金融システム研究フォーラム報告用資料(2009/7/3)

S. Morris and H.S. Shin (2009) “Illiquidity Component of Credit Risk”

Speaker: Kohei Kawaguchi and Yusuke Narita

Overview

"[T]he fate of Bear Sterns was the result of
a lack of confidence, not a lack of capital"---C. Cox

The problem was...

On the *liabilities side* rather than the asset side &
Liquidity based rather than solvency based

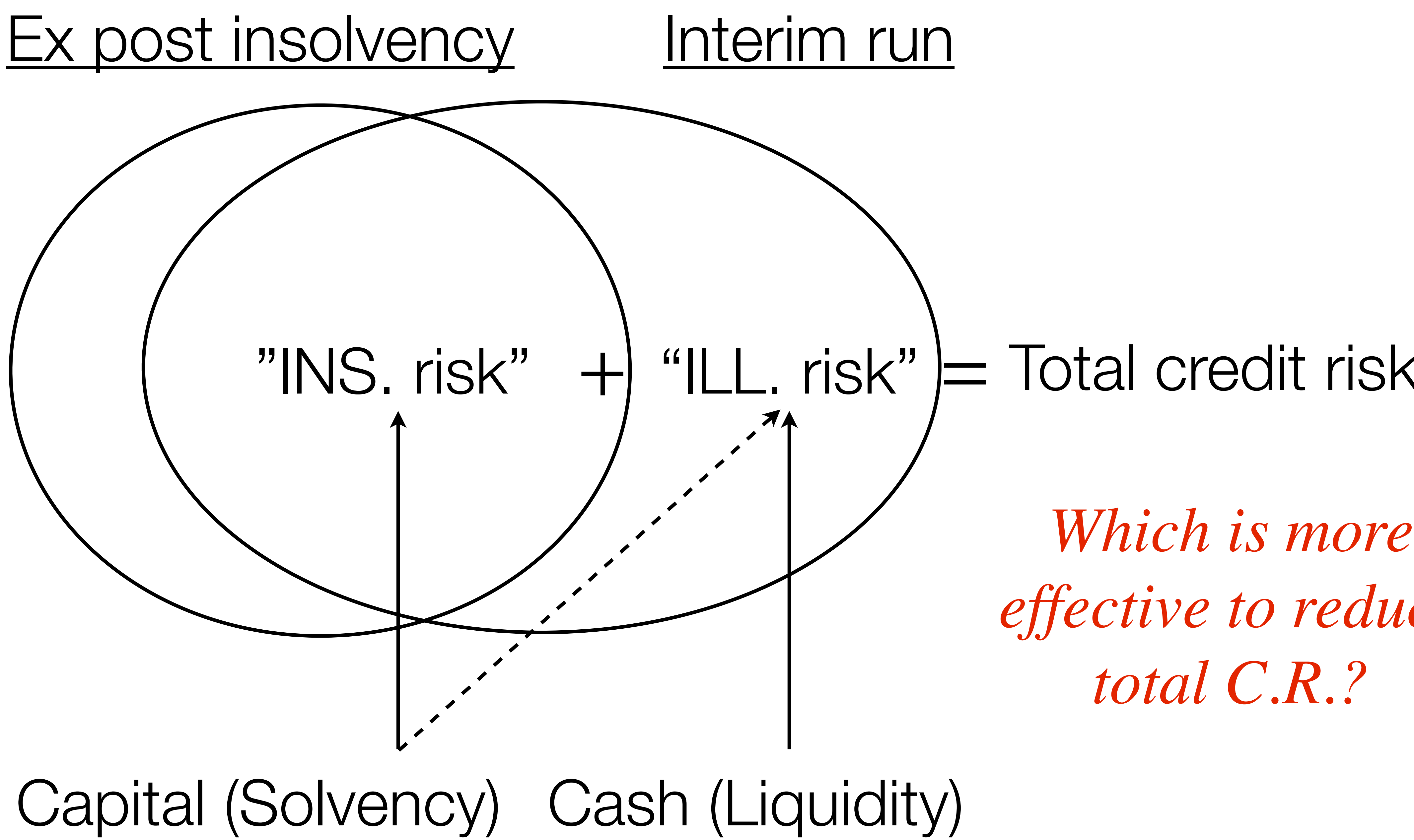
↓↑

Conventional approaches to bank capital regulation aim to...

