NBER WORKING PAPER SERIES

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Working Paper 29835
http://www.nber.org/papers/w29835

NATIONAL BUREAU OF ECONOMIC RESEARCH
1050 Massachusetts Avenue
Cambridge, MA 02138
March 2022

We thank Changhua Yu, Jiao Wang, and participants at China International Conference in Macroeconomics and the seminar at Peking University. Juanyi Xu acknowledges support from the government of Hong Kong Special Administrative Region through the General Research Grant (16602420). Devereux acknowledges support from the Social Sciences and Humanities Research Council of Canada. The views expressed herein are those of the authors and do not necessarily reflect the views of the National Bureau of Economic Research.

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NBER Working Paper No. 29835
March 2022
JEL No. F3,F4

ABSTRACT

We explore how consumption heterogeneity affects the international transmission mechanism of monetary shocks and the choice of optimal monetary policy in an open economy. Incorporating two types of agents (Ricardian versus Keynesian) into a standard open economy macro model, we find that there are sizeable ranges of household heterogeneity in which monetary policy become ineffective, but this depends sensitively on the interaction of aggregate demand and relative price effects. We derive the global optimal monetary policy with household heterogeneity under alternative pricing regimes. PPI targeting is still the optimal monetary policy under PCP and can restore the economy to the efficient equilibrium. Under LCP, however, the presence of consumption heterogeneity and currency misalignment implies that CPI inflation targeting is no longer optimal in most cases. Finally, we show that when fiscal instruments such as an import tax and export subsidy are introduced, both currency misalignment and consumption heterogeneity can be eliminated, and even under LCP, PPI targeting is the optimal monetary rule.

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1 Introduction

Over the last few decades, income and wealth inequality have increased considerably in both developing countries and advanced economies. For example, according to a survey by the US Census Bureau, the Gini coefficient in the United States was 0.49 in 2018, rising from 0.43 in 1990, indicating an increase in income inequality. Likewise, according to the World Inequality Database, the share of the top 10 percent of US income earners went from 37 percent in 1990 to 45 percent in 2020. Similar trends have been seen in other OECD countries. Our focus is particularly on the question of how monetary policy interacts with inequality, and how the presence of heterogeneity impacts on the optimal monetary policy problem in an open economy.

This question has already attracted considerable attention among policy makers and academic researchers. A growing literature now incorporates heterogeneity into New-Keynesian macro models (see references below). This literature argues that monetary policy mostly operates via general equilibrium effects on the labour market, rather than the standard intertemporal substitution channels which are key to the first generation New Keynesian models. In these models, households that are constrained in their access to financial markets have high marginal propensities to consume and their spending reacts strongly to changes in disposable income.

In this paper, we explore how heterogeneity impacts on the international transmission of monetary policy shocks in a two-country open economy macro model, and in addition, how the presence of heterogeneity affects the design of optimal monetary policy. Heterogeneity in our model is introduced in a simple analytically tractable manner. We assume two types of households, namely, "Ricardian" and "Keynesian" consumers, with a constant share over time. Ricardian consumers can smooth consumption over time by borrowing and lending freely in financial markets, as in the standard New Keynesian model. Keynesian consumers however are hand-to-mouth agents who do not have access to financial markets and therefore can only consume their disposable labor income every period. Our model is thus a version of a Two-Agent New Keynesian model (TANK) as in Debortoli and Gali (2018).

In the absence of price stickiness, heterogeneity has no material impact in the model. The heterogeneity of Ricardian consumption to Keynesian consumption negatively responds to the output gap. In face of an output gap increase, Keynesian households consume more than Ricardian households. However, even with household heterogeneity, our solution to the flexible price equilibrium is exactly the same as the efficient allocation in the standard new open macro model (see for instance Clarida Gali and Gertler, 2002). There is no consumption heterogeneity in equilibrium. This is because in the flexible price case, Ricardian agents, who hold all equity in the economy, receive a zero dividend, and consume the same amount as Keynesian agents.[1]

When we consider the sticky price case, the situation differs considerably. First, when we consider the PCP case, we find that household heterogeneity does matter for the transmission mechanism of monetary shocks. We find that there exists a range of household heterogeneity, in which expansionary monetary policy is ineffective, which we call a "monetary trap".[2] Also, consumption heterogeneity itself introduces a new channel of international spillovers of monetary shocks under PCP. The effect could be positive or negative, depending on the degree of household heterogeneity.

Intuitively, household heterogeneity has two effects in the open economy. The first is the aggregate consumption effect. Domestic output is determined by aggregate consumption in both the home and foreign market and the relative price of home goods. In face of a domestic interest rate cut, home Ricardian consumption increases, which raises foreign Ricardian consumption through a risk-sharing condition. However, the effect on aggregate consumption depends critically on the composition of households. When there are more Ricardian households, aggregate consumption will increase with Ricardian consumption, which increases output. However, when there are more Keynesian households, aggregate consumption may fall, reducing output.

[1]To achieve an efficient allocation, a production subsidy is used to eliminate monopolistic distortion, as usual in the literature. When prices are flexible, the markup is constant and will be fully eliminated.

[2]This result is similar to Bilbiie (2008).

The second effect is the relative price or terms of trade effect. When there is an interest rate cut in the home economy, the relative price of home goods to foreign goods falls, which also increases domestic output. We show that as the share of Keynesian households increases beyond a certain threshold, the relative price effect dominates, and the negative impact of monetary policy on output becomes positive again. Hence, the range in which monetary policy becomes ineffective depends on the share of Keynesian household and the interaction of the two effects.

The spillover of domestic monetary policy also depends on the heterogeneity and interaction of these two effects. In face of an interest rate cut in the home economy, due to the risk-sharing condition, foreign Ricardian consumption increases as much as the home Ricardian consumption, which also increases aggregate consumption and then output. However, when the share of home Keynesian household is small, the consumption effect is positive and dominates the negative terms of trade effect, so we observe a positive spillover effect of monetary policy. But when the share of home Keynesian households is large enough, both effects are negative, so the spillover effect is negative. In this case, household heterogeneity generates the "beggar-thy-neighbor"effect emphasized by Betts and Devereux (2000).

We also consider a PCP case with asymmetric household heterogeneity. In such a setting, when the sizes of Keynesian household in both countries are small, we can find that both home and foreign expansionary monetary shock can raise domestic output, while the impact of monetary shocks on output is amplified by the size of Keynesian households in both countries.

The results change considerably in the case of LCP. With symmetric household heterogeneity, there is a threshold such that when the share of Keynesian households is below this threshold, an expansionary monetary shock in the home or foreign country will increase home output, but beyond this threshold, the effect is negative for both home or foreign monetary shocks. Unlike the PCP case however, we find that further increases in the share of Keynesian households do not restore the positive impact of monetary shocks. This is because the impact of monetary shocks in the

LCP case is fully determined by the aggregate consumption effect, and independent of the relative price effect.

We then solve for the global optimal monetary policy with consumption heterogeneity. Interestingly, PPI targeting is still the optimal monetary policy under PCP and can restore the economy to the efficient equilibrium. This is because consumption heterogeneity proportionally responds to the output gap, and once the output gap is closed, then heterogeneity will not be a concern for monetary authorities. However, under LCP, due to currency misalignment, the output gap cannot be closed. Consequently, consumption heterogeneity becomes a new distortion faced by policy makers, and this makes CPI inflation targeting no longer optimal in most cases. There is only one exception. when there is no home bias, the relative output between two countries is independent of monetary policy and currency misalignment. In such a case, the central bank will choose CPI to eliminate currency misalignment, which helps to achieve efficient world output and inflation stability.

Finally, we show that both currency misalignment and consumption heterogeneity can be eliminated when fiscal instruments such as an import tax and export subsidy are introduced under LCP. With the help of fiscal instruments, the optimal monetary policy will target PPI again.

Our paper is closely related to two strands of literature. The first is the work on new Keynesian open-economy macroeconomic model. Two papers are considered as benchmark for comparison, Clarida, Gali and Gertler (2002, hereafter CGG) and Engel (2011). CGG develops the canonical model for open-economy monetary policy analysis in the Keynesian framework. They show that if price setting is based on PCP, the central bank should target producer inflation (PPI). Engel (2011) examines optimal monetary policy under the LCP case and argues that due to currency misalignment, CPI targeting should be optimal. These two papers assume a Calvo mechanism for price setting, which differs from a large set of papers in which prices are sticky, but are set one period in advance. For example, see Devereux and Engel (2003), Corsetti and Pesenti (2005). Our papers incorporate two types of agents in their model and study how household heterogeneity affect the choice of optimal monetary policy. We derive the role of consumption heterogeneity in the policy maker's loss function, and find that when the output gap cannot be closed, consumption heterogeneity should be a concern for the central bank as well.

The second one is the literature of the heterogenous agent model that studies the redistributive effects of fiscal and monetary policies. For example, Kaplan, Moll, and Violante (2018) embed heterogeneous agents and incomplete markets into the New Keynesian workhorse model. However, in the standard heterogeneous agent model, households face idiosyncratic labor income shocks that cannot be fully insured against. As a result, there exists a non-degenerate time-varying wealth distribution that needs to be tracked, as well as difficulties arising from the presence of occasionally binding borrowing constraints. To avoid the computation difficulty, Debortoli and Gali (2018) assess the comparative advantage of a simpler alternative heterogeneous agent model, namely the Two Agents New Keynesian model, in understanding aggregate dynamics relative to heterogeneous agent models. In this model, heterogeneity of households is characterized by two types of households, namely, "Ricardian" and "Keynesian" consumers, with a constant share over time. Similarly, Bilbiie (2008) introduces two types of households (asset holder vs non-asset holder) in dynamic general equilibrium and develops a simple analytical framework for monetary policy analysis. He argues that low asset participation may lead to inverted aggregate demand logic and it also affect the aggregate dynamics and stability of economy. Different from their work, we focus on the implication of consumption heterogeneity for monetary policy in international dimensions.

The study of the heterogeneous agent model in an open economy is still in its infancy. Auclert, Rognile, Souchier and Straub (2021) introduce heterogeneous households to a New-Keynesian small open economy model and find that heterogeneous households amplify the real income channel of exchange rates: the rise in import prices from a depreciation lowers households' real incomes, and leads them to cut back on spending. Ferra, Mitman and Romei (2020) build a Heterogeneous-Agent

New-Keynesian small open model economy that experiences a current account reversal. They find that the contraction is more severe when households are leveraged and owe debt in foreign currency. Guo, Ottonello and Perez (2020) develop a heterogeneous-agent New Keynesian small open economy model, in which households differ in their income, wealth, and real and financial integration with international markets. They show that there is a trade-off between aggregate stabilization and inequality in consumption responses to external shocks. However, these papers are based on small open economy model, and do not explicitly investigate the role of heterogeneity in the optimal monetary problem in such a setting. Different from their work, we investigate the role of household heterogeneity in the international monetary transmission mechanism in a two-country model, and explore the optimal monetary policy in an environment of heterogeneity.

The rest of the paper is organized as follows. Section 2 present a two- agent two-country model; Section 3 discusses the flexible price equilibrium; Section 4 and 5 study how household heterogeneity affect the transmission mechanism of monetary shock under PCP and LCP respectively. Section 6 explores the optimal monetary policy under both PCP and LCP. Section 7 concludes.

