger losses than those in long-term debt.

This specific moment is important since the variation of haircuts across maturity length is a component of the term premia of sovereign bonds. Therefore, it affects the maturity choice of sovereign debt. Considering the frequent occurence of rollover crises in emerging markets, which often involves the accumulation of short-term debt, economists should provide the theoretical explanation for this one source of term premium.

Despite this importance of the variation of haircuts across maturities, the quantitative sovereign debt models[2] developed so far have not provided the appropriate framework to analyze this moment. Arellano and Ramanarayanan (2012) develop the model with endogenous maturity choices but they miss the debt renegotiation after sovereign default. Yue (2010) constructs the model with debt renegotiation, where only short-term debt is available for sovereign. Moreover, she assumes that the sovereign is supposed to repay cash to the external creditor in debt renegotiation and regains

Table 1: Sturzenegger and Zettelmeyer (2008)

	Corr(Haircut,Maturity Length)
Russia	-0.56
Ukraine	-0.97
Pakistan	-0.96
Ecuador	-0.90
Argentina	0.30
Urugray	-0.46

[1]See Table 2 in Sturzenegger and Zettelmeyer (2008) for more details. They calculated the haircut with the following definition:

$$H_{NPV} \equiv 1 - \frac{NPV(new, r_{new})}{NPV(old, r_{new})},$$

where $NPV(new, r_{new})$ and $NPV(old, r_{new})$ are net present values of new and old debt instruments respectively. And the discount rate is the interest rate of the newly issued debt, r_{new}.

[2]The models in this literature are largely based on the Eaton-Gersovitz model. The piorneers are Aguiar and Gopinath (2006) and Arellano (2008)

the access to the international financial market only if it finishes cash repayment.

In this context, I develop a version of the quantitative sovereign debt models similar to Arellano and Ramanarayanan (2012) and Yue (2010) to fill the gap of the two models to provide the theoretical foundation on the empirical fact that the short-term debt suffers from more severe loss in debt restructurings than the long-term debt. In particular, I allow the sovereign to exchange the set of old debt prior to default with that of newly issued debt in debt renegotiation. This novelty is essential to explain the facts mentionend in Table 1.

The primary result shows that my model can capture the negative association between haircut and maturity length observed in data. There are two counter forces determining this correlation. On the one hand, the sovereign would like to issue long-term debt rather than short-term debt in debt restructuring. The sovereign can pay back the long-term debt at market price and therefore, the repayment for the long-term debt depends on the market price. This market price is decreasing function of the current endowment realization and debt levels, both of which increase the likelihood of default. Therefore, the repayment for the long-term debt shrinks prior to crisis. This implies that the intertemporal marginal rate of substitution of the sovereign is negatively correlated with the long-term debt repayment. By contrast, the repayment for short-term debt is always constant. That is why the sovereign prefers long-term debt to short-term debt.

On the other hand, the external creditor are reluctant to receive the set of newly issued bonds mostly composed of long-term sovereign bonds. This force can rely on the argument of Arellano and Ramanarayanan (2012); the bias toward long-term debt increases the default incentive of the sovereign. This biased composition toward long-term debt decreases that price. Therefore, the external creditor demands moderate amount of short-term and long-term debt in debt renegotiation. Under the plausible calibration, the first force driven by the sovereign dominates the second one by the external creditor. For all of these reasons, my model can replicate the negative relationship between haircut and maturity length.

My model can successfully explain the correlation of haircut with maturity length without any complicated setting of debt restructuring. It results from the optimal behaviors of the sovereign and the external creditor in incomplete market. Notably, that negative relationship arises neither from the ability to repay debt nor from the rich static structure of bargaining game. Rather, it reflects the forward looking behaviors taken after default period by the sovereign. This emphasizes the importance of the exchange of the set of old debt prior to default with that of newly issued debt to analize the outcome of debt restructurings unlike Yue (2010).

The rest of the article proceeds as follows. Section II presents the model. Section III describes the calibration and results. Section IV concludes.

II. Model Mechanics

II.A. Model Environment

There are two groups of agents in the model, one in the domestic economy (sovereign and identical households) and another in the international financial market (external creditors). The sovereign acts as a social planner and makes several decisions on behalf of the continuum of identical households including consumption, debt issurance, and default decision. The sovereign is assumed to have adequate policy instruments to implement a series of decision making mentioned above. The economy just receives the stochastic endowment and does not engage in production, which is standard in the literature. The time is discrete and one unit of time is meant to be a year.

