# Capital Adequacy Requirements and Financial Friction in a Neoclassical Growth Model

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March 20, 2017

**Abstract**

I introduce financial market friction into a neoclassical growth model. I consider a moral hazard problem between bankers and workers in the macroeconomic model. Using the model, this study analyzes how capital adequacy requirements for banks affect the economy. I show that there is a case in which policy institution should not change the minimum capital adequacy requirements in order to improve the steady-state level of consumption when the economy experiences a recession. This result implies that counter-cyclical capital requirements, that is, relaxing the rule when there is recession, is not always optimal for consumers. The condition for the above case depends on the combinations of parameters, such as the degree of financial friction, discount factor, and initial net worth of banks. Moreover, I show that when a negative shock on productivity occurs, deregulation has a good effect on the economy in only the country in which the financial market develops sufficiently.

**Keywords:** Capital adequacy requirements, Financial Intermediaries, Macro-prudential policies

**JEL Classification Codes:** E44, G21, G28

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# 1 Introduction

Since the financial crisis, macro-prudential policies that aim to make the financial sector more resilient have been discussed all over the world. Above all, many researchers and policy institutions have discussed capital adequacy requirements. The Bank for International Settlements (BIS) revised the Basel II market risk framework in 2009. Moreover, the BIS revised the minimum capital requirements for market risk framework in 2016. Basel III is intended to strengthen bank capital requirements. Then, these reforms will raise minimum capital adequacy ratio. Generally, there is agreement that minimum capital adequacy ratios should be pro-cyclical.[1] Against this background, there are many studies on the analysis of capital adequacy requirements in macroeconomic models after the 2007–2008 financial crisis. Benigno et al. (2010), Bianchi and Mendoza (2010), Kannan et al.(2012), Angeloni and Faia (2013), Unsal (2013), Angelini et al. (2014), Medina and Roldos (2014), Baker and Wurgler (2015), and Collard et al (2017) study capital adequacy requirements as one type of macro-prudential policy using recent macroeconomic models.[2] Undertaking empirical research, Furfine (2000) and Francis and Osborne (2012) study capital adequacy requirements for banks in the US and European countries.

The current study examines how capital adequacy requirements affect the economy in a macroeconomic model. I introduce financial friction into the macro economy with bankers and workers. Using this model, I find that policy institutions should not change minimum capital adequacy ratios even if the economy experiences a recession in the country whose financial market has developed sufficiently. In a country whose financial market has not developed sufficiently, policy institutions should pull up the minimum capital adequacy ratio. This policy makes the capital adequacy ratio determined in market equilibrium equal to the minimum capital adequacy ratio and thereby realizes higher levels of consumption and output.

The model I develop is a simple macroeconomic model with financial market friction. Following

[1] Some studies examine whether the leverage ratio is counter-cyclical. For instance, He and Krishnamurthy (2008) and Gertler and Kiyotaki (2015) show that the capital adequacy ratio is pro-cyclical, that is, the leverage ratio is counter-cyclical. On the other hand, Adrian and Shin (2009) show that the leverage ratio is pro-cyclical. The current study shows the former case.

[2] Capital adequacy requirements existed before the financial crisis. Rochet (1992), Blum and Hellwig (1995), and Barth et al. (2004) study it in macroeconomic models.

Gertler and Kiyotaki (2010), I introduce financial market friction into a macroeconomic model. Gertler et al. (2012), Gertler and Kiyotaki (2015), and Aoki et al. (2016) extend the model of Gertler and Kiyotaki (2010) in order to analyze bank runs, the monetary policies in emerging countries, and macro-prudential policies, respectively. Gertler et al. (2012) is relevant to the current study since they analyze macro-prudential policies; however, they do not analyze capital adequacy requirements. To incorporate capital adequacy requirements in such a model and examine how these requirements affect the level of workers' investment, bankers' investment, aggregate capital, and thereby consumption, this study simplifies the model of Gertler and Kiyotaki (2010).[3] In the current model, there is only one production sector and there is no capital goods sector. Owing to this simplification, using the current model, I cannot analyze how the economic crisis affects the economy through changes of asset prices. However, in the current model, I can analyze how a productivity shock affects consumption and output, since the productivity shock changes the spread of lending returns and deposit costs, and this shock affects both the production sector and the banking sector independently, as well as the above studies.[4] This effect does not emerge in a neoclassical growth model with no financial market frictions. In the current model, I analyze how capital adequacy requirements affect macroeconomic variables, such as investment, consumption, and output, using the neoclassical growth model with financial market friction.

The remainder of this paper is organized as follows. Section 2 presents a basic structure of the model. It describes the behavior of households and banks, final goods production, and market equilibrium. Section 3 describes the model's dynamic system and Section 4 analyzes the properties of a steady state. Concluding remarks are offered in Section 5.

## 2 Model

I consider a closed economy in which time is discrete. Following Gertler and Kiyotaki (2010), I introduce financial market friction into a macroeconomic model.[5] There is a representative household

[3]Technically, incorporating capital adequacy requirements in such a model leads to more cases of dynamic systems.

[4]From this view, the current study adopts the spirit of the macroeconomic model with financial market friction, as in Kiyotaki and Moore (1997).

[5]Gertler et al. (2012), Gertler and Kiyotaki (2015), and Aoki et al. (2016) introduce financial market friction into a macroeconomic model following Gertler and Kiyotaki (2010).

with a continuum of members of measure unity. The members consist of bankers and workers. I explain the behavior of the households in the following subsection.

## 2.1 Households

Workers supply labor to a final good sector. They use their wage earnings for savings and consumption. They save by depositing their assets with bankers and managing the capital market. I assume there is a disadvantage of workers relative to bankers in the financing business.[6] Moreover, workers are lenders in the capital market.[7] Specifically, in order to manage capital in the capital market, workers require the following extra management costs while bankers do not:

$$f\left(K^h\right) = \frac{1}{2} \cdot \omega \cdot \left(K^h(t)\right)^2, \tag{1}$$

where $K^h$ represents capital holdings by workers at the end of period $t$ and $\omega > 0$ is a parameter reflecting the disadvantage of workers relative to bankers in the financing business. Workers are under the following no-arbitrage condition:

$$r\,(t+1) \cdot K^h(t) - f\left(K^h\right) = r^d\,(t+1) \cdot K^h(t), \tag{2}$$

where $r\,(t+1)$ is a rental price in the capital market and $r^d\,(t+1)$ represents the returns on deposits. The left-hand side of equation (2) represents the returns on managing capital in the capital market. The right-hand side of equation (2) represents the returns on deposits with banks.

Rewriting equation (2), I obtain

$$r\,(t+1) - r^d\,(t+1) = \frac{\omega}{2} \cdot K^h(t). \tag{3}$$

Next, I describe the representative households' problem. At each period, with probability $\sigma$, bankers move back to the household and with the same probability $\sigma$, workers become the new bankers. Therefore, the ratio of workers to bankers is constant and thus, the total population is constant in this model. When a banker becomes a worker, the banker brings the net worth of banking to the household. When a worker becomes a banker, the representative household gives

[6] Gertler and Kiyotaki (2015) and Aoki et al. (2016) adopt the same assumption and the same function of management cost.

[7] I assume the cost is infinity when workers are borrowers in the capital market.

a part of its savings to the new banker for start-up funds. I consider a moral hazard problem in which bankers misbehave instead of investing their capital. This capital consists of deposits from workers and the bankers' own net worth.[8] Thus, workers do not deposit all of their assets but lend them in the capital market even if they need extra management costs of capital.

The representative households maximize its expected utility subject to a budget constraint, as follows:[9]

$$E_0 \sum_{t=0}^{\infty} \beta^t \ln C(t), \tag{4}$$

subject to

$$\begin{aligned} & C(t) + f\left(K^h(t)\right) + S(t) \\ =& w(t) \cdot L(t) + r(t) \cdot K^h(t-1) + r^d(t) \cdot D(t-1) \\ +& (1-\sigma) \cdot \left[r(t) \cdot K^b(t-1) - r^d(t) \cdot D(t-1)\right] - (1-\sigma) \cdot \lambda \cdot S(t), \end{aligned} \tag{5}$$

with

$$S(t) = K^h(t) - (1-\delta) \cdot K^h(t-1) + D(t) - D(t-1), \tag{6}$$

where $C(t)$ is consumption in period $t$; $\beta \in (0,1)$ denote the discount factor; $S(t)$ represents savings in period $t$; $w(t)$ is the wage in period $t$; $L(t)$ is the labor supply in period $t$; $D(t)$ represents deposits in period $t$; $K^b(t)$ is the capital investment by bankers; $\lambda \in (0,1)$ is the proportion of savings used for the net worth of new bankers; and $\delta \in (0,1)$ is the rate of depreciation of capital. Here, $1-\sigma \in (0,1)$ is the probability that bankers become workers and the probability that workers become bankers. I assume that $L(t)$ is 1 hereafter.

The first-order conditions for consumption, the capital investment of workers and the deposits, imply

$$\frac{q(t)}{q(t+1)} = \frac{C(t+1)}{\beta \cdot C(t)}, \tag{7}$$

$$\frac{q(t)}{q(t+1)} = \frac{(1-\delta) \cdot (1 + (1-\sigma) \cdot \lambda) + r(t+1)}{1 + (1-\sigma) \cdot \lambda + \omega \cdot K^h(t)}, \tag{8}$$

$$\frac{q(t)}{q(t+1)} = \frac{(1 + (1-\sigma) \cdot \lambda) + \sigma \cdot r^d(t+1)}{1 + (1-\sigma) \cdot \lambda}. \tag{9}$$

[8] I describe the moral hazard problem in detail in the next subsection.

[9] I assume a logarithmic utility function for simplification.

From equations (3) and (9), I obtain

$$\frac{q(t)}{q(t+1)} = \frac{(1+(1-\sigma)\cdot\lambda)+\sigma\cdot r(t+1)-\frac{\sigma}{2}\cdot\omega\cdot K^h(t)}{1+(1-\sigma)\cdot\lambda}. \tag{10}$$

From equations (8) and (10), I obtain

$$\frac{\omega^2}{2}\cdot\sigma\left(K^h(t)\right)^2+\left[-\sigma\cdot\omega\cdot r(t+1)-\left(1-\frac{\sigma}{2}\right)\cdot\omega\cdot(1+(1-\sigma)\cdot\lambda)\right]\cdot K^h(t)$$
$$+(1+(1-\sigma)\cdot\lambda)\left[r(t+1)\cdot(1-\sigma)-\delta\cdot(1+(1-\sigma)\cdot\lambda)\right]=0. \tag{11}$$

## 2.2 Banks

Bankers maximize the expected value of their own net worth. The problem of a banker who exits the bank at period $j$ and brings net worth back to the household is

$$\max V(t) = E_t\left[\sum_{j=1}^{\infty}\beta^t\cdot\sigma^{j-1}\cdot(1-\sigma)\cdot n(t+j)\right], \tag{12}$$

subject to

$$n(t)+d(t)=k^b(t), \tag{13}$$

and

$$n(t)=r(t)\cdot k^b(t-1)-r^d(t)\cdot d(t-1), \tag{14}$$

where $n(t)$ is the net worth of each banker at the end of period $t$, $d(t)$ represents funds from households' deposits of each banker at the end of period $t$, and $k^b(t)$ represents the investment of each banker at the end of period $t$. Equations (13) and (14) are constraints on the flow of funds. Equation (13) represents the balance sheet condition while equation (14) is the revolution of a banker's net worth.

I consider the following moral hazard problem, following Gertler and Kiyotaki (2010). After bankers collect deposits from workers, bankers can leave the bank with the funds and divert a part of the funds for their private benefit. A proportion of the funds that can be diverted is $\theta \in (0,1)$. Thus, the incentive comparative constraint can be written as

$$V(t) \geq \theta\cdot k^b(t). \tag{15}$$

The light-hand side of equation (15) is the value of investment of bankers' funds. The right-hand side of equation (15) is the value of diverting these funds.

I introduce capital adequacy requirements into this model. The rule is that bankers must keep the ratio of their net worth to risky assets, that is, investment must be larger than $\overline{\phi}$. Let $\phi_t$ denote the capital adequacy ratio, $\phi_t \equiv \frac{n(t)}{k^b(t)}$. Formally, the capital adequacy requirements in this model are described as follows:

$$\phi_t \geq \overline{\phi}. \tag{16}$$

Generally, the value of the banker at the end of period $t$ satisfies the Bellman equation:

$$V(t) = E_t \left[\beta \cdot (1-\sigma) \cdot n(t+1) + \beta \cdot \sigma \cdot V(t+1)\right]. \tag{17}$$

As in Gertler and Kiyotaki (2010), to solve the decision problem, I guess that the value function is linear, as follows:

$$V(t) = \iota(t) \cdot k^b(t) + \nu(t) \cdot d(t), \tag{18}$$

where $\iota(t) > 0$ is the marginal cost of the banker's investment and $\nu(t) > 0$ is the marginal cost of deposits.

