Information Contamination, Market Crashes, and Overshooting*

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This paper provides the model of endogenous stock markets crashes with overshooting. Unlike typical macroeconomic and finance models, an economy can experience a boom and bust without any aggregate exogenous shocks nor successive idiosyncratic shocks. Under the boom, the economy spontaneously moves toward a critical phase, exceeds the critical point and a crash with overshooting will take place. What matters for this story is a herding behavior of irrational agents and contamination of information of a price signal. Irrational agents take Ss-type behavior under ambiguous situations and this makes a portion of traders escape from a stock market, which may lead to the further escape of other traders as well as a tremendous price drop. This highlights the new key variable which causes the crash and would be one of the compelling explanations of mysterious sources of the aggregate phase transitions.
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I. Introduction

Stock market crashes are fascinating events to the most of the economists as well as practitioners. According to the efficient market hypothesis, only a disclosure of drastic information can cause tremendous swings of prices. In macrofinance models, a small aggregate productivity shock can be amplified by frictions and cause a huge fluctuation. However, we usually cannot specify such informational shocks, or, in general, we do not know what the source of aggregate shock would be. Whether aggregate phase transitions happen or not totally depends on exogenous aggregate shocks in typical models but the source of them is such a mystery.

This paper provides a totally different view of the market crash. Collapses of the markets are fundamentally due to an unstable configuration of key variables such as price informativeness. The novel point of this paper is the fact that the economy *spontaneously* moves toward the critical phase and jumps into the crash

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phase. There are no exogenous aggregate shocks nor successive idiosyncratic shocks. Only one idiosyncratic shock pushes a start button and the economy starts moving. Once the story begins, the stock market represents stylized characteristics of the booms such as herding market participation and a tremendous increase in the asset price. Of course, the market is doomed in many cases. The crash of the market is waiting; a huge portion of traders escape from the market and a free fall of the asset price, which may overshoot, takes place.

Inspired by the field of Self-Organized Criticality (SOC) investigated by Bak et al. (1987), I call this kind of crash as a self-organized market crash (SOMC) hereafter. The main purpose and the result of my model are remarkably highlighted by the following statement about Great Depression in 1929 by Sornette (2004).

> *At that time (1929), a growth and prosperity on Wall Street and Main Street ended with the crash of the market. Before the crash, the market experienced a boom which is said to be driven by the participation into the market of largely uninformed investors.*

It can be interpreted that the boom before the crash in 1929 might be caused by the herding behavior of agents who did not have sophisticated information and behaved irrationally based on rumors. Then, somehow the state of the information had changed and the market collapsed. In this paper, I will make clear what the fundamental power that causes this change in the informational state would be.

The first key factor is informativeness of the asset price. The recognition of the importance of the informativeness of price has been at the root of economic thinking for a long time in the field of finance. Ever since the declaration of the importance of price information by the seminal papers such as Grossman (1976), Grossman and Stiglitz (1980), and Kyle (1985), literature has been working on the asset market with noise traders to investigate what kind of information asset prices might transmit. These works are focusing on the informativeness of a price under the noisy rational expectations equilibria (REE) in which the price would reveal only imperfect information about fundamentals because of the noise traders. The literature on market microstructure also investigated the possibility of price information under Nash equilibria as well as strategic REE. It is a natural conjecture and actually proved in the literature that the informativeness of the price depends on a share of informed traders in the market compared to the noise trader, i.e., how much of the noise the informed agents can absorb. Not only does this phenomenon bite the decision over being informed or uninformed, it also should be a reason for inaction (non-participation) if there is a (psychological) cost of trading. The fraction of the informed and uninformed agents in the market are endogenously determined in most of the existing literature and also the Kyle-type models shed light on the information dynamics with the long-lived information context. However, the consequences of the fluctuations of the price informativeness in a dynamic sense have not been paid enough attention in the context of market booms and busts.

What should be underlined here is the fact that precision of information can be crucial when traders decide to participate or not if they have a cost of being active or if it takes a cost to recognize information. I attribute this role to Knightian uncertainty and ambiguity aversion in a sense of Knight (1921) and Gilboa and Schmeidler (1989), which has been a way to represent "uncertain" situations rather than risks in the usual sense. In the economy, aside from sophisticated agents, there are heterogeneous ambiguity averse agents. They may not have any sophisticated information but have a doubt about returns of the asset. It is derived that they make a pessimistic evaluation when they make inferences[1]. As a consequence, ambiguity averse agents take a Ss-type strategy; they will find it optimal not to participate in the market under certain conditions. More precisely, in the CARA-Normal world, their optimal portfolio is given by the form

$$x = \alpha(\inf_{\mu \geq \mu_L} E[v|y] - P)\mathbb{I}_{\{\inf_{\mu \geq \mu_L} E[v|y] > P\}},$$

where v is a return, y is a set of information, $\mu = E[v]$, and μ_L is the worst case mean. $\mathbb{I}$ denotes an indicator function. Thus, they evaluate the expected return by using the worst case scenario given the available information set. If the worst case scenario is extremely good compared to the current price, they will trade with a long position. Otherwise, they will not trade. This tells us that observing what kind of information, y, which might include the equilibrium price signal, would be our first concern.

The contamination of the information plays a significant role in the story. At date t, suppose that there are a sophisticated insider who knows the persistent fundamentals v_{t+1}, naive traders A who can use the signal of the *past* price only, and naive traders B who also have a noisy signal s_{t+1}. The price is not fully revealing the information because of the noise, but I consider the situation such that the trading of naive traders itself brings a noise into the price[2]. It will be shown that the information set above implies the price at date t contains a signal of the form $q_t = v_{t+1} + \gamma s_{t+1}$, where γ represents the ratio of the measure of trader B relative to the sophisticated insider. If the agents can use q_t at date t, then there is no informational friction because they basically do not agree to disagree. The no-trade theorem of Milgrom and Stokey (1982) would prevail. However, if there is a time lag and naive agent cannot analyze the price right away as in the model of Hellwig (1982), then q_t would be a noisy signal of persistent fundamentals v_{t+2} for date $t+1$ traders and the main noise comes from s_{t+1} with the coefficient γ. The similar arguments of Kalman filter and Kalman gain can be applied here. Since γ is endogenously determined by the participation of naive traders B at date t, their behavior has an intertemporal effect on the participation decision of traders A and even on the decision of B themselves in

[1] This pessimistic subjective belief is not an assumption, but a derived result from the set of Savage type axioms including ambiguity aversion analyzed by Gilboa and Schmeidler (1989).

[2] See Han et al. (2016) for the detail of the endogenous noise trading.

the next period.

If the number of naive trader B is zero in the initial period, then the signal q_t that agent at date $t+1$ will use is clean and precise since γ is zero. Then, seeing this signal, A and B at date $t+1$ will decide to trade because their worst case posterior expectation is more likely to be revised upward by using the relatively clear signal $q_t = v_{t+1}$ (to infer v_{t+2}). In turn, the participation of B precipitates information contamination, i.e., it will make q_{t+1} noisier because γ will be higher. This clearly affects the participation behavior of $t+2$ traders. SOMC's implication is such that naive traders who rely on the price signal will keep trading as long as the value of γ is low and q transmits a precise signal enough to make naive traders confident and willing to trade.

As we can imagine, the value of γ can be endogenously time varying. What if there are many B traders and participation of trader B_1 would make other naive traders, say $B_2, B_3, \cdots$ willing to invest? In this situation with cascading participation, γ would be increasing over time. This can have a positive impact on the price as long as γ does not exceed the threshold mentioned above. If the γ hits the critical point, however, then the all naive traders would stop trading simultaneously and this would make the tremendous price drop through the demand effect. In the model, these procedures are derived from the optimization problem of each trader. Moreover, the crash is self-organized because we can cause the sequential participation and the boom/bust only by switching the behavior of a single agent, namely, trader B_1.

The key assumption in the story is a lagged information. As discussed by Geonka (2003) and Hellwig (1982), this timing assumption is relevant when we think of a trading taking place in a real time. In the real economy, the inferiority of information set of some traders also comes from a speed of information processing. One of the important evidence shown by Ozsoylev et al. (2014) is the fact that agents can have a faster access to information if they are located close to the center of the network, and investors located around a periphery are suffering from informational lag. As discussed in Subsection III.D, delayed information can be a source of the noise when the information set includes the price signal and allows the economy steer clear of the no-trading theorem of Milgrom and Stokey (1982). By making use of this property of information that agents can refer, the evolution of the informativeness of the price is derived in a system of closed form (though non-linear) difference equations, which have a similar property as the typical Kalman filter. It is possible to derive the critical value of informativeness of the price that triggers the escape of the Knightian agents.

Moreover, the model replicates one of the most typical features of the market crisis: the escape followed by the further escape and overshooting of asset prices. The overshooting price is one of the most difficult phenomena to show for theoretical models of the financial crisis. Like Hirano and Yanagawa (Forthcoming), in most of the macroeconomics models with stochastic asset price bubbles, asset prices cannot be worse than the benchmark economy, i.e., the bubble component cannot be negative or cannot hurt fundamental part of the asset price. On the other hand, in my model, asset prices can overshoot. A simple demand effect implies that overshooting will take place if the demand volume at the crisis point is smaller than the demand at the initial period

In Section 2, I briefly summarize related literature. Section 3 provides the description of the model economy. We consider a simple three periods example of escaping behavior in Section 4. Section 5 characterizes decisions and the equilibrium in the infinite horizon economy, and dynamic properties are provided. Section 6 puts forth some of the empirical implications (very preliminary), and Section 7 concludes the discussions.

II. Related Literature

One of the main results of the paper is the endogenous market crash. As the citation in Introduction, the Econ/Physics book by Sornette (2004) provides a lot of insights. He views the market crashes in a different way from the mainstream of macroeconomics and finance. According to his perspective, what matters for the market crashes would be accumulation and aggregation of micromotives/individual behaviors. It might be rational or irrational, the quality and type of the information matters, and these micro and local movements create chaotic phenomena through non-linear local interactions. Event hough the mechanism I will specify in this research is different from the points he put forth, the background of the idea is crucially alike.

