Robust Monetary Policy Under Deep Parameter Uncertainty

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1 Introduction

How to conduct monetary policy in the face of uncertainty has always been an interest to both practitioners and academic researchers. In the day-to-day implementation of monetary policy, policymakers must realize that every model are a simplification, necessarily incomplete, and stuck in the world of uncertainty. In general, they have to set their instrument without knowing the "true model" of underlying economy or how their policy will work on the variables which they care about. This difficulty raises the questions about how best to achieve control over uncertainty and in particular, what constitutes the "optimal" or "robust" monetary policy in the face of uncertainty.

One of the seminal works in this area, Brainard (1967), proposed the common intuition that uncertainty about the impact of monetary policy should require a more cautious policy. His result suggests that it is often optimal for policymakers to change their instrument by less than would be optimal if all parameters were correctly known. This result has been called the "Brainard conservatism principle" and found wide acceptance from both theoretical and empirical points of view.[1]

Recently, however, there has been a growing body of literature challenging this principle. Using backward-looking model, Söderström (2002) argues that uncertainty about inflation persistence leads the policymakers to pursue more aggressive policy. Kimura and Kurozumi (2007) has considered the same type of uncertainty with forward looking model and found that robust policy calls for a stronger response of the interest rate against the demand shock. Giannoni (2002) has introduced uncertainty for the several

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[1]See Blinder (1997) and Goodhart (1999), for example.

structural paramters of the forward-looking model, concluded that the robust rule for the model involves a stronger response of the interest rate, again compared to the case in the absence of uncertainty.

While Brainard's principle is often believed to be persuasive among practitioners,[2] these studies imply that the opposite can also be true in the certain settings. As pointed out by Onatski and Williams (2003), the implication of parameter uncertainty seems to be very sensitive to different assumptions about the models and uncertainty attached within. So far, there is no clear answer to whether robust policy rules in the presence of uncertainty generally should be aggressive or cautious.

The aim of this paper, therefore, is to clarify the true factor which determines the stance of robust monetary policy under parameter uncertainty. In other words, we attempt to reveal in what situations Brainard's lesson is worth listening, and vice versa. To this end, we focus on the two aspects which may be crucially important for the condition of robust policy being more or less aggressive. The first element we expect to affect the stance of robust monetary policy is the attitude of the policymaker toward uncertainty. Previous studies have suggested the two distinct approaches to describe the policymaker's action against uncertaitny: the Bayesian and the minimax approaches. In the Bayesian approach, the policymaker is assumed to have a belief about the true value of the uncertain parameter in the form of a prior distribution. By minimizing the expected loss based on this prior distribution, it tries to obtain the average result with the all possible cases. In the previous studies, Brainard (1967), Söderström (2002), and Kimura and Kurozumi (2007) have adopted the Bayesian approch. In contrast, the policymaker may want to prepare for "once in a century" crisis, whose probability of happening is very low, but once it occurs, brings serious damage to the economy. In this situation, the more realistic action may be to give up considering all possbilities and conduct the policy that works reasonably well only in the worst possible case. This is called the minimax approach, which has been used in Giannoni (2002), Giannoni (2007), Hansen and Sargent (2002), Hansen and Sargent (2012). While which approach the real policymakers will find appealing depends on their preferences, analyzing whether or not the difference between these two approaches make their policy recommendations different is important.

As the second aspect, we focus on the role of the uncertain parameters in the model. Which parameters are uncertain and in what route the parameters contribute to the overall uncertainty in the model must mirror the part of underlying economy which the policymaker concerns about. We divide

[2]For example, Alan Blinder has written that the Brainard result was "never far from my mind when I occupied the Vice Chairman's office at the Federal Reserve." and "Still, I find these new anti-Brainard results both puzzling and troubling. Though my confidence in the conclusion has been shaken by recent research, my gut still tells me that Brainard was right in practice." See Blinder (1998) and Blinder (2000).

the source of uncertainty in the reference model into the two categories: the policy impact unvertainty and the dynamics uncertainty. The first category is what Brainard (1967) originally considered, uncertainty about the degree to which monetary policy instrument can stabilize the target variables. Another type of uncertainty, the dynamics uncertainty, is related to how long the effect of demand or supply disturbances on the target variables survive. By classifying uncertainty based on the role of parameters in this way, we succeed to understand the essential reason why different papers make different conclusions about the property of robust policy against parameter uncertainty.

To characterize how these two factors affect the stance of robust monetary policy, this paper prepares three dynamic macroeconomic models. All of which are appropriately designed to include either (or both) of the two categories of parameter uncertainty. The first model is a simple New Keynesian model, which does not involve any intrinsic inertia of the target variables. By introducing parameter uncertainty into this model, we can investigate the effect of the policy impact uncertainty independently. The second model is an ad-hoc hybrid New Keynesian model, which is constructed to include the persistence of the key variables and to see the dynamics uncertainty independently. Finally, for the third model, we turn to examine the micro-founded hybrid New Keynesian model. Since structural parameters in this model are tied to the move of a single deep parameter, uncertainty of one deep parameter spreads to the several parameters simultaneously. This feature allows us to deal with a very rich structure of uncertainty, which affects both the policy impact and the dynamics of target variables, even though we focus on every single deep parameter separately. Compared to the second ad-hoc model, the two categories of parameter uncertainty are mixed in the "natural" way that can withstand the Lucas critique thanks to the micro-foundations, so we can analyze which categories crucially determines the stance of robust monetary policy when the policymaker faces the policy impact and the dynamics uncertainty at the same time.

Our results are threefold. First, the property of robust monetary policy critically depends on the role of uncertain parameters in the policymaker's model. In particular, when the uncertain parameter is related to the policy impact, cautious policy should be conducted. In contrast, if it is related to the inertia or dynamics of the key variables, then the policymaker will benefit from aggressive policy. Second, the Bayesian and the minimax approaches lead to quite similar policy recommendations with the same category of uncertainty, even with the complex micro-founded model. Third, considering the micro-foundation of parameter uncertainty is important. When we consider the mixed effect of the policy impact and the dynamics uncertainty, the relative importance between the two sources will be the key determinant of the policy's stance. In the micro-founded model, it is naturally controlled by the relative influence of the uncertain deep parameter on each structural parameter in the model. In somewhat surprisingly, the Bayesian and the minimax policy show the similar performance with the micro-founded mixed uncertainty, compared to the case with the single category of uncertainty. The Bayesian policy works well even with the extreme value of the uncertain parameter which is targeted by the minimax policymaker. The minimax policy shows the average performance even when the target parameter value does not realize. While we tend to consider the policymaker can deal with uncertainty from the single factor more easily rather than from the complexly mixed factors, our simulation implies the opposite intuitons. Against the mixed uncertainty based on the micro-foundations, policymakers can conduct the policy which succeeds to generate the average result with the broader range of the economy, with doing a certain level of damage control.

The paper is broadly related to the literature which examines the property of robust monetary policy under parameter uncertainty. In contrast to the previous studies which limit their focus to the particular source of uncertainty and the particular approach to derive the policy, our work successfully integrate the various results of them to reveal why and how Brainard's wisdom seems to decrease effectiveness and should be re-evaluated.

The rest of the paper is organized as follows. Section 2 reviews the two approaches for finding robust policy and introduces the two categories of parameter uncertainty. Section 3 conducts several numerical simulations with the three dynamic models, to find the true factor which determines the stance of robust monetary policy under parameter uncertainty. Finally, Section 4 concludes.

2 Robust Policy for Parameter Uncertainty: Two Approaches and Two Categories of Uncertainty

This section introduces the two factors that may affect a property of robust monetary policy for parameter uncertainty. First, we review two approaches for finding robust monetary policy: the Bayesian and the minimax approaches. Then as the second factor, the possibility that the different source of uncertainty may call for the different policy will be discussed.

2.1 Bayesian and Minimax Approaches

Recent studies have considered two approaches, the Bayesian and the minimax approaches, for modeling the polciymaker's action to make its policy be "robust" against parameter uncertainty. To illustrate how each approach defines the "robustness" of monetary policy in an intuitive way, we consider the problem of deriving the Bayesian and the minimax Taylor rule with a standard New Keynesian model.

In what follows, we adopt the conventional notations: π_t denotes inflation rate at time t, x_t denotes output at time t, i_t denotes the short term nominal interest rate at time t, $\varepsilon_{\dots,t}$ denotes an error term where '...' part is replaced by "d" or "s" corresponding to the demand or supply shock, and $E_t[\cdot]$ denotes expectations conditional on the information set at time t. All variables with lower case letters denote percent deviations from the zero-inflationary steady state of the model.

The standard New Keynesian model is typically represented by three equations: an IS curve, a Phillips curve and a monetary policy rule. The IS equation, which relates spending decisions to the real interest rate, is given by

$$x_t = E_t x_{t+1} - \sigma(i_t - E_t\pi_{t+1} - r_t^n), \tag{2.1}$$

and the Phillips curve is given by

$$\pi_t = \kappa x_t + \beta E_t \pi_{t+1} + \varepsilon_{s,t}, \tag{2.2}$$

where the natural rate of interest r_t^n represents the real interest rate that equates output to its flexible price equilibrium level. To simplify the discussion, here we consider the case where the natural rate is fixed at the steady state level, so the policymaker needs to worry about only the supply disturbance, $\varepsilon_{s,t}$. For ease of exposition, we assume that this supply disturbance is i.i.d. with its variance σ_s^2.

In the following analysis, we will generally focus on a simple Taylor-type rule such as

$$i_t = \psi_\pi \pi_t + \psi_x x_t, \tag{2.3}$$

and consider the policymaker's problem to determine the policy coefficients ψ_π and ψ_x with the loss function given by

$$L_0 = E_0 \sum_{t=0}^{\infty} \beta^t [\pi_t^2 + \lambda_x x_t^2 + \lambda_i i_t^2], \tag{2.4}$$

where λ_x and λ_i are positive weights on the output gap and interest rate deviation, respectively. However, in this section, we follow Woodford (1999) and Giannoni (2002) to directly derive the coefficient ϕ in the reduced-form equation of the equilibrium interest rate, which can be expressed as

$$i_t = \phi \varepsilon_{s,t}. \tag{2.5}$$

While we can determine the values of Taylor rule coefficents ψ_π and ψ_x from ϕ easily, the reduced form coefficient is informative enough to characterize the difference between the Bayesian and the minimax approaches.