2 A two-country model with household heterogeneity

Our benchmark model extends the existing NOEM literature by introducing household heterogeneity. The PCP model is standard and similar to the classic Clarida, Gali and Gertler (2002) model. The LCP model is standard and similar to Engel (2011). The difference in each case is that we model two types of agent in each country.

Following Bilbiie (2008) and Debortoli and Gali (2018), we adopt a Two-Agent New Keynesian (TANK) model, in which the *within group* difference for different type of agent is ignored but the difference between *two group* of agents are emphasized.

2.1 Household

There is a continuum of households $[0,1]$, all having the same utility function in the home country. A constant measure $1-n$ of households, is labelled Ricardian and have unconstrained access to the financial markets. A fraction n of households, referred to as Keynesian just consume their labor income and lump-sum transfers each period.

Let $s \in \{R, K\}$ specify the household type (Ricardian and Keynesian). The utility is

$$E\sum_{t=0}^{\infty}\beta^t[\ln(C_t^s)-\eta\frac{(L_t^s)^{1+\omega}}{1+\omega}] \tag{1}$$

$$C_t^s=[C_{h,t}^s]^{\frac{v}{2}}[C_{f,t}^s]^{1-\frac{v}{2}} \tag{2}$$

We assume that home bias v is identical in both types of agent. This implies that $P_t^s=P_t$, so the consumption price index is identical for both types of households. So we omit the superscript "R" or "K" for the price variables. The demand of the Home and Foreign good is

$$C_{h,t}^s = \frac{v}{2}\frac{P_tC_t^s}{P_{hht}} \tag{3}$$

$$C_{f,t}^s = (1-\frac{v}{2})\frac{P_tC_t^s}{P_{fht}} \tag{4}$$

The consumption index $C_{ht}^s=[\int_0^1 C_{ht}^s(i)^{\frac{\varepsilon-1}{\varepsilon}}di]^{\frac{\varepsilon}{\varepsilon-1}}$ and $C_{ft}^s=[\int_0^1 C_{ft}^s(i)^{\frac{\varepsilon-1}{\varepsilon}}di]^{\frac{\varepsilon}{\varepsilon-1}}$ And demand for varieties:

$$C_{ht}^s(i) = (\frac{P_{hht}(i)}{P_{hht}})^{-\varepsilon}C_{ht}^s;\ C_{ht}^{s*}(i)=(\frac{P_{hft}^*(i)}{P_{hft}^*})^{-\varepsilon}C_{ht}^{s*} \tag{5}$$

$$C_{ft}^s(i) = (\frac{P_{fht}(i)}{P_{fht}})^{-\varepsilon}C_{ft}^s;\ C_{ft}^{s*}(i)=(\frac{P_{fft}^*(i)}{P_{fft}^*})^{-\varepsilon}C_{ft}^{s*} \tag{6}$$

2.1.1 Ricardian Households

Ricardian households have assess to both state-contingent bonds in the domestic market and foreign market. They are also equity holders who claim the ownership of firms. Their period budget constraint is

$$\begin{aligned} & P_t^R C_t^R + B_{t+1} + Q_t^e S_t^R + \sum_{\zeta^{t+1} \in Z_{t+1}} Q(\zeta^{t+1}|\zeta^t) D_{hh}(\zeta^{t+1}) + \varepsilon_t \sum_{\zeta^{t+1} \in Z_{t+1}} Q^*(\zeta^{t+1}|\zeta^t) D_{hf}^*(\zeta^{t+1}) \\ = \; & W_t L_t^R + (1 + i_{t-1}) B_t + (D_t^e + Q_t^e) S_{t-1}^R + D_{hh}(\zeta^t) + \varepsilon_t D_{hf}^*(\zeta^t) \end{aligned} \tag{7}$$

B_{t+1} and S_t^R are the holding of domestic non-state-contingent bonds and equity. Q_t^e and D_t^e are the price and dividend of the equity. i_{t-1} is the domestic bond's nominal interest rate. ε_t is the nominal exchange rate. $D_{hh}(\zeta^{t+1})$ and $D_{hf}^*(\zeta^{t+1})$ are Home's holding of the state-contingent domestic and foreign security.[3] Here we are assuming complete international financial markets. L_t^R is the labor supplied by the Ricardian household and $W_t L_t^R$ is wage income.

Let Λ_t represent the Lagrangian multiplier associated with the flow budget constraint. The stochastic discount factor is $\Lambda_{t,t+i} = \beta^i \frac{\Lambda_{t+i}}{\Lambda_t} = \beta(\frac{C_{t+i}^R}{C_t^R})^{-1} \frac{P_t}{P_{t+i}}$. The first order conditions of the Ricardian household are:

$$\Lambda_{t,t+1} \frac{P_{t+1}}{P_t} = \beta \frac{C_t^R}{C_{t+1}^R} \tag{8}$$

$$\frac{1}{1+i_t} = E_t \Lambda_{t,t+1} \tag{9}$$

$$Q_t^e = E_t \left[\Lambda_{t,t+1} (D_{t+1}^e + Q_{t+1}^e) \right] \tag{10}$$

$$W_t \frac{1}{C_t^R P_t} = \eta (L_t^R)^\omega \tag{11}$$

In a symmetric world, we obtain the risk-sharing condition as follows:

$$\frac{1}{C_t^R P_t} = \frac{1}{S_t C_t^{R*} P_t^*} \tag{12}$$

We define the real exchange rate as:

$$RER_t = \frac{S_t P_t^*}{P_t}$$

[3] The profit variations and asset income variation is the source of heterogeneity and is the key for our analysis.

2.1.2 Keynesian Households

The Keynesian households don't have access to financial markets. They consume their labor income transfers each period. The budget constraint is

$$P_t C_t^K = W_t L_t^K \tag{13}$$

where C_t^K and L_t^K denote consumption and labor supply for Keynesian households. We abstract away from idiosyncratic shocks as in Bilbiie (2008) and Debortoli and Gali (2017). The household maximizes (1) subject to the (13). Given the utility function, the optimal labor supply by Keynesian households is:

$$W_t \frac{1}{C_t^K P_t} = \eta (L_t^K)^{\omega} \tag{14}$$

This labor supply equation, together with the budget constraint of Keynesian household, implies that their labor supply is constant $L_t^K = \eta^{-\frac{1}{1+\omega}}$.

2.1.3 Heterogeneity

In equilibrium, aggregate consumption is

$$C_t = (1-n)C_t^R + nC_t^K \tag{15}$$

Here we define a key variable in our analysis: the index of heterogeneity between the Ricardian and Keynesian households.

$$H_t \equiv \frac{C_t^R}{C_t} = (1 - n + n\frac{C_t^K}{C_t^R})^{-1}. \tag{16}$$

Then the Euler equation of Ricardian households can be rewritten as

$$\frac{1}{1+i_t} = E_t \beta (\frac{C_{t+1}^R}{C_t^R})^{-1} \frac{P_t}{P_{t+1}} = E_t \beta (\frac{C_{t+1}}{C_t})^{-1} (\frac{H_{t+1}}{H_t})^{-1} \frac{P_t}{P_{t+1}} \tag{17}$$

Meanwhile, the risk sharing condition, expressed in terms of aggregate consumption is rewritten as:

$$\frac{(C_t H_t)^{-1}}{P_t} = \frac{(C_t^* H_t^*)^{-1}}{S_t P_t^*} \tag{18}$$

H_t is therefore directly related to real exchange rate dynamics.

2.2 Production

We consider two types of pricing strategies; producer currency pricing (PCP), and local currency pricing (LCP). We start with PCP.

2.2.1 Producer Currency Pricing Case

With PCP, the terms of trade is

$$Q_t = \frac{S_t P^*_{fft}}{P_{hht}} \tag{19}$$

Each firm i in the home economy has the following production technology.

$$Y_t(i) = Z_t L_t(i) \tag{20}$$

where $Z_t = exp(\theta_t)$ is a country-specific productivity shock, and θ_t is distributed with mean zero and variance σ^2_θ.

We assume the Dixit-Stiglitz consumption structure. Each firm produces a differentiated good facing a downward-sloping individual demand curve and chooses its optimal price along the demand curve. As in the standard NK literature, firms adjust prices following the standard Calvo mechanism. In the home country, a firm may reset its prices with probability $1-\kappa$ each period. When the firm resets price under PCP, it sets a price in home currency, $P^o_{hh,t}(i)$ for sales in the home market, and the price in the foreign market becomes $P^{o*}_{hf,t}(i) = \frac{P^o_{hh,t}(i)}{S_t}$ in foreign currency.

Specifically, a firm chooses $P^o_{hh,t}(i)$ to maximize the following objective function,

$$E_t \sum_{j=0}^{\infty} \kappa^j \Lambda_{t,t+j}[((1+\tau)P^o_{hht}(i) - MC_{t+j}(i))Y_t(i)]$$

where $Y_t(i) = C_{ht}(i) + C^*_{ht}(i)$, and $C_{ht}(i)$ and $C^*_{ht}(i)$ represent the demand for the home good i from the home and foreign markets, respectively, and $\Lambda_{t,t+j} = \beta^j(\frac{C^R_{t+j}}{C^R_t})^{-1}(\frac{P_t}{P_{t+j}})$ is the stochastic discount factor. $\tau = \frac{1}{\lambda-1}$ is a subsidy imposed by the government to eliminate the steady state monopolistic distortion. We detrend the price by the average price level:

$$\frac{P_{hht}^{o}(i)}{P_{hht}} = \frac{E_t \sum_{j=0}^{\infty} (\beta\kappa)^j \frac{C_{t+j}^{R-1}}{C_t^{R-1}} (\frac{P_{hht+j}}{P_{hht}})^{\lambda} mc_{t+j} Y_{t+j}}{E_t \sum_{j=0}^{\infty} (\beta\kappa)^j \frac{C_{t+j}^{R-1}}{C_t^{R-1}} (\frac{P_{hht+j}}{P_{hht}})^{\lambda-1} Y_{t+j} p_{hht+j}} \tag{21}$$

The firm in the foreign country faces an analogous problem, and chooses $P_{ff,t}^{*o}$ in terms of foreign currency and $P_{fh,t}^{o}$ in terms of home currency.