In terms of the role of the price as an information intermediary, this paper is related to the long list of papers which deal with the equilibrium implication of noisy rational expectations equilibria and price informativeness. Seminal works are done by Grossman (1976), Grossman and Stiglitz (1980), Hellwig (1980), Kyle (1985) and De Long et al. (1990). Most of the literature, however, did not pay attention to dynamic characteristics of the noise and the price informativeness. Kyle (1985) analyzed the dynamic sequence of the one-shot auction to see how long-lived information might be incorporated into the price in a dynamic sense. Also, Wang (1993) provided a continuous-time dynamic model of Grossman-Stiglitz type economy and showed that the existence of asymmetric information can be a reason for the excess volatility, a higher risk premium, and the negative autocorrelations, but more drastic phenomena, like market crashes, were not analyzed.

In the field of finance as well as macroeconomics, a growing literature[3] has been analyzing the application of Knightian uncertainty. The seminal work by Dow and da Costa Werlang (1992) is the first paper that shows Ss-type inaction

[3] There are many documented evidence for irrationality and overconfidence in markets. For example, Odean (1998) and Daniel et al. (1998) investigate effects of overconfidence on the market. The survey by Vissing-Jorgensen (2003) summarizes behavioral and psychological interpretations of market anomalies including non-participation behavior.

property of the optimal portfolio under ambiguity aversion. A recent paper by Easley and O'Hara (2009) adopted ambiguity aversion into a simple setting of a heterogeneous traders model and provided the sharp explanation on the market (non-) participation which I will use in my model.

Epstein and Schneider (2008) and Leippold et al. (2008) considered the information acquisition and resolution of ambiguity by learning. These papers put forth the ambiguity as one of the explanations of puzzling facts such as the risk premium puzzle and excess volatility but did not go through the dynamic characteristics of information, market participations and the price. Mele and Sangiorgi (2010) constructed the model to investigate the effect of ambiguity on information value in a context of Grossman-Stiglitz model and showed that there will be information complementarity. They also considered the possibility of dramatic price swings as a result of the changes in the information value, but the crucial parameter to get the price swing was still the degree of ambiguity, which has aggregate and exogenous characteristics. Caballero and Krishnamurthy (2005) is another paper that looked at the flight to quality as a result of ambiguity aversion but the mechanism is not endogenized and the structure of the crisis is totally different from my model.

Synchronization is also one of the important but mysterious phenomena observed in the real market[4]. Abreu and Brunnermeier (2002) develops the interesting model of "a synchronized risk". In their model, the portfolio adjustment is costly and thus moving alone to get the arbitrage is not profitable if a single agent's order cannot change the price. As a result, traders will wait until enough amount of other traders start moving. The synchronization behavior in my model is much simpler in a sense that each trader will just imitate her neighbors' behavior, but the fact that agents are not knowing the meaning of the key parameter is common in two models. Also, Abreu and Brunnermeier (2003) applied the theory of the currency attack of Obstfeld (1996) to the model of bubbles. In their model, there is a threshold of the degree of synchronized behavior that leads to a collapse of the bubble. Thus, arbitrageurs try to ride the bubble until the synchronization with other agents will exceed that threshold. This structure is similar to my model. In my model, agents' (unconsciously) coordinated participation causes the information contamination which in turn will be the reason of escape from the market. The different point from the currency attack argument might be the fact that the accumulation of information contamination and the threshold of the contamination which leads to the crisis are endogenously determined as the result of optimizations of traders. Also, the price dynamics is endogenously determined as the equilibrium consequence,

[4] The imitation behavior of the traders is the well-observed phenomenon in the experimental economics such as Boissevain (1979), Darke and Freedman (1997) and Heath and Gonzalez (1995). The literature showed a trader is more likely to rely on the information from her neighbors and, as a result, she imitates what their neighboring agents are doing if her private information is not reliable enough. See Orléan (1995) for the theoretical arguments. Also, Ozsoylev, Walden, Yavuz and Bildik (2014) shows the evidence for the existence of the network of traders' group, and my model introduces this idea.

while it is exogenously given in Abreu and Brunnermeier (2003). Moreover, my model can be interpreted as an explanation of fads, fashions, and herding. The mainstream of the discussion focuses on how information cascade takes place as in my model. Bikhchandani et al. (1992) and Banerjee and Fudenberg (2004) proposed the model in which each agent relies on the signal from her neighbors rather than her private (and might be correct) information. Also, my model intends to investigate how the market crash occurs due to the information contamination caused by the herding behavior, while in the previous papers, fads and fashions are the direct consequence of the information cascade. Therefore, my focus is on the mutual reactions of information and traders' behavior, while the previous literature only considered one of them at most. In this sense, my model provides more detailed discussions on what is crucial for the market to move into the crises phase.

III. The Model

The core of the model is similar to the multi-periods Kyle (1985) model, but the property of difference equations as well as the behavior of agents are totally different from his model because there is no long-lived information and the model is constructed by infinite horizon with OLG agents. Even though the forward-looking insider is not assumed to exist as in Kyle-type models, lagged information reference of each trader allows us to specify the dynamics of information.

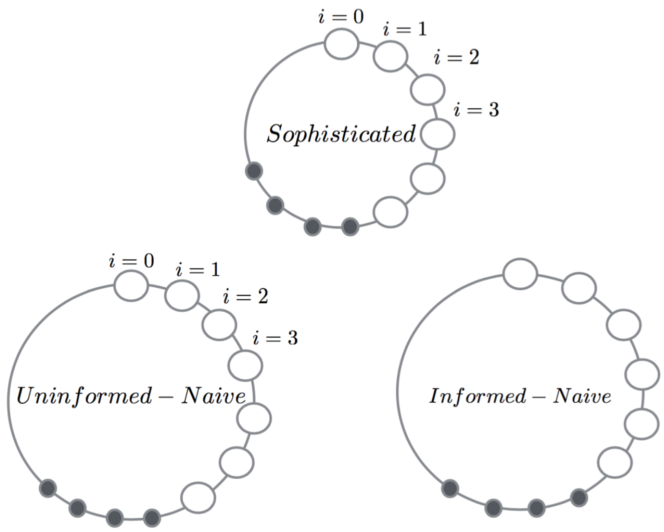
I consider a discrete time OLG economy[5], which is characterized by three types of (groups of) agents: there are sophisticated agents (S-agent), informed-naive agents (I-agent) and uninformed-naive agents (U-agent). The difference between these agents will be specified later but basically, it comes from the information they rely on and from the degree of ambiguity aversion.

Assume that these types of agents are constructing three groups respectively. Group-$J \in \{S, I, U\}$ has a mass λ_J of type-J agents and agents $i \in J$ in a group-J are connected by a network as shown in Figure 1. It will be clear that the network matters only for group $J = U$ and thus I will describe S and I agents as representative OLG traders. The measure of the representative S-trader is normalized to one and the representative I-trader is supposed to have measure λ_I. Each agent lives three periods: phase I, II and III, and the behavior in each phase will be specified in the next subsection.

They are trading two types of asset. A risk free saving with constant interest payment $r = 0$, and a risky asset with fixed supply x. By holding the risky asset from date t to $t+1$, it pays $v_{t+1} = \theta_{t+1} + u_{t+1}$. The evolution of the fundamental

[5] The assumption of overlapping generations model makes the analysis much easier because we can avoid the time inconsistency problem which is a typical complication with the ambiguity aversion preference and MEU. Also, this casts a difference from Kyle's long-lived information model.

FIGURE 1. HETEROGENEOUS TRADERS



component θ is given by the mean reverting AR (1) process,

$$\theta_{t+1} = \rho\theta_t + (1-\rho)\mu + \varepsilon_{t+1},\ \rho \in (0,1) \tag{1}$$

where μ denotes the mean component and ε represents a stochastic shock. Assume that u and ε follow normal distributions independently with variance σ_u^2 and σ_e^2. In my model, I will consider the risky asset as the sequence of one period risky investment opportunity and thus there is no resale[6]. This simplification gives us a straightforward insight into the mechanism of simultaneous escape and allows us to derive analytically simple results. I will use the tuple notation to denote each agent, e.g., the agent i in group-j at date t facing the phase I is denoted by $(i, I, t)_j$.

A. *Timing and Available Information*

The entire structure of the information flow and heterogeneity of an available set of information are described in this subsection.

Information Heterogeneity

It is natural to think that there are potentially three types of information available for naive agents: the price signal, neighbors portfolio, and some other privately created information. I assume that S and I-agents have a costless access to the

[6] There is always the problem of the possibility to resale in this literature. I follow the traditional sequential auction setting used in the literature of market microstructure such as Kyle (1985) and Glosten and Milgrom (1985).

private source of information as well as the price signal. We can also think of the private signals as information obtained from investment brokers or information on newspapers and so on. For $i \in \{S, I\}$, let y_t^i be a heterogeneous information vector at date t for trader-i. The specification of S and I says

$$y_t^i = (s_t^i, q_{t-1})$$

where s_t^i is a private signal and q_t is a information content of the price at date t. The reason why y_t contains q_{t-1} instead of q_t is explained later. In order to make the model as simple as possible, I assume that S-trader cannot see the price information. This does not change the essential implication of the model, but just makes the parameter region of the crash larger. Also, I consider the private signal as a noisy signal about the fundamental of the risky asset,

$$s_t^i = \theta_{t+1} + z_{t+1}^i$$

where z^i is normally distributed noise with variance $\sigma_{z_i}^2$, correlation $\rho_{S,I}$, and is independent of other noises. Therefore, the differences between S and I-trader come from the private signals (and also from the ambiguity aversion).

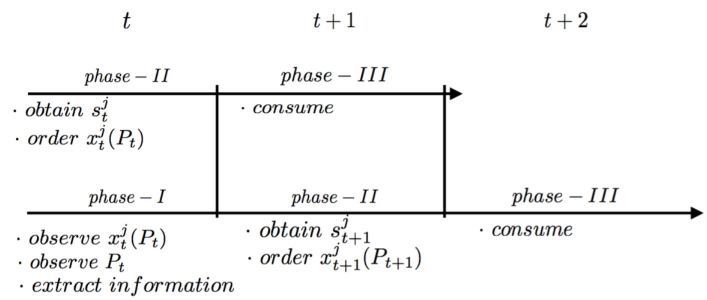
On the other hand, the distinctions between I-agent and U-agents come from the inferior information set of U-agents. It is natural to think that there are potential investors (such as households) who do not pay attention, or who cannot interpret market signals unlike professional investors or investment bankers. I categorize these agents by assuming that U-traders do not have any private information and cannot see (or interpret) the market price directly. According to the general classification of the information sets mentioned above, the only signal they can use will be neighbor's behavior. Thus, if x denotes the portfolio order, the information set of the U-trader with index i is given by

$$y_t^i = x_{t-1}^{U,i-1}$$

as long as this value is non-zero, i.e., her neighbor is active.