Given the absence of any intrinsic inertia in the model, the reduced-form solution of the rational expectation equilibrium will only involve the current shock $\varepsilon_{s,t}$, which implies $E_t\pi_{t+1} = E_t x_{t+1} = 0$. Thus, the reduced form solution can be expressed as

$$i_t = \phi\varepsilon_{s,t}, \tag{2.6}$$

$$x_t = -\sigma\phi\varepsilon_{s,t}, \tag{2.7}$$

$$\pi_t = (1 - \kappa\sigma\phi)\varepsilon_{s,t}. \tag{2.8}$$

When the all model parameters are known with certainty, the policymaker directly minimize the reduced-form loss calculated as

$$E[\pi_t^2 + \lambda_x x_t^2 + \lambda_i i_t^2] = \sigma_s^2 \left[1 - 2\kappa\sigma\phi + \left\{(\kappa^2 + \lambda_x)\sigma^2 + \lambda_i\right\}\phi^2\right], \tag{2.9}$$

and the optimal policy coefficient is derived as

$$\phi^* = \frac{\kappa\sigma}{(\kappa^2 + \lambda_x)\sigma^2 + \lambda_i}. \tag{2.10}$$

Next, we consider the case where the policymaker cannot observe the true value of the parameter σ. In this situation, the policymaker will give up finding the optimal policy for the model and instead, seek to the robust monetary policy, i.e., the policy whose result is not much influenced by parameter uncertainty. Although the robust monetary policy may not be optimal for the true economy the policymaker lives in, it may avoid seriously bad outcomes under any rational expectation equilibrium. In this sense, the robust policy can be the second best choice for the policymaker who faces such kind of parameter uncertainty. With the New Keynesian model above, we now consider how the Bayesian and the minimax approaches deal with σ's uncertainty.

2.1.1 The Bayesian Approach

Under the Bayesian approach, the policymaker is assumed to have a prior belief about the true value of uncertain parameter in the form of a distribution. Then the Bayesian policymaker determines its policy by minimizing the average loss, whose weight is based on the prior distribution.

We assume here that the policymaker has the prior distribution of unknown parameter σ in its mind, as its mean is $E[\sigma]$ and its variance is $V[\sigma]$. Then, under the Bayesian approach, the objective function of the policymaker is given by

$$E^B[L_0] = E^B\left\{E_0 \sum_{t=0}^{\infty} \beta^t [\pi_t^2 + \lambda_x x_t^2 + \lambda_i i_t^2]\right\}, \tag{2.11}$$

where $E^B[\cdot]$ denotes expectations based on the prior distribution of the policymaker. The reduced-form loss (2.9) is repalced by

$$E^B[\pi_t^2 + \lambda_x x_t^2 + \lambda_i i_t^2] = \sigma_s^2 \left[1 - 2\kappa E[\sigma]\phi + \left\{(\kappa^2 + \lambda_x)(E[\sigma]^2 + V[\sigma]) + \lambda_i\right\}\phi^2\right], \tag{2.12}$$

and the Bayesian policy coefficient ϕ^B can be expressed as

$$\phi^B = \frac{\kappa E[\sigma]}{(\kappa^2 + \lambda_x)(E[\sigma]^2 + V[\sigma]) + \lambda_i}. \tag{2.13}$$

As it is clear from (2.13), the Bayesian policy is critically affected by the prior distribution of the policymaker. As the degree of subjective uncertainty about σ increases (i.e., $V[\sigma]$ increases), ϕ_B will decrease, which implies the policymaker will respond less aggressively to the supply shock.

The reason why the Bayesian approach recommends more cautious policy for σ's uncertainty becomes clear from the reduced form solutions (2.6) - (2.8). The subjective expectations about squared deviation of each variable is written as

$$E^B[x_t^2] = E^B[\sigma^2]\phi^2\sigma_s^2 = (E[\sigma]^2 + V[\sigma])\phi^2\sigma_s^2,$$
$$E^B[\pi_t^2] = E^B[(1-\kappa\sigma\phi)^2]\sigma_s^2 = \left\{1 - 2\kappa E[\sigma]\phi + \kappa^2(E[\sigma]^2 + V[\sigma])\phi^2\right\}\sigma_s^2.$$

The above equations imply that the uncertainty about σ, which is represented by $V[\sigma]$, increases the subjective variances of output and inflation. The more aggressive policy (i.e., the bigger ϕ) will amplify the uncertainty effect which $V[\sigma]$ has on the subjective variances. If there is no uncertainty about σ, there is no gain for the policymaker from suppressing the response of its policy to the disturbance. However, when the policymaker faces substantial uncertainty about σ, the Bayesian policymaker will succeed to dampen the uncertainty effect by conducting cautious policy compared with the case of no parameter uncertainty.

The above example illustrates how the "robustness" of monetary policy is defined under the Bayesian approach. For the Bayesian policy maker, robust monetary policy is the policy which succeeds to reduce the uncertainty effect arised from parameter uncertainty. When a response of interest rate to exogenous shocks amplifies the uncertainty effect, more cautious policy will be conducted and vice versa.

2.1.2 The Minimax Approach

Another approach is the minimax policy, which is based on the idea that the objective of the policymaker under parameter uncertainty is to minimize the maximum loss in its concern. When the policymaker adopts the minimax approach, it needs not to have well-defined prior beliefs about the uncertain parameter. Instead, the policymaker is supposed to have an assumption about the conceivable range of the value which uncertain parameter can take. About the uncertain parameter σ, we assume that the policymaker's concern can be represented as the range

$$\sigma \in [\underline{\sigma}, \bar{\sigma}]. \tag{2.14}$$

Given the specific policy coefficent, the policymaker can search the worst case parameter, which will cause the maximum loss within the concerned range. Then it selects the policy that will be optimal for the model with this worst case parameter. With the New Keynesian model above, such procedure can be expressed as the minimax problem like

$$\min_{\phi} \max_{\sigma \in [\underline{\sigma}, \bar{\sigma}]} E \left\{ E_0 \sum_{t=0}^{\infty} \beta^t [\pi_t^2 + \lambda_x x_t^2 + \lambda_i i_t^2] \right\}. \tag{2.15}$$

First, we have to search the worst case parameter as a solution of the maximization problem given by

$$\max_{\sigma \in [\underline{\sigma}, \bar{\sigma}]} \sigma_s^2 \left[1 - 2\kappa\sigma\phi + \left\{ (\kappa^2 + \lambda_x)\sigma^2 + \lambda_i \right\} \phi^2 \right], \tag{2.16}$$

where ϕ is fixed at the specific value. While there are two cases about the solution of (2.16) since it is a quadratic function of σ, under the standard calibration, the minimum σ in (2.16) causes the worst case for the policymaker.[3] Then the minimax policy coefficient ϕ^M is calculated as

$$\phi^M = \frac{\kappa\underline{\sigma}}{(\kappa^2 + \lambda_x)(\underline{\sigma})^2 + \lambda_i}. \tag{2.17}$$

Whether (2.17) indicates more cautious or aggressive policy is rather complex compared with the Bayesian case. In the later section, we confirm that (2.17) becomes an increasing function of σ under the standard calibration, so more cautious policy is generally selected.

For the minimax policymaker, dealing with the worst case is the most important issue and any information about the likelihood of uncertain parameter values, whether or not it is subjective or objective, is not relevant for determining its policy. Indeed, the above procedure and (2.17) imply that only the worst case parameter is at the heart of the minimax approach. A value of the worst case parameter depends on the functional form of loss function about the uncertain parameter, or more fundamentally, the model itself.

2.2 Policy Impact and Dynamics Uncertainty

In the preceding part, the difference between the two approaches to find robust monetary policy are reviewed. We now turn our attention to the role of uncertain parameter which it plays in the economic model. In particular, we categorize model parameters into the two types: the policy impact parameter and the dynamics parameter. Even under the same approach, introducing uncertainty into the different parameter may call for the different policy.

[3] When $\sigma < \frac{\kappa}{(\kappa^2+\lambda_x)\phi}$, $\underline{\sigma}$ will maximize (2.16).

2.2.1 Policy Impact Uncertainty

In the previous part, we introduce the parameter σ's uncertainty into the New Keynesian model. While the parameter σ represents the intertemporal elasticity of substitution, from the policymaker's perspective, it also represents the slope of the IS curve, which critically affects the ability of the policymaker's instrument (i.e., the nominal interest rate) to control the output gap. In the remainder of this paper, we call the parameters which determine the effect of policy instrument on the target variables as the "policy impact parameters", and uncertainty about such parameters as the "policy impact uncertainty".

Under the Bayesian approach, the parameter σ's uncertainty results in the more cautious policy because it reduces the uncertainty effect arised from the policy impact uncertainty. On the other hand, the minimax policymaker generally concerns about the case where the policy impact is extremely weak. Its concern affects trade-off facing the policymaker by increasing the weight given to interest rate stabilization relative to inflation and output gap stabilization, makes the policymaker conduct the more cautious policy.

In summary, while the reasons are different, the both approaches recommend the policymaker to reduce a response of interest rate to the supply shock under the simple New Keyenesian model. In the later section, we verify that the result applies to more general cases.

2.2.2 Dynamics Uncertainty

To characterize the second category of parameter uncertainty, that is, the "dynamics uncertainty", we introduce the new model which can be considered as an alternative version of the New Keynesian model.