Let $\mu(t)$ be defined such that $\mu(t) \equiv \iota(t) - \nu(t)$. Substituting this definition and equation (13) into equation (18), I obtain

$$V(t) = \mu(t) \cdot k^b(t) + \nu(t) \cdot n(t). \tag{18'}$$

Substituting equation (18') into (15), I obtain

$$\phi(t) \geq \frac{\theta - \mu(t)}{\nu(t)}, \tag{19}$$

where $\phi(t) \equiv \frac{n(t)}{k^b(t)}$.

Equation (19) is binding if $0 < \mu(t) < \theta$, and thus, equations (16) and (19) yield

$$\frac{\theta - \mu(t)}{\nu(t)} = \hat{\phi}(t) \equiv \left\{ \begin{array}{lll} \phi(t), & \text{when} & \phi(t) > \overline{\phi}, \\ \overline{\phi}, & \text{when} & \phi(t) \leq \overline{\phi}, \end{array} \right\}. \tag{20}$$

From equation (20) and the definition of $\mu(t)$, I obtain

$$\iota(t) = \theta + \nu(t) \cdot \left[1 - \hat{\phi}(t)\right]. \tag{21}$$

I arrange the flow constraint on funds. First, equation (14) can be rewritten as

$$d(t) = \frac{r(t+1)}{r^d(t+1)} \cdot k^b(t) - \frac{n(t+1)}{r^d(t+1)}. \tag{22}$$

Then, substituting equation (22) into equation (13), I obtain

$$n(t+1) = \Big[r(t+1) - r^d(t+1)\Big] \cdot k^b(t) + r^d(t+1) \cdot n(t). \tag{23}$$

Let $\Omega(t+1)$ be the marginal value of net worth at period $t+1$. After I combine the conjectured value function (18'), the Bellman equation (17), and equation (23), I verify that the value function is linear in $k^b(t)$ and $n(t)$ if $\mu(t)$ and $\nu(t)$ satisfy[10]

$$\mu(t) = \beta \cdot \Omega(t+1) \cdot \Big[r(t+1) - r^d(t+1)\Big], \tag{24}$$

$$\nu(t) = \beta \cdot \Omega(t+1) \cdot r^d(t+1), \tag{25}$$

where

$$\Omega(t+1) \equiv (1-\sigma) + \frac{\sigma \cdot \mu(t+1)}{\hat{\phi}(t+1)} + \sigma \cdot \nu(t+1). \tag{26}$$

From equations (24), (25), and the definition of $\mu(t)$, I obtain

$$\iota(t) = \beta \cdot \Omega(t+1) \cdot r(t+1). \tag{27}$$

Substituting equations (25) and (27) into equation (21), I obtain

$$\beta \cdot \Omega(t+1) \cdot \Big[r(t+1) - \Big(1 - \hat{\phi}(t)\Big) \cdot r^d(t+1)\Big] = \theta. \tag{28}$$

$\hat{\phi}(t)$ is given by equation (20).

From equations (21) and (26), and the definition of $\mu(t)$, I obtain

$$\Omega(t+1) = \frac{(1-\sigma) \cdot \hat{\phi}(t+1) + \sigma \cdot \theta}{\hat{\phi}(t+1)}, \forall t. \tag{29}$$

Substituting equations (25), (27), and (29) into equation (21), I obtain the relationship between the return of investment $r(t+1)$ and the payments for deposits $r^d(t+1)$, as follows:

$$r(t+1) = \left[\frac{\theta \cdot \hat{\phi}(t+1)}{\beta \cdot \Big((1-\sigma) \cdot \hat{\phi}(t+1) + \sigma \cdot \theta\Big)}\right] + \Big(1 - \hat{\phi}(t)\Big) \cdot r^d(t+1). \tag{30}$$

In equilibrium, the solution of the maximization problem for the banker, (30) and the no-arbitrage condition (3) must be satisfied. Thus, from equations (3) and (30), I obtain the return of capital

[10] See Appendix A.

investment $r(t+1)$ as the function of the capital adequacy ratio, $\hat{\phi}(t)$ and $\hat{\phi}(t+1)$, where $\hat{\phi}(\cdot)$ is given by equation (20):

$$r(t+1) = -\left(\frac{1-\hat{\phi}(t)}{\hat{\phi}(t)}\right) \cdot \frac{\omega}{2} \cdot K^h(t) + \frac{\theta}{\beta \cdot \left((1-\sigma) \cdot \hat{\phi}(t+1) + \sigma \cdot \theta\right)} \cdot \left(\frac{\hat{\phi}(t+1)}{\hat{\phi}(t)}\right), \tag{31}$$

where $\omega > 0$ and $0 < \sigma < 1$.

Since in equilibrium the optimal conditions for workers and bankers, equations (11) and (31), are satisfied, I obtain the following equation:[11]

$$\hat{\phi}(t+1) = \begin{cases} \Psi\left(K^h(t), \hat{\phi}(t)\right), & \text{when} \quad \phi(t) > \overline{\phi}, \\ \overline{\phi}, & \text{when} \quad \phi(t) \leq \overline{\phi}, \end{cases}, \tag{32}$$

where

$$\Psi\left(K^h(t), \hat{\phi}(t)\right) \equiv \frac{\beta \cdot \sigma \cdot \theta \cdot \Gamma\left(K^h(t), \hat{\phi}(t)\right)}{\theta \cdot \omega \cdot \hat{\phi}(t) \cdot K^h(t) - (1-\sigma) \cdot (1 + (1-\sigma) \cdot \lambda) - \beta \cdot (1-\sigma) \cdot \Gamma\left(K^h(t), \hat{\phi}(t)\right)}, \tag{33}$$

with

$$\Gamma\left(K^h(t), \hat{\phi}(t)\right) \equiv \frac{\omega^2 \cdot \sigma}{2} \cdot \left(K^h(t)\right)^2 - \frac{\omega}{2} \cdot K^h(t) \cdot (1+(1-\sigma) \cdot \lambda) \cdot [(1-\sigma) + \hat{\phi}(t)] - \delta \cdot \hat{\phi}(t) \cdot (1+(1-\sigma) \cdot \lambda)^2.$$

Equations (32) and (33) imply that the capital adequacy ratio at period $t+1$, $\hat{\phi}(t+1) = \frac{n(t+1)}{k^b(t+1)}$ depends on the capital adequacy ratio at period $t$, $\hat{\phi}(t) = \frac{n(t)}{k^b(t)}$ and the capital investment of workers, $K^h(t)$ under the capital adequacy requirements, $\hat{\phi}(t) > \overline{\phi}$.

## 2.3 Final Goods Producer

The final goods are produced by capital and labor. The production function is as follows:

$$Y(t) = A \cdot K(t)^{\alpha} \cdot L(t)^{1-\alpha}, \tag{34}$$

where $Y(t)$ is aggregate output at period $t$, A is a parameter of aggregate productivity, $K(t)$ is aggregate capital used for production at period $t$, and $L(t)$ is aggregate labor supply at period $t$ with $\alpha \in (0,1)$. For simplicity, I normalize the number of workers in each period as one, $L(t) = 1$.

[11] The derivation of (32) is given in appendix B.

Perfect competition prevails in the final goods sector. I take the final good as a numeraire. The optimal conditions of the profit maximization are

$$r(t) = \alpha \cdot \frac{Y(t)}{K(t)}, \tag{35}$$

$$w(t) = \alpha \cdot Y(t). \tag{36}$$

### 2.4 Market Equilibrium

Output is consumed, invested, or used to pay the cost of managing the household's capital, as follows:

$$Y(t) = C(t) + I(t) + f\left(K^h(t)\right), \tag{37}$$

with

$$I(t) = K(t+1) - K(t) + \delta \cdot K(t). \tag{38}$$

The market equilibrium for capital ownership implies

$$K(t) = K^h(t-1) + K^b(t-1), \tag{39}$$

where $K(t)$ is the aggregate capital in period $t$, $K^h(t-1)$ is the aggregate capital holdings of workers at the end of $t-1$, and $K^b(t-1)$ is the aggregate capital holdings of bankers at the end of $t-1$ with $K^b(t-1) \equiv \int k^b(t-1)$.

The competitive equilibrium is described by the four state variables, $\left(K^h(t), K^b(t), D(t), N(t)\right)$, the three price variables $\left(w(t), r(t), r^d(t)\right)$, and the two control variables $(Y(t), C(t))$.

## 3 Dynamic System

To describe the dynamic system of the model, I define the ratio of bankers' lending and aggregate capital as $\eta(t) \equiv \frac{K^b(t-1)}{K(t)}$, the ratio of workers' capital holdings and aggregate capital as $1-\eta(t) \equiv \frac{K^h(t-1)}{K(t)}$, the ratio of consumption and aggregate capital as $x(t) \equiv \frac{C(t-1)}{K(t)}$, and the ratio of bankers' net worth and aggregate capital as $B(t) \equiv \frac{N(t-1)}{K(t)}$ with $N(t) \equiv \int n(t)$.

The no-arbitrage condition (1), the relationship among the return of investment, $r(t)$, the capital adequacy ratio, $\hat{\phi}(t)$ (31), the goods market-clearing condition (37), the capital accumulation (38), and the definitions of $x(t)$, $\eta(t)$ and $B(t)$ yield [12]

$$-\left(\frac{B(t+1)}{\eta(t+1)}\cdot\frac{\eta(t)}{B(t)}\right)\cdot\left[\frac{\theta\cdot(1+\alpha\beta\cdot x(t))}{\alpha\beta\left[(1-\sigma)\cdot\frac{B(t+1)}{\eta(t+1)}+\sigma\theta\right]}\right]+\frac{K(t+1)}{K(t)}+\frac{\omega}{2}\cdot\frac{(1-\eta(t+1))^2\cdot(K(t+1))^2}{K(t)}$$

$$=\frac{x(t)\cdot\left[\beta\cdot(1-\delta)\cdot(1+(1-\sigma)\lambda)-\frac{\beta\cdot\omega}{2}\cdot(1-\eta(t))\cdot K(t)\cdot\left(\frac{1-\frac{B(t)}{\eta(t)}}{\frac{B(t)}{\eta(t)}}\right)\right]}{(1+(1-\sigma)\lambda)+\omega\cdot(1-\eta(t))\cdot K(t)}.$$

$$-\frac{\omega}{2\alpha}\cdot(1-\eta(t))\cdot K(t)\cdot\frac{\left(1-\frac{B(t)}{\eta(t)}\right)}{\frac{B(t)}{\eta(t)}}+(1-\delta). \tag{40}$$

From the optimization conditions for the representative households' problem, (7) and (8), the relationship among the return of investment, $r(t)$, the capital adequacy ratio, $\hat{\phi}(t)$ (31), and the definitions of $x(t)$, $\eta(t)$ and $B(t)$, I obtain[13]

$$\frac{x(t+1)}{x(t)}\cdot\frac{K(t+1)}{K(t)}\cdot[(1+(1-\sigma)\lambda)+\omega\,(1-\eta(t))\,K(t)]-\frac{\theta\frac{B(t+1)}{\eta(t)}\frac{\eta(t+1)}{B(t)}}{(1-\sigma)\frac{B(t+1)}{\eta(t+1)}+\sigma\theta}$$

$$=\beta(1-\delta)\,(1-(1-\sigma)\lambda)+\beta\cdot\left(\frac{\frac{B(t)}{\eta(t)}-1}{\frac{B(t)}{\eta(t)}}\cdot\frac{\omega}{2}\,(1-\eta(t))\,K(t)\right). \tag{41}$$

From the no-arbitrage condition (3), the flow constraint of funds for bankers, (23), the relationship between $r(t+1)$ and $K^h(t)$, (31), and the definitions of $x(t)$, $\eta(t)$ and $B(t)$, I obtain[14]

$$\frac{K(t+1)}{K(t)}\cdot\frac{\eta(t+1)}{\eta(t)}\cdot\beta\cdot\left[(1-\sigma)\frac{B(t+1)}{\eta(t+1)}+\sigma\theta\right]=\theta. \tag{42}$$

Note that $\frac{B(t+1)}{\eta(t+1)}=\hat{\phi}(t)$, the capital adequacy ratio.

Taking a one-period lag of equation (32) and using the definitions of $x(t)$, $\eta(t)$ and $B(t)$, I obtain

$$\frac{B(t+1)}{\eta(t+1)}=\left\{\begin{array}{ll}\Psi\left(\eta(t),B(t),K(t)\right), & \text{when } \frac{B(t)}{\eta(t)}>\overline{\phi},\\ \overline{\phi}, & \text{when } \frac{B(t)}{\eta(t)}\leq\overline{\phi},\end{array}\right\}, \tag{43}$$

where

$$\Psi\left(\eta(t),B(t),K(t)\right)$$
$$\equiv\frac{\beta\cdot\sigma\cdot\theta\cdot\Gamma\left(\eta(t),B(t),K(t)\right)}{\theta\cdot\omega\cdot\frac{B(t)}{\eta(t)}\cdot(1-\eta(t))\cdot K(t)-(1-\sigma)\cdot(1+(1-\sigma)\cdot\lambda)-\beta\cdot(1-\sigma)\cdot\Gamma\left(\eta(t),B(t),K(t)\right)}, \tag{44-1}$$

[12] See Appendix C.

[13] See Appendix D.

