市場の「合理性」と資産価格モデル

青山学院大学大学院国際マネジメント研究科
福井義高

平成26年8月27日

Asset Pricing Is Paradoxical

On the one hand, the theory is so persuasive that it is widely believed to be correct, to the point that business and both the executive and jurisdictional parts of government appeal to it. Yet there is little evidence that the theory explains the past, let alone that it predicts the future.

↓

We will rather be interested in: *why?* a deeper understanding of the workings of financial markets, not just a statistical description of their history.（Bossaerts 2002）

Relative vs. Absolute Pricing

In the former, we price one security given the prices of others, while in the latter, we price each security by reference to fundamental sources of risk（Cochrane 2005）

- Relative Pricing
  - 無裁定条件の下での価格付け
  - 原資産価格が「間違って」いても構わない
- Absolute Pricing
  - 均衡概念に基づく資産評価モデル構築と検証
  - 「適切」なリスクとリターンの関係

Statistical Description ≠Asset Pricing *Theory*

- 裁定機会がなければ、mean-variance optimal portfolio（Sharpe ratioの上限）が存在
  - ➢Single beta modelは必ず見つかる
    - ベンチマークを理論で特定しなければ、有益ではあってもstatistical description/summaryにとどまる
  - ➢Fama-French model等マルチファクター・モデルも同様

Unless there is a theoretical reason why the factor portfolios “work,” it should not be referred to as an asset-pricing model.（Bossaerts 2002）

モデルの出発点

- Time-separableな期待効用最大化
  - 効用をもたらすのは消費

$$U_t = E\left[\sum_{i=t}^{\infty} \delta^{i-t} \tilde{u}(c_i) \middle| x \right]$$

- 資産価格(グロス)リターンが満たすべき条件

$$\delta E\left[\frac{\frac{\partial \tilde{u}(c')}{\partial c'}}{\frac{\partial \tilde{u}(c)}{\partial c}} R_n \middle| x \right] = 1$$

実証における特定化

- 集計問題は無視
  - 代表的個人の存在を仮定し、マクロ消費データ使用

$$\delta E\left[\frac{\frac{\partial \tilde{u}(c'_A)}{\partial c'_A}}{\frac{\partial \tilde{u}(c_A)}{\partial c_A}} R_n \middle| x\right] = 1$$

Pricing KernelとExcess Return

- Pricing kernel特定の必要性

$$A \equiv \delta \frac{\dfrac{\partial \tilde{u}(c'_A)}{\partial c'_A}}{\dfrac{\partial \tilde{u}(c_A)}{\partial c_A}} \qquad E\left[AR_n \middle| x \right] = 1$$

- Excess returnのかたちで書けば

$$E\left[R_n - R_F \middle| x \right] = -\mathrm{cov}\left(\frac{A}{E\left[A \middle| x \right]}, R_n \middle| x \right)$$

代表的モデル

- CAPM

$$E\left[R_n - R_F \left| x \right. \right] = \beta_{n,x}^{M} E\left[R_M - R_F \left| x \right. \right]$$

- CCAPM

$$\tilde{u}(c_A) = \frac{(c_A)^{\gamma+1}}{\gamma+1} \qquad A \equiv \delta \left(\frac{c'_A}{c_A} \right)^{\gamma} \qquad E\left[\delta \left(\frac{c'_A}{c_A} \right)^{\gamma} R_n \left| x \right. \right] = 1$$

- Rubinstein（1976）Model (log utility)

$$\gamma = -1 \rightarrow \tilde{u}(c_A) = \log c_A \qquad E\left[\frac{1}{R_M} R_n \left| x \right. \right] = 1$$

合理的期待の意味

- Lucas（1978）型合理的期待
  ＝ Radner（1972）型合理的期待＋ correct belief
- Radner型：consistent but can be “wrong”
- Correct belief：客観的確率分布＝主観的確率分布

$$E^m\left[R_n - R_F \middle| x\right] = -\mathrm{cov}^m\left(\frac{A}{E\left[A \middle| x\right]}, R_n \middle| x\right)$$

$$\downarrow$$

$$E\left[R_n - R_F \middle| x\right] = -\mathrm{cov}\left(\frac{A}{E\left[A \middle| x\right]}, R_n \middle| x\right)$$

定常性の追加

- 市場データの「弱点」: 実現したサンプル（パス）は一つ
- Lucas（1978）は定常性（＋mixing）すなわち

  時系列平均＝アンサンブル平均

  を仮定し、市場データを用いた実証を正当化
- Efficient Market Hypothesis＝correct belief ＋定常性

$$\frac{1}{T}\sum_{t=1}^{T}\left[R_{n,t}-R_{F,t}\right]\sim\left(\frac{1}{T}\sum_{t=1}^{T}R_{n,t}\frac{A_t}{\left[\frac{1}{T}\sum_{t=1}^{T}A_t\right]}-\left[\frac{1}{T}\sum_{t=1}^{T}R_{n,t}\right]\left[\frac{1}{T}\sum_{t=1}^{T}\frac{A_t}{\left[\frac{1}{T}\sum_{t=1}^{T}A_t\right]}\right]\right)$$