First, to simplify the discussion, we temporary apparts from the forward-looking expectaions and transforms the Phillips curve (2.2) as

$$\pi_t = \kappa x_t + \gamma \pi_{t-1} + \varepsilon_{s,t}, \tag{2.18}$$

where we change the notation of the coefficient on the expectation of inflation rate to avoid confusion. Furthermore, we adopt the output gap x_t as the instrument variable. In other words, we assume that the policymaker can completely control the output gap by moving interest rates. Then the policy rule can be expressed as

$$x_t = -\phi \varepsilon_{s,t}, \tag{2.19}$$

and the policymaker's loss function is reduced to

$$L_0 = E_0 \sum_{t=0}^{\infty} \beta^t [\pi_t^2 + \lambda_x x_t^2]. \tag{2.20}$$

Since the model involves intrinsic inertia of the inflation rate, the policymaker has to take into account how the past supply shocks until today affect inflation in the current period. By solving the model, the inflation rate π_t is represented as the moving average of the current and past exogenous shocks as

$$\pi_t = (1 - \kappa\phi) \sum_{j=0}^{\infty} \gamma^j \varepsilon_{s,t-j}. \tag{2.21}$$

We consider the case where the parameter γ is uncertain to examine how the "dynamics uncertainty" affects the policy stance.

Under the Bayesian approach, the policymaker is assumed to have the prior distribution about the value of γ and its target can be expressed as

$$E^B[\pi_t^2] \simeq \frac{(1-\kappa\phi)^2 \sigma_s^2}{1 - (E[\gamma]^2 + V[\gamma])}, \tag{2.22}$$

where $E[\gamma]$ and $V[\gamma]$ are the mean and variance of the prior distribution, respectively. When uncertainty about the persistence of the inflation rate (i.e., $V[\gamma]$) increases, (2.22) indicates that conducting the more aggressive policy is better for choking down the uncertainty effect.

On the other hand, the variance of inflation rate

$$E[\pi_t^2] = \frac{(1-\kappa\phi)^2 \sigma_s^2}{1-\gamma^2} \tag{2.23}$$

becomes the highest level when the value of γ is the highest, that is, in the case with the highest inflation persistence. In other words, for the minimax policymaker, the highest priority task is dealing with the case where the inflationary or deflationary shocks survive long and control of inflation via monetary policy becomes severely difficult. As is clear from (2.23), making the interest rate respond stronger against the shock is the best way to do it.

In summary, against the dynamics uncertainty, the policymaker tends to place more weight on the stabilization of the target variables and relatively less weight on the stabilization of the instrument, then conduct more aggressive policy. We confirm whether or not it can become the general statement in the next section.

3 Numerical Simulations with Three Models

This section conducts several numerical simulations with the three models we prepare below, to investigate the effects of the two approaces and the two categories of parameter uncertainty on the stance of robust monetary policy.

3.1 Common settings

Before presenting the details about our models, we outline here the settings which are common between all simulations.

As in many studies of monetary policy, we assume that the objective of the policymaker is to minimize the mean-squared deviation of inflation and the output gap from the targets. The loss function of the policymaker is given as

$$E_0 \sum_{t=0}^{\infty} \beta^t [\pi_t^2 + \lambda_x x_t^2 + \lambda_i i_t^2], \tag{3.24}$$

which is exactly the same as in Section 2. Note that this loss function implicitly assumes that the target levels of three variables are the steady steate values, 0.

When we consider that the policymaker's fundamental objective is to maximize households' welfare, we should interpret such a loss function as a second-order approximation to the expected utility of the representative household as in Woodford (1999). In such a case, the weights λ_x and λ_i are derived as the functions of deep parameters, and uncertain deep parameters make the policymaker face uncertainty about the loss function's weights as well as about the structural parameters of the model. In the present paper, however, we assume that the preference parameters of the policymaker are kept fixed regardless of the values of deep parameters, and also correctly known to the all agents in the model even if they face parameter uncertainty on the structural equations. While the assumption here is put for simplicity, it is still consistent with the situation in reality that policymakers like the central bank have a policy target independent of the true social welfare.[4]

The policymaker controls a short term nominal interest rate as a policy instrument. To make the analysis as simple as possible, we assume that the policymaker credibly commits to the simple Taylor-type rule such as

$$i_t = \psi_\pi \pi_t + \psi_x x_t. \tag{3.25}$$

Therefore, if the values of all parameters in the model are publicly known, the policymaker's problem is to determine the coefficients ψ_π and ψ_x to minimize the unconditional expectation of the loss (3.24) subject to the structural equations. When the policymaker has ambiguity about the true values of parameters in the model, it gives up conducting the optimal policy in the usual manner, and instead conducts the Bayesian or minimax policy introduced in the previous section.

The main objective of our simulations is to analyze how the two approaches and the two categories of parameter uncertainty affect the stance of robust monetary policy. To examine this, we want to see the changes of the policies which are derived by the two approaches as uncertainty about

[4]In contrast, Kimura and Kurozumi (2007) explicitly consider the effect of deep parameter uncertainty on the loss functional weights and show that the effect makes the Bayesain robust policy more responsive to the exogenous shock. However, their result also implies that even without this effect, the Bayesian policy requires aggressive response to the deep parameter uncertainty.

unknown parameter increases. The simple method is to assume the policymaker's prior distribution in some form, increase the variance of the distribution, and check the shifts of Taylor rule coefficients of each policy. However, as pointed out in Section 2, the minimax method does not reflect such changes of the distribution since the minimax policymaker concerns about only the worst case and does not care the likelihood of the value of the uncertain parameter.

To see the changes of both the Bayesian and the minimax policy under the same condition, we try to replicate the increase of uncertainty by spreading the support of the subjective distribution. In particular, in our simulations, the policymaker is assumed to have a uniform distributed prior belief about the uncertain parameter which can be expressed as

$$\theta \sim U(E[\theta] - \Delta, E[\theta] + \Delta), \tag{3.26}$$

where θ denotes the uncertain parameter and $E[\theta]$ is the mean of distribution. When the policymaker's belief is based on the above uniform distribution, expanding the interval Δ has the same meaning as the variance of the prior distribution increases. By considering symmetric expansion of the support with fixing the middle value of the interval, we can successfully analyze the effect of the mean preserving spread of the distribution on the robust optimal policies under the two approaches.

For the Bayesian policymaker, the widen canditate interval indicates that ambiguity about the exact parameter value increases. On the other hand, the minimax policymaker will worry about the risk that the damege from the mistaken policy becomes more critical. Such difference in a view about increasing uncertainty may or may not generate interesting divergence of the policy's response against the mean preserving spread exercise. Each robust policy is compared to the optimal policy when there is no uncertainty and the value of uncertain parameter is fixed to be the mean of the subjective distribution. All comparisons between the policies are conducted in the Taylor coefficients of each policy.

In our simulations, we prepare the three alternatives of New Keynesian model: a simple, an ad-hoc hybrid, and a micro-founded hybrid New Keynesian model. All of the three models are designed to include either (or both) of the two categories of parameter uncertainty introduced in Section 2.

The first model is a simple New Keynesian model, which does not involve any intrinsic inertia of the target variables. By introducing parameter uncertainty into this model, we can investigate the effect of the policy impact uncertainty independently. The second model is an ad-hoc hybrid New Keynesian model, which is constructed to include the persistence of the key variables and to see the dynamics uncertainty independently. Finally, for the third model, we turn to examine the micro-founded hybrid New Keynesian model. Since structural parameters in this model are tied to the move of a single deep parameter, uncertainty of one deep parameter spreads to the several parameters simultaneously. This feature allows us to deal with a very rich structure of uncertainty, which affects both the policy impact and the dynamics of target variables, even though we focus on every single deep parameter separately. Compared to the second ad-hoc model, the two categories of parameter uncertainty are mixed in the "natural" way that can withstand the Lucas critique thanks to the micro-foundations, so we can analyze which categories crucially determines the stance of robust monetary policy when the policymaker faces the policy impact and the dynamics uncertainty at the same time.

3.2 Simple New Keynesian Model

For the first simulation, we consider a simple New Keynesian model with the white noise shocks as

$$x_t = E_t x_{t+1} - \sigma(i_t - E_t \pi_{t+1}) + \varepsilon_{d,t},$$
$$\pi_t = \kappa x_t + \beta E_t \pi_{t+1} + \varepsilon_{s,t},$$

where the shocks $\varepsilon_{d,t}$ and $\varepsilon_{s,t}$ are i.i.d. processes with thier variance σ_d^2 and σ_s^2. In these equations, σ denotes the elasticity of substitution, β denotes the discount factor, and κ denotes the slope of the Phillips curve.

We introduce uncertainty into the parameter σ as the exercise in Section 2. The parameter σ affects the ability of the nominal interest rate to control the target variables. A smaller value of σ weakens the direct impact of the interest rate to the output gap. Since the model does not involve any inertial dyanamics of inflation and the output gap, we can see the effect of the policy impact uncertainty independently. If we consider explicit micro-foundations of the model, the uncertain σ also makes the parameter κ uncertain since κ is the decreasing function of σ. For ease of exposition, we postpone the discussion of such micro-founded uncertainty until the third model and consider the independent uncertainty on the paramter σ.

About the baseline parameters, we calibrate the model using the parameter values estimated by Levin et al. (2005). The calibration result is reported in Table 1. About the weight on the output gap and the interest rate deviations in the policymaker's loss function, we follow Rudebusch (2001) and Williams (2006) to set the $\lambda_x = 1$ and $\lambda_i = 0.1$.

Parameter	Description	Value
Structural		
β	discount factor	0.99
κ	slope of NKPC	0.13
Shock process		
σ_d^2	variance of demand shock	1.0
σ_s^2	variance of supply shock	1.0
Loss function		
λ_x	weight on output gap	1.0
λ_i	weight on nominal interest rate	0.1

Table 1: Calibrated parameter values

In the numerical example considered here, the policymaker is assumed to have the prior distribution over the interval $[1.0-\Delta, 1.0+\Delta]$, which implies the mean of the prior distribution is set to 0.1. Figure 1 shows the Taylor rule coefficients of the robust monetary policies under the two approaches as a function of uncertainty, Δ. The candidate interval of σ is spreading from $[1.0, 1.0]$ to $[0.8, 1.2]$. In the Figure 1, the left-hand graph depicts the Taylor coefficient on the inflation rate, and the right-hand graph draws it on the output gap. The dotted lines represent the optimal policy coefficient when σ is fixed at 1.0, while the solid and chain lines correspond to the Bayesian and the minimax policy, respectively. On the other hand, Figures 2 shows the relative losses with the two robust policies when uncertainty about σ is maximam (i.e., $\Delta = 0.2$), to the optimal policy for the model with $\sigma = 1.0$.

