Domestic Policies and Sovereign Default[*]

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Abstract

This paper incorporates fiscal and monetary policies into a model of sovereign default. In addition to the standard impatience vs default-risk tradeoff faced by governments when choosing debt, distortionary policy instruments introduce an intertemporal tradeoff, which may moderate or amplify the incentives to accumulate debt. Taxation, the money growth rate and currency depreciation all increase with the level of debt. The model reproduces standard business cycle statistics, the response of spreads, inflation and growth to terms-of-trade shocks, and the cyclical properties of fiscal and monetary policies in emerging markets.

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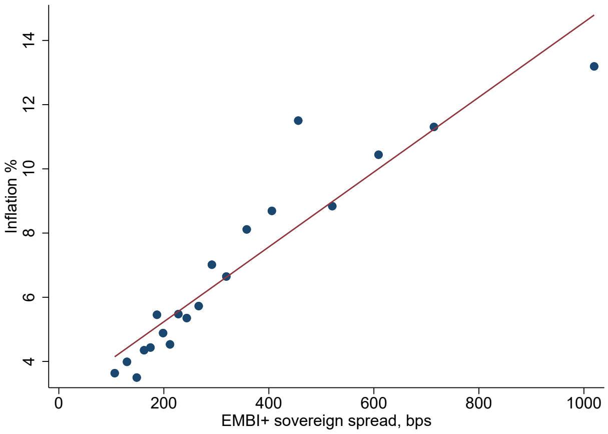
1 Introduction

A now large literature, spanned by the work of Eaton and Gersovitz (1981), Aguiar and Gopinath (2006) and Arellano (2008), studies recurrent sovereign debt crises by contending that emerging countries underinsure against negative shocks by overborrowing during booms. It is also widely understood, though mostly ignored by this literature, that emerging markets have traditionally experienced high inflation, especially during times of debt crises. This fact can be seen in Figure 1, which displays the correlation between the EMBI spread and inflation for emerging markets countries since 1990.

Figure 1: Inflation and Sovereign Spreads in Emerging Markets



Note: This figure is a binscatter. Each dot is the mean for a bin containing the same number of observations. We removed outliers from both EMBI+ spread and inflation series. Outliers are defined as values below the 3rd percentile or above the 97th percentile.

We demonstrate that once a standard sovereign default model is extended to incorporate *domestic policies*, there is a close connection between debt crises and inflation. In our framework, domestic fiscal and monetary policies in emerging countries interact with the availability of external credit and the possibility of default. As in the standard model, debt accumulation is promoted by relative impatience and hindered by default risk. When fiscal and monetary policy instruments are distortionary, we find that the government actively relies on seigniorage to finance expenditure and faces an additional intertemporal tradeoff, that may moderate or amplify the incentives to accumulate debt. We also show that distortions increase with debt holdings, as the higher probability of default implies a higher service cost; thus, the money growth rate, the tax rate and the exchange rate all increase with debt.

In a quantitative version of the model, calibrated to match long-term averages of emerging Latin American countries, we find that sovereign debt spreads, inflation and output growth react in an empirically plausible way to shocks to the terms of trade. The model also reproduces standard business cycle statistics as well as the cyclical properties of fiscal and monetary policies. We also study a version of our model in which the government has access to lump-sum taxes. In this case, implemented monetary policy is no longer distortionary and the equilibrium allocation coincides with the one that would obtain from a real economy. Debt accumulation results from the standard trade-off between relative impatience and default-risk. The version with lump-sum taxes is thus a variant of a standard sovereign default model, in the tradition of Eaton and Gersovitz (1981). Importantly, we show that this *EG version* of our model cannot match the levels of nominal variables, such as inflation and currency depreciation, as well as the empirical correlation of output with debt spreads, exports and the trade balance.

Our framework consists of a tradable-nontradable (TNT) small open economy (as in Uribe and Schmitt-Grohé, 2017, §8), extended to include production, money and sovereign default. Firms produce both non-tradable goods and exported goods; agents consume non-tradable goods and imported goods. Consumers need money to finance their purchases of non-tradable goods, which gives rise to a demand for fiat money.[1] The government provides a valued public good and makes transfers to individuals; expenditures are financed with labor taxes, money creation and external debt. Government debt is issued in foreign currency to foreign risk-neutral investors. In the event of default, the government enjoys a haircut on its external liabilities but suffers temporary exclusion from financial markets and a productivity loss. We further assume that the government's inability to commit extends to all future policy actions.

We derive necessary first-order conditions to characterize government policy.[2] We show that the decision of how much debt to issue depends on three channels. The first involves distortion-smoothing: debt allows the government to trade-off intertemporally how severely the balance of payments restricts its policy. If the domestic economy is impatient relative to the rest of the world, then this factor provides incentives to accumulate debt. The second channel reflects the negative impact of more debt, which leads to a higher default premium and, thus, counter the desire to accumulate debt. The third channel arises because fiscal and monetary policies are distortionary. Higher debt tomorrow leads to larger future distortions when repaying and a larger default probability, both of which affect the demand for money today and, hence, the government's current budget constraint. This factor may be positive or negative, depending on agents' preferences, and thus may amplify or moderate the incentives to issue debt.

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[1] Though we model domestic government liabilities as fiat money, one could also interpret them more generally to include debt issued domestically in local currency as long as this debt is liquid to some extent.

[2] Arellano, Mateos-Planas, and Ríos-Rull (2019) also derive the Generalized Euler Equation to solve and characterize a sovereign default model.

We conduct several exercises to evaluate the model's ability to capture critical mechanisms observed in emerging markets. First, model-generated data are compared with data since 1980 for seven Latin American economies. In the presence of expected fluctuations in the terms of trade, the model replicates standard business cycle statistics and, more importantly, the cyclical properties of fiscal and monetary policies. Next, impulse responses to terms-of-trade shocks are estimated in the data and the model. The dynamics of the EMBI spread, inflation and real GDP all have the same signs and similar magnitudes in the model and the data. Then, we show that in the EG version of our model, when the government has access to lump-sum taxes, the model is no longer able to replicate the empirical correlation of output with spreads, exports and the trade balance. We therefore conclude that the presence of distortionary taxation is necessary to explain the average level of nominal variables, such as inflation and currency depreciation, as well as the cyclical regularities of real and external variables. Finally, feeding the model with the actual evolution of the terms of trade and government expenditure to replicate Argentina's experience during 2005-2017, the model broadly replicates the dynamics of key macroeconomic variables. We then use this simulation to run the policy counterfactuals explained above.

Literature review

The literature on sovereign default has evolved from the framework developed by Eaton and Gersovitz (1981) to quantitative models that account for stylized facts about business cycles in emerging countries (Aguiar and Gopinath, 2006; Arellano, 2008). Although recent models have added realistic features, such as long-term debt (Hatchondo and Martinez, 2008; Hatchondo, Martinez, and Sosa-Padilla, 2016) and sovereign-debt restructuring (Dvorkin, Sánchez, Sapriza, and Yurdagul, 2021; Yue, 2010), there are few papers concerned with the role of fiscal policy and almost no work on monetary policy.

In terms of fiscal policy analysis, Cuadra, Sánchez, and Sapriza (2010) show how a desirable counter-cyclical fiscal policy is reversed by including debt with a risk of default. The critical difference is that we introduce money, which significantly extends the scope of the analysis and complicates the environment by adding an intertemporal optimization problem for households, which the government needs to take into account when formulating policy. Bianchi, Ottonello, and Presno (2019) also argue that pro-cyclical fiscal policy is a property of countries with a high risk of default. They show that this is true even in a model with nominal rigidities and significant Keynesian stabilization gains. More recently, Anzoategui (2019) studies the effect of alternative fiscal rules.

Our mechanism is in line with the history of Latin America described in Kehoe, Nicolini, and Sargent (2020) "... countries in Latin America have been plagued by economic crises. The specific symptoms of each crisis have been very different: high inflation rates, balance of payments crises followed by large devaluations, banking crises, defaults on government debt, ... Our fundamental hypothesis is that, despite their different manifestations, all economic crises in Latin America have been the result of poorly designed or poorly implemented macro-fiscal policies."

Concerning monetary policy, there is recent work studying the currency composition of debt and inflation (Ottonello and Perez, 2019; Sunder-Plassmann, 2018). These papers show that debt denominated in local currency raises incentives to dilute debt repayment through inflation. Closer to our work, Arellano, Bai, and Mihalache (2020) analyze the interaction of sovereign default risk with a monetary policy rule in a cashless economy. They argue that the model rationalizes the positive co-movements of sovereign spreads with domestic nominal rates and inflation in Brazil. Their work complements ours since they study the case in which central bankers in emerging markets can commit to a Taylor rule. In contrast, we assume that both fiscal and monetary policies are discretionary and useful to finance government spending. We also study the case when the government can commit to a monetary policy rule.

Another important aspect of our model is the role of nominal exchange rates. In this regard, our work connects with Na, Schmitt-Grohé, Uribe, and Yue (2018), which point to the link between devaluations and default. In a model with downward nominal wage rigidity, they show that an optimal exchange rate devaluation occurs in periods of default, lowering the real value of wages to reduce unemployment. Their paper and ours both show how to recover a "real" economy as in Eaton and Gersovitz (1981). In Na, Schmitt-Grohé, Uribe, and Yue (2018), the key is an optimal devaluation to undo the wage rigidity, while in our model, it is the availability of unconstrained lump-sum taxation.

Finally, our paper is also related to work in closed economies, e.g., Díaz-Giménez, Giovannetti, Marimón, and Teles (2008) and Martin (2009, 2011), among others. These papers study government policy without commitment in monetary economies. Unlike our work, they do not consider the role of sovereign default risk. However, these papers share important similarities in terms of the intertemporal tradeoffs faced by the government.

The paper is structured as follows. Section 2 describes the environment and characterizes the monetary equilibrium. Section 3 formulates the problem of the government, characterizes policies and derives the theoretical results. Section 5 studies the main quantitative properties of our model by focusing on an economy without anticipated aggregates shocks. Section 6 studies an economy with expected shocks to the terms of trade and evaluates the empirical plausibility of the model by comparing model-simulated data with actual data.

2 Model

2.1 Environment

We study a small open economy populated by a large number of identical infinitely-lived agents with measure 1. Time is discrete. Throughout the paper, we make use of recursive notation, denoting next-period variables with a prime.

Preferences, endowments and technology

There are three private goods and one public good in the economy. First, there is a non-tradable good that is consumed and produced domestically, their quantities being denoted c^N and y^N, respectively. Second, there is tradable imported good that is consumed domestically but not produced. Let c^T denote the consumption of this imported good. Third, there is a tradable exported good that is not consumed domestically and is only produced to be exported. Let y^T denote the production of this exported good. Finally, the government can transform non-tradable output y^N one-to-one into a public good, g.

The representative household is endowed with one unit of time each period, which can be either consumed as leisure, ℓ, or supplied in the labor market, h. Thus, $\ell + h = 1$.

Preferences are represented by a time-separable, expected discounted utility. Let the period utility be given by

$$u(c^N, c^T) + v(\ell) + \vartheta(g),$$

where u, v and ϑ are strictly increasing, strictly concave, C^2 and satisfy standard boundary conditions. Let $\beta \in (0, 1)$ denote the discount factor. In what follows, u_j denotes the partial derivative of u with respect to the consumption good c^j, with $j = \{N, T\}$, and v_ℓ denotes the derivative of v with respect to $\ell = 1 - h$. We assume that cross derivatives are zero; i.e., $u_{NT} = u_{TN} = 0$.

There is an aggregate production technology that transforms hours worked, h, into non-tradable output, y^N, and exported goods, y^T. This technology is represented by a cost function $F : \mathbb{R}^2_+ \to \mathbb{R}_+$, which is strictly increasing, strictly convex and homogeneous of degree 1. Given

h, feasible levels of (y^N, y^T) must satisfy

$$F(y^N, y^T) - h \leq 0, \tag{1}$$

where F_j is the partial derivative of F with respect to y^j, $j = \{N, T\}$.

Market structure

Agents can exchange both tradable and non-tradable goods, as well as domestic currency (fiat money), while trading of other financial assets will be restricted to the government. Let M^d denote individual money holdings. Prices are denominated in domestic currency (i.e., pesos) and given by P^X, P^M and P^N for exports, imports and non-tradable goods, respectively. Let W denote the nominal wage in units of domestic currency.

The nominal exchange rate E is defined as the units of domestic currency necessary to purchase one unit of foreign currency (i.e., pesos per dollar). We assume that the law of one price holds for tradable goods and so $P^X = Ep^T$ and $P^M = E$, where p^T is the (potentially time-varying) international price of exported goods, while the international price of imported goods is assumed to be constant and normalized to 1. Given these assumptions, p^T also stands for the terms of trade.

In order to study a stationary environment, we normalize nominal variables by the stock of the money supply, M. Let μ denote the growth rate of the money supply and $M' = (1+\mu)M$ denote its law of motion. We define the corresponding normalized variables as $p^N = P^N/M$, $w = W/M$, $e = E/M$ and $m = M^d/M$.

To motivate a role for fiat money, we assume that households face a cash-in-advance constraint when purchasing non-tradable goods:

$$p^N c^N \leq m. \tag{2}$$

That is, (normalized) expenditure on non-tradable goods, $p^N c^N$, cannot exceed (normalized) money balances available at the beginning of the period, m.

Government and the balance of payments

The government provides a public good, g, which is transformed one-to-one from non-tradable output. It may also make lump-sum transfers to households. Let γ be the real value (in units of the non-tradable good) of government transfers. We assume that transfers are exogenous, non-negative and represent a non-discretionary redistributive policy.

To finance its expenditure, the government may tax labor income wh at rate τ, increase the money supply at rate μ, and issue debt in international credit markets. Debt takes the form of one-period discount bonds that pay one unit of foreign currency and trade at the price q, also denominated in foreign currency. Let B denote the value of maturing debt and qB' the funds collected from issuing new debt B', both expressed in foreign currency units.

We consolidate the fiscal and monetary authority and write the government budget constraint in (normalized) units of domestic currency as follows[3]:

$$p^N(g+\gamma)+eB \leq \tau wh+\mu+eqB'. \tag{3}$$

The balance of payments, expressed in units of foreign currency, implies

$$p^T y^T - c^T = B - qB', \tag{4}$$

where the left-hand side of (4) is the trade balance, while the right-hand side is the change in the country's net asset position plus implicit debt interest payments.

Combining (3) and (4) we can express the government budget constraint as the relationship between the external sector (the trade balance) and the public sector (the primary surplus plus seigniorage):

$$\tau wh - p^N(g+\gamma)+\mu - e(p^T y^T - c^T) \geq 0. \tag{5}$$

2.2 The problem of the representative firm

Local firms produce non-tradable and tradable goods by hiring labor according to the technology represented by F. Constant returns to scale imply that we can assume that the industry behaves as if there were a representative firm that solves the static problem

$$\max_{y^N,y^T,h} \; \{p^N y^N + ep^T y^T - wh\}$$

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[3] As we argue in Section 4.3, when lump-sum taxes are available, the government sets distortionary taxes equal to zero and follows the optimal monetary policy (the Friedman rule). In this case, the model becomes Ricardian: the government budget constraint solves for lump-sum taxes and places no further restrictions on government policy.

subject to (1). The necessary and sufficient first-order conditions imply expressions for the wage and exchange rate as functions of (y^N, y^T, p^N, p^T) as follows:

$$\begin{aligned} w &= \frac{p^N}{F_N}, && (6) \\ e &= \frac{p^N}{p^T}\frac{F_T}{F_N}. && (7) \end{aligned}$$

2.3 The problem of the representative household

The endogenous state of the economy consists of the amount of maturing foreign debt, B, and an indicator function $\mathcal{I}$, which specifies whether the government is in default ($\mathcal{I} = D$) or not ($\mathcal{I} = P$). As we shall explain below, the default state may last several periods while the country is excluded from international credit markets. Agents know the government's default state before making any decisions at the beginning of every period. The exogenous state of the economy is summarized by s and known at the beginning of each period. The state s may include any variable that evolves stochastically over time, e.g., the terms of trade, p^T. The set of all possible realizations for the stochastic state is S. Note that we are allowing for the possibility that state variables may depend on the default state.

Agents know the laws of motion of all aggregate state variables. All prices and government policies are perceived by agents as being functions of the aggregate state. This dependence is omitted to simplify notation. The period budget constraint of the household is

$$p^N c^N + ec^T + m'(1+\mu) \leq (1-\tau)wh + m + p^N\gamma, \tag{8}$$

where, as mentioned above, p^N, w e, m are all normalized by the aggregate money supply at the beginning of the period. As also mentioned above, purchases of non-tradable goods are subject to the cash-in-advance constraint, (2).

The individual state variable is the household's (normalized) money balances at the beginning of the period, m. Let $V(m, B, \mathcal{I}, s)$ denote the agent's value function as a function of individual and aggregate state variables. Let $\mathbb{E}[V(m', B', \mathcal{I}', s')|B, \mathcal{I}, s]$ be the conditional expected value of the agent's value function in the next period, given current aggregate state $(B, \mathcal{I}, s)$.

The problem of the representative household is

$$V(m, B, \mathcal{I}, s) = \max_{(c^N, c^T, m', h)} u(c^N, c^T) + v(1-h) + \vartheta(g) + \beta\mathbb{E}\left[V(m', B', \mathcal{I}', s')|B, \mathcal{I}, s\right]$$

subject to (2) and (8). As derived in Appendix A.1, the solution to this problem is characterized by

$$\begin{aligned} \frac{(1-\tau)wu_T}{e} &= v_\ell, & (9)\\ \frac{(1+\mu)u_T}{e} &= \beta\mathbb{E}\left[\frac{u_N'}{p^{N\prime}}\Big| B,\mathcal{I},s\right] & (10) \end{aligned}$$

plus constraints (2) and (8).

Conditions (9) and (10) show how policies distort households' choices. The tax rate introduces an intra-temporal wedge between the marginal utilities of consumption of tradable goods and leisure, while the money growth rate introduces an inter-temporal wedge, as it distorts the substitution between current consumption of tradable goods and future consumption of non-tradable goods.

2.4 Monetary equilibrium

Since all agents are identical, c^N, c^T and h should be interpreted as referring to aggregate quantities from now on.

The resource constraint in the non-tradable sector is

$$c^N + g = y^N. \tag{11}$$

From (1) and (11), labor is a function of non-tradable (private) consumption, public expenditures and the production of tradables; i.e.,

$$h = F(c^N + g, y^T). \tag{12}$$

All agents enter the period with the same money balances, m. Market clearing implies that $m = m' = 1$. Without loss of generality, the cash-in-advance constraint (2) is satisfied with equality.[4] Then, in equilibrium

$$p^N = \frac{1}{c^N}. \tag{13}$$

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[4] If the cash-in-advance constraint is slack, then the price level p^N is, in general, indeterminate. A standard assumption is to take the limiting case, when the constraint is satisfied with equality.

The equilibrium wage can be derived by combining (6) and (13):

$$w = \frac{1}{c^N F_N}. \tag{14}$$

Similarly, the equilibrium exchange rate follows from (7) and (13):

$$e = \frac{1}{c^N p^T} \frac{F_T}{F_N}. \tag{15}$$

Finally, the Lagrange multiplier associated with the cash-in-advance constraint must be non-negative, which implies the following equilibrium condition:

$$\frac{u_N}{u_T} - \frac{p^T F_N}{F_T} \geq 0. \tag{16}$$

If (16) is positive, then the cash-in-advance constraint (2) binds; if is equal to zero, then the cash-in-advance constraint is slack. Condition (16) reflects an inefficiency wedge created by monetary policy, as the marginal rate of substitution between non-tradable and tradable goods is larger than the corresponding marginal rate of transformation. Note that, although monetary policy creates this intra-temporal wedge, similar to a tax on non-tradable consumption, it actually operates through an inter-temporal channel, as reflected by (10).

3 Government policy

3.1 Government budget constraint in a monetary equilibrium

Below, we formulate the problem of the government following the *primal approach*. That is, we solve for allocation and debt choices that are implementable in a monetary equilibrium. In order to proceed, we need to use the equilibrium conditions derived above, to replace prices (p^N, w, e) and policies (μ, τ) in the government budget constraint (5).

