

December 18, 2009

Incentives in Hedge Funds

(Forthcoming)

Hitoshi Matsushima

Faculty of Economics, University of Tokyo

December 13, 2009

What is HF? (1)

Financial Intermediation: cf. Bank, MF

Private Partnership: Investor (Unsophisticated Investor)

Manager (Manager + Sophisticated Investors)

What is HF? (2)

Investor:

1 Unit of Fund

No Withdrawal

Manager:

M Units of Self-Fund

Run Investor's Fund and Part of Self-Fund

Low Transparency

Weak Regulation

Dynamical Investment Strategy: cf. Buy-and-Hold

What is HF? (3)

Return for Investor's Fund: $x \in [0, \infty)$

Alpha: $x - 1$

Manager's Fee Scheme: $y : [0, \infty) \rightarrow [-M, \infty]$

$y(x) \in [-M, \infty)$

Maximal Penalty: $w(y) \equiv \max[-\min_{x \in [0, \infty)} y(x), 0] \geq 0$

What is HF? (4)

Real Fee Scheme:

2:20

Asymmetry

No Penalty, $w(y) = 0$

Convexity

High-Powered

Criticisms:

Positive Penalty $w(y) > 0$, Linear, Symmetry, Fulcrum

Lower-Powered

Warren Buffet

HF Problem (1)

Fake Alpha: Performance Mimicry

Option Trading Induces Market Failure

Hidden Action (Moral Hazard): Skilled Manager Mimics High Performance

Hidden Information (Lemon): Unskilled Enters and Mimics Skilled

Real HF: Much Lower Alpha than Expected

HF Problem (2)

Capital Decimation Partners (CDP)

Andrew Lo (2001): **2:20 Scheme Motivates CDP**

Foster-Young (FY, 2008): **In Theory, Incentive Scheme is Impossible to Design**

Press Releases: **FT (18/3/08), NYT (3/8/08), etc**

“Ideal HF Never Survives. More Transparency!”

My Research (1)

Q: Why Is FY so Pessimistic?

A: Incentives in HFs are Special:

Maximal Penalty $w(y)$ Must be Put in Escrow: Unmanageable

Skilled is Deprived of Self-fund Management: Distortion, Lemon

My Research (2)

Q: Is FY True?

A: No!

FY Depends on 'No Capital Gain Tax (CGT)'

My Research (3)

Incentive Fee Scheme: $y(x) \in [-M, \infty), x \in [0, \infty)$

Five Requirements:

Skilled Manager's Entry

Unskilled Manager's Exit

Investor's Entry

Welfare Improvement

Skilled Manager's Performance Mimicry

My Research (4)

Q: Does Incentive Scheme Exist?

What Condition?

How to Design?

Welfare-Optimal?

How to Implement?

Results (1)

- **CGT Matters:**

No CGT

Incentive Scheme Never Exists!

Allow Positive CGT

Incentive Scheme Does Exist!

- **Shape of Incentive Scheme:**

Positive Penalty $w(y) > 0$, Linear,

Symmetry, Fulcrum ...

Results (2)

- **Welfare-Optimal CGT Rate: Low Enough to Keep Skilled Manager's Entry Binding**
- **Implementation: Manager Prefers Over-Solvency**
'Tax on Manager's Fee' Matters
Tax Rate on Fee Must be Bigger than CGT Rate

Manager's Types: Skilled and Unskilled (1)

Skilled Manager:

Investment Strategy:

Action $a \in [0, \infty)$

Alpha a

Non-pecuniary Cost $c(a)$: Increasing, Convex, ...