External creditors are risk neutral and inhabit the perfectly competitive international financial market. They can get access to the riskfree world interest rate and in addition, they purchase two types of bonds, short-term and long-term, issued by the sovereign. The economic environment does not allow the sovereign to transact the state contingent bonds with the external creditors, so that the international financial market is incomplete.

Before exploring the details of the model, let me explain the timing of events in the model. This is described in Figure 1 and Figure 2. I believe this makes it easier for readers to understand the model mechanics. First of all, I discuss the sequence of decision making in good financial standing, which means that the sovereign can get access to the international financial market and borrow from the external creditors. At the beginning of the period, the sovereign decides whether to default or not, given the current debt levels and realized endowment. If it does not default, it follows the left arrow in Figure 1 and determines the level of debt it issues in the current period and the present consumption level. Them the sovereign enters the next period in good financial standing. By contrast, if the sovereign decides to default, it moves on to the debt renegotiation with the external creditors. In the debt restructuring, these two combatants settle on the amount of exempted debt. The consumption level is always equal to the current realization of output since the sovereign is excluded from the international financial market. Moreover, the sovereign loses some amounts of endowment as the output cost of default. And it proceeds to the next period in bad financial standing, which will be explained immediately.

Now I reach the position to discuss the timing of events in bad financial standing. Basically, there is nothing for the sovereign to do; it just consumes the current endowment incuring the output cost of default since it cannot get access to the international financial market. In the next period, it regains the access to it in good financial standing with the exogenous probability θ and it stays in bad financial standing with probability $1 - \theta$.

II.B. Sovereign

II.B.1. Continuing to Repay

Sovereign chooses consumption and debt issuances so as to maximize the lifetime utility of the representative household when it decides not to default in the current period in good financial standing. Considering the breifness of the presentation and the connection to the quantitative method, I express the decision problem of the sovereign in recursive representation thereafter.

The decision problem of the sovereign in continuing to repay is identical to that in Arellano and Ramanarayanan (2012). Given that it does not default in the present period, the Bellman equation of the sovereign is described as

$$v^c(b_S, b_L, y) = \underset{c, b'_S, b'_L, l, c}{\text{Max}} \{u(c) + \beta E_y v^g(b'_S, b'_L, y')\} \tag{1}$$

s.t.

$$c = y - b_S - b_L + q_s(b'_S, b'_L, y)b'_S + q_L(b'_S, b'_L, y)l, \tag{2}$$

$$b'_L = \delta b_L + l. \tag{3}$$

where the set of current state variables are given by the current short-term debt b_S, the long-term debt b_L, and the present realization of endowment y. $v^c(b_S, b_L, y)$ in the right hand side of (1) is the value of continuing to repay in good financial standing given the current state variables. This value comes from the two sources in the right hand side of (1): the instantaneous utility from the current consumption $u(c)$ and the expected discount value of good financial standing in the next period $\beta E_y v^g(b'_S, b'_L, y')$. Here $0 < \beta < 1$ is the discount factor and prime denotes the variables in the next period. v^g is the value in good financial standing defined later on. E_y is the mathematical expectation operator for the endowment in the next period conditional on the current endowment y.

The budget constraint is given by equation (2). The current consumption level is determined by the three components: the present endowment y, the debt repayment $-b_S - b_L$, and the revenue from the newly issued short-term and long-term debt $q_s(b'_S, b'_L, y)b'_S + q_L(b'_S, b'_L, y)l$. $q_s(b'_S, b'_L, y)$ and $q_L(b'_S, b'_L, y)$ denote the price of short-term and long-term debt respectively. l is the amount of the long-term debt issued in the current period.

Equation (3) describes the law of motion of long-term debt. The sovereign must repay one unit of endowment as the principal of one unit of long-term debt, but the only fraction $0 < 1-\delta < 1$ of the long-term debt matures in this period. The sovereign has to repay δ unit of consumption as the coupon of one unit of long-term debt. This specification is popular in the literature since it helps to reduce the dimension of the state variables. This particular parameter δ is closely related to the duration of the long-term debt[3]. The duration of the long-term debt is calculated as

$$D = \frac{1 + r^*}{1 - \delta + r^*}, \tag{4}$$

where r^* denotes the constant per-period yield delivered by the bond. If δ is equal to zero, this bond collapses to the short-term bond.

