UNIVERSITY OF TOKYO

1st Finance Junior Workshop Program

Monetary Policy and Welfare Issues in the Economy with Shifting Trend Inflation

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1. Introduction
Exercises

- This paper studies a standard New Keynesian model with Calvo price setting, shifting trend inflation and without full price indexation
- Two assumptions: Central Bank
  - Set a positive inflation target
  - But lack commitment to a fixed inflation target (trend inflation behaves as a shock)
- Objectives:
  - Quantify the welfare cost and inefficiency sources of shifting trend inflation using the U.S data

Rationales

Necessities of Study on Trend Inflation

- Trend Inflation: Central Bank's inflation target and private sector's long run expectation.
- Restrictive assumptions: inflation target must be zero and full price indexation

➔ empirically unrealistic because

  - Exceedingly rare (the authority always sets the positive inflation target)
  - Misleading conclusions (e.g. "divine coincidence", a highly non-linear and positive slope of the long-run NKPC…)
  - Washing out implications of micro-foundations (price dispersion, marginal markup, discounting)
  - Welfare's issue: Inappropriateness of standard welfare function (Woodford, 2003)

Rationales

Necessities of Study on Shifting Trend Inflation

- Shifting Trend Inflation: the authority lacks a commitment to pursue a fixed inflation target→ target can change (Policy Implementation Inconsistency)
- Few papers pay attentions to time-varying property of trend inflation to indicate its necessities:
  - Cogley (2008): a fitting data problem from monetary regimes.
  - Nakata (2014): changes in welfare of representative agents.
  - Consistent to reality

Research Questions

- How does the Policy Implementation Inconsistency (shifting trend inflation) affects the economy and brings about consequences?
- What are components causing economy to deviate from its efficiency (the inefficient sources) and how they magnify the welfare cost?

Limitations of Closely Related Papers

- Woodford (2003): the approach is not consistent
  - Curvature of loss function (small as of zero inflation) sharply increase as trend inflation grows
  - Steady-state variable are still dynamic, while exogenous shocks remain stable at their mean

  →thus the standard welfare loss function underestimates welfare under positive trend inflation.
- Alves (2012, 2014): find new approach but abstract the property of shifting trend inflation
- Nakata (2014): successes in computing the welfare function But not control the radius of convergence and the appropriateness of local approximation is wholly problem specific

Contributions

- Show the exact magnitude of persistence for trend inflation
- Indicate different channels that shifting trend inflation affects the economy
- Find a new approach to derive the inefficiency sources

2. Overview of the Model

- 4 sectors:
  - Household
  - Intermediate Producers
  - Final Producers
  - Monetary Authority
- structural shocks
  - Technology(ϵ_{zt}), Cost-push (ϵ_{θ_t}), government expenditure (ϵ_{gt}), interest rate (ϵ_{Rt}), or money growth(ϵ_{mt}), and shock to trend inflation($\epsilon_{\overline{\pi,t}}$).

Household sector

Households solve a problem that how they maximize their utility with respect to a given budget constraint

$$\underset{B_t,M_t,C_t,h_t}{\text{Max}} E_o \sum_{t=0}^{\infty} \beta^t \left[\ln(C_t - \gamma C_{t-1}) + \ln\left(\frac{M_t}{P_t}\right) - \frac{\omega}{1+v} H_t^{\,1+v} \right]$$

Such that: $P_t C_t + \frac{B_t}{r_t} + M_t = M_{t-1} - P_t T_t + B_{t-1} + W_t h_t + D_t$

Final goods-producing firm

The competitive final good producing firms solve a problem that how they maximize their profit with a given technology

- θ_t: price elasticity of demand for intermediate goods.
- The cost-push shock: $\ln(\theta_t) = (1 - p_\theta)\ln(\theta) + p_\theta \ln(\theta_{t-1}) + \epsilon_{\theta_t}$
- The competitive final good producing firms maximize the profit:

$$\text{Max } \pi_t^I = P_t \left[\int_0^1 Y_t(i)^{\frac{\theta_t - 1}{\theta_t}} di \right]^{\frac{\theta_t}{\theta_t - 1}} - \int_0^1 P_t(i) Y_t(i) di$$

The constant-return-to scale technology: $\left[\int_0^1 Y_t(i)^{\frac{\theta_t - 1}{\theta_t}} di \right]^{\frac{\theta_t}{\theta_t - 1}} \geq Y_t$