As is clear from Figure 1, when uncertainty of σ increases, both the Bayesian and the minimax policy become less and less aggressive. This figure is consistent with the "Brainard uncertainty principle", that is, the effect of monetary policy is uncertain, it is optimal for policymakers to change their instrument by less than would be optimal if all parameters were correctly known.

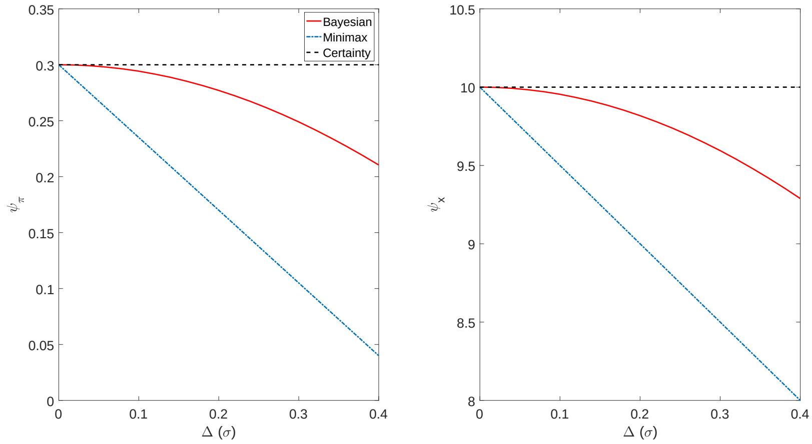

Figure 1: Robust policy rules as the fuction of uncertainty, Δ

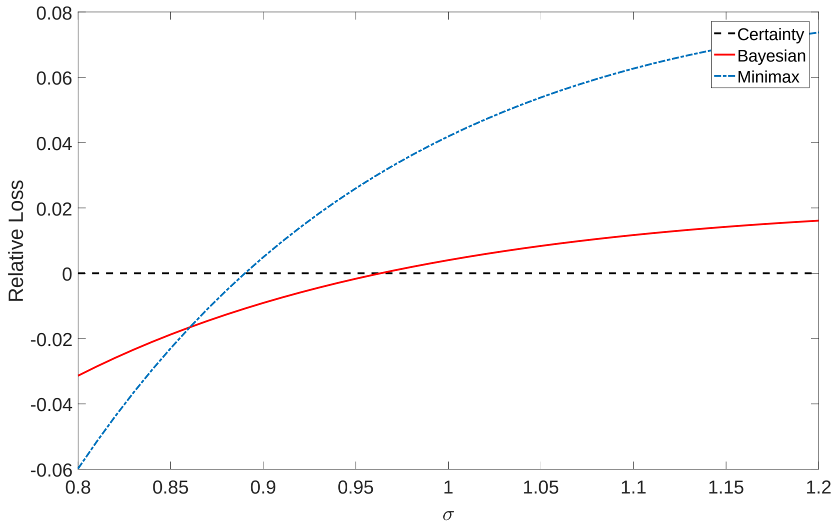

Figure 2: Relative (percentage) loss with robust policy rules

The intuition for the result is rather simple. As pointed out in Section 2, for the Bayesian policymaker, the more cautious policy is better for reducing the uncertainty effect, that is, the policy impact uncertainty makes the policymaker's subjective uncertainty about inflation and the output gap larger. As Figure 2 shows, the Bayesian policymaker can obtain the average result with any realizations of the paramter σ by conducting moderate policy. On the other hand, the target for the minimax policymaker is the case where the policy impact is extremely weak, i.e, the parameter σ takes

0.8. Figure 2 shows that the minimax policy gives the best performance in such situations at the risk of relatively large loss when σ takes larger values than 0.9.

3.3 Ad-hoc Hybrid New Keynesian Model

For the next simulation, we consider an ad-hoc hybrid New Keynesian model as

$$
\begin{aligned}
x_t &= \phi x_{t-1} + (1-\phi)E_t x_{t+1} - \sigma(i_t - E_t \pi_{t+1}) + \varepsilon_{d,t}, \\
\pi_t &= \theta \pi_{t-1} + (1-\theta)E_t \pi_{t+1} + \kappa x_t + \varepsilon_{s,t},
\end{aligned}
$$

where the setting about the shocks is the same as in the previous model. In this model, the parameters ϕ and θ determine the degree of the target variables depending on their own past values. Since the model has no micro-foundations and there are no correlations between the structural parameters, uncertainty can be introduced separately. To see the effect of uncertainty about the dynamics of the target variables independently, we consider uncertainty only about the two parameters.

The parameter setting is almost the same as in the previous model. The baseline calibration is summrized in Table 1. About the uncertain paramters ϕ and θ, we assume that the mean of the prior distribution for each parameter is 0.5, and the maximal Δ is 0.2.

Figure 3 shows the changes of the Bayesian and the minimax policy while the candidate intervals of ϕ are spreading from $[0.5, 0.5]$ to $[0.3, 0.7]$. In contrast to the case where the policy impact parameter is uncertain, increasing uncertainty about the dynamics of the output gap makes both of the robust policies be more and more aggressive. By taking relatively aggressive policy, the Bayesian policymaker tries to dampen the current fluctuations of the output gap and inflation rate to decrease uncertainty in the future. On the other hand, as Figure 4 shows, the minimax policymaker always prepares for the non-inertial economy (i.e., the lower bound of ϕ).

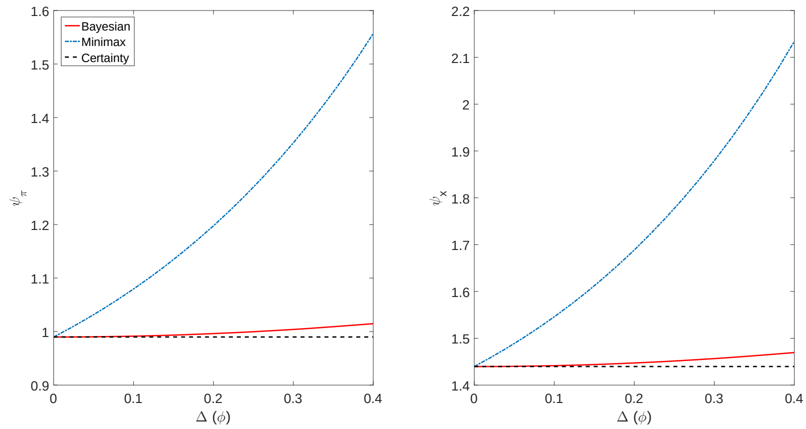

Figure 3: Robust policy rules as the fuction of uncertainty, Δ

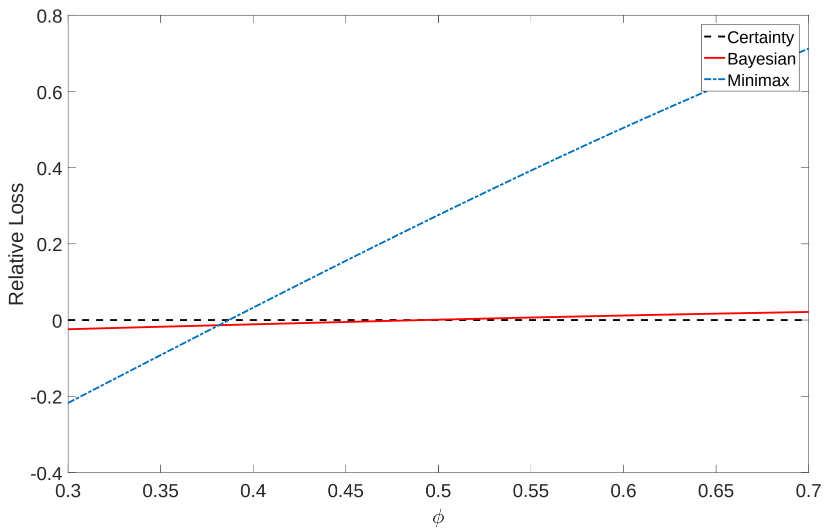

Figure 4: Relative (percentage) loss with robust policy rules

Figure 5 shows both of the robust policy rules while the candidate intervals of θ are spreading from $[0.5, 0.5]$ to $[0.3, 0.7]$. As the case where ϕ is uncertain, increasing uncertainty about the dynamics of inflation calls for more and more aggressive policy. However, the intuition for the result is entirely different, especially about the minimax policy. Figure 6 shows that the minimax policymaker's target is the maximum θ, which represents the economy with extremely inertial inflation. The strategy of the Bayesian and the minimax policymaker is to take aggressive policy and dampen the fluctuation as soon as possible to avoid the damage from the prolonged effect of the demand and supply disturbances.