It will be clear that the neighbor's portfolio at date t contains the same signal as the price at date t. Hence, the agents in group-I and S never use the neighbor's portfolio as a source of information. Therefore, even if we allow S and I-traders to see the neighbors' portfolio, all S and I-agents have the same set of information within their group. Thus, each can be expressed as a representative agent. On the other hand, there is heterogeneity among the agents in group-U. If the agent i in group-U is making non-zero investment at date t, then the agent $j = i, i+1$ can see her neighbor's portfolio, say x_t, and thus the information set available at date $t+1$ is $\mathcal{F}_{t+1}^j = \sigma(\{x_t\})$, where $\sigma(\cdot)$ represents $\sigma-$algebra. In contrast, other agents with $j \neq i, i+1$ do not have any information since their neighbors are not active, $x = 0$, and cannot extract any information from inactive

FIGURE 2. BEHAVIOR OF OLG AGENTS



behavior. Thus, $\mathcal{F}_{t+1}^j = \sigma(\phi)$.

Timing

Note that each generation of investors will make their investment only in phase II, and they will consume the return of investment at the end of phase III. In order to prepare for the investment, agents seek for information during phase I. For example, let $x_t^{U,i}$ be the portfolio of agent $i \in U$ at t, then the agents $j = i, i+1$ in their phase I observe this market order and make use of this signal when they are in phase II at date $t+1$ to infer the return v_{t+2}.

The fact that the information that agents- $(i+1, II, t+1)$ are using is out of date at date $t+1$ is crucial in this model. This can be interpreted as the limited speed of information processing. Also, as discussed by Geonka (2003) and Hellwig (1982), this timing assumption is relevant when we think of a trading taking place in a real time[7]. The flow of behavior of a typical agent is described as Figure 2. If the agent is S or I, then they do not have to observe x_t^j, while if she is U, then she needs to look up neighbor's x_t^j to extract the signal of P_t because she cannot observe P_t nor obtain s_t.

Finally, the value of v_{t+1} is actually not observable for agents until the end of $t+1$, i.e., after all phase II agents at $t+1$ finish making their investment decisions. Also, I assume that each agent does not know the history of the realized $\{\theta_t\}_{t=-\infty}$[8]. This last assumption only makes a difference in quantitative values and can be easily relaxed.

[7] This timing assumption can also be discussed in the context of Grossman-Stiglitz paradox of REE models. Competitive agents can infer information from the equilibrium price while they do not take into account the effect of their order on the informativeness of the price. See Dubey, Geanakoplos and Shubik (1987) about the clear explanations and one of the solutions to the GS paradox.

[8] These timing assumptions with respect to the availability of v_{t+1} information is crucial in my model. If the realization of the return becomes observable before the decision making is done, then the participation of uninformed naive agents does not bring any noise since they can back up the insider information from the price signal (if available). However, it is unlikely that all naive agents have a real-time access to the realized information and can back up the insider information. Thus, the naive agents who make their decision under this assumption can be interpreted as those who do not have a sufficiently fast access to the information. Even if we drop this assumption, we can still get our result by modifying the model so that U-agents have their own

B. Behavior of Sophisticated Agent

I consider the representative sophisticated agent with a normalized mass of unity. Each generation has CARA utility with the absolute risk parameter τ. Assume that they don't discount the future.

Let us focus on the optimization of the trader in her phase II at date t who is making a portfolio order x_t^S. By CARA-Normal property, the certainty equivalent level is

$$u_t^S = (E[v_{t+1}|s_t^S] - P_t)x_t^S - \frac{\tau}{2}(x_t^S)^2 Var(v_{t+1}|s_t^S).$$

The optimal position of the S-agents is a typical MV portfolio such as

$$x_t^S = \frac{v_t^S - P_t}{\tau\sigma^2_{v|s_t^S}},$$

where $v_t^S \equiv E[v_{t+1}|s_t^S]$. For later use, let $\alpha^S \equiv 1/\tau\sigma^2_{v|s^S}$, which makes

$$x_t^S = \alpha^S(v_t^S - P_t). \tag{2}$$

This optimal portfolio is the case for all generations. If we relax the assumption and allow her to see the price signal, then α^S will be time varying.

C. Behavior of Naive Agents

They also have CARA utility with the same risk parameter τ. They have the aversion toward ambiguity in the sense of Gilboa and Schmeidler (1989) and take Max-Min expected utility (MEU) which is axiomatized in their paper.

Each naive agent has a doubt about the mean component of the dividend process, μ, and evaluates her expected utility by the subjective belief which yields the worst case scenario. Let $\mathcal{M}_j = [\mu_L^j, \mu_H^j]$, $j \in \{I, U\}$ be the set of possible means which induces the multiple prior for each agent. Following arguments assume that there is no heterogeneity with respect to $\mathcal{M}_j$ but, in the later section, I will make it heterogeneous across the groups, $\mathcal{M}_I \neq \mathcal{M}_U$.

The optimization problem of naive agents $(i, II, t)_j$ is given by

$$\sup_{x_t^{j,i}} \inf_{\mu \in \mathcal{M}} u_t^{j,i},$$

where

$$u_t^{j,i} = (E[v_{t+1}|\mathcal{F}_t^j] - P_t)x_t^{j,i} - \frac{\tau}{2}(x_t^{j,i})^2 Var(v_{t+1}|\mathcal{F}_t^j)$$

source of information which is a noisier version of the insider information. Endogenous noise trading can be also supported by some literature such as Huddart et al. (2001).

is a certainty equivalence conditional on the lagged information set of type j agents, $\mathcal{F}_t^j = \sigma(\{y_t^j\})$.

Let $\underline{v}_t^j(\mathcal{F}_t) \equiv \inf_\mu E[v_{t+1}|\mathcal{F}_t^j]$ and $\bar{v}_t^j(\mathcal{F}_t) \equiv \sup_\mu E[v_{t+1}|\mathcal{F}_t^j]$ be the worst and best case posterior expectation conditional on information set y_t (I will omit $\mathcal{F}$ in $\underline{v}(\cdot)$ if there is no fear of confusion). The optimal portfolio of an agent $(t, II, i)_j$ given her information set $\mathcal{F}_t^i$ is

$$x_t^{j,i} = \begin{cases} \frac{\underline{v}_t^j - P_t}{\tau\sigma^2_{v|\mathcal{F}_t^j}} & if\ \underline{v_t}^j > P_t \\ 0 & if\ P_t \in [\underline{v}_t^j, \bar{v}_t^j] \\ \frac{\bar{v}_t^j - P_t}{\tau\sigma^2_{v|\mathcal{F}_t^j}} & if\ \bar{v}_t^j < P_t \end{cases} . \tag{3}$$

Therefore, the naive agents take the Ss-type strategy. I call the situation such that $x = 0$ (resp: $x \neq 0$) as “being outside (inside) of the market” or “non-participation (participation)”. This property is a typical inaction property also shown by Dow and da Costa Werlang (1992) and Easley and O’Hara (2009), and will take a crucial role in the following discussion.

Intuitively, they take the non-participation strategy if and only if the degree of ambiguity is not extreme. If the worst case return is sufficiently high, they will take a long position because the expected net return is positive even though they are considering the ambiguity (the first line). On the other hand, if the best case return is sufficiently low, it is optimal for them to short. If the degree of ambiguity is moderate, it is hard for them to decide which position is profitable and they end up with being trading zero amount.

In order to make our analysis as simple as possible, the possibility of short selling by naive agents is eliminated by assuming $\mu_H^j = \infty$ for all j. Under this assumption, the third case of (3) does not happen a.s. Also, as in Mele and Sangiorgi (2010), the discussion hereafter assumes that the set of prior depends on the information set of naive agents[9]. Specifically, suppose that $\mathcal{M} = \mathbb{R}$ if the agent does not have any signal, while $\mathcal{M}$ shrinks to $\mathcal{M}_j$ described above if she has a signal. This assumption guarantees that naive traders will not participate if they do not have any signals, while they may or may not participate, according to (3) if they get some signals. We can relax this assumption by imposing one additional condition as in the footnote 12 for each proposition in later sections, but the implication of the results do not change.

[9] Mele and Sangiorgi (2010) also consider the resolution of ambiguity by the signal acquisition. In their setting, not only does the signal reduce the risk, it also resolves the ambiguity from a compact set of multiple priors into a singleton.

D. Filtered Worst Case Scenario, Compensation, and Noise

Filtered Worst Case Expectation and Compensation

As shown in later section, the informativeness of the price or neighbors' behaviors plays a crucial role when naive agents decide to participate or not. To see the idea, suppose that a naive agent has a signal $q = \theta + \xi$, $\xi \sim N(0, \sigma_\xi^2)$ when she infer the expected value of θ. If not conditioning on the signal, her worst case belief gives $\inf E[\theta] = \mu_L$. On the other hand, the conditional worst case expectation is $\inf E[\theta|\theta+\xi] = \inf[\mu + \frac{\sigma_\theta^2}{\sigma_\theta^2+\sigma_\xi^2}(\theta+\xi-\mu)] = (1-b)\mu_L + b(\theta+\xi)$. Of course, it depends on the realization of the signal, but if we have $\xi = 0$ and $\theta = \mu$, then the filtered worst case expectation is a weighted average of the distorted belief μ_L and the true value μ. Thus, the distorted worst case expectation is *compensated* by the information signal. In this sense, the signal will reduce the risk and this works as if the naive agent is compensated and gets optimistic compared to the case with no signal. More generally, if the normally distributed signal q is unbiased, $E[q] = \mu$, the expected value of compensated posterior worst case expectation is

$$E[\inf E[v|q]] = \beta\mu + (1-\beta)\mu_L$$

where $\beta = Cov(q,v)/Var(q)$. The key takeaway is the fact that, if the signal is more precise, then she will get more compensated and get a higher upward revision.

Noise

In this model, the existence of exogenous noise traders has not been assumed so far. In the usual context, the equilibrium notion of the economy cannot stand the absence of noise trader because the economy will end up with being the non-trading equilibrium of Milgrom and Stokey (1982). However, in this model, the agents potentially agree to disagree because of the ambiguity averse agents having the distorted subjective priors. Moreover, even though the noise traders are not assumed, the assumption of the lagged information makes the economy stay away from the non-trading equilibrium. Market orders do not incorporate the current price information and the agent cannot infer the information of other traders[10]. The underlying force working here is discussed more in the context of

[10] The main reasoning of no-trading theorem relies on the general equilibrium notion with rational agents. If two traders got different realizations of private information and they agree to trade at the price, the equilibrium price reveals the private information of the opponent of each trader. Then there is no informational friction left and incentive to trade is eroded. What is assumed here is the agents can use the information of price right away (or even before the realization of equilibrium price!). But this is unrealistic as discussed in Dubey, Geanakoplos and Shubik (1987). Approaches such as Nash equilibrium can steer clear of this REE paradox. They viewed lagged reference is natural because "*price formation is modeled as a process, the pooling and transmission of information takes time as any process must. So when public information is revealed it can only be used in the next time period*".

market microstructure such as Dubey, Geanakoplos and Shubik (1987).