2.2.2 Local Currency Pricing Case

When we have LCP, the terms of trade is

$$Q_t = \frac{P_{fht}}{S_t P_{hft}^{*}} \tag{22}$$

And the deviations from the law of one price (LOOP) for the home and foreign good are:

$$d_t = \frac{S_t P_{hft}^{*}}{P_{hht}}, d_t^{*} = \frac{P_{fht}}{S_t P_{fft}^{*}}$$

As in the PCP case, a firm may reset its prices with probability $1-\kappa$ each period. But now the firm sets two prices, $P_{hh,t}^{o}(i)$ in home currency for sales in the home market, and $P_{hf,t}^{o*}(i)$ in foreign currency for sales in the foreign market. In general, $P_{hf,t}^{o*}(i)$ is not equal to $\frac{P_{hh,t}^{o}(i)}{S_t}$. Specifically, the firm chooses $P_{hh,t}^{o}(i)$ and $P_{hf,t}^{o*}(i)$ to maximize the following objective function,

$$E_t \sum_{j=0}^{\infty} \kappa^j \beta_{t,t+j} [((1+\tau) P_{hht}^{o}(i) - MC_{t+j}(i)) C_{h,t+j}(i) + ((1+\tau) S_{t+j} P_{hf,t}^{o*}(i) - MC_{t+j}(i)) C_{h,t+j}^{*}(i)]$$

Where $C_{ht}(i)$ and $C_{ht}^{*}(i)$ are the demands for the home good i from the home and foreign markets, respectively. The optimal pricing equations are:

$$\frac{P^{o}_{hht}(i)}{P_{hht}} = \frac{E_t \sum_{j=0}^{\infty}(\beta\kappa)^j \frac{C_{t+j}^{R-\rho}}{C_t^{R-\rho}}(\frac{P_{hht+j}}{P_{hht}})^{\lambda} mc_{t+j} C_{ht+j}}{E_t \sum_{j=0}^{\infty}(\beta\kappa)^j \frac{C_{t+j}^{R-\rho}}{C_t^{R-\rho}}(\frac{P_{hht+j}}{P_{hht}})^{\lambda-1} C_{ht+j} p_{hht+j}} \tag{23}$$

$$\frac{P^{o*}_{hft}(i)}{P^{*}_{hft}} = \frac{E_t \sum_{j=0}^{\infty}(\beta\kappa)^j \frac{C_{t+j}^{R-\rho}}{C_t^{R-\rho}}(\frac{P^{*}_{hft+j}}{P^{*}_{hft}})^{\lambda} mc_{t+j} C^{*}_{ht+j}}{E_t \sum_{j=0}^{\infty}(\beta\kappa)^j e_{t+j} \frac{C_{t+j}^{R-\rho}}{C_t^{R-\rho}}(\frac{P^{*}_{hft+j}}{P^{*}_{hft}})^{\lambda-1} C^{*}_{ht+j} p^{*}_{hft+j}} \tag{24}$$

The firm in the foreign country faces an analogous problem, and chooses $P^{*o}_{ff,t}$, in terms of foreign currency and $P^{o}_{fh,t}$, in terms of home currency.

2.3 Monetary policy

In the model, monetary authorities use the nominal interest rate as policy instrument. The policy rule will be specified below.

2.4 Market Clearing Conditions and Equilibrium

The goods market:

$$Y_t = \frac{v}{2}\frac{P_t C_t}{P_{hht}}\Delta_{hh,t} + (1-\frac{v}{2})\frac{P^*_t C^*_t}{P^*_{hft}}\Delta^*_{hf,t} \tag{25}$$

$$Y^*_t = \frac{v}{2}\frac{P^*_t C^*_t}{P^*_{fft}}\Delta^*_{ff,t} + (1-\frac{v}{2})\frac{P_t C_t}{P_{fht}}\Delta_{fh,t} \tag{26}$$

Where $\Delta_{xx,t} = \int(\frac{P_{xx}(i)_t}{P_{xx,t}})^{-\lambda}di$ is a price dispersion term, $xx \in \{hh, hf, ff, fh\}$.

The bond market:

$$(1-n)D_{hh}(s^t) + (1-n^*)D_{fh}(s^t) = 0 \tag{27}$$

$$(1-n^*)D^*_{ff}(s^t) + (1-n)D^*_{hf}(s^t) = 0 \tag{28}$$

The equity market:

Ricardian agents are identical and the asset market is complete. We have

$$S_t^R = \frac{1}{1-n},\ S_t^R = \frac{1}{1-n^*} \tag{29}$$

The labor market clearing conditions are:

$$L_t = (1-n)L_t^R + nL_t^K \tag{30}$$

$$L_t^* = (1-n^*)L_t^{*R} + n^*L_t^{*K} \tag{31}$$

Under PCP, the dividend on equity is:

$$D_t^e = \int_0^1 D_t^e(i)di = \int_0^1 (1+\tau)P_{hht}(i)Y_t(i)di - MC_t\int_0^1 Y_t(i)di = ((1+\tau)P_{hht} - MC_t\Delta_{hh,t})Y_t \tag{32}$$

$$D_t^{e*} = ((1+\tau)P_{fft}^* - MC_t^*\Delta_{ff,t}^*)Y_t^* \tag{33}$$

In the LCP case, the dividend is:

$$D_t^e = ((1+\tau)P_{hht} - MC_t\Delta_{hh,t})\frac{v}{2}\frac{P_tC_t}{P_{hht}} + ((1+\tau)P_{hft}^*S_t - MC_t\Delta_{hf,t}^*)(1-\frac{v}{2})\frac{P_t^*C_t^*}{P_{hft}^*} \tag{34}$$

$$D_t^{e*} = ((1+\tau)P_{fft}^* - MC_t^*\Delta_{ff,t}^*)\frac{v}{2}\frac{P_t^*C_t^*}{P_{fft}^*} + ((1+\tau)P_{fht}/S_t - MC_t^*\Delta_{fh,t})(1-\frac{v}{2})\frac{P_tC_t}{P_{fht}} \tag{35}$$

Note that the dividend is zero in the flexible price equilibrium, however, it is not zero because the markup is not constant in the sticky price model.

3 The Flexible Price Equilibrium

We first look at the flexible price equilibrium. We normalize the wage, marginal cost, prices and dividend by the CPI price level in the relevant country. The normalized variables are labelled as lower case letters. That is, $w_t = \frac{W_t}{P_t}$, $mc_t = \frac{MC_t}{P_t}$, $p_{hht} = \frac{P_{hht}}{P_t}$,and $d_t^e = \frac{D_t^e}{P_t}$. In our notation, variables with hat refer to the deviation of the log of corresponding variables from steady state, and variables with a superscript "fb"are defined as variables in flexible price equilibrium. To derive the equilibrium allocation, we log-linearize the flexible price model around the steady state, which is given by

$$\begin{aligned} 1+i^{ss} &= \frac{1}{\beta}, \qquad S=1 \\ D^{e,ss} &= 0,, \qquad H^{ss}=1 \end{aligned}$$

$$Y = C^{ss} = C^{R,ss} = C^{K,ss} L^{R,ss} = L^{K,ss} = \eta^{-\frac{1}{1+\omega}}$$

In this steady state level, the dividend is zero, the allocation is efficient, and agents are homogeneous. The solution details are available in the Technical Appendix. For the convenience of presentation, we focus on the case with no home bias ($v = 1$).

In the flexible price equilibrium, the price set by a firm is a constant markup over marginal cost, so the real wage is simply given by

$$\widehat{w}_t^{fb} - \theta_t = \widehat{p}_{hht}^{fb}$$

Given the definition of the terms of trade $Q_t = \frac{S_t P^*_{fft}}{P_{hht}}$, we also have $\widehat{p}_{hht}^{fb} = -\frac{1}{2}\widehat{q}_t^{fb}$. From Ricardian and Keynesian households' labor supply functions, we have

$$\begin{aligned} \widehat{C}_t^{K,fb} &= \widehat{w}_t^{fb} \\ \widehat{C}_t^{R,fb} &= \widehat{w}_t^{fb} - \omega \widehat{L}_t^{R,fb} \end{aligned}$$

The above two equations, together with the definition of heterogeneity $\widehat{H}_t^{fb} \equiv \widehat{C}_t^{R,fb} - \widehat{C}_t^{fb}$, the production function $\widehat{Y}_t^{fb} = \theta_t + \widehat{L}_t^{fb}$, the aggregation of labor $\widehat{L}_t^{fb} = (1-n)\widehat{L}_t^{R,fb}$, and the definition of aggregate consumption $\widehat{C}_t^{fb} = n\widehat{C}_t^{K,fb} + (1-n)\widehat{C}_t^{R,fb}$, deliver a relationship between output and heterogeneity:

$$\widehat{H}_t^{fb} = \frac{-n}{1-n}\omega(\widehat{Y}_t^{fb} - \theta_t) \tag{36}$$

Equation (36) is a key relationship. Intuitively, It says that the heterogeneity of Ricardian to Keynesian consumption is negatively related to output gap. That is, in face of an increase in output, Keynesian households consume more than Ricardian households. More importantly, this relationship is derived from the household side and holds whatever pricing policy is in place.

In the flexible price equilibrium, the real exchange rate is a constant and thus $\widehat{e}_t^{fb} = 0$. Therefore, the risk-sharing condition becomes

$$\widehat{C}_t^{R,fb} = \widehat{C}_t^{R*,fb} \tag{37}$$

or $\widehat{C}_t^{fb} + \widehat{H}_t^{fb} = \widehat{C}_t^{*,fb} + \widehat{H}_t^{*fb}$.

Using the labor supply function of Ricardian households and the production function, we have

$$\widehat{Y}_t^{fb} = [\theta_t + \frac{1-n}{\omega}(\theta_t - \frac{1}{2}\widehat{q}_t - \widehat{C}_t^{R,fb})] \tag{38}$$

Using the goods market clear condition and risk-sharing condition, we have,

$$\widehat{Y}_t^{fb} = \widehat{C}_t + \frac{1}{2}(\widehat{q}_t + \widehat{H}_t - \widehat{H}_t^*) = \widehat{C}_t^{R,fb} + \frac{1}{2}(\widehat{q}_t - \widehat{H}_t - \widehat{H}_t^*) \tag{39}$$

The risk-sharing condition (37) and equation (38) and (39) and their corresponding equations in the foreign country can be used to solve for the 5 variables, $\widehat{Y}_t^{fb}$,$\widehat{Y}_t^{fb*}$, $\widehat{C}_t^{R,fb}$, $\widehat{C}_t^{*,fb}$, and $\widehat{q}_t^{fb}$. The solution to the flexible price equilibrium are summarized in the Proposition 1.

Proposition 1 *With household heterogeneity ($n > 0, n^* > 0$), the solution to the flexible price equilibrium is identical to the efficient equilibrium in the standard new open macro model (CGG).*

$$\begin{aligned}
\widehat{H}_t^{fb} &= \widehat{H}_t^{*,fb} = 0 \\
\widehat{q}_t^{fb} &= \widehat{Y}_t^{fb} - \widehat{Y}_t^{fb*} = (\theta_t - \theta_t^*) \\
\widehat{Y}_t^{fb} &= \theta_t, \qquad\qquad \widehat{Y}_t^{fb*} = \theta_t^* \\
\widehat{C}_t^{fb} &= \widehat{C}_t^{*,fb} = \widehat{C}_t^{R,fb} = \widehat{C}_t^{R*,fb} = \frac{1}{2}(\theta_t + \theta_t^*)
\end{aligned}$$

The proof is presented in the Technical Appendix section 1.3. There is no consumption heterogeneity in the flexible price equilibrium, and since the markup is eliminated by constant production subsidy and the dividend is zero, this means that households are indifferent as to their asset holdings.

4 Transmission mechanism of monetary shocks under PCP

We now discuss the PCP case with sticky prices. As in the traditional models, monetary policy has a real effect through the Euler equation. And as in the NOEM literature, the terms of trade externality works through the goods market clearing condition. Our analysis of monetary policy transmission will also focus on the Euler equation and the market clearing condition, but we highlight the role of heterogeneity in the solution. It is noteworthy that the equilibrium conditions from the household side are identical to the ones in the flexible price equilibrium. Particularly, our analysis relies on the relationship between heterogeneity and the output gap is represented as:

$$\widehat{H}_t = \frac{-n\omega}{1-n}(\widehat{Y}_t - \theta_t) \tag{40}$$

$$\widehat{H}_t^* = \frac{-n^*\omega}{1-n^*}(\widehat{Y}_t - \theta_t^*) \tag{41}$$

We first focus on a symmetric case where $n = n^*$ and then discuss a more general asymmetric case where $n \neq n^*$.