Side Contracting (CDP):

$F : [0, \infty) \rightarrow [0, 1] \in \Phi(a)$

$E[z | F] = a + 1$

$y(z) - c(a)$

Manager's Types: Skilled and Unskilled (2)

Unskilled Manager:

No Investment Strategy:

$$a = 0$$

Only Performance Mimics:

$$F : [0, \infty) \rightarrow [0, 1] \in \Phi(0)$$

$$E[z | F] = 1$$

$$y(z) - c(0) = y(z)$$

Other Characters

Investor: $x - y(x)$

Third Party: **Option Buyer, Arbitrageur**

Escrow Service: **Make Manager Solvent to Penalty and Side Contract**

Government: **CGT Rate** $t \geq 0$

Tax on Fee Rate $\tau \geq 0$

CDP: Performance Mimicry (1)

By Selling Covered Options, Unskilled Manager Mimics 'Alpha $a = \frac{1}{1-p}$ with Prob. $1-p$ '

Put Investor's Fund in Escrow

Give it to Buyer if S&P 500 Stock Index Declines 20%: Prob. p

Give Nothing Otherwise: Prob. $1-p$

Option Price p

Put p in Escrow and Sell Options Further, ...

Total Option Sale: $1 + p + p^2 + \dots = \frac{1}{1-p}$

Total Return: $\frac{1}{1-p}$ Prob. $1-p$

0 Prob. p

CDP: Performance Mimicry (2)

By Using CDP, Even Skilled Manager Can Mimic High Performance

CDP Implements Any Side Contract

Requirements & : Linearity, Symmetry

Requirement : Skilled Manager's Entry

Outside Opportunity:

Manage Self-Fund M

Select $a = \tilde{a}(1-t)$ to Maximize $(1-t)a - c(a)$

$$\bar{V}(t) \equiv M\{(1-t)\tilde{a}(1-t) - c(\tilde{a}(1-t))\}$$

Payoff:

Put $w(y)$ in Escrow, Unmanageable

$$V(y, \tau, t) \equiv \min[(1-\tau)y(a^*(y, \tau) + 1), y(a^*(y, \tau) + 1)] - c(a^*(y, \tau)) \\ + \{M - w(y)\}\{(1-t)\tilde{a}(1-t) - c(\tilde{a}(1-t))\}$$

Entry $V(y, t, \tau) \geq \bar{V}(t)$:

$$\min[(1-\tau)y(a^*(y, \tau) + 1), y(a^*(y, \tau) + 1)] - c(a^*(y, \tau)) \\ \geq w(y)\{(1-t)\tilde{a}(1-t) - c(\tilde{a}(1-t))\}$$

Requirement : Unskilled Manager's Exit

Outside Opportunity: Zero

Payoff: $\max_{F \in \Phi} E[\min[(1 - \tau)y(z), y(z)] | F]$

Exit: $\max_{F \in \Phi} E[\min[(1 - \tau)y(z), y(z)] | F] \leq 0$

Requirement : Investor's Entry

Outside Opportunity: Zero

**payoff: $U(y,t,\tau) \equiv \min[(1-t)\{a^*(y,\tau) - y(a^*(y,\tau) + 1)\},$
 $a^*(y,\tau) - y(a^*(y,\tau) + 1)]$**

Entry $U(y,t,\tau) \geq 0$: $a^*(y,\tau) \geq y(a^*(y,\tau) + 1)$

Requirement : Welfare Improvement

Status Quo:

No HF, $t = \tau = 0$

$$\bar{S} \equiv M\{\tilde{a}(1) - c(\tilde{a}(1))\}$$

Social Surplus:

$$S(y, t, \tau) \equiv a^*(y, \tau) - c(a^*(y, \tau))$$

$$+ \{M - w(y)\}\{\tilde{a}(1-t) - c(\tilde{a}(1-t))\}$$

$$S(y, t, \tau) > \bar{S}$$

$$a^*(y, \tau) - c(a^*(y, \tau)) + \{M - w(y)\}\{\tilde{a}(1-t) - c(\tilde{a}(1-t))\}$$

$$> M\{\tilde{a}(1) - c(\tilde{a}(1))\}$$

Requirement : Skilled Manager's Mimicry

With $a \in A$ and $F \in \Phi(a)$, Skilled Manager Receives;

$$E[\min[(1-\tau)y(z), y(z)] | F] - c(a) \\ + \{M - w(y)\} \{(1-t)\tilde{a}(1-t) - c(\tilde{a}(1-t))\}$$