There is only one source of uncertainty, endowment shock in this model. This stochastic process is specified as an AR(1) process:

$$log(y') = \rho log(y) + \epsilon \tag{5}$$

, where ϵ is an exogenous standard normal shock to the output.

$$E[\epsilon^2] = \eta^2. \tag{6}$$

ρ captures the persistence of the shock to output and it is assumed to be close to unity, which is typically observed in emerging markets.

II.B.2. Bad Financial Standing

Next I describe the Bellman equation in bad financial standing, where the sovereign is not allowed to borrow in the international financial market. Therefore, there is nothing for the sovereign to do under this status. The Bellman equation in bad financial standing is given by

$$v^b(b_S, b_L, y) = u(c) + \beta\theta E_y v^g(b_S, b_L, y') + \beta(1-\theta) E_y v^b(b_S, b_L, y'), \tag{7}$$

where

$$c = y - L(y). \tag{8}$$

v^b is the current value in bad financial standing, and θ is an exogenous probablity with which the sovereign restores the access to financial market and moves on to the next period in good financial standing. Therefore, the second term in the right hand side of equation (10) is the discounted expected value of the next period in good financial standing, while the third term is that in bad financial standing. $L(y)$ is an output loss function in bad financial standing, which is standard in this literature[4]. Note that I assume that the sovereign is not required to repay debt in bad

[3]See Hatchondo and Martinez (2009) for the more detailed description of this specification of long-term debt. Also see Chatterjee and Eyigungor (2012) for the similar type of the specification.

[4]See Mendoza and Yue (2012) for the theoretical explanation for this adhoc default cost function. They assume that the intermediate goods production sector cannot import a part of imported inputs since they require the trade credit which is unavailable in bad financial standing. Because these imported inputs cannot be perfectly substituted by other inputs, the economy suffers from the efficiency loss in production after default.

financial standing, while Yue (2010) assumes that the sovereign must repay all the debt arrears before regaining access to the international financial market. This difference is crucial in debt renegotiation since my specification introduces the forward looking variables (asset prices) to the bargaining game.

II.B.3. Good Financial Standing

I am now in the position to integrate the value functions described so far and provide the Bellman equation in good financial standing. The sovereign chooses to default at the begining of the period if the value in bad financial standing exceeds that in continuing to repay debt. Therefore, the Bellman equation in good financial standing is expressed as

$$v^g(b_S, b_L, y) = \underset{repay, default}{\text{Max}} \{v^c(b_S, b_L, y), v^b(b_S^r(b_S, b_L, y), b_L^r(b_S, b_L, y).y)\}, \tag{9}$$

where $b_S^r(b_S, b_L, y)$ and $b_L^r(b_S, b_L, y)$ are determined in debt renegotiation, which is mentioned lator on. I define the default indicator function as the policy function of equation (9).

$$d(b_S, b_L, y) = \begin{cases} 1 & (v^c < v^b) \\ 0 & (otherwise) \end{cases} \tag{10}$$

If the sovereign chooses to default, it takes unity. Otherwise, it takes zero.

II.C. Debt Renegotiation

In this subsection, I formulate the debt renegotiation between the sovereign and the external creditor in a different way from Yue (2010). I allow the sovereign to exchange the set of older debt prior to default with that of newly issued debt in debt restructuring. By contrast, Yue (2010) assumes that the debt renegotiation determines the debt arrears owing to the external creditor and the sovereign must finish this repayment to restore the right to borrow in the international financial market. This difference is crucial as noted earilier since my formulation introduces the sovereign bond prices (forward looking variables) into the bargaining game. Moreover, the sovereign must consider the risk of short-term and long-term debt in good financial standing since they will repay them after default periods. The model is successful in capturing the negative relationship between haircut and maturity length because of this novel aspect in modeling debt renegotiation as explained in detail later.