4.1 Symmetric household heterogeneity ($n = n^*$)

In the PCP setting with no home bias ($v = 1$), purchasing power parity (PPP) holds. This fact implies that $\widehat{e}_t = 0$ and $\widehat{C}_t^R = \widehat{C}_t^{*,R}$. The goods market clearing condition is:

$$\widehat{Y}_t = \widehat{C}_t + \frac{1}{2}(\widehat{q}_t + \widehat{H}_t - \widehat{H}_t^*) = \widehat{C}_t^R - \widehat{H}_t + \frac{1}{2}(\widehat{q}_t + \widehat{H}_t - \widehat{H}_t^*)$$

In the PCP case, the real marginal cost is:

$$\widehat{mc}_t - \widehat{p}_{hh,t} = \widehat{w}_t - \theta_t - \widehat{p}_{hh,t} = (1+\omega)\left(\widehat{Y}_t - \theta_t\right) + \frac{1}{2}(\widehat{H}_t^* - \widehat{H}_t) \tag{42}$$

Following, CGG, we define $\widehat{r}_t = \widehat{i}_t - E_t\pi_{hh,t+1}$ and $\widehat{r}_t^* = \widehat{i}_t^* - E_t\pi_{ff,t+1}^*$, we have the Euler equation:[4]

$$\widehat{C}_t^R = E_t(\widehat{C}_{t+1}^R) - [\widehat{r}_t - \frac{1}{2}E_t\Delta\widehat{q}_{t+1}] \tag{45}$$

where we define $\Delta x_t = x_t - x_{t-1}$ as the first difference of a variable x_t, i.e., $\Delta\widehat{q}_{t+1} = \widehat{q}_{t+1} - \widehat{q}_t$ in the above equation. As shown in the Technical Appendix, we solve for the difference equation for output output in terms of real interest rate and productivity shocks.

$$\widehat{Y}_t = E_t\widehat{Y}_{t+1} - \frac{1+\delta}{2\delta}\widehat{r}_t - \frac{1-\delta}{2\delta}\widehat{r}_t^* + \frac{1-\delta}{2\delta}(E_t\Delta\theta_{t+1} + E_t\Delta\theta_{t+1}^*) \tag{46}$$

$$\widehat{Y}_t^* = E_t\widehat{Y}_{t+1}^* - \frac{1+\delta}{2\delta}\widehat{r}_t^* - \frac{1-\delta}{2\delta}\widehat{r}_t + \frac{1-\delta}{2\delta}(E_t\Delta\theta_{t+1} + E_t\Delta\theta_{t+1}^*) \tag{47}$$

where $\delta = 1 - \frac{n\omega}{1-n}$. From the Euler equation in the flexible price equilibrium, we can have the domestic natural interest rate as: $\widehat{r}_t^{fb} = E_t(\widehat{Y}_{t+1}^{fb}) - \widehat{Y}_t^{fb} = E_t\Delta\theta_{t+1}$. Therefore, we rewrite the above equations as their deviation from flexible price equilibrium:

$$\widehat{Y}_t - \widehat{Y}_t^{fb} = E_t(\widehat{Y}_{t+1} - \widehat{Y}_{t+1}^{fb}) - \frac{1+\delta}{2\delta}(\widehat{r}_t - \widehat{r}_t^{fb}) - \frac{1-\delta}{2\delta}(\widehat{r}_t^* - \widehat{r}_t^{*fb}) \tag{48}$$

Similarly, using the Euler equation in the foreign country, we have

$$\widehat{Y}_t^* - \widehat{Y}_t^{fb*} = E_t(\widehat{Y}_{t+1}^* - \widehat{Y}_{t+1}^{fb*}) - \frac{1+\delta}{2\delta}(\widehat{r}_t^* - \widehat{r}_t^{*fb}) - \frac{1-\delta}{2\delta}(\widehat{r}_t - \widehat{r}_t^{fb}) \tag{49}$$

For ease of analysis, we consider an nominal interest rate rule, by which the domestic real interest rate equals its natural rate and monetary shock.

$$\widehat{i}_t = E_t\pi_{hh,t+1} + u_t + \widehat{r}_t^{fb}$$

$$\widehat{i}_t^* = E_t\pi_{ff,t+1}^* + u_t^* + \widehat{r}_t^{*fb}$$

[4] The dynamics of CPI inflation and PPI inflation are determined by

$$\pi_t = \pi_{hh,t} + \frac{1}{2}\Delta\widehat{q}_t \tag{43}$$

$$\pi_t^* = \pi_{ff,t}^* - \frac{1}{2}\Delta\widehat{q}_t \tag{44}$$

where monetary shocks u_t, u_t^* are i.i.d. We can the solve the equilibrium allocations under PCP, as follows:

$$
\begin{aligned}
\widehat{Y}_t - \widehat{Y}_t^{fb} &= -\frac{1+\delta}{2\delta}u_t - \frac{1-\delta}{2\delta}u_t^* \\
\widehat{Y}_t^* - \widehat{Y}_t^{fb*} &= -\frac{1+\delta}{2\delta}u_t^* - \frac{1-\delta}{2\delta}u_t \\
\widehat{q}_t &= \widehat{Y}_t - \widehat{Y}_t^* = \widehat{Y}_t^{fb} - \widehat{Y}_t^{fb*} + u_t^* - u_t \\
\widehat{C}_t^R &= \widehat{C}_t^{R*} = (\frac{1}{2} - \frac{n\omega}{1-n})(\widehat{Y}_t + \widehat{Y}_t^*) + \frac{n\omega}{1-n}(\theta_t + \theta_t^*) \\
\widehat{H}_t &= \frac{-n\omega}{1-n}(\widehat{Y}_t - \theta_t), \quad \widehat{H}_t^* = \frac{-n\omega}{1-n}(\widehat{Y}_t^* - \theta_t^*)
\end{aligned}
$$

Based on the above equilibrium allocation, we establish the following Proposition.

Proposition 2 *In a two-agent and two-country economy, when the share of Keynesian households n is within the range of $(\frac{1}{\omega+1}, \frac{1}{\frac{\omega}{2}+1})$, there exists a monetary trap, in which an expansionary monetary policy reduces output.*

The proof is straightforward and the details is available in the Technical Appendix section 2.1.2. To have an expansionary effect of a monetary shock on output, we require $-\frac{1+\delta}{2\delta} < 0$, which is equivalent to $n < \frac{1}{\omega+1}$ or $n > \frac{1}{\frac{\omega}{2}+1}$.Alternatively, when $\frac{1}{\omega+1} < n < \frac{1}{\frac{\omega}{2}+1}$, an unexpected interest rate cut reduces output.

This result compares with Bilbiie (2008). Bilbiie (2008) shows that, in a closed economy, when the share of Keynesian households is larger than a threshold, there exists an "inverted aggregated demand logic"(IADL), which makes monetary policy ineffective. However, this result is revised in an open economy. That is, when the share of Keynesian households is large enough, monetary policy can work again. The intuition behind this result is simple. In a closed economy, output is only driven by aggregate consumption. In face of an interest rate cut, Ricardian consumption increases, but its effect on aggregate consumption depends critically on the composition of households. When there are more Ricardian households, aggregate consumption will increase with Ricardian consumption, which consequently stimulates output.

However, when there are more Keynesian households, aggregate consumption may fall, leading to less output.

In an open economy, however, there are two effects. The first effect is the aggregate consumption effect, as before. Domestic output is determined by the aggregate consumption in both the home and foreign market and the relative price of home goods. In face of a domestic interest rate cut, domestic Ricardian consumption increases, which raises foreign Ricardian consumption through the risk-sharing condition. However, their effect on aggregate consumption depends critically on the composition of households. When there are more Ricardian households, aggregate consumption will increase with Ricardian consumption, which increases output. However, when there are more Keynesian households, the aggregate consumption may fall, which reduces output. The second effect is the relative price effect or terms of trade effect. When there is an interest rate cut in the home economy, the relative price of home goods to foreign goods falls, which also increases domestic output.

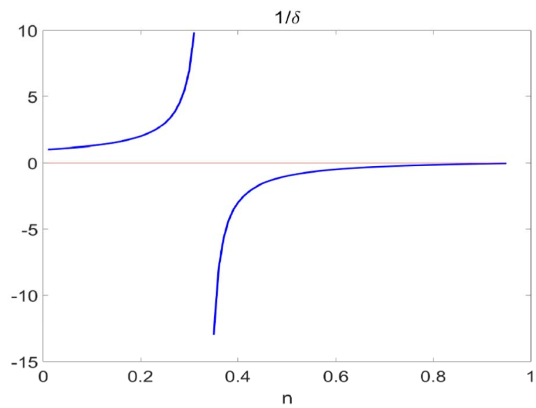
Thus, the effect of monetary policy on the domestic output is affected not only by the aggregate consumption effect but also the terms of trade effect. When the share of Keynesian households is small, both the aggregate consumption effect and terms of trade effect raise output. In the middle range, the aggregate consumption effect turns to negative and dominates the terms of trade effect. However, when n increases again, the aggregate consumption effect is still negative but small and will be dominated by the terms of trade effect, which leads the economy to exit the monetary trap.

To show the result clearly, we rewrite the goods market clearing condition (assuming no productivity shocks) as

$$\widehat{Y}_t = \frac{1}{\delta}\widehat{C}_t^R + \frac{1}{2}\widehat{q}_t \tag{50}$$

When n is small, both aggregate consumption effect and the terms of trade effect raise output. In the range of the policy trap, the aggregate consumption effect is negative and dominates the terms of trade effect. However, when n increases again, the aggregate consumption effect is negative but small and will be dominated by the terms of trade effect (illustrated in Figure 1), which leads to an exit from the monetary trap.

Figure 1: The role of heterogeneity



How large is the monetary trap? The range for the trap depends on the value of inverse elasticity of labor supply ω and given by $\frac{\omega}{(1+w)(2+\omega)}$. There is a large gap between the microeconometric estimates of the labor supply elasticity (0 - 0.5) and the values used by macroeconomists to calibrate general equilibrium models (2 – 4). We set $\omega = 1/2$, so that the elasticity of labor supply will be 2, which is a low value used for the calibration in general equilibrium models. In this case, the range of the monetary trap is from 0.67 to 0.8, which may have substantial impact on the conduct of monetary policy.

Proposition 3 *For any $n > 0$, there are positive or negative spillover effects of home monetary shocks. When $n < \frac{1}{\omega+1}$, there is a positive spillover effect of a home monetary shock to the foreign country. When $n > \frac{1}{\omega+1}$, the spillover effect is negative.*

The proof is in the Technical Appendix section 2.1.2. As shown by CGG, there is no spillover effect of a monetary shock under the standard PCP case ($n = 0$)

unless the elasticity of intertemporal substitution is greater than one. However, in the presence of Keynesian households, this result is revised. As shown in the foreign country's good market clearing conditions, the terms of trade effect is negative for foreign output.

$$\widehat{Y}_t^* = \frac{1}{\delta}\widehat{C}_t^{R*} - \frac{1}{2}\widehat{q}_t \tag{51}$$

In face of an interest rate cut in the home economy, due to the risk-sharing condition, foreign Ricardian consumption would increase as much as the home Ricardian consumption, which also increases aggregate consumption and then output. When $n < \frac{1}{\omega+1}$, the consumption effect is positive and dominates the negative terms of trade effect, so we observe a positive spillover effect of monetary policy. But when $n > \frac{1}{\omega+1}$, both effects are negative, so the spillover effect is negative. In this case, household heterogeneity generates the "beggar-thy-neighbor" effect emphasized by Betts and Devereux (2000).