To obtain an expression for the tax rate, combine (9), (14) and (15):

$$\tau = 1 - \frac{v_\ell}{u_T} \frac{F_T}{p^T}. \tag{17}$$

Similarly, the money growth rate can be written by combining (10), (13) and (15):

$$\mu = \frac{\beta \mathbb{E}\left[u'_N c^{N\prime} | B, \mathcal{I}, s\right]}{u_T c^N p^T (F_N / F_T)} - 1. \tag{18}$$

Using (13)–(18) we obtain the government budget constraint in a monetary equilibrium:

$$u_T[c^T - \gamma p^T(F_N/F_T)] - v_\ell F(c^N + g, y^T) + \beta \mathbb{E}\left[u_N' c^{N\prime} | B, \mathcal{I}, s\right] \geq 0, \tag{19}$$

which depends on $(c^N, c^T, y^T, g, c^{N\prime})$.

3.2 Repayment and default

Suppose the government is currently not excluded from international credit markets. At the beginning of any such period, the government decides between repaying (P) and defaulting (D) on its debt. If it decides to default, then debt is set to zero. Define

$$\hat{\mathcal{V}}(B, s, \varepsilon^P, \varepsilon^D) = \max\{V^P(B, s) + \varepsilon^P, V^D(s) + \varepsilon^D\},$$

where $V^P(B, s)$ and $V^D(s)$ denote the values of repayment and default, respectively, which are defined in detail below. Notice that the decision to repay or default is also influenced by random additive shocks to utility. Next, we explain how these shocks are used to obtain some useful expressions under particular assumptions on their distribution.

Assume that ε^P and ε^D are independently, identically distributed extreme value (Gumbel or type I extreme value) shocks. The difference between these two shocks will affect the default decision. The assumptions on the distribution of the shocks imply that this difference has mean zero and is distributed logistic; i.e., $\varepsilon = \varepsilon^P - \varepsilon^D$ follows

$$F(\varepsilon) = \frac{\exp[\varepsilon/\kappa]}{1 + \exp[\varepsilon/\kappa]},$$

where $\kappa > 0$ is the scale parameter of the distribution, which will be useful to control the variance of the ε shocks.

Let $\mathcal{P}(B, s)$ be the probability of repayment for any given (B, s), which can be expressed as

$$\mathcal{P}(B, s) = \Pr(V^P(B, s) - V^D(s) \geq -\varepsilon).$$

This probability has a simple expression given the assumptions on ε^i. Following McFadden (1974), this integral results in a closed-form expression:

$$\mathcal{P}(B, s) = \frac{\exp[V^P(B, s)/\kappa]}{\exp[V^P(B, s)/\kappa] + \exp[V^D(s)/\kappa]}, \tag{20}$$

which, in turn, implies

$$\frac{\partial \mathcal{P}(B,s)}{\partial B} = \frac{\partial V^P(B,s)}{\partial B}\frac{\mathcal{P}(B,s)(1-\mathcal{P}(B,s))}{\kappa}. \tag{21}$$

Next, we can derive a closed-form expression for the expectation of the value function with respect to the utility shocks:

$$\mathcal{V}(B,s) = E_\varepsilon[\hat{\mathcal{V}}(B,s,\varepsilon^P,\varepsilon^D)] = \kappa \ln\left\{\exp[V^P(B,s)/\kappa] + \exp[V^D(s)/\kappa]\right\}.$$

Using this expression, we can easily see that

$$\frac{\partial \mathcal{V}(B,s)}{\partial B} = \mathcal{P}(B,s)\frac{\partial V^P(B,s)}{\partial B}. \tag{22}$$

Expressions (21) and (22) will be useful to characterize the choice of debt as long as V^P is differentiable.

3.3 Problem of the government

Every period, the government first decides on whether to repay or default on its debt. After that, it implements policy for the period, taking into account the response of private domestic agents and international lenders and the government policies it expects to be implemented in the future. A period policy consists of choices on the amount of future debt, the money growth rate, the tax rate and government expenditure.

If the government decided to default, then the country is excluded from international credit markets. It regains access at the beginning of the period with probability δ; hence, $1/\delta$ is the expected duration of exclusion. The country reenters credit markets with a renegotiated level of debt $B^d \geq 0$, which is exogenous. This assumption implies that debt haircuts are increasing in the level of defaulted debt. While in default, $\mathcal{I} = D$, the country experiences a productivity penalty, $\Omega(s)$, which generally depends on the exogenous state of the economy.

If the government is currently repaying, the probability that it will remain in repayment status tomorrow is given by $\mathcal{P}(B', s')$ for any given (B', s'), as derived above. On the other hand, if the government is currently in default, the probability that it will transition to repayment status tomorrow is given by $\delta\mathcal{P}(B^d, s')$ for any given s'. To compute these probabilities we need to know the value functions $V^P(B,s)$ and $V^D(s)$, which we will derive below.

In equilibrium, zero expected profits by risk-neutral international lenders implies that

$$Q(B',s)B' = \frac{\mathbb{E}\left[\mathcal{P}(B',s')|s\right]B' + \mathbb{E}\left[(1-\mathcal{P}(B',s'))Q^d(s')|s\right]B^d}{1+r}, \tag{23}$$

where

$$Q^d(s') \equiv \delta Q(B^d,s') + \frac{(1-\delta)\mathbb{E}[Q^d(s'')|s']}{1+r}$$

for all s'. The first term in (23) reflects the expected debt repayment, while the second term reflects the expected debt recovery. $Q^d(s')$ stands for the price an investor would pay to earn B^d in the period the defaulting country reenters international credit markets. Note that, although $Q^d(s')$ is an endogenous object, as it depends on $Q(B^d,s')$, the government takes it as given. Using (23) and the expression for $Q^d(s')$ we obtain the following recursion:

$$Q^d(s) = \frac{\delta\mathbb{E}\left[\mathcal{P}(B^d,s')|s\right] + \mathbb{E}\left[(1-\delta\mathcal{P}(B^d,s'))Q^d(s')|s\right]}{1+r} \tag{24}$$

for all s.

Taking the derivative of (23) with respect to B' and combining with (21) implies

$$\frac{\partial[Q(B',s)B']}{\partial B'} = \mathbb{E}\left\{\frac{\mathcal{P}(B',s')}{1+r}\left[1+\frac{\partial V^P(B',s')}{\partial B'}\frac{(1-\mathcal{P}(B',s'))(B'-Q^d(s')B^d)}{\kappa}\right]\middle| s\right\}. \tag{25}$$

As explained above, we follow the primal approach to formulate the government's problem. Hence, we use equilibrium conditions to express domestic prices, the money growth rate, and the tax rate as functions of current and future allocations. Every period, the government then chooses a debt level (when repaying) and domestic policies that implement the allocation (c^N, c^T, y^T, g). These choices need to satisfy the balance of payment, (4), the government budget constraint, (19), and the non-negativity constraint, (16).

When the government is in the repayment state, $\mathcal{I} = P$, its policies are a function of the state (B,s); let the relevant policy functions be denoted by $\{\mathcal{B}, \mathcal{C}^N, \mathcal{C}^T, \mathcal{Y}^T, \mathcal{G}\}$. While in the default state, $\mathcal{I} = D$, its policies are a function of the state s; let the relevant policy functions be denoted by $\{\bar{\mathcal{C}}^N, \bar{\mathcal{C}}^T, \bar{\mathcal{Y}}^T, \bar{\mathcal{G}}\}$.

Given these policy functions, we can define the value functions $V^P(B,s)$, $V^D(s)$ and $\mathcal{V}(B,s)$

as follows:

$$\begin{aligned}
V^P(B,s) &= u(\mathcal{C}^N(B,s),\mathcal{C}^T(B,s)) + v(1-F(\mathcal{C}^N(B,s)+\mathcal{G}(B,s),\mathcal{Y}^T(B,s))) + \vartheta(\mathcal{G}(B,s)) \\
&\quad + \beta\mathbb{E}[\mathcal{V}(\mathcal{B}(B,s),s')|s], \\
V^D(s) &= u(\bar{\mathcal{C}}^N(s),\bar{\mathcal{C}}^T(s)) + v(1-F(\bar{\mathcal{C}}^N(s)+\bar{\mathcal{G}}(s),\bar{\mathcal{Y}}^T(s))) + \vartheta(\bar{\mathcal{G}}(s)) \\
&\quad + \beta\mathbb{E}[\delta\mathcal{V}(B^d,s') + (1-\delta)V^D(s')|s], \\
\mathcal{V}(B,s) &= \kappa \ln\left\{\exp[V^P(B,s)/\kappa] + \exp[V^D(s)/\kappa]\right\},
\end{aligned}$$

for all (B,s). Using (20) and (23) these functions imply expressions for $\mathcal{P}(B,s)$ and $Q(B,s)$.

3.3.1 Repayment

The problem of the government in the repayment state is

$$\max_{(B',c^N,c^T,y^T,g)} u(c^N,c^T) + v(1-F(c^N+g,y^T)) + \vartheta(g) + \beta\mathbb{E}[\mathcal{V}(B',s')|s] \tag{PP}$$

subject to

$$\begin{aligned}
p^T y^T - c^T + Q(B',s)B' - B &= 0, &(26)\\
u_T c^T - \gamma u_T p^T (F_N/F_T) - v_\ell F(c^N+g,y^T) + \beta\mathbb{E}\left[u'_N c^{N\prime}|P,s\right] &= 0, &(27)\\
u_N - u_T p^T (F_N/F_T) &\geq 0. &(28)
\end{aligned}$$

The constraints in the government's problem correspond to the balance of payment, (4), the government budget constraint, (19), and the non-negativity constraint, (16). Note that the expectation term in the government budget constraint is conditioned on the current state being repayment ($\mathcal{I} = P$); hence, the relevant transition probabilities are $\mathcal{P}(B',s')$ for repayment and $1-\mathcal{P}(B',s')$ for default, for all (B',s').

3.3.2 Default

The problem of the government in the default state is

$$\max_{(c^N,c^T,y^T,g)} u(c^N,c^T) + v(1-F(c^N+g,y^T)) + \vartheta(g) + \beta\mathbb{E}[\delta\mathcal{V}(B^d,s') + (1-\delta)V^D(s')|s] \tag{DP}$$

subject to

$$
\begin{aligned}
p^T y^T - c^T &= 0, \qquad (29)\\
u_T c^T - \gamma u_T p^T (F_N/F_T) - v_\ell F(c^N + g, y^T) + \beta \mathbb{E}[u_N' c^{N\prime} | D, s] &= 0, \qquad (30)\\
u_N - u_T p^T (F_N/F_T) &\geq 0. \qquad (31)
\end{aligned}
$$

and where total factor productivity, embedded in the cost function $F(y^N, y^T)$, is reduced by a default penalty, $\Omega(s)$.

In this case, note that the balance of payments is simply the trade balance, as the government is excluded from international credit markets. The expectation term in the government budget constraint is conditioned on the current state being default ($\mathcal{I} = D$); hence, the relevant transition probabilities are $\delta \mathcal{P}(B^d, s')$ for repayment and $1 - \delta \mathcal{P}(B^d, s')$ for default, for all s'.

4 Domestic policy and debt choice

4.1 Domestic policy

We begin by characterizing domestic policy, i.e., the government's choice for μ, τ and g. Given the formulation of the government's problem using the primal approach, we begin with the choices for the allocations (c^N, c^T, y^T, g), given state (B, s), repayment status $\mathcal{I} = \{P, D\}$ and a debt policy $\mathcal{B}(B)$.

To simplify some of the notation below, let $\Gamma(c^N, c^T, y^T, g; s) \equiv u_T p^T (F_N/F_T)$, which is an expression that shows up in the government budget and non-negativity constraints. Note that $\Gamma_T = d\Gamma/dc^T = \Gamma \times (u_{TT}/u_T) < 0$, while the convexity of F implies that $\Gamma_N = \Gamma_g = \Gamma \times (F_{NN}/F_N - F_{NT}/F_T) > 0$ and $\Gamma_y = \Gamma \times (F_{NT}/F_N - F_{TT}/F_T) < 0$. Also define $\Phi \equiv v_\ell - v_{\ell\ell} F(c^N + g, y^T) > 0$.

Since the problems in repayment and default are functionally identical with respect to (c^N, c^T, y^T, g), we focus on (PP)–(28). Let ξ, λ and ζ be the Lagrange multipliers associated with the constraints (26), (27) and (28), respectively. The necessary first-order conditions with respect to (c^N, c^T, y^T, g) are

$$\begin{aligned}
u_N - v_\ell F_N - \lambda(F_N\Phi + \gamma\Gamma_N) + \zeta(u_{NN} - \Gamma_N) &= 0, & (32)\\
u_T - \xi + \lambda(u_T + u_{TT}c^T - \gamma\Gamma_T) - \zeta\Gamma_T &= 0, & (33)\\
-v_\ell F_T + \xi p^T - \lambda(F_T\Phi + \gamma\Gamma_y) - \zeta\Gamma_y &= 0, & (34)\\
-v_\ell F_N + \vartheta_g - \lambda(F_N\Phi + \gamma\Gamma_g) - \zeta\Gamma_g &= 0. & (35)
\end{aligned}$$

Our first result relates to the provision of public goods.

Proposition 1. *If $\zeta = 0$ then $u_N = \vartheta_g$; if $\zeta > 0$ then $u_N > \vartheta_g$.*

Proof. See Appendix A.3. □

When the non-negativity constraint (28) is slack, i.e., $\zeta = 0$, the provision of public goods is optimal, in the sense that the marginal utilities of non-tradable goods consumption is equal to the marginal utility of public good consumption. Recall that the public good is transformed on-to-one from the non-tradable good. In contrast, when the non-negativity constraint (28) binds, i.e., $\zeta > 0$, there is an over-provision of public goods. In this case, the government is at a corner and can no longer use monetary policy to increase the consumption of non-tradable goods.

We now characterize conditions under which the solution is interior, i.e., $\zeta = 0$ and so (28) does no bind. Thus, suppose $\zeta = 0$ and focus first on the case with no transfers, $\gamma = 0$. Using (32) and (34) to solve for the Lagrange multipliers we obtain

$$\begin{aligned}
\lambda &= \frac{u_N - v_\ell F_N}{F_N\Phi},\\
\xi &= \frac{u_N F_T}{p^T F_N},
\end{aligned}$$

and so (33) implies

$$F_T\Phi\left(\frac{u_N}{u_T} - \frac{p_T F_N}{F_T}\right) = p^T(u_N - v_\ell F_N)\left(1 + \frac{u_{TT}c^T}{u_T}\right). \tag{36}$$

The term in brackets in the left-hand side is non-negative by (28) and so both sides of (36) are non-negative. The derivation of (36) relied on $\zeta = 0$ and we now verify under which conditions this assumption holds.

Proposition 2. *Assume that $\gamma = 0$. Then, $\zeta = 0$, and so the non-negativity constraint* (28) *is* *slack if and only if* $\frac{-u_{TT}c^T}{u_T} \leq 1$.

Proof. See Appendix A.3. $\square$

If $\frac{-u_{TT}c^T}{u_T} < 1$, then (28) is satisfied with strict inequality; when $\frac{-u_{TT}c^T}{u_T} = 1$, then (28) is satisfied with equality but still slack. In both of these cases, $\zeta = 0$. In contrast, when $\frac{-u_{TT}c^T}{u_T} > 1$, then (28) binds and so, $\zeta > 0$. When (28) is satisfied with strict inequality, the marginal rate of substitution between non-tradables and tradables is not equal to their marginal rate of transformation.

Why does only the curvature of the utility from the tradable good matter? The key is the cash-in-advance constraint, which relates the consumption of non-tradables to real balances. Suppose instead there was no cash-in-advance constraint (and hence, no fiat money circulating in the economy) and that the government taxed the consumption of non-tradable goods, say at rate τ^N. In this case, a standard result is that if $\frac{u_{NN}c^N}{u_N} = \frac{u_{TT}c^T}{u_T}$ then $\tau^N = 0$. The government would tax labor income, but not distort the relative consumption of the two goods. If instead $\frac{u_{NN}c^N}{u_N} \neq \frac{u_{TT}c^T}{u_T}$, then τ^N would adjust appropriately to account for the different elasticities of substitution.

When transfers are positive, $\gamma > 0$, the analysis is more involved (see Appendix A.2). We use the knife-edge case, $\gamma = 0$ and $\frac{-u_{TT}c^T}{u_T} = 1$ which implies (28) is slack and satisfied with equality, to understand the impact of transfers.

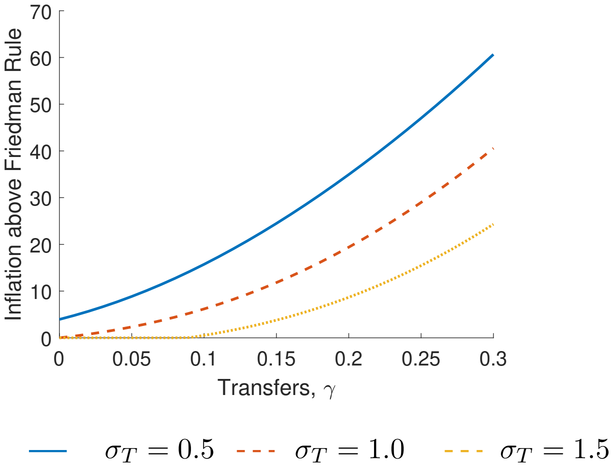
Proposition 3. *Assume that* $\gamma > 0$. *(i) If* $\frac{-u_{TT}c^T}{u_T} \leq 1$ *then, the non-negativity constraint* (28) *is satisfied with strict inequality. (ii)* $\zeta = 0$ *and* (28) *is satisfied with equality for some* $\frac{-u_{TT}c^T}{u_T} > 1$.

Proof. See Appendix A.3. $\square$

When $\gamma > 0$ and $\frac{-u_{TT}c^T}{u_T} \leq 1$ the non-negativity constraint (28) is satisfied with strict inequality, which implies that the cash-in-advance constraint binds. By continuity, (28) will now also be slack for some range when $\frac{-u_{TT}c^T}{u_T} > 1$ and this range increases as γ increases. In fact, the knife-edge case, when $\zeta = 0$ and (28) is satisfied with equality, now happens at some $\frac{-u_{TT}c^T}{u_T} > 1$. In other words, positive transfers enlarge the set of primitives consistent with a binding cash-in-advance constraint.

Figure 2 provides a visual representation of these results. We now assume preferences with a constant elasticity-of-substitution; in particular, $\frac{-u_{TT}c^T}{u_T} = \sigma_T$. When $\sigma_T < 1$, (28) is satisfied

Figure 2: Transfers and Inflation



with strict inequality and so, the cash-in-advance constraint binds. Thus, monetary policy is inefficient and inflation is above optimal. This inefficiency worsens as transfers γ increase. When $\sigma_T = 1$, (28) is slack and equal to zero for $\gamma = 0$ but is satisfied with strict inequality for $\gamma > 0$; thus, inflation increases in γ, as monetary policy becomes progressively more inefficient. When $\sigma_T > 1$, (28) binds for γ small enough, but eventually becomes slack and then inflation starts increasing with γ.