CAPM実証結果①

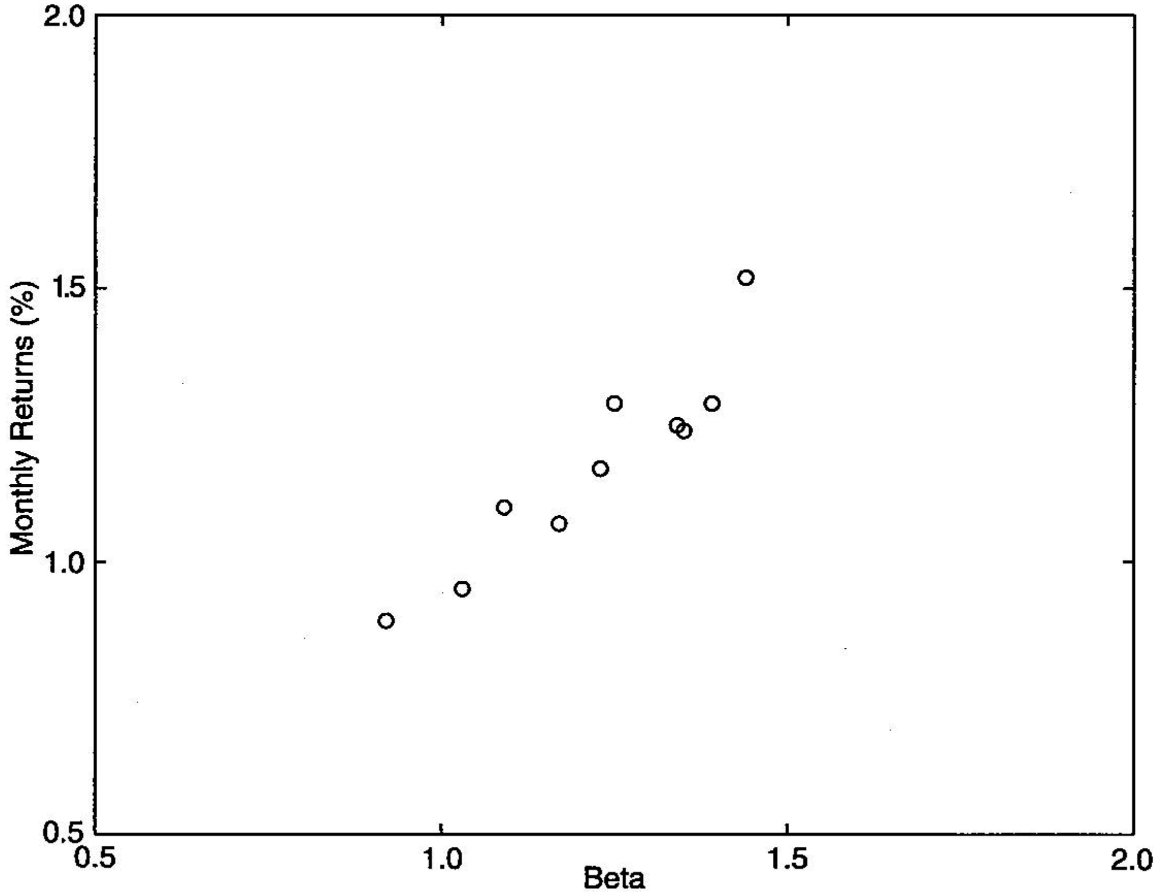

Figure 3.1

CAPM evidence: size-sorted portfolios of U.S. stock, July 1962 to December 1990.

CAPM実証結果②

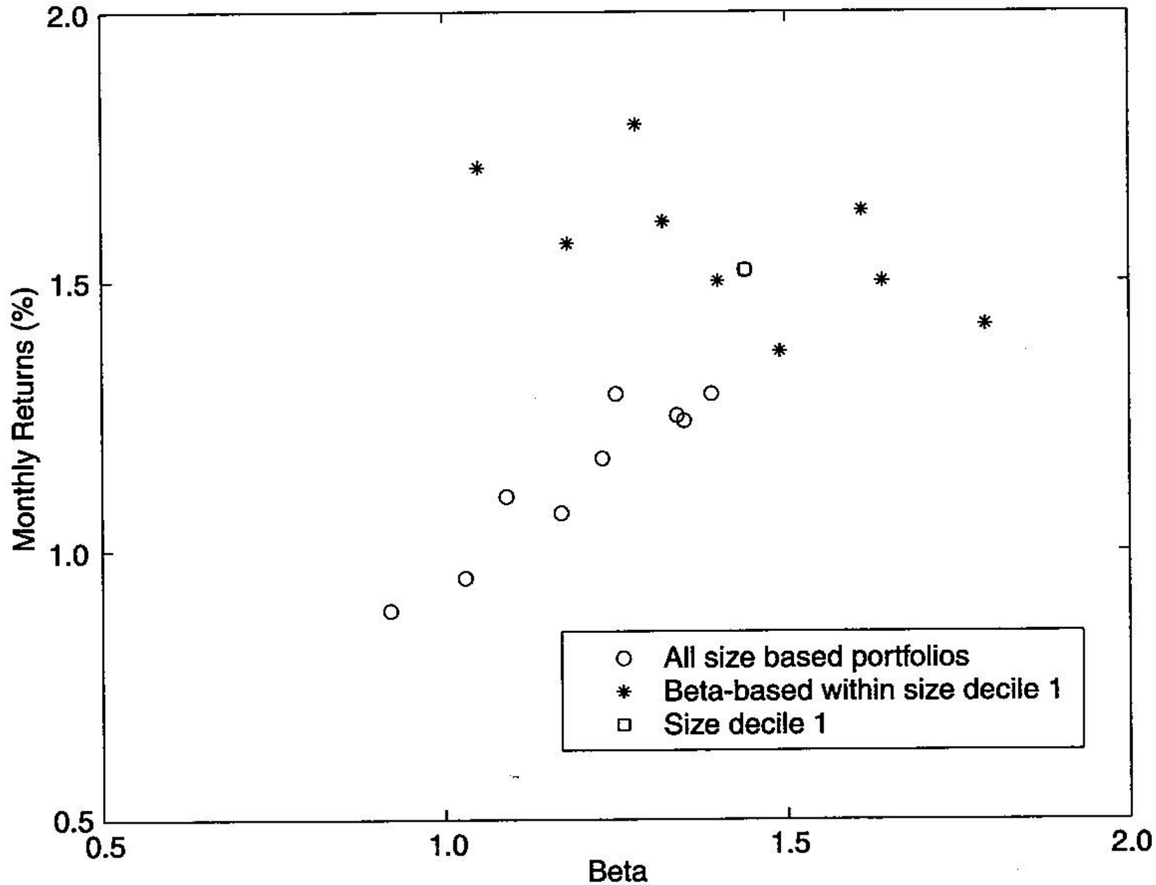

Figure 3.2

CAPM evidence: size-sorted and beta-sorted portfolios of U.S. stock, July 1962 to December 1990. Beta-based within size decile 1.