We also analyze the case with home bias ($v \neq 1$). We leave the details of derivations in the Technical Appendix and summarize the results in the following proposition.

Proposition 4 *In the case with home bias $v \neq 1$, when the share of Keynesian households n is within the range of $(\frac{1}{\omega+1}, \frac{1}{\frac{v\omega}{2}+1})$, there exists a monetary trap, in which an expansionary monetary policy reduces output. For $v > 1$ when $n < \frac{1}{\omega+1}$ or $n > \frac{1}{\omega(v-1)+1}$, there is a positive spillover effect of the home monetary shock for the foreign country. However, for $v < 1$, the spillover effect is positive only when $n < \frac{1}{\omega+1}$.*

The proof is shown in the Technical Appendix section 2.1.3. When $v \neq 1$, PPP does not hold in the PCP setting, and $\widehat{e}_t = (v-1)\widehat{q}_t$. In such a setting, the effects of relative price changes on aggregate consumption includes changes in both the real exchange rate and terms of trade. Hence, the second threshold will depend on the value of home bias v. For the spillover effect of monetary shocks, when $v > 1$, the real exchange rate and terms of trade are positively correlated, however, the relationship will be reversed when v is below 1.

4.2 Asymmetric household heterogeneity ($n \neq n^*$)

Now we discuss the case with asymmetric household heterogeneity where $n \neq n^*$ but no home bias $v = 1$. For simplicity, we only consider monetary shocks. As shown in the Technical Appendix, we solve for equilibrium allocations as below.

$$\widehat{Y}_t = E_t\widehat{Y}_{t+1} - \frac{\delta^* + 1}{\delta + \delta^*}\widehat{r}_t - \frac{1 - \delta^*}{\delta + \delta^*}\widehat{r}^*_t \tag{52}$$

$$\widehat{Y}^*_t = E_t\widehat{Y}^*_{t+1} - \frac{\delta + 1}{\delta + \delta^*}\widehat{r}^*_t - \frac{1 - \delta}{\delta + \delta^*}\widehat{r}_t \tag{53}$$

Using this solution, we establish the following proposition.

Proposition 5 *For relatively low values of parameters n and w, that is, $n < \frac{1}{1+w}$ and $n^* < \frac{1}{1+w}$, the following set of results hold:*

(a) The interest rate cut in either home or foreign countries increases domestic output;

(b) The impact of domestic monetary shocks on domestic output increases with the size of domestic Keynesian households (n); the impact of domestic monetary shocks on domestic output remains constant and is unaffected by foreign's household heterogeneity when there are zero domestic Keynesian households ($n = 0, \delta = 1$);

(c) There are positive spillover effects of monetary shocks as long as there are Keynesian households in the source country; The spillover effect can be increased by the size of Keynesian households in both source country or host country ; the spillover of monetary shocks is more sensitive to the size of Keynesian households in the source country than that in the host country.

The proof is straightforward and the details is available in the Technical Appendix section 2.1.2. Given $n < \frac{1}{1+w}$ and $n^* < \frac{1}{1+w}$, we have $0 < \delta = 1 - \frac{n\omega}{1-n} \leq 1$

and $0 < \delta^* = 1 - \frac{n^*\omega}{1-n^*} \leq 1$. Since $0 < \delta \leq 1$ and $0 < \delta^* \leq 1$, we have $\frac{\delta^*+1}{\delta+\delta^*} > 0$ and $\frac{1-\delta^*}{\delta+\delta^*} > 0$. This implies that an interest rate cut in either home or foreign countries will raise domestic output, but the size of effects depend critically on δ and δ^*. As discussed above, δ and δ^* captures the impact of global consumption on output. Facing expansionary monetary shocks, Keynesian consumption increases more than Ricardian consumption. Consequently, the more Keynesian households in the global economy, the larger the is the impact of global consumption on output. Relatively speaking, for the output effect of domestic monetary shocks, the size of domestic Keynesian households (n) is more important than the size of foreign Keynesian households (n^*). When there is no domestic Keynesian household ($n = 0, \delta = 1$), the output effect of domestic monetary shocks is not affected by the size of foreign Keynesian households, but the reverse does not hold.

In this general asymmetric case, as long as there are Keynesian households in the source country of the shocks, we can find positive spillover effects of monetary shocks, while the effects can be amplified by the sizes of Keynesian households in both countries. This result reveals an interesting new perspective on the international transmission of monetary policy; household heterogeneity acts as a mechanism to generate spillover effects of monetary shocks.

5 Transmission mechanism of monetary shocks under LCP

We now turn to the model under the LCP case. The household side of LCP, including consumption and labor supply, is identical to that of PCP case. We still have equation (40) and (41) in the LCP case. Meanwhile, we rely on the Euler equations as well as the goods market clearing condition to analyze the effect of monetary policy shocks. In the LCP case however, the deviation from the law of one price influences the equilibrium allocation. Therefore, we need to use the Phillips curves to characterize the dynamics of deviations from LOOP. This part differs from the PCP case and complicates the solution. We first consider the symmetric case ($n = n^*$) with no home bias ($v = 1$) and keep our discussion closely related to Engel (2011), but we highlight the role of household heterogeneity. We will then extend the analysis to the asymmetric case ($n \neq n^*$).

5.1 Symmetric household heterogeneity ($n = n^*$)

In the LCP setting, the law of one price does not hold and the real exchange rate $\widehat{e}_t \neq 0$. Then the risk-sharing condition is

$$\widehat{C}_t^R = \widehat{C}_t^{*,R} + \widehat{e}_t \tag{54}$$

The movement in the real exchange rate creates a gap between home Ricardian consumption and foreign Ricardian consumption. When the real exchange rate depreciates, foreign Ricardian consumption will increase less than home Ricardian consumption. This will lead to different transmission mechanism of monetary shocks under LCP than under PCP.

Log-linearizing the deviation from the law of one price, terms of trade, and the real exchange rate, we obtain

$$\widehat{d}_t = \widehat{p}^*_{hf,t} + \widehat{e}_t - \widehat{p}_{hh,t} \tag{55}$$

$$\widehat{d}^*_t = \widehat{p}_{fh,t} - \widehat{e}_t - \widehat{p}^*_{ff,t} \tag{56}$$

$$\widehat{q}_t = -\widehat{q}^*_t = \widehat{p}_{fh,t} - \widehat{p}^*_{hf,t} - \widehat{e}_t \tag{57}$$

$$\widehat{e}_t = \frac{1}{2}(\widehat{d}_t - \widehat{d}^*_t) \tag{58}$$

Meanwhile, from the definition of $\widehat{q}_t$ and $\widehat{d}_t$, we connect the change of $\widehat{d}_t$ with the inflation dynamics.

$$\Delta\widehat{q}_t + \Delta\widehat{d}_t = \pi_{fh,t} - \pi_{hh,t}, \tag{59}$$

Similarly,

$$\Delta\widehat{q}^*_t + \Delta\widehat{d}^*_t = \pi^*_{hf,t} - \pi^*_{ff,t} \tag{60}$$

Using the real marginal cost derived in the Technical Appendix, the New Keynesian Phillips Curves in the LCP setting is given by:

$$
\begin{aligned}
\pi_{hh,t} &= \widetilde{\kappa}\left[(1+\omega)\widehat{Y}_t - (1+\omega)\theta_t + \frac{1}{2}\widehat{d}_t + \frac{1}{2}(\widehat{H}_t^* - \widehat{H}_t)\right] + \beta E_t \pi_{hh,t+1} && (61)\\
\pi_{hf,t}^* &= \widetilde{\kappa}\left[(1+\omega)\widehat{Y}_t - (1+\omega)\theta_t - \frac{1}{2}\widehat{d}_t + \frac{1}{2}(\widehat{H}_t^* - \widehat{H}_t)\right] + \beta E_t \pi_{hf,t+1}^* && (62)\\
\pi_{ff,t}^* &= \widetilde{\kappa}\left[(1+\omega)\widehat{Y}_t^* - (1+\omega)\theta_t^* + \frac{1}{2}\widehat{d}_t^* - \frac{1}{2}(\widehat{H}_t^* - \widehat{H}_t)\right] + \beta E_t \pi_{ff,t+1}^* && (63)\\
\pi_{fh,t} &= \widetilde{\kappa}\left[(1+\omega)\widehat{Y}_t^* - (1+\omega)\theta_t^* - \frac{1}{2}\widehat{d}_t^* - \frac{1}{2}(\widehat{H}_t^* - \widehat{H}_t)\right] + \beta E_t \pi_{fh,t+1} && (64)
\end{aligned}
$$

The deviation of the law of one prices $\widehat{d}_t$ and $\widehat{d}_t^*$ has a direct impact on real marginal costs in LCP, which differs from those in PCP. Meanwhile, assuming the symmetric condition $n = n^*$ implies that the difference in consumption heterogeneity is (the difference of equation (40) and (41):

$$
\widehat{H}_t - \widehat{H}_t^* = \frac{-n\omega}{1-n}[(\widehat{Y}_t - \theta_t) - (\widehat{Y}_t^* - \theta_t^*)] \qquad (65)
$$

Using the difference between equation (59), (60), and the NKPC equations, we obtain

$$
\begin{aligned}
2\Delta\widehat{q}_t + \Delta\widehat{d}_t - \Delta\widehat{d}_t^* &= \pi_{fh,t} - \pi_{hh,t} - \pi_{hf,t}^* + \pi_{ff,t}^* && (66)\\
&= \widetilde{\kappa}\left[2(1+\omega)(\widehat{Y}_t^* - \widehat{Y}_t) - 2(1+\omega)(\theta_t^* - \theta_t) - 2(\widehat{H}_t^* - \widehat{H}_t)\right]\\
&\quad +\beta E_t\left[2\Delta\widehat{q}_{t+1} + \Delta\widehat{d}_{t+1} - \Delta\widehat{d}_{t+1}^*\right]
\end{aligned}
$$

The next step is to show that $2\Delta\widehat{q}_t + \Delta\widehat{d}_t - \Delta\widehat{d}_t^*$ can be expressed as the output difference of the two countries. First, $\Delta\widehat{d}_t - \Delta\widehat{d}_t^*$ can be replaced by $2\Delta\widehat{e}_t$ according to equation (58). Second, we utilize the goods market clearing conditions and risk-sharing condition to show how $\Delta\widehat{q}_t + \Delta\widehat{e}_t$ is connected with the output difference.