Intermediate Goods-Producing Firms

A positive steady state inflation

- A monopolistic competitive firm produces an intermediate good using a linear production technology: $Z_t\, h_t(i) \geq Y_t(i)$
- The aggregate technology shock: $\ln(Z_t) = \ln(z) + p_z \ln(Z_{t-1}) + \epsilon_{zt}$
- A Calvo Model: a fraction η of firms cannot optimize prices, but can update it

  $P_t(i) = \left(\pi_{t-1}^{\mu}\bar{\pi}_{t-1}^{1-\mu}\right)^{\chi} P_{t-1}(i)$, where $\bar{\pi}_t$: Authority's inflation target
- $(1-\eta)$ Intermediate-goods producing firms set the price P_t^* to optimize profit

$$\max_{Y_{i,t},P_{i,t}} E_t \sum_{j=0}^{\infty} \frac{\lambda_{t+j}}{\lambda_t}\eta^j \left\{ \frac{P_{i,t}^*\left(\bar{\pi}_t^{\chi j}\right)^{1-\mu}\left(\pi_{t-1,t+j-1}^{\chi}\right)^{\mu}}{P_{t+j}} Y_{i,t+j} - \frac{W_{t+j}}{P_{t+j}}\left(\frac{Y_{i,t+j}}{Z_{t+j}}\right)\right\}$$

$$\text{s.t } Y_{i,t+j} = \left[\frac{P_{i,t}^*\left(\bar{\pi}_t^{\chi j}\right)^{1-\mu}\left(\pi_{t-1,t+j-1}^{\chi}\right)^{\mu}}{P_{t+j}}\right]^{-\theta} Y_{t+j}$$

$$\pi_{t,t+j} = \left(\frac{P_{t+1}}{P_t}\right)\left(\frac{P_{t+2}}{P_{t+1}}\right)\ldots\left(\frac{P_{t+j}}{P_{t+j-1}}\right) \qquad for\ j = 1,2,3\ldots$$

Intermediate Goods-Producing Firms

A positive steady state inflation

The first order condition

$$\frac{P_{i,t}^*}{P_t} = \frac{\theta}{\theta-1} \frac{E_t \sum_{j=0}^{\infty} (\beta\eta)^j \Lambda_{t+j} \frac{W_{t+j}}{P_{t+j}} \frac{Y_{t+j}}{Z_{t+j}} Y_{i,t+j} \left[\frac{P_{i,t}^* \left(\bar{\pi}_t^{\chi j}\right)^{1-\mu} \left(\pi_{t-1,t+j-1}^{\chi}\right)^{\mu}}{P_{t+j}} \right]^{-\theta}}{E_t \sum_{j=0}^{\infty} (\beta\eta)^j \Lambda_{t+j} Y_{i,t+j} \left[\frac{P_{i,t}^* \left(\bar{\pi}_t^{\chi j}\right)^{1-\mu} \left(\pi_{t-1,t+j-1}^{\chi}\right)^{\mu}}{P_{t+j}} \right]^{1-\theta}}$$

$$\rightarrow \frac{P_{i,t}^*}{P_t} = \frac{\theta}{\theta-1} \frac{No_t}{De_t}$$

- $$No_t = E_t \sum_{j=0}^{\infty} (\beta\eta)^j \Lambda_{t+j} \frac{W_{t+j}}{P_{t+j}} \frac{Y_{t+j}}{Z_{t+j}} Y_{i,t+j} \left[\frac{P_{i,t}^* \left(\bar{\pi}_t^{\chi j}\right)^{1-\mu} \left(\pi_{t-1,t+j-1}^{\chi}\right)^{\mu}}{P_{t+j}} \right]^{-\theta}$$

$$\rightarrow No_t = w_t + \beta\eta \left(\bar{\pi}_t^{-\chi\theta}\right)^{1-\mu} \left(\pi_t^{-\chi\theta}\right)^{\mu} E_t\{\pi_{t+1}^{\theta} No_{t+1}\}$$

- $$De_t = E_t \sum_{j=0}^{\infty} (\beta\eta)^j \Lambda_{t+j} Y_{i,t+j} \left[\frac{P_{i,t}^* \left(\bar{\pi}_t^{\chi j}\right)^{1-\mu} \left(\pi_{t-1,t+j-1}^{\chi}\right)^{\mu}}{P_{t+j}} \right]^{1-\theta}$$

$$\rightarrow De_t = 1 + \beta\eta \left(\bar{\pi}_t^{\chi(1-\theta)}\right)^{1-\mu} \left(\pi_t^{\chi(1-\theta)}\right)^{\mu} E_t\{\pi_{t+1}^{\theta-1} De_{t+1}\}$$