Following Yue (2010), I use the Nash bargaining game to model the debt renegotiation between the sovereign and the external creditor.

$$\underset{b_S', b_L'}{\text{Max}} \left[\left(v^b(b_S^r, b_L^r, y) - v^a(y) \right)^{\alpha} \times \left(q_S^b(b_S^r, b_L^r, y) b_S^r + q_L^b(b_S^r, b_L^r, y) b_L^r \right)^{1-\alpha} \right] \tag{11}$$

s.t.

$$b_S^r \leqq b_S, \quad b_L^r \leqq b_L \tag{12}$$

The objective function is the weighted average of the payoffs given to the sovereign and the external creditor. The payoff for sovereign is the difference between the lifetime value when it agrees on the settlement, $v^b(b_S^r, b_L^r, y)$ and it stays in autarkey, $v^a(y)$. On the other hand, the payoff for the external creditor is given by the aggregate value of newly issued debts. q_S^b and q_L^b are prices of the short-term and long-term bonds in bad financial standing. The sovereign has the bargaining power α, while the external creditor possesses $1-\alpha$. The two constraints in equation (12) basically say that the restructured debt must not exceed the debt arrears prior to default.

II.D. Asset Pricing

In this subsection, I describe the equilibrium conditions for asset prices in the model. There are four types of assets: the short-term and long-term bonds issued by the sovereign in good and bad financial standing. Since the external creditors are risk-neutral and the international financial market is perfectly competitive, the expected return for each of the sovereign bonds must be equalized to the risk-free real world interest rate, r.

First, the price of the short-term sovereign bond in good financial standing is given by

$$q_S(b'_S, b'_L, y) = \frac{1}{1+r}\left[E_y[1-d(b'_S, b'_L, y')] + E_y[d(b'_S, b'_L, y')(b_S^r/b'_S)q_S^b(b_S^r, b_L^r, y')]\right]. \tag{13}$$

The expected return for this asset is decomposed into two parts: the expected principal repayment in good financial standing and expected short-term debt recovered in debt renegotiation after the default event.

Second, the price of the long-term sovereign bond in good financial standing is provided by

$$\begin{aligned} q_L(b'_S, b'_L, y) = \frac{1}{1+r}\Big\{ & E_y[(1-d(b'_S, b'_L, y'))(1+\delta q_L(b''_S(b'_S, b'_L, y'), b''_L(b'_S, b'_L, y'), y'))] \\ & + E_y[d(b'_S, b'_L, y)(b_L^r/b'_L)q_L^b(b_S^r, b_L^r, y')]\Big\}. \end{aligned} \tag{14}$$

Again, the expected return for the long-term bond is composed of two parts. The first term in equation (14) is the return when the sovereign does not decide to default in the next period. The second term is the term debt recovered in debt restructuring after default.

Third, the price of the short-term debt in bad financial standing is calculated as

$$\begin{aligned} q_S^b(b_S, b_L, y) = \frac{1}{1+r}\Big\{ & (1-\theta)E_y[q_S^b(b_S, b_L, y')] \\ & + \theta E_y[(1-d(b_S, b_L, y'))] + \theta E_y[d(b_S, b_L, y')(b_S^r/b_S)q_S^b(b_S^r, b_L^r, y')]\Big\} \end{aligned} \tag{15}$$

The first term in the bracket on the left hand side is the expected price of the sovereign debt when the sovereign stays in bad financial standing in the next period with probability '$1-\theta$. The remaining two terms describe the returns when the sovereign moves on to the next period in good financial standing with probability θ. There are two cases; in the first case, the sovereign will choose to continue to repay so that the expected return is $\theta E_y[(1-d(b_S,b_L,y'))]$. In the second case, the sovereign will decide to default again. The expected return in this case is given by $\theta E_y[d(b_S,b_L,y')(b^r_S/b_S)q^b_S(b^r_S,b^r_L,y')]$.

Finally, the price of the long-term debt in bad financial standing is expressed as

$$\begin{aligned} q^b_L(b_S,b_L,y) = \frac{1}{1+r}\Big\{ &(1-\theta)E_y[q^b_L(b_S,b_L,y')] \\ &+\theta E_y[(1-d(b_S,b_L,y'))(1+\delta q_L(b'_S(b_S,b_L,y'),b_L(b_S,b_L,y'),y')] \\ &+\theta E_y[d(b_S,b_L,y')(b^r_L/b_L)q^b_L(b^r_S,b^r_L,y')]\Big\} \end{aligned} \tag{16}$$

This price has the similar structure to the price of the short-term bond in bad financial standing mentioned above. The first term in the bracket on the right hand side is the expected return when the sovereign stays in bad financial standing in the next period. The second term is the expected return when the sovereign goes into the next period in good financial standing and decides to continue to repay. The last term is the expected return when the sovereign moves on to the next period in good financial standing and then decides to default immediately.

II.E. Equilibrium

Now I define the Recursive Markov equilibrium to close the model.