The log-linearized goods market clearing condition is

$$
\begin{aligned}
\widehat{Y}_t &= \frac{1}{2}(\widehat{C}_t - \widehat{p}_{hht}) + \frac{1}{2}(\widehat{C}_t^* - \widehat{p}_{hft}^*) \\
&= \frac{1}{2}\widehat{C}_t + \frac{1}{2}(\widehat{C}_t + \widehat{H}_t - \widehat{H}_t^* - \widehat{e}_t) - \frac{1}{2}(\widehat{p}_{hft}^* + \widehat{p}_{hht}) \\
&= \frac{1}{2}\widehat{C}_t + \frac{1}{2}(\widehat{C}_t + \widehat{H}_t - \widehat{H}_t^* - \widehat{e}_t) + \frac{1}{2}(\widehat{q}_t + \widehat{e}_t)
\end{aligned} \tag{67}
$$

The last equation tells us how the terms of trade affect the country's output and consumption. Without consumption heterogeneity ($\widehat{H}_t = \widehat{H}_t^* = 0$), we have a standard equation that characterizes the term of trade externality in the LCP setting. That is, an improvement of home's term of trade can help to increase the country's consumption without increasing the country's production. With consumption heterogeneity, we see a gap between the Ricardian household's consumption (which is characterized by full risk sharing) and the aggregate consumption. Analogously, we can obtain the log-linearized version of the foreign goods market clearing conditions. And these two market clearing conditions can be simplified to express the terms of trade as the output difference between Home and Foreign countries.

$$
\widehat{q}_t + \widehat{e}_t = \widehat{Y}_t - \widehat{Y}_t^* \tag{68}
$$

where $\widehat{q}_t + \widehat{e}_t$ captures the relative prices of home goods in both home and foreign markets. In the LCP case, the relative prices includes two components: terms of trade and the real exchange rate. Using equation (68), we rewrite the equation (66) as a difference equation of relative output:

$$
\begin{aligned}
&\Delta(\widehat{Y}_t - \widehat{Y}_t^*) \\
= \ &\widetilde{\kappa}\left[(1+\omega)(\widehat{Y}_t^* - \widehat{Y}_t) - (1+\omega)(\theta_t^* - \theta_t) - (\widehat{H}_t^* - \widehat{H}_t)\right] + \beta E_t \Delta(\widehat{Y}_{t+1} - \widehat{Y}_{t+1}^*)
\end{aligned} \tag{69}
$$

Equation (65) shows that $\widehat{H}_t^* - \widehat{H}_t$ is also a linear function of $\widehat{Y}_t^* - \widehat{Y}_t$. The above equation implies that $\widehat{Y}_t^* - \widehat{Y}_t$ is independent of monetary policies. In other words, we have $\widehat{Y}_t - \widehat{Y}_t^* = 0 + t.i.p$, where t.i.p is a function of productivity shocks but independent of monetary policy shocks. Engel (2011) also reports the similar result.

Moreover, using equation (59), (60), and NKPC equations, we have

$$\Delta(\widehat{d}_t + \widehat{d}_t^*) = \pi_{fh,t} + \pi_{hf,t}^* - \pi_{hh,t} - \pi_{ff,t}^* = \widetilde{\kappa}\left[-(\widehat{d}_t + \widehat{d}_t^*)\right] + \beta E_t \Delta(\widehat{d}_{t+1} + \widehat{d}_{t+1}^*) \quad (70)$$

Given the facts that both relative output and relative household heterogeneity are independent of monetary policy, we find that $\widehat{d}_t + \widehat{d}_t^*$ is independent of the monetary policy as well. That is, $\widehat{d}_t + \widehat{d}_t^* = 0 + t.i.p.$

Next, we use the Euler equation below to solve for the difference equation of output in terms of monetary policy shocks:

$$\widehat{C}_t^R = E_t(\widehat{C}_{t+1}^R) - [\widehat{r}_t - \frac{1}{2}E_t\Delta\widehat{q}_{t+1} - \frac{1}{2}E_t\Delta\widehat{d}_{t+1}] \quad (71)$$

where $\widehat{r}_t = \widehat{i}_t - E_t\pi_{hh,t+1}$ is the real interest rate. And we can further use the goods market condition to relate $\widehat{C}_t^R$ with the output.

$$\begin{aligned}\widehat{C}_t^R &= \widehat{Y}_t - \frac{1}{2}(\widehat{Y}_t - \widehat{Y}_t^* - \widehat{e}_t + \widehat{H}_t - \widehat{H}_t^*) + \widehat{H}_t \quad (72)\\ &= (\frac{1}{2} - \frac{n\omega}{1-n})(\widehat{Y}_t + \widehat{Y}_t^*) + \frac{1}{2}\widehat{e}_t + \frac{n\omega}{1-n}(\theta_t + \theta_t^*)\end{aligned}$$

Using the above equation to replace $\widehat{C}_t^R$ in Euler equation (71), we obtain

$$\widehat{Y}_t = E_t\widehat{Y}_{t+1} + (\frac{\delta-1}{\delta+1})E_t\Delta\widehat{Y}_{t+1}^* - \frac{2}{\delta+1}\widehat{r}_t + (\frac{1-\delta}{\delta+1})(E_t\Delta\theta_{t+1} + E_t\Delta\theta_{t+1}^*) + \frac{1}{\delta+1}E_t\Delta\widehat{d}_{t+1} \quad (73)$$

where $\delta = 1 - \frac{n\omega}{1-n}$, which are the same parameters as in the PCP case. Finally, using home and foreign Euler equations and the risk sharing condition, we have

$$0 = E_t\Delta(\widehat{q}_{t+1} + \widehat{e}_{t+1}) - [\widehat{r}_t - \widehat{r}_t^*] + \frac{1}{2}E_t\Delta\widehat{d}_{t+1} - \frac{1}{2}E_t\Delta\widehat{d}_{t+1}^*$$

Since both $\Delta(\widehat{q}_{t+1} + \widehat{e}_{t+1}) = \Delta(\widehat{Y}_{t+1} - \widehat{Y}_{t+1}^*)$ and $\widehat{d}_t + \widehat{d}_t^*$ are independent of the monetary policy, we can have

$$E_t\Delta\widehat{d}_{t+1} = -E_t\Delta\widehat{d}_{t+1}^* = [\widehat{r}_t - \widehat{r}_t^*] + t.i.p \quad (74)$$

Now we replace $\widehat{Y}_t - \widehat{Y}_t^* = 0 + t.i.p$ and replace $E_t\Delta\widehat{d}_{t+1} = [\widehat{r}_t - \widehat{r}_t^*] + t.i.p$ in equation (73), and express home output in terms of monetary shocks and productivity shocks.

$$\widehat{Y}_t = E_t(\widehat{Y}_{t+1}) - \frac{1}{2\delta}\widehat{r}_t - \frac{1}{2\delta}\widehat{r}_t^* + \Theta \quad (75)$$

where Θ is functions of productivity terms. Using the interest rate rules specified in the PCP case, we have

$$\widehat{Y}_t - \widehat{Y}_t^{fb} = E_t(\widehat{Y}_{t+1} - \widehat{Y}_{t+1}^{fb}) - \frac{1}{2\delta}\widehat{\mu}_t - \frac{1}{2\delta}\widehat{\mu}_t^* + \Theta \tag{76}$$

Note that in the LCP case, even without monetary shocks, the interest rate rules cannot close the output gap as in the PCP case. While we cannot solve for the analytical solution for the output, we can clearly show how the monetary shocks affect the output from equation (76). We establish the following Proposition.

Proposition 6 *With household heterogeneity, under LCP, when* $n < \frac{1}{w+1}$*, an expansionary monetary shock in the home or foreign country can increase home output; However, when* $n > \frac{1}{w+1}$*, an expansionary monetary shock in home or foreign country reduces home output.*

The proof is in the Technical Appendix section 2.2.1. It can seen from equation (76). Given the results from the PCP case, it is not surprising that as the share of Keynesian households increases, the effect of a monetary shock on output will be reversed. However, in contrast to the PCP case, we do not have the reversal as n increases further, so we no longer find that monetary policy becomes effective for higher values of n. We explain the results using the following relationship that is derived from home goods market clearing condition,

$$\widehat{Y}_t = \frac{1}{\delta}(\widehat{C}_t^R - \frac{1}{2}\widehat{e}_t) + \frac{1}{2}(\widehat{q} + \widehat{e}_t) \tag{77}$$

There are also two effects of monetary shocks on output. The first is the aggregate consumption effect. In face of an interest rate cut in the home country, domestic Ricardian consumption increases; However, in the LCP case, foreign Ricardian consumption increases less than Home Ricardian consumption due to the real exchange rate depreciation. Here, $\widehat{C}_t^R - \frac{1}{2}\widehat{e}_t$ captures the total change in global Ricardian consumption and $\frac{1}{\delta}$ measures the response of aggregate consumption to Ricardian consumption.

The second effect is the change in the relative prices of home goods in both home and foreign countries. In the PCP case, this effect is just the terms of trade. In the LCP case, it includes changes in both terms of trade and the real exchange rate. As shown in the above analysis however, $\widehat{q}+\widehat{e}_t$ is independent of monetary policy. In other words, the second effect is shut down in the LCP case. Without the second effect, the effect of monetary shocks on the output could be either positive or negative, depending on $\frac{1}{\delta}$. Furthermore, since $\widehat{Y}_t-\widehat{Y}_t^*$ is independent of monetary policy, in face of a monetary shock in the home country, home and foreign output must respond in the same manner. Compared with PCP case, the LCP case looks more like the one in closed economy discussed in Bilbiie (2008).

The general case with $v\neq 1$ with $n=n^*$ is discussed in the Technical Appendix section 2.2.2. Assuming no productivity shocks and that the economy is in its steady state before the shocks come in period t, we can solve for the home output as below

$$
\begin{aligned}
\widehat{Y}_t &= E_t\widehat{Y}_{t+1}-\frac{1}{2}E_t\Delta\widehat{Y}^d_{t+1}-\frac{1}{2\delta}\widehat{r}_t-\frac{1}{2\delta}\widehat{r}^*_t \qquad (78)\\
&= E_t\widehat{Y}_{t+1}+\frac{1}{2}(1-\lambda_1)\beta\lambda_1 v\widetilde{\kappa}(v-1)(\widehat{r}_t-\widehat{r}^*_t)-\frac{1}{2\delta}\widehat{r}_t-\frac{1}{2\delta}\widehat{r}^*_t
\end{aligned}
$$

where $E_t\Delta\widehat{Y}^d_{t+1}=(\lambda_1-1)\beta\lambda_1 v\widetilde{\kappa}(v-1)(\widehat{r}_t-\widehat{r}^*_t)$.As shown in the appendix, the dynamics of the relative output $\widehat{Y}^d_t=\frac{1}{2}(\widehat{Y}_t-\widehat{Y}^*_t)$ is simply determined by the term of currency misalignment m_t. When $v=1$, the relative output will not be affected by currency misalignment m_t and will be independent of monetary policy. When $v\neq 1$, there is another effect of a monetary shock on output, which is through expected changes in relative output. This effect could be positive or negative, depending on home bias. However, for reasonable parameters, we can show this effect is extremely small and dominated by the direct effect $(-\frac{1}{2\delta})$. This implies that our results from the case with no home bias $(v=1)$ also holds in more general cases.