4.2 Debt policy

We now characterize debt choice in the event the government decides to repay its inherited debt. The necessary first-order condition of problem (PP) with respect to B' is

$$\beta \frac{\partial \mathbb{E}[\mathcal{V}(B', s')|s]}{\partial B'} + \xi \frac{[\partial Q(B', s)B']}{\partial B'} + \lambda \beta \frac{\partial \mathbb{E}\left[u'_N c^{N\prime} | P, s\right]}{\partial B'} = 0. \tag{37}$$

As reflected by the three terms in (37), debt choice affects the continuation value for the government and how tightly the balance of payment and government budget constraints bind. We can further characterize this equation using some of the expressions derived above. The envelope condition of problem (PP) implies $\frac{\partial V^P(B,s)}{\partial B} = -\xi$. Hence, using (21)–(25), we obtain

$$\begin{aligned} \frac{\partial \mathbb{E}[\mathcal{V}(B', s')|s]}{\partial B'} &= -\mathbb{E}[\mathcal{P}(B', s')\xi' | s] \\ \frac{\partial Q(B', s)B'}{\partial B'} &= \mathbb{E}\left\{ \frac{\mathcal{P}(B', s')}{1+r} \left[1 - \frac{(1-\mathcal{P}(B', s'))(B' - Q^d(s')B^d)\xi'}{\kappa} \right] \Big| s \right\}. \end{aligned}$$

The last term in (37) requires a bit more work. Given that $\mathcal{P}(B', s')$ is the probability of transitioning from $\mathcal{I} = P$ to $\mathcal{I}' = P$ for all (B', s'), we can write

$$\mathbb{E}\left[u'_N c^{N\prime}|P, s\right] = \mathbb{E}\left[\mathcal{P}(B', s')u'_N \mathcal{C}^{N\prime} + (1 - \mathcal{P}(B', s'))\bar{u}'_N \bar{\mathcal{C}}^{N\prime}|s\right],$$

where $u'_N \mathcal{C}^{N\prime}$ corresponds to the repayment state tomorrow, $\mathcal{I}' = P$, and $\bar{u}'_N \bar{\mathcal{C}}^{N\prime}$ corresponds to the default state tomorrow, $\mathcal{I}' = D$. Note that the expectation on the right-hand side (only conditional on s) is taken with respect to s'. We can take the derivative of the expression above with respect to B' to obtain

$$\frac{\partial \mathbb{E}\left[u'_N c^{N\prime}|P, s\right]}{\partial B'} = \mathbb{E}\left[\mathcal{P}(B', s')(u'_N + u'_{NN}\mathcal{C}^{N\prime})\mathcal{C}^{N\prime}_B + (u'_N \mathcal{C}^{N\prime} - \bar{u}'_N \bar{\mathcal{C}}^{N\prime})\mathcal{P}'_B|s\right],$$

where $\mathcal{C}^{N\prime}_B$ and $\mathcal{P}'_B$ denote the derivatives of $\mathcal{C}^N(B', s')$ and $\mathcal{P}(B', s')$ with respect to B'. Recall that, when in default, allocations are not a function of B, i.e., $\bar{C}^N(s)$ and so $\bar{C}^{N\prime}_B = 0$. From (21), we have an analytical expression for $\mathcal{P}'_B$ and from the envelope condition, $\frac{\partial V^P(B,s)}{\partial B} = -\xi$. Thus, we obtain

$$\frac{\partial \mathbb{E}\left[u'_N c^{N\prime}|P, s\right]}{\partial B'} = \mathbb{E}\left\{\mathcal{P}(B', s')\left[(u'_N + u'_{NN}\mathcal{C}^{N\prime})\mathcal{C}^{N\prime}_B - \frac{(u'_N \mathcal{C}^{N\prime} - \bar{u}'_N \bar{\mathcal{C}}^{N\prime})(1 - \mathcal{P}(B', s'))\xi'}{\kappa}\right]\Big|s\right\}.$$

We can now write the equation characterizing debt choice as

$$\begin{aligned}&\mathbb{E}\left\{\mathcal{P}(B', s')\left[\left(\frac{\xi}{1+r} - \beta\xi'\right) - \frac{\xi(1 - \mathcal{P}(B', s'))(B' - Q^d(s')B^d)\xi'}{\kappa(1+r)}\right]\Big|s\right\}\\ &+ \lambda\beta\mathbb{E}\left\{\mathcal{P}(B', s')\left[(u'_N + u'_{NN}\mathcal{C}^{N\prime})\mathcal{C}^{N\prime}_B - \frac{(u'_N \mathcal{C}^{N\prime} - \bar{u}'_N \bar{\mathcal{C}}^{N\prime})(1 - \mathcal{P}(B', s'))\xi'}{\kappa}\right]\Big|s\right\} = 0. \quad (38)\end{aligned}$$

Note that if there are no aggregate shocks (other than the extreme value shocks, ε), then $\mathcal{P}(B', s')$ only depends on B' and is always positive; hence, the term $\mathcal{P}(B', s')$ that multiplies all the expressions in square brackets in (38) can be eliminated.

The Generalized Euler Equation (38) highlights three channels in the government's debt decision. The first channel, $\mathbb{E}\{\mathcal{P}(B', s')[(1+r)^{-1}\xi - \beta\xi']|s\}$, corresponds to distortion smoothing: debt allows the government to trade off intertemporally how tightly the balance of payments binds. The distortion-smoothing term has an intrinsic bias since the government is relatively impatient as $\beta(1 + r) < 1$. In other words, this term would not be zero if the government kept expected distortions constant over time, i.e., set $\xi = E[\xi'|s]$. This relative impatience motivates debt accumulation in the sovereign default literature.

The second channel, $-[\kappa(1 + r)]^{-1}\xi\mathbb{E}\left\{\mathcal{P}(B', s')\left[(1 - \mathcal{P}(B', s'))(B' - Q^d(s')B^d)\xi'\right]|s\right\}$, captures the default premium: more debt leads to a higher probability of default and, hence, a higher interest rate. The default-premium term reflects the added financial cost due to default risk. This term, as is standard in the literature, moderates debt accumulation. $B_d > 0$ counters this effect, as lenders take into account that they partially recover their loan after a default event.

The third channel, $\lambda\beta\mathbb{E}\{\mathcal{P}(B', s')[(u'_N + u'_{NN}\mathcal{C}^{N\prime})\mathcal{C}_B^{N\prime} - (u'_N\mathcal{C}^{N\prime} - \bar{u}'_N\bar{\mathcal{C}}^{N\prime})(1-\mathcal{P}(B', s'))(\xi'/\kappa)]|s\}$, reflects an intertemporal tradeoff due to the fact that fiscal and monetary policies are distortionary. Higher debt leads to a change in policies tomorrow, which affects the demand for money today and, hence, how tightly the current government budget constraint binds.

The second and third channels described above reflect time-consistency problems in the government's problem due, respectively, to the possibility of default and the use of distortionary domestic policy instruments. In both cases, the government tomorrow does not internalize the effects of its actions on the tradeoffs faced by the government today—*bygones are bygones*. The default channel is central to standard sovereign default models, but the distortionary-policies channel is absent. Below, we analyze this latter channel in more detail.

4.2.1 Intertemporal tradeoff due to distortionary policies

There are two parts in the distortionary-policies term in (38), and we will analyze each in turn.

We follow Martin (2011) to interpret the expression $\mathcal{P}(B', s')(u'_N + u'_{NN}\mathcal{C}^{N\prime})\mathcal{C}_B^{N\prime}$. The envelope condition from the household's problem implies $V_m = u_N/p^N$ (see derivation in Appendix A.1). When using equilibrium condition (13) we obtain $V_m = u_N\mathcal{C}^N$, which states the equilibrium value of entering the period with an additional unit of domestic currency. We can further establish how this value changes with debt: $dV_m/dB = (u_N + u_{NN}\mathcal{C}^N)\mathcal{C}_B^N$. Hence, first part of the distortionary-policies term in (38) reflects how a change in debt directly affects the demand for money and, therefore, how tightly the government budget constraint binds. Note that this first part of the channel is multiplied by $\mathcal{P}(B', s')$, implying that it only operates within repayment states today and tomorrow (recall that $\bar{\mathcal{C}}_B^N = 0$).

The level of debt affects the level of implemented distortions since the policy instruments available to the government are distortionary; these expected distortions, in turn, affect the demand for money. Two opposing forces determine how higher debt and distortions affect the demand for money in equilibrium: a substitution effect and an income effect. The substitution effect dictates that larger distortions should lead to lower consumption and lower demand for money to finance non-tradable goods. The income effect induces households to want to mitigate the drop in consumption due to larger distortions; hence, it increases the demand for money. Which effect dominates depends on the curvature of u, more specifically, the sign of $u_N + u_{NN}c^N$. If this term is positive (negative), the substitution (income) effect dominates.

The second part of the term is the expression $(u'_N\mathcal{C}^{N\prime} - \bar{u}'_N\bar{\mathcal{C}}^{N\prime})\mathcal{P}'_B$, where $\mathcal{P}'_B = -\mathcal{P}(B', s')(1 - \mathcal{P}(B', s'))(\xi'/\kappa) < 0$. As explained above, $V_m = u_N c^N$; hence, this term reflects the impact of debt choice on the current money demand, through the change in the repayment probability. As the government issues more debt, it lowers the probability of repayment, $\mathcal{P}'_B < 0$. This matters to domestic households since the value of an extra unit of money depends on whether the government repays or defaults, as policies (and distortions) are different in each case. Again, the sign of $u'_N\mathcal{C}^{N\prime} - \bar{u}'_N\bar{\mathcal{C}}^{N\prime}$ depends on the curvature of u. Assuming that $\mathcal{C}^{N\prime} > \bar{\mathcal{C}}^{N\prime}$ (an assumption we verify numerically), the difference is positive (negative) if the substitution (income) effect dominates.

So, how does the distortionary-policies channel operate? First, issuing more debt today alters future fiscal and monetary policies in the repayment state, as well as the probability of repayment. Second, since domestic policy instruments are distortionary, anticipated changes in these future policies alter the marginal value of money tomorrow and, hence, households' current money-holding decisions. Third, the change in future repayment probability also alters the expected marginal value of money tomorrow, as policies differ if the government repays or defaults. Fourth, these changes in the current demand for money affect the real value of domestic government liabilities and hence, the government's budget constraint. Importantly, this effect is not internalized by the government tomorrow, which results in a time-consistency problem, as the government values current debt issuance differently today and tomorrow.

The distortionary-policies channel alters how the other two components in (38) are traded off when the government decides how much debt to issue. The effect may be positive, zero, or negative, depending on the assumptions on preferences, thus altering the standard tradeoff in sovereign debt choice. For example, if the utility is logarithmic, then $u_N + u_{NN}\mathcal{C}^N = u'_N\mathcal{C}^{N\prime} - \bar{u}'_N\bar{\mathcal{C}}^{N\prime} = 0$, and so there is no time-consistency problem due to the interplay between debt policy and the demand for money. In this case, the government would trade off its relative impatience with the default risk premium; i.e., the desire to accumulate debt is moderated by the extra financial cost of supporting it due to the higher default probability. Suppose instead that $u_N + u_{NN}\mathcal{C}^N < 0$, which also implies $u'_N\mathcal{C}^{N\prime} - \bar{u}'_N\bar{\mathcal{C}}^{N\prime} < 0$, as argued above. Given that $\mathcal{C}^N_B < 0$ (since higher debt implies larger distortions and, hence, lower consumption) and $\mathcal{P}^N_B < 0$ (as shown above), the distortionary-policies term would now be positive, countering the default premium term and reinforcing the relative impatience term. That is, when the income effect dominates the substitution effect in the preference for the non-tradable good (and the demand for money), the distortionary-policies channel provides additional incentives to accumulate debt. In the opposite case, $u_N + u_{NN}\mathcal{C}^N > 0$, which implies $u'_N\mathcal{C}^{N\prime} - \bar{u}'_N\bar{\mathcal{C}}^{N\prime} > 0$, the distortionary-policies channel mitigates the incentive to accumulate debt.

4.3 Comparison to real models of sovereign default

Here, we show that our setting encompasses the celebrated Eaton and Gersovitz (1981) economy. Suppose that lump sum, unconstrained taxes $\mathcal{T}$ are available. In such a case, the government budget constraint (3) becomes $p^N(g+\gamma) + eB \leq \mathcal{T} + \tau wh + eqB'$. Proceeding as we did to derive (19), lump sum taxes can be written in terms of allocations as follows:

$$u_T[c^T - \gamma p^T(F_N/F_T)] - v_\ell F(c^N + g, y^T) + \beta\mathbb{E}\left[u'_N c^{N\prime}|\mathcal{I}, s\right] + \mathcal{T} \geq 0 \tag{39}$$

for $\mathcal{I} = \{P, D\}$. The problem of the government in the repayment state is (PP) subject to (26), (28) and (39) for $\mathcal{I} = P$. Similarly, when in default, the problem of the government is (DP) subject to (29), (31) and (39) for $\mathcal{I} = D$.

Consider now the following centralized version of the government's problem, where the only constraint is the balance of payments. In the repayment state, the problem of the government is

$$\max_{(B', c^N, c^T, y^T, g)} \; u(c^N, c^T) + v(1 - F(c^N + g, y^T)) + \vartheta(g) + \beta\mathbb{E}[\mathcal{V}(B', s')|s] \tag{PPEP}$$

subject to $p^T y^T - c^T + Q(B', s)B' - B = 0$. The problem of the government in the default state is defined similarly. The solution to problem (PPEP), denoted $(\hat{B}', \hat{c}^N, \hat{c}^T, \hat{y}^T, \hat{g})$, will be referred to as the *EG real allocation.* The associated lump-sum taxes necessary to finance this allocation are given by

$$\hat{\mathcal{T}} = -\hat{u}_T\hat{c}^T + \gamma\hat{u}_T p^T(\hat{F}_N/\hat{F}_T) + v_\ell F(\hat{c}^N + \hat{g}, \hat{y}^T) - \beta\mathbb{E}\left[\hat{u}'_N\hat{c}^{N\prime}|P, s\right] \tag{40}$$

We now establish the following equivalence result between the two problems.

Proposition 4. *Given lump sum, unconstrained taxes* $\hat{\mathcal{T}}$ *given by* (40)*, the EG real allocation* $(\hat{B}', \hat{c}^N, \hat{c}^T, \hat{y}^T, \hat{g})$ *solves the problem* (PP)*, with the corresponding value of default,* $V^D(s)$*, that solves* (DP)*, and the constraints* (28)*,* (31) *and* (39)*, for* $\mathcal{I} = \{P, D\}$*, are slack.*

Proof. See Appendix A.3. ☐

This result implies that when lump-sum taxes are available, the EG real allocation can be decentralized as a competitive equilibrium as follows. The government finds it optimal to (i) set the distortionary tax rate τ equal to zero and (ii) conduct monetary policy μ so that the cash-in-advance constraint in the household's problem does not bind, i.e., implement the Friedman rule given by

$$\hat{\mu} = \frac{\beta \mathbb{E}\left[\hat{u}'_N \hat{c}^{N\prime} | B, \mathcal{I}, s\right]}{\hat{u}_N \hat{c}^N} - 1. \tag{41}$$

Lump-sum taxes adjust so that under these policies the government budget constraint is satisfied with no need for distortions. Hence, in this version of the model, the government budget constraint is no longer a restriction in the government's problem. In effect, the policy regime becomes Ricardian.

We can write the analog of the Generalized Euler Equation (38) for the case with unconstrained lump-sum taxes as follows:

$$\mathbb{E}\left\{\mathcal{P}(B', s')\left[\frac{u_T}{1+r} - \beta u'_T - \frac{u_T B'(1-\mathcal{P}(B', s'))u'_T}{\kappa(1+r)}\Big| s\right]\right\} = 0. \tag{42}$$

In this case, the multiplier of the balance of payment constraint, ξ, is equal to u_T.[5] We can see that, with lump-sum taxes, the government trades off distortion-smoothing (plus relative impatience) and the default premium. The intermporal tradeoff due to distortionary policies is absent since the government budget constraint is automatically satisfied with lump-sum taxes and, thus, is no longer a binding restriction to policy implementation.

We now argue that the EG real allocation cannot be decentralized in the absence of lump-sum taxes, i.e., cannot be implemented if only distortionary policy instruments are available. To show this result, we focus on the case when $\hat{\mu} \leq 0$, which is isomorphic to requiring a non-negative implicit real interest rate on a domestic bond denominated in non-tradable goods.

Proposition 5. *Suppose that $\hat{\mu}$, given by* (41)*, is non-positive. If lump-sum, unconstrained taxes are not available, then there are no feasible monetary and fiscal policies that decentralize the EG real allocation.*

Proof. See Appendix A.3. ☐

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[5] Note that with distortionary taxes, ξ is not equal to u_T in general; from (33), this would require either the government budget constraint to be slack, i.e., $\lambda = 0$, or $\lambda(u_T + u_{TT}c^T) - (\gamma\lambda + \zeta)\Gamma_T = 0$. This last case obtains when $u_T + u_{TT}c^T = \gamma = 0$, while it cannot happen when $u_T + u_{TT}c_T > 0$ and may be possible when $u_T + u_{TT}c^T < 0$.

To grasp the idea behind this result, observe that in order to implement the EG real allocation, the government budget constraint must be $\hat{e}[\hat{Q}(B', s)B' - B] = \hat{p}^N(\hat{g} + \gamma) - \hat{\mu}$. Since $\hat{g} + \gamma \geq 0$ and implementing the EG real allocation requires $\hat{\mu} \leq 0$, the left-hand side of this expression is strictly positive; hence, the government would need to run a Ponzi scheme, with $B' > B$. This would lead to $\hat{Q}(B', s) = 0$ in finite time and, thus, a contradiction.

5 An economy with unanticipated aggregate shocks

In this section, we study the behavior of key variables in a version of our economy with no expected aggregate shocks other than the extreme value shocks, ε. We start by calibrating and analyzing an economy with no shocks, i.e., when $s = \bar{s} \in S$ in all periods. Then, we study the effects of unanticipated shocks followed by perfect foresight dynamics. Most of the insights derived from this analysis will carry over to an economy with expected shocks, such as the one we study Section 6.

5.1 Functional forms

The utility functions for consumption and leisure are, respectively,

$$\begin{aligned} u(c^N, c^T) &= \alpha^N \frac{\left(c^N\right)^{1-\sigma^N}}{1-\sigma^N} + \alpha^T \frac{\left(c^T\right)^{1-\sigma^T}}{1-\sigma^T}, \\ v(\ell) &= \alpha^H \frac{\ell^{1-\varphi}}{1-\varphi}. \end{aligned}$$

We let $\sigma^N = \sigma^T = \sigma$, which implies that $1/\sigma$ represents both the intratemporal elasticity of substitution between c^N and c^T and the intertemporal elasticity of substitution.

The utility function for the public good is

$$\vartheta(g) = \alpha^G \ln g,$$

which is a standard representation in the optimal taxation literature and close to empirical estimates.[6]

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[6] A more general representation with constant relative risk aversion, $\alpha^G(g^{1-\nu} - 1)/(1-\nu)$, converges to log utility as ν approaches 1. Nieh and Ho (2006) estimate values of ν around 0.8. Azzimonti et al. (2016), among others, use log utility for the public good. See also the discussion in Debortoli and Nunes (2013).

The function describing the labor requirement for production is

$$F(y^N, y^T) = \frac{\left[(y^N)^\rho + (y^T)^\rho\right]^{1/\rho}}{A},$$

where ρ determines how costly it is to change the composition of y^N and y^T that is produced, in terms of labor units, and A is a measure of labor productivity. Below, we will analyze the response of the economy to shocks to A.

Finally, we assume that the economy experiences a drop in productivity when the government is in default. Following the work of Arellano (2008), we allow this penalty to vary with the state of the economy. Productivity, while in the default state, takes the following form:

$$A^{def} = A - \Omega(s) \tag{43}$$

with

$$\Omega(s) = \max\left\{\omega_1 + \omega_2 \frac{(s - \bar{s})}{\bar{s}}, 0\right\}, \tag{44}$$

where $\omega_1 > 0$ is the intercept and the slope parameter ω_2 makes the default cost a function of the shock, s. The value of the shock $\bar{s}$ represents the steady-state value of the shock s. Note that when the economy has no shocks ($s = \bar{s}$), as the economy we study in this section, the cost of default is determined only by ω_1.

5.2 Calibration

Table 1 shows the values of the parameters set externally. The risk-free interest rate is set at an annual 3%, in line with the average real interest rate of the world since 1985 in King and Low (2014). We calibrate the value of φ to 1.50 so that the Frisch elasticity is one-half on average.[7] Considering the duration of a default episode from Das et al. (2012) and the length of exclusion after restructuring from Cruces and Trebesch (2013), we choose an expected period of exclusion after a default of 6 years, which implies $\delta = 1/6$.[8]

We set $\sigma_N = \sigma_T = 0.5$. As shown in the previous section, $\sigma_T < 1$ is a sufficient condition for the non-negativity constraint in the government's problem to be satisfied with strict inequality (it is also necessary when transfers γ are zero). In addition, $\sigma_N < 1$ implies the distortionary-policies channel has a negative sign, mitigating the incentives to accumulate debt. This choice

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[7] We can calibrate this parameter externally because we target the value of h.

[8] Here, the duration of exclusion is exogenous. The country reenters with no debt after exclusion. See Dvorkin et al. (2021) for a model of endogenous restructuring.

Table 1: Exogenous parameters

Parameter	Description	Value	Basis
r	risk-free rate	0.03	long-run average
φ	curvature of leisure	1.50	Frisch elasticity
δ	reentry probability	0.17	exclusion duration
α^T	preference share for c^T	1.00	normalization
σ^N	curvature of c^N	0.50	see appendix B.3
σ^T	curvature of c^T	0.50	see appendix B.3
ρ	elasticity of substitution between y^N and y^T	1.50	see appendix B.3
p^T	terms of trade	1.00	normalization

implies that imported goods are gross substitutes for non-tradable goods, as in the estimates of Ostry and Reinhart (1992).[9]

The value of ρ, which determines the elasticity of substitution between y^N and y^T in the cost function, is set to 1.5. A number larger than 1 guarantees that F is convex and, thus, ensures that the production possibilities frontier is concave.