Definition (Recursive Markov Equilibrium)

The recursive Markov equilibrium for this economy is defined as a set of policy functions for (*i*) the sovereign's consumption $c(b_S,b_L,y)$, (*ii*) the sovereign's asset holdings $\{b'_S(b_S,b_L,y),b'_L(b_S,b_L,y)\}$, and (*iii*) the default decision $d(b_S,b_L,y)$; and (*iv*) the asset prices $\{q_S(b_S,b_L,y),q_L(b_S,b_L,y),$ $q^b_S(b_S,b_L,y),q^b_L(b_S,b_L,y)\}$; and (*v*) the outcomes of the debt renegotiation $\{b^r_S(b_S,b_L,y),b^r_L(b_S,b_L,y)\}$ such that;

1. Given the asset price functions $\{q_S(b_S,b_L,y),q_L(b_S,b_L,y)\}$ and the outcomes of the debt renegotiation $\{b^r_S(b_S,b_L,y),b^r_L(b_S,b_L,y)\}$, the consumption policy function $c(b_S,b_L,y)$, asset holdings policy functions $\{b'_S(b_S,b_L,y),b'_L(b_S,b_L,y)\}$, and default indicator function $d(b_S,b_L,y)$ solves the sovereign's optimization problems.

2. Taken the asset price functions $\{q^b_S(b_S,b_L,y),q^b_L(b_S,b_L,y)\}$ as given, the outcomes of the debt renegotiation $\{b^r_S(b_S,b_L,y),b^r_L(b_S,b_L,y)\}$ solve the Nash bargaining game in debt renegotiation.

3. Given the sovereign's asset holdings $\{b'_S(b_S, b_L, y), b'_L(b_S, b_L, y)\}$, (iii) the default decision $d(b_S, b_L, y)$, and the outcomes of the debt renegotiation $\{b^r_S(b_S, b_L, y), b^r_L(b_S, b_L, y)\}$, the asset price functions $\{q_S(b_S, b_L, y), q_L(b_S, b_L, y), q^b_S(b_S, b_L, y), q^b_L(b_S, b_L, y)\}$ are consistent with creditor's expected zero profits.

III. Results

In this section, I parametize some functions in subsection A and calibrate the model in subsection B and then I provide the main result from the simulation in subsection C.

III.A. Parameterization and Calibration

The model is solved numerically to yield the prediction for the maturity structure of haircuts in debt restructuring. Therefore, it is necessary to parametize functions in the model.

I use the following CRRA utility function in the model simulation.

$$u(c) = \frac{c^{1-\gamma}}{1-\gamma}, \tag{17}$$

where γ is the relative risk aversion parameter and the inverse of intertemporal substitution parameter. This specification is standard in the quantitative sovereign debt models.

The output cost function is nonlinear in the sense that it includes the max operator.

$$L(y) = max\{a_0 + a_1 y + a_2 y^2, 0\} \tag{18}$$

This specification implies that the default cost is higher when the current endowment is higher. This can lead to the larger amount of debt in equlibrium[5].

III.B. Calibration

I mainly follow the calibration from Arellano and Ramarayanan (2012). Table 2 describes all the parameter values in the model. They used Brazilian data to estimate their model. I changed the parameter values of the discount factor β and the value of parameters determining the duration of long-term debt δ. Moreover, Moreover, I chose the parameter value of the bargaining power of the sovereign in debt renegotiation α, which is absent from Arellano and Ramarayanan (2012).

[5]This nonlinearity of the output loss function is also closely related to the volatility of spread, which is defined as the difference between the one-period expected return of sovereign bond and risk-free world real interest rate. This functional form helps the model capture the volatility of spread when the variance of endowment is sufficiently high. However, this creates too strong link between spread and output, which is unrealistc in data. For more information, see Aguiar et.al (2016).