Serve only as a buffer against the shortfall
in the *asset side* of BS

Overview (Cont.)

A classification of equilibria by the global game method...



Which is more effective to reduce total C.R.?

Overview (Cont.)

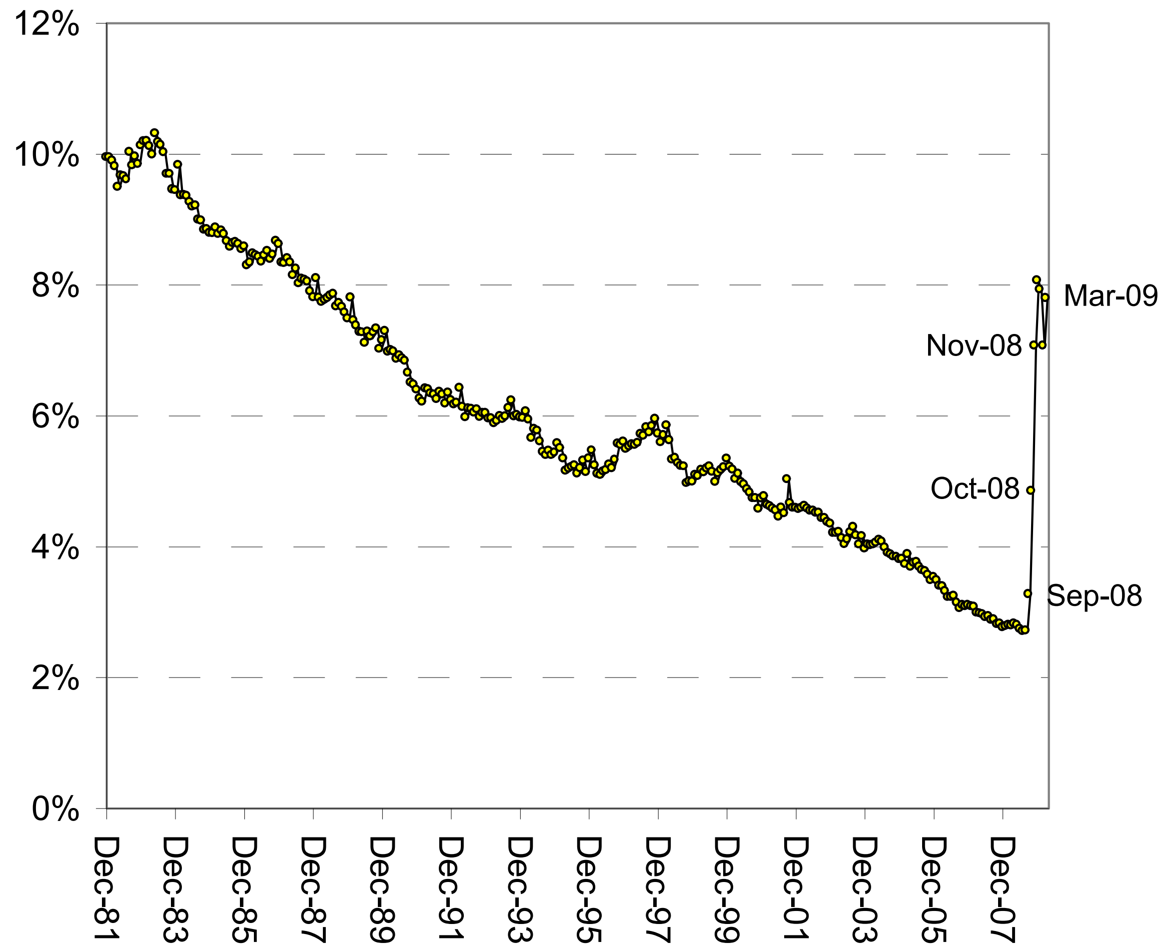
This paper shows...