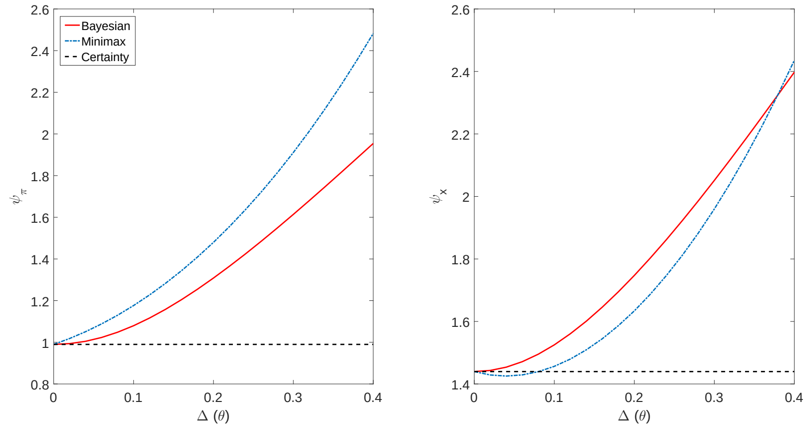

Figure 5: Robust policy rules as the fuction of uncertainty, Δ

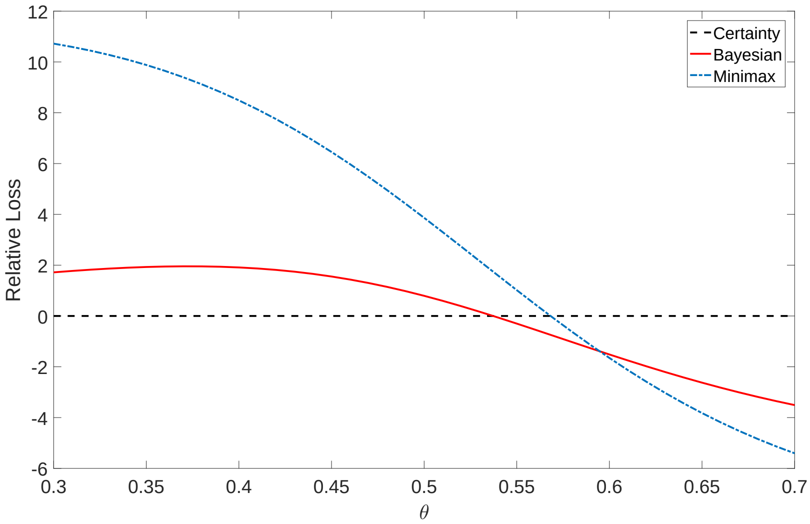

Figure 6: Relative (percentage) loss with robust policy rules

3.4 Micro-founded Hybrid New Keynesian Model

As the final simulation, we consider uncertainty about a deep parameter of the micro-founded hybrid New Keynesian model. The model is a simplified version of Smets and Wouters (2003), Smets and Wouters (2007) or Christiano et al. (2005), which involves the household's habit formation and firm's rule of thumb pricing. The structural equations of the model are given as

$$x_t = \frac{h}{1+h}x_{t-1} + \frac{1}{1+h}E_t x_{t+1} - \frac{1-h}{(1+h)\sigma_c}(i_t - E_t\pi_{t+1}) + \varepsilon_{d,t}, \tag{3.27}$$

$$\begin{aligned}\pi_t = &\frac{\omega}{\phi+\omega[1-\phi(1-\beta)]}\pi_{t-1} + \frac{\beta\phi}{\phi+\omega[1-\phi(1-\beta)]}E_t\pi_{t+1} \\ &+ \frac{(1-\omega)(1-\beta\phi)(1-\phi)}{\phi+\omega[1-\phi(1-\beta)]}\left(\sigma_l + \frac{\sigma_c}{1-h}\right)x_t \\ &- \frac{(1-\omega)(1-\beta\phi)(1-\phi)}{\phi+\omega[1-\phi(1-\beta)]}\frac{\sigma_c h}{1-h}x_{t-1} + \varepsilon_{s,t},\end{aligned} \tag{3.28}$$

where h denotes the degree of the representative household's habit persistence, ω dentoes the ratio of backward looking firms, and σ_c is the intertemporal elasticity of substitution.[5]

We introduce uncertainty into these three deep parameters, h, ω and σ_c. About the policy impact, the larger values of the deep parameters h and σ_c make the IS curve be flatter, i.e., weaken the effect of the nominal interest on the output gap. In addition, the slope of the Phillips curve, which is related to the trade-off between inflation and the output gap, is affected by ω. On the other hand, the dynamics of the ouput gap and the inflation rate are affected by h and ω, respectively. As the deep parameters act on the several structural parameters complexly, we can not clearly distinguish the role of the parameters into the two categories as with the first and second model. All results in the simulation with this micro- founded model are the products of the mixed effect of uncertainty about the policy impact and the dynamics of the target variables. This feature allows us to analyze which categories can be the crucial factor to determine the stance of robust monetary policy when the policymaker faces the two types of uncertainty at the same time.

Table 2 summerizes the calibrated parameter settings which is fixed and known to both the policymaker and the private agents. Table 3 shows the setup about the uncertain deep parameters h, ω, and σ.

[5]The derivations of the all structural equations of the model are summerized in the Appendix A.

Parameter	Description	Value
Deep parameter		
β	discount factor	0.99
σ_l	inverse of elasticity of labor supply	1.5
ϕ	probability of no price revision	0.8
Shock process		
σ_d^2	variance of demand shock	1
σ_s^2	variance of supply shock	1
Loss function		
λ_x	weight on output gap	1
λ_i	weight on nominal interest rate	0.1

Table 2: Calibrated parameter values

Parameter	Description	Mean and Δ
h	degree of habit persistence	$E[h] = 0.5, \Delta = 0.3$
ω	ratio of backward looking firms	$E[\omega] = 0.5, \Delta = 0.3$
σ	inverse of elasticity of substitution	$E[\sigma] = 1.0, \Delta = 0.3$

Table 3: Uncertain deep parameters

Figures below report the change of robust optimal policies while the candidate interval of each unknown parameter are expanding, i.e., the variance of each parameter are increasing. As the previous part, the horizontal dotted lines denote the optimal Taylor rule's coefficient for the model when the values of the uncertain parameter is equal to the mean of the prior distributuion. The solid and chain lines correspond to the Bayesian and the minimax policy coefficients, respectively.

Figure 7 indicates that both of the robust policy rules become less and less aggressive when uncertainty of h increases. In the micro-founded model, the deep parameter h affects the policy effectiveness by moving the slope of IS curve and the Phillips curve. At the same time, it affects the dynamics of the output gap by changing the coefficient on the past and future output gaps. The results so far imply that for the Bayeisan and minimax policymakers, it is better to select moderate policy when the policy impact is uncertain, and aggressive policy when the dynamics of the key variables is uncertain. Then the result in Figure 7 implies that at least for h, the effect of policy impact uncertainty dominates that of the uncertainty about the dynamics of the output gap.

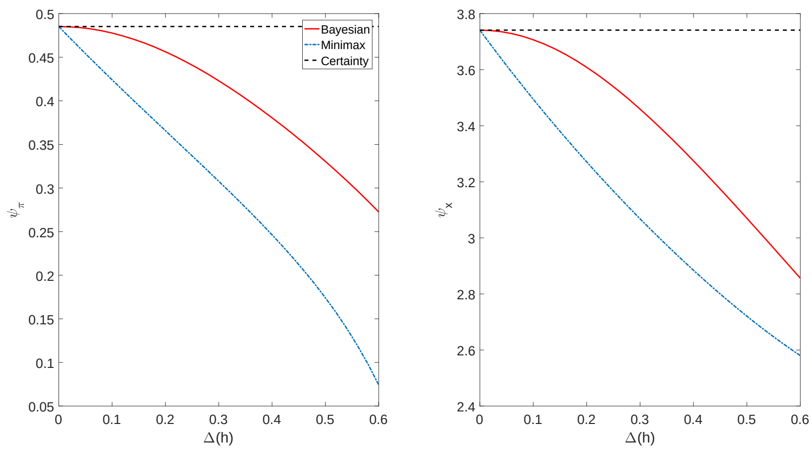

Figure 7: Robust policy rules as the fuction of uncertainty, Δ

About the minimax policymaker's target, Figure 8 shows the interesting result. When we consider the ad-hoc hybrid model, the policymaker's concern is the possibility of non-inertial, i.e., highly forward-looling, IS curve. In contrast, the minimax policy here shows the best performance for the upper bound of h, which implies the policymaker's target is opposed to the case with the ad-hoc model. The point is that under the micro-founded model, the higher h also implies the flatter IS curve, which indicates the weaker impact of monetary policy. The minimax policymaker here places emphasis on coping with the environment where the power of monetary policy is weak, at the risk of being hurt by the realization of non-inertial IS curve.

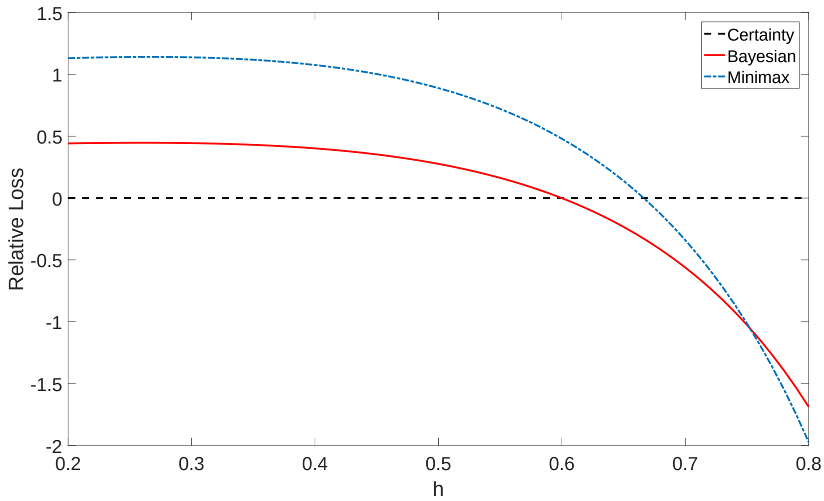

Figure 8: Relative (percentage) loss with robust policy rules

In Figure 9, as ω's uncertainty increases, both of the robust policy rules initially show stronger and stronger reponse against the inflation rate. However, when the uncertainty of ω becomes significantly large, the minimax policy changes its direction to make the response weaker. On the other hand, the Taylor coefficient on the output gap becomes smaller and smaller under the same condition.

When ω becomes uncertain, the dynamics of inflation and the trade off between inflation and the output gap become uncertain at the same time. Against the demand or supply disturbance, the policymaker tries to indirectly control the output gap to influence the inflation rate. In the New Keynesian model, the impact of this output control is represented by the slope of the Phillips curve. In this sense, the slope of the Phillips curve can be viewed as the policy impact parameter and the parameter ω's uncertainty can make the policy impact uncertain.