[14] See Appendix E.

with

$$
\begin{aligned}
&\Gamma\left(\eta(t), B(t), K(t)\right) \\
&\equiv \frac{\omega^2 \cdot \sigma}{2} \cdot ((1-\eta(t)) \cdot K(t))^2 - \frac{\omega}{2} \cdot (1-\eta(t)) \cdot K(t) \cdot (1+(1-\sigma)\cdot\lambda) \cdot [(1-\sigma) + \frac{B(t)}{\eta(t)}] \\
&- \delta \cdot \frac{B(t)}{\eta(t)} \cdot (1+(1-\sigma)\cdot\lambda)^2. \qquad (44\text{-}2)
\end{aligned}
$$

The above equations (40), (41), (42), and (43) are the dynamic systems that describe the economy. The variables determined by this system are the ratio of bankers' lending and aggregate capital as $\eta(t) \equiv \frac{K^b(t-1)}{K(t)}$, the ratio of workers' capital holdings and aggregate capital as $1-\eta(t) \equiv \frac{K^h(t-1)}{K(t)}$, the ratio of consumption and aggregate capital as $x(t) \equiv \frac{C(t-1)}{K(t)}$, and the ratio of bankers' net worth and aggregate capital as $B(t) \equiv \frac{N(t-1)}{K(t)}$.

## 4 Steady-State Analysis

I consider the steady-state economy. Let $y_{ss}$ denote the level of steady state of variable $y$. Equation (42) yields the steady-state level of capital adequacy ratio, $\phi_{ss}$:

$$
\frac{B_{ss}}{\eta_{ss}} = \phi_{ss} \equiv \begin{Bmatrix} \frac{\theta(1-\beta\sigma)}{\beta(1-\sigma)}, & \text{when } \frac{\theta(1-\beta\sigma)}{\beta(1-\sigma)} > \overline{\phi} \\ \overline{\phi}, & \text{when } \frac{\theta(1-\beta\sigma)}{\beta(1-\sigma)} \leq \overline{\phi}, \end{Bmatrix}. \qquad (45)
$$

Since $\theta$ is a parameter reflecting the degree of financial friction of the economy, equation (45) yields the following lemma 1.[15]

**Lemma 1** *The capital adequacy ratio in the steady state $\phi_{ss}$ is higher in the economy with larger financial friction $\theta$.*

In addition, this lemma implies that the leverage ratio in the steady state $\frac{1}{\phi_{ss}}$ is lower in the economy with larger financial friction $\theta$.

The level of households' investment in the steady state, $K^h_{ss}$, must satisfy (41) and (45) with $x(t) = x(t+1) = x_{ss}$ and $(1-\eta(t)) \cdot K(t) = (1-\eta(t+1)) \cdot K(t+1) = K^h_{ss}$ and $\frac{B(t)}{\eta(t)} = \frac{B(t+1)}{\eta(t+1)} = \phi_{ss}$

[15] Since $\beta < 1$, $\sigma \in (0,1)$, and $\theta > 0$, $\phi_{ss} \in (0,1)$ is satisfied in equation (45).

From (41) and (45), I obtain[16]

$$K_{ss}^h = \left\{ \begin{array}{ll} \frac{2\theta\cdot(1-\beta\sigma)\cdot[\beta-(1+(1-\sigma)\cdot\lambda)(1-\beta(1-\delta))]}{\omega\cdot[(2-\beta)\cdot(1-\beta\sigma)\cdot\theta+\beta^2\cdot(1-\sigma)]} \equiv \hat{K}_{ss}^h, & \text{when} \quad \frac{\theta(1-\beta\sigma)}{\beta(1-\sigma)} > \overline{\phi} \\ \frac{\frac{\theta}{(1-\sigma)\overline{\phi}+\sigma\theta}-(1+(1-\sigma)\cdot\lambda)(1-\beta(1-\delta))}{\omega\left(1+\frac{\beta}{2}\frac{1-\overline{\phi}}{\overline{\phi}}\right)} \equiv \overline{K}_{ss}^h, & \text{when} \quad \frac{\theta(1-\beta\sigma)}{\beta(1-\sigma)} \leq \overline{\phi}, \end{array} \right\}. \tag{46}$$

**Lemma 2** *In the steady state, the capital adequacy ratio $\phi_{ss}$ and the households' management capital $K_{ss}^h$ are given by equations (45) and (46), respectively. Let $\lambda_0$, $\beta_0$, $\tilde{\phi}$, $\lambda_1$ and $\beta_1$ be such that $\beta_0 \equiv \frac{1}{1+(1-\delta)(1-\sigma)}$, $\lambda_0 \equiv \frac{\beta\cdot[1+(1-\delta)(1-\sigma)]-1}{(1-\beta(1-\delta))\cdot(1-\sigma)}$, $\tilde{\phi} \equiv \frac{\theta\cdot[\sigma-(1+(1-\sigma)\lambda)\cdot(1-\beta\cdot(1-\delta))]}{(1+(1-\sigma)\lambda)\cdot(1-\beta\cdot(1-\delta))\cdot(1-\sigma)}$, $\lambda_1 \equiv \frac{\sigma+\beta(1-\delta)-1}{(1-\sigma)\cdot(1-\beta\cdot(1-\delta))}$ and $\beta_1 \equiv \frac{1-\sigma}{1-\delta}$.The conditions for the existence are as follows:*
*(i) when $\frac{\theta(1-\beta\sigma)}{\beta(1-\sigma)} > \overline{\phi}$, $\lambda < \lambda_0$ and $\beta > \beta_0$.*
*(ii) when $\frac{\theta(1-\beta\sigma)}{\beta(1-\sigma)} \leq \overline{\phi} < \tilde{\phi}$, $\lambda < \lambda_1$ and $\beta > \beta_1$ and $\delta < \sigma$.*

**Proof.** See Appendix G. ■

Lemma 2 implies that when the minimum capital adequacy ratio $\overline{\phi}$ is excessively high, the households' management capital $K_{ss}^h$ in the steady state does not exist. Moreover, Lemma 2 implies that the households' management capital $K_{ss}^h$ in the steady state exists if the initial net worth of a new banker, $\lambda$ is sufficiently small and the discount factor $\beta$ is sufficiently high. When the initial net worth of a new banker, $\lambda$ is sufficiently small and the discount factor $\beta$ is sufficiently high, the saving rate is high. These conditions for the existence of $K_{ss}^h$ does not depend on a parameter of financial friction, $\theta$.[17]

Since $\frac{B(t+1)}{\eta(t+1)} = \frac{B(t)}{\eta(t)} = \phi_{ss}$, $(1-\eta(t))\cdot K(t) = K_{ss}$, $\eta(t)\cdot K_{ss} = K_{ss}^b$ in the steady state, equation (40) yield

$$K_{ss}^b = \frac{\sigma\cdot K_{ss}^h\cdot\left[\beta\omega\cdot[\sigma\theta+(1-\sigma)\phi_{ss}]\cdot K_{ss}^h-\theta\phi_{ss}^2\right]+(1+(1-\sigma)\lambda)\cdot\phi_{ss}\cdot[(1-\sigma)(1-\phi_{ss})-\delta\beta\sigma\theta]}{\beta\cdot[(1-\sigma)+\phi_{ss}]\cdot[\sigma\theta+(1-\sigma)\cdot\phi_{ss}]}, \tag{47}$$

where $\phi_{ss}$ is given by equation (45) and $K_{ss}^h$ is given by equation (46).

Since $K_{ss}^b$ depends on $K_{ss}^h$ fron equation (47), I examine the relationship between the level of households' investment and the level of bankers' investment in the steady state.

**Lemma 3** *First, consider the case in which $\overline{\phi} < \frac{\theta(1-\beta\sigma)}{\beta(1-\sigma)} \equiv \hat{\phi}_{ss}$.*
*Let $\hat{\lambda}_2$ be such that $\hat{\lambda}_2 \equiv \frac{4\beta(1-\sigma)^2\cdot[\beta-(1-\beta(1-\delta))]-\theta^2(1-\beta\sigma)\cdot[(2-\beta)(1-\beta\sigma)\theta+\beta^2(1-\sigma)]}{4\beta(1-\sigma)^2\cdot(1-\beta(1-\delta))}$. The amount of*

[16] Appendix F.

[17] Whether the capital adequacy requirements are satisfied depend on a parameter of financial friction $\theta$.

*bankers' lending $\hat{K}^b_{ss}$ increases as the amount of households' investment $\hat{K}^h_{ss}$ increases, if $\hat{\lambda}_2 \geq \lambda$.*
*Second, consider the case in which $\frac{\theta(1-\beta\sigma)}{\beta(1-\sigma)} \leq \overline{\phi}$.*
*Let $\overline{\phi}_2$ be such that $\overline{\phi}_2 \equiv \frac{\theta\cdot[(4-\beta)-4\beta\sigma\cdot(1+(1-\sigma)\lambda)\cdot(1-\beta(1-\delta))]}{2\cdot\left[\theta\cdot\left(1-\frac{\beta}{2}\right)+(1-\sigma)\cdot 2\beta\cdot(1+(1-\sigma)\lambda)(1-\beta(1-\delta))\right]}$. The amount of bankers' lending $\overline{K}^b_{ss}$ increases as the amount of workers' investment $\overline{K}^h_{ss}$ increases, if $\overline{\phi}_1 \geq \overline{\phi}$.*

**Proof.** Appendix H. ■

As the investment of households $K^h_{ss}$increases, the interest rate spread $(r_{ss} - r^d_{ss})$widens from equation (3); however, the interest rate spread widens, the investment of bankers does not increase, if $\lambda$ is sufficiently large or the minimum capital adequacy ratio $\overline{\phi}$ is sufficiently high. Lemma 3 implies that the amount of lending of bankers $K^b_{ss}$ increases as the investment of households $K^h_{ss}$ increases if a parameter of the initial net worth of new bankers, $\lambda$ is sufficiently small, when the steady state level of capital adequacy ratio is higher than the minimum capital adequacy ratio. Lemma 3 implies that the amount of lending of bankers $K^b_{ss}$ increases as the investment of households $K^h_{ss}$ increases if he minimum capital adequacy ratio, $\overline{\phi}$is sufficiently low, when the steady state level of capital adequacy ratio is higher than the minimum capital adequacy ratio, $\hat{\phi}_{ss} = \overline{\phi}$.

Since $K_{ss} = K^b_{ss} + K^h_{ss}$, equations (46) and (47) determine the level of aggregate capital in the steady state, $K_{ss}$. As the amount of households' management capital $K^h_{ss}$ increases, the management cost increases from equation (1). As the amount of households' management capital $K^h_{ss}$ increases, the amount of bankers' lending $K^b_{ss}$ increases from lemma 3 and then the increase of $K^h_{ss}$decreases the amount of aggregate capital $K_{ss}$.

**Lemma 4** *First, consider the case in which $\overline{\phi} < \frac{\theta(1-\beta\sigma)}{\beta(1-\sigma)} \equiv \hat{\phi}_{ss}$.*
*Let $\hat{\theta}_2$ be such that satisfies the following equation for a positive value:*

$$\sigma\theta^2(1-\beta\sigma)^2 - \beta(1-\sigma)(1-\beta\sigma)\theta = \beta^2(1-\sigma)^3.$$

*The aggregate capital $\hat{K}_{ss}$ increases as the households' investment $\hat{K}^h_{ss}$ increases, if $0 < \theta < \hat{\theta}_2$.*
*Second, consider the case in which $\frac{\theta(1-\beta\sigma)}{\beta(1-\sigma)} \leq \overline{\phi}$. Let $\overline{\theta}_1$ be such that $\overline{\theta}_1 \equiv \frac{\beta(1-\sigma)}{\sigma}$*
*Let $\overline{\phi}_4$ be such that satisfies the following equation for a positive value:*

$$(\theta\sigma - \beta(1-\sigma)) \cdot (\overline{\phi})^2 - \beta \cdot [\sigma\theta + (1-\sigma)^2] \cdot \overline{\phi} = \beta\sigma\theta(1-\sigma).$$

*The aggregate capital $\overline{K}_{ss}$ increases as the households' investment $\overline{K}^h_{ss}$ increases, if $\overline{\theta}_1 > \theta$ or $\overline{\theta}_1 \leq \theta$ and $0 < \overline{\phi} < \overline{\phi}_4$.*

**Proof.** Appendix I. ■

Lemma 4 implies that the aggregate capital $K_{ss}$ increases as the investment of households $K^h_{ss}$ increases if a parameter of financial friction, $\theta$ is sufficiently small, regardless of the minimum capital adequacy ratio, $\overline{\phi}$i . Lemma 4 implies that the aggregate capital $K_{ss}$ increases as the investment of households $K^h_{ss}$ increases if the financial system is not sufficiently developed and the minimum capital adequacy ratio, $\overline{\phi}$is sufficiently low, when the steady state level of capital adequacy ratio is higher than the minimum capital adequacy ratio, $\hat{\phi}_{ss} = \overline{\phi}$.