- Improving liquidity may be *more effective* than increasing solvency when there is *great uncertainty* or *low liquidity*.
- This result is robust to assumptions about economic environment

→ *New approach to bank capital regulation !*

Empirical Eviences

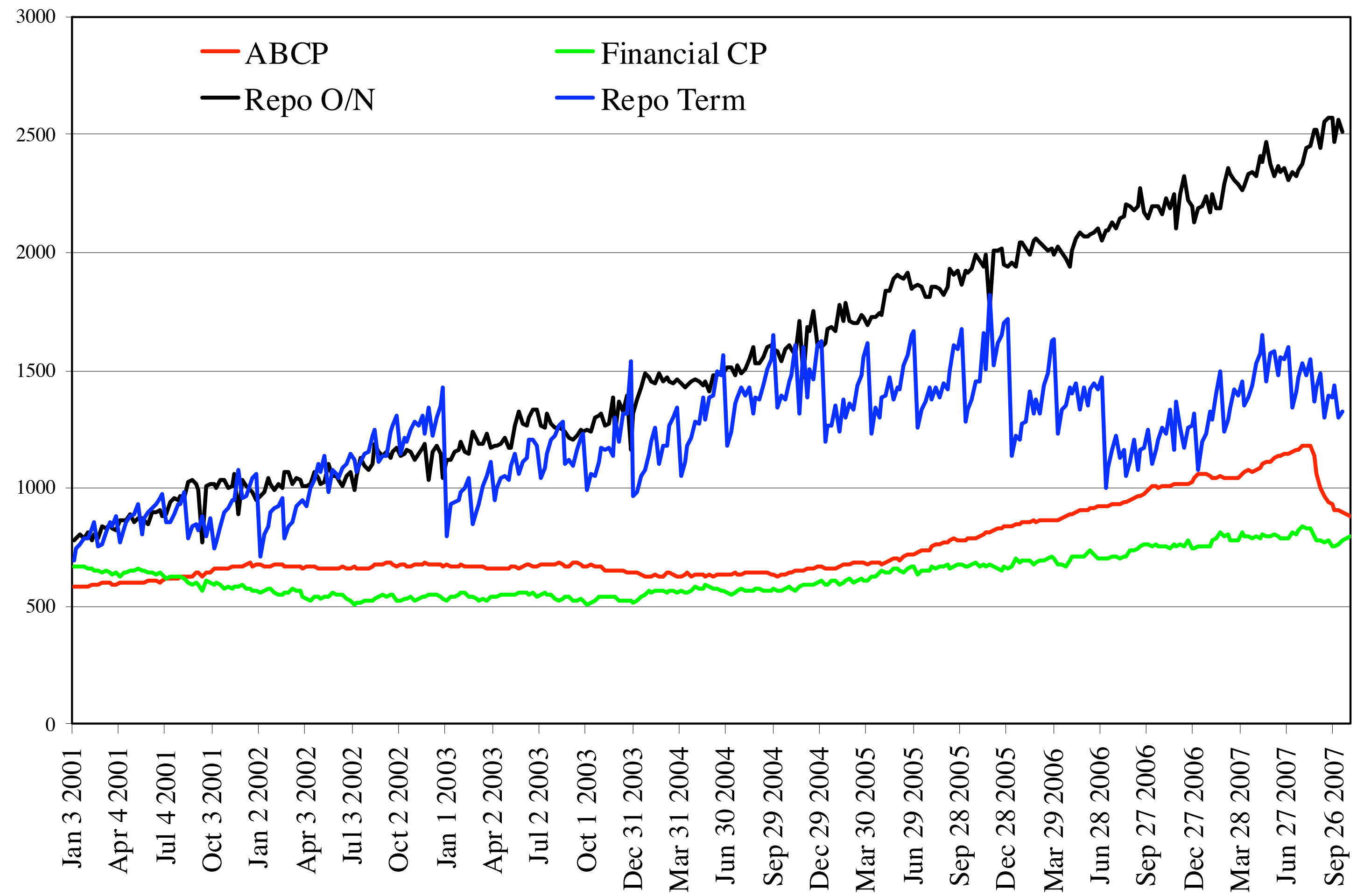
1st trend: Decline in cash holding by banks

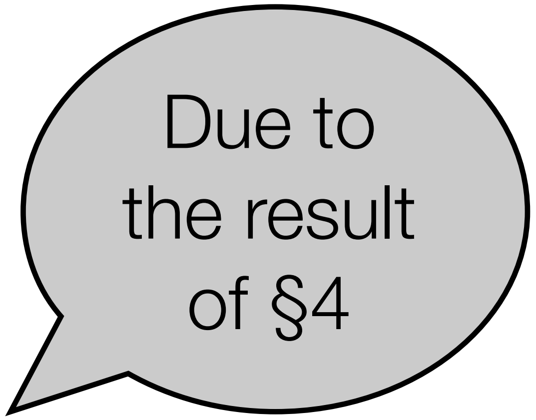


→Decreased liquidity (Increased ILL component of C.R.) ?