- Price dispersion: $s_t = (1-\eta)\left(p_{i,t}^*\right)^{-\theta} + \eta\left(\bar{\pi}_{t-1}^{-\chi\theta}\right)^{1-\mu} \left(\pi^{-\chi\theta}\right)^{\mu} \pi^{\theta} s_{t-1}$

Intermediate Goods-Producing Firms
Positive vesus Zero Inflation Target

- The evolution of price depends on the previous inflation rate and the trend inflation
- The future expected inflation rates enter on both No_t and De_t, thus have effects on the future variables. Price-setting becomes more "forward-looking" so inflation does
- Implications of microfoundations (price dispersion term, marginal markup, discounting) appears when trend inflation is positive but disappear when it is zero.

The authority
Monetary Policy

- **The Taylor rule:** $\frac{r_t}{r} = \left(\frac{r_{t-1}}{r}\right)^{p_r} \left[\left(\frac{\pi_t}{\bar{\pi}_t}\right)\left(\frac{y_t}{y}\right)^{p_y}\right]^{1-p_r} \delta_r e^{rt}$
  - Where $\bar{\pi}_t$ is trend inflation, $y_t = \frac{Y_t}{Z_t}$, r and y are deterministic levels of r_t and y_t
- **Two properties of trend inflation:**
  - Increase over time
  - High persistence
- **The model under two different assumptions on the process of trend inflation**
  - $\ln \bar{\pi}_t = (1-\rho_\pi)\ln \bar{\pi}^* + \rho_\pi \ln \bar{\pi}_{t-1} + \epsilon_{\overline{\pi,t}}$

    (capture the second property and high probability of negative number)
  - $\ln[\bar{\pi}_t - 1] = (1-\rho_\pi)\ln[\bar{\pi}^* - 1] + \rho_\pi \ln[\bar{\pi}_{t-1} - 1] + \epsilon_{\overline{\pi,t}}$

    (capture both properties)

The authority
Fiscal Policy

- The government budget resource is represented as

$$\frac{M_{t-1}}{P_t} + B_t + P_t G_t = P_t T_t + \frac{B_{t+1}}{r_t} + \frac{M_t}{P_t}$$

- Government expenditure is financed by lump-sum taxes and seigniorage as follows

$$G_t = T_t + M_t - \frac{M_{t-1}}{\pi_t}$$

- Let g_t denote the the government spending growth and we have

$$G_t = \left(1 - \frac{1}{g_t}\right) Y_t \; where \; g_t > 1\text{: the gov expenditure growth}$$

- Where g_t is an AR(1) process

$$\ln(g_{t+1}) = (1 - p_g)\ln(g) + p_\theta \ln(g_t) + \epsilon_{\theta_t}$$

Market Clearing Conditions

- The market clearing condition in the labor market can be expressed as

$$H_t = \int H_t(i)di$$

- The market clearing condition in the good market

$$Y_t = C_t + G_t \rightarrow Y_t = C_t + \left(1 - \frac{1}{g_t}\right) Y_t \rightarrow C_t = \frac{1}{g_t} Y_t$$

- Finally, the zero net supply of bond is

$$B_t = 0$$

Welfare Cost Computation

The compensation variation in consumption that enhances the welfare of a typical household in one economy to make them as better-off as others in another economy

$$E\left\{\sum_{t=0}^{\infty} \beta^t u\left(\left(1+\frac{wc}{100}\right) C_{A,t}, H_{A,t}, m_{A,t}\right)\right\} = \left\{\sum_{t=0}^{\infty} \beta^t u\left(C_{B,t}, H_{B,t}, m_{B,t}\right)\right\}$$

Where $C_{A,t}$, $H_{A,t}$, $m_{A,t}$ are consumption, labor supply and money growth in the economy with $\sigma_{\overline{\pi}} > 0$ and $C_{B,t}$, $H_{B,t}$, $m_{B,t}$ are in economy with $\sigma_{\overline{\pi}} = 0$. .