CAPM実証結果③

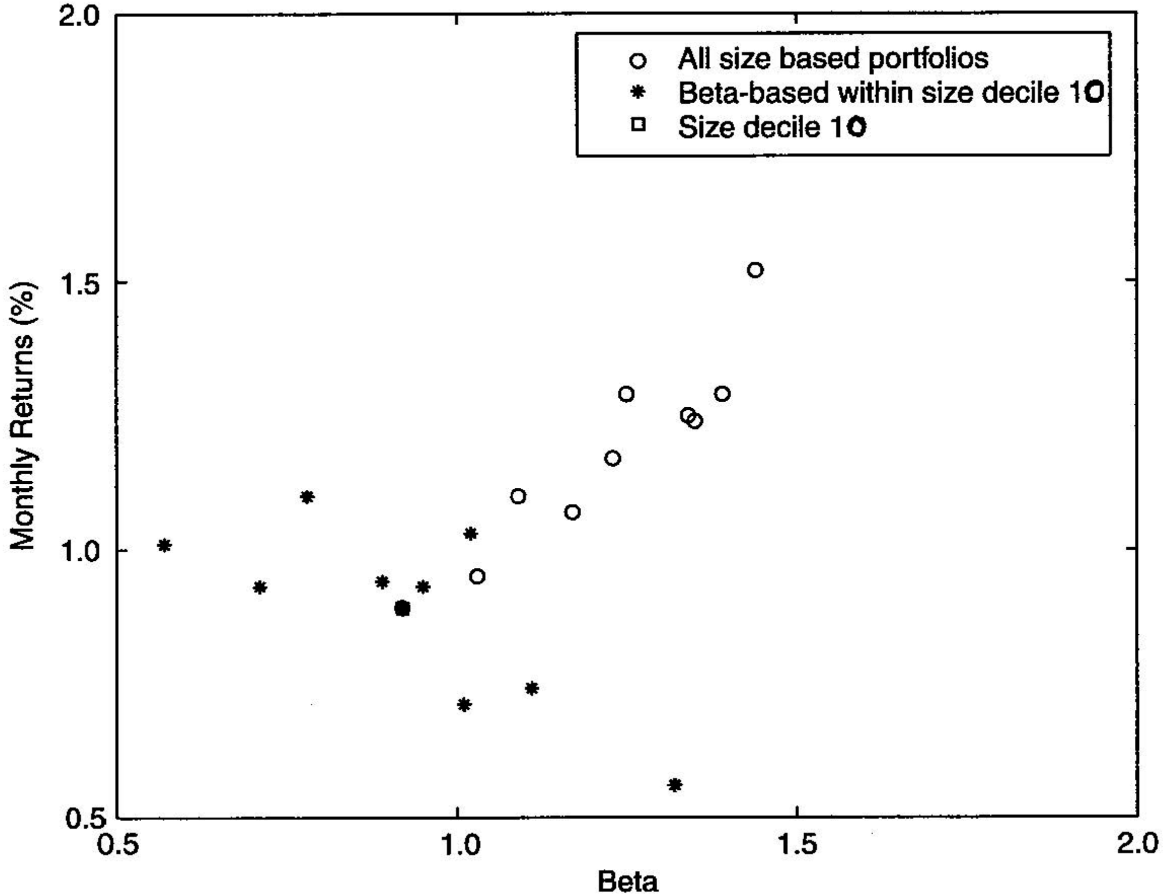

Figure 3.3

CAPM evidence: size-sorted and beta-sorted portfolios of U.S. stock, July 1962 to December 1990. Beta-based within size decile 10.

CCAPM：無条件

- 多くの場合、conditioning informationを無視

$$E\left[AR_n \middle| x\right] = \delta E\left[\left(\frac{c'_A}{c_A}\right)^{\gamma} R_n \middle| x\right] = 1$$

ではなく、

$$\delta E\left[\left(\frac{c'_A}{c_A}\right)^{\gamma} R_n\right] = 1$$

を仮定し、

$$\delta \frac{1}{T}\sum_{t=1}^{T}\left(\frac{c_{A,t}}{c_{A,t-1}}\right)^{\gamma} R_{n,t} \sim 1$$

を用いて検証

CCAPM:条件付き

- Instrumentsを用いることで条件付きモデルも検証可能

$$\delta E\left[\left(\frac{c'_A}{c_A}\right)^{\gamma} R_n x_b \middle| x\right] = x_b$$

$$\delta E\left[\left(\frac{c'_A}{c_A}\right)^{\gamma} R_n x_b\right] = E[x_b]$$

$$\delta \frac{1}{T}\sum_{t=1}^{T}\left(\frac{c_{A,t}}{c_{A,t-1}}\right)^{\gamma} R_{n,t} x_{b,t} \sim \frac{1}{T}\sum_{t=1}^{T} x_{b,t}$$

Equity Risk Premium Puzzle

- 市場リスクプレミアムには上限（Sharpe ratio）

$$\frac{\sigma(A)}{E[A]} \geq \frac{E[R_M - R_F]}{\sigma(R_M)}$$

- ところが、右辺が0.5前後なのに、「常識」的危険回避度のもとでは、左辺は一桁小さい数値
- 「非常識」な危険回避度を想定すると、今度は無リスク利子率が高くなり過ぎる
- Mehra & Prescott（1985）が指摘して以来、数々の解決策が提示されるも、危険回避度と異時点間代替率の連動がネックに

変動するリスク･プレミアアム

- ストック価格＝フロー／割引率
  - ➢分子のフローの時系列は安定しているのに、ストック価格は大きく変動
- Correct beliefを仮定すると、分母の割引率変動すると考えるしかない
  - ➢ 無リスク利子率は比較的安定しているので、市場リスク･プレミアアム変動を「説明」せねばならない