Empirical Evidences (Cont.)

2nd trend: Shortening maturity of bank liabilities





→Paradoxically increased ILL component of C.R.?

Roadmap

Model

A decomposition of C.R.

--- *Ex ante / Interim*

--- *INS component / ILL component*

Analytical expressions for decomposed C.R.

BS comparative statics

--- *Solvency*

--- *Liquidity*

--- *Liquid asset vs Safe asset*

Policy implication

Model: BS

<u>Liabilities side</u>

- S, L : Face value of short- and long-term debt
- E : That of equity
- r_S, r_L : Promised gross rate of return for S and L

<u>Asset side</u>

- $i \in \{0, 1, ..., N\}$: Set of categories of assets
- A_i : Total face value of assets in category i
- $\theta_t = \theta_{t-1} + \sigma_t \epsilon_t \ (\epsilon_t \sim F_t, E(\epsilon_t) = 0, \epsilon_t \perp\!\!\!\perp \epsilon_{t'})$

  : Value of fundamentals at date t
- $\alpha_i + \beta_i \theta_t$: Asset i's value (per unit face value) at date t

Model: Timing Structure of the Game

Solvent or not?

Date 0

Asset	Liabilities
	S
$\sum_{i=0}^{N} A_i (\alpha_i + \beta_i \theta_0)$	L
	E_0

Date 1

Sufficiently many S-holders decide to roll over?

Yes → **Date 2**

Asset	Liabilities
	Sr_S
$\sum_{i=0}^{N} (\alpha_i + \beta_i \theta_2) A_i$	Lr_L
	E_2

No →

Model: Insolvency at Date 2

The bank is *solvent* at date 2 if

$$\sum_{i=0}^{N} (\alpha_i + \beta_i \theta_2) A_i \geq S r_S + L r_L$$

Total asset — Total debt

Equivalent condition for fundamentals is

$$\theta_2 \geq \frac{S r_S + L r_L - \sum_{i=0}^{N} \alpha_i A_i}{\sum_{i=0}^{N} \beta_i A_i} \equiv \theta^*$$

Call this the *solvency point*.

Model: Run at Date 1

Interim liquidation

$\lambda_i\left(\alpha_i+\beta\theta_0\right)A_i(\lambda_i\in[0,1])$: Liquidation value of asset i

$A^*\equiv\sum_{i=0}^{N}\lambda_i(\alpha_i+\beta_i\theta_0)A_i$: Total liquidation value

Run occurs at date 1 if

The proportion of S-holders *not* rolling over $>$ A^*/S

Model: Run at Date 1 (Cont.)

Assumption

$$q \equiv \frac{r^* S}{r_S A^*} < 1 < \frac{S}{A^*}$$

r^* : Gross rate of return of outside opportunity

Remark

- This implies $r^* A^* < r^* S < r_S A^* < r_S S$.
- ∃ 1-to-1 mapping from $\frac{S}{A^*}$ to q.

→ Call q *illiquidity index*.

Model: Timing Structure of the Game

$\theta_2 \geq \theta^*?$

<u>Date 0</u>

Asset	Liabilities
	S
$\sum_{i=0}^{N} A_i (\alpha_i + \beta_i \theta_0)$	L
	E_0

<u>Date 2</u>

Asset	Liabilities
	Sr_S
$\sum_{i=0}^{N} (\alpha_i + \beta_i \theta_2) A_i$	Lr_L
	E_2

Given θ_1, A^*, r^*

<u>Date 1</u>

More than (1- $\frac{A^*}{S}$)×100% of S-holders decide to roll over?