As pointed above, the reponses of the robust policies vary with the key variables, so we have to devide the discussion by the type of the shock. If the policymaker wants to put down the demand shock, the effect of uncertain slope of the Phillips curve is more important than that of the inflation dynamics. It is the reason why the Bayesian and minimax policy show the weaker response against the move of the output gap. In contrast, to control the damage of the supply shock, the policymaker has to concern about the dynamics of inflation rate because it directly reflects the behavior of the disturbance. Then it is natural for the Bayesian policymaker to select the aggressive policy and try to kill the uncertain effect on the inflation dynamics quickly. In contrast, when the possibility of the parameter ω's realization becomes significantly broad, the minimax policymaker starts to consider the possibility of the extremely flattening Phillips curve, which implies that monetary policy through the control of the output gap has totally no power. Then it partially gives up dealing with the supply disturbance, makes the policy reponse a bit weaker.

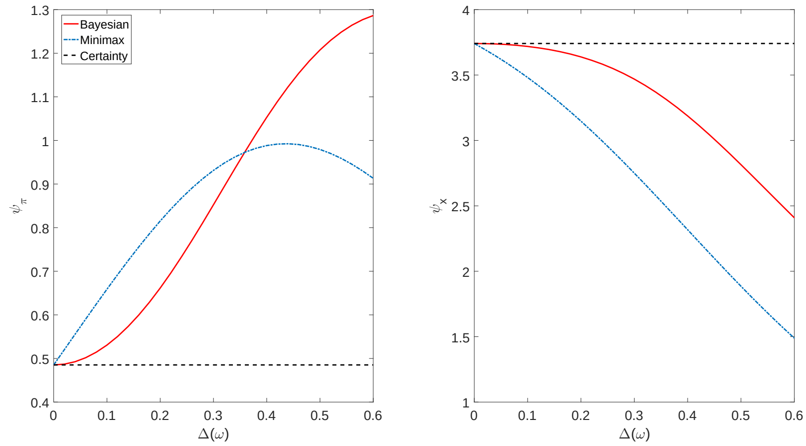

Figure 9: Robust policy rules as the fuction of uncertainty, Δ

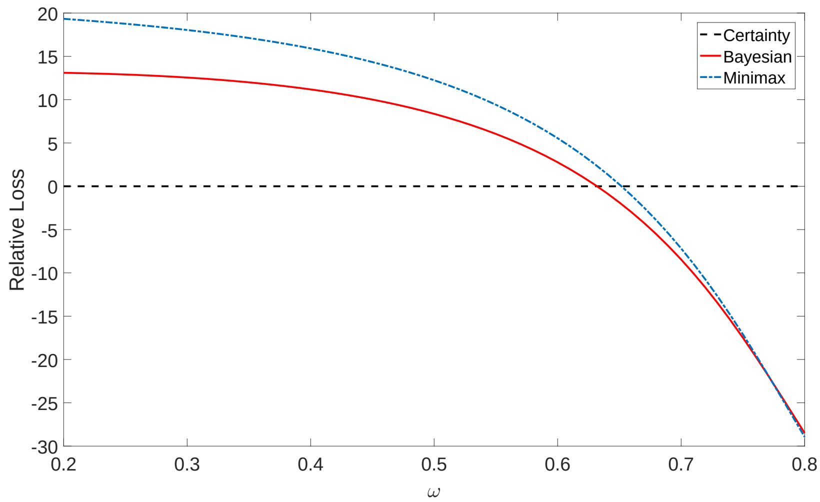

Figure 10: Relative (percentage) loss with robust policy rules

The result of the parameter σ_c's uncertainty supports the intuition which we have accumulated so far. With our model, the uncertainty of σ_c is related to only the policy impact, so both the Bayesian and minimax policymaker choose the more cautious policy in Figure 11. As we check with the first model, the minimax policy's target is the highest realized value of σ_c, which represents the weakest policy impact situation.

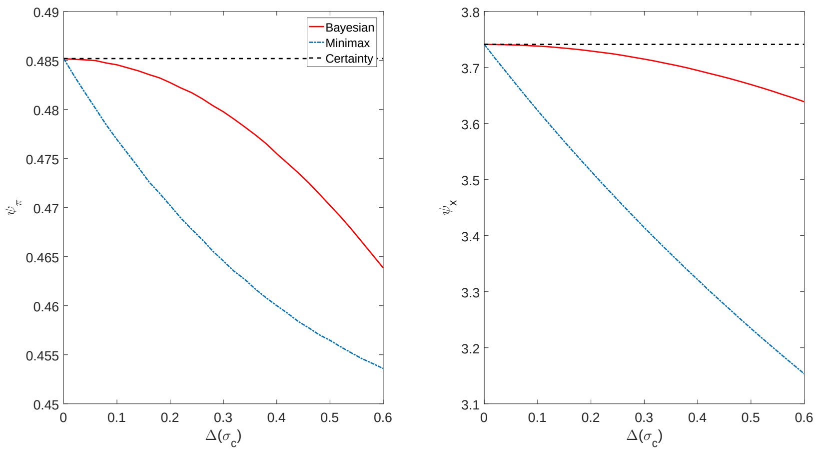

Figure 11: Robust policy rules as the fuction of uncertainty, Δ

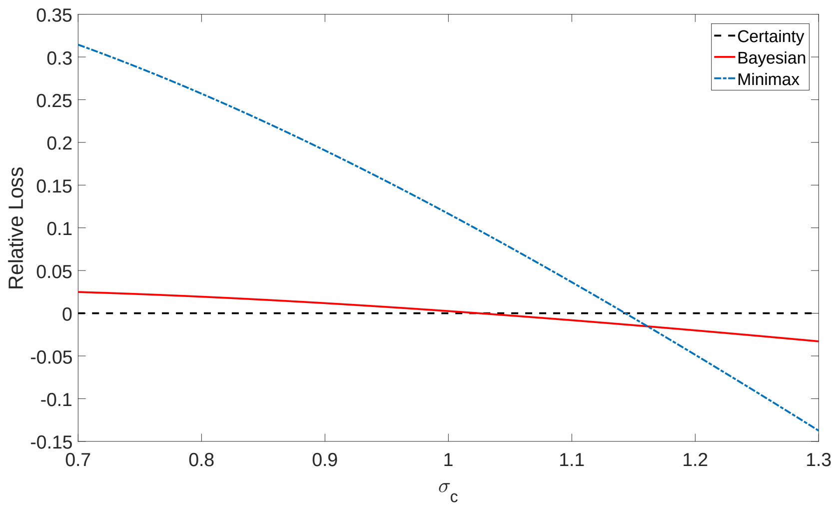

Figure 12: Relative (percentage) loss with robust policy rules

3.5 Discussions

Our simulations with the three New Keynesian model shows the interesting implications about the factor of determining the stance of robust monetary policy. When we examine the Bayesian and the minimax policy with the same category of parameter uncertainty, their responses always coincide, even under the complex micro-founded model. It implies that the policy stance is generally determined by the point about which the policymaker faces uncertainty in the economy, and irrelevant to whether its object is to obtain the average result or to prepare the disastrous case. When policymakers faces uncertainty about the effect of monetary policy, the more cautious policy should be conducted, as the "Brainard uncertainty principle" said. On the other hand, when the dynamics of the key variables such as the inflation rate or the output gap is uncertain, policymakers should select the more aggressive policy compared to the case without uncertainty.

How about the situation where the two categories of uncertainty are concerned at the same time? With the simulation about uncertainty of the deep paramters h and ω under the micro-founded model, we can see the naturally mixed effect of the policy impact and the dynamics uncertainty. Compared to the results with the single uncertainty which are depicted by Figure 2, 4, 6, and 12, we notice that the Bayesian and the minimax policy show the similar performance with this mixed uncertainty, see Figure 8 and 10. The Bayesian policy works well even with the extreme value of the uncertain parameter which is targeted by the minimax policymaker. The minimax policy shows the average performance even when the target parameter value does not realize. It is somewhat surprising result, because we tend to consider the policymaker can deal with uncertainty from the single factor more easily rather than from the complexly mixed factors. Our simulation with the third model implies that when the several factors of uncertainty is mixed in the micro-founded fashion, policymakers can conduct the policy which succeeds to generate the average result with the broader range of the economy, with doing a certain level of damage control.

4 Concluding Remarks

To find the true factor which determines the stance of robust monetary policy under parameter uncertainty, this paper has focused on the two aspects, the two approaches for finding robsut policy and the two categories of paramter uncertainty. We have conducted the numerical simulations with the three types of New Keynesian models, which are designed to include the policy impact and the dynamics uncertainty. When there is only the former type of uncertainty, both the Bayesian and the minimax policy show less aggressive responce, consistent with “Brainard uncertainty principle”. In contrast, against the latter type of uncertainty, the robust policymakers persue more aggressive policy. The result implies that the policy stance is generally determined by the point about which the policymaker faces uncertainty in the economy, and irrelevant to whether the policymaker takes the Bayesian or the minimax approach.

When the two categories of uncertainty are mixed, the relative importance between the two sources will be the key determinant of the policy's stance. In the micro-founded model, it is naturally controlled by the relative influence of the uncertain deep parameter on each structural parameter in the model. In somewhat surprisingly, the Bayesian and the minimax policy show the similar performance with the micro-founded mixed uncertainty, compared to the case with the single category of uncertainty. The Bayesian policy works well even with the extreme value of the uncertain parameter which is targeted by the minimax policymaker. The minimax policy shows the average performance even when the target parameter value does not realize. The result brings a good news for the real policymakers that concern about the complexly mixtured uncertainty, because it implies that if the mixed uncertainty is based on the micro-foundations, they can conduct the policy which succeeds to generate the average result with the broader range of the economy, with doing a certain level of damage control.

References

Amato, J. and T. Laubach (2003) “Rule-Of-Thumb Behavior and Monetary Policy,” *European Economic Review* 47, 791–831.

Blinder, A. (1998) *Central banking in theory and practice*: Mit press.