Habit Persistence

- Power utilityのまま、期待効用にhabitを導入
  - 現時点の効用に過去の消費パターンが影響（Campbell & Cochrane 1999)

$$\tilde{u}(c_A) = \frac{(c_A - c_A^H)^{\gamma+1}}{\gamma+1}$$

- 危険回避度と異時点間代替率が切り離され、大きなリスク・プレミアムと低い利子率が「説明」できる

しかし、

A dose of scepticism is needed when interpreting the accomplishments: it took about twenty years to fit a consumption-based model to what turns out to be a very small historical data set…（Bossaerts 2002）

Non-Time Separable Utility

- 標準的でない効用関数の導入

$$U_t = \left[c_t^{\gamma+1} + \delta \left[E_t(U_{t+1}^{\alpha+1}) \right]^{\frac{\gamma+1}{\alpha+1}} \right]^{\frac{1}{\gamma+1}}$$

- これをもとに、割引率とキャッシュフローの二つのファクターに分離した「良いベータ・悪いベータ」モデル (Campbell & Vuolteenaho 2004)

データ選択と定常性

- Survivorship biasやselection biasがあると、クリーンな理論検証はできない
- データ特性が非定常であることと、用いた時系列期間が「典型的」でないというPeso problem（Danthine & Donaldson 1999）は、

$$\frac{1}{T}\sum_{t=1}^{T}\left[R_{n,t}-R_{F,t}\right]\sim\left(\frac{1}{T}\sum_{t=1}^{T}R_{n,t}\frac{A_t}{\left[\frac{1}{T}\sum_{t=1}^{T}A_t\right]}-\left[\frac{1}{T}\sum_{t=1}^{T}R_{n,t}\right]\left[\frac{1}{T}\sum_{t=1}^{T}\frac{A_t}{\left[\frac{1}{T}\sum_{t=1}^{T}A_t\right]}\right]\right)$$

を用いて理論が検証できないという点で同じ

否定されているのは理論、補助仮説？

- データの制約から、代表的個人を仮定した集計量ベースの検証にならざるを得ない
- さらに、追加的に仮定される、定常かつcorrect belief（Lucas 1978）の下ではlearningは存在しない
- Lucas型合理的期待の仮定の下、パラメータを追加することで「説明」するより、定常あるいはcorrect beliefを仮定しない理論の実証は可能か？
- そもそも、投資家の合理性として、時系列で次々と与えられる情報を正しく理解することを超えて、あらゆる事象の事前確率を正しく予測ことまで要求すべきか？

Field Dataの限界と実験の可能性

Many problems can plague test on field data… Tests rely on a host of auxiliary assumptions that are not necessary part of asset-pricing theory…

Economic experiments provide a unique opportunity to evaluate whether and when the predictions of the theory obtains…

We will demonstrate that asset-pricing theory correctly predicts the direction of price movement in experimental markets. (Bossaerts 2002)

実験経済学からわかること

- 必ずおカネを払うことで実験を「真剣」勝負に
  - 対象者である学生の時給を勘案
- 理論の知識に影響されないように実験設定
  - 対象者が実験の裏をかくことを防ぐ
- 市場データと違い、パラメータは自分で設定するので、理論のクリーンなテストが可能

実験の概要

Table 4.1

Typical Payoff Matrix

Security	State X	State Y	State Z
A	170	370	150
B	160	190	250
Notes	100	100	100

Table 4.2

List of Experiments and Their Parameters

	Experiment	UCLA		Yale2		Stanford	
	Yale 1	Group I	Group II	Group I	Group II	Group I	Group II
Number of subjects	30	23	21	8	11	22	22
Signup reward (F)	0	0	0	0	0	175	175
Allocation							
A	4	5	2	5	2	9	1
B	4	4	7	4	7	1	9
Notes	0	0	0	0	0	0	0
Cash (F)	400	400	400	400	400	400	400
Loan (F)	1900	2000	2000	2000	2000	2500	2400
Exchange rate ($/F)	0.03	0.03	0.03	0.03	0.03	0.04	0.04