Steady-State Distortions
Social Planner

- Q_t: the optimal consumption
- Social planer maximize Social welfare function under the frictions associated monetary trade and sluggish price adjustments

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[\ln(Q_t - \gamma Q_{t-1}) + \ln\left(\frac{\mathrm{M}_t}{\mathrm{P}_t}\right) - \int_0^1 \omega \frac{n_t(i)^{1+v}}{1+v} di \right]$$

- The aggregate feasibility constraint: $Z_t \left[\int_0^1 n_t(i)^{\frac{\theta_t}{\theta_t - 1}} di \right]^{\frac{\theta_t}{\theta_t - 1}} \geq g_t Q_t$ (Market Clearing Condition)
- Compare to Household's problem:

$$\underset{\mathrm{B}_t, \mathrm{M}_t, \mathrm{C}_t, h_t}{\mathrm{Max}} E_o \sum_{t=0}^{\infty} \beta^t \left[\ln(C_t - \gamma C_{t-1}) + \ln\left(\frac{\mathrm{M}_t}{\mathrm{P}_t}\right) - \frac{\omega}{1+v} H_t^{\;1+v} \right]$$

$$\mathrm{C}_t + \frac{B_t}{r_t} + \mathrm{M}_t = M_{t-1} - \mathrm{P}_t \mathrm{T}_t + \mathrm{B}_{t-1} + W_t h_t + D_t \mathrm{P}_t$$

Steady-State Distortions

- The inefficiency sources is defined here as components making the consumption deviate from its efficient amount in the steady state. In particular,

$$c = \left\{\frac{1}{\omega}\left(\frac{z-\beta\gamma}{z-\gamma}\right)\frac{w}{s^{v}g^{v}}\right\}^{\frac{1}{1+v}} \textit{deviate } q = \left[\frac{g^{(1-\theta)/\theta}}{\omega}\frac{z-\beta\gamma}{z-\gamma}\right]^{\frac{1}{1+v}} \text{by}$$

$$\frac{w}{s^{v}g^{v+\frac{1-\theta}{\theta}}} = \frac{1}{g^{v+\frac{1-\theta}{\theta}}} * \frac{1}{\mu_m} * \left[\frac{1-\eta\beta\pi^{(1-\chi)(-\theta)}}{1-\eta}\right]\left[\frac{1-\eta}{1-\eta\pi^{(1-\chi)(\theta)}}\right]\left[\frac{1-\eta\pi^{(1-\chi)(\theta-1)}}{1-\eta}\right]^{\frac{1+\theta v}{1-\theta}}$$

- The source of inefficiency:
  - $g^{v+\frac{1-\theta}{\theta}}$: inefficient government expenditure (d_f)
  - $\frac{1}{\mu_m}$: the monopolistic competition distortion (d_m)
  - the non-optimal inflation target (d_i)

Proposition 1: If the price is fully flexible ($\eta = 1$), or a price indexation is unit ($\chi = 1$), or when the zero-inflation target is considered ($\bar{\pi}_t = 1^{0.25}$), the optimal consumption can be obtained if $g = \left(\frac{1}{\mu_m}\right)^{\frac{\theta}{v\theta+1-\theta}} = \left(\frac{\theta-1}{\theta}\right)^{\frac{\theta}{v\theta+1-\theta}}$

3. Parametrization Values
GMM, SMM, and Bayesian

- Observable variables
  - Quarterly data seasonally adjusted: 1982Q4:2015Q1
  - GDP growth; GDP Deflator; 3-month treasury bill rate

Table 2: Estimated Parameters from Different Methods

	β	ρ_Z	δ_Z	$\rho_{\overline{\Pi}}$	$\delta_{\overline{\Pi}}$
GMM	0.9974 (0.0414)	0.8000 (0.3903)	0.0992 (0.0121)	**0.9950** (0.0820)	0.098 (0.0067)
Bayesian Estimation	0.9999	0.7523	0.1390	**0.9949**	0.05