Since in the steady state $x(t+1) = x(t) = x_{ss}$ and $K(t+1) = K(t) = K_{ss}$, equation (40) yields

$$x_{ss} = \frac{\frac{\omega}{2} \cdot K^h_{ss} \cdot \left[\left(\frac{1-\phi_{ss}}{\phi_s s}\right) + \frac{K^h_{ss}}{K_{ss}}\right] - (1-\delta) - \left(\frac{\theta}{\alpha\beta\cdot[(1-\sigma)\cdot\phi_{ss}+\sigma\theta]}\right)}{\beta\cdot(1-\delta)\cdot(1+(1-\sigma)\cdot\lambda) - \frac{\beta\cdot\omega}{2}\cdot K^h_{ss}\cdot\left(\frac{1-\phi_{ss}}{\phi_{ss}}\right) + \left(\frac{\theta}{(1-\sigma)\cdot\phi_{ss}+\sigma\theta}\right)}, \tag{48}$$

where $\phi_{ss}$ is given by equation (45) and $K^h_{ss}$ is given by equation (46).

Since $x$ is the ratio of consumption and aggregate capital, $x_{ss} \equiv \frac{C_{ss}}{K_{ss}}$, multiplying both sides of equation (48) by $K_{ss}$ yields

$$C_{ss} = \frac{\left[\frac{\omega}{2} \cdot K^h_{ss} \cdot \left(\frac{1-\phi_{ss}}{\phi_s s}\right) - (1-\delta) - \left(\frac{\theta}{\alpha\beta\cdot[(1-\sigma)\cdot\phi_{ss}+\sigma\theta]}\right)\right]\cdot K_{ss} + \frac{\omega}{2}\cdot\left(K^h_{ss}\right)^2}{\beta\cdot(1-\delta)\cdot(1+(1-\sigma)\cdot\lambda) - \frac{\beta\cdot\omega}{2}\cdot K^h_{ss}\cdot\left(\frac{1-\phi_{ss}}{\phi_{ss}}\right) + \left(\frac{\theta}{(1-\sigma)\cdot\phi_{ss}+\sigma\theta}\right)}, \tag{49}$$

where $\phi_{ss}$ is given by equation (45) and $K^h_{ss}$ is given by equation (46) and $\phi_{ss}$ and $K^h_{ss}$ can be described as the functions of parameters. The following proposition 1 shows the properties of the steady state level of consumption, $C_{ss}$.

**Proposition 1** *The steady-state level of consumption increases as the steady-state level of households' investment,* $\frac{dCss}{dK^h ss} > 0$ *regardless of the level of minimum capital adequacy ratio,* $\overline{\phi}$*.*

**Proof.** Appendix J. ■

Since I obtain the steady state level of three variables; $\eta_{ss} \equiv \frac{K^b_{ss}}{K_{ss}}$, $x_{ss} \equiv \frac{C_{ss}}{K_{ss}}$ and $B_{ss} \equiv \frac{N_{ss}}{K_{ss}}$, I examine how the capital adequacy requirements affect the economy in the model. Let $\hat{y}_{ss}$ and $\overline{y}_{ss}$ denote the steady state level of variable $y$ when $\phi_{ss} = \frac{\theta(1-\beta\sigma)}{\beta(1-\sigma)}$ and the steady state level of variable $y$ when $\phi_{ss} = \overline{\phi}$. First, I investigate the case where there is no productivity shock on the economy. First, I examine how the minimum capital adequacy ratio, $\overline{\phi}$ affect the investment of households $K^h_{ss}$ when $\phi_{ss} = \overline{\phi}_{ss}$.

**Lemma 5** *Consider the case in which* $\frac{\theta(1-\beta\sigma)}{\beta(1-\sigma)} \leq \overline{\phi}$.

*Let* $\overline{\theta}_2$ *be such that* $\overline{\theta}_2 \equiv \frac{(1+\beta)^2+\sigma[(1+(1-\sigma)\lambda)(1-\beta(1-\delta)]\cdot[(1+\beta)(1-\sigma)+(1-\sigma)+\sigma^2(1+(1-\sigma)\lambda)\cdot(1-\beta(1-\delta))]}{4\sigma\cdot[1-\sigma\cdot(1+(1-\sigma)\lambda)\cdot(1-\beta(1-\delta))]}$.

*Let* $\overline{\phi}_5$ *and* $\overline{\phi}_6$ *be such that satisfies the following equation:* $Z_1 \cdot (\overline{\phi}^2) + Z_2 \cdot \overline{\phi} = Z_3$ *where*

$Z_1 \equiv \theta - (1-\sigma)(1+(1-\sigma)\lambda)(1-\beta(1-\delta))$; $Z_2 \equiv -\theta(1+\beta) - 2\sigma\theta \cdot (1+(1-\sigma)\lambda)(1-\beta(1-\delta))$;

$Z_3 \equiv \frac{\sigma\theta^2\cdot[\sigma\cdot(1+(1-\sigma)\lambda)(1-\beta(1-\delta))-1]}{1-\sigma}$.

*The households' investment* $\overline{K}_{ss}^h$ *increases as the minimum capital adequacy ratio* $\overline{\phi}$ *is higher, if* $\overline{\theta}_2 > \theta$ *or* $\overline{\theta}_2 \leq \theta$ *and* $\overline{\phi}_5 < \overline{\phi} < \overline{\phi}_6$.

**Proof.** Appendix K. ■

Lemma 5 implies that if the economy faces large financial friction, $\theta$, the households' investment $\overline{K}_{ss}^h$ increases as the minimum capital adequacy ratio $\overline{\phi}$ is higher. If the the economy faces small financial friction, $\theta$, the households' investment $\overline{K}_{ss}^h$ increases as the minimum capital adequacy ratio $\overline{\phi}$ is higher for a range, $\overline{\phi}_5 < \overline{\phi} < \overline{\phi}_6$.

Since Propositions 1 shows that $\frac{dC_{ss}}{dK_{ss}} > 0$ regardless of the range of parameters. Using this results and (49), I investigate the conditions that $\hat{C}_{ss}$ under the capital adequacy requirements $\phi_{ss} > \overline{\phi}$ is larger than $\overline{C}_{ss}$ with the minimum capital adequacy ratio $\overline{\phi}$ equals to the steady state capital adequacy ratio. The following lemma 6 and proposition 2 summarizes the above arguments.Using this results and (48), I investigate the conditions that $\hat{K}_{ss}$ under the capital adequacy requirements $\phi_{ss} > \overline{\phi}$ is larger than $\overline{K}_{ss}$ with the minimum capital adequacy ratio $\overline{\phi}$ equals to the steady state capital adequacy ratio. The following lemma and proposition summarizes the above arguments.

**Lemma 6** *(i)If the proportion of savings used for the net worth of new bankers* $\lambda$ *is sufficiently large and the degree of financial friction* $\theta$ *is sufficiently small , then the aggregate capital under the capital adequacy requirements* $\phi_{ss} > \overline{\phi}$*,* $\hat{K}_{ss}^h$ *is higher than* $\overline{K}_{ss}^h$ *with the minimum capital adequacy ratio* $\overline{\phi}$ *equals to the steady state capital adequacy ratio.*

*(ii) If the proportion of bankers,*$1-\sigma$ *and the discount rate,*$\beta$ *are close to* $0$ *and* $2 > \overline{\phi}$*, then the aggregate capital under the capital adequacy requirements* $\phi_{ss} > \overline{\phi}$*,* $\hat{K}_{ss}^h$ *is higher than* $\overline{K}_{ss}^h$ *with the minimum capital adequacy ratio* $\overline{\phi}$ *equals to the steady state capital adequacy ratio.*

**Proof.** Appendix L.

■

**Proposition 2** *(i)If the proportion of savings used for the net worth of new bankers $\lambda$ is sufficiently large, then the aggregate capital under the capital adequacy requirements $\phi_{ss} > \overline{\phi}$, the steady state level of consumption,$\hat{C}^h_{ss}$ is higher than $\overline{C}^h_{ss}$ with the minimum capital adequacy ratio $\overline{\phi}$ equals to the steady state capital adequacy ratio.*
*(ii) If the proportion of bankers,$1-\sigma$ and the discount rate,$\beta$ are close to 0 and $2 > \overline{\phi}$, then the aggregate capital under the capital adequacy requirements $\phi_{ss} > \overline{\phi}$, the steady state level of consumption,$\hat{C}_{ss}$ is higher than $\overline{C}_{ss}$ with the minimum capital adequacy ratio $\overline{\phi}$ equals to the steady state capital adequacy ratio.*

**Proof.** Proposition 1 and Lemma 6 yields $\frac{dC_{ss}}{dK^h_{ss}} > 0$, the condition for $\hat{K}^h_{ss} > \overline{K}_{ss}$ is same as the one for $\hat{C}_{ss} > \overline{C}_{ss}$. ■

Proposition 2 implies that if the economy faces the large financial friction $\theta$, the policy institution lowers the minimum capital adequacy ratio $\phi$ in order to the steady state capital adequacy ratio become stuck at the minimum ratio if the policy institution aims to raise the steady state consumption level and thus the growth rate of the economy.

Finally, let us consider the case where a negative productivity shock occurs at that economy. Substituting the relationship between the interest rate $r_{ss}$ and the capital holdings by workers $K^h_{ss}$, (31), $\phi_{ss}$ given by equation (45) and $K^h_{ss}$ given by equation (48) into the relationship between $r_{ss}$ and the aggregate capital $K_{ss}$ (35), the aggregate capital in the steady state $K_{ss}$ can be determined:

$$K^b_{ss} + K^h_{ss} = (\alpha A)^{\frac{1}{1-\alpha}} \cdot \left[ -\left( \frac{1-\phi_{ss}}{\phi_{ss}} \right) \frac{\omega}{2} K^h_{ss} + \frac{\theta}{\beta[(1-\sigma)\phi_{ss} + \sigma\theta]} \right]^{\frac{-1}{1-\alpha}}, \tag{50}$$

where $K^h_{ss}$ is given by (46) and $K^b_{ss}$ is given by (47).

From equation and (50), I obtain

$$\frac{dK^h_{ss}}{dA} = \left( \frac{dK^h_{ss}}{dK_{ss}} \right)^{-1} \cdot \frac{1}{1-\alpha} \cdot K_{ss} \cdot \frac{1}{A}. \tag{51}$$

Equation (51) tells us that the negative productivity shock decreases the steady state level of households' capital investment if it decreases the steady state level of aggregate capital. Proposition 1 and lemma 4 show that how the households' capital investment $K^h_{ss}$ affect the aggregate capital $K_{ss}$ and then the consumption $C_{ss}$. Thus, the following proposition 3 shows that how the negative productivity shock affects the consumption $C_{ss}$.

**Proposition 3** *First, consider the case in which* $\overline{\phi} < \frac{\theta(1-\beta\sigma)}{\beta(1-\sigma)} \equiv \hat{\phi}_{ss}$.
*Let* $\hat{\theta}_2$ *be such that satisfies the following equation for a positive value:*

$$\sigma\theta^2(1-\beta\sigma)^2 - \beta(1-\sigma)(1-\beta\sigma)\theta = \beta^2(1-\sigma)^3.$$

*The aggregate capital* $\hat{C}_{ss}$ *decreases as the productivity* $A$ *declines, if* $0 < \theta < \hat{\theta}_2$.
*Second, consider the case in which* $\frac{\theta(1-\beta\sigma)}{\beta(1-\sigma)} \leq \overline{\phi}$. *Let* $\overline{\theta}_1$ *be such that* $\overline{\theta}_1 \equiv \frac{\beta(1-\sigma)}{\sigma}$
*Let* $\overline{\phi}_4$ *be such that satisfies the following equation for a positive value:*

$$(\theta\sigma - \beta(1-\sigma)) \cdot (\overline{\phi})^2 - \beta \cdot [\sigma\theta + (1-\sigma)^2] \cdot \overline{\phi} = \beta\sigma\theta(1-\sigma).$$

*The aggregate capital* $\overline{C}_{ss}$ *decreases as the productivity* $A$ *declines, if* $\overline{\theta}_1 > \theta$ *or* $\overline{\theta}_1 \leq \theta$ *and* $0 < \overline{\phi} < \overline{\phi}_4$.

**Proof.** Equation (51) applies to Proposition 1. Then, I obtain the above results. ■

Propositions 4 implies that when a negative shock on productivity occurs, deregulation have a good effect on the economy in only the country where the financial market sufficiently develops. Since if the economy faces the large financial friction $\theta$, the policy institution highers the minimum capital adequacy ratio $\phi$ in order to the steady state capital adequacy ratio become stuck at the minimum ratio if the policy institution aims to raise the steady state consumption level and thus the growth rate of the economy. One of the key mechanism behind this result is the decreases of bankers' net worth in the non-production sector at the negative productivity shock. Because of financial frictions, the decreases of the aggregate capital has an ambiguous effect on the consumption in this model. If there is no financial friction, when the economy experience a recession with decreases of aggregate capital, the investment and consumption decline.

# 5 Concluding Remarks

I introduce financial market frictions into a simple macroeconomic model. Using the current model, this study analyzes how capital adequacy requirements for banks affect the economy. I show that in the economy with larger financial frictions, policy institution should not change the minimum capital adequacy requirements in order to improve the steady state level of consumption when the economy faces a negative productivity shock. This result implies that counter-cyclical capital requirements, that is, relaxing the rule when recession is not always optimal for consumers. The condition for the above case depends on the combinations of parameters such as the degree of financial friction, discount factor and initial net worth of banks. Moreover, I show that when a negative shock on productivity occurs, deregulation have a good effect on the economy in only the country where the financial market sufficiently develops. Because of financial frictions, the decreases of the aggregate capital has an ambiguous effect on the economy in this model.