実験結果：理論値に収束①

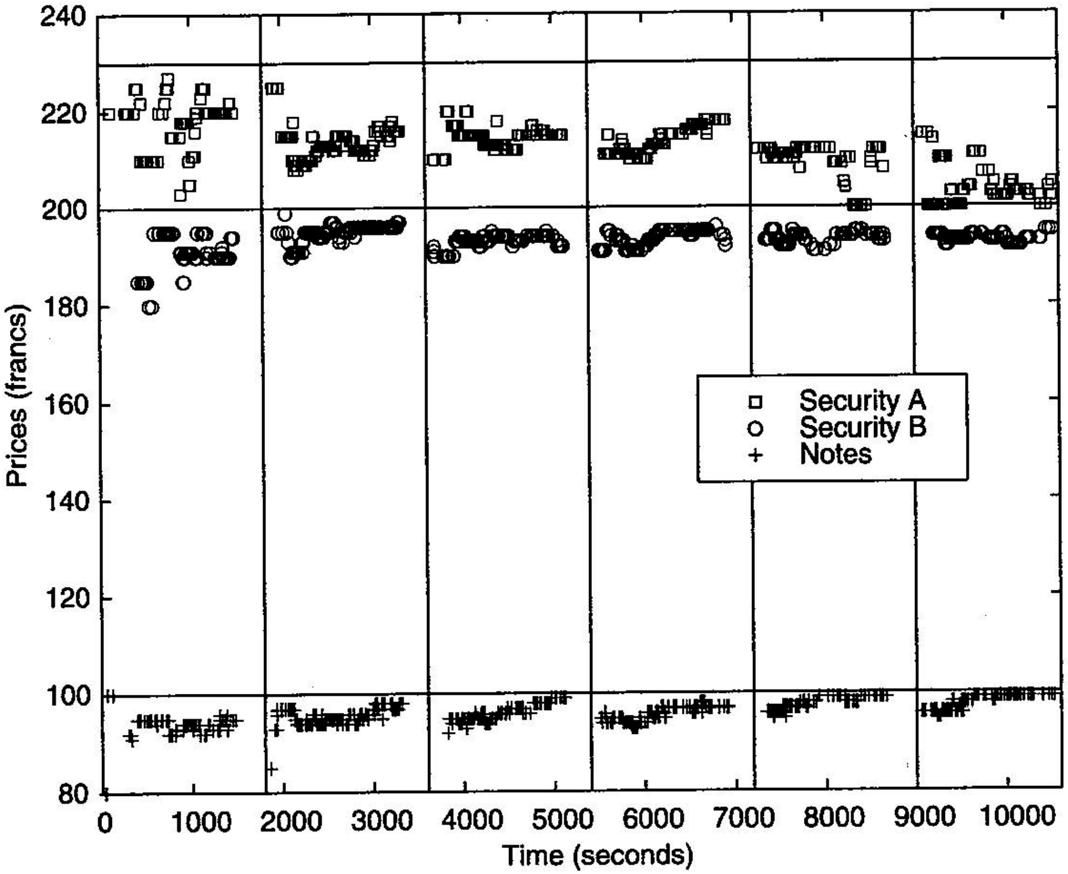

Figure 4.2

Transaction prices in experiment Yale1. Vertical lines delineate periods; horizontal lines show expected payoffs.

実験結果：理論値に収束②

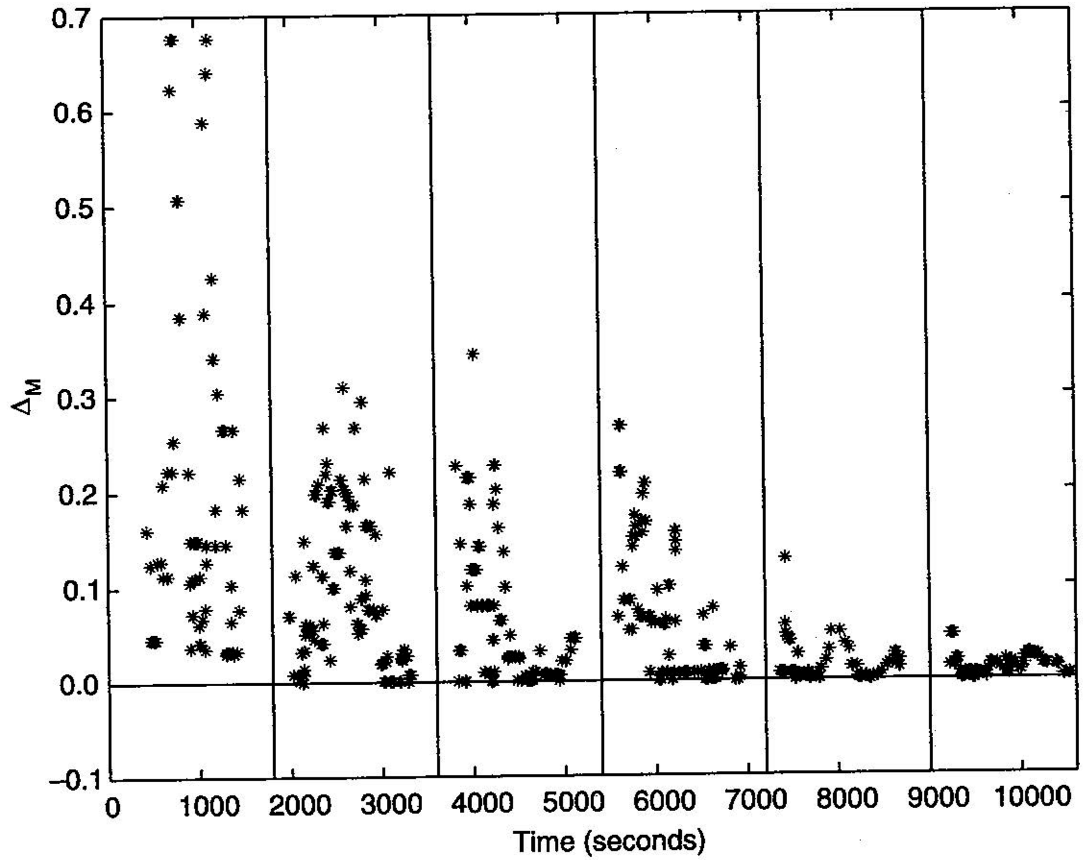

Figure 4.3

Distance from CAPM equilibrium (Δ_M) in experiment Yale1. Vertical lines delineate periods.