Parametrization Values

Benchmark Model

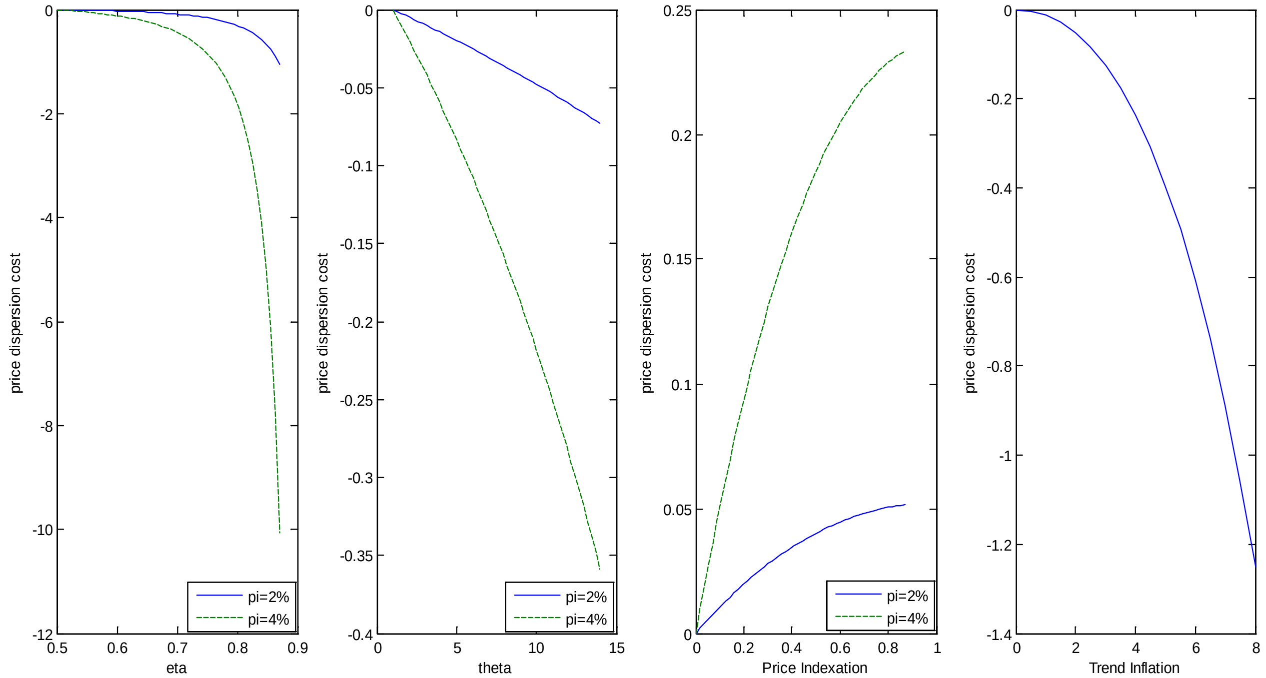
Parameter	Description	Calibrated Value
β	The discount factor	0.9974
γ	Consumption habit	0.81
z	The steady state of technology shock	1.00
ω	Labor supply disutility	1.00
v	Inverse Frisch elasticity of labor supply	1.59
θ	Elasticity of substitution	10.0
$1\text{-}g^{-1}$	Steady state share of Government expenditure	
p_z	AR(1) coefficient for technology shock	0.80
p_g	AR(1) coefficient for government spending shock	0.98
$100\delta_z$	Standard deviation of technology shock	1.10
$100\delta_g$	Standard deviation of government spending shock	0.55
Monetary Policy (The interest rate rule)		
ϕ_π	Taylor coefficient on the inflation gap	1.92
ϕ_y	Taylor coefficient on the output gap	0.10
p_r	AR(1) coefficient for monetary shock	0.81
$100\delta_r$	Standard deviation of monetary shock	0.25
Monetary Policy (The money growth rule)		
p_{gm}	The persistence of money growth	0.81
$100p_{em}$	AR(1) coefficient of monetary shock	0.25
s1	Impacts of inflation	1.92
s2	Impact of output	0.10
Calvo Price Setting		
η	Probability of not being able to optimize	[0.6, 0.65, 0.7, 0.75]

4.1. Transmission Mechanism

The cost of price dispersion

- The cost of price dispersion by $\tilde{z}_t = z_t / s_t$ (an effective aggregate productivity)