The assumption of lagged information makes the price signal at date t a noisy signal of v_{t+1}. The argument is basically same as the theory of Kalman filter. To grasp the idea, we consider the period $t = 0$ and assume that there are two traders: the informed agent with a noisy signal $s_t = \theta_{t+1} + z_{t+1}$ (who cannot see the current price) and the uninformed naive trader. Both can see the lagged price information. As we can see from (1), P_0 would be the linear function of s_0 because the private information informed trader should be perfectly reflected in the price. For simplicity, assume that there are no naive agents at $t = 0$ investing. Then the agent $(\cdot, II, 1)$ will trade conditioning on $q_0 = \theta_1 + z_1$ which they obtained in the previous period. Therefore, at $t = 1$, their inference is

$$E[v_2|q_0] = E[\theta_2 + u_2|\theta_1 + z_1] = E[\rho\theta_1 + (1-\rho)\mu + \varepsilon_2 + u_2|\theta_1 + z_1].$$

First, the filtering shows that the agent cannot back up θ_2 perfectly because of ε_2. Second, since z_1 is not observable until the end of $t = 1$, then this would add more noise. In the next period $t = 1$, this implies that the signal q_1 would be a linear function of $\theta_2 + z_2$, which comes from insiders, and θ_1, which comes from naive traders. Then, the signal q_1 is a linear function of θ_2, z_2, and ε_2. It can be interpreted as the noisy signal of v_3, that is, the agent needs to infer $\theta_3 + u_3 = \rho\theta_2 + \cdots$ given a signal which is a linear function of θ_2, z_2, and ε_2, because the realization of $v_2, z_2,$ and θ_2 is assumed to be not yet observable when she makes her decision at date 2. Hence, the lagged information creates the noise when it is referred before the realization of the return.

E. Pseudo-Stochastic Variables and Pseudo-Stochastic GE

Finally, the new concept of equilibria is introduced. This concept allows us to consider the stochastic model as if it is deterministic keeping the properties of stochastic equilibria in our hands.

Suppose that $y \sim N(\bar{y}, \hat{\sigma}_y^2)$, and $\hat{\sigma}_y$ is an ambiguous parameter for ambiguity averse agents. Suppose that the set of priors is characterized by $\hat{\sigma}_y \in S = [\underline{\sigma}_y, \bar{\sigma}_y]$. For simplicity assume that $0 < \underline{\sigma}_y$. Under these specification, even if the true distribution of y is degenerate ($\sigma_y = 0$ and $y = \bar{y}$ a.s.), the ambiguity averse agent anticipates that y is non-degenerate since the worst case scenario in her mind is achieved by the truly stochastic y.

DEFINITION 1: *Consider a normal random variable. A degenerate stochastic variable (or constant stochastic variable) y is pseudo-stochastic variable if its variance is ambiguous for ambiguity averse agents and the worst case scenario is achieved by a subjective distribution with a positive variance.*

The pseudo-stochastic variables can be interpreted as the situation in which the rational agents know that the variables are degenerate and there are no stochastic risks, while irrational agents fear the random realization of the results since they do not know the true specification of the variables. The heterogeneous belief of this kind depends on irrationality of the subset of agents who agree to disagree, and if all agents are fully rational, they get rid of disagreement and the phenomena above must be eliminated. Of course, this assumption would be somewhat unrealistic and extreme, but yields the clear insights of the model by making the analysis much simpler. This also helps us with separating the effect of aggregate stochastic shocks from the fundamental effects of the idiosyncratic initial trigger shock under the scenario of SOMC. The effect of information contamination would be salient.

To get the notion of equilibrium under the pseudo-stochastic variables, I define the following concept of the equilibrium.

DEFINITION 2: *Pseudo-stochastic general equilibrium (PSGE) is the version of a general equilibrium, in which specified random variables are degenerate in the sense of Definition 1*[11].

F. General Equilibrium

Given the initial condition of θ and initial trigger shock (initial information set), which I will define later, the equilibrium with a state $\{\theta_t\}_{t=0}^{\infty}$ is defined by the sequence of the quantities $\{x_t^S, x_t^I, (x_t^{U,j})_{j=0}^{\infty}\}_{t=0}^{\infty}$, information sets y and the price $\{P_t\}_{t=0}^{\infty}$ which satisfy the optimal portfolio given by (2) and (3), and clear the market such that

$$x = x_t^S + \lambda_I x_t^I + \lambda_U \sum_{i=0} x_t^{U,i},$$

where the amount of supply is fixed and λ_j represents the measure of each type of naive agents.

IV. Simple Example of Escape

I will say agents "escape" if traders have been trading positive amount up to some point t, but they stop trading at $t+1$. In this section, the mechanism of escape is specified under PSGE.

As we saw in Introduction and (3), the informativeness of the neighbors' portfolio and that of the price would be our first concern. Note that, given the signal y which is unbiased, whether an agent trades or not is totally determined in PSGE by

$$\inf_{\mu \geq \mu_L} E[v|y] = \beta\mu + (1-\beta)\mu_L \gtreqless P. \tag{4}$$

[11] In general, we need to be more precise about the learning of the distribution because it is unnatural that people think some variables as stochastic even though they keep observing constant realization in every period. However, considering the i.i.d property of shocks in this model and the OLG structure, it might be not extreme to omit the arguments about the learning. Moreover, there is a possibility that the realization and an observation of (pseudo-)stochastic variables lead to a confidence erosion of ambiguity averse agents, which means they get confused more and the set of the possible posterior is even worse that the prior in a sense of its size. See Nishimura and Ozaki (2008) for a learning and confidence erosion.

Since β is determined by the past price signal and the past signal is set as a result of previous generation's behavior, we can see the importance of the participation behavior pertaining to the behavior of the next generation. By specifying the dynamics of information-related variables, we could check if naive agents are in the market or not in each period. This section is devoted to a simple three periods economy, where only S and U type naive agents are trading, in order to accentuate the fundamental points of the arguments, namely, simultaneous escape behavior. After that, we will consider an infinite horizon economy with I-type naive agents. Because of its simplicity, this example may or may not predict a huge price movement as in the real world. One can skip this section and go to Section V if the mechanism of the escape and the price informativeness a la Grossman-Stiglitz are clear enough.

A. Three periods example

The economy has three periods, $t = 0, 1, 2, 3$, and assume that $\lambda_I = 0$, i.e. there is no I-agent. In order to investigate the dynamics of the economy, we need to specify the idiosyncratic initial trigger shock with respect to the information set of U-agents. Moreover, I assume that $\sigma_{z_S} = 0$, i.e., S-agent has perfect information.

Suppose that at $t = 0$, only agent $(i = 0, II, t = 0)_U$ gets the noisy signal of the risky asset before she makes her decision. Let the signal take the form $q_{-1} = \theta_0 + z$, where $z \sim N(0, \sigma_z^2)$ and may be correlated with θ_1. There are phase II of S-agent and U-agent with index $i = 0$ who has information at date $t = 0$. Then, (3) implies

$$x_0^U = \begin{cases} \frac{\underline{v}_0 - P_0}{\tau\sigma^2_{v|q_{-1}}} & if\ \underline{v}_0 > P_0 \\ 0 & o/w \\ \frac{\bar{v}_0 - P_0}{\tau\sigma^2_{v|q_{-1}}} & if\ \bar{v}_0 < P_0 \end{cases},$$

with

$$\underline{v}_0 = \mu_L(1 - \rho\beta_0) + \rho\beta_0(\theta_0 + z)$$

($\bar{v}_0$ has μ_H instead of μ_L). Let $\alpha_0^U \equiv 1/\tau\sigma^2_{v|q_{-1}}$. Then, we will get following Lemma, which provides the condition for the participation of the initial agents $i = 0$ who got the shock. The equilibrium price under the participation of $i = 0$ and non-participation of $i \neq 0$ agents is given by

$$P_0 = \frac{\alpha^S + \lambda_U\alpha_0^U}{\alpha^S + \lambda_U\alpha_0^U\beta_0} q_0 + \frac{1}{\alpha^S + \lambda_U\alpha_0^U\beta_0}[\lambda_U\alpha_0^U\beta_0(\rho z - (1-\rho)\mu_L) - x],$$

where q_0 and β_0 are given in Lemma below.

LEMMA 1: *(i): Under the setting above, the initial U-naive agent $i = 0$ with an idiosyncratic trigger shock trades a positive amount, i.e., participates in the market with* *long position and $i \neq 0$ traders do not invest iff*

$$\frac{\theta_1 - \frac{x}{\alpha^S} - \rho\beta_0(\theta_0 + z)}{1 - \rho\beta_0} < \mu_L < P_0$$

where

$$\beta_0 = \frac{\sigma_\theta^2}{\sigma_\theta^2 + \sigma_z^2}.$$

(ii): Under PSGE, the conditions in (i) are reduced to,

$$\mu - \frac{x}{(1 - \rho\beta_0)\alpha^S} < \mu_L < \frac{\alpha^S + \lambda_U \alpha_0^U \beta_0(\mu - 1)}{\alpha^S + \lambda_U \alpha_0^U \beta_0(1 + \beta_0(1 - \rho))} \equiv \mu_{up}. \tag{5}$$

This Lemma says the participation behavior crucially depends on the informativeness of the initial shock and thus σ_z^2. Once we get the initial trigger with σ_z which satisfies the condition above, then the boom/bust takes place as I will show later. Otherwise, nothing happens. I will come back to the intuition of this condition after I specified the analytical dynamics of the behavior in the degenerate case in a later section, but the inequality (4) and (5) suggest that the naive agent will participate if and only if the compensation coefficient β_0 is sufficiently high in a degenerate case. Also (4) tells us that the degree of ambiguity $-\mu_L$ has to be low enough for a naive agent with information to be compensated and start trading (RHS of (5)).

In addition, we get the following result with respect to the price signal.