Yes

No

Results from the Global Game Method

Result 1: In the limit as $u_j \to 0$, $\forall \theta_1$

Each S-holder j's belief on the proportion of S-holders *not* rolling over $= U[0,1]$

Result 2: Let $s_j : \Theta_1 \to$ {Roll over, Not} be a S-holder j's strategy.
In any eqrm., each j follows a strategy s.t. $\exists \theta^{**}$

$$s_j(\theta_1) = \begin{cases} \text{Roll over} & \text{if } \theta_1 \geq \theta^{**} \\ \text{Not} & \text{if } \theta^{**} > \theta_1 \end{cases}$$

In addition, there is *unique eqrm.* & associated θ^{**}.

What is θ^{**}?

Characterization of Run Point

θ^{**} satisfies the following "indifference condition."

Prob(No run) Prob(Solvancy| No run)

$$\frac{A^*}{S}\left(1 - F_2\left(\frac{\theta^* - \theta^{**}}{\sigma_2}\right)\right) r_S = r^*$$

Gross expected rate of return of rolling over

Gross rate of return of not rolling over)

Equivalently,

$$\theta^{**} = \theta^* - \sigma_2 F_2^{-1}(1 - q)$$

Run occurs if $\mathcal{INS}_1(\theta_1) \geq q$,
which justify the term "illiquidity index."

A Decomposition of C.R.: Interim

Interim ILS risk

$$\mathcal{INS}_1(\theta_1) = \text{Prob}\{\theta_2 < \theta^* | \theta_1\}$$

$$= F_2\left(\frac{\theta^* - \theta_1}{\sigma_2}\right) \;(\because \epsilon_2 \sim F_2\;)$$

Remark

- This denotes the prob. that *there is no run at date 1 & the bank becomes insolvent at date 2*

A Decomposition of C.R.: Interim (Cont.)

Interim ILL risk

$=1-\mathcal{INS}_1(\theta_1)$

$$\mathcal{ILL}_1(\theta_1) = \begin{cases} 1 - F_2\left(\dfrac{\theta^* - \theta_1}{\sigma_2}\right) & \text{if } \theta_1 \leq \theta^{**} \\ 0 & \text{if } \theta_1 > \theta^{**} \end{cases}$$

Remark

- This denotes the prob. that *a run occurs at date 1*

the bank would not have been insolvent in the absence of a run

Interim total C.R.

Run

$$\mathcal{C}_1(\theta_1) = \mathcal{INS}_1(\theta_1) + \mathcal{ILL}_1(\theta_1) = \begin{cases} 1, \text{ if } \theta \leq \theta^{**} \\ F_2\left(\frac{\theta^* - \theta_1}{\sigma_2}\right), \text{ if } \theta > \theta^{**} \end{cases}$$

Not run but insolvency

A Decomposition of C.R.: Ex Ante

Ex ante ILS risk

$$\mathcal{INS}_0(\theta_0) = \int_{\theta_1=-\infty}^{\infty} \underbrace{F_2\left(\frac{\theta^* - \theta_1}{\sigma_2}\right)}_{=\mathcal{INS}_1(\theta_1)} \overbrace{\frac{1}{\sigma_1} f_1\left(\frac{\theta_1 - \theta_0}{\sigma_1}\right)}^{=\text{Prob}\{\theta_0 + \sigma_1\epsilon_1 = \theta_1 | \theta_0\}} d\theta_1$$

Ex ante ILL risk

$$\mathcal{ILL}_0(\theta_0) = \int_{\theta_1=-\infty}^{\theta^* - \sigma_2 F_2^{-1}(1-q)} \underbrace{\left(1 - F_2\left(\frac{\theta^* - \theta_1}{\sigma_2}\right)\right)}_{=\mathcal{ILL}_1(\theta_1) \text{ if } \theta_1 \leq \theta^{**}} \frac{1}{\sigma_1} f_1\left(\frac{\theta_1 - \theta_0}{\sigma_1}\right) d\theta_1$$

A Decomposition of C.R.: Ex Ante (Cont.)