Table 2: Calibration of the Model

Parameter	Value	Description	Sources
σ	2	RRA	Common value in business cycle
β	0.85	Yearly discount factor	Close value used in the literature
r	0.032	World Interest Rate	Arellano and Ramarayanan (2012)
θ	0.17	Probability of reentry	Arellano and Ramarayanan (2012)
a_0	-0.955$\bar{y}$	Parameter of output loss function	Arellano and Ramarayanan (2012)
a_1	1.0	Parameter of output loss function	Arellano and Ramarayanan (2012)
a_2	0.0	Parameter of output loss function	Arellano and Ramarayanan (2012)
ρ	0.9	Serial correlation of $ln(y_t)$	Arellano and Ramarayanan (2012)
η	0.017	Std. dev. of innonvation ϵ_t	Arellano and Ramarayanan (2012)
α	0.7	The bargaining power of the debtor	Value close to Yue(2010)
δ	0.5	The geometric coupon rate	Duration is set to two years (tentative)

The values of discount factor β and the bargaining power α are very close to those adopted in the literature. Relatively low discount factors are common in this literature. And Yue (2010) chooses $\alpha = 0.72$ in calibration. I acknowledge the value of δ is too low. This low value results in the lower duration of long-term debt around two years, while the common durations in the literature are much higher. Therefore, I must set this parameter value much higher in line with the literature in my future research. However, I believe that the implication of the model for the haircuts is robust to the change in this parameter value.

To check the varidity of the model, Table 3 describes some of the simulated and empirical moments. Although I do not estimate the model, it well capture the empirical moments except for the slope of the spread curve. Here spread is defined as the difference between the expected one-period return to the sovereign bond and the risk free world interest rate. In the model, the spread curve is downward sloping while in the data, it is upward sloping. This failure comes from the variation of the haircut as explained in the next section. However, I do not include all the sources of the term-premium. Therefore, it is my future agenda to include other causes of term premia. The relative size of the standard deviation of trade balance to that of output is too high in the model. But if the duration of long-term debt determined by δ is extended, this matching will be improved. This is because the sovereign will be less exposed to the large fluctuation of bond prices which drives the variation of trade balance. The model can predict the relative size of consumption to that of output close to the corresponding moments in data. It is also successful in matching the moments related to debt with the corresponding empirical counterparts.

III.C. Results

I simulated the model, generating an artificial time series of endowment by random generator and

Table 3: Business Cycle Moments

	Data	Model
Mean spread of short-term debt	2.9	3.9
Mean spread of long-term debt	4.8	2.4
SD(trade balance)/SD(y)	0.36	1.53
SD(c)/SD(y)	1.1	0.98
All debt: $(q_S b_S + q_L b_L)/y$	0.3	0.22
Short: $q_S b_S/(q_S b_S + q_L b_L)$	0.13	0.17
Long: $q_L b_L/(q_S b_S + q_L b_L)$	0.87	0.82

Table 4: Theoretical Moments for Haircuts

	Percentages
Mean haircut of short-term debt	89%
Mean haircut of long-term debt	49%
SD(haircut of short-term debt)	31%
SD(haircut of long-term debt)	8%

then I take the averages and standard deviations of each series. I report the results related to haircuts in Table 4.

The model predicts the negative relationship between haircut and maturity length, which is consistent with what Table 1 reports. On average, the haircut of short-term debt is more severe than that of long-term debt. There are two driving forces to affect the maturity structure of haircuts in debt restructuring: the risk aversion of the sovereign and the forward looking behavior taken by the external creditor.

First of all, the sovereign prefers long-term debt to short-term debt since long-term debt plays the role of insurance. More specifically, the sovereign needs to repay $1 + \delta q_L(b'_S, b'_L, y)$ to pay back one unit of debt issued in the previous period. $q_L(b'_S, b'_L, y)$ is an increasing function of endowment y as shown in Figure 3, where $B'_l = -b'_L$. The line paited orange dipicts the price of long-term bond when the endowment is relatively high, while the blue curve dipicts that when the endowment is comparatively low. This monotonicity comes from the fact that the higher the endowment is, the larger default incentive is. The consumption level is lower in the periods of lower endowment so that the debt repayment for the long-term debt is negatively correlated with the pricing kernel. On the other hand, the sovereign must repay the fixed amount of consumption for the short-term debt. The correlation of the stochastic discount factor with that repayment is nil. That is why the sovereign is more willing to issue long-term debt in debt renegotiation.

Second, the external creditor takes the future actions taken by the sovereign into consideration in debt restructuring since their payoff is the function of the bond prices (forward looking variables) in my specification. If the sovereign issues the larger amount of long-term debt than short-term debt, it is more likely to default after it reenters the international financial market as shown in Arellano and Ramarayanan (2012). This biased holdings toward long-term debt, therefore, decrease the price of sovereign bonds, reducing the payoff of the external creditor in debt renegotiation. Quantitatively, the first force dominates the second one on average. Therefore, the model captures the negative association between haircut and maturity length on average.