LEMMA 2: *Under the condition (5) in Lemma 1, the price signal at the initial period is given by*

$$q_0 = \theta_1 + \gamma_0 \varepsilon_1, \tag{6}$$

$$\gamma_0 = \frac{-\lambda_U \alpha_0^U \beta_0}{\alpha^S + \lambda_U \alpha_0^U \beta_0}.$$

If there is no noise, the price must reveal the insider information. If the agents are allowed to use the current price signal and the information acquisition is costly, then Grossman-Stiglitz paradox would be omnipresent. However, the best thing that agents can do is just to get the signal for the usage of the next period. Thus, even if the price signal at t is perfect for the investment decision at date t, it will no longer be true at date $t + 1$. Moreover, not only will this lagged information be imperfect in order to infer the return in the current period, it also brings the noise into the market. This is why we have the second term in (6). Note that the absolute value of γ_0 positively depends on λ_U since λ_U represents the mass of the investors in the market who are relying on the noisy signal rather than the insider information.

Suppose that the condition (5) holds. Then at $t = 1$, group $i = \{0, 1\}$ of U-agents have the signal which is equivalent to q_0 because the portfolio of $i = 0$ at $t = 0$ reveals the same information as the price to her neighbors. Note that the S-agent keeps going with (2). Then, the each agent faces the same problem, and the optimal portfolio again takes the form of (3) with

$$\underline{v}_1 = (1 - \rho\beta_1)\mu_L + \rho\beta_1(\theta_1 + \gamma_0\varepsilon_1),$$

given

$$\beta_1 = \frac{1 + (1 - \rho^2)\gamma_0}{1 + (1 - \rho^2)\gamma_0(2 + \gamma_0)}, \tag{7}$$

(same form for $\bar{v}_1$ with μ_H instead of μ_L). Also the price information of this period is given by

$$q_1 = \begin{cases} \theta_2 + \gamma_1\varepsilon_2 & \text{if } U \text{ participate} \\ \theta_2 & \text{otherwise} \end{cases},$$

$$\gamma_1 = \frac{-2\lambda_U\alpha_1\beta_1}{\alpha^S + 2\lambda_U\alpha_1\beta_1},\ \alpha_1 = \tau^{-1}\sigma^{-2}_{v_2|q_0}$$

because we now have $i = 0, 1$ agents who are investing in the market.

Compared to the previous period, the signal can be noisier because of the larger number of uninformed agents who bring the noisy signal into the market. The participation decision is described by

$$(1 - \rho\beta_1)\mu_L + \rho\beta_1(\theta_1 + \gamma_0\varepsilon_1) > \theta_2 - \frac{x}{\alpha^S} \tag{8}$$

for $i = 0, 1$ agents[12]. It is clear that the participation decision at the current date depends on the information variables in the *previous* period. Also, if we consider the PSGE, we can see the price for date $t = 0$ and 1 can be reduced to $P_t = P(t; \lambda_U)$ with

$$P(t; \lambda_U) = \frac{\alpha^S\mu + (t + 1)\lambda_U\alpha_t^U[\mu - \beta_t(1 - \rho)\mu_L] - x}{\alpha^S + (t + 1)\lambda_U\alpha_t^U\beta_t}.$$

Note that we have a recursive structure in γ, α, β. Therefore, in the final period, $t = 2$, we only have to consider the same form of parameters. Suppose that the U-agents with $i = 0, 1$ were trading in the previous period with a long position,

[12] If we relax the assumption about $\mathcal{M}$ to be $\mathbb{R}$ for U-traders without signals, we need to impose the following condition to make them stay away from the market,

$$\mu_L < P_1 = \frac{\alpha^S + 2\lambda_U\alpha_1^U}{\alpha^S + 2\lambda_U\alpha_1^U\beta_1}q_1 - \frac{1}{\alpha^S + 2\lambda_U\alpha_1^U\beta_1}[2\lambda_U\alpha_1^U\beta_1(\rho\gamma_0\varepsilon_1 + (1 - \rho)\mu_L) + x].$$

i.e., (5) and (8) are satisfied. Then, at $t = 2$, $i = 0, 1$ and 2 agents can use $q_1 = \theta_2 + \gamma_1 \varepsilon_2$ as a signal to get the optimal portfolio. This implies U-agents at date 2 participate in the market if and only if

$$(1 - \rho\beta_2)\mu_L + \rho\beta_2(\theta_2 + \gamma_1\varepsilon_2) > \theta_3 - \frac{x}{\alpha^S} \tag{9}$$

for GE case and

$$\mu - \frac{x}{(1 - \rho\beta_2)\alpha^S} < \mu_L$$

for PSGE case, where

$$\beta_2 = \frac{1 + (1 - \rho^2)\gamma_1}{1 + (1 - \rho^2)\gamma_1(\gamma_1 + 2)}.$$

Thus, we could get the conditions such that the behavior of the U-agents at $t = (0, 1, 2)$ is such that (In,In,Out) for $i = 0$ agent, (Out,In,Out) for $i = 1$ agent, and (Out,Out,Out) for other agents. This can be interpreted as herding through $t = 0, 1$ and escape from the market in the final period. It is clear that the aggregate shock such as θ and ε matter in this setting. In order to get the "without aggregate shock" property of SOMC, I have been considering the case where the most of the stochastic variables are degenerate while keeping the model meaningful.

B. *Escape in the three periods example*

In order to declare the analytical result without any aggregate shock, we impose the assumption that all stochastic variables except u are pseudo-stochastic variables. Then we have $\theta \to \mu$, $\varepsilon \to 0$ a.s. By looking back on the previous results and the conditions (4), (7) and (8), we notice the following conditions are necessarily and sufficient conditions for U-agents to herd and escape as described in the last subsection.

PROPOSITION 1: *If there is a trigger shock specified above, index $i \in \{0, 1\}$ of U-agents sequentially participate in the market through $t = 0, 1$, and then stop trading at $t = 2$ iff the following conditions are satisfied.*

$$\mu_{low} \equiv \mu - \min_{t=0,1}\left\{\frac{x}{(1 - \rho\beta_t)\alpha^S}\right\} < \mu_L < \mu_{up}. \tag{10}$$

To understand the economic intuition of this endogenous escape, we can refer to the informativeness of the equilibrium price, which is defined by the following quantity in lines with a typical context of noisy REE models.

DEFINITION 3: *Informativeness of the price signal q is defined as the precision of the* *signal compared to the insider information.*

$$\eta(q_t) = \log \frac{Var(v_{t+2}|q_t)}{Var(v_{t+2}|\theta_{t+2})}.$$

From the perspective of agents at date $t+1$, who need to refer to the lagged information, the variable above records the amount of risk about the realization of the risky asset for agents who only observe the price signal q_t, compared to the risk remaining when one knows the insider information θ_{t+2}. The informativeness of the price η is typically positive when q cannot resolve the same amount of risk as the insider information due to the noise, that means $Var(v_{t+2}|q_t) > Var(v_{t+2}|\theta_{t+1})$ and $\eta > 0$. As the information contents of the price would be more precise, η goes down and converges to 0. Hereafter, I also use the term "price informativeness" by referring $\sigma_u^2\eta(q_t)$ since the dynamics of it will be the matter of focus and the precision of the insider information is not time varying.

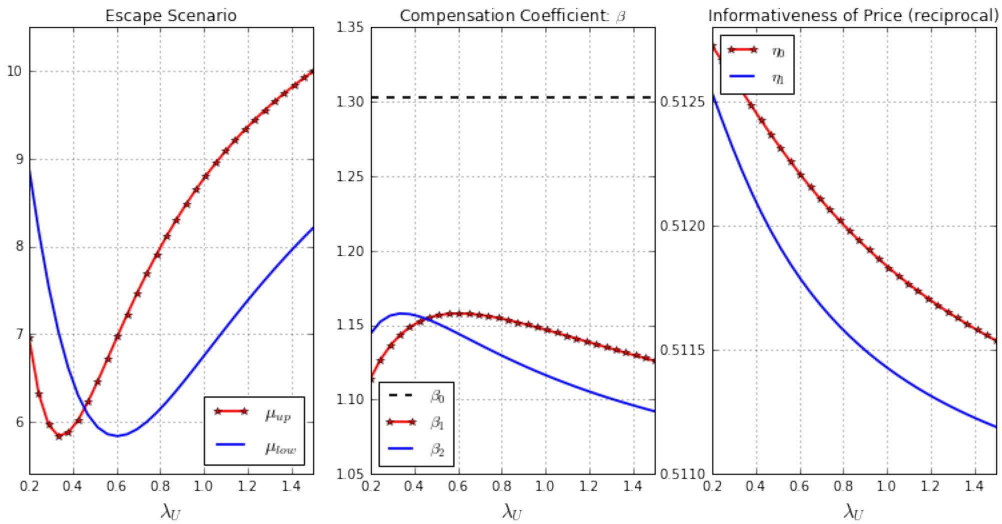
Note that, in the previous example, the (normalized) price informativeness under the condition that the U-agents are participating at $t = 1$ would be such that

$$\eta(q_t) = -\log(\tau\alpha_{t-1}^U) - \log(\sigma_u^2).$$

Since we are considering the degenerated case with pseudo-stochastic variables, the U-agent will be more compensated and gets more optimistic (relative to the worst case scenario) if the precision of the signal is higher. Thus, in the first period, if we had a less noisy signal, then it made the U-agent with $i = 0$ confident enough and she decides to trade. However, in turn, this makes the price signal to which the next generation ($i = 0, 1$) will refer much noisier. As a result, under the parameter condition in (7), this effect leads to erosion of the confidence of the U-agent and kicks her out from the market. This behavior of the U-agent has an effect on the price and the portfolio of the S-agent.

As shown in Figure 3, the informativeness of price q_{t-1} decreases from $t = 1$ to $t = 2$ (note that $t = 0$ is given). We can see from the figure that a higher value of λ_U precipitates information contamination within a period as well as across the periods. This is consistent with the intuition that a larger number of uninformed traders contaminates the price signal more. As a result, this leads to the drop of the compensation coefficients and it gets more difficult for U-agents with the price information to be compensated enough to make a trade. This scenario holds only if the value of λ_U is large enough (there is no region for $\mu \in [\mu_{low}, \mu_{up}]$ if λ_U is low). Around the small value of λ_U, three periods are not long enough for the participation of U-traders so that it contaminate the price signal to make the next generation reluctant to trade.