I examine the steady state in the economy; however, I have not examine the transition of the economy. This work is left for future work. Moreover, for future works, I calibrate this model and compare the real economy. Recently, many researchers and institutions have interests on the discussion about Basel III. To discuss this, I extend this model to open economy, This is also left for future works.

# Appendix

## A Derivation of (24), (25) and (26)

Substituting equation (18') into left-hand side of $V(t)$ in (17) and the left-side hand of $V(t+1)$ in (17), I obtain

$$\mu(t)k^b(t) + \nu(t)n(t) = \beta(1-\sigma)n(t+1) + \beta\sigma\mu(t+1)k^b(t+1) + \beta\sigma\nu(t+1)n(t+1).$$

Since $\frac{n(t+1)}{k^b(t+1)} \equiv \hat{\phi}(t+1)$ from (20), substituting this definition into the above equation, I obtain

$$\mu(t)k^b(t) + \nu(t)n(t) = \beta(1-\sigma)n(t+1) + \beta\sigma\mu(t+1)\frac{n(t+1)}{\hat{\phi}(t+1)} + \beta\sigma\nu(t+1)n(t+1).$$

Substituting $n(t+1)$ given by equation (23) into the above equation, I obtain

$$\mu(t)k^b(t) + \nu(t)n(t) = \beta\Omega(t+1)\left[r(t+1) - r^d(t+1)\right]k^b(t) + \beta\Omega(t+1)r^d(t+1)n(t), \tag{52}$$

where

$$\Omega(t+1) \equiv (1-\sigma) + \sigma\mu(t+1)k^b(t+1) + \sigma\nu(t+1). \tag{26}$$

Since the coefficient of the left-hand side in (52) is equivalent to the coefficient on the right-hand side on (52), if the guess is correct, I obtain the following equations:

$$\mu(t) = \beta\Omega(t+1)\left[r(t+1) - r^d(t+1)\right], \tag{24}$$

$$\nu(t) = \beta\Omega(t+1)r^d(t+1), \tag{25}$$

where

$$\Omega(t+1) \equiv (1-\sigma) + \frac{\sigma\mu(t+1)}{\hat{\phi}(t+1)} + \sigma \cdot \nu(t+1). \tag{26}$$

# B Derivation of (32)

Substituting equation (31) into equation (11), I obtain

$$\begin{aligned}
&\frac{\omega^2 \cdot \sigma}{2} \cdot \left(K^h(t)\right)^2 - \left(\frac{2-\sigma}{2}\right) \cdot \omega \cdot (1+(1-\sigma)\cdot\lambda) \cdot K^h(t) - \delta \cdot (1+(1-\sigma)\cdot\lambda)^2 \\
&+\sigma \cdot \omega \cdot K^h(t) \cdot \left(\frac{1-\hat{\phi}(t)}{\hat{\phi}(t)}\right) \cdot \frac{\omega}{2} \cdot K^h(t) - \frac{\sigma \cdot \omega \cdot K^h(t) \cdot \theta \cdot \hat{\phi}(t+1)}{\beta \cdot [(1-\sigma)\cdot\hat{\phi}(t+1) + \theta\cdot\theta] \cdot \hat{\phi}(t)} \\
&+\frac{(1+(1-\sigma)\cdot\lambda)\cdot(1-\sigma)\cdot\theta\cdot\hat{\phi}(t+1)}{\beta \cdot [(1-\sigma)\cdot\hat{\phi}(t+1) + \sigma\theta] \cdot \hat{\phi}(t)} - (1+(1-\sigma)\cdot\lambda)\cdot(1-\sigma)\cdot\left(\frac{1-\hat{\phi}(t)}{\hat{\phi}(t)}\right) \cdot \frac{\omega}{2} \cdot K^h(t) \\
&=0
\end{aligned}$$

Rearranging the above equation, I obtain

$$\begin{aligned}
&\frac{1}{\hat{\phi}(t)} \cdot \frac{\omega^2 \cdot \sigma}{2} \cdot \left(K^h(t)\right)^2 - \frac{\omega}{2} \cdot (1+(1-\sigma)\cdot\lambda) \cdot \frac{[(1-\sigma)+\hat{\phi}(t)]}{\hat{\phi}(t)} \cdot K^h(t) \\
&-\omega \cdot K^h(t) \cdot \left(\frac{\sigma\cdot\theta\cdot\hat{\phi}(t+1)}{\beta\cdot\left[(1-\sigma)\cdot\hat{\phi}(t+1)+\sigma\cdot\theta\right]}\right) - \delta\cdot(1+(1-\sigma)\cdot\lambda)^2 + \frac{(1+(1-\sigma)\cdot\lambda)\cdot(1-\sigma)\cdot\theta\cdot\hat{\phi}(t+1)}{\beta\cdot[(1-\sigma)\cdot\hat{\phi}(t+1)+\sigma\cdot\theta]\cdot\hat{\phi}(t)} \\
&=0
\end{aligned}$$

The above equation can be rewritten as

$$\hat{\phi}(t+1) = \frac{\beta\left[(1-\sigma)\cdot\hat{\phi}(t+1)+\sigma\cdot\theta\right]\cdot\Gamma\left(K^h(t),\hat{\phi}(t)\right)}{\theta\cdot\left[\omega\cdot\hat{\phi}(t)\cdot K^h(t)-(1-\sigma)\cdot(1+(1-\sigma)\cdot\lambda)\right]},$$

where

$$\Gamma\left(K^h(t),\hat{\phi}(t)\right) \equiv \frac{\omega^2\cdot\sigma}{2}\cdot\left(K^h(t)\right)^2 - \frac{\omega}{2}\cdot K^h(t)\cdot(1+(1-\sigma)\cdot\lambda)\cdot[(1-\sigma)+\hat{\phi}(t)] - \delta\cdot\hat{\phi}(t)\cdot(1+(1-\sigma)\cdot\lambda)^2.$$

The above equation can be rewritten as

$$\hat{\phi}(t{+}1)\Psi\left(K^h(t),\hat{\phi}(t)\right) \equiv \frac{\beta\cdot\sigma\cdot\theta\cdot\Gamma\left(K^h(t),\hat{\phi}(t)\right)}{\theta\cdot\omega\cdot\hat{\phi}(t)\cdot K^h(t)-(1-\sigma)\cdot(1+(1-\sigma)\cdot\lambda)-\beta\cdot(1-\sigma)\cdot\Gamma\left(K^h(t),\hat{\phi}(t)\right)}, \tag{33}$$

with

$$\Gamma\left(K^h(t),\hat{\phi}(t)\right) \equiv \frac{\omega^2\cdot\sigma}{2}\cdot\left(K^h(t)\right)^2 - \frac{\omega}{2}\cdot K^h(t)\cdot(1+(1-\sigma)\cdot\lambda)\cdot[(1-\sigma)+\hat{\phi}(t)] - \delta\cdot\hat{\phi}(t)\cdot(1+(1-\sigma)\cdot\lambda)^2.$$

Since $\hat{\phi}(t+1)$ is determined by equation (33) as long as the capital adequacy requirement (16) is satisfied, that is, as far as $\hat{\phi}(t+1) > \overline{\phi}$, I obtain equation (32).

# C Derivation of (40)

From (35), I obtain

$$Y(t) = \frac{1}{\alpha}\cdot r(t)\cdot K(t). \tag{35'}$$

Taking a one period lag of equation (31), substituting this into equation (35), and dividing it by $K(t)$, I obtain

$$\frac{Y(t)}{K(t)} = \frac{1}{\alpha}\cdot\left[\frac{\hat{\phi}(t-1)-1}{\hat{\phi}(t-1)}\right]\cdot\frac{\omega}{2}\cdot K^h(t-1) + \frac{\theta}{\alpha\beta\cdot\left[(1-\sigma)\cdot\hat{\phi}(t)+\sigma\theta\right]}\cdot\left(\frac{\hat{\phi}(t)}{\hat{\phi}(t-1)}\right). \tag{35''}$$

From (38), I obtain

$$\frac{I(t)}{K(t)} = \frac{K(t+1)}{K(t)} - 1 + \delta. \tag{38'}$$

The definition of $x(t)$ yields

$$\frac{C(t)}{K(t)} = \frac{C(t-1)}{K(t)}\cdot\frac{C(t)}{C(t-1)} = x(t)\cdot\frac{C(t)}{C(t-1)}. \tag{53}$$

Moreover, equations (7) and (8) yield

$$\frac{C(t)}{C(t-1)} = \beta\cdot\frac{q(t-1)}{q(t)} = \frac{\beta\cdot(1-\delta)\cdot(1+(1-\sigma)\lambda)+\beta\cdot r(t)}{(1+(1-\sigma)\lambda)+\omega\cdot K^h(t-1)}. \tag{54}$$

Substituting (54) into (53), I obtanin

$$\frac{C(t)}{K(t)} = \beta\cdot x(t)\cdot\left[\frac{(1-\delta)\cdot(1+(1-\sigma)\lambda)+r(t)}{(1+(1-\sigma)\lambda)+\omega\cdot K^h(t-1)}\right], \tag{55}$$

where $r(t)$ ia given by the function of $K^h(t-1)$and $\hat{\phi}(t),\hat{\phi}(t-1)$ in equation (31).

Substituting (35'), (38'), (55) and (1) into the market clearing condition on the goods market and dividing it by $K(t)$, I obtain

$$\frac{1}{\alpha}\cdot r(t) = \beta\cdot x(t)\cdot\left[\frac{(1-\delta)\cdot(1+(1-\sigma)\lambda)+r(t)}{(1+(1-\sigma)\lambda)+\omega\cdot K^h(t-1)}\right] + \frac{K(t+1)}{K(t)} - 1 + \delta + \frac{\omega}{2}\cdot\frac{\left(K^h(t)\right)^2}{K(t)}. \tag{56}$$

where $r(t)$ ia given by the function of $K^h(t-1)$and $\hat{\phi}(t), \hat{\phi}(t-1)$ in equation (31).

Substituting $r(t)$ given by (31) into (56) and using the definition of $1-\eta(t)$, (56) can be rewritten as

$$\beta \cdot x(t) \cdot \left[ \frac{-\left(\frac{1-\hat{\phi}(t-1)}{\hat{\phi}(t-1)}\right) \cdot \frac{\omega}{2} \cdot k^h(t-1) + \frac{\theta}{\beta\cdot[(1-\sigma)\cdot\hat{\phi}(t)+\sigma\theta]} \cdot \frac{\hat{\phi}(t)}{\hat{\phi}(t-1)} + (1-\delta)\cdot(1+(1-\sigma)\lambda)}{(1+(1-\sigma)\lambda) + \omega \cdot K^h(t-1)} \right]$$

$$+ (1-\delta) + \frac{\theta}{\alpha\beta\cdot[(1-\sigma)\cdot\hat{\phi}(t)+\sigma\theta]} \cdot \frac{\hat{\phi}(t)}{\hat{\phi}(t-1)} - \frac{\omega}{2\alpha} \cdot K^h(t-1) \cdot \frac{\left(1-\hat{\phi}(t-1)\right)}{\hat{\phi}(t-1)}$$

$$= \frac{K(t+1)}{K(t)} + \frac{\omega}{2} \cdot \frac{(1-\eta(t+1))^2 \cdot (K(t+1))^2}{K(t)}. \tag{56'}$$

The left-hand side of equation (56') is described by $x(t)$, $\hat{\phi}(t-1), \hat{\phi}(t), \eta(t)$ and $K(t)$ and the right-hand side of equation (56') is described by $K(t+1), K(t)$ and $\eta(t+1)$.

Using the definition of $\hat{\phi}(t)$, (56') can be rewritten as

$$-\left(\frac{B(t+1)}{\eta(t+1)} \cdot \frac{\eta(t)}{B(t)}\right) \cdot \left[\frac{\theta\cdot(1+\alpha\beta\cdot x(t))}{\alpha\beta\left[(1-\sigma)\cdot\frac{B(t+1)}{\eta(t+1)}+\sigma\theta\right]}\right] + \frac{K(t+1)}{K(t)} + \frac{\omega}{2} \cdot \frac{(1-\eta(t+1))^2 \cdot (K(t+1))^2}{K(t)}$$

$$= \frac{x(t)\cdot\left[\beta\cdot(1-\delta)\cdot(1+(1-\sigma)\lambda) - \frac{\beta\cdot\omega}{2}\cdot(1-\eta(t))\cdot K(t)\cdot\left(\frac{1-\frac{B(t)}{\eta(t)}}{\frac{B(t)}{\eta(t)}}\right)\right]}{(1+(1-\sigma)\lambda)+\omega\cdot(1-\eta(t))\cdot K(t)}.$$

$$-\frac{\omega}{2\alpha}\cdot(1-\eta(t))\cdot K(t)\cdot\frac{\left(1-\frac{B(t)}{\eta(t)}\right)}{\frac{B(t)}{\eta(t)}} + (1-\delta). \tag{40}$$

The left-hand side of equation (40) is described by $x(t)$, $B(t+1), \eta(t+1)$ and $K(t)$ and the right-hand side of equation (56') is described by $x(t), K(t), B(t)$ and $\eta(t)$.