Ex ante total C.R.

$$\mathcal{C}_0(\theta_0) = \mathcal{INS}_0(\theta_0) + \mathcal{ILL}_0(\theta_0)$$

Mere substitution →

$$= \int_{\theta_1=-\infty}^{\infty} F_2\left(\frac{\theta^* - \theta_1}{\sigma_2}\right)\frac{1}{\sigma_1} f_1\left(\frac{\theta_1 - \theta_0}{\sigma_1}\right) d\theta_1 + \int_{\theta_1=-\infty}^{\theta^* - \sigma_2 F_2^{-1}(1-q)} \left(1 - F_2\left(\frac{\theta^* - \theta_1}{\sigma_2}\right)\right)\frac{1}{\sigma_1} f_1\left(\frac{\theta_1 - \theta_0}{\sigma_1}\right) d\theta_1$$

Rearrangement of the integration →

$$= F_1\left(\frac{\theta^* - \theta_0 - \sigma_2 F_2^{-1}(1-q)}{\sigma_1}\right) + \int_{\theta_1=\theta^* - \sigma_2 F_2^{-1}(1-q)}^{\infty} F_2\left(\frac{\theta^* - \theta_1}{\sigma_2}\right)\frac{1}{\sigma_1} f_1\left(\frac{\theta_1 - \theta_0}{\sigma_1}\right) d\theta_1$$

Analytical Expressions for Interim C.R.

<u>Assumption</u>: *Noise w/ uniform dist.*

$$\epsilon_t \sim_{\text{i.i.d.}} U[-1/2, 1/2]$$

We can relax
this assumption
(§4.2)

Equivalent assumption for the c.d.f.s is

$$F_1(x) = F_2(x) = \begin{cases} 0, \text{ if } x \leq -\frac{1}{2} \\ x + \frac{1}{2}, \text{ if } -\frac{1}{2} \leq x \leq \frac{1}{2} \\ 1, \text{ if } \frac{1}{2} \leq x \end{cases}$$

Analytical Expressions for Interim C.R. (Cont.)

Run point

$$\theta^{**} = \theta^* + \sigma_2 \left(q - \frac{1}{2} \right)$$

Interim ILS risk

↑ *Mere substitution* ↓

$$\mathcal{INS}_1(\theta_1) = \begin{cases} 1, \text{ if } \theta_1 \leq \theta^* - \frac{1}{2}\sigma_2 \\ \frac{1}{2} + \frac{1}{\sigma_2}(\theta^* - \theta_1), \text{ if } \theta^* - \frac{1}{2}\sigma_2 \leq \theta_1 \leq \theta^* + \frac{1}{2}\sigma_2 \\ 0, \text{ if } \theta^* + \frac{1}{2}\sigma_2 \leq \theta_1 \end{cases}$$

Analytical Expressions for Interim C.R. (Cont.)