実験結果：収束後、再び乖離

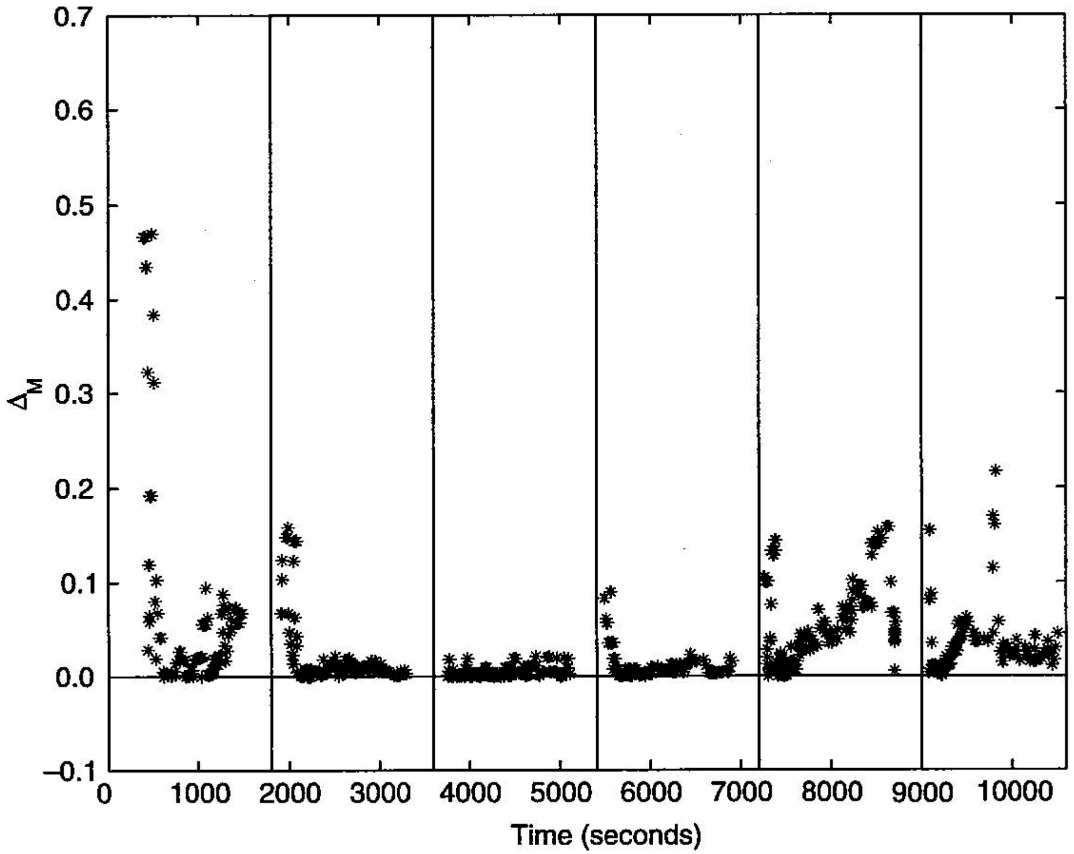

Figure 4.5

Distance from CAPM equilibrium (Δ_M) in experiment UCLA. Vertical lines delineate periods.

実験結果:「誤った」期待形成

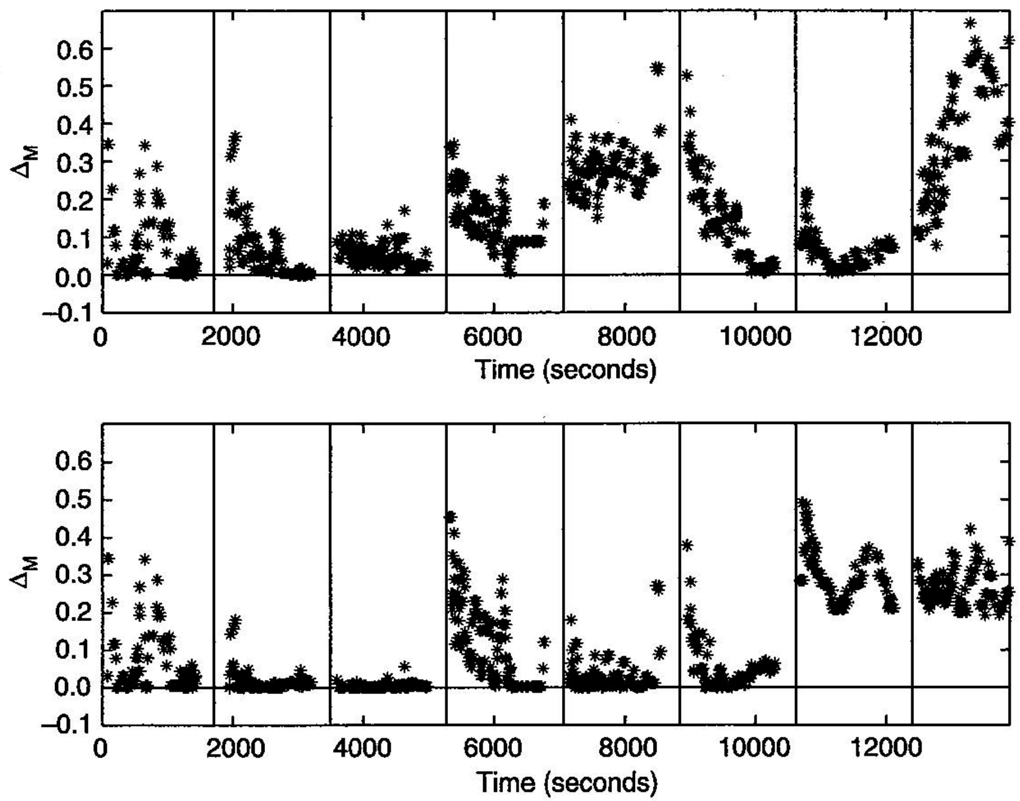

Figure 4.8

Distance from CAPM equilibrium (Δ_M) in experiment Stanford. Top panel, based on announced likelihood of states. Bottom panel, based on falsely interpreting the drawing of states as taking place without replacement. Vertica lines delineate periods.

おおむね理論は支持されるが．．．

- 時間はかかるものの、理論が予測する均衡値におおむね収束

しかし、

It is surprising how hard it is to model subjects’ beliefs even in such simple experiments…

It is inappropriate to build an empirical methodology of tests of asset-pricing models entirely on the extreme assumption of correct priors…

Consequently, the time is ripe to consider changes to the empirical methodology that allow for biases in the markets’ belief, even if only partial.

実験経済学の問題点

Expected-utility maximizers are (almost everywhere) arbitrarily close to risk neutral when stakes are arbitrarily small... the approximate risk-neutrality prediction holds not just for negligible stakes, but for quite sizable and economically important stakes.

Expected-utility theory may well be a useful model of the taste for very-large-scale insurance. Despite its usefulness, however, there are reasons why it is important for economists to recognize how miscalibrated expected-utility theory is as an explanation of modest-scale risk aversion.