——— (2000) “Critical issues for modern major central bankers,” *European Central Bank Conference on "Monetary policy-making under uncertainty"*, 64–74.

Blinder, A. S. (1997) “What Central Bankers Could Learn from Academics–and Vice Versa,” *Journal of Economic Perspectives* 11, 3–19.

Brainard, W. C. (1967) “Uncertainty and the Effectiveness of Policy.”

Christiano, L. J., M. Eichenbaum, and C. L. Evans (2005) “Nominal Rigidities and the Dynamic Effects of a Shock to Monetary Policy,” *Journal of Political Economy* 113, 1–45.

Galí, J. and M. Gertler (1999) “Inflation dynamics: A structural econometric analysis,” *Journal of Monetary Economics* 44, 195–222.

Giannoni, M. P. (2002) “Does Model Uncertainty Justify Caution? Robust Optimal Monetary Policy in a Forward-Looking Model,” *Macroeconomic Dynamics* 6, 111–144.

——— (2007) “Robust Optimal Monetary Policy in a Forward-looking Model with Parameter and Shock Uncertainty,” *Journal of Applied Econometrics* 22, 179–213.

Goodhart, C. A. E. (1999) “Central Bankers and Uncertainty,” *Bank of England Quarterly Bulletin* 39, 102–114.

Hansen, L. P. and T. J. Sargent (2002) “Robust control of forward-looking models,” *Journal of Monetary Economics* 50, 581–604.

——— (2012) “Three types of ambiguity,” *Journal of Monetary Economics* 59, 422–445.

Kimura, T. and T. Kurozumi (2007) “Optimal monetary policy in a microfounded model with parameter uncertainty,” *Journal of Economic Dynamics and Control* 31, 399–431.

Levin, A. T., A. Onatski, J. Williams, and N. M. Williams (2005) “Monetary Policy Under Uncertainty in Micro-Founded Macroeconometric Models,” *NBER Macroeconomics Annual* 20, 229–312.

Onatski, A. and N. Williams (2003) “Modeling Model Uncertainty,” *Journal of the European Economic Association* 1, 1087–1122.

Rudebusch, G. D. (2001) “Is the Fed Too Timid? Monetary Policy in an Uncertain World,” *Review of Economics and Statistics* 83, 203–217.

Smets, F. and R. Wouters (2003) “An Estimated Dynamic Stochastic General Equilibrium Model of the Euro Area,” *Journal of the European Economic Association* 1, 1123–1175.

——— (2007) “Shocks and frictions in US business cycles: A Bayesian DSGE approach,” *American Economic Review* 97, 586–606.

Söderström, U. (2002) “Monetary Policy With Uncertain Parameters,” *Scandinavian Journal of Economics* 104, 125–145.

Williams, J. C. (2006) “Robust Estimation and Monetary Policy with Unobserved Structural Change,” *Economic Review*, 1–16.

Woodford, M. (1999) “Optimal Monetary Policy Inertia,” *NBER Working Paper* No. 7261.

Appendix

A Micro-founded Hybrid New Keynesian Model

About this section, we mainly refer to Galí and Gertler (1999), Amato and Laubach (2003) and Smets and Wouters (2007).

A.1 Households

The representative household derives utility from consumption C_t and disutility from hours worked N_t. In particular, the household seeks to maximize the following utility function;

$$\max_{C_t, N_t, B_{t+1}, M_t} \quad E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \frac{(C_t - H_t)^{1-\sigma_c}}{1-\sigma_c} - \frac{N_t^{1+\sigma_l}}{1+\sigma_l} \right\},$$

where β is the discount factor, σ_c is the inverse of the intertemporal elsticity of substitution, and σ_l is the inverse of Frish elasticity.

The only thing we depart from the standard purely forward looking problem is inclusion of habit formation setting. In this utility function, H_t denotes a external habit stock, which is exogenously given to the household. We additionally asuume that this external habit stock is propotonal to the aggregate past consumption as $H_t = hC_{t-1}$. When the habit persistence parameter h is high, the household gives much weight to smoothing between current and past consumption.

The household has access to a domestic bond market where nominal government bonds B_t are traded that pay (net) interest i_t. Furthermore, it receives nominal wage W_t and aggregate residual nominal profits Π_t from the firms. Thus, the household's budget constraint is of the form

$$P_t C_t + B_t \leq W_t N_t + \Pi_t + (1 + i_{t-1}) B_{t-1}. \tag{A-29}$$

Given the budget constraint (A-29), a lagrangian function associated with the dynamic optimization problem for the representative household can be formulated as follows.

$$\mathcal{L} = E_0 \sum_{t=0}^{\infty} \beta^t \Big[\frac{(C_t - H_t)^{1-\sigma_c}}{1-\sigma_c} - \frac{N_t^{1+\sigma_l}}{1+\sigma_l} + \lambda_t \{ W_t N_t + \Pi_t + (1 + i_{t-1}) B_{t-1} - P_t C_t - B_t \} \Big].$$

where λ is a Lagrange multiplier. The first order necessary conditions are given by

$$\lambda_t = (C_t - H_t)^{-\sigma_c}, \tag{A-30}$$

$$\lambda_t = \beta E_t \left[(1+i_t) \frac{P_t}{P_{t+1}} \lambda_{t+1} \right], \tag{A-31}$$

$$N_t^{\sigma_l} = \lambda_t \frac{W_t}{P_t}, \tag{A-32}$$

The household's behavior is restricted by these conditions and budget constraint (A-29) in each period t.

A.2 Firms

Final Good Producer

A representative firm aggregates intermediate goods $Y_t(j)$ and produce final goods Y_t according to the production function

$$Y_t = \left(\int_0^1 Y_t(j)^{\frac{\epsilon-1}{\epsilon}} dj \right)^{\frac{\epsilon}{\epsilon-1}}, \tag{A-33}$$

where ϵ is the elasticity of substitution between intermediate goods.

Final good firm solves profit maximization problem to choose a combination of intermediate goods as

$$\max_{Y_t(j)} P_t \left(\int_0^1 Y_t(j)^{\frac{\epsilon-1}{\epsilon}} dj \right)^{\frac{\epsilon}{\epsilon-1}} - \int_0^1 P_t(j) Y_t(j) dj.$$

Under this profit maximizing problem, the relative demand function for each intermediate good is given by

$$Y_t(j) = \left(\frac{P_t(j)}{P_t} \right)^{-\epsilon} Y_t. \tag{A-34}$$

With the production function (A-33) and the demand function (A-34), the aggregate price index can be represented as

$$P_t = \left(\int_0^1 P_t(j)^{1-\epsilon} dj \right)^{\frac{1}{1-\epsilon}}. \tag{A-35}$$

Intermediate Good Producers

There exists a continuum of intermediate firms indexed by $j \in [0,1]$. Each intermediate firm hires labor from the household and produces the intermediate good according to a constant returns to scale production as

$$Y_t(j) = A_t N_t(j),$$

where A_t is a productivity factor which is common between all intermediate firms.

Whether firms can adjust their prices freely or not, once the price is given, they solve the cost minimization problem subject to the demand function (A-34). Then their cost minimization problem can be written as

$$\min_{N_t(j)} \; W_t N_t(j) \quad \text{s.t.} \quad A_t N_t(j) \geq \left(\frac{P_t(j)}{P_t}\right)^{-\epsilon} Y_t.$$

Under these settings, the real marginal cost of the intermediate firms is

$$mc_t = \frac{W_t}{P_t}\frac{1}{A_t}, \tag{A-36}$$

and the real profit for the firm j is given by

$$\frac{\Pi_t(j)}{P_t} = \frac{P_t(j)}{P_t}\left(\frac{P_t(j)}{P_t}\right)^{-\epsilon} Y_t - mc_t\left(\frac{P_t(j)}{P_t}\right)^{-\epsilon} Y_t.$$

If the firms can adjust their price flexibly, all firms set the price as a constant markup over current marginal cost as

$$\frac{P_t(j)}{P_t} = \frac{\epsilon}{\epsilon-1} mc_t.$$

About the price setting of the intermediate firms, we follow Amato and Laubach (2003) and Galí and Gertler (1999) in adapting modified Calvo price setting. In each period, only a fixed fraction $1-\phi$ of the intermediate firms can re-optimize their prices, while remaining ϕ must keep them. The firms who are offered the price changing opportunity are drawn independent of the time and their pricing history. Of these firms, a share $(1-\omega)$ is a 'forward looking' type, who sets prices optimally in standard Calvo fashion. The remaining 'backward looking' type, of measure ω, follows a rule-of-thumb which is based on the aggregate price index in previous period.