Interim ILL risk

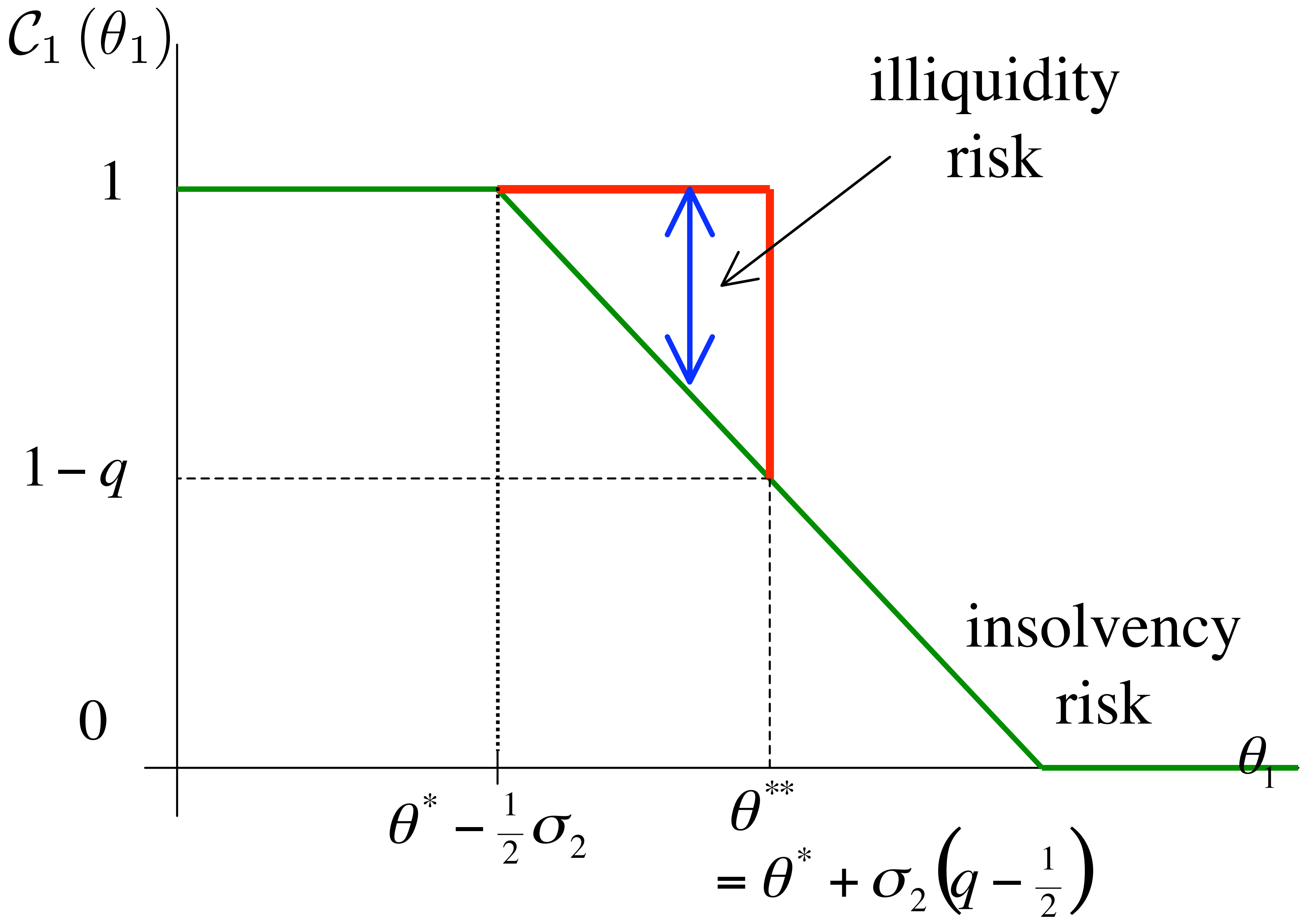
$$\mathcal{ILL}_1(\theta_1) = \begin{cases} 0, \text{ if } \theta_1 \leq \theta^* - \frac{1}{2}\sigma_2 \\ \frac{1}{2} - \frac{1}{\sigma_2}(\theta^* - \theta_1), \text{ if } \theta^* - \frac{1}{2}\sigma_2 \leq \theta_1 \leq \theta^* + \sigma_2\left(q - \frac{1}{2}\right) \\ 0, \text{ if } \theta_1 > \theta^* + \sigma_2\left(q - \frac{1}{2}\right) \end{cases}$$

Interim total C.R.

↑ *Mere substitution* ↓

$$\mathcal{C}_1(\theta_1) = \begin{cases} 1, \text{ if } \theta_1 \leq \theta^* + \sigma_2\left(q - \frac{1}{2}\right) \\ \frac{1}{2} + \frac{1}{\sigma_2}(\theta^* - \theta_1), \text{ if } \theta^* + \sigma_2\left(q - \frac{1}{2}\right) \leq \theta_1 \leq \theta^* + \frac{1}{2}\sigma_2 \\ 0, \text{ if } \theta^* + \frac{1}{2}\sigma_2 \leq \theta_1 \end{cases}$$

Analytical Expressions for Interim C.R. (Cont.)



Analytical Expressions for Ex Ante C.R.