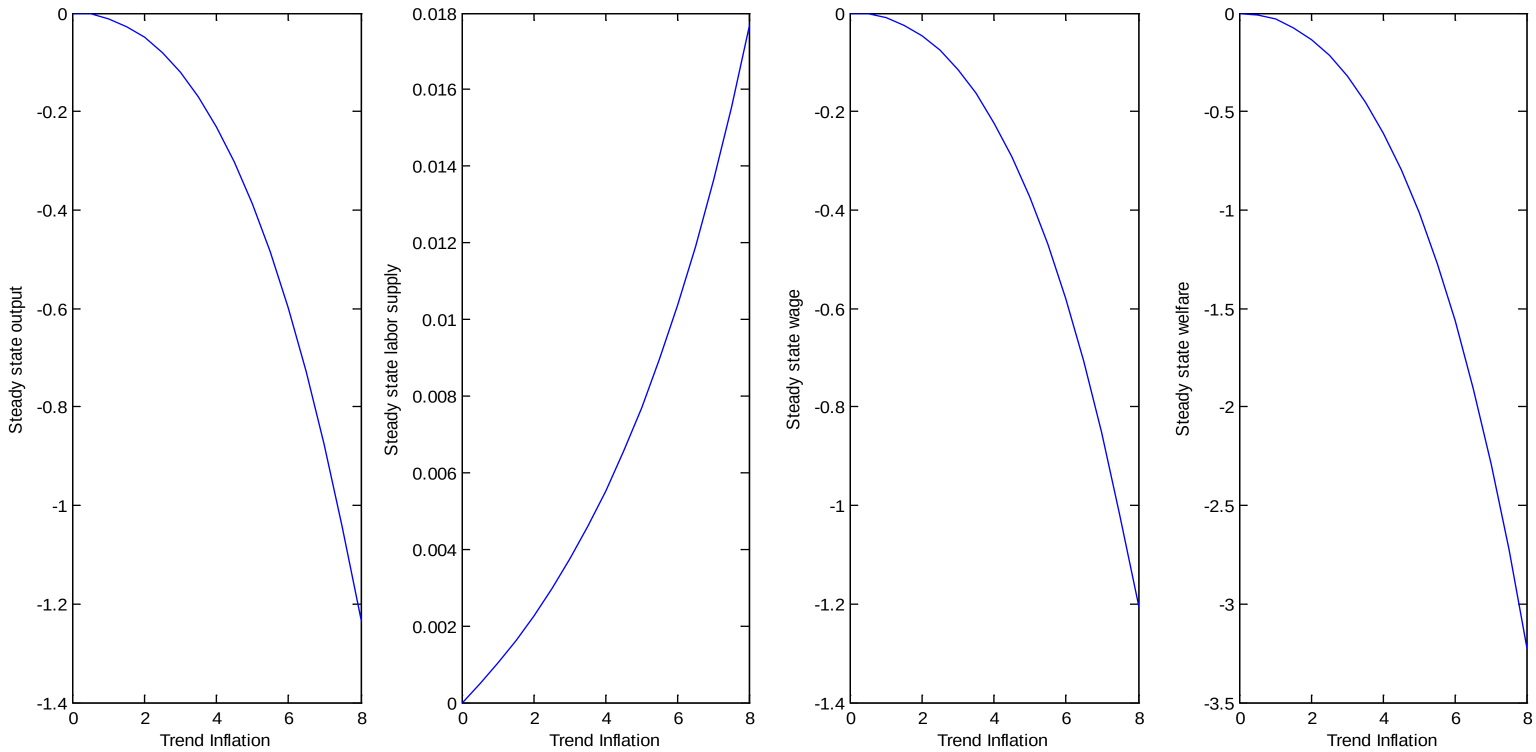
$$s(\overline{\pi}_t) = \frac{(1-\eta)^{\frac{1}{1-\theta}}}{\left(1-\eta\overline{\pi}_t^{\,(1-\chi)(\theta)}\right)\left(1-\eta\overline{\pi}_t^{\,(1-\chi)(\theta-1)}\right)^{\frac{\theta}{1-\theta}}}$$



4.1. Transmission Mechanism
Steady-State Variables

- Changes in trend inflation affect the steady state, which leads to a change in the point around which the model is log-linearly approximated