In Appendix B.3 we support our choices of σ and ρ by studying the reaction of macroeconomic variables to shocks to the terms of trade. Though we find that many results are robust to the choice of parameter values, the reaction of spreads, inflation, output and exports favor our benchmark calibration.

Finally, p^T, which denotes the terms of trade, is set to 1. Our calibration strategy, described below, is such that all external variables are pinned down by other parameters. As such, any value of p^T delivers the same observables, except for the exchange rate, which we do not target. In Section 6 we allow the terms of trade to evolve stochastically and calibrate the stochastic process for $\ln p_t^T$ to match the data.

The remaining parameters are calibrated *jointly* to match a set of long-run averages. We use data collected by the World Bank for Argentina, Brazil, Chile, Colombia, Mexico, Peru and Uruguay for the period 1991–2018 because of data availability. A significant fraction of the parameters can be calibrated in this simplified setup without shocks, reducing the time it takes to calibrate the model and allowing the model to fit the targets exactly. We will mention the critical parameter that reproduces each moment as we explain the choice of targets for exposition. Table 7 in Appendix B.2 shows the marginal reaction of moments to parameters.[10]

We set A so that the steady state real GDP is equal to 1, making some statistics easier to

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[9] However, the estimates in Ostry and Reinhart (1992) are in the range of 1.22-1.27 and our calibration implies an elasticity equal to 2. In Appendix B.3, we study how our results change when setting $\sigma^N = \sigma^T = 1.5$, which implies that the goods are complements with an elasticity of 0.66.

[10] The corresponding expressions for each target are presented in Appendix B.1.

Table 2: Calibration parameters and targets

Parameter	Value	Statistic	Target/Non-stochastic Model
A	1.4575	Real GDP	1.000
β	0.8675	Inflation, %	3.800
γ	0.1082	Transfers/GDP	0.117
α^N	2.6888	Exports/GDP	0.209
α^H	0.9265	Employment/Population	0.587
α^G	0.4240	Gov. Consumption/GDP	0.133
B^d	0.1854	Debt/GDP	0.185
ω_1	0.0228	Haircut, Share of Debt	0.305
κ	0.0235	Default, %	0.700

read. The value of the discount factor β helps the model produce an annual 3.8% inflation rate. The parameter γ matches the ratio of transfers to GDP, which in the data average 11.7%. The value of α^H allows the model to hit the long-run average for the employment-to-population ratio, 0.59. The weight in the utility of the government consumption good, α^G, delivers government consumption over GDP of 13.3%. The parameter α^N takes a value that allows the model to reproduce the ratio of exports to GDP, which is 21% in the data.

The values of ω_1 and B^d determine the costs and benefits of default, respectively. Thus, they are used to reproduce the implied haircut obtained by the country in default, which is 30.5% (Dvorkin, Sánchez, Sapriza, and Yurdagul, 2021), and the external debt-to-GDP ratio, which is 18.5% in our data set.

The scale parameter in the distribution of taste shocks, κ, determines the risk of sovereign default in the steady state and is calibrated to reproduce a default rate of 0.7% annual. We choose a default rate target that is lower than the more typical 2% since in this version of the model, default only occurs due to the extreme value "non-fundamental" shocks.[11]

5.3 Equilibrium policy

Figure 3 shows equilibrium policies in the repayment and default states as functions of beginning-of-period debt. These policy functions are computed using the benchmark calibration from Table 2.[12]

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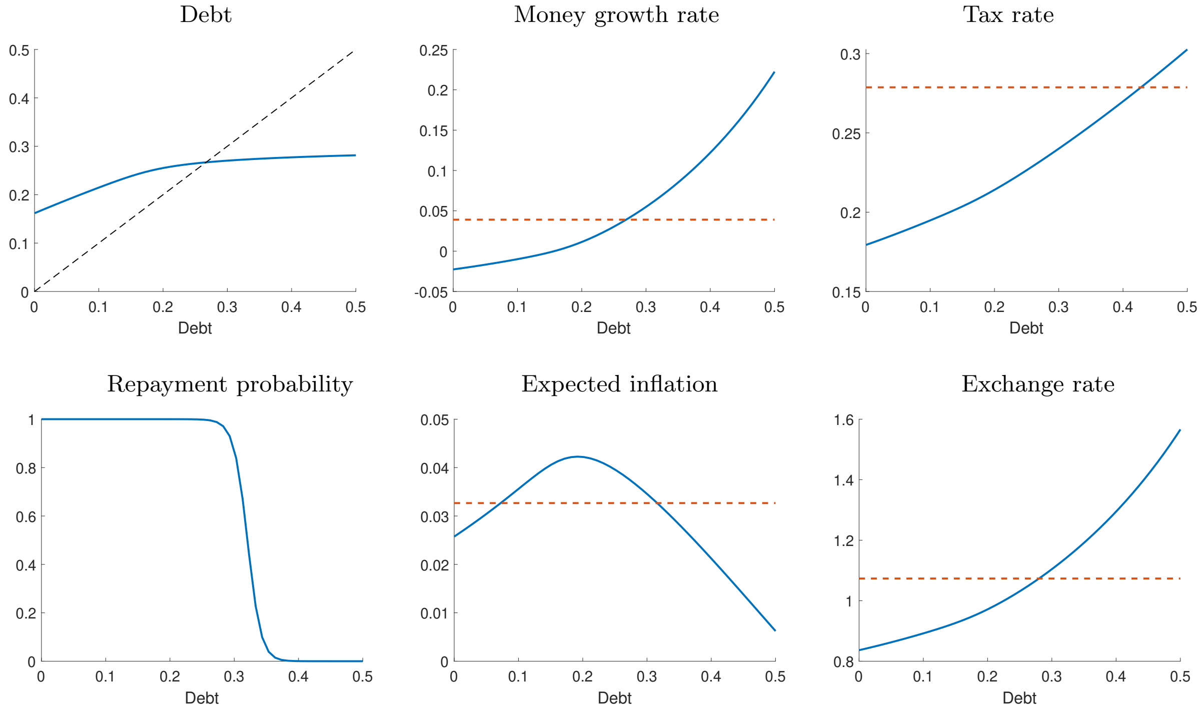
[11] See the numbers calculated by Tomz and Wright (2013) for different sets of countries. Alternatively, we could target the average EMBI for these countries. Matching the average for this period, about 300 basis points, would require a higher value of κ and a smaller value of β. That calibration, which we also experimented with, yields similar results.

[12] The equilibrium is solved globally using the equations derived above. The algorithm uses 100 equally spaced gridpoints between 0 and 1 for debt. Note that using as few as 10 gridpoints results in visually indistinguishable (interpolated) equilibrium functions. Figure 3 shows debt up to 0.5 since the probability of repayment is essentially

Figure 3: Equilibrium policies as functions of debt



Note: Policy functions in repay are solid lines, while in default are the dashed ones.

As we can see from Figure 3, conditional on repayment, debt choice, the money growth rate, the tax rate and the exchange rate are all increasing in debt. In contrast, the repayment probability is decreasing in debt as the incentives to default become larger. Higher debt implies a higher interest rate due to a lower repayment probability and so requires larger distortions to finance it. Hence, domestic policy involves faster money printing and higher taxation as debt grows.

Policy remains constant while the country is in default. The difference between the policy functions in default and repayment is informative. For example, if the country starts with $b = 0.4$ (roughly 50% above the steady state) and the government decides to repay its debt, which is extremely unlikely, the money growth rate is about 12%; in contrast, if it decides to default, the money growth rate declines to 4%.

Conditional on repaying today, expected inflation between today and tomorrow (expected, since there are two possible states tomorrow, repayment or default) is non-monotonic in debt. It is increasing for low values of debt and decreasing for higher values of debt. Note that inflation, measured as the increase in the price level of consumption goods, has two components:

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zero for higher values. We use cubic splines to interpolate the evaluation of future policies and their derivatives. As a reference of precision, the sum of squared residuals for the Generalized Euler Equation is $7e^{-21}$. See also Appendix B.3 for a comparison with alternative calibrations and Appendix B.4 for the policy functions of the model with anticipated shocks.

non-tradable and imported. The expected inflation in nontradable goods increases with debt because the money growth rate increases with debt. However, expected inflation in imported goods is decreasing in debt for large debt values. This pattern occurs because conditional on repayment, an increasing proportion of debt must be repaid, resulting in exchange rate appreciation. Combining these two forces results in the non-monotonicity of overall expected inflation as a function of debt.[13]

In terms of allocations, Figure 13 in Appendix B.3 shows that in the benchmark economy, c^N, c^T, g, y^N and h are decreasing in debt, while y^T is increasing in debt. Again this follows from higher debt implying larger domestic policy distortions (higher μ and τ) and a higher exchange rate (which discourages imports and promotes exports).

5.4 Dynamics after an unexpected shock

We now study dynamics to further illustrate the model's inner working and understand the impact of different shocks. We consider the effect of an unexpected shock at $t = 0$; the shock is then assumed to gradually die out following a known process that ends at date $t = T$.[14] Hence, once the process for a particular parameter is determined, the model is solved backwards from $t = T$ to $t = 0$, assuming that at $t = T + 1$ the economy returns to normal. We then simulate the path of the economy starting from its steady state at $t = -1$ and assuming that the extreme value shocks are small enough to not trigger default. For a given parameter x, we assume an after-shock value, x_0, and then a process $x_t = \bar{x} + \rho_x(x_{t-1} - \bar{x})$, where $\bar{x}$ corresponds to its pre-shock (steady state) value (see Table 2).

We consider adverse shocks to the (log of) terms of trade p^T and productivity A. The sizes of the shocks were chosen such that all of them imply a similar impact on inflation.[15] For the persistence, we set $\rho_p = 0.8803$, as estimated in Appendix C.2, and $\rho_A = 0.9$ so that both shocks have similar duration. In all cases, we assume $\omega_{2,x} = 0$ so that the simulations do not include the added impact of changing the cost of default due to an adverse shock. Figure 4 shows the simulated path for selected variables in response to the three alternative shocks described above.

All shocks considered imply similar dynamics: real output falls, while debt over GDP, inflation, depreciation, spreads and the primary deficit increase.[16] We can see that shocks to

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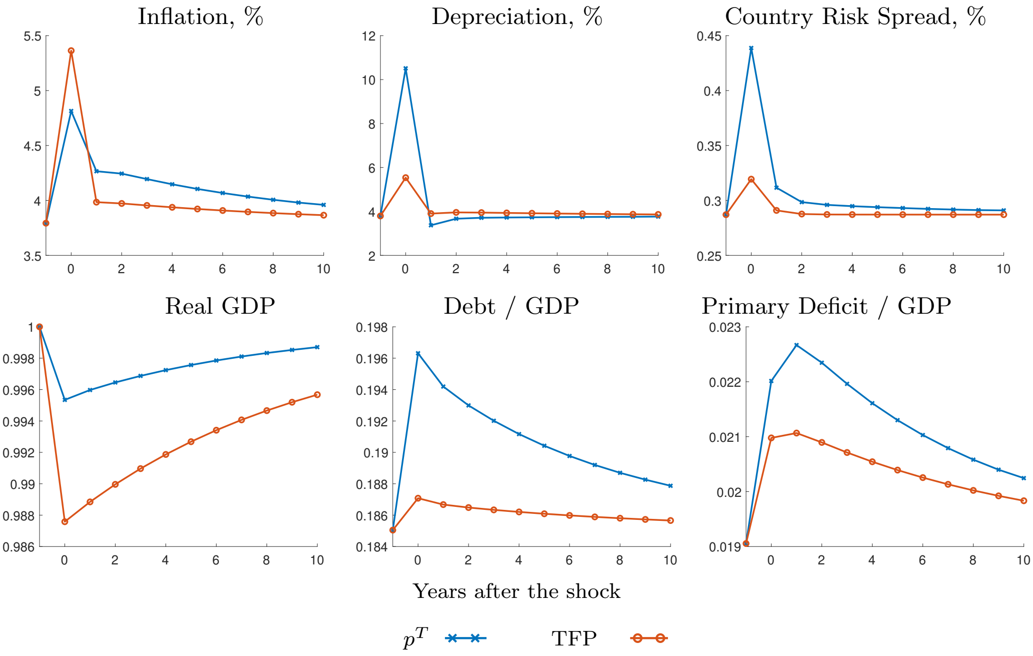
[13] A similar result is obtained if we instead define inflation as the increase in the GDP deflator.

[14] In later sections, we analyze the economy assuming these shocks are fully anticipated.

[15] We set $\ln p_0 = -0.0756$ and $A_0 = \bar{A} \times 0.99$.

[16] We define spreads as the yield of the bond maturing this period, given the level of debt, B, and the realization of shocks, s, but before the realization of the extreme value shocks ε, and minus the yield of a risk-free bond. That is, $\text{Spread}(B, s) = \left(\mathcal{P}(B, s) + (1 - \mathcal{P}(B, s))\, \frac{B^d}{B} Q^d(s)\right)^{-1} - 1$.

Figure 4: Dynamics following unexpected adverse shocks



the terms of trade have a significant impact on currency depreciation, spreads, and debt over GDP (and this last one mainly through the effects of the depreciation). Shocks to productivity affect mainly real GDP and inflation.

It is worth noting that, though debt over GDP increases, debt in dollars actually decreases very slightly in response to these shocks. If we allowed for $\omega_2 \neq 0$ (as we do in Section 6), then there would be an additional reason to lower the debt in response to adverse shocks, as the incentives to default and, hence, the spreads would increase further.[17] In the next section we incorporate these shocks, one at a time, and compute two full models of sovereign default that are compared directly with data on emerging markets.

6 Models with anticipated aggregate shocks

This section analyzes a two versions of our economy with shocks that evolve stochastically over time. One has shocks to terms of trade and the other to TFP. We start from the benchmark calibration of the previous section and modify it so that the models with anticipated shocks predicts a higher default rate and fits the targeted levels of debt and haircuts. Then, we show how model-simulated data compare with the actual data from the seven countries used in the calibration.

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[17] For this mechanism to work we would need $\omega_{2,p} > 0$ and $\omega_{2,A} > 0$.

6.1 Recalibration

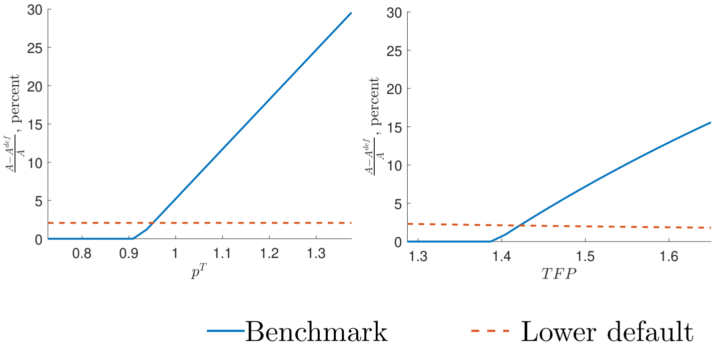
The calibration of these two models uses the same parameters as in Table 2 except for two: the value of debt after default, B^d, and the value of the cost of default, ω_1. The value of debt after default, B^d, is so that the models reproduce the same targeted median haircuts. Likewise, the new value of ω_1 is set to obtain the same Debt/GDP ratio. Table 3 presents the results.

Table 3: Recalibrated and new parameters and targets

Parameter	Model, shocks to		Statistic	Target	Model, shocks to	
	p^T	TFP			p^T	TFP
B^d	0.149	0.160	Debt/GDP	0.185	0.173	0.169
ω_1	0.087	0.068	Haircut/Debt	0.305	0.257	0.230
ω_2	0.955	1.450	Default, %	2.000	2.140	2.010
ρ_s	0.880	0.863	Estimation details in Appendix C.2.			
σ_s	0.076	0.031	Estimation details in Appendix C.2.			

The third parameter calibrated is ω_2, which from (44) determines how the cost of default changes as the economy deteriorates. This parameter played no role in the non-stochastic economy but is critical for determining the economy's default rate with aggregate shocks. We set is so that the model replicates a default rate of 2%. The blue lines in Figure 5 display the total cost of default as a function of the shocks. With the choice of ω_2, this cost is zero for some value below the steady state. Since that point the cost of default increases linearly with the value of the shock.

Figure 5: Cost of default in terms of reduction in TFP



Model with TOT shocks

Model with TFP shocks

We assume the terms of trade follow the process $\ln(p^T_{t+1}) = \rho_p \ln(p^T_t) + \varepsilon_{t+1}$, where $\varepsilon \sim N(0, \sigma_p^2)$. We assume TFP also follows the same AR1 process and we estimated the values of ρ_{tfp} and σ_{tfp}. The values used and presented in Table 3 are the averages from estimations of these processes for each of the seven countries we use for our calibration. We also show a series for commodity prices yield similar results than our terms of trade process.[18]

Figures 15 and 16 in the appendix present the policy functions for the model with p^T shocks.

6.2 Alternative specifications

In the next sections, we will compare our result with two alternative models to understand what drivers the results. Recall that our benchmark model has two key component: (1) domestic policies and (2) sovereign default. These two alternative models will be used to evaluate the importance of each of these components for the results.

Model with lump-sum taxation: This is a model in which the only difference with our benchmark is that we allow for unconstrained lump-sum taxation as in Proposition 4. In that framework, the government does not need to use distortionary policy instruments. Thus, it will be useful to show which findings derived from or benchmark model are obtained from adding distortionary fiscal and monetary policies.

Model with lower sovereign default risk: This model is exactly our benchmark model but recalibrated to have significantly lower sovereign default risk. We change the default cost function such that the government is not forced to reduce debt in bad times. The new cost of default is displayed by the dashed red lines in Figure 5. The key difference with the benchmark calibration is that the cost of default is independent of the value of the shock.[19]

To illustrate how this economy is recalibrated to have "lower default risk" works, Figure 6 shows the relationship between spreads and the amount of debt issued changes after a shock to terms-of-trade shocks—a similar behavior is observed for TFP shocks. The left panel shows the benchmark calibration. The blue line is before the shock and the dashed red line is after the shock. The dots mark the level of debt chosen by the government in each case. Note that after the shock hits, this functions moves upwards, and the government must reduce debt (even at spreads of 500 basis point it would not be able to obtained 15.5% of the previous period GDP). The is the key mechanism at work in models of sovereign default in the tradition of Arellano (2008).

The right panel shows our economy with "lower default risk'.' Given the same shock, the relationship between spreads and debt moves much less, so the size of the deleveraging is much

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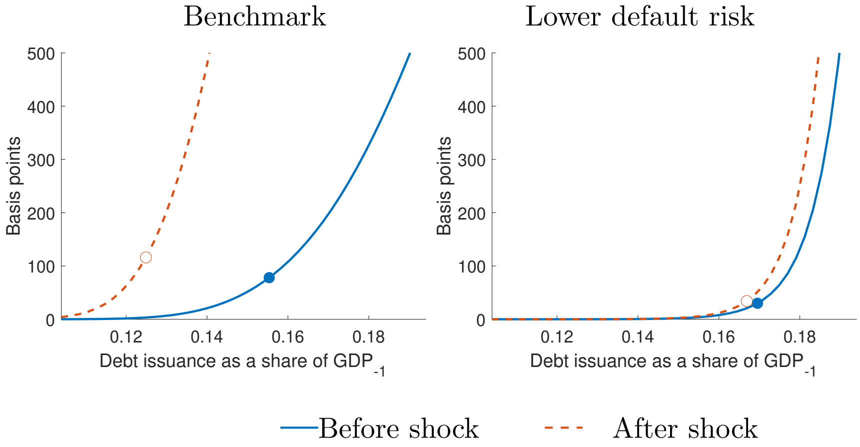
[18] More details on Appendix C.2.

[19] The recalibration of the model is such that the level of default is the same than in the benchmark economy.

Figure 6: Spreads as a function of debt, benchmark vs lower default



smaller. Recall that in this case the cost of default is independent of the state of the economy. This calibration of the default cost, which is closer Aguiar and Gopinath (2006), would be useful to understand which of the results in our benchmark model are driven by the sovereign default risk.

7 Inflation and Currency Depreciation

In this section, we focus on the dynamics of inflation during debt crises. First, we follow the same methodology in the data and in the model to select a crisis episode. An episode of sovereign debt crisis is identified with a sudden increase in sovereign debt spreads. We follow the definition of "aggregate bond spreads are unusually large" in Calvo et al. (2006b) that considers spikes on spreads exceeding two standard deviations above the prevailing sample mean. Table 4 shows the both in the data and the model there is significantly higher inflation during debt crisis. Inflation increased by 6.8 percentage points in the year after the crisis on average in all the episodes since 1997. When the sample is restricted to Latin America, which is closer to the countries used in the calibration of our model, inflation also increased after debt crises but by 5.1 percentage points. The, we follow the same procedure in the data simulated from the model. We found similar patterns but different magnitudes for the two shocks. In the model with terms-of-trade shocks, the increase in inflation after a debt crisis was 4.4 percent points, and in the model with TFP shocks it was 10.1 percentage points.