（Rabin 2000）

ポストIPOアノマリー

- IPOの後、投資家は「失望」するのが常
- 倒産があり得る上場企業株式は定義上、非定常
- 将来の倒産確率を平均的に正しく予測することを「合理性」にふくめるのは酷
- 事後に倒産せず存在している企業だけを対象に実証
- サンプルにwinners（非倒産企業）しか含まれないという「バイアス」があっても問題ない実証方法の存在

（Bossaerts 2001; 2002; 2004）

CAR下方バイアス

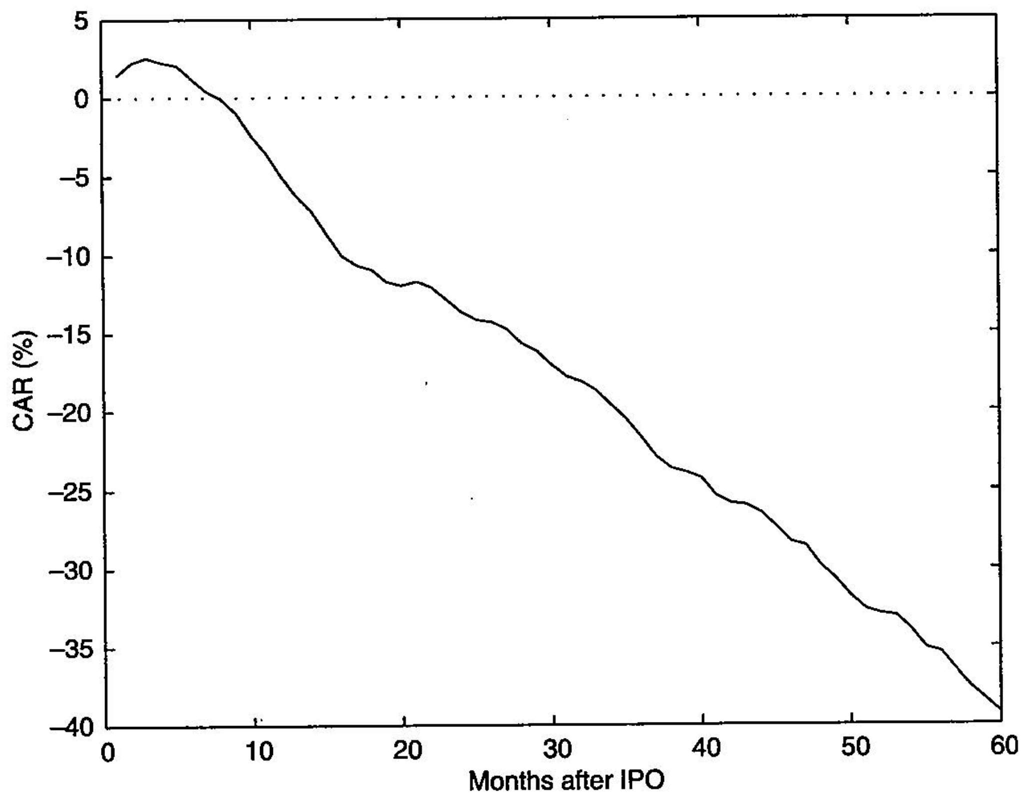

Figure 6.1

Cumulative average monthly returns (CAR) on 4,848 U.S. IPOs in the period 1975–95, in excess of the cumulative average monthly return on the value-weighted CRSP index, excluding the first day of trading.

Efficient Learning Market

- 定常性もcorrect beliefも仮定せず、correct/efficient learningのみ仮定
- 最初の見込み（事前確率）はまちがっていても、ベイズ・ルールで事前確率を正しく修正

$$\lambda_t(V) = \frac{l_t(x_t \mid x_{t-1}, V)\lambda_{t-1}(V)}{\int l_t(x_t \mid x_{t-1}, v)\lambda_{t-1}(v)dv}$$