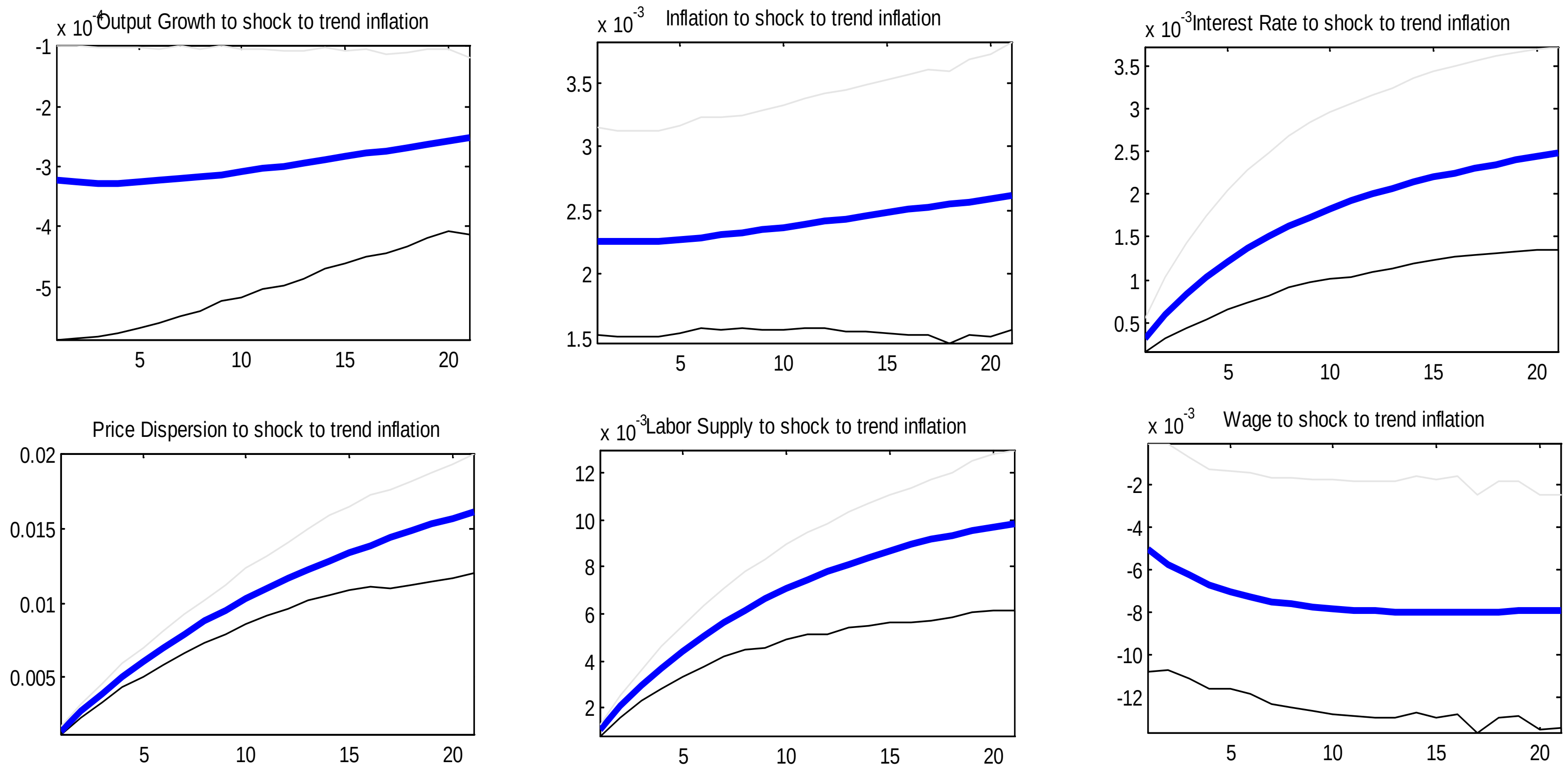
→ the log-linear dynamics of the model alter