The last column of Table 4 presents the results for nominal currency depreciation around these episodes. We find that in the data there is a significant depreciation during sovereign debt crises. Note that this depreciation is larger than inflation both in the data and in the model, suggesting a real depreciation during the year of the crisis. The deference between depreciation and inflation is particularly large in the model with term-of-trade shocks, which should not be surprising given that this shock induces a real depreciation. Finally, note that debt crises in the data are a combination of potentially multiple shocks, such as terms of trade and/or productivity shocks, among others. In the next subsection, we further analyze the differences in the dynamics after specific terms of trade or productivity shocks.

Table 4: Change in inflation during crisis year

	Mean change in	
	Inflation rate	Nominal depreciation
All countries since 1997	6.83	8.60
Latin America since 1997	5.09	6.98
Benchmark model, TOT shocks	4.43	16.83
Benchmark model, TFP shocks	10.12	13.04

Details about the construction of this event study are in the appendix.

Next, we compare the reaction of inflation to a shock to the terms of trade (top right plot in Figure 7), which is useful to test our assumptions for the inclusion of money in the model. The results suggest that a 10% fall in the terms of trade in the data implies an increase in inflation of slightly less than 2 percentage points.[20] The model's reaction is initially more considerable than in the data, close to 3 percentage points in this case, and also less persistent. The similarity between the data and the model is reassuring of the modeling choice of inflation. In line with inflation, the top-right plot shows the impact on nominal devaluation. In the data and in the model, the devaluation is close to 10 percentage points in the period of the shock, with the impact being a bit larger in the model. In the following periods, the devaluation is actually negative in the model, indicating that there is an overshooting in the period of the shock.

The impact on inflation and devaluation after a TFP shock, presented in the bottom panels of Figure 7, is positive in the data and the model. But there is a clear difference in the dynamics. The estimation in the data indicates the effect lasts long (at least 3 years), while in the model after one year inflation returns to the pre-shock levels. However, note that cumulative inflation during the first 3 years is similar, as the initial response of inflation is significantly larger in the model.

Lastly, we would like to identify which components of the model are important to obtain our results. Since in the model without distortionary policy (with lump sum taxation), the cash-

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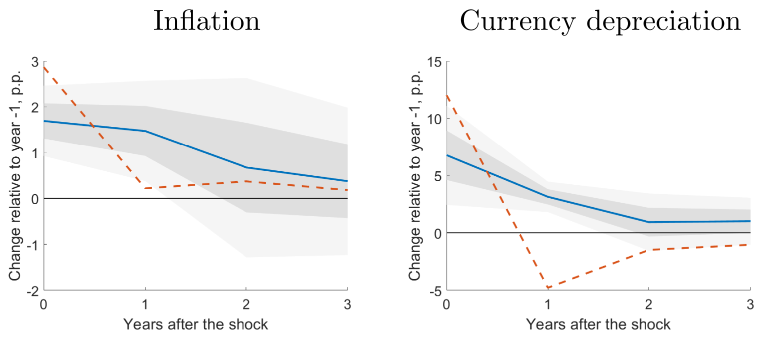
[20] We truncated inflation at 50% annual. Otherwise, hyperinflation episodes in Argentina and Brazil dominate the value of any statistic.

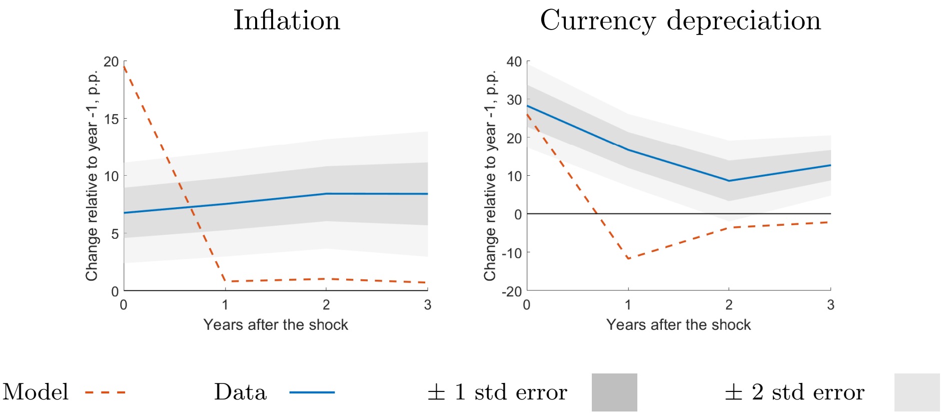
Figure 7: Effect of shocks on inflation and devaluation

Response of variables to a 10% negative p^T shock


Inflation

Currency depreciation

Response of variables to a 10% negative TFP shock


Inflation

Currency depreciation

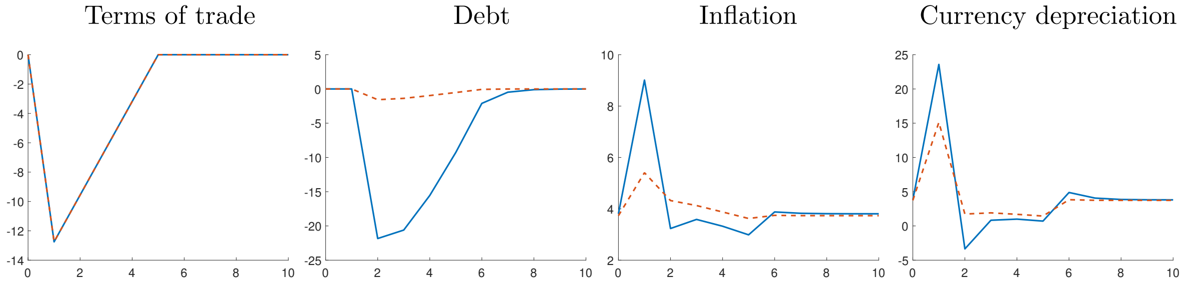
Note: See Appendix C.4 for more information on the regressions.

in-advance constraint is not binding, we do not characterize the behavior of nominal variables such as inflation or devaluation. Thus, we focus the rest of this section in the comparison with the model specification with lower risk of default.

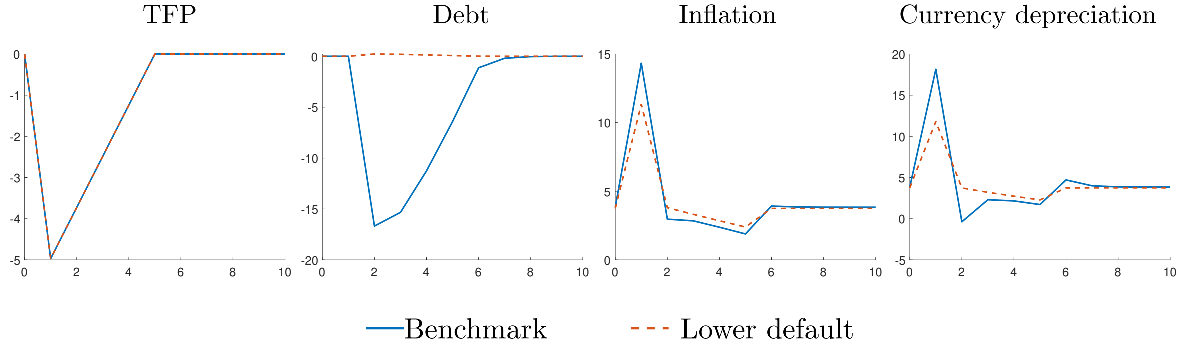
Figure 8 show the evolution of shocks, debt, inflation and nominal currency depreciation after shocks to terms of trade in the first row, and productivity in the second row. The solid blue line represents the benchmark model and the dashed-red line the model with lower default. In both cases we choose as initial debt the associated steady state level when the exogenous shock is at its mean for many periods. By construction, after the shock, in the economy with lower default risk there is a significantly smaller reduction in debt. This highlights an essential feature of sovereign default risk: countries are forced to repay debt to the rest of the world after a negative shock. The reaction of inflation and currency depreciation is significantly smaller both for terms of trade and productivity shocks. This shows that it is important to model sovereign default risk to understand the response of nominal variables such as inflation and currency depreciation.

Figure 8: Evolution of Inflation and Depreciation after a shock

Response to a p^T shock



Response to a TFP Shock



To quantify the importance of sovereign risk for nominal variables we computed the variance of inflation and currency depreciation in simulated time series with the models. In the model with p^T shocks, the low default risk model generates a variance of inflation that is only 21 percent of what it is in the benchmark model and a variance of nominal currency depreciation that it is 35 percent of what it is in the benchmark model. Thus, most of the fluctuations in inflation and depreciation in the model with p^T comes from sovereign default risk. Similar ratio are observed for the models with TFP shocks. In that case, this ratio of variances is 57 percent for inflation and 31 percent for nominal currency depreciation, again suggesting that sovereign default risk is very important for accounting for the dynamics of inflation and currency depreciation.

8 Emerging Markets Business Cycles

A standard practice is to compare key macroeconomic statistics in the model and the data. To do this comparison, we use time series for the seven countries in our sample. We detrend GDP and consumption using a band-pass filter as in Christiano and Fitzgerald (2003) to separate a time series into trend and cyclical components.[21] Table 5 shows the results for the data, the

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[21] We set the minimum and maximum periods of oscillation of cyclical component at 2 and 16 years, respectively.

benchmark model, and the two alternative models. Fluctuations in output are larger in the benchmark model with TFP shocks than in the one with term of trade shocks. Computing the ratio of the volatility in the model to the volatility in the data, and using variances instead of standard deviation such that one can add the contribution of each model, we obtain the the model with terms of trade accounts for 17% of the volatility in the data and the model with TFP shocks for 83%.[22] In the empirical literature, the percent of fluctuations in output accounted for by terms-of-trade shocks varies between about 10% to 40% depending mostly on the country and period considered (Drechsel and Tenreyro, 2017; Schmitt-Grohé and Uribe, 2018). An important feature for the relevance of terms-of-trade shocks is risk of default, since in the model with lower default risk the standard deviation of output is much smaller (0.005).

The benchmark model also generates significant variation in the trade balance-to-GDP ratio and spreads. As shown in Table 5, the standard deviation of these variables is much lower with lower risk of default, so that is the key feature to reproduce these moments.

Table 5: Business Cycle Statistics

	Data	Model with p^T shocks			Model with TFP shocks		
		bench.	lower default	lump-sum taxation	bench..	lower default	lump-sum taxation
Std. Dev. (y)	0.038	0.016	0.005	0.018	0.035	0.035	0.030
Std. Dev. (trade bal./Y)	0.035	0.017	0.003	0.016	0.015	0.002	0.013
Std. Dev. (spreads)	3.923	3.303	0.112	3.415	2.315	0.034	3.194
Std. Dev. (exports/Y)	0.052	0.021	0.015	0.022	0.015	0.004	0.015
Correlation(trade bal./Y, y)	-0.357	-0.177	-0.360	0.249	-0.492	-0.061	-0.304
Correlation(spreads,y)	-0.362	-0.073	-0.602	0.058	-0.187	0.218	-0.213
Correlation(exports/Y,y)	-0.178	-0.140	0.347	0.124	-0.556	-0.371	-0.288

Note: Data for Argentina, Brazil, Chile, Colombia, Mexico, Peru and Uruguay from 1980 to 2018. Y refers to nominal GDP; c and y are the cyclical components of real consumption and real GDP per capita, respectively. See appendix C.1 for details.

Though not targeted in the calibration, the benchmark models reproduce the correlations between output and other relevant variables we observe in the data. For example, the benchmark models replicate two salient properties of emerging markets, namely, that the ratio of trade balance-to-GDP, exports-to-GDP, and bond spreads are counter-cyclical. Here, note that the model with lump-sum taxation and p^T generates counterfactual correlations. The benchmark models also reproduce the negative correlation between the ratio of and economic activity.

Next, we analyze how spreads and real GDP respond to shocks to the terms of trade, p^T, and TFP, A. Importantly, none of these responses were targeted in our calibration of the

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[22] Note these two numbers do not need to add to 100%. For example, because there may be other shocks that affect output or because these shocks may be correlated.

model. The estimation in the data uses local projections, following Jordà (2005). To compare these with the model, we perform the same estimation on simulated time series. First, the top two panels of Figure 9 presents the responses to a negative 10% shock to the terms of trade. This analysis resembles the work of Drechsel and Tenreyro (2017), who estimate the contemporaneous response of Argentina's spread to terms-of-trade shocks. They argue that it is reasonable to assume that international commodity prices are exogenous to developments in Argentina's economy. They find that "a 10 percent deviation of commodity prices from their long-run mean can move Argentina's real interest spread by almost 2 percentage points" (i.e., 200 basis points). We use a shorter time series but we include seven countries and analyze the impact not just on spreads but also on inflation, real GDP, and the nominal exchange rate. To capture the differences in the international prices that are relevant for each country, we use terms of trade instead of commodity prices.

The top left plot in Figure 9 shows the response of the EMBI spread to a terms-of-trade shock. In the data, we find that a 10% decline in the terms of trade increases the EMBI spread by about 50 basis points, an effect that persists over the next year and then declines to zero.[23] The dashed red line corresponds to the same estimation using model-simulated data. The response of the model is larger in the first period but lower in the second period. Notably, the overall response has the same sign and a similar magnitude.

The top-right panel of Figure 9 displays the impact of a terms-of-trade shock on real GDP. Among other things, this comparison helps validate our choices for the curvature of the utility function $u(c^N, c^T)$ and the elasticity of substitution between y^T and y^N in the production cost function. In the data, the effect is significantly different from zero but has large standard errors. On average, real GDP growth falls by close to 1 percentage point the year of a 10% fall in the terms of trade. Replicating such an effect in the model is challenging. Kehoe and Ruhl (2008) show that in a multi-sector model, the first-order effect of changes in the terms of trade on real GDP is zero. Our model's structure allows for a novel mechanism, as policy distortions need to increase to repay the sovereign debt when the terms of trade deteriorate. This mechanism generates a 0.5 percentage points decline the year of the shock, which is smaller than the point estimate in the data but within 2 standard errors.[24]

The bottom panels of Figure 9 present the same analysis but for TFP shocks. The overall message does not change. The impact of TFP shocks on the EMBI spread and real GDP has

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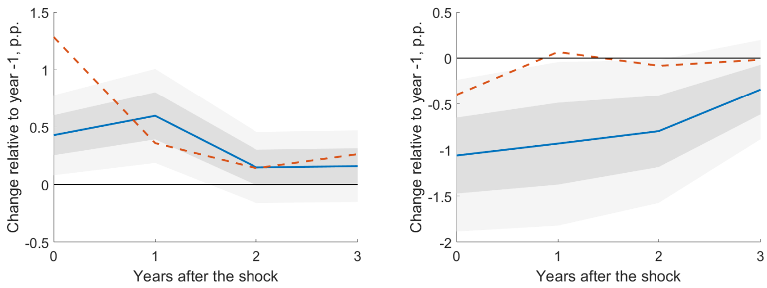
[23] Our estimates are more conservative than Drechsel and Tenreyro (2017) because we include more countries with fewer debt crises than Argentina. If we consider only Argentina the estimates are more similar.

[24] In the literature, there exist other channels to generate a larger effect of terms-of-trade shocks on output. See Kohn et al. (forthcoming).

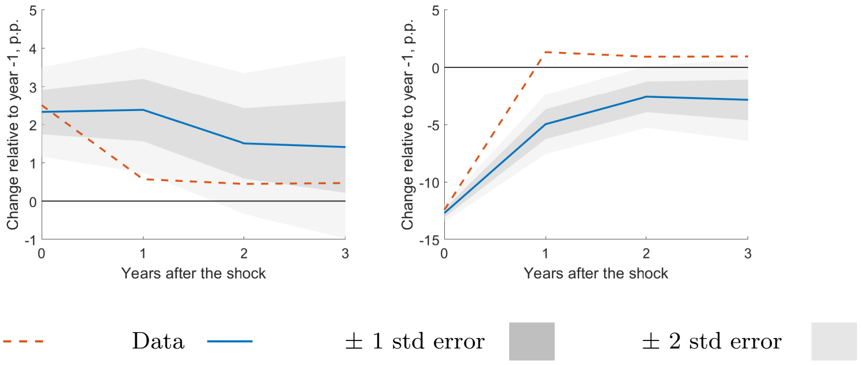
Figure 9: Effect of shocks on EMBI and GDP growth

Response of variables to a 10% negative p^T shock


EMBI

GDP growth

Response of variables to a 10% negative TFP shock


EMBI

GDP growth

Note: See Appendix C.4 for more information on the regressions.

the same sign and similar magnitude in the model and the data. However, the magnitudes are significantly larger. Note that we use a 10% in both case but such a change on TFP is more rare than in p^T. As we show in Table 10 in the appendix, the standard deviation for p^T is 7.5% and for TFP it is 3.1%. The bottom-left plot of Figure 9 shows that a 10% decline in TFP increases the EMBI spread between 200 and 300 basis points in the model and the data. This effect seems to last longer on the data than in the model, although the error bands increases as the number of years after the shock increase. The bottom-right panel of Figure 9 shows the dynamics of growth are remarkably well reproduced by the model.

9 Cyclical properties of domestic policies

This subsection asks three questions: How do monetary and fiscal policies vary over the business cycles when the government cannot commit to future policies? Does the model capture the cyclicality of policy in emerging markets? And what are the key features to shape the dynamics of domestic policies?

First, we study the cyclicality of monetary policy. Following Figure 2 in Vegh and Vuletin (2015), we compute the correlations between the cyclical components of the inflation tax and real GDP. They define the inflation tax as inflation/(1+inflation). We detrend the series using Christiano and Fitzgerald (2003), as in the previous subsection. This allows us to compare the model and the average of the data for the seven countries in our sample. The results, presented in Table 6, show that in recessions, when real output falls, the inflation tax increases both in the model and in the data. In our model with terms-of-trade shocks, this occurs because a terms of trade deterioration requires a real exchange rate depreciation that is implemented by the government with a nominal devaluation that causes inflation (mostly coming from the price of imports). Importantly, note that a similar pattern emerges with recessions induced by TFP shocks.

Table 6: Policy over the business cycle

	Data	Model, shocks	
		p^T	TFP
Corr. (real expenditure, y)	0.260	0.467	0.906
Corr. (personal income tax, y)	-0.150	-0.122	-0.440
Corr. (inflation tax, y)	-0.214	-0.526	-0.655

Note: Data are the average of Argentina, Brazil, Chile, Colombia, Mexico, Peru and Uruguay. The variable y is the cyclical component of real GDP per capita.

Second, we analyze the cyclicality of fiscal policy. In this case, we consider two variables that are government choices in our model: expenditures and labor taxes. Following Figure 1 in Vegh and Vuletin (2015), we proceed as with the inflation tax and compute the correlation between the cyclical components of real government expenditure and real GDP, both in the model and the data. Similarly, we study the behavior of taxation by looking at the cyclical components of the personal income tax rate and real GDP. The results, presented in Table 6, show that government expenditure decreases and taxes increase during bad times—the government follows “austerity policies” during recessions. This policy, which is optimal in our models because the government must repay debt to the rest of the world in “bad times,” is typical of emerging markets. Fiscal policy’s procyclicality stands in stark contrast with the behavior in developed economies, where fiscal policy is generally countercyclical (Frankel et al., 2013). For example, in Japan, the correlation between the cyclical component of output and real government expenditure was -0.2 during the same period.

The fact that the models reproduce these correlations is a valuable validation of the most significant contribution of our model: incorporating domestic policies in a model of sovereign default. Clearly, of the two features in our model, introducing domestic policies into the model is essential to be able to analyze its dynamics. The rest of this section studies how important is to include sovereign default risk to reproduce the dynamics of domestic policies. We do this by presenting the results of a simulated path for key variables after a shock.

Figure 10 shows the behavior of taxes and the money growth rate after the shock to terms of trade displayed in the top-left panel. Recall from Figure 8 that after the shock, in the economy with lower default risk there is a significantly smaller reduction in debt. The implications of this smaller deleveraging on domestic policies are very significant. As displayed in the bottom panels, taxes and money growth are much less pro-cyclical in the model with lower default risk. A very similar pattern is shown in Figure 11 for the dynamics following a TFP shock.