No Incentive to CDP:

$$\forall a \in A \forall F \in \Phi(a):$$

$$V(y, t, \tau) \geq E[\min[(1-\tau)y(z), y(z)] | F] - c(a) \\ + \{M - w(y)\} \{(1-t)\tilde{a}(1-t) - c(\tilde{a}(1-t))\}$$

No CGT: Lemon (1)

Theorem 1 (Lemon): *Suppose CGT Rate $t = 0$. Then, There Exists No Fee Scheme that Satisfies Skilled's Entry , Unskilled's Exit , and Welfare Improvement .*

No CGT: Lemon (2)

Outline of Proof: Suppose $a > 0$ is only available, $c(a) = c$

By CDP, Unskilled Earns

$a + 1$	with Prob. $\frac{1}{a + 1}$
0	with Prob. $\frac{a}{a + 1}$

Unskilled Exit :

$$\frac{1}{a + 1} y(a + 1) + \frac{a}{a + 1} y(0) \leq 0$$

$$y(a + 1) - w(y)a \leq 0$$

No CGT: Lemon (3)

Skilled's Outside Opportunity: $M(a - c)$

Skilled's Payoff: $y(a + 1) - c + \{M - w(y)\}(a - c)$

Skilled Exit + Welfare Improvement : $y(a + 1) - w(y)a \geq \{1 - w(y)\}c > 0$

Contradiction!

Positive CGT: Existence

Theorem 2: *There exist $(t, \tau) \in [0, 1]^2$ and $y \in Y^*(\tau)$ that satisfy .*

Outline of Proof:

CGT Makes Compatible with

Skilled Has Incentive to Save CGT

Unskilled Has No Such Incentive No Skill to Earn Alpha

‘Low-Powered + Large Fund’ is More Efficient Than ‘High-Powered + Small Fund’

Welfare Optimization (1)

Specify Fee Scheme

$$y = y^{k,\tau}$$

$$(1 - \tau)y^{k,\tau}(x) = k(x - 1) \quad \text{for all } x \in [1, \infty)$$

$$y^{k,\tau}(x) = k(x - 1) \quad \text{for all } x \in [0, 1)$$

$$0 \leq k \leq \min[E, 1 - \tau], \quad w(y^{k,\tau}) = k$$

Specify CGT Rate

$$t = \hat{t}(k)$$

$$\text{Binding} \quad : \quad k\tilde{a}(k) - c(\tilde{a}(k)) = w(y)\{(1 - t)\tilde{a}(1 - t) - c(\tilde{a}(1 - t))\}$$

Welfare Optimization (2)

Specify (k^*, t^*, τ^*)

$$k = k^* \text{ maximizes } \tilde{a}(k) - c(\tilde{a}(k)) - (M - k)\{\tilde{a}(1 - \hat{t}(k)) - c(\tilde{a}(1 - \hat{t}(k)))\}$$

$$t^* \equiv \hat{t}(k^*)$$

$$\tau^* \equiv 1 - k^*$$

Tax Rate on Fee τ^* is Greater than CGT Rate t^*

Welfare Optimization (3)

Theorems 3&5&6: (y^{k^*}, t^*, τ^*) Satisfies $S(y, t, \tau) > S(y^{k^*}, t^*, \tau^*)$. There exists No (y, t, τ) that Satisfies $S(y, t, \tau) > S(y^{k^*}, t^*, \tau^*)$, and

$$S(y, t, \tau) > S(y^{k^*}, t^*, \tau^*).$$

No Tax on Fee: Useless HF

Skilled Manager Has Very Strong Incentive to Save CGT: Best Prefers $y^{\min[M, 1-\tau], \tau}$

High Tax Rate on Fee Impedes Over-Solvency

Theorem 4 (Useless HF): If $M > 1$ and $\tau = 0$, then

$$\bar{S}(\mathbf{0}) > S(y^{\min[M, 1-\tau], \tau}, t, \tau) = S(y^{1,0}, t, \mathbf{0}).$$