FIGURE 3. DYNAMICS AND COMPARATIVE STATICS



V. Full Dynamic Model of Market Crash

The basic environment is the same as the three periods example, but we assume $\lambda_I > 0$. My goal in this section is to describe the market crash and overshooting of the price without any aggregate shocks nor successive idiosyncratic shocks. Therefore, after calculating the equilibrium dynamics, I will make all stochastic variables, except u, pseudo-random variables. The full stochastic model can be considered as an extension and we can derive an endogenous probability of the crash (hazard probability) as well as its dynamic property. It will be clear that the role played by I-trader, who did not exist in the previous section, is fundamental to replicate the realistic price movements around the crises.

The procedure of the analysis is as follows. First, I will consider the equilibrium where $i = \{0, 1, 2, 3, \cdots, m\}$ of U-agents are participating at date t. This yields the local dynamics of the key variables and will be used to derive the critical value for escape. Next, I will show the entire dynamics of variables in the case with an imitation behavior, i.e., if the group i was investing at date t, then the group $i+1$ also start trading in the next period. This information cascade continues until the information is *reset* by the market crash.

A. Dynamics of the Economy

I focus on the PSGE. Suppose that $i \in \{0, 1, 2, \cdots, m\}$ of U-agents, I-agents and S-agent are participating in the market at date t. We go back to the initial setting and thus S and I-trader have the signal s_t^s and s_t^I. Behavior and the optimal market order of each agent are given by the following proposition.

PROPOSITION 2: *Given P_t and available information sets, the optimal portfolio for each type of trader is given by*

$$x_t^S = \alpha^S(\theta_{t+1} - P_t),$$

$$x_t^I = \alpha_t^I[(1-\beta_t^z - \rho\beta_t^q)\mu + (\beta_t^z + \rho\beta_t^q)\mu_L^I - P_t]\chi_t^I,$$
$$x_t^{U,i} = \alpha_t^U[\rho\beta_t^U\mu + (1-\rho\beta_t^U)\mu_L^U - P_t]\chi_t^U,$$

where the compensation coefficients β_t^j are determined by a typical linear filtering problem given by Proposition 3 below. Also, $\chi_t^j = \mathbb{I}_{\{j-trading\}}$ is an indicator function of participation of trader-j. The investment coefficient α_t^j is given by

$$\alpha_t^j \equiv \tau^{-1}Var(v_{t+1}|y_t^j).$$

As induced from the example in the previous section, I guess the form of the price signal at t would be

$$q_t = \theta_{t+1} + \gamma_t^s z_{t+1}^s + \gamma_t^I z_{t+1}^I + \gamma_t^e \varepsilon_{t+1}, \tag{11}$$

with some coefficient function $\gamma_t : \mathbb{T} \to \mathbb{R}$. Intuitively, the first and the second terms of the signal are noise contributions by S and I-trader respectively. The last term is a noise brought by the traders who are relying on the past signal, i.e., I and U-traders.

Let $\mathcal{A}$ denote the entire set U-agents, and the subset $\mathcal{A}_{t+1} \subseteq \mathcal{A}$ for each $t+1$ represent the subset of U-traders who have the information q_t. Presumably, we have that $\mathcal{A}_t = \{0, 1, 2, \cdots, m\}$. Hence, by the assumption of the signal acquisition, we have $\mathcal{A}_{t+1} = \mathcal{A}_t \cup \{(m+1)\}$.

The market clearing condition at date t is given by

$$x = x_t^S + \lambda_I x_t^I + \sum_{i \in \mathcal{A}_t} \lambda_U x_t^U,$$

and this yields the price signal of date $t+1$ such that

$$q_{t+1} = \theta_{t+2} + \gamma_{t+1}^s z_{t+2}^s + \gamma_{t+1}^I z_{t+2}^I + \gamma_{t+1}^e \varepsilon_{t+2}.$$

This implies that, given $(\gamma_t^j)_{j\in\{s,I,e\}}$, we can calculate the optimal portfolio and $\{(\beta_{t+1}^j)_{j\in\{q,z,U\}}, (\alpha_{t+1}^j)_{j\in\{I,U\}}\}$. Then, together with the market clearing condition, we can get q_{t+1} as well as $(\gamma_{t+1}^j)_{j\in\{s,e,I\}}$. As a consequence, entire evolutions of variables can be given by difference equations system as in Kyle (1985). The exact specification is in Proposition 5. Now I define the "reset" or "crash" in the following sense.

DEFINITION 4: *Consider the situation such that the no U-agents are in the market at dates $t < t_0 - 1$ and the idiosyncratic shock makes $i = 0$ trader trade in the market at $t = t_0 - 1$. We define the time of the "crash" or equivalently the "reset" of the market by T^* such that*

$$T^* = \inf\{t \geq t_0 | \chi_t^I \chi_t^{U,i} = 0 \ \forall i \in \mathcal{A}\}.$$

Then, the initial guess (11) can be shown to be correct as long as the economy is in the "pre-reset" periods.

PROPOSITION 3: *Given the initial shock to U-trader, the information content of price P_t takes the form such as (11).*

Therefore, the informativeness of price is time dependent and this would be critical for the economy when she moves toward the crash as we have seen in the previous example. Moreover, the dynamics of the coefficients are given by the following proposition.

PROPOSITION 4: *For each $t < T^*$, given the initial conditions as well as the initial trigger shock to $i = 0$ U-trader, the dynamics of the key variables are recursively given by the following non-linear difference equations, i.e., for q_{t-1} as a linear function of $(\gamma_{t-1}^j)_{j \in \{s,I,e\}}$ as in (11), we have*

$$\begin{pmatrix} \beta_t^z \\ \rho \beta_t^q \end{pmatrix} = \begin{pmatrix} \sigma^2_{s_t^I}, & \sigma_{s_t^I, q_{t-1}} \\ \sigma_{s_t^I, q_{t-1}}, & \sigma_{q_{t-1}^2} \end{pmatrix}^{-1} \begin{pmatrix} \sigma_{s_t^I, v_{t+1}} \\ \sigma_{q_{I_{t-1}}. v_{t+1}} \end{pmatrix}, \tag{12}$$

$$\beta_t^U = \frac{Cov(q_{t-1}, v_{t+1})}{\rho Var(q_{t-1})}, \tag{13}$$

$$\alpha_t^I = \tau^{-1} Var(v_{t+1} | (s_t^I, q_{t-1})), \tag{14}$$

$$\alpha_t^U = \tau^{-1} Var(v_{t+1} | q_{t-1}), \tag{15}$$

$$\gamma_t^s = \frac{\alpha^S \beta}{\alpha^S \beta + \lambda_I \alpha_t^I (\beta_t^z + \beta_t^q) + (t+1) \lambda_U \alpha_t^U \beta_t^U}, \tag{16}$$

$$\gamma_t^I = \frac{\lambda_I \alpha_t^I \beta_t^z}{\alpha^S \beta + \lambda_I \alpha_t^I (\beta_t^z + \beta_t^q) + (t+1) \lambda_U \alpha_t^U \beta_t^U} \tag{17}$$

$$\gamma_t^e = -\frac{\lambda_I \alpha_t^I \beta_t^q + (t+1) \lambda_U \alpha_t^U \beta_t^U}{\alpha^S + \lambda_I \alpha_t^I (\beta_t^z + \beta_t^q) + (t+1) \lambda_U \alpha_t^U \beta_t^U}. \tag{18}$$

PROOF:

See Appendix.

Note that the result above holds under the condition that all agents are taking a long position until the reset which is guaranteed by the assumption of $\mu_H = \infty$. The difference equation system is highly non-linear and we cannot make analytical proposals as for the property of the variables. However, numerical simulation of this system gives us an interesting example of booms and crises. Note that this system does not have to be a stable one and I do not analyze the uniqueness of the solution. We need to impose a parameter setting to make a model stable for the S and I equilibrium, but, as we will see later, S, I and U-traders system can be exploding.

Intuitions of difference equations are quite simple. Compensation coefficients are determined by the linear filtering problem of Gaussian random variables and α can be interpreted as a degree of risk each agent will be exposed after she saw the set of information. These variables basically reflect how informative the price would be. As for the noise coefficients γ, the denominator expresses the total number of traders weighted by the portfolio coefficients and compensation coefficients. The numerator of each γ is also the total number of traders who are related to the noise. Since z^S is held only by S agent, the weight γ^S has $\alpha^S\beta$, which is the coefficients of the portfolio of S trader. Note that the noise ε is associated with both I and U-traders.

B. Herding and Escape

As in the three period example, we consider the participation behavior of the $(i, II, t+1)_U$ agents with $i \in \mathcal{A}_{t+1}$, given that agents $\{(i, II, t)_U; i \in \mathcal{A}_t\}$ took non-zero position. In this model, there are two possible situations as a benchmark scenario: an equilibrium with S and I-trader or an equilibrium with S-trader only. I will focus on the first case to replicate the interesting and realistic situation of a crisis.

PROPOSITION 5: *Given that U-agents $\{(i, II, t); i \in \mathcal{A}_t\}$ and I-agent took a non-zero position at date t, U-agents $\{(i, II, t+1); i \in \mathcal{A}_{t+1}\}$ and I-agent in the next generation (i): take non-zero position if and only if*

$$\min\{\underline{v}^I_{t+1}(s^I_{t+1}, q_t), \underline{v}^U_{t+1}(q_t)\} > P_{t+1},$$

(ii): I-trader stops trading while U-traders keep trading iff

$$\underline{v}^I_{t+1}(s^I_{t+1}, q_t) < P_{t+1} < \underline{v}^U_{t+1}(q_t),$$

and (iii): both traders stop trading iff

$$\max\{\underline{v}^I_{t+1}(s^I_{t+1}, q_t), \underline{v}^U_{t+1}(q_t)\} < P_{t+1}.$$

Intuitively, β^j_t governs how naive agents are compensated by looking at the past price information (see (4) and Proposition 2). By construction, if the neighbors were trading in the previous period, today's phase II agents know the past price information. In the degenerate case, the ex-post realization of the signal is constant μ. Thus, the signal puts the upward improvement on the prior expectation and its weight is controlled by β^j_t. Therefore, as the weight β^j_t to μ gets larger, the more improved the filtered worst case expectation would be. This implies (as if) more optimistic expectation is formed under the precise signal and therefore naive-agents are more likely to take a long position. Also, the effect of the ambiguity on the posterior expectation is clear from Proposition 2.