Figure 10: Response to a TOT Shock in a Simulated Path

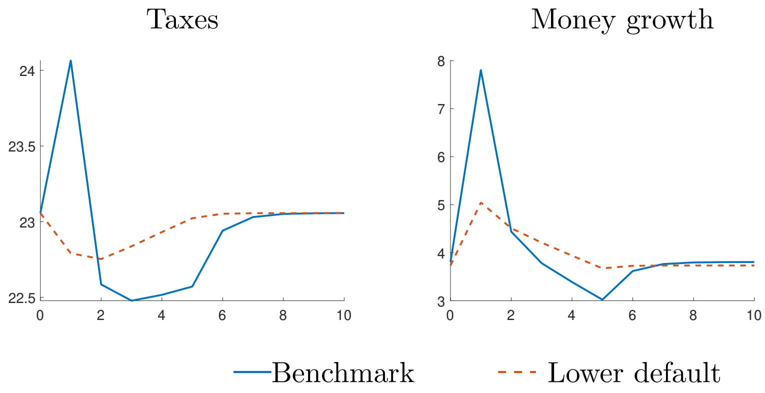
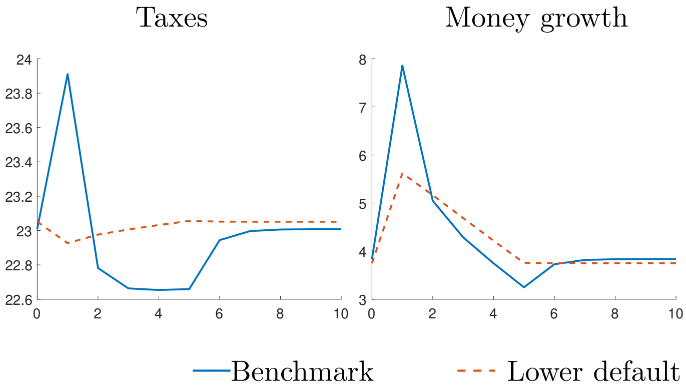


Figure 11: Response to a TFP Shock in a Simulated Path



10 Concluding remarks

Emerging economies experience recurrent debt crises, in part, due to their tendency to overborrow during good times. Their fragility to adverse shocks is likely also a consequence of inadequate economic policy frameworks; for example, a lack of fiscal discipline and an excessive reliance on seigniorage and currency depreciation.

Our paper connects domestic policies to sovereign default and economic outcomes. We modeled fiscal and monetary policies as being inherently distortionary and assumed the government lacks commitment to both external credit repayment and the conduct of its domestic policies. Our framework led to new insights on the tradeoffs faced by governments when deciding their level of indebtedness, the probability of repayment and the determination of domestic policies. We then showed that the model is able to reproduce standard business cycle statistics and the dynamic responses of policies and macroeconomic aggregate to terms-of-trade shocks in emerging markets.

In follow-up research, we use the setup developed here to analyze the fiscal and monetary policy response to the COVID-19 pandemic in emerging markets. We model the pandemic as a "perfect storm" combining four unexpected shocks and decompose their individual contribution to policy and economic outcomes.

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Appendix

A Theory

A.1 Derivations

In order to characterize the solution to the household's problem, let χ and ψ denote the Lagrange multipliers associated with constraints (8) and (2), respectively. The necessary first-order conditions with respect to (c^N, c^T, h, m') for an interior solution are

$$\begin{aligned} u_N - p^N(\chi + \psi) &= 0, & (45) \\ u_T - e\chi &= 0, & (46) \\ -v_\ell + \chi(1-\tau)w &= 0, & (47) \\ \beta\mathbb{E}\left[V_m'|\mathcal{I}, s\right] - \chi(1+\mu) &= 0, & (48) \end{aligned}$$

where V_m denotes the partial derivative of V with respect to the individual state variable, m. The corresponding envelope condition implies that $V_m = \chi + \psi$. From (45) and (46) we can solve for the Lagrange multipliers,

$$\begin{aligned} \chi &= \frac{u_T}{e}, & (49) \\ \psi &= \frac{u_N}{p^N} - \frac{u_T}{e}. & (50) \end{aligned}$$

Replacing these expressions in (47) and (48) yields (9) and (10). Using (7) to replace e in (50) and imposing $\psi \geq 0$ yields (16).

The government budget constraint

Take the government budget constraint (5), multiply both sides by $F_N c^N$ and use (12), (13), (14) and (15) to obtain

$$\tau F(y^N, y^T) + F_N(\mu c^N - g - \gamma) - (F_T/p^T)(p^T y^T - c^T) \geq 0.$$

Next, replace the tax rate, τ, using (17) and the money growth rate, μ, using (18) to obtain the government budget constraint in a competitive equilibrium,

$$[1-(F_T/p^T)(v_\ell/u_T)]F(c^N+g, y^T) - F_N(c^N+g+\gamma) + \beta(F_T/p^T)\mathbb{E}\left[u_N' c^{N\prime}|\mathcal{I}, s\right]/u_T - (F_T/p^T)(p^T y^T - c^T) \geq 0.$$

Since $F(y^N, y^T) = F_N y^N + F_T y^T = F_N(c^N + g) + F_T y^T$ we obtain

$$(F_T/p^T)\left\{c^T - (v_\ell/u_T)F(c^N + g, y^T) + \beta\mathbb{E}\left[u'_N c^{N\prime}|\mathcal{I}, s\right]/u_T\right\} - \gamma F_N \geq 0,$$

which after multiplying both sides by $u_T(p^T/F_T)$ implies (19).

A.2 Domestic policy with transfers

Using (32) and (34) to solve for the Lagrange multipliers we obtain

$$\begin{aligned}\lambda &= \frac{u_N - v_\ell F_N}{F_N\Phi + \gamma\Gamma_N},\\ \xi &= \frac{u_N F_T\Phi + \gamma[v_\ell F_T\ \Gamma_N + (u_N - v_\ell F_N)\Gamma_y]}{p^T(F_N\Phi + \gamma\Gamma_N)},\end{aligned}$$

and so (33) implies

$$F_T\Phi(u_N - \Gamma) - \gamma\left[(u_T p^T - v_\ell F_T)\Gamma_N - (u_N - v_\ell F_N)\Gamma_y\right] = p^T(u_N - v_\ell F_N)(u_T + u_{TT}c^T - \gamma\Gamma_T), \tag{51}$$

where we used the definition of Γ to simplify the expression.

A.3 Proofs

Proof of Proposition 1. Given $\Gamma_N = \Gamma_g$, (32) and (35) imply $u_N - \vartheta_g = -\zeta u_{NN} \geq 0$. If $\zeta = 0$ then $u_N = \vartheta_g$; if $\zeta > 0$ then $u_N > \vartheta_g$. □

Proof of Proposition 2. Note that (28) implies $u_N - \Gamma \geq 0$. Since $\Phi > 0$, the left-hand side of (36) is non-negative. Next, $\lambda > 0$ implies $u_N - v_\ell F_N > 0$. Hence, $u_N - \Gamma > 0$ if and only if $u_T + u_{TT}c^T > 0$, while $u_N - \Gamma = 0$ if and only if $u_T + u_{TT}c^T = 0$. If preferences are such that $u_T + u_{TT}c^T < 0$, then (36) cannot be satisfied—a contradiction. In this case, $\zeta > 0$ and therefore, (28) binds. □

Proof of Proposition 3. Suppose $\zeta = 0$. From (33) we can write $\xi = u_T + \lambda(u_T + u_{TT}c^T - \gamma\Gamma_T)$ and then rearrange (34) as follows

$$F_T(v_\ell + \lambda\Phi) - p^T u_T = p^T\lambda[u_T + u_{TT}c^T - \gamma(\Gamma_T + \Gamma_y)]. \tag{52}$$

From (32) $F_T(v_\ell + \lambda\Phi) = (u_N - \lambda\gamma\Gamma_N)(F_T/F_N)$. Thus, (52) implies

$$u_N(F_T/F_N) - p^T u_T = p^T \lambda[u_T + u_{TT}c^T + \gamma[\Gamma_N(F_T/F_N) - \Gamma_T - \Gamma_y]. \tag{53}$$

Recall that $\Gamma_N > 0$, $\Gamma_T < 0$ and $\Gamma_y < 0$. Hence, if $\frac{-u_{TT}c^T}{u_T} \leq 1$, then $u_T + u_{TT}c^T \geq 0$ and so, the right hand-side of (53) is strictly positive. Then, $u_N(F_T/F_N) - p^T u_T > 0$ and (i) follows.

For (ii), suppose (28) is satisfied with equality while $\zeta = 0$. Then (53) implies $u_T + u_{TT}c^T = -\gamma[\Gamma_N(F_T/F_N) - \Gamma_T - \Gamma_y] < 0$ and so, $\frac{-u_{TT}c^T}{u_T} > 1$. □

Proof of Proposition 4. Consider the EG real allocation $(\hat{B}', \hat{c}^N, \hat{c}^T, \hat{y}^T, \hat{g})$ that solves the problem (PPEP) where lump-sum, unconstrained taxes $\mathcal{T}$ make the government budget constraint becomes (40). In order to prove this result, we first solve the problem (PPEP) and then we construct the monetary policy and taxes $(\hat{\mu}, \hat{\tau})$ as well as the prices $(\hat{p}^N, \hat{e}, \hat{w})$ that support this allocation as an equilibrium in our setting.

The necessary first-order conditions characterizing the EG real allocation are

$$\begin{aligned} \hat{u}_N &= \hat{v}_\ell \hat{F}_N, && (54)\\ \hat{v}_\ell \hat{F}_T &= p^T \hat{u}_T, && (55)\\ \hat{\vartheta}_g &= \hat{v}_\ell \hat{F}_N, && (56) \end{aligned}$$

which imply $\frac{\hat{u}_N \hat{F}_T}{p^T} = \hat{u}_T \hat{F}_N$; i.e., the non-negative constraint (28), which we ignore to derive the EG real allocation, is satisfied with equality. The balance of payment implies

$$p^T \hat{y}^T - \hat{c}^T + \hat{Q}(B', s)\hat{B}' - \hat{B} = 0. \tag{57}$$

We now construct the policies and prices that support the EG real allocation, we have that the price of non-tradable goods and wages are determined by

$$\begin{aligned} \hat{p}^N &= \frac{1}{\hat{c}^N}, \\ \hat{w} &= \frac{\hat{p}^N}{\hat{F}_N}, \end{aligned}$$

while the exchange rate is determined by

$$\hat{e} = \frac{\hat{p}^N}{p^T} \frac{\hat{F}_T}{\hat{F}_N},$$

The monetary policy has to be tailored so that

$$\hat{\mu} = \frac{\beta \mathbb{E}\left[\hat{u}_N' \hat{c}^{N\prime} | B, \mathcal{I}, s\right]}{\hat{u}_T \hat{c}^N p^T (\hat{F}_N / \hat{F}_T)} - 1,$$

as it has to decentralize money holdings such that $m' = m = 1$. Since $\frac{\hat{u}_N \hat{F}_T}{p^T} = \hat{u}_T \hat{F}_N$, we obtain

$$\hat{\mu} = \frac{\beta \mathbb{E}\left[\hat{u}_N' \hat{c}^{N\prime} | B, \mathcal{I}, s\right]}{\hat{u}_N \hat{c}^N} - 1. \tag{58}$$

On the other hand, taxes are given by

$$\hat{\tau} = 1 - \frac{\hat{v}_\ell}{\hat{u}_T} \frac{\hat{F}_T}{p^T} = 0.$$

Finally, lump-sum transfers are designed to make the budget constraint of the government (40) hold so that

$$\hat{\mathcal{T}} = \hat{p}^N(\hat{g} + \gamma) - \hat{\mu} + \hat{e}(p^T \hat{y}^T - \hat{c}^T).$$

□

Proof of Proposition 5. As shown in Proposition 4, the real EG allocation implies zero labor taxes when monetary policy is given by (41). Thus, combining the balance of payments with the government budget constraint when lump-sum taxes are not available implies

$$\hat{e}[\hat{Q}(\hat{B}', s)\hat{B}' - B] = \hat{p}^N(\hat{g} + \gamma) - \hat{\mu}, \tag{59}$$

i.e., a non-linear first-order difference equation in domestic debt, B.

First, consider a steady state in an environment with no aggregate shocks s. Since $\hat{g} + \gamma \geq 0$ and $\hat{\mu} \leq 0$ (see (58) above), imply $\hat{e} > 0$ and $\hat{Q}(\hat{B}', s) < 1$, it follows that any steady state would require $B < 0$; i.e., the government must accumulate a sufficiently large amount of assets to finance its expenditures. This asset position would never be reached, as (59) implies that the amount of B is strictly increasing and positive when the initial stock of debt is positive.

Consider now the general case in a stochastic environment. Observe that since $\hat{p}^N(\hat{g} + \gamma) - \hat{\mu} \geq 0$, then $B > 0$ implies that $B' > 0$ as

$$\frac{\hat{B}'}{(1+r)} \geq \hat{Q}(\hat{B}', s)\hat{B}' = B + \frac{\hat{p}^N}{\hat{e}}(\hat{g} + \gamma) - \frac{\hat{\mu}}{\hat{e}}, \tag{60}$$

and so $\hat{B}' \geq (1+r)B + (1+r)\left(\frac{\hat{p}^N}{\hat{e}}(\hat{g}+\gamma) - \frac{\hat{\mu}}{\hat{e}}\right)$. Therefore, as $r > 0$, the sequence of debt for this allocation is strictly increasing and unbounded as long as $B_0 > 0$. We argue that this cannot be an equilibrium path. To see this, define $\bar{y}^T$ as the unique solution to $F(0, \bar{y}^T) = 1$, i.e., the highest level of the tradable good that can be produced as $h = 1$ and $\hat{y}^N = \hat{c}^N + \hat{g} = 0$.

Let $\bar{p^T} = \max p^T$ and conjecture that $Q(B', s) = 0$ for all $B' \geq \frac{(1+r)}{r}\bar{p^T}\bar{y}^T$ and all s. From the balance of payments, non-default tradable consumption can be written as

$$\hat{c}^T = \hat{Q}(\hat{B}', s)\hat{B}' + p^T\hat{y}^T - B \leq \max_{B'}\{\hat{Q}(\hat{B}', s)\hat{B}'\} + \bar{p}^T\bar{y}^T - B \leq \frac{1}{1+r}\frac{(1+r)}{r}\bar{p^T}\bar{y}^T + \bar{p}^T\bar{y}^T - B. \tag{61}$$

Therefore, if $B = \frac{(1+r)}{r}\bar{y}^T$, then (61) implies that non-default tradable consumption cannot be positive and leads to a contradiction, as the EG allocation is interior and consequently the outcome must be default; i.e., $Q\left(\frac{(1+r)}{r}\bar{y}^T, s\right) = 0$ for all s. Therefore, since Q is decreasing, $Q(B', s) = 0$ for all $B' \geq \frac{(1+r)}{r}\bar{y}^T$ and all s and validates the conjecture.

To conclude the proof, observe that as the sequence of debt would be strictly increasing and unbounded, it would be larger than $\frac{(1+r)}{r}\bar{y}^T$ in finite time and thus contradicts (60) since $Q(B', s) = 0$ for all $B' \geq \frac{(1+r)}{r}\bar{y}^T$ and all s . $\square$

B Quantitative Results

B.1 Definition of macroeconomic aggregates

- Nominal GDP (in pesos, normalized by the money stock),

$$Y_t = e_t p_t^T y_t^T + p_t^N y_t^N.$$

- GDP in foreign currency (USD),

$$Y_t^{USD} = p_t^T y_t^T + \frac{1}{e_t} p_t^N y_t^N.$$

- The GDP deflator (in pesos, normalized by the money stock)

$$P_t^y = \left(\frac{e p^T y^T}{Y}\right) e_t p_t^T + \left(\frac{p^N y^N}{Y}\right) p_t^N$$

- Real GDP,

$$Y_t^R = \frac{Y_t}{P_t^y}.$$

- Consumption expenditures (in pesos, normalized by the money stock),

$$C_t = e_t c_t^T + p_t^N c_t^N.$$

- Consumption price index (in pesos, normalized by the money stock),

$$P_t^c = \left(\frac{ec^T}{C}\right) e_t + \left(\frac{p^N c^N}{C}\right) p_t^N.$$

- Inflation, measured as the change in the consumption price index,

$$\pi_t = \frac{P_t^C}{P_{t-1}^C}(1+\mu_{t-1}) - 1.$$

- Currency depreciation

$$\varDelta_t = \frac{e_t}{e_{t-1}}(1+\mu_{t-1}) - 1$$

Note that inflation and currency depreciation are corrected by the money growth rate, since prices are normalized by the money stock.

B.2 Identification

To provide a heuristic proof of identification, we compute how each parameter would change if we change one target at a time by 10 percent. The results, presented in Table 7, justify the link between parameters and targets mentioned in the calibration section. The first column shows how each parameter change when we target a default rate 10 percent larger, i.e., 1.1 percent instead of 1 percent. Note that the more significant change is for κ. By increasing κ 9.68 percent and adjusting all the parameters (except ω_1) very slightly, the model can replicate all the targets perfectly. Thus, we selected κ as the critical parameter to get the default rate.

In the second column of Table 7, we present the percent change in each parameter that would allow the model to replicate a debt to GDP ratio 10 percent larger. In addition to the change in κ, which we already show is key to replicating the default rate, the most substantial change is in B_d followed by ω_1. Clearly, these parameters are important to determine debt because they determine the benefits and costs of default. We pick B_d for debt because its adjustment

Table 7: Percent change in each parameter when a target is increased by 10 percent

	Target increased by 10 percent								
	Default	Debt	Haircut	G	Hours	Exports	Inflation	Transfers	Real GDP
κ	**9.68**	8.99	6.07	-11.79	57.65	6.39	7.34	-22.34	4.88
B_d	0.00	**10.00**	-4.39	0.00	0.00	-4.13	0.00	0.00	10.00
ω_1	3.39	9.33	**5.85**	-4.95	22.31	5.95	2.74	-9.61	0.00
α_g	0.02	0.23	0.01	**11.72**	-4.22	-7.92	-0.27	3.17	4.88
α_h	0.01	0.07	0.00	-1.80	**-27.77**	-6.86	0.33	-1.52	4.88
α_n	0.02	0.23	0.01	0.53	-4.22	**-9.32**	-0.27	3.17	0.00
β	-0.01	-0.09	0.00	-1.54	4.40	1.25	**0.64**	-3.07	0.00
γ	0.00	0.00	0.00	0.00	0.00	-0.15	0.00	**10.00**	10.00
A	0.00	0.00	0.00	0.00	-9.09	-1.03	0.00	0.00	**10.00**

Note: Each number represents the percentage change in the parameter when the target is increased by 10 percent.

is larger and highlights ω_1 for matching haircuts because it is the larger adjustment to match the haircut among the remaining parameters.

Continuing with the same logic, we connect each parameter in Table 7 with a moment.