The firms who get the chance to update and actually update their prices try to maximize expected future profit as

$$\max_{P_t(j)} \; E_t \sum_{s=0}^{\infty} (\beta\phi)^s \frac{\lambda_{t+s}}{\lambda_t}\left[\frac{P_t(j)}{P_{t+s}}\left(\frac{P_t(j)}{P_{t+s}}\right)^{-\epsilon} Y_t - mc_t\left(\frac{P_t(j)}{P_{t+s}}\right)^{-\epsilon} Y_t\right].$$

The solution to this maximization problem is given by

$$P_t^f = \frac{\epsilon}{\epsilon-1}\frac{E_t \sum_{s=0}^{\infty} (\beta\phi)^s \lambda_{t+s} mc_{t+s} P_{t+s}^{\epsilon} Y_{t+s}}{E_t \sum_{s=0}^{\infty} (\beta\phi)^s \lambda_{t+s} mc_{t+s} P_{t+s}^{\epsilon-1} Y_{t+s}}. \tag{A-37}$$

On the other hand, rule-of-thumb price setters adjust their prices to

$$P_t^b = P_{t-1}^* \frac{P_{t-1}}{P_{t-2}}, \tag{A-38}$$

where P_{t-1}^* is the aggregate index of the price chosen by fraction ϕ of firms, who receives price-change signal in period $t-1$. This newly chosen price index is expressed as

$$P_t^* = (1-\omega)P_t^f + \omega P_t^b. \tag{A-39}$$

Finally, under the index equation (A-35), aggregate price dynamics evolves as

$$P_t = \left\{(1-\phi)(P_t^*)^{1-\epsilon} + \phi P_{t-1}^{1-\epsilon}\right\}^{\frac{1}{1-\epsilon}}. \tag{A-40}$$

A.3 Market Clearing Condition

We impose the market clearing condition for the final good market. It requires the supply of final goods to be equal to the demand of it, which in this simple model, is only for the household's consumption. Thus, the market clearing condition can be written as follows.

$$Y_t = C_t. \tag{A-41}$$

A.4 Log-linearization

Marginal utility of consumption

$$\lambda_t = (C_t - hC_{t-1})^{-\sigma_c}$$

log linearization:

$$\tilde{\lambda}_t = -\sigma_c \left(\frac{1}{1-h}\tilde{c}_t - \frac{h}{1-h}\tilde{c}_{t-1} \right) \tag{A-1}$$

Bond demand

$$\lambda_t = \beta E_t (1+i_t) \frac{P_t}{P_{t+1}} \lambda_{t+1}$$

log linearization:

$$\tilde{\lambda}_t = \tilde{i}_t + E_t(\tilde{\lambda}_{t+1} - \tilde{\pi}_{t+1}) \tag{A-2}$$

Labor supply

$$N_t^{\sigma_l} = \lambda_t \frac{W_t}{P_t}$$

log linearization:

$$\sigma_l \tilde{N}_t = \tilde{\lambda}_t + \tilde{w}_t \tag{A-3}$$

A.5 Firms

Final good production

$$Y_t = \left(\int_0^1 Y_t(j)^{\frac{\epsilon-1}{\epsilon}} dj \right)^{\frac{\epsilon}{\epsilon-1}}$$

log linearization:

$$\tilde{y}_t = \int_0^1 \tilde{y}_t(j) dj \tag{A-4}$$

Intermediate good production

$$Y_t(j) = A_t N_t(j),$$

log linearization:

$$\tilde{y}_t(j) = \tilde{A}_t + \tilde{N}_t(j) \tag{A-5}$$

Marginal cost of intermediate production

$$mc_t = \frac{W_t}{P_t} \frac{1}{A_t}$$

log linearization:

$$\widetilde{mc}_t = \tilde{w}_t - \tilde{A}_t \tag{A-6}$$

Forward-looking firm's price setting

$$P_t^f = \frac{\epsilon}{\epsilon - 1} \frac{E_t \sum_{s=0}^{\infty} (\beta\phi)^s \lambda_{t+s} mc_{t+s} P_{t+s}^{\epsilon} Y_{t+s}}{E_t \sum_{s=0}^{\infty} (\beta\phi)^s \lambda_{t+s} mc_{t+s} P_{t+s}^{\epsilon-1} Y_{t+s}}.$$

log linearization:

$$\tilde{p}_t^f = (1 - \beta\phi) E_t \sum_{s-0}^{\infty} (\beta\phi)^s (\tilde{m}c_{t+s} + \tilde{p}_{t+s})$$

quasi differencing:

$$\tilde{p}_t^f = (1 - \beta\phi)(\tilde{m}c_t + \tilde{p}_t) + \beta\phi E_t \tilde{p}_{t+1}^f \tag{A-7}$$

Backward-looking (rule-of-thumb) firm's price setting

$$P_t^b = P_{t-1}^* \frac{P_{t-1}}{P_{t-2}},$$

log linearization:

$$\tilde{p}_t^b = \tilde{p}_{t-1}^* + \tilde{\pi}_{t-1} \tag{A-8}$$

Newly chosen price index

$$P_t^* = (1-\omega)P_t^f + \omega P_t^b.$$

log linearization:

$$\tilde{p}_t^* = (1-\omega)\tilde{p}_t^f + \omega \tilde{p}_t^b \tag{A-9}$$

Aggregate price dynamics

$$P_t = \left\{(1-\phi)(P_t^*)^{1-\epsilon} + \phi P_{t-1}^{1-\epsilon}\right\}^{\frac{1}{1-\epsilon}}.$$

log linearization:

$$\frac{\phi}{1-\phi}\tilde{\pi}_t = \tilde{p}_t^* - \tilde{p}_t \tag{A-10}$$

Market clearing condition

$$Y_t = C_t.$$

log linearization:

$$\tilde{y}_t = \tilde{c}_t \tag{A-11}$$

A.6 Derivation of linear structural equations

A.6.1 Intertemporal IS Curve

Inserting (A-1) into (A-2),

$$\tilde{c}_t = \frac{h}{1+h}\tilde{c}_{t-1} + \frac{1}{1+h}E_t\tilde{c}_{t+1} - \frac{1-h}{(1+h)\sigma_c}(i_t - E_t\pi_{t+1}).$$

With the market clearing condition,

$$\tilde{y}_t = \frac{h}{1+h}\tilde{y}_{t-1} + \frac{1}{1+h}E_t\tilde{y}_{t+1} - \frac{1-h}{(1+h)\sigma_c}(i_t - E_t\pi_{t+1}).$$

By subtracting the log-deviation of natural output $\tilde{y}_t^f$ from both sides, we obtain

$$x_t = \frac{h}{1+h}x_{t-1} + \frac{1}{1+h}E_t x_{t+1} - \frac{1-h}{(1+h)\sigma_c}(i_t - E_t\pi_{t+1}). \tag{A-12}$$

where $x_t = \tilde{y}_t - \tilde{y}_t^f$.

Hybrid New Keynesian Phillips Curve

Combining (A-10) and (A-9), we get

$$\frac{\phi}{1-\phi}\tilde{\pi}_t = (1-\omega)(\tilde{p}_t^f - \tilde{p}_t) + \omega(\tilde{p}_t^b - \tilde{p}_t). \tag{A-13}$$

Plugging (A-9) into (**??**),

$$\tilde{p}_t^b - \tilde{p}_t = -\tilde{\pi}_t + \frac{1}{1-\phi}\tilde{\pi}_{t-1} \tag{A-14}$$

From (A-7),

$$\tilde{p}_t^f - \tilde{p}_t = (1-\beta\phi)\tilde{mc}_t - \beta\phi\tilde{p}_t + \beta\phi E_t\tilde{p}_{t+1}^f.$$

By leading the equation (A-13) one period and inserting (A-14),

$$E_t\tilde{p}_{t+1}^f = \frac{\phi+\omega(1-\phi)}{(1-\phi)(1-\omega)}E_t\tilde{\pi}_{t+1} - \frac{\omega}{(1-\phi)(1-\omega)}\tilde{\pi}_t + E_t\tilde{p}_{t+1} \tag{A-15}$$

Substituting for (A-15) into (A.6.1),

$$\tilde{p}_t^f - \tilde{p}_t = \left[1 + \frac{\phi+\omega(1-\phi)}{(1-\phi)(1-\omega)}\right]\beta\phi E_t\tilde{\pi}_{t+1} - \frac{\beta\phi\omega}{(1-\phi)(1-\omega)}\tilde{\pi}_t + (1-\beta\phi)\tilde{mc}_t$$

Inserting (A-14) and (A-15) into (A-13),

$$\tilde{\pi}_t = \frac{\omega}{\phi+\omega[1-\phi(1-\beta)]}\tilde{\pi}_{t-1} + \frac{\beta\phi}{\phi+\omega[1-\phi(1-\beta)]}E_t\tilde{\pi}_{t+1} + \frac{(1-\omega)(1-\beta\phi)(1-\phi)}{\phi+\omega[1-\phi(1-\beta)]}\tilde{mc}_t \tag{A-16}$$

At the flexible price equilibrium, the intermediate firm will set the price as

$$\frac{p_t(j)}{P_t} = \frac{\epsilon}{\epsilon-1}mc_t,$$

and since all firms set the same price,

$$1 = \frac{\epsilon}{\epsilon-1}mc_t$$

must hold. Combining this and the FOC of household, we get

$$\frac{(N_t^f)^{\sigma_l}}{\lambda_t} = \frac{\epsilon-1}{\epsilon}A_t. \tag{A-17}$$

By combining log-linearization of (A-17)

$$\sigma_l\tilde{N}_t^f - \tilde{\lambda}_t = \tilde{A}_t,$$

(A-1) and the market clearing condition, we obtain

$$\sigma_l \tilde{N}_t^f + \sigma_c \left(\frac{1}{1-h} \tilde{y}_t^f - \frac{h}{1-h} \tilde{y}_{t-1}^f \right) = \tilde{A}_t.$$

At the flexible price equilibrium, the intermediate firm's production function can be expressed as follows.

$$\tilde{y}_t^f = \tilde{A}_t + \tilde{N}_t^f$$

Then (A.6.1) can be transformed as

$$\left(\sigma_l + \frac{\sigma_c}{1-h} \right) \tilde{y}_t^f - \sigma_c \frac{h}{1-h} \tilde{y}_{t-1}^f = (1+\sigma_l) \tilde{A}_t. \tag{A-18}$$

On the other hand, marginal cost of the firm under the ordinary (i.e., not flexible price) equilibrium can be written as

$$\tilde{mc}_t = \sigma_l \tilde{N}_t - \tilde{\lambda}_t - \tilde{A}_t$$

and again by using the intermediate ans final good firm's production function, we obtain

$$\tilde{y}_t = \tilde{A}_t + \tilde{N}_t.$$

By combining these equations and (A-1), we get

$$\tilde{mc}_t = \left(\sigma_l + \frac{\sigma_c}{1-h} \right) \tilde{y}_t - \sigma_c \frac{h}{1-h} \tilde{y}_{t-1} - (1+\sigma_l) \tilde{A}_t. \tag{A-19}$$

By inserting (A-18) into (A-19),

$$\tilde{mc}_t = \left(\sigma_l + \frac{\sigma_c}{1-h} \right) x_t - \sigma_c \frac{h}{1-h} x_{t-1}. \tag{A-20}$$

Finally, by combining (A-16) and (A-20), hybrid New Keynesian Phillips Curve can be derived.