PROPOSITION 6: *If naive agents are facing a more ambiguous risky asset/ having a higher degree of ambiguity aversion, then the escape is more likely to happen.*

A lower μ_L (or a greater $\mu - \mu_L$, in a relative sense) implies a more ambiguous situation or a stronger aversion toward ambiguity[13]. Thus, Proposition 6 is obvious from Proposition 2 and Proposition 5. The result is quite intuitive. A lower $\mu - \mu_L$ makes the escape more likely since the naive-agents need to be compensated more in order to stay in the market.

If $T^* \in (0, \infty)$ a.s., then we want to think of it as a boom and bust of the economy. Aside from the price movement, we have shown by Proposition 5 that the number of U-agents who are trading in the market keeps growing during $t < T^*$ as long as the condition (i) in Proposition 5 is satisfied. This herding behavior is a typical characteristic of the boom we can see in the real economy. If we let m_j be the number of the j-trader trading in the market, the population of the traders in the market is determined endogenously and evolves as the following sequence as long as (i) in Proposition 5 keeps holding.

$$
\begin{aligned}
(m_S, m_I, m_U)_{t<t_0} &= (1, \lambda_I, 0), \\
(m_S, m_I, m_U)_{t=t_0} &= (1, \lambda_I, \lambda_U), \\
(m_S, m_I, m_U)_{t<t_0} &= (1, \lambda_I, 2\lambda_U), \\
\vdots \qquad & \qquad \vdots
\end{aligned}
$$

We know from Proposition 6 that this sequence of population of traders determines the information variables (α, β, γ) and the participation behavior of each generation.

Also, the root of these herding and imitation are basically the lack of a reliable information. When the irrational agents do not have credible and crude information, they tend to imitate the neighboring agents if they are taking some actions. These characteristics are widely observed in the laboratory experiments as well as the real market (See the papers cited in Section II for more details).

Moreover, one of the novel points of this model is a simultaneous escape of naive-agents from the market. During the boom, the herding behavior of uninformed naive-agents accumulates the noise. Before the market is reset, each naive-agent does not care about the effect of her own behavior on her neighbor, and therefore she (her group) keeps trading until the information contamination will be severe enough to cross the critical point. This can be thought as the net-

[13] The model with MEU does not allow us to separate these two concepts, but the smooth ambiguity does. See Klibanoff et al. (2005) for more detail of arguments about the separability.

work externality in the sense of Acemoglu et al. (2015) and other literature of network theory.

Proposition 5 says there are several scenarios we can consider by using this model. Suppose that the initial trigger shock for $i = 0$ agent is precise enough to make her trade at date $t = t_0$. Then, the most interesting scenario is as follows.

DEFINITION 5: *The crisis scenario is characterized by the crash time T^* in Definition 4 which satisfies following conditions.*

(i) For date $t < T^*$, cascade occurs and all U-traders in $\mathcal{A}_t$ and I-trader are trading, i.e.,

$$P_t < \min\{\underline{v}_t^I, \underline{v}_t^U\}.$$

(ii) At date $t = T^*$, participation of U-traders contaminates q_t enough to make I-traders stop trading, i.e.,

$$\underline{v}_t^I < P_t < \underline{v}_t^U.$$

(iii) At $t = T^* + 1$, the escape of I-trader makes q_t less precise and leads to escape of U-traders, i.e.,

$$\max\{\underline{v}_t^I, \underline{v}_t^U\} < P_t.$$

Let this scenario be labeled as the crisis scenario. Now we are in a position to think about the role of I-trader more in details.

C. *Escape Leads to the Further Escape: Overshooting*

As shown in the previous section, escape behavior can be described without I-traders. The role of I-trader is attributed to replicating the realistic price movements during crises. In the real world, a market crash is defined by a huge drop in a market price. In the model only with U-traders, however, we cannot obtain that kind of drop. The reason is clear; as the share of U-traders, $(t+1)\lambda_U$ approaches to the critical point, we must have $\underline{v}^U - P$ close to zero. This implies that the portfolio order by U-traders is almost zero and has almost no effect on the price. Even if U-traders escaped, the change in the price would be small and there is no tremendous price drop, nor overshooting. In the real world, on the other hand, a huge price drop is sometimes a result of sequential selling which is caused by the previous selling by other traders. In other words, escape behavior of some traders can lead to the further escape of other agents. To describe this feature, I-trader plays an important role.

Note that, in the initial setting, both S and I-trader have a noisy signal. Thus we can consider the following situation.

(i): An equilibrium price with S and I-trader is more precise than the price formed by S-trader only. Also,

(ii): I-trader will escape from the market before U-traders will.

Under these two conditions, information contamination caused by U-traders kicks I-trader out first by (ii), but the escape of I-trader contaminates the price signal further by (i). Then, this can push $\underline{v}$ of U-traders downward discretely and U-traders will also escape in the next period. Thus, participation behavior of U-traders would be a trigger of the escape of I-trader, and this can be a trigger of the second round escape by U-traders themselves. Escape of I-trader does not make much of difference in the price level following the logic above. However, the second round escape can create a huge drop, which may overshoot compared to the initial price level.

The situation in (ii) can be found easily if we set $\mu_L^I < \mu_L^U$, i.e., informed naive trader is more ambiguity averse/ facing more ambiguous situation than uninformed naive traders. In this model, informed vs uninformed is not necessarily associated with more ambiguous vs less ambiguous. We can think of the situation such as the agents get more confused if they get more information. Even though the signal can resolve the risk, this cannot always reduce ambiguity in the sense of Knight. In line with a robust control model described by Hansen and Sargent (2001), I-trader takes more robust decision than U-traders.

As for the situation (i), we can think about the case where the noisy signal of S-trader is not precise enough compared to the signal of I-trader, or they might be negatively correlated. To see this, we can compare the price informativeness under the equilibrium with S and I-trader and that with S trader only. Let a superscript $k \in \{I, S\}$ denote variables under each equilibrium, and define the difference of informativeness of prices under these two equilibria by

$$\Delta_t = \log \frac{Var(v_{t+1}|q_{t-1}^S)}{Var(v_{t+1}|q_{t-1}^I)},$$

where two equilibrium price signals are given by $q_t^S = \theta_{t+1} + z_{t+1}^S$ and q_t^I is identical to (11). Therefore,

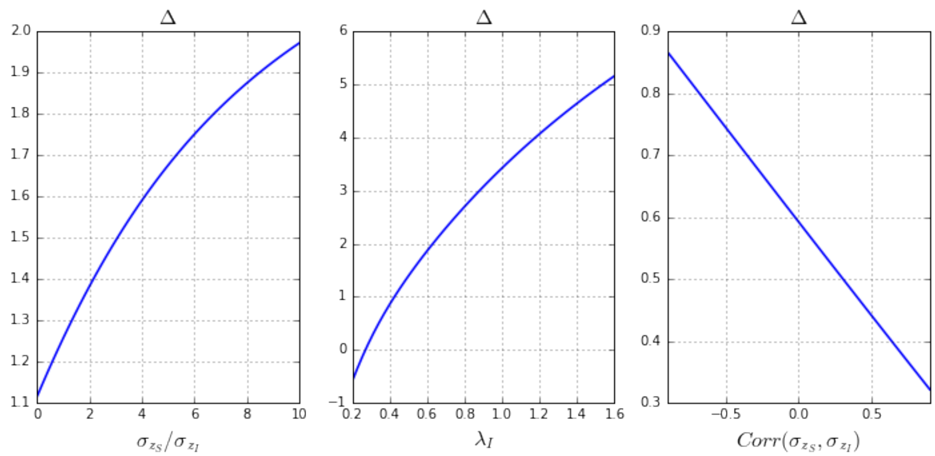
$$Var(q_t^S) = \sigma_\theta^2 + \sigma_{z_S}^2, \tag{19}$$

$$Var(q_t^I) = \sigma_\theta^2 + 2\gamma_t^e \sigma_e^2 + 2\gamma_t^S \gamma_t^I \sigma_{z_S} \sigma_{z_I} Corr_{s,I} + \sum_{i \in \{S,I,e\}} (\gamma_t^i)^2 \sigma_i^2. \tag{20}$$

We can check that participation of I-trader can improve the price information ($\Delta > 0$) or can harm it ($\Delta < 0$). Figure 4 shows how this effect varies according to the (relative) variance of signals, λ_I, and the correlation between z^I and z^s.

A higher Δ_t implies that q_t is more precise under S and I-trader's equilibrium. The figure shows a higher volatility of S-trader's signal compared to I-trader's signal makes the participation of I-trader meaningful in the sense of price informativeness. Also, if these two shocks are negatively and perfectly correlated, these two noise components would be canceled out and the price will be more

FIGURE 4. EFFECT OF I-TRADER ON Δ_t



precise. We can find the similar patterns even if we allow U-traders to participate.

The key takeaway is the fact that *there is a parameter region where the price signal will be less precise (contaminated) if I-trader stops trading*. This fact makes it possible for the crises scenario described in the previous subsection to realize, i.e., herding of U-traders contaminates the price signal, kicks I-trader out from the market, and this leads to further contamination of the price signal which finally makes U-traders inactive.

By making use of this property, the model replicates the price dynamics which is consistent with the main characteristics of booms and busts in the real economy.