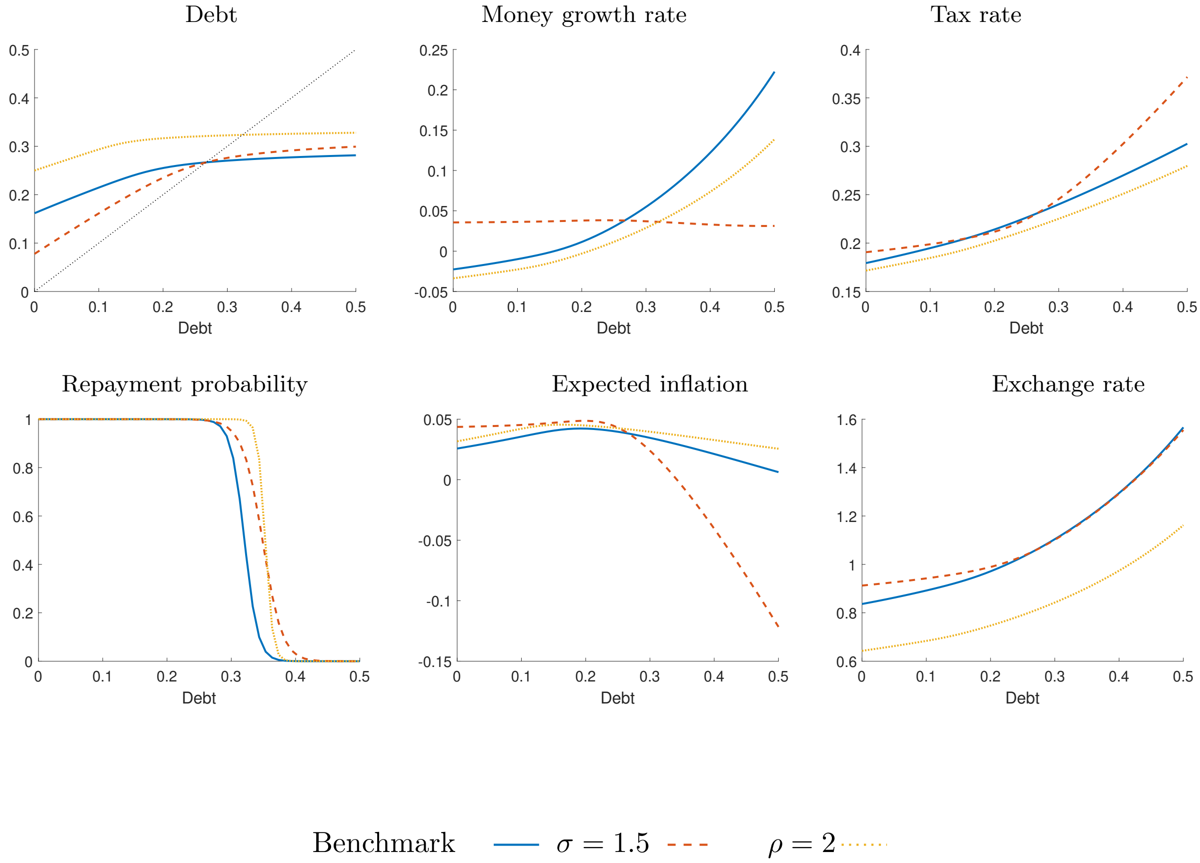
B.3 The choice of ρ and σ

This section discusses the choice of $\sigma_N = \sigma_T = 0.5$ and $\rho = 1.5$ by comparing the results for alternative parameters. In particular, we consider $\sigma_N = \sigma_T = 1.5$ and $\rho = 2$. Recall that we set $\sigma_N = \sigma_T = 0.5$ because $\sigma_T < 1$ is sufficient for the non-negativity constraint in the government's problem to be satisfied with strict inequality (it is also necessary when transfers γ are zero). The value of ρ determines the elasticity of substitution between y^N and y^T in the cost function and is set to 1.5. A number larger than 1 ensures that the production possibilities frontier is concave.

To be able to perform this comparison, we re-calibrate the model without terms-of-trade shocks twice to make sure that the model with $\sigma = 1.5$ and the one with $\rho = 2$ fit the targets well. Next, Figure 12 shows how policies and allocations change with the alternative calibrations.

The economy calibrated with $\sigma = 1.5$ shows some important qualitative differences with the benchmark economy. First, the money growth rate is decreasing in debt over the relevant range, while expected inflation takes a faster dive as debt increases. Though not shown, μ becomes increasing for sufficiently high debt, but this occurs for a range of debt that makes the government likely to default. In terms of allocations, the one critical difference is that hours is increasing rather than decreasing in debt. These differences serve to identify which calibration of the model fits the data better. More on this below.

Figure 12: Policies as functions of debt, alternative calibrations



The economy calibrated with $\rho = 2$ is qualitatively similar to the benchmark economy. Differences are quantitative, though they are calibrated to match the same steady state statistics. In this sense, varying ρ, i.e., the elasticity of substitution between nontradable output and exports in the cost function, changes how the economy behaves outside the steady state and, thus, determine how it reacts to shocks.

We now study dynamics to further illustrate the model's inner working and the differences across different calibration. In Figure 14, we consider here the reaction to unexpected shocks to terms of trade. When we consider the economy calibrated with $\sigma = 1.5$ we see important differences: real GDP and exports actually increase in response to a fall in export prices. This is a critical source of identification for which calibration is preferred; as we describe in more detail below, the behavior of output and exports favors the benchmark calibration. Another interesting difference is the behavior of inflation. In the benchmark economy, inflation increases and exhibits persistence; in the economy calibrated with $\sigma = 1.5$, inflation first increases above and then falls below the steady state, thus displaying more erratic dynamics and less persistence.

The economy calibrated with $\rho = 2$ is qualitative similar to the benchmark economy. In this case, identification of the preferred value for ρ is quantitative. As Figure 14 shows, the response

Figure 13: Equilibrium allocations when repaying as functions of debt, alternative calibrations

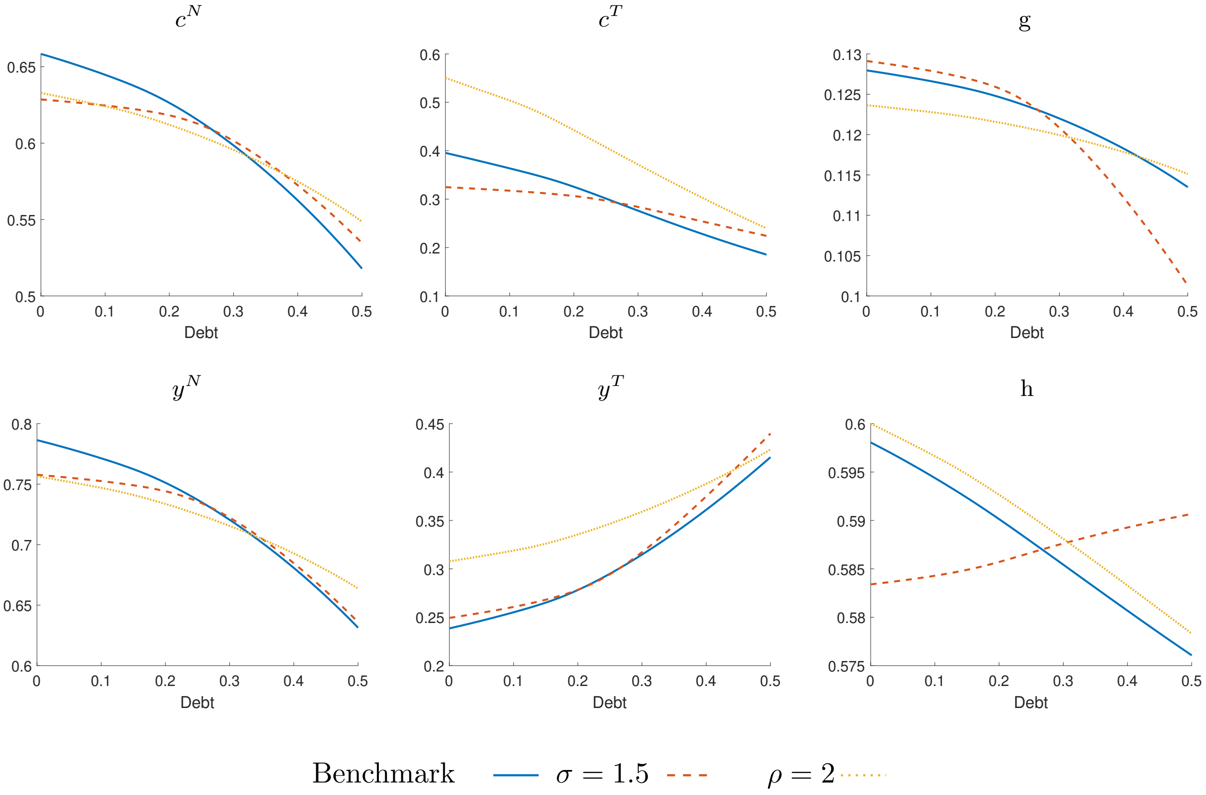
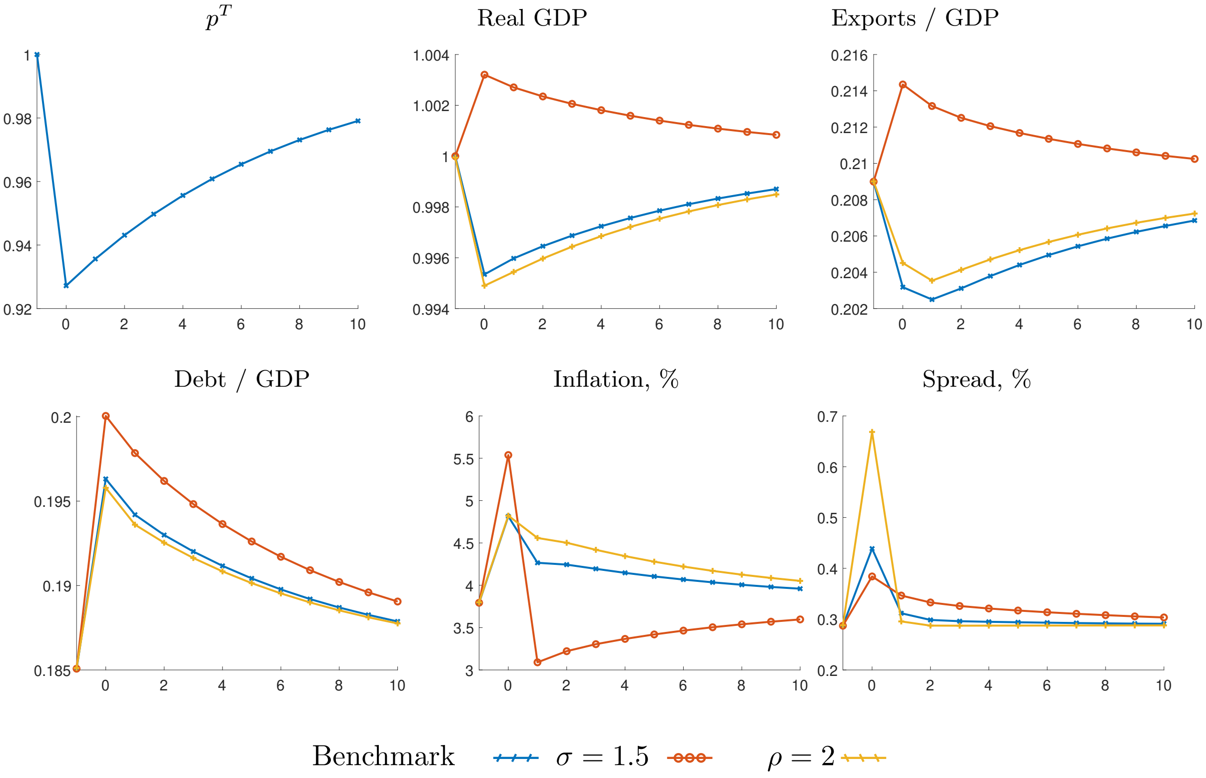


Figure 14: Dynamics following unexpected adverse shock to p^T, alternative calibrations



of the spread is much more pronounced when ρ is calibrated to a higher value.

We also re-calibrated the model with terms-of-trade shocks to evaluate the non-targeted statistics in Tables 5-6. We present all the moments in Table 8, where we added at the bottom the average absolute distance to the moments. This last statistic is revealing of how better our preferred calibration fits these moments. This measure of the fit of non-targeted moments in twice as large for the economy with $\rho = 2$ and a bit more in the economy with $\sigma = 1.5$.

In particular, we find that the economy with $\sigma = 1.5$ does a worse job fitting these moments because it generates a positive correlation of trade balance/Y with output, a negative correlation of real expenditure with output, no correlation of real consumption and real output, and almost acyclical inflation. In the case of the economy with $\rho = 2$, the most poorer fit is due to the fact that it generates a positive correlation of exports/Y with output and pro-cyclical tax rates.

Table 8: Business Cycles and Policy Statistics for Alternative Parameters

	Data	Benchmark	$\rho = 2.0$	$\sigma = 1.5$
Std. Dev. (y)	0.038	0.016	0.006	0.008
Std. Dev. (trade balance/Y)	0.035	0.017	0.008	0.007
Std. Dev. (c) / Std. Dev. (y)	1.193	2.478	3.981	1.772
Std. Dev. (spreads)	3.923	3.303	2.097	1.213
Std. Dev. (exports/Y)	0.052	0.021	0.014	0.012
Std. Dev. (depreciaton)	0.196	0.102	0.086	0.102
Correlation(trade balance/Y, y)	-0.357	-0.177	-0.288	0.275
Correlation(c,y)	0.846	0.589	0.723	-0.064
Correlation(spreads,y)	-0.362	-0.073	-0.132	0.260
Correlation(exports/Y,y)	-0.178	-0.141	0.108	0.302
Correlation(depreciation,y)	-0.252	-0.226	-0.316	0.278
Correlation(depreciation,spreads)	0.431	0.205	0.261	0.291
Corr. (inflation tax, y)	-0.214	-0.526	-0.649	0.107
Corr. (real expenditure, y)	0.260	0.467	0.709	-0.275
Corr. (personal income tax, y)	-0.171	-0.122	0.237	0.364
Average absolute distance to data	-	0.244	0.470	0.546

Note: Data is the average of the numbers Argentina, Brazil, Chile, Colombia, Mexico, Peru and Uruguay. The variable y is the cycle of GDP. The inflation tax is defined as in Vegh and Vuletin (2015): inflation/ (1 + inflation).

Finally, we analyze how our conclusions about the performance of the EG version of our model change when we recalibrate the model for different values for ρ and σ. The results are presented in Table 9. The main result is that our conclusion are robust to the values of ρ and σ. In particular, the correlations of trade balance, exports, and debt spreads with real GDP have a positive sign, contrary to what we observe in the data.

Table 9: The Role of Distortionary Policies, Alternative Parameters

	Data	Benchmark	Recalibrated EG version	
			$\rho = 2$	$\sigma = 1.5$
Std. Dev. (y)	0.038	0.016	0.007	0.008
Std. Dev. (trade balance/Y)	0.035	0.017	0.007	0.007
Std. Dev. (c) / Std. Dev. (y)	1.193	2.478	3.186	1.403
Std. Dev. (spreads)	3.923	3.303	2.394	0.958
Std. Dev. (exports/Y)	0.052	0.021	0.017	0.013
Correlation(trade balance/Y, y)	-0.357	-0.177	0.249	0.508
Correlation(c,y)	0.846	0.589	0.396	-0.249
Correlation(spreads,y)	-0.362	-0.073	0.034	0.460
Correlation(exports/Y,y)	-0.178	-0.141	0.321	0.459

B.4 Policy functions in an economy with terms-of-trade shocks

The equilibrium is solved globally, using the equations derived above. The algorithm uses 21 equally spaced gridpoints for debt, between 0 and 1.5, and 21 gridpoints for p^T estimated with the Tauchen method with a bandwidth of 2 (i.e., a multiple 2 of the unconditional standard deviation). To compute expectations we interpolate policy functions with a modified Akima piecewise cubic Hermite interpolation in a dense grid of 20001 equally spaced points in debt and 501 equally spaced points in p^T for which we estimate its corresponding transition matrix with the Tauchen method.[25] We experimented we different grid sizes, different interpolations schemes (e.g., linear), and different ways of computing the expectations (e.g., computing its corresponding integral). The final choice of grid points and methods is the most efficient allocation of computing time. For example, either computing the integral, or increasing the size of the grids, deliver the same solution but requires more computing time.

Figures 15 and 16 show some policies and allocations as function of the repayment status, debt and terms of trade. For example, the top-left figure shows the policy function of debt issuances as a function current debt (in the horizontal axis) and terms of trade (ranging from blue for lower terms of trade to yellow for higher terms of trade). The dashed lines correspond to the allocations when the economy is in autarky. Figures 17 and 18 show policies and allocations as functions of the repayment status, debt and tfp.

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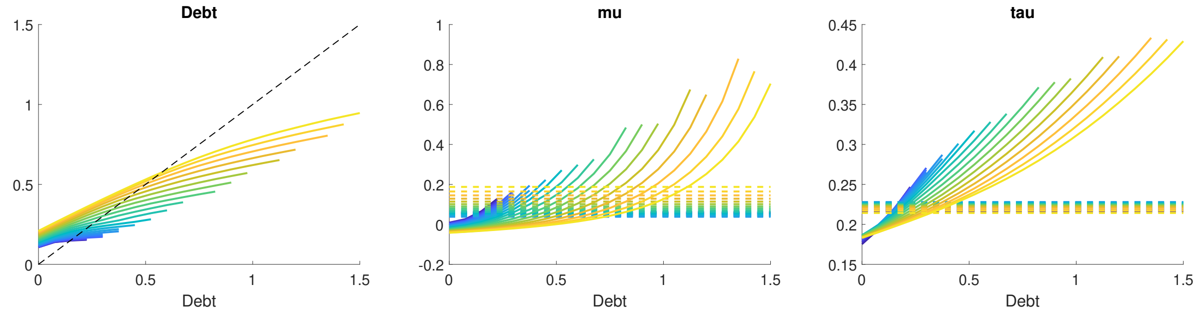
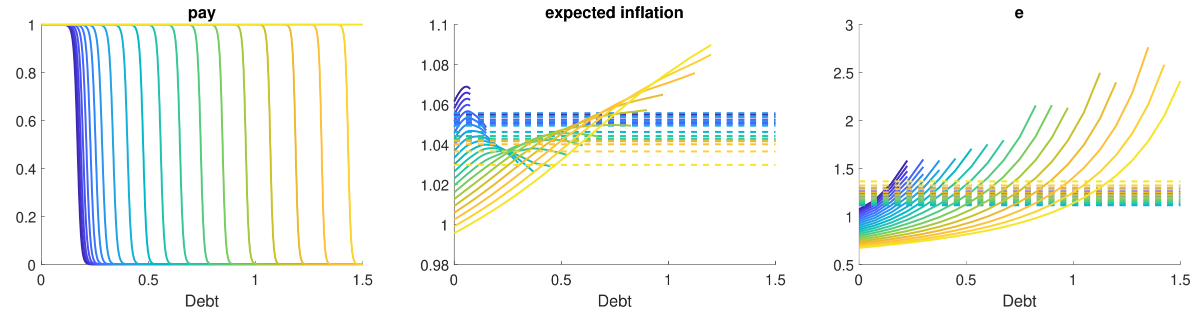
[25] The interpolated value at a query point is based on a piecewise function of polynomials with degree at most three evaluated using the values of neighboring grid points in each respective dimension. The Akima formula is modified to avoid overshoots.

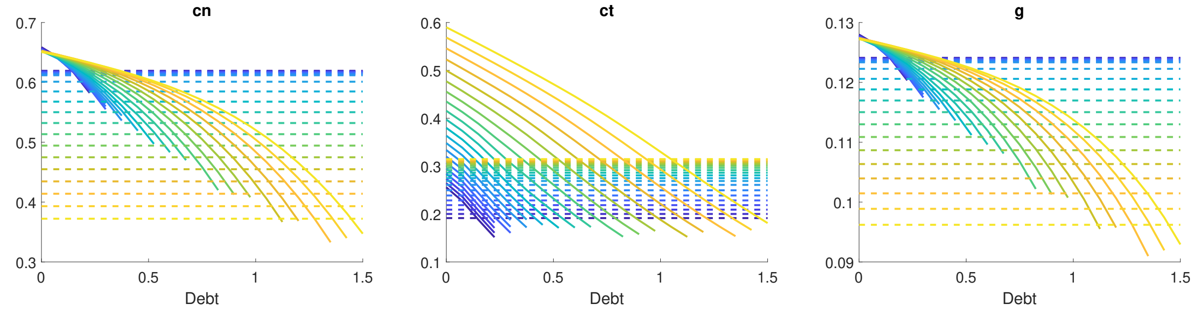
Figure 15: Policies as function of debt, p^T, and repayment status

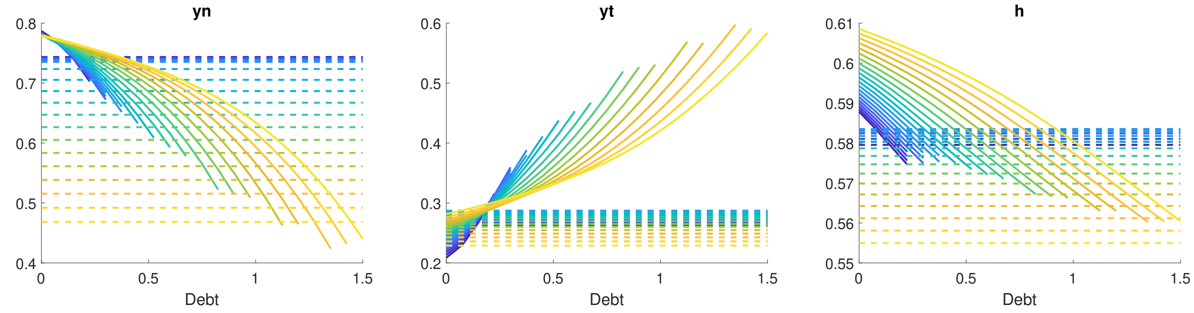

Debt Money growth rate Tax rate


Repayment probability Expected inflation Exchange rate

Note: Policy functions in repay are solid lines, while in default are the dashed ones. The different lines ranging from blue to yellow correspond to states with higher terms of trade.