Notably, this maturity structure is determined not by the current situation in which the sovereign renegotiates the debt restructuring with the external creditor, but the future situation where the sovereign will regain access to the international financial market. This forward looking aspect in haircut cannot be introduced in the form of cash repayment like Yue (2010) since the sovereign brings no debt arrear after regaining access to the international financial market by repaying all debt arrears determined in debt restructuring. The result here implies that it might be not be innocuous to formulate debt renegotiation in the form of either bond exchange as in my model or cash repayment as in Yue (2010) to analyze the implication of haircut.

IV. Conclusions

I developed a extended version of the quantitative sovereign debt models similar to Arellano and Ramanarayanan (2012) and Yue (2010) to provide the theoretical explanation for the negative correlation of haircut with maturity length frequently observed in data. This difference in haircut across maturities is a source of term premium of sovereign bonds. Since debt crises often involves rollover crises in emerging markets and they are caused partly because of the maturity structure of debt biased toward the short-term instruments, it is important to analyze this source which affects the maturity choices by sovereigns.

My specification of debt renegotiation allows the sovereign to exchange the old debt instruments prior to default with the newly issued debt instruments. This introduces the possibility for haircuts to be affected by the future actions taken by the sovereign after bad financial standing periods. This new insight is the key to capture the negative relationship between haircut and maturity length; the sovereign prefers long-term debt since it is less risky from its perspective and the external creditor tries to avoid the large holdings of long-term debt which leads to the higher default incentive. My modification of the bargaining game in debt restructuring emphasizes the forward looking aspect of maturity structure of haircut.

I acknowledge that the model should be formally estimated by the simulated method of moments, while I tentatively calibrate the model, adopting the parameter values used in the literature. However, I believe that the implication of the model for the maturity structure of haircuts will be robust even if I estimate the model. This is because the implication comes from the inherent nature of long-term debt.

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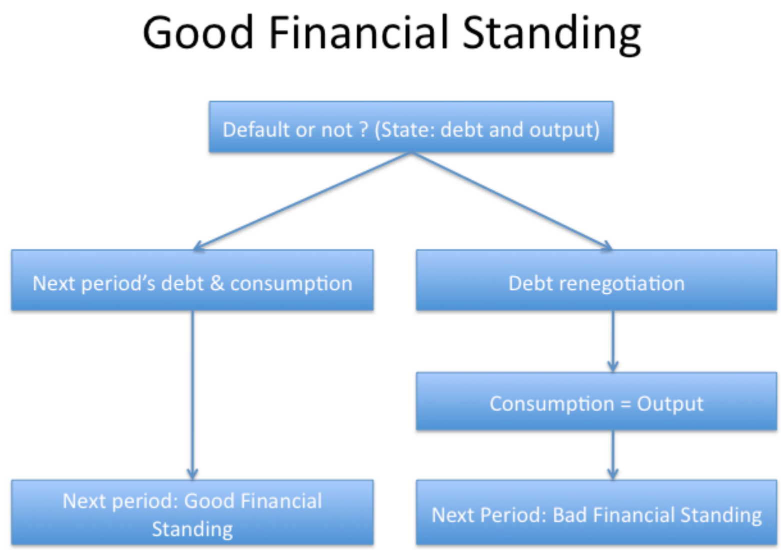