- 加重修正リターンがゼロという、パラメータ推計不要（ただし、対数効用を仮定）の条件が導出できる

$$E\left[\frac{P_t - P_{t-1}}{P_t}V \middle| x_{t-1}, V > 0\right] = 0$$

加重修正リターンゼロ

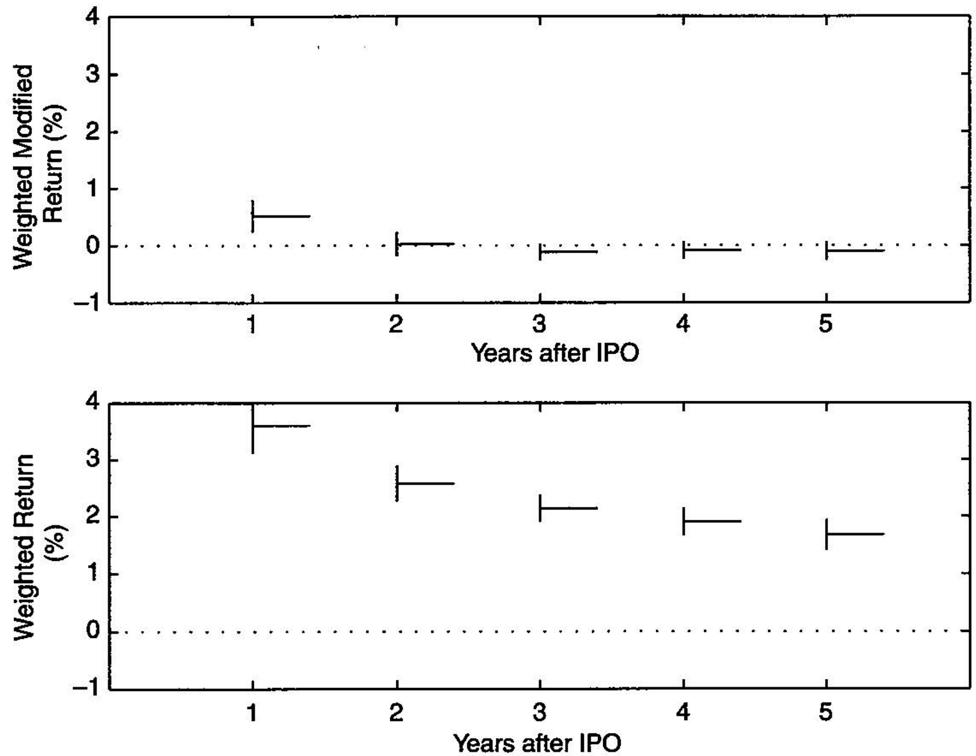

Figure 6.2

One-month weighted average risk-adjusted returns (horizontal bars) and 95 percent confidence intervals (vertical bars) on 4,848 U.S. IPOs in the period 1975–95, for different horizons, winners only. Top panel, weighted modified return. Bottom panel, usual return. Modified return is computed with end-of-month risk-adjusted price as basis. Risk adjustment is based on Rubinstein's model.

ELMの応用: Risk Premium Puzzle

- Bossaertsは実証ファイナンス研究を「廃業」したものの、学界大御所のSargentがELMを米国株式市場長期時系列に応用（Cogley and Sargent 2008）
- 1929年以降の大恐慌で投資家は過度に悲観的なbelief（prior）形成
- 時間選好率＝無リスク利子率=2.3%とリスク中立の過仮定の下でも、Risk Premium Puzzleを「解決」するシミュレーション結果を提示

Mean Excess Returns

Table 7
Mean excess returns

	Full sample	First half	Second half
Data: 1934–2003	**0.0680**	**0.0990**	**0.0360**
Prior: 1890–1933			
Beta	0.0090	0.0153	0.0027
$2 \log B = 5$	0.0317	0.0485	0.0148
$2 \log B = 10$	0.0515	0.0747	0.0283
Prior: 1919–1933			
Beta	0.0267	0.0425	0.0109
$2 \log B = 5$	0.0421	0.0566	0.0277
$2 \log B = 10$	**0.0511**	**0.0654**	**0.0369**

Standard Deviations

Table 8
Standard deviation of excess returns

	Full sample	First half	Second half
Data: 1934–2003	**0.166**	**0.177**	**0.151**
Prior: 1890–1933			
Bayesian	0.130	0.161	0.066
$2 \log B = 5$	0.174	0.226	0.071
$2 \log B = 10$	0.241	0.305	0.115
Prior: 1919–1933			
Bayesian	0.147	0.179	0.080
$2 \log B = 5$	0.128	0.146	0.078
$2 \log B = 10$	**0.136**	**0.148**	**0.092**

Sharpe Ratios

Table 9
Sharpe ratios for excess returns

	Full sample	First half	Second half
Data: 1934–2003	**0.409**	**0.559**	**0.235**
Prior: 1890–1933			
Bayesian	0.069	0.095	0.041
$2 \log B = 5$	0.182	0.215	0.208
$2 \log B = 10$	0.214	0.245	0.246
Prior: 1919–1933			
Bayesian	0.182	0.237	0.136
$2 \log B = 5$	0.329	0.388	0.355
$2 \log B = 10$	**0.376**	**0.442**	**0.401**

終わりに：合理的に学ぶ投資家

An econometrician who conditions on less information than consumers still draws correct inferences about the magnitude of risk prices…しかし、

An econometrician who mistakenly conditions on more information than consumers makes false inferences about the magnitude of risk prices.（Hansen and Sargent 2010）

- 情報から「正しく」学ぶという点では合理的な投資家？

It appears to be more appropriate to model financial markets in the image of a Bayesian learners who may at times hold mistaken expectations, but who updates beliefs correctly.

（Bossaerts 2002）

参照文献

Bossaerts, P. 2001. IPO Post-Issue Markets: Questionable Predilections But Diligent Learners? *Review of Economics and Statistics* 83 (2): 334-347.

Bossaerts, P. 2002. *The Paradox of Asset Pricing*. Princeton, U.S.A.: Princeton University Press.

Bossaerts, P. 2004. Filtering Returns for Unspecified Biases in Priors When Testing Asset Pricing Theory. *Review of Economic Studies* 71 (1): 63-86.

Campbell, Y. J., and J. H. Cochrane. 1999. By Force of Habit: A Consumption-Based Explanation of Aggregate Stock Market Behavior. *Journal of Political Economy* 107 (2): 205-251.

Campbell, J. Y., and T. Vuolteenaho. 2004. Bad Beta, Good Beta. *American Economic Review* 94 (5): 1249-1275.

Cochrane, J.H. 2005. *Asset Pricing*, Revised Edition. Princeton, U.S.A.: Princeton University Press.

Cogley, T., and T. J. Sargent. 2008. The Market Price of Risk and the Equity Premium: A Legacy of the Great Depression? *Journal of Monetary Economics* 55 (3): 454-476.

Danthine, J.-P., and J. B. Donaldson. 1999. Non-Falsified Expectations And General Equilibrium Asset Pricing: The Power of the Peso. *Economic Journal* (October) 109 (458): 607-635.

Hansen, L. P., and T. J. Sargent. 2010. Fragile Beliefs and the Price of Uncertainty. *Quantitative Economics* 1 (1): 129-162.

Lucas, R. E. 1978. Asset Prices in an Exchange Economy. *Econometrica* 46 (6): 1429-1445.

Mehra, R., and E. C. Prescott. 1985. The Equity Premium: A Puzzle. *Journal of Monetary Economics* 15 (2): 145-161.

Rabin, M. 2000. Risk Aversion and Expected-Utility Theory: A Calibration Theorem. *Econometrica* 68 (5): 1281-1292) .

Radner, R. 1972. Existence of Equilibrium of Plans, Prices, and Price Expectations in a Sequence of Markets. *Econometrica* 40 (2): 289-303.

Rubinstein, M. 1976. The Valuation of Uncertain Income Streams and the Pricing of Options. *Bell Journal of Economics* 7 (2): 407-425.