# D Derivation of (41)

From the definition of $x(t)$, I obtain

$$\frac{x(t+1)}{x(t)} \equiv \frac{\frac{C(t)}{K(t+1)}}{\frac{C(t-1)}{K(t)}} = \frac{C(t)}{\beta\cdot C(t-1)}\cdot\beta\cdot\left(\frac{K(t+1)}{K(t)}\right)^{-1}. \tag{57}$$

Substituting (7) and (8) into (57), I obtain

$$\frac{x(t+1)}{x(t)} = \frac{q(t-1)}{q(t)}\cdot\beta\cdot\left(\frac{K(t+1)}{K(t)}\right)^{-1}. \tag{57'}$$

Substituting (31) into (57'), I obtain

$$\frac{x(t+1)}{x(t)} = \beta \cdot \left(\frac{K(t+1)}{K(t)}\right)^{-1} \cdot \frac{(1-\delta)\cdot(1+(1-\sigma)\lambda) + \left(\frac{\hat{\phi}(t-1)-1}{\hat{\phi}(t-1)}\right)\cdot\frac{\omega}{2}\cdot K^h(t-1) + \frac{\theta\cdot\frac{\hat{\phi}(t)}{\hat{\phi}(t-1)}}{\beta\cdot[(1-\sigma)\cdot\hat{\phi}(t)+\sigma\theta]}}{(1+(1-\sigma)\lambda)+\omega\cdot K^h(t-1)}. \tag{57''}$$

Multiplying $\frac{K(t+1)}{K(t)}$ by the both sides of equation (57") and using $\hat{\phi}(t-1) \equiv \frac{B(t)}{\eta(t)}$ and $K^h(t-1) = (1-\eta(t))\cdot K(t)$, I obtain

$$\begin{aligned}&\frac{x(t+1)}{x(t)}\cdot\frac{K(t+1)}{K(t)}\cdot[(1+(1-\sigma)\lambda)+\omega\,(1-\eta(t))\,K(t)] - \frac{\theta\frac{B(t+1)}{\eta(t)}\frac{\eta(t+1)}{B(t)}}{(1-\sigma)\frac{B(t+1)}{\eta(t+1)}+\sigma\theta}\\ &= \beta(1-\delta)\,(1-(1-\sigma)\lambda) + \beta\cdot\left(\frac{\frac{B(t)}{\eta(t)}-1}{\frac{B(t)}{\eta(t)}}\cdot\frac{\omega}{2}\,(1-\eta(t))\,K(t)\right).\end{aligned} \tag{41}$$

# E Derivation of (42)

The constraint of flow of funds of bankers (23) can be rewritten as the following equation for the aggregate variables:

$$N(t+1) = \left[r(t+1) - r^d(t+1)\right]\cdot K^b(t) + r^d(t+1)\cdot N(t). \tag{23'}$$

Substituting equation (3) into equation (23'), I obtain

$$N(t+1) = \frac{\omega}{2}\cdot K^h(t)\cdot K^b(t) + r(t+1)\cdot N(t) - \frac{\omega}{2}\cdot K^h(t)\cdot N(t). \tag{23''}$$

Taking a one lag of (23") and using $N(t) = B(t)\cdot K(t)$ and $K^b(t-1) = \eta(t)\cdot K(t)$, I obtain

$$B(t+1)\cdot K(t+1) = \frac{\omega}{2}\cdot K^h(t-1)\cdot\eta(t)\cdot K(t) + r(t)\cdot B(t)\cdot K(t) - \frac{\omega}{2}\cdot K^h(t-1)\cdot B(t)\cdot K(t). \tag{58}$$

Dividing the both sides of (58) by $K(t)$, I obtain

$$B(t+1)\cdot\frac{K(t+1)}{K(t)} = \frac{\omega}{2}\cdot K^h(t-1)\cdot\eta(t) + r(t)\cdot B(t) - \frac{\omega}{2}\cdot K^h(t-1)\cdot B(t). \tag{58'}$$

From the definition of $\hat{\phi}(t)$, $B(t) = \eta(t)\cdot\hat{\phi}(t-1)$. Substituting it into (58'), I obtain

$$\eta(t+1)\cdot\hat{\phi}(t)\cdot\frac{K(t+1)}{K(t)} = \frac{\omega}{2}\cdot K^h(t-1)\cdot\eta(t) + r(t)\cdot\eta(t)\cdot\hat{\phi}(t-1) - \frac{\omega}{2}\cdot K^h(t-1)\cdot\eta(t)\cdot\hat{\phi}(t-1). \tag{58''}$$

Multiplying the both sides of (58") by $\frac{1}{\eta(t)\cdot\hat{\phi}(t-1)}$ , I obtain

$$\frac{\eta(t+1)}{\eta(t)}\cdot\frac{\hat{\phi}(t)}{\hat{\phi}(t-1)}\cdot\frac{K(t+1)}{K(t)}=\frac{\omega}{2}\cdot K^h(t-1)\cdot\left(\frac{1-\hat{\phi}(t-1)}{\hat{\phi}(t-1)}\right)+r(t). \tag{59}$$

Substituting (31) into (59), I obtain

$$\frac{\eta(t+1)}{\eta(t)}\cdot\frac{\hat{\phi}(t)}{\hat{\phi}(t-1)}\cdot\frac{K(t+1)}{K(t)}=\frac{\omega}{2}\cdot K^h(t-1)\cdot\left(\frac{1-\hat{\phi}(t-1)}{\hat{\phi}(t-1)}\right)+\left(\frac{\hat{\phi}(t-1)-1}{\hat{\phi}(t-1)}\right)\cdot\frac{\omega}{2}\cdot K^h(t-1)+\frac{\theta\cdot\frac{\hat{\phi}(t)}{\hat{\phi}(t-1)}}{\beta\cdot[(1-\sigma)\cdot\hat{\phi}(t)+\sigma\theta]}. \tag{59'}$$

Substituting $\hat{\phi}(t)=\frac{B(t+1)}{\eta(t+1)}$ into (59'), I obtain

$$\frac{\eta(t+1)}{\eta(t)}\cdot\frac{K(t+1)}{K(t)}=\frac{\theta}{\beta\cdot[(1-\sigma)\cdot\frac{B(t+1)}{\eta(t+1)}+\sigma\theta]}.$$

$$\leftrightarrow\frac{K(t+1)}{K(t)}\cdot\frac{\eta(t+1)}{\eta(t)}\cdot\beta\cdot\left[(1-\sigma)\frac{B(t+1)}{\eta(t+1)}+\sigma\theta\right]=\theta. \tag{42}$$

# F Derivation of (46)

In the steady state, $\frac{x(t+1)}{x(t)}=1$, $\frac{K(t+1)}{K(t)}=1$, $(1-\eta(t))\cdot K(t)=K^h_{ss}$, $\frac{B(t+1)}{B(t)}\cdot\frac{\eta(t+1)}{\eta(t)}=1$, and $\frac{B(t+1)}{\eta(t+1)}=\phi_{ss}$. Then, substituting these into (41), I obtain

$$(1+(1-\sigma)\lambda)+\omega\cdot K^h_{ss}-\frac{\theta}{(1-\sigma)\cdot\phi_{ss}+\sigma\theta}=\beta\cdot(1-\delta)\cdot(1+(1-\sigma)\lambda)+\beta\cdot\left(\frac{\phi_{ss}-1}{\phi_{ss}}\right)\cdot\frac{\omega}{2}\cdot K^h_{ss}$$

$$\leftrightarrow\omega\cdot\left(1-\frac{\beta}{2}\cdot\left(\frac{\phi_{ss}-1}{\phi_{ss}}\right)\right)\cdot K^h_{ss}=\frac{\theta}{(1-\sigma)\cdot\phi_{ss}+\sigma\theta}-(1+(1-\sigma)\lambda)\cdot(1-\beta(1-\delta))$$

$$\leftrightarrow K^h_{ss}=\frac{\frac{\theta}{(1-\sigma)\overline{\phi}+\sigma\theta}-(1+(1-\sigma)\cdot\lambda)(1-\beta(1-\delta))}{\omega\left(1+\frac{\beta}{2}\frac{1-\overline{\phi}}{\overline{\phi}}\right)}$$

From (45), $\frac{\beta}{2}\cdot\frac{1-\phi_{ss}}{\phi_{ss}}=\frac{\beta^2(1-\sigma)-\beta\theta(1-\beta\sigma)}{2\theta(1-\beta\sigma)}$, $(1-\sigma)\phi_{ss}+\sigma\theta=\frac{\theta}{\beta}$ when $\phi_{ss}=\frac{\theta(1-\beta\sigma)}{\beta(1-\sigma)}$ or $\frac{\beta}{2}\cdot\frac{1-\phi_{ss}}{\phi_{ss}}=\frac{\beta}{2}\cdot\frac{1-\overline{\phi}_{ss}}{\overline{\phi}_{ss}}$ when $\phi_{ss}=\overline{\phi}$. Substituting into the above equations into the above equation in each case, I obtain

$$K^h_{ss}=\begin{Bmatrix}\frac{2\theta\cdot(1-\beta\sigma)\cdot[\beta-(1+(1-\sigma)\cdot\lambda)(1-\beta(1-\delta))]}{\omega\cdot[(2-\beta)\cdot(1-\beta\sigma)\cdot\theta+\beta^2\cdot(1-\sigma)]}\equiv\hat{K}^h_{ss}, & \text{when} \quad \frac{\theta(1-\beta\sigma)}{\beta(1-\sigma)}>\overline{\phi} \\ \frac{\frac{\theta}{(1-\sigma)\overline{\phi}+\sigma\theta}-(1+(1-\sigma)\cdot\lambda)(1-\beta(1-\delta))}{\omega\left(1+\frac{\beta}{2}\frac{1-\overline{\phi}}{\overline{\phi}}\right)}\equiv\overline{K}^h_{ss}, & \text{when} \quad \frac{\theta(1-\beta\sigma)}{\beta(1-\sigma)}\leq\overline{\phi},\end{Bmatrix}. \tag{46}$$

# G Proof of Lemma 2

**Proof.** First, I consider the case in which $\frac{\theta(1-\beta\sigma)}{\beta(1-\sigma)} > \overline{\phi}$. From equation (46), for the existence of $K_{ss}^h$ for a positive value, the following condition must be satisfied;

$$\frac{2\theta\cdot(1-\beta\sigma)\cdot[\beta-(1+(1-\sigma)\cdot\lambda)(1-\beta(1-\delta))]}{\omega\cdot[(2-\beta)\cdot(1-\beta\sigma)\cdot\theta+\beta^2\cdot(1-\sigma)]} > 0.$$

The above inequality is satisfied if

$$\begin{aligned}
&[\beta-(1+(1-\sigma)\cdot\lambda)(1-\beta(1-\delta))] > 0\\
&\leftrightarrow \lambda < \frac{\beta\cdot[1+(1-\delta)\cdot(1-\sigma)]-1}{(1-\beta\cdot(1-\delta))\cdot(1-\sigma)}\\
&\leftrightarrow \lambda < \hat{\lambda}_1,
\end{aligned}$$

where $\hat{\lambda}_1 \equiv \frac{\beta\cdot[1+(1-\delta)\cdot(1-\sigma)]-1}{(1-\beta\cdot(1-\delta))\cdot(1-\sigma)}$.

Since $\lambda > 0$, $\hat{\lambda}_1 > 0$ must be satisfied;

$$\begin{aligned}
&\hat{\lambda}_1 > 0\\
&\leftrightarrow \beta\cdot[1+(1-\delta)\cdot(1-\sigma)]-1 > 0\\
&\leftrightarrow \beta > \frac{1}{1+(1-\delta)\cdot(1-\sigma)} \equiv \hat{\beta}_1.
\end{aligned}$$

Since $\delta > 0$ and $\sigma > 0$, $\hat{\beta}_1$ satisfies $0 < \hat{\beta}_1 < 1$.

if $\lambda \leq \frac{\beta-(1+\beta(1-\sigma))(1-\beta(1-\delta))}{(1-\sigma)(1+\beta(1-\sigma))(1-\beta(1-\delta))} \equiv \hat{\lambda}_1$.