5.2 Asymmetric household heterogeneity ($n \neq n^*$)

We also consider an asymmetric case ($n \neq n^*$) under LCP, in which there is no home bias ($v = 1$) and no productivity shocks. Again, we assume that the economy is in its steady state before the shocks come in period t. As shown in the Technical Appendix, the solution to the dynamic equation for output in the home country is given by

$$\widehat{Y}_t = E_t\widehat{Y}_{t+1} - \frac{(2-\delta^*)}{\delta+\delta^*}(\lambda_1+1)\frac{\beta}{1-B}\lambda_1\widetilde{\kappa}(\widehat{r}_t+\widehat{r}_t^*) - \frac{1}{\delta+\delta^*}(\widehat{r}_t+\widehat{r}_t^*) \tag{79}$$

where $(\lambda_1+1)\frac{\beta}{1-B}\lambda_1\widetilde{\kappa}$ is a function of parameters, which is empirically small. We establish the following proposition.

Proposition 7 *Under LCP, for reasonable low values of parameters n and w, that is, when $n < \frac{1}{1+w}$ and $n^* < \frac{1}{1+w}$, both domestic and foreign expansionary monetary shock will increase domestic output. The impacts also increase with the size of both domestic and foreign Keynesian households (n, n^*).*

The proof is in the Technical Appendix section 2.2.3. The results differ from those under PCP case. There are always spillover effects of monetary shocks under LCP. Whether the spillover effects exist or not doesn't depend on the size of Keynesian households in the source country. That is, even when $n^* = 0$ ($\delta^* = 1$), we still have spillover effects. It should be noted that since $(\lambda_1+1)\frac{\beta}{1-B}\lambda_1\widetilde{\kappa}$ is small, the spillover effects are mainly determined by the second term $\frac{1}{\delta+\delta^*}(\widehat{r}_t+\widehat{r}_t^*)$. This implies that the spillover effects of monetary shocks are almost identical to their impact on domestic output.

6 Optimal Monetary policy under PCP and LCP

In this section, we investigate optimal monetary policy under both PCP and LCP in a global coordination game. For the convenience of presentation, we focus on the case with no home bias ($v = 1$) first and then discuss more general case ($v \neq 1$) .

6.1 Optimal monetary policy under PCP

As shown in the Technical Appendix, we can derive the global planner's objective function ($v = 1$) with household heterogeneity:

$$L_{h,0} + L_{f,0}^{*} = \left\{ \begin{array}{c} \frac{1+\omega}{2}(\widehat{Y}_t - \theta_t)^2 - \frac{1}{2}\frac{1-n}{n}\left[(\frac{1}{\omega})^2\right](\widehat{H}_t)^2 + \frac{1+\omega}{2}(\widehat{Y}_t^* - \theta_t^*)^2 \\ -\frac{1}{2}\frac{1-n^*}{n^*}\left[(\frac{1}{\omega})^2\right](\widehat{H}_t^*)^2 + \frac{1}{4}(\widehat{H}_t^* - \widehat{H}_t)^2 + \frac{\lambda}{2\kappa}\pi_{hh,t}^{*2} + \frac{\lambda}{2\kappa}\pi_{ff,t}^{*2} \end{array} \right\}$$

In additional to the welfare loss from the output gap and inflation, there are losses due to household heterogeneity. The planner chooses $\left\{\widehat{Y}_t, \widehat{Y}_t^*, \pi_{hh,t}, \pi_{ff,t}^*, \widehat{q}_t\right\}$ to minimize the global loss, subject to the following NKPCs and resource constraint:

$$\pi_{hh,t} = \widetilde{\kappa}\left[(1+\omega)\,\widehat{Y}_t - (1+\omega)\,\theta_t + \frac{1}{2}(\widehat{H}_t^* - \widehat{H}_t)\right] + \beta E_t \pi_{hh,t+1} \tag{80}$$

$$\pi_{ff,t}^* = \widetilde{\kappa}\left[(1+\omega)\,\widehat{Y}_t^* - (1+\omega)\,\theta_t^* - \frac{1}{2}(\widehat{H}_t^* - \widehat{H}_t)\right] + \beta E_t \pi_{ff,t+1}^* \tag{81}$$

$$\widehat{q}_t = \widehat{Y}_t - \widehat{Y}_t^*$$

Note that we can replace $\widehat{H}_t$ and $\widehat{H}_t^*$ using the output gap in the loss function. We establish the proposition on optimal monetary policy under PCP.

Proposition 8 *The solution to the global planner's problem under PCP restores the economy to the flexible price equilibrium. PPI inflation stabilization can close both the output gap and eliminate the consumption heterogeneity even when $n \neq n^*$.*

We show the detailed derivations in the Technical Appendix section 4. This finding shows that when there is optimal monetary policy and monetary authorities in both countries can fully achieve PPI inflation targeting, then the consumption heterogeneity has no implications for monetary policy. This because consumption heterogeneity is proportional to the output gap. Once the output gap is closed, then consumption heterogeneity will be eliminated as well. As shown in the Technical Appendix, we find that the solution of the Nash game is identical to that of the coordination game and there is no monetary policy coordination gain. This mirrors the results in CGG. They show that there is no policy gain for the case with log utility since there is no direct term of trade effect in the loss function. But we note also that the results also hold in the general case $v \neq 1$, which is discussed in the Technical Appendix.

6.2 Optimal monetary policy under LCP

For ease of analysis, we focus on the case with symmetric household heterogeneity ($n = n^*$) and without home bias ($v = 1$). The global planner's loss function with household heterogeneity under LCP is given by:

$$L_0 = \left\{ \begin{array}{c} \frac{1+\omega}{2}(\widehat{Y}_t - \theta_t)^2 + \frac{1+\omega}{2}(\widehat{Y}_t^* - \theta_t^*)^2 - \frac{1}{2}\frac{1-n}{n}(\frac{1}{\omega})^2 \left[(\widehat{H}_t)^2 + (\widehat{H}_t^*)^2\right] \\ +\frac{1}{8}(\widehat{d}_t + \widehat{H}_t^* - \widehat{H}_t)^2 + \frac{1}{8}(\widehat{d}_t^* - \widehat{H}_t^* + \widehat{H}_t)^2 \\ +\frac{\varepsilon}{2\widetilde{\kappa}}\left[\frac{1}{2}\pi_{hh,t}^2 + \frac{1}{2}\pi_{fh,t}^2 + \frac{1}{2}\pi_{hf,t}^{*2} + \frac{1}{2}\pi_{ff,t}^{*2}\right] \end{array} \right\} \tag{82}$$

The constraints are represented by the planner 's four NKPCs and terms of trade equation $\widehat{q}_t + \widehat{e}_t = \widehat{Y}_t - \widehat{Y}_t^*$. Note that $\widehat{H}_t$ and $\widehat{H}_t^*$ can be replaced by the relative output $\widehat{Y}_t - \widehat{Y}_t^*$, which is independent of monetary policy.

We employ the method of Engel (2011) to solve the optimization problem. The loss function is rewritten as below:

$$L_0 = \left\{ \begin{array}{c} \left[1+\omega - \frac{n}{1-n}\right](\widehat{Y}_t^d - \widehat{Y}_t^{d,fb})^2 + \left[1+\omega - \frac{n}{1-n}\right](\widehat{Y}_t^W - \widehat{Y}_t^{W,fb})^2 \\ +\frac{1}{8}[\widehat{m}_t + \widehat{z}_t + \frac{-2n\omega}{1-n}(\widehat{Y}_t^d - \widehat{Y}_t^{d,fb})]^2 + \frac{1}{8}[\widehat{z}_t - \widehat{m}_t - \frac{-2n\omega}{1-n}(\widehat{Y}_t^d - \widehat{Y}_t^{d,fb})]^2 \\ +\frac{\varepsilon}{\widetilde{\kappa}}\left[(\pi_t^d)^2 + (\pi_t^W)^2\right] + t.i.p \end{array} \right\} \tag{83}$$

where

$$\widehat{Y}_t^d = \frac{1}{2}\widehat{Y}_t - \frac{1}{2}\widehat{Y}_t^*$$
$$\widehat{Y}_t^W = \frac{1}{2}\widehat{Y}_t + \frac{1}{2}\widehat{Y}_t^*$$
$$\widehat{m}_t = \frac{1}{2}(\widehat{d}_t - \widehat{d}_t^*) = \widehat{e}_t$$
$$\pi_t^d = \frac{1}{2}(\pi_t - \pi_t^*) = \frac{1}{2}\left[\frac{1}{2}(\pi_{hh,t} + \pi_{fh,t}) - \frac{1}{2}(\pi_{hf,t}^* + \pi_{ff,t}^*)\right]$$
$$\pi_t^W = \frac{1}{2}(\pi_t + \pi_t^*) = \frac{1}{2}\left[\frac{1}{2}(\pi_{hh,t} + \pi_{fh,t}) + \frac{1}{2}(\pi_{hf,t}^* + \pi_{ff,t}^*)\right]$$

Following Engel (2011), we define the "relative" and "world" value for output and inflation. Here, $\widehat{Y}_t^{d,fb} = \frac{1}{2}(\theta_t - \theta_t^*)$ and $\widehat{Y}_t^{W,fb} = \frac{1}{2}(\theta_t + \theta_t^*)$ represent the efficient relative and world value for output in the flexible price equilibrium. m_t is currency misalignment, which measures the average deviation of consumer prices in the foreign country from consumer prices in the home country, and $z_t = \frac{1}{2}(\widehat{d}_t + \widehat{d}_t^*)$ captures the relative price differences, which is independent of monetary policy.[5] The constraints can also be rewritten as below:

$$\pi_t^d = \frac{1}{2}\widetilde{\kappa}\widehat{m}_t + \beta E_t \pi_{t+1}^d$$

$$\pi_t^W = \widetilde{\kappa}\,(1+\omega)\,(\widehat{Y}_t^W - \widehat{Y}_t^{W,fb}) + \beta E_t \pi_{t+1}^W$$

We consider the *timeless perspective* monetary policy. The choice variables for the global planner are $\left\{\widehat{Y}_t^W, \pi_t^d, \pi_t^W, m_t\right\}$.

Let $\xi_{1,t}$ and $\xi_{2,t}$ to be the shadow prices. The optimal monetary policy is characterized by the following first order conditions:

[5] To rewrite the loss function, we use the following relationship

$$(2\pi_t^d)^2 + (2\pi_t^W)^2 + 2(\pi_{fh,t} - \pi_{hh,t})^2 + 2(\pi_{hf,t}^* - \pi_{ff,t}^*)^2 = 4\left(\pi_{hh,t}^2 + \pi_{fh,t}^2 + \pi_{hf,t}^{*2} + \pi_{ff,t}^{*2}\right)$$

where $\pi_{fh,t} - \pi_{hh,t} = \pi_{hf,t}^* - \pi_{ff,t}^* = \Delta\widehat{q}_t + \Delta\widehat{d}_t$. and $\pi_{fh,t} - \pi_{hh,t}$ and $\pi_{hf,t}^* - \pi_{ff,t}^*$ are independent of policy choices.

$$2\left[1+\omega-\frac{n}{1-n}\right](\widehat{Y}_t^W-\widehat{Y}_t^{W,fb})+\widetilde{\kappa}\,(1+\omega)\,\xi_{2,t}=0 \tag{84}$$

$$2\frac{\varepsilon}{\widetilde{\kappa}}\pi_t^d+\xi_{1,t}-\xi_{1,t-1}=0 \tag{85}$$

$$2\frac{\varepsilon}{\widetilde{\kappa}}\pi_t^W+\xi_{2,t}-\xi_{2,t-1}=0 \tag{86}$$

$$m_t+\widetilde{\kappa}\xi_{1,t}=0 \tag{87}$$

We establish the following Proposition on the optimal monetary policy under LCP based on the above solution.