We can relax these assumptions (§4.1)

Assumption 1: High ex ante noise ($\sigma_1 \geq \sigma_2$)

Assumption 2: Mild θ_0 ($\theta_0 \in \left[\theta^* - \frac{1}{2}(\sigma_1 - \sigma_2), \theta^* + \frac{1}{2}(\sigma_1 - \sigma_2)\right]$)

Ex ante ILS risk

$$\mathcal{INS}_0(\theta_0) = \int_{-\frac{1}{2}}^{\frac{1}{2}} \mathcal{INS}_1(\theta_0 + \sigma_1\epsilon) f_1(\epsilon) d\epsilon = \frac{1}{2} + \frac{\theta^* - \theta_0}{\sigma_1}$$

$\Updownarrow$ *Mere substitution*

Ex ante ILL risk

$$\mathcal{ILL}_0(\theta_0) = \int_{-\frac{1}{2}}^{\frac{1}{2}} \mathcal{ILL}_1(\theta_0 + \sigma_1\epsilon) f_1(\epsilon) d\epsilon = \frac{\sigma_2}{2\sigma_1} q^2$$

Analytical Expressions for Ex Ante C.R. (Cont.)

Ex ante total C.R.

$$\mathcal{C}_0(\theta_0) = \mathcal{INS}_0(\theta_0) + \mathcal{ILL}_0(\theta_0)$$

$$= \frac{1}{2} + \frac{\theta^* - \theta_0}{\sigma_1} + \frac{\sigma_2}{2\sigma_1} q^2 \quad \leftarrow \textit{Mere substitution}$$

Interpretation

- Illiquidity as a source of total C.R. ($q \uparrow \Rightarrow C_0(\theta_0) \uparrow$)
- Great interim uncertainty as a source ($\sigma_2/\sigma_1 \uparrow \Rightarrow C_0(\theta_0) \uparrow$)
- Interaction between illiquidity (q) & uncertainty (σ_2/σ_1)
- How to interpret $\sigma_1 \uparrow \Rightarrow C_0(\theta_0) \downarrow$?

BS Comparative Statics: Solvency & Liquidity

W.r.t. solvency

$$\sigma_1 \frac{d\mathcal{C}_0}{d\theta^*} = 1$$

Recall that

$$\mathcal{C}_0\left(\theta_0\right) = \frac{1}{2} + \frac{\theta^* - \theta_0}{\sigma_1} + \frac{\sigma_2}{2\sigma_1} q^2$$

W.r.t. liquidity

$$\sigma_1 \frac{d\mathcal{C}_0}{dq} = \sigma_2 q$$

Interpretation

- When σ_2 or q is large, improving liquidity may be *more effective* than increasing solvency to reduce C.R..

BS Comparative Statics: Each Asset

Notation

$$E_0 = \sum_{i=0}^{N} A_i \left(\alpha_i + \beta_i \theta_0\right) - S r_S - L r_L$$: Total value of equity at date 0

$$V = \sum_{i=0}^{N} A_i \beta_i$$: Measure of variability of assets

$\overline{r}_i = \alpha_i + \beta_i \theta_0$: Gross expected rate of return of asset i

Comparative statics w.r.t. liquidity

$$\sigma_1 \frac{d\mathcal{C}_0}{dA_i} = -\frac{\overline{r}_i}{V} + \frac{E_0 \beta_i}{V^2} - \frac{\sigma_2 q^2 \lambda_i \overline{r}_i}{A^*}$$

BS Comparative Statics: Liquid vs Safe Asset

Let *i*=0 be cash category, i.e., $\alpha_0 = 1, \beta_0 = 0 (\Rightarrow r_0 = 1)$ & $\lambda_0 = 1$.

$$\sigma_1 \frac{d\mathcal{C}_0}{dA_0} = -\frac{1}{V} - \sigma_2 \frac{q^2}{A^*}$$

Let *i*=*k* be a riskless but completely illiquid asset category, i.e., $r_k = R$ & $\lambda_k = 0$.

$$\sigma_1 \frac{d\mathcal{C}_0}{dA_k} = -\frac{R}{V}$$

The multiplier of cash is

$$\frac{A^* + \sigma_2 V q^2}{A^* R}$$

When $\sigma_2 V q^2$ is large, *liquid asset is more important than safe asset*.

Policy Implication

The results of this paper give support to...

- *A cash holding requirement for banks*
- *Requirement for the record of ultra-short term liabilities*

However, it is still unclear how INS risk & ILL risk interact.

Discussion

To evaluate the impact of “new approach” on total welfare...

- *Cost of policy implementation*
- *Bank’s optimization behavior*
- *Some welfare criterion*

must be built in the model.