Second, I consider the case in which $\frac{\theta(1-\beta\sigma)}{\beta(1-\sigma)} \leq \overline{\phi}$. From equation (46), for the existence of $K_{ss}^h$ for a positive value, the following condition must be satisfied;

$$\overline{K}_{ss}^h = \frac{\frac{\theta}{(1-\sigma)\overline{\phi}+\sigma\theta}-(1+(1-\sigma)\cdot\lambda)(1-\beta(1-\delta))}{\omega\left(1+\frac{\beta}{2}\frac{1-\overline{\phi}}{\overline{\phi}}\right)} > 0.$$

Since $\omega\left(1+\frac{\beta}{2}\frac{1-\overline{\phi}}{\overline{\phi}}\right) > 0$, the above inequality can be rewritten as follows:

$$\begin{aligned}
&\frac{\theta}{(1-\sigma)\overline{\phi}+\sigma\theta}-(1+(1-\sigma)\lambda)(1-\beta(1-\delta)) > 0\\
&\leftrightarrow \frac{\theta\cdot[\sigma-(1+(1-\sigma)\lambda)\cdot(1-\beta\cdot(1-\delta))]}{(1+(1-\sigma)\lambda)\cdot(1-\beta\cdot(1-\delta))\cdot(1-\sigma)} > \overline{\phi}\\
&\leftrightarrow \overline{\phi}_1 > \overline{\phi},
\end{aligned}$$

where $\overline{\phi}_1 \equiv \frac{\theta\cdot[\sigma-(1+(1-\sigma)\lambda)\cdot(1-\beta\cdot(1-\delta))]}{(1+(1-\sigma)\lambda)\cdot(1-\beta\cdot(1-\delta))\cdot(1-\sigma)}$.

Since $\overline{\phi} > 0$, the following conditions must be satisfied in the steady state;

$$[\sigma - (1 + (1-\sigma)\lambda)\cdot(1-\beta\cdot(1-\delta))] > 0$$
$$\lambda < \frac{\sigma + \beta(1-\delta) -}{(1-\sigma)\cdot(1-\beta\cdot(1-\delta))} \equiv \overline{\lambda}_1.$$

For $\lambda < 1$ and $\beta < 1$, $\beta > \frac{(1-\sigma)}{(1-\delta)} \equiv \overline{\beta}_1$ and $\delta < \sigma$ must be satisfied.

Because I consider the case in which $\frac{\theta(1-\beta\sigma)}{\beta(1-\sigma)} \leq \overline{\phi}$, $\overline{\phi}_1 > \overline{\phi}$ must be satisfied.

$$\overline{\phi}_1 > \overline{\phi}$$

if

$$\lambda < \frac{\beta\sigma - (1+\beta\cdot(1-\sigma))\cdot(1-\beta\cdot(1-\delta))}{(1+\beta\cdot(1-\sigma))\cdot(1-\beta\cdot(1-\delta))\cdot(1-\sigma)} \equiv \overline{\lambda}_1.$$

Since $\hat{\lambda}_1 > \lambda$ and $\hat{\lambda}_1 < \overline{\lambda}_1$, $\overline{\lambda}_1 > \lambda$ is satisfied.

Hence, $\phi_{ss} = \overline{\phi}$, $\overline{K}^h_{ss}$ exists if $\frac{\theta\cdot(1-\beta\sigma)}{\beta\cdot(1-\sigma)} \geq \overline{\phi} < \overline{\phi}_1$, $\lambda < \overline{\lambda}_1$, $\beta > \overline{\beta}_1$ and $\delta < \sigma$.

■

## H Proof of Lemma 3

**Proof.** Differentiating equation (47) with respect to $K^h_{ss}$, I obtain

$$\frac{dK^b_{ss}}{dK^h_{ss}} = \frac{\sigma\cdot\left[2\beta\omega K^h_{ss}\cdot(\sigma\theta + (1-\sigma)\phi_{ss}) - \theta(\phi_{ss})^2\right]}{\beta\cdot[(1-\sigma)+\phi_{ss}]\cdot[\sigma\theta + (1-\sigma)\cdot\phi_{ss}]}. \tag{60}$$

From (60), $\frac{dK^b_{ss}}{dK^h_{ss}} > 0$ if

$$2\beta\omega K^h_{ss} > \frac{\theta\cdot(\phi_{ss})^2}{\sigma\theta + (1-\sigma)\cdot\phi_{ss}}. \tag{60'}$$

First, I consider the case in which $\frac{\theta(1-\beta\sigma)}{\beta(1-\sigma)} > \overline{\phi}$. Substituting $K^h_{ss} = \hat{K}^h_{ss}$ and $\phi_{ss} = \hat{\phi}_{ss}$ from (45) and (46) into the condition (60'), I obtain

$$2\beta\omega\hat{K}^h_{ss} > \frac{\theta\cdot(\hat{\phi}_{ss})^2}{\sigma\theta + (1-\sigma)\cdot\hat{\phi}_{ss}}$$
$$\leftrightarrow \frac{4\beta(1-\sigma)^2\cdot[\beta-(1-\beta(1-\delta))] - \theta^2(1-\beta\sigma)\cdot[(2-\beta)(1-\beta\sigma)\theta + \beta^2(1-\sigma)]}{4\beta(1-\sigma)^2\cdot(1-\beta(1-\delta))} > \lambda. \tag{60-1}$$

Let $\hat{\lambda}_2$ be such that $\hat{\lambda}_2 \equiv \frac{4\beta(1-\sigma)^2\cdot[\beta-(1-\beta(1-\delta))]-\theta^2(1-\beta\sigma)\cdot[(2-\beta)(1-\beta\sigma)\theta+\beta^2(1-\sigma)]}{4\beta(1-\sigma)^2\cdot(1-\beta(1-\delta))}$. From (60-1), $\frac{d\hat{K}^b_{ss}}{d\hat{K}^h_{ss}} > 0$ if $\hat{\lambda}_2 > \lambda$.

Second, I consider the case in which $\frac{\theta(1-\beta\sigma)}{\beta(1-\sigma)} \leq \overline{\phi}$. Substituting $K^h_{ss} = \overline{K}^h_{ss}$ and $\phi_{ss} = \overline{\phi}_{ss}$ from (45) and (46) into the condition (60'), I obtain

$$
\begin{aligned}
& 2\beta\omega\overline{K}^h_{ss} > \frac{\theta \cdot (\overline{\phi}_{ss})^2}{\sigma\theta + (1-\sigma)\cdot\overline{\phi}_{ss}} \\
\leftrightarrow & \frac{\theta \cdot [(4-\beta) - 4\beta\sigma \cdot (1+(1-\sigma)\lambda)\cdot(1-\beta(1-\delta))]}{2\cdot\left[\theta\cdot\left(1-\frac{\beta}{2}\right) + (1-\sigma)\cdot 2\beta\cdot(1+(1-\sigma)\lambda)(1-\beta(1-\delta))\right]} > \overline{\phi}.
\end{aligned}
\tag{60-2}
$$

Let $\overline{\phi}_2$ be such that $\overline{\phi}_2 \equiv \frac{\theta\cdot[(4-\beta)-4\beta\sigma\cdot(1+(1-\sigma)\lambda)\cdot(1-\beta(1-\delta))]}{2\cdot\left[\theta\cdot\left(1-\frac{\beta}{2}\right)+(1-\sigma)\cdot 2\beta\cdot(1+(1-\sigma)\lambda)(1-\beta(1-\delta))\right]}$.From (60-2), $\frac{d\overline{K}^b_{ss}}{d\overline{K}^h_{ss}} > 0$ if $\overline{\phi}_2 > \overline{\phi}$. ■

# I Proof of Lemma 4

**Proof.** Since $K_{ss} = K^h_{ss} + K^b_{ss}$, equation (47) yields

$$
\begin{aligned}
K_{ss} = & \frac{K^h_{ss}\cdot\beta\cdot[(1-\sigma)+\phi_{ss}]\cdot[\sigma\theta+(1-\sigma)\cdot\phi_{ss}]}{\beta\cdot[(1-\sigma)+\phi_{ss}]\cdot[\sigma\theta+(1-\sigma)\cdot\phi_{ss}]} \\
& + \frac{\sigma\cdot K^h_{ss}\cdot\left[\beta\omega\cdot[\sigma\theta+(1-\sigma)\phi_{ss}]\cdot K^h_{ss} - \theta\phi^2_{ss}\right] + (1+(1-\sigma)\lambda)\cdot\phi_{ss}\cdot[(1-\sigma)(1-\phi_{ss}) - \delta\beta\sigma\theta]}{\beta\cdot[(1-\sigma)+\phi_{ss}]\cdot[\sigma\theta+(1-\sigma)\cdot\phi_{ss}]}.
\end{aligned}
\tag{61}
$$

Differentiating equation (61) with respect to $K^h_{ss}$, I obtain

$$
\frac{dK_{ss}}{dK^h_{ss}} = \frac{\beta\cdot[\sigma\theta+(1-\sigma)\phi_{ss}]\cdot[(1-\sigma)+\phi_{ss}+2\sigma\omega K^h_{ss}] - \theta\sigma\cdot(\phi_{ss})^2}{\beta\cdot[(1-\sigma)+\phi_{ss}]\cdot[\sigma\theta+(1-\sigma)\cdot\phi_{ss}]}.
\tag{62}
$$

(62') implies that $\frac{dK_{ss}}{dK^h_{ss}} > 0$ if

$$
K^h_{ss} > \frac{\theta\sigma(\phi_{ss})^2 - \beta\cdot[\sigma\theta+(1-\sigma)\phi_{ss}]\cdot[(1-\sigma)+\phi_{ss}]}{2\beta\sigma\omega\cdot[\sigma\theta+(1-\sigma)\cdot\phi_{ss}]}.
\tag{62'}
$$

Let $\Lambda$ be such that $\Lambda \equiv \frac{\theta\sigma(\phi_{ss})^2-\beta\cdot[\sigma\theta+(1-\sigma)\phi_{ss}]\cdot[(1-\sigma)+\phi_{ss}]}{2\beta\sigma\omega\cdot[\sigma\theta+(1-\sigma)\cdot\phi_{ss}]}$. Then, (62') is satisfied if

$$
\begin{aligned}
& \Lambda < 0 \\
\leftrightarrow & \theta\sigma(\phi_{ss})^2 - \beta\cdot[\sigma\theta+(1-\sigma)\phi_{ss}]\cdot[(1-\sigma)+\phi_{ss}] < 0 \\
\leftrightarrow & (\theta\sigma - \beta(1-\sigma))\cdot(\phi)^2_{ss} - \beta\cdot[\sigma\theta+(1-\sigma)^2]\cdot\phi_{ss} < \beta\sigma\theta(1-\sigma).
\end{aligned}
\tag{63}
$$

First, consider the case in which $\overline{\phi} < \frac{\theta(1-\beta\sigma)}{\beta(1-\sigma)} \equiv \hat{\phi}_{ss}$.

Substituting $\hat{\phi}_{ss}$ into (63), I obtain

$$
\sigma\theta^2(1-\beta\sigma)^2 - \beta(1-\sigma)(1-\beta\sigma)\theta < \beta^2(1-\sigma)^3.
\tag{63-1}
$$

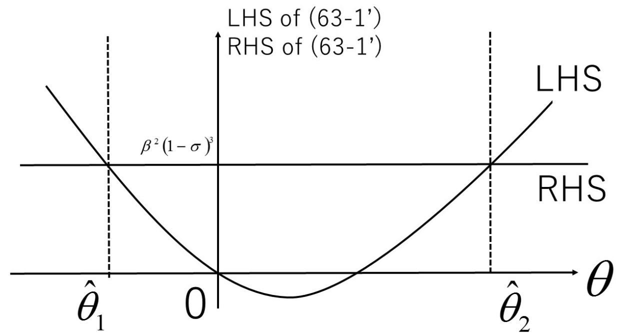


Figure 1: Equation (63-1')

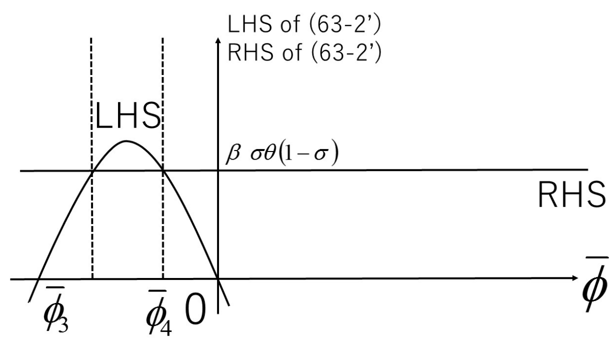


Figure 2: Equation (63-2') with $\overline{\theta}_1 > \theta$

Let $\hat{\theta}_2$ be such that satisfies the following equation for a positive value:

$$\sigma\theta^2(1-\beta\sigma)^2 - \beta(1-\sigma)(1-\beta\sigma)\theta = \beta^2(1-\sigma)^3. \tag{63-1'}$$

Similarly, let $\hat{\theta}_1$ be such that satisfies equation (63-1') for a negative value.

As shown in Figure 1, (63-1) is satisfied if $\theta < \hat{\theta}_2$. Since $\theta > 0$, $\frac{dK_{ss}}{dK_{ss}^h} > 0$ if $0 < \theta < \hat{\theta}_2$.