Proposition 9 *When there is household heterogeneity ($n>0$) but with no home bias ($v=1$),CPI inflation is the optimal monetary policy under LCP.*

The proof is in the Technical Appendix section 5.1. When $n=0$, we have $z_t=0, \widehat{H}_t=\widehat{H}_t^*=0$, and the model will have the optimal solution $\pi_t^d=\pi_t^W=\widehat{Y}_t^W-\widehat{Y}_t^{W,fb}=0$, which is the one shown in Engel (2011). In this case, the optimal monetary policy is CPI inflation targeting, which can achieve an efficient level of world output but cannot ensure that both home and foreign will have the efficient allocations. When $n>0$, there is household heterogeneity. However, the terms in heterogeneity can be replaced by relative output and the world output. Since $v=1$, relative output is not affected by currency misalignment m_t and is independent of monetary policy, so the central bank sets $\pi_t^d=\pi_t^W=\widehat{Y}_t^W-\widehat{Y}_t^{W,fb}=0$, but cannot achieve $\widehat{Y}_t^d-\widehat{Y}_t^{d,fb}=0$. With PCP, the output gap is closed , so the policy maker does not need to take consumption heterogeneity $\widehat{H}_t-\widehat{H}_t^*$ into consideration. With LCP, the output gap cannot be closed, but relative output is independent of monetary policy, so policy makers still can ignore consumption heterogeneity and choose CPI targeting as optimal monetary policy. As shown in the Technical Appendix, we can solve for the output below

$$\widehat{Y}_t = \overline{\lambda}_1 \widehat{Y}_{t-1} + \frac{1}{2}(\Delta\theta_t + \Delta\theta_t^*) + \frac{1}{2}\beta\overline{\lambda}_1 v\widetilde{\kappa}(1+\omega+\frac{n\omega}{1-n})E_t\sum_{i=0}^{\infty}(\beta\overline{\lambda}_1)^i(\theta_{t+i}-\theta_{t+i}^*)$$

The dynamics of home output follows AR(1) process with persistence $\overline{\lambda}_1$. In the dynamic process, there are two productivity terms that affect the output. The first term is the change of efficient world output, $\frac{1}{2}(\Delta\theta_t + \Delta\theta_t^*)$; The second term is the sum of expected future efficient relative output. From the second term, we can show that the equilibrium level of output will be affected by household heterogeneity ($\frac{n\omega}{1-n}$) since it amplifies the impact of relative productivity on output.

It should be noted that, as shown in the Technical appendix, this proposition also holds in the case with asymmetric household heterogeneity. For the details, refer to the Technical Appendix.

Now we consider a more general case with home bias. To highlight the role of home bias, we assume symmetric household heterogeneity ($n = n^*$) but with home bias ($v \neq 1$). We establish the following proposition.

Proposition 10 *When there is household heterogeneity ($n > 0$) and home bias ($v \neq 1$),CPI inflation is not the optimal monetary policy under the LCP case.*

The details of the proof can be seen in the Technical Appendix section 5.2. When $v \neq 1$, the dynamic of relative output $\widehat{Y}_t^d$ will be affected by both productivity shocks and currency misalignment.

$$\begin{aligned} & \Delta\widehat{Y}_t - \Delta\widehat{Y}_t^* + (v-1)(\Delta\widehat{H}_t - \Delta\widehat{H}_t^*) \\ = \; & v\widetilde{\kappa}\begin{bmatrix} (1+\omega)(\widehat{Y}_t^* - \widehat{Y}_t) - (1+\omega)(\theta_t^* - \theta_t) \\ +(v-1)m_t - (2-v)(\widehat{H}_t^* - \widehat{H}_t) \end{bmatrix} \\ & +\beta E_t\left[\Delta\widehat{Y}_{t+1} - \Delta\widehat{Y}_{t+1}^* + (v-1)(\Delta\widehat{H}_{t+1} - \Delta\widehat{H}_{t+1}^*)\right] \end{aligned} \tag{88}$$

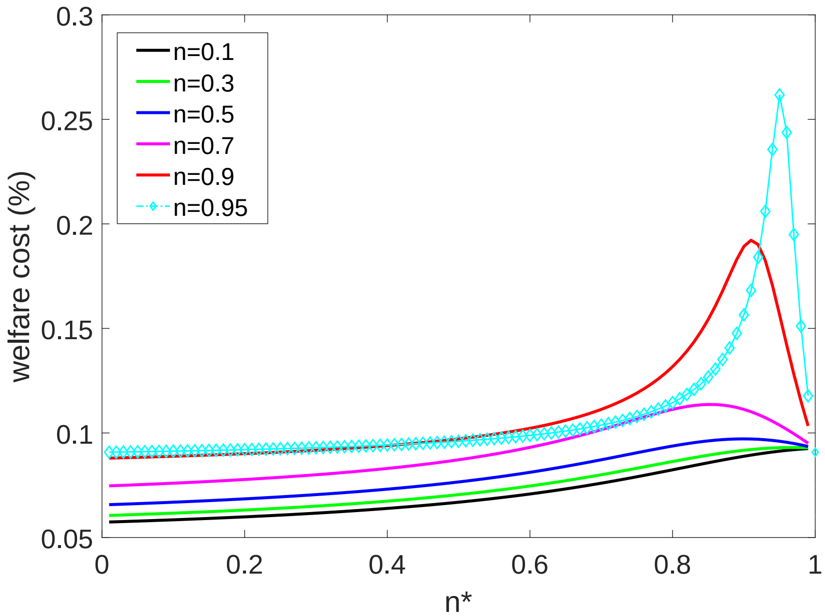
As shown in the Technical Appendix section 5.2, as long as the relative output deviates from its efficient level, currency misalignment cannot be zero anymore, which implies that $\pi_t^d = \pi_t^W = \widehat{Y}_t^W - \widehat{Y}_t^{W,fb} = 0$ cannot be the optimal solution. Consequently, CPI cannot be the optimal policy under LCP. This differs from the results in Engel (2011) that even with home bias, CPI is still the optimal monetary policy. Engel (2011) emphasizes the importance of home bias $v \neq 1$ but only discusses the case where the elasticity of labor supply $\omega = 0$. In our case, home bias $v \neq 1$, household heterogeneity $n > 0$, elasticity of labor supply $\omega > 0$ are essential for results. Note that consumption heterogeneity is given by $\widehat{H}_t = \frac{-n\omega}{1-n}(\widehat{Y}_t - \theta_t)$, this implies that consumption heterogeneity will disappear when either the output is closed or $n\omega = 0$.

Table 1: Parameter values (Baseline)

Parameter	Description	Value
β	discount factor	0.99
η	preference weight on labor	2
ρ	inverse of elasticity of intertemporal substitution	2
ω	Inverse of the Frische elasticity in Labor supply	1
κ	degree of price stickiness	0.75
λ	elasticity of substitution across individual goods	11
v	home country size and weight on home product	1.5
n	the fraction of household whose is "hand-to-mouth"	0.21
ρ_θ	persistence of productivity shock	0.9
σ	productivity shock size	0.01

In the Technical Appendix section 5.3, we also present the solution to the planner's problem for the general case with $n \neq n^*$. The complicated system does not allow us to definitively describe the separate roles of n or n^*. We then apply the Ramsey approach to evaluate the welfare cost numerically when specifying different values of n and n^*. The policy instruments in the government problems are the four variables on inflation dynamics. The calibrated parameters are presented in Table

Figure 2: The welfare cost



1[6]. Figure 2 illustrates the role of n and n^* in the global welfare cost[7]. The changes in n and n^* have two direct welfare implications. First, when n and n^* increase, more households are excluded from the financial market, which reduces welfare. Sec-

[6]We follow Engel (2011) and Fujiwara and Wang (2017) in choosing the basic value of parameters. We assume that each period is one quarter; The discount factor is 0.99, while the preference weight on labor disutility is 2. The risk aversion or the parameter governing the elasticity of intertemporal substitution is 2, the degree of price stickiness is 0.75 so that the average duration of price change is 4 quarters, the elasticity of substitution across individual goods is 11 so that the markup is 10%, the home bias parameter v is 1, the persistence of the productivity shock is 0.9, and the labor supply elasticity is 1.7.

[7]In an economy with shocks, the welfare cost is measured as the percentage change of deterministic steady state consumption that is required to compensate the household such that they maintain the utility level as in the steady state.

ond, the changes in n and n^* also affect the consumption heterogeneity. Under LCP, consumption heterogeneity can not be eliminated by optimal monetary policy, so there is welfare loss caused by consumption heterogeneity. However, the relationship between n and n^* and consumption heterogeneity are non-monotonic. When both n and n^* are zero, the model is reduced to the standard representative agent model in NOEM with complete international market structure, in which the global welfare loss is the smallest. Then for most cases, the global welfare loss increases with n and n^*, which is driven by the financial market effect. When both n and n^* approach unit, the model is reduced to a standard two-country model without any financial market access. As shown in the figure, this model does not deliver the largest global welfare loss. Given a particular n, when n^* increases, there is a welfare loss peak before n^* reaches unit. This is because when n^* is close to 1, consumption heterogeneity starts to fall, which may reduce welfare loss.

Chen, Devereux, Shi and Xu (2020) show that the currency misalignment under LCP can be corrected by fiscal instruments such as state-contingent import tax and export subsidy. It is natural to ask if the results will be affected by the presence of consumption heterogeneity. We investigate this issue in the Technical Appendix section 6 and summary our findings in the following proposition.

Proposition 11 *Under LCP, when the global planner is allowed to choose state-contingent import tax and export subsidy for both countries, there will be no currency misalignment and consumption heterogeneity, optimal monetary policy will be PPI targeting.*

The details of proof can be seen in the Technical Appendix section 6.5. When the tax and subsidy are introduced, currency misalignment will be corrected. Consequently, the economy will be restored to flexible price equilibrium. In such an equilibrium, the output gap is closed, which in turn eliminates consumption heterogeneity.

7 Conclusion

This paper incorporates household heterogeneity (Ricardian versus Keynesian) into a standard two-country open economy macro model and studies how consumption heterogeneity affects the transmission mechanism of monetary policy and the choice of optimal monetary policy in open economy. We find that, under both PCP and LCP, there will be a sizeable range of consumption heterogeneity in which monetary policy becomes ineffective. Meanwhile, the spillover effects of monetary shocks in an open economy also change dramatically and could be positive or negative under both PCP and LCP cases, depending on the degree of consumption heterogeneity. For the choice of optimal monetary policy, with consumption heterogeneity, PPI is still the optimal monetary policy under PCP and restores the economy to the efficient equilibrium. However, under LCP, consumption heterogeneity will create a new distortion that cannot be corrected by monetary policy. We show that only in the case without home bias, CPI is the optimal monetary policy. However, in most cases, CPI can no longer be an optimal monetary policy. Finally, we show that when fiscal instruments such as import tax and export subsidy are introduced, both currency misalignment and consumption heterogeneity can be eliminated, and the central bank will target PPI again.

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