Figure 1: Timing of Events in Good Financial Standing

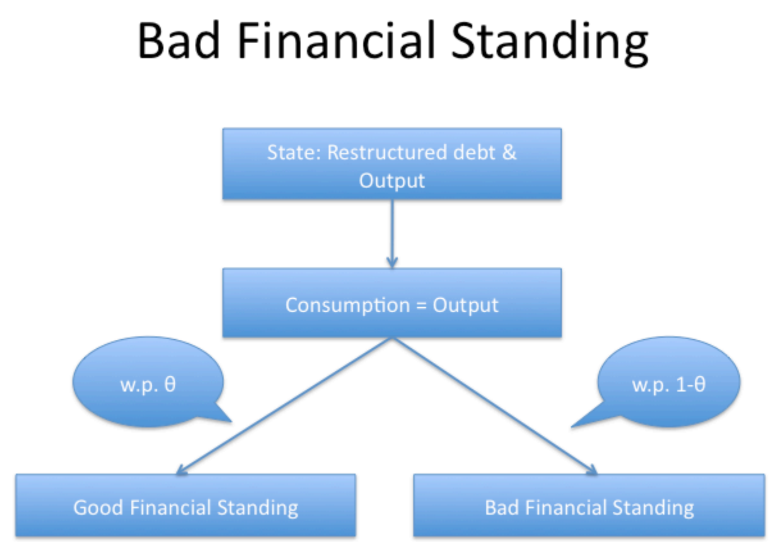

Figure 2: Timing of Events in Bad Financial Standing

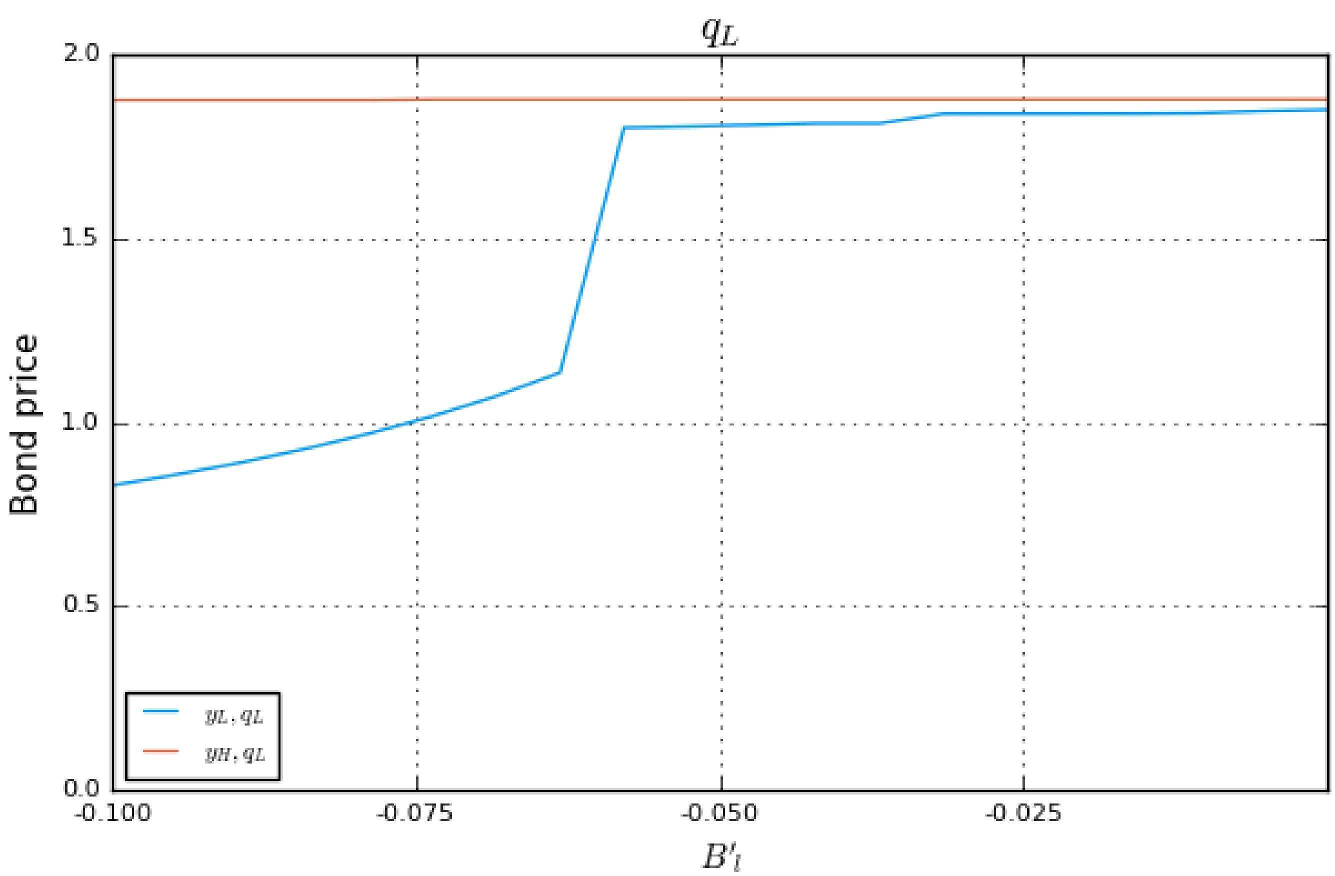

Figure 3: Long-Term Bond price Function in Good Financial Standing