Second, consider the case in which $\frac{\theta(1-\beta\sigma)}{\beta(1-\sigma)} \leq \overline{\phi}$. Substituting $\overline{\phi}_{ss}$ into (63), I obtain

$$(\theta\sigma - \beta(1-\sigma)) \cdot (\overline{\phi})^2 - \beta \cdot [\sigma\theta + (1-\sigma)^2] \cdot \overline{\phi} < \beta\sigma\theta(1-\sigma). \tag{63-2}$$

Let $\overline{\phi}_3$ and $\overline{\phi}_4$ be such that satisfies the following equation with $\overline{\phi}_3 < \overline{\phi}_4$ :

$$(\theta\sigma - \beta(1-\sigma)) \cdot (\overline{\phi})^2 - \beta \cdot [\sigma\theta + (1-\sigma)^2] \cdot \overline{\phi} = \beta\sigma\theta(1-\sigma). \tag{63-2'}$$

The left-hand side of (63-2') is convex upwards if $(\theta\sigma - \beta(1-\sigma)) < 0$ as shown in Figure 2. The left-hand side of (63-2') is convex downwards if $(\theta\sigma - \beta(1-\sigma)) > 0$ as shown in Figure 3. Let $\overline{\theta}_1$ be such that $\overline{\theta}_1 \equiv \frac{\beta(1-\sigma)}{\sigma}$. Then, $(\theta\sigma - \beta(1-\sigma)) < 0$ can be rewritten as $\overline{\theta}_1 > \theta$.

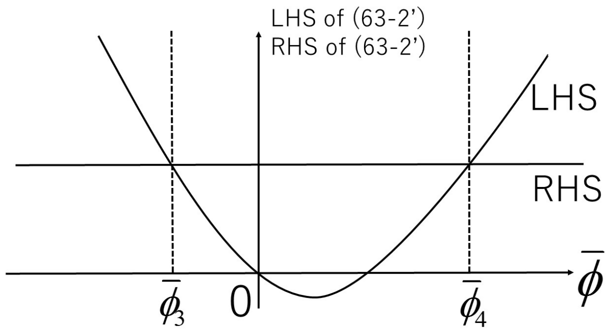


Figure 3: Equation (63-2') with $\overline{\theta}_1 \leq \theta$

As shown in Figures 2 and 3, $\frac{d\overline{K}_{ss}}{d\overline{K}^h_{ss}} > 0$, if $\overline{\theta}_1 > \theta$ or $\overline{\theta}_1 \leq \theta$ and $0 < \overline{\phi} < \overline{\phi}_4$.

■

# J Proof of Proposition 1

**Proof.** Differentiating equation (49) with respect to $K^h_{ss}$, I obtain

$$\frac{dCss}{dK^h ss} = \frac{\left[\frac{\omega}{2}\cdot\left(\frac{1-\phi_{ss}}{\phi_{ss}}\right)\cdot K^h_{ss} - (1-\delta) - \left(\frac{\theta}{\alpha\beta[(1-\sigma)\phi_{ss}+\sigma\theta]}\right)\right]\cdot\frac{dK}{dK^h_{ss}}}{\beta(1-\delta)(1+(1-\sigma)\lambda) - \frac{\beta\omega}{2}K^h_{ss}\cdot\left(\frac{1-\phi_{ss}}{\phi_{ss}}\right)+\left(\frac{\theta}{(1-\sigma)\phi_{ss}+\sigma\theta}\right)} + \frac{\frac{\beta\omega}{2}\cdot\left(\frac{1-\phi_{ss}}{\phi_{ss}}\right)\cdot\left(\left[\frac{\omega}{2}\cdot K^h_{ss}\cdot\left(\frac{1-\phi_{ss}}{\phi_s s}\right) - (1-\delta) - \left(\frac{\theta}{\alpha\beta\cdot[(1-\sigma)\cdot\phi_{ss}+\sigma\theta]}\right)\right]\cdot K_{ss} + \frac{\omega}{2}\cdot\left(K^h_{ss}\right)^2\right)}{\left[\beta(1-\delta)(1+(1-\sigma)\lambda) - \frac{\beta\omega}{2}K^h_{ss}\cdot\left(\frac{1-\phi_{ss}}{\phi_{ss}}\right)+\left(\frac{\theta}{(1-\sigma)\phi_{ss}+\sigma\theta}\right)\right]^2}. \tag{64}$$

Since $\left(\left[\frac{\omega}{2}\cdot K^h_{ss}\cdot\left(\frac{1-\phi_{ss}}{\phi_s s}\right) - (1-\delta) - \left(\frac{\theta}{\alpha\beta\cdot[(1-\sigma)\cdot\phi_{ss}+\sigma\theta]}\right)\right]\cdot K_{ss} + \frac{\omega}{2}\cdot\left(K^h_{ss}\right)^2\right)$ is positive due to $C_{ss}$ is non-negative in equation (49), the second term of the right-hand side in equation (64) is positive.

The first-term of the right-hand side in equation (65) is positive if

$$\underbrace{\left[\frac{\omega}{2}\cdot\left(\frac{1-\phi_{ss}}{\phi_{ss}}\right)\cdot K^h_{ss} - (1-\delta) - \left(\frac{\theta}{\alpha\beta[(1-\sigma)\phi_{ss}+\sigma\theta]}\right)\right]}_{(*)}\cdot\frac{dK}{dK^h_{ss}} > 0. \tag{65}$$

Substituting $K^h_{ss}$ and $\phi_{ss}$ from (45) and (46) into the term $(*)$ of (65), I confirm this term is negative. Due to the condition of existence of $K^h_{ss}$ from lemma 2, $\beta$ is sufficiently large such that $\beta_1 < \beta$ and then the term $(*)$ of (65) is close to 0. Thus, I obtain the inequality (65). Because the first term of (64) is sufficiently small and close to 0, and the second term of (64) is positive regardless of the minimum capital adequacy ratio $\overline{\phi}$, I obtain $\frac{dC_{ss}}{dK^h_{ss}} > 0$. ■

## K Proof of Lemma 5

**Proof.** Consider the case in which $\overline{\phi} < \frac{\theta(1-\beta\sigma)}{\beta(1-\sigma)} \equiv \hat{\phi}_{ss}$. Differentiating equation (46) with respect to $\overline{\phi}$, I obtain

$$\frac{d\overline{K}_{ss}^{h}}{d\overline{\phi}} = \frac{\left(\frac{-\theta(1-\sigma)}{(1-\sigma)\overline{\phi}+\sigma\theta}\right)\cdot\omega\cdot\left(1+\frac{\beta}{2}\frac{(1-\overline{\phi})}{\overline{\phi}}\right) - \frac{\beta\omega}{2}\left(\frac{-1}{(\overline{\phi})^2}\right)\left(\frac{\theta(1-\sigma)}{(1-\sigma)\overline{\phi}+\sigma\theta} - (1+(1-\sigma)\lambda)(1-\beta(1-\delta))\right)}{\omega^2\cdot\left[1+\frac{\beta}{2}\frac{1-\overline{\phi}}{\overline{\phi}}\right]}. \tag{66}$$

(66) implies that $\frac{d\overline{K}_{ss}^{h}}{d\overline{\phi}} > 0$ if

$$(\theta-(1-\sigma)(1+(1-\sigma)\lambda)(1-\beta(1-\delta)))\cdot(\overline{\phi}^2) + (-\theta(1+\beta) - 2\sigma\theta\cdot(1+(1-\sigma)\lambda)(1-\beta(1-\delta)))\cdot\overline{\phi}$$
$$> \frac{\sigma\theta^2\cdot[\sigma\cdot(1+(1-\sigma)\lambda)(1-\beta(1-\delta))-1]}{1-\sigma}. \tag{66'}$$

Let $Z_1$, $Z_2$,and$Z_3$, be such that $Z_1 \equiv \theta-(1-\sigma)(1+(1-\sigma)\lambda)(1-\beta(1-\delta))$; $Z_2 \equiv -\theta(1+\beta) - 2\sigma\theta\cdot(1+(1-\sigma)\lambda)(1-\beta(1-\delta))$; $Z_3 \equiv \frac{\sigma\theta^2\cdot[\sigma\cdot(1+(1-\sigma)\lambda)(1-\beta(1-\delta))-1]}{1-\sigma}$. Due to the condition for existence of $\overline{K}_{ss}^{h}$ from (46), $Z_1$ takes a positive value. Since $\beta \in (0,1)$ and $\sigma\cdot(1+(1-\sigma)\lambda)(1-\beta(1-\delta))-1] < \beta\sigma < 1$, $Z_2 < 0$ and $Z_3 < 0$.

Let $\overline{\phi}_5$ and $\overline{\phi}_6$ be such that satisfies the following equation: $Z_1\cdot(\overline{\phi}^2) + Z_2\cdot\overline{\phi} = Z_3$ with $\overline{\phi}_5 < \overline{\phi}_6$. As shown in Figure 4, (64') is satisfied if $\frac{(Z_2)^2}{-4Z_1} > Z_3$ or $\frac{d\overline{K}_{ss}^{h}}{d\overline{\phi}} > 0$ if $\frac{(Z_2)^2}{-4Z_1} \leq Z_3$ and $\overline{\phi}_5 \leq \overline{\phi} \leq \overline{\phi}_6$.

$\frac{(Z_2)^2}{-4Z_1} > Z_3$ can be rewritten as follows:

$$\frac{(1+\beta)^2+\sigma[(1+(1-\sigma)\lambda)(1-\beta(1-\delta)]\cdot[(1+\beta)(1-\sigma)+(1-\sigma)+\sigma^2(1+(1-\sigma)\lambda)\cdot(1-\beta(1-\delta))]}{4\sigma\cdot[1-\sigma\cdot(1+(1-\sigma)\lambda)\cdot(1-\beta(1-\delta))]} > \theta.$$

Let $\overline{\theta}_2$ be such that $\overline{\theta}_2 \equiv \frac{(1+\beta)^2+\sigma[(1+(1-\sigma)\lambda)(1-\beta(1-\delta)]\cdot[(1+\beta)(1-\sigma)+(1-\sigma)+\sigma^2(1+(1-\sigma)\lambda)\cdot(1-\beta(1-\delta))]}{4\sigma\cdot[1-\sigma\cdot(1+(1-\sigma)\lambda)\cdot(1-\beta(1-\delta))]}$. Then, $\frac{d\overline{K}_{ss}^{h}}{d\overline{\phi}} > 0$ if $\overline{\theta}_2 > \theta$ or $\frac{d\overline{K}_{ss}^{h}}{d\overline{\phi}} > 0$ if $\overline{\theta}_2 < \theta$ and $\overline{\phi}_5 \leq \overline{\phi} \leq \overline{\phi}_6$.

■

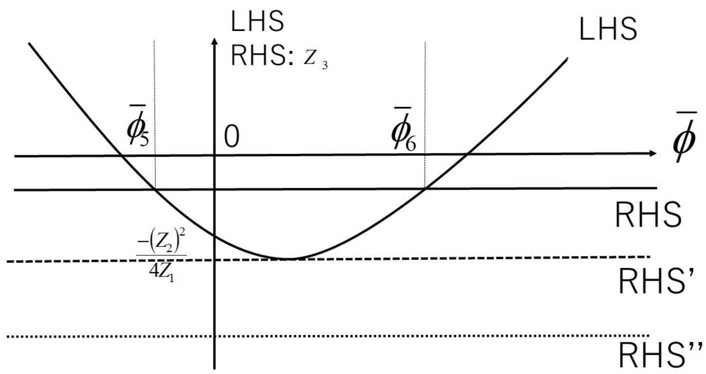


Figure 4: Equation $Z_1 \cdot (\overline{\phi}^2) + Z_2 \cdot \overline{\phi} = Z_3$

# L Proof of Lemma 6

**Proof.** From (45) and (46), I obtain

$$\hat{K}^h_{ss} - \overline{K}^h_{ss}$$
$$= 2\theta\omega(1-\beta\sigma)[\beta - (1+(1-\sigma)\lambda][1-\beta(1-\delta)]\left(1+\frac{\beta}{2}\left(\frac{1-\overline{\phi}}{\overline{\phi}}\right)\right)$$
$$- \frac{\theta\omega[(2-\beta)(1-\beta\sigma)\theta + \beta^2(1-\sigma)]}{(1-\sigma)\overline{\phi}+\sigma\theta} + (1+(1-\sigma)\lambda)(1-\beta(1-\delta))\omega[(2-\beta)(1-\beta\sigma)\theta + \beta^2(1-\sigma)] > 0$$
$$\leftrightarrow (1+(1-\sigma)\lambda)(1-\beta(1-\delta)) > \frac{\overline{\phi}[(2-\beta)(1-\beta\sigma)\theta + \beta^2(1-\sigma)] + \beta(1-\beta\sigma)(2\overline{\phi}+\beta(1-\overline{\phi}))\cdot[(1-\sigma)\overline{\phi}+\sigma\theta]}{\overline{\phi}(1-\beta\sigma)(4-\beta) + \beta[1-\overline{\phi}(1-(1-\sigma)\beta))\cdot[(1-\sigma)\overline{\phi}+\sigma\theta]}. \tag{67}$$

(67) implies that if $\lambda$ is sufficiently large and $\theta$ is sufficiently low in the left-hand side, the above inequality is satisfied.

If $\sigma \to 1$ and $\beta \to 0$, then (67) can be rewritten as

$$2 > \overline{\phi}. \tag{67'}$$

Thus, if $\lambda$ is sufficiently large and $\theta$ is sufficiently low such that (67) is satisfied, or if $\sigma \to 1$, $\beta \to$ and $2 > \overline{\phi}$, then $\hat{K}^h_{ss} - \overline{K}^h_{ss} > 0$. ■

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