4.1. Transmission Mechanism

A Shock to Trend Inflation

- The shock persistently distorts the economy



Note: Bayesian Model using the U.S data

4.1. Transmission Mechanism

Summary

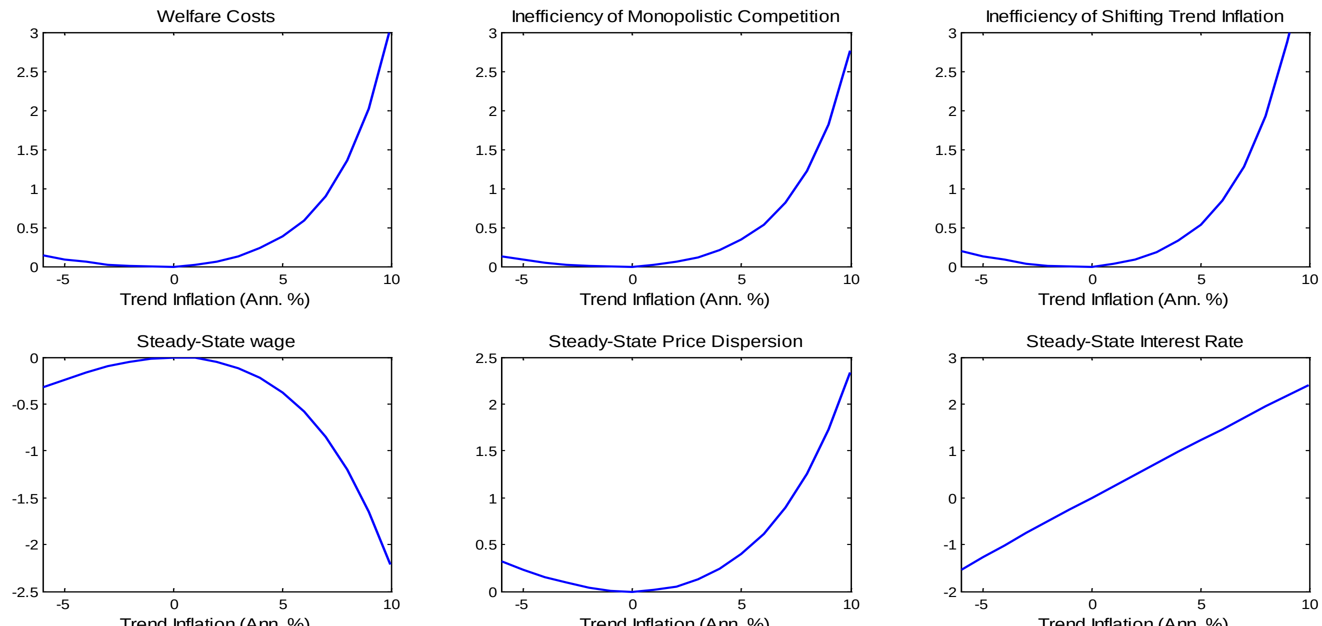
1. A rise in trend inflation directly causes price dispersion augment, and then a reduction in an effective aggregate productivity occurs.
2. The results illustrating changes of steady-state variables due to shifting trend inflation show that the more inputs are required to produce a given amount of output when output and consumption diminish. Hence, the welfare cost is a direct consequence of more working while salary and consumption decrease.
3. It will put burden on the society by distorting the environment for the economic growth, such as a persistent increase in inflation and interest rate, and price dispersion while wage relentlessly reduces

4.2. Welfare Cost and Inefficiency Sources Computations

Constant Positive Trend Inflation

	Welfare	Welfare Cost	d_f	d_m	d_i
$\overline{\Pi} = 1.00^{0.25}$	-894.844				
$\overline{\Pi} = 1.06^{0.25}$	-895.459	0.595%	0.496%	0.535%	0.845%

- Can Deflation be good? Probably if
  - The deflation leads to a small level of price dispersion
  - As long as the deflation is set, if the central bank mandates negative interest rates



4.2. Welfare Cost and Inefficiency Sources Computations

Shifting Trend Inflation

	Welfare	Welfare Cost	d_f	d_m	d_i
$\ln(\bar{\Pi}_t/\bar{\Pi}^*) = \rho_{\bar{\Pi}} \ln(\bar{\Pi}_{t-1}/\bar{\Pi}^*) + \epsilon_{\Pi,T}$					
All shocks	-895.315				
$\sigma_{\Pi} = 0$	-894.589	0.09%	0.119%	0.082%	0.130%
Without Business Cycle Fluctuation	-890.896	0.571%	0.477%	0.514%	0.812%
$\ln\left((\bar{\Pi}_t - 1)/(\bar{\Pi}^* - 1)\right) = \rho_{\bar{\Pi}} \mathrm{lln}\left((\bar{\Pi}_{t-1} - 1)/(\bar{\Pi}^* - 1)\right) + \epsilon_{\Pi,T}$					
All shocks	-895.883				
$\sigma_{\Pi} = 0$	-894.590	0.166%	0.215%	0.149%	0.235%
Without Business Cycle Fluctuation	-891.464	0.569%	0.475%	0.512%	0.809%

5. Conclusions

- The theory on the mechanism:
  - a rise in price dispersion causing a larger difference between output and labor hours
  - a reduction in an effective aggregate productivity
  - a decrease in consumption and wage but an increase in labor hours
  - the effect of distorting an improving path of output growth while amplifying an expansion of inflation and labor supply
- The trend inflation source signified the welfare cost the most significantly
- The high-trend-inflation economy is more elastic to changes as opposed to the low-trend inflation economy

Thank You

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