PROPOSITION 7: *Let $\beta_t^I \equiv \beta_t^z + \rho\beta_t^q$. Under some parameter region, $\exists T^*$ which describes the crises scenario in Definition 5, i.e., (i): For $t < T^*$,*

$$\min_{j \in \{I, \mathcal{A}_t\},} \{\underline{v}_t^j\} > P_t$$

(ii) At date T^*, we have

$$\underline{v}_t^U > P_t > \underline{v}_t^I,\ U = \mathcal{A}_t,$$

which are equivalent to,

$$\rho\beta_{T^*}^U \mu + (1 - \rho\beta_{T^*}^U)\mu_L^U > (1 - \beta_t^I)\mu_L^I + \beta_t^I \mu,$$

$$\alpha^S(\mu - \underline{v}_{T^*}^I) + (t+1)\lambda_U \alpha_{T^*}^U (\underline{v}_{T^*}^U - \underline{v}_{T^*}^I) \geq x,$$

and

$$\alpha^S(\mu - \underline{v}_{T^*}^U) < x.$$

(iii) At date $T^* + 1$,

$$\max_{j \in \{I, \mathcal{A}_t\},} \{\underline{v}_t^j\} < P_t$$

or equivalently,

$$\beta_{T^*+1}^N < \frac{1}{\rho}[1 - \frac{x}{\alpha^S(\mu - \mu_L^N)}],$$

$$\underline{v}_{T^*+1}^P < P_{T^*+1},$$

where, given γ,

$$\beta_t^U = \begin{cases} \frac{\sigma_\theta^2 + \gamma_{t-1}^e \sigma_e^2}{\sigma_\theta^2 + 2\gamma_{t-1}^e \sigma_e^2 + 2\gamma_{t-1}^S \gamma_{t-1}^I \sigma_{z_S} \sigma_{z_I} Corr_{s,I} + \sum_{i \in \{S,I,e\}} (\gamma_{t-1}^i)^2 \sigma_i^2} & if\ t = T^* \\ \frac{\sigma_\theta^2 + \gamma_{t-1}^e \sigma_e^2}{\sigma_\theta^2 + (\gamma_{t-1}^S)^2 \sigma_{z_S}^2 + (\gamma_{t-1}^e)^2 \sigma_e^2 + 2\gamma_{t-1}^e \sigma_e^2} & if\ t = T^* + 1 \end{cases}.$$

Proposition 7 says we can find the parameter region in which the price drop will overshoot as in Figure 5. The mechanism is clear. An initial condition for the I-trader (if she did not participate in the previous period) makes I-trader keep trading and the economy evolves toward the steady state with S and I-trader, which would be the benchmark. Then the idiosyncratic shock hits the randomly picked U-trader and the cascade takes place, making U-traders sequentially participate. This flow contaminates the price signal and makes I-trader stop trading at some point. Then, the price signal gets contaminated more and U-traders also stop trading, and eventually, the economy converges to the equilibrium only with S-trader.

PROPOSITION 8: *Given T^* in Proposition 7 and under the crises scenario with overshooting, the price evolves as follows.*

(21)
$$P_t = \begin{cases} \frac{\alpha^S \mu + \lambda_I \alpha_t^I \underline{v}_t^I + (t+1)\lambda_U \alpha_t^U \underline{v}_t^U - x}{\alpha^S + \lambda_I \alpha_t^I + (t+1)\lambda_U \alpha_t^U} & if\ t \leq T^* \\ \frac{\alpha^S \mu + (t+1)\lambda_U \alpha_t^U \underline{v}_t^U - x}{\alpha^S + (t+1)\lambda_U \alpha_t^U} & if\ t = T^* \\ \mu - x/\alpha^S & if\ t \geq T^* + 1. \end{cases}$$

PROOF:

See Appendix.

Figure 5 provides the illustrative example of the crises scenario. As long as all types of traders are trading, the price would evolve according to P^{all} in the figure and it keeps increasing since the demand also increases as a number of U-traders keeps going up. This sequential participation contaminates the price signal and therefore, compensation coefficient as well as the worst case expectations of both I and U-traders decline[14]. Then, I define T^* as the timing of the switch of $\underline{v}_t^I > P_t$, when I-trader stops trading. As shown in Figure 4, this can contaminate

[14] Note that the amount of the market order of each trader decreases as the price goes up and the worst

FIGURE 5. CRISIS SCENARIO

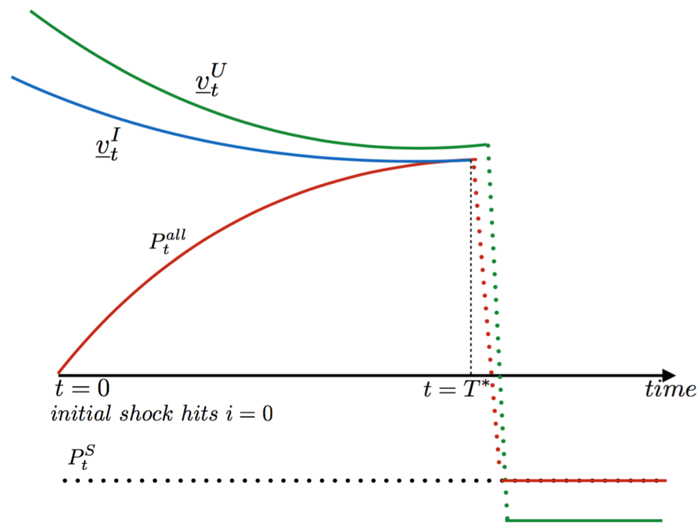
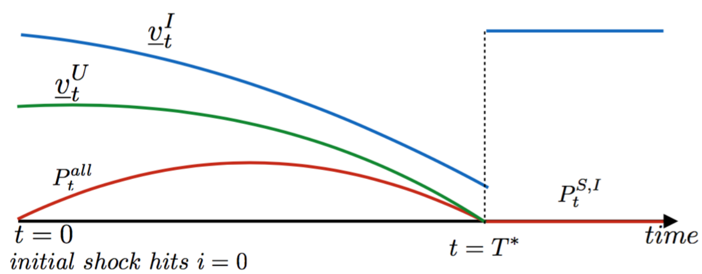


FIGURE 6. SOFT-LANDING SCENARIO



the price information further and make $\underline{v}_t^U$ jump down. Under some parameter region, this drop exceeds the price drop and thus the lower price will be an equilibrium, $P_t^{all} \to P_t^S$.

Also, we can think about the situation where U-traders escape faster than I-trader as described in Figure 6. In this case, the economy goes back to the initial steady state, and there is no tremendous price drop since U-traders stop trading after the price decreases and becomes close to $\underline{v}^U$. Thus, we can think of this situation as a soft-landing. Even though a boom takes place, the price does not jump down or overshoot. It gradually goes back to the initial point drawing a hump-shaped trajectory by construction. In this scenario, I-trader will not stop

case scenario goes down. However, the total amount of trading done by U-traders can go up if the effect of the increase in the number of U-traders dominates the negative effects on the individual's portfolio.

trading because, once U-traders stop trading, the informativeness of the price goes back to the initial level, where I-trader is compensated enough to trade.

Overall, this model could separate the two scenarios, one with a tremendous price drop with overshooting and one with a soft-landing, by looking at how active each type of agents might be.

VI. Empirical Implications and Discussions (preliminary)

This model provides at least two empirical implications. First, we can make a distinction of the booms which will lead to the crises and the one which will lead to soft-landing. Note that, in order for the model to replicate the crisis with overshooting price, we are more likely to have $x_t^I < x_t^U$ in terms of individual level portfolio. On the other hand, if we got $x_t^I > x_t^U$, then U-traders escape from the market faster than I-trader, which means we will not experience the huge drop in the price. This highlights the highly active uninformed traders described in Sornette (2004) and the citation in Introduction of this paper. In terms of this classification, if we could measure the information trade, heterogeneous degrees of ambiguity as well as their portfolio as in Collin-Dufresne and Fos (2015), then we can make a distinction of overshooting scenario and soft-landing scenario. What might be crucial here is the fact that the total investment activity by U-traders is more likely to dominate the activity of I-trader in both cases. However, if we could look at the individual activity or market order, then checking if each informed trader is more active than individual U-trader would be a clew to know whether the boom is harmful or not.

Moreover, this model suggests that price could be more volatile than we have expected in the model with only rational agents or homogeneous ambiguity averse agents. In the literature of the ambiguity aversion such as Epstein and Schneider (2008) and Mele and Sangiorgi (2010), it is shown that the existence of ambiguity averse trader makes the price more volatile since the price reacts more sharply to the informational shock under the equilibrium with ambiguity averse traders. Thus, ambiguity aversion itself can contribute to the volatile price. However, in my model, it is shown that heterogeneity in terms of ambiguity aversion can create further price fluctuation. If we consider the heterogeneous ambiguity averse traders with different sets of available information, the participation of the one type of traders can make other types of traders reluctant to trade. Then, this escape can lead to further escape as shown in the model and can be a source of overshooting price. In this sense, this model suggests that we have to incorporate the feedback type of participation behavior which makes the prices more volatile.

VII. Conclusion

Markets can crash without any aggregate nor successive idiosyncratic shocks. This model provides one of the critical situations in which the initial idiosyncratic trigger shock can create the market booms and busts. The key factors of the entire story are the herding participation of irrational uninformed agents, who crucially need to have information, and the informativeness of the price signal. If the market is completely occupied by the sophisticated agents who have an insider information, then the price informativeness is extremely high. Thus, if this information is accidentally leaked to one of the irrational agents (say agent $i = 0$), she gets compensated (would be optimistic) and decides to start trading. This initial agent's behavior can be seen from her neighbors $i = 1$, which can be interpreted as further information leakage from the initial agent. The participation of irrational/uninformed agents itself adds the noise to the insider information. Therefore, up to some moment, irrational agents who get a signal are compensated enough to trade since the number of participating irrational agents is sufficiently small. So they start trading. However, their participation itself contaminates the information of the price and they find the signal they will use in the next period not reliable enough, and stop trading, i.e., simultaneously escape from the market. This sudden escape from the market creates the crash of the price under some parameter conditions.

This model further contributes to the discussion of overshooting as well as escape (selling) behavior which leads to further escape (selling). As described by Sornette (2004), uninformed traders' participation may contaminate the price signal, and this can make another agent feel uncomfortable to trade. If this agent has been contributing to make precise information, her escape makes the situation worse; the price signal will be more contaminated. Then, this first round escape can cause the second round escape by uninformed naive traders, which induces a discrete drop of demand and can be a trigger of overshooting.

Overall, we could see how ambiguity aversion affects the booms and busts of the market. Not only can it make the informativeness of the price a source of the booms, but it also can be a self-fulfilling reason for the market crashes. The Ss type portfolio strategy is the key mechanism and also makes the information flow much more important compared to the existing literature. We can know how the boom will end by focusing on the time trajectory of informativeness of the price, as well as the portfolio of each type of trader.

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Appendix A

The following values of parameters give the examples of crises scenario and the soft-landing scenario.

Appendix B

TABLE A1—EXAMPLE PARAMETER VALUES FOR CRITICAL SCENARIOS

parameter	values: crises scenario	values: soft-landing	
λ_I	2.30	0.01	measure of I-trader
λ_U	0.01	0.80	measure of U-traders
σ_e^2	1.30	0.10	variance of ε
σ_u^2	1.10	0.30	variance of u
$\sigma_{z^S}^2$	20.0	0.01	variance of z^S
$\sigma_{z^I}^2$	20.0	0.01	variance of z^I
ρ	0.78	0.90	persistence of θ
$\rho_{S,I}$	0.80	0.50	$Corr(z^S, z^I)$
τ	3.00	3.00	absolute RA
μ	28.0	1.50	mean of μ
μ_L^I	3.10	1.40	worst case μ of I
μ_L^U	9.50	1.20	worst case μ of U