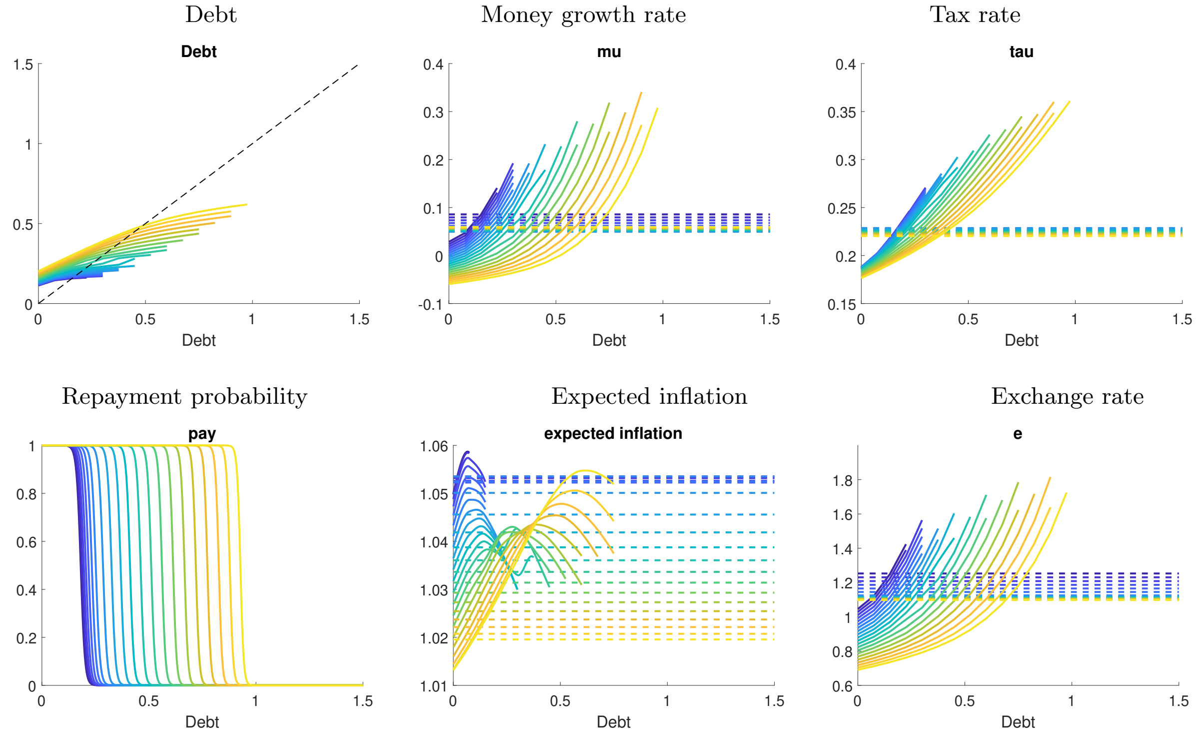
Figure 16: Equilibrium allocations as function of debt, p^T, and repayment status


c^N c^T g


y^N y^T h

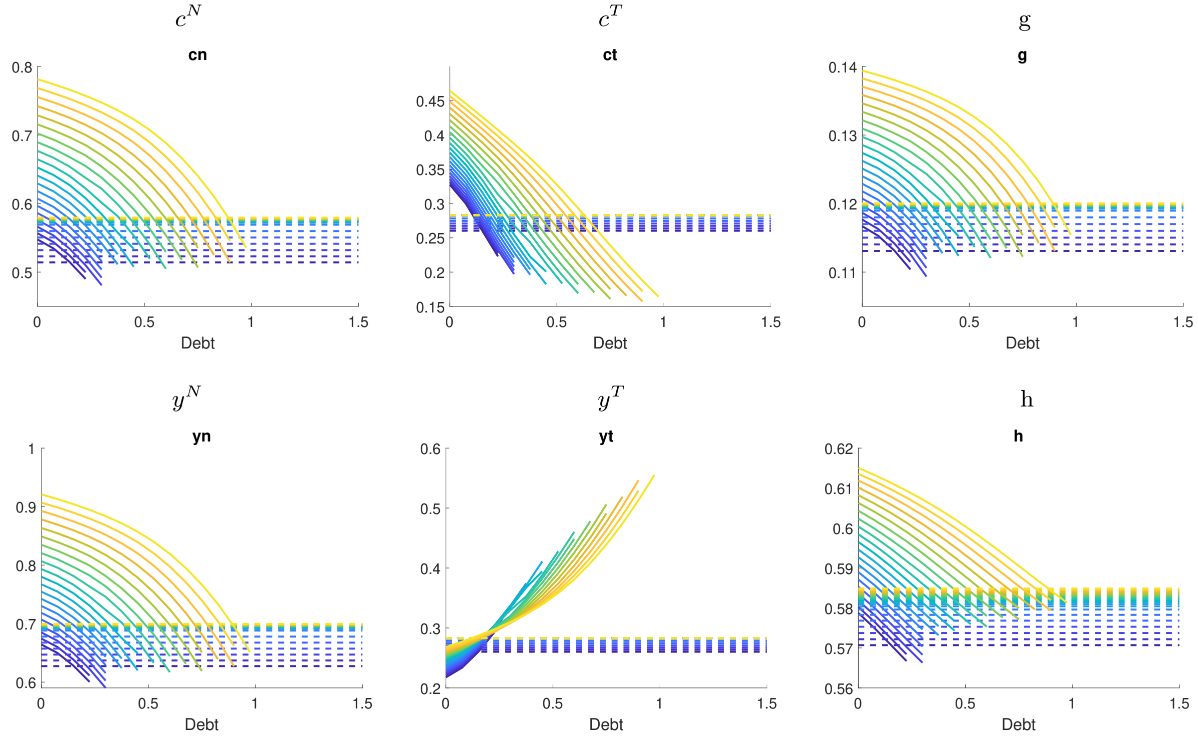
Note: Policy functions for repayment are solid lines, while in default are the dashed ones. The different lines ranging from blue to yellow correspond to states with increasingly higher terms of trade.

Figure 17: Policies as function of debt, TFP, and repayment status



Note: Policy functions in repay are solid lines, while in default are the dashed ones. The different lines ranging from blue to yellow correspond to states with higher tfp.

Figure 18: Equilibrium allocations as function of debt, tfp, and repayment status



Note: Policy functions for repayment are solid lines, while in default are the dashed ones. The different lines ranging from blue to yellow correspond to states with increasingly higher tfp.

C Data

C.1 Data Sources

This section lists the sources for all the variables used in the main body of the paper.

Variables in Table 2:

- “Inflation” is Inflation, consumer prices (annual %) from the World Bank. Indicator Code FP.CPI.TOTL.ZG.
- “Transfers/GDP” constructed as the product of two series from the World Bank. Subsidies and other transfers (% of expense) with indicator code GC.XPN.TRFT.ZS and Expense (% of GDP) with indicator code GC.XPN.TOTL.GD.ZS.
- “Exports/GDP” is Exports of goods and services (% of GDP) from the World Bank. Indicator code NE.EXP.GNFS.ZS.
- “Employment/Population” is Employment to population ratio, 15+, total (%) (modeled ILO estimate). Indicator code SL.EMP.TOTL.SP.ZS.
- “Gov. Consumption/GDP” is General government final consumption expenditure (% of GDP) from the World Bank. Indicator code NE.CON.GOVT.ZS.
- “Debt/GDP” is Public External Debt (%GDP) computed using the ratio of the following two variables from the World Bank. External debt stocks, public and publicly guaranteed (PPG) (DOD, current US$) with indicator code DT.DOD.DPPG.CD and GDP (current US$) with indicator code NY.GDP.MKTP.CD.
- “Haircut, Share of Debt” is the median “SZ haircut, HSZ” in Table 2 of Dvorkin et al. (2021).
- “Default rate” is obtained from Tomz and Wright (2013). They construct a database of 176 sovereign entities spanning 1820 to 2012. The frequency of default is sensitive to the sample being analyzed. They mention that their findings are “similar to the 2% default probability that is a target for many calibrated versions of the standard model,” which is the number we use as well. The unconditional probability of a country with positive debt (a borrower) defaulting on debts owed to commercial creditors is 1.7% per year. Nevertheless, this probability is higher in developing countries. Note also in Figure 2 of Tomz and Wright (2013) that in a typical year, there are no defaults or there is one

country in default. We considered this fact when calibrating a significantly lower default rate in the model with only ε shocks.

The sources for variables used in Table 5 are:

- "Real GDP growth" is GDP per capita (constant LCU) from the World Bank. Indicator Code NY.GDP.PCAP.KN.
- "Trade balance" is Trade balance (% GDP) computed using two variables from the World Bank. Trade (% of GDP) with indicator code NE.TRD.GNFS.ZS and the variable Exports of goods and services (% of GDP) mentioned above.
- "Consumption" is Consumption per capita constructed using two World Bank Variables. Households and NPISHs Final consumption expenditure (constant LCU) with indicator code NE.CON.PRVT.KN and Total Population with indicator code SP.POP.TOTL.
- "Spreads" is the J.P. Morgan Emerging Markets Bond Spread (EMBI+) obtained from the World Bank. Indicator Indicator Id: EMBIG.

The additional sources for Table 6 is Vegh and Vuletin (2015). For real expenditure and inflation tax, the numbers in the column "Data" corresponds to the averages for the countries in our sample of the correlations reported by Vegh and Vuletin (2015). The only difference with the numbers that they report and what we do with the simulated data from the model is that the detrending method is different. We use these numbers because the series made publicly available are already detrended. For taxes, we use the file they made available, "data_AEJEP.dta", and using the variable "individual_tr," we follow the same detrending procedure to make the correlation more comparable. For this variable, Vegh and Vuletin (2015) present in Figure 11 the correlation in growth rates; i.e., the correlation between the change in the personal income tax rate and GDP growth. If we follow that procedure, our results confirm the similarity of the model and the data—we obtain -0.2319 in the model and -0.1009 in the data.

Figure 4 uses only a new variable: the terms of trade index. We use data from ECLAC - CEPALSTAT, Economic Indicators and statistics, External sector. The index is called "terms of trade and purchasing power of exports". The same data is used to estimate the autoregressive process for terms of trade in Appendix C.2.

C.2 Estimation of a stochastic process for the terms of trade and productivity

We use data on terms of trade as described above and the time series of commodity prices used by Drechsel and Tenreyro (2017). Before estimating the autoregressive process, we take logs of the series and subtract the mean. Table 10 presents the results. The time period is 1980 to 2019. The coefficients ρ_p and σ_p are both similar for all the seven countries, so we use the average in our benchmark calibration. It is reassuring that the estimation results for the commodity price index presented at the bottom of Table 10 are also quite similar to the average.

For productivity we use the Penn World Table version 10.0, variable *rtfpna* (TFP at constant national prices). Before estimating the autoregressive process, we take logs of the series and subtract the linear trend. Table 10 presents the results. The time period is the same as for terms of trade, 1980 to 2019. The coefficients ρ_{tfp} and σ_{tfp} are both similar for all the seven countries, so we use the average in our benchmark calibration.

Table 10: Estimation of process for shocks: p^T and tfp

		p^T		TFP	
	Number of years	ρ_p	σ_p	ρ_{tfp}	σ_{tfp}
Argentina	40	0.9302	0.0608	0.7759	0.0433
		(0.0567)	(0.0064)	(0.1072)	(0.0062)
Brazil	40	0.8742	0.0656	0.8158	0.0270
		(0.0759)	(0.0072)	(0.0575)	(0.0027)
Colombia	40	0.9174	0.0851	0.8796	0.0139
		(0.0588)	(0.0095)	(0.0905)	(0.0015)
Mexico	40	0.8216	0.0702	0.8765	0.0251
		(0.1339)	(0.0036)	(0.0556)	(0.0023)
Chile	40	0.9154	0.1022	0.9066	0.0289
		(0.0868)	(0.0106)	(0.0906)	(0.0024)
Peru	40	0.9327	0.0735	0.9171	0.0444
		(0.0728)	(0.0063)	(0.0533)	(0.0040)
Uruguay	40	0.7707	0.0718	0.8712	0.0369
		(0.0928)	(0.0072)	(0.0432)	(0.0031)
Average		0.8803	0.0756	0.8632	0.0314
Commodity price index	36	0.8757	0.0910		
		(0.0965)	(0.0134)		

Note: Standard errors in parenthesis.

We did not de-trend the series before estimating the stochastic process so as to include long-duration cycles in the terms of trade (often referred to as "super-cycles") in our quantitative exercises and keep the model and the data more comparable. We have also estimated these stochastic processes after de-trending the time series for terms of trade. The main difference is that the resulting value of ρ_p is smaller, which implies that shocks are less persistent.

To get a simple idea of the role of persistence, we present in Figure 19 the response to an unexpected shock to the terms of trade with three values of persistence: the benchmark, $\rho_p = 0.8803$, a higher value, $\rho_p = 0.98$, and a lower value $\rho_p = 0.8$. The dynamics after the shock are perfect-foresight, as in the exercises in Section 5.4. Changing ρ_p mainly affects the persistence of real GDP and fiscal variables (debt over GDP and primary deficit over GDP); it also affects spreads on impact. The reason for this last result is that a more persistent fall in terms of trade limits the capacity for debt repayment more severely. Interestingly, the persistence of inflation, currency depreciation and spreads are not altered significantly.

C.3 Event Study: Data-Model Comparison

C.3.1 EMBI+ events

- We use quarterly data on EMBI+ Sovereign Spread data from Bloomberg to select the episodes of debt crisis. The series are in basis points and cover 13 emerging market countries and extend from 1997Q1 to 2021Q4, depending on the country.

- Following Calvo et al. (2006a), episodes are initially flagged if the EMBI+ spread is larger than the sample mean plus 2 standard deviations. The sample mean and standard deviation are calculated without spread observations above the 95th percentile to avoid increasing these values with extreme observations.

- Episodes are dropped if the EMBI spread never reaches 500 basis points or if the peak is small relative to previous country events (e.g. Ukraine in 2020)

- Episodes are dropped if the spread is within 100 basis points in the prior year without continual increases (up-down scenario). This is done to limit volatile events. For example, assume 2007Q3 is flagged as an event start for a country with a spread of 750 basis points. If the spread in 2007Q1 is 700 basis points and then drops to 600 basis points in 2007Q2 and then increases to 750 basis points in 2007Q3 then we remove the flag. Given the flag is time t=0, this procedure is applied to t=-4, t=-3, and t=-2, i.e., the quarters in the year prior to the event flag, except the one directly before. We do not include the quarter directly before the flag in question (t=-1) since it cannot be determined if the spread declined between the two values.

- To these episodes we added others for which four conditions are satisfied: the spreads increase to over 500 basis points, there are no "up-down" patterns, the peak is not small relative to previous country events, and there is a known event that occured (the 2007-09 GFC shock). This adds events in 2008 for Peru, Phillipines, and Russia.

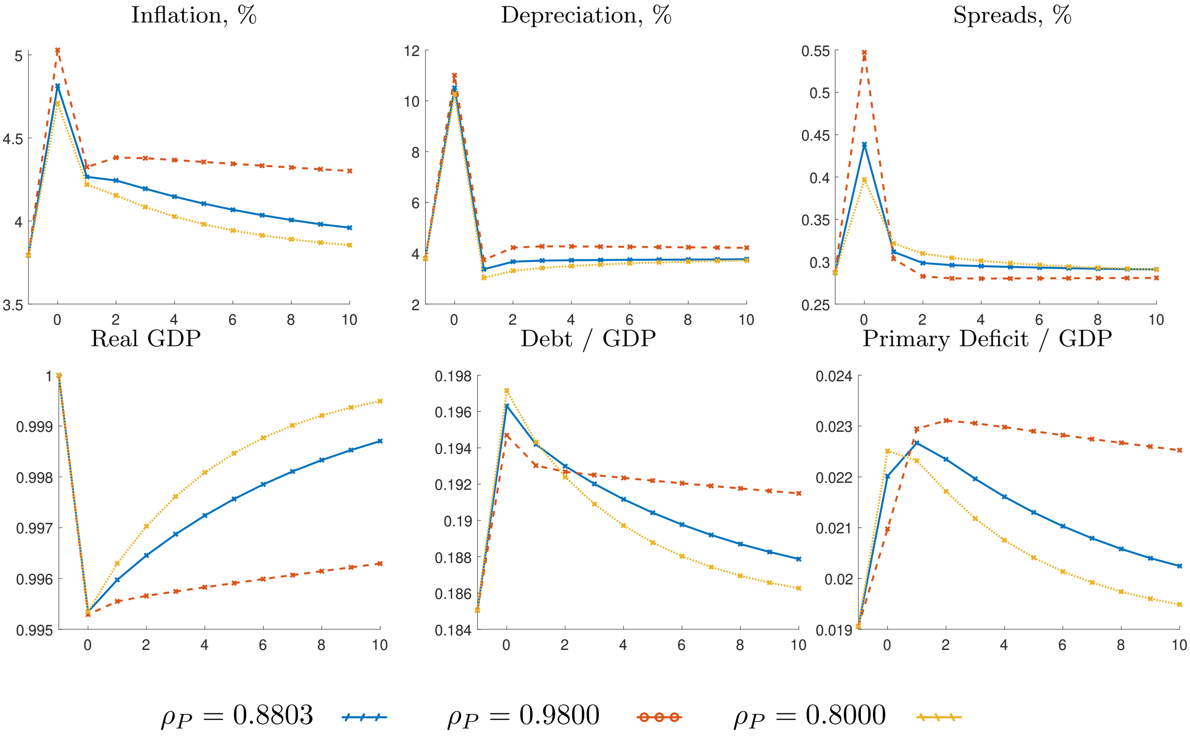
- Once we have an episode, quarters leading to that episode are flagged as part of the event if spreads are significantly increasing at that point. This is measured by if the change in spread from previous quarter is greater than 90 basis points or the year-over-year growth rate is greater than 75 basis points. This is done to better capture the beginnings of some crises.
- If there are only 1 to 3 unflagged quarters between flagged quarters for a country, we count those two flags as the same event.
- After we have the episodes selected, the event start (t=0) is marked as the first quarter flagged. For example, if an episode has flags from 2005Q1 to 2007Q2 then t=0 would be 2005Q1.
- The inflation series are end-of-quarter consumer price index values retreived from either IMF International Financial Statistics (IFS) or the country's statistical agency via Haver Analytics, depending on availability. The indices are used to calculate year-over-year percent inflation. Croatia, Mexico, and Ukraine inflation data are from IFS. Argentina inflation data are from Cavallo and Bertolotto (2016). All others are from the Emerge database in Haver Analytics.
- For the table, we presented the difference in inflation betwen period -1 and period 0. Since the model is yearly and the data is quarterly, we take average of inflation for quarters -4 through -1 of each event to be the equivalent to period -1 in the yearly model values. Likewise, we consider the average inflation of quarter 0 through 3 in the data to be equivalent to period 0 in the model.
- The above measure of inflation is truncated at +/- 50 percentage points
- For the model, we adjust the criteria that the spread must be at least 500 basis points. We take the average spread in selected events in the data from the quarter of the event (t=0) to the third quarter after the event (t=3). The minimum of these averages is 379 basis points, which we use for the model lower threshold for event selection.

C.3.2 Reinhart, Reinhart, Rogoff, and Tresbech events

Havard Business School provides yearly data on inflation, banking crises, systemic crises, currency crises, and domestic and external debt crises collected by Reinhart et al.. The data begin in 1800 and extend to 2016.

- We exclude years in which only an inflation crisis occurs.
- We also exclude years in which prior-year inflation was greater than 100%.
- The event start (t=0) is the first year in which a crisis is flagged.
- Inflation is based to period t=-1 and truncated at +/- 50 percentage points
- Since these data are yearly, we take the based inflation in t=0 to be the percentage point change in inflation during the crisis year.

Figure 19: Shocks to the terms of trade: the role of persistence


Inflation, %

Depreciation, %

Spreads, %

Real GDP

Debt / GDP

Primary Deficit / GDP

C.4 Local projections

We consider three alternative left-hand-side variables: inflation, EMBI spreads, and GDP growth (i.e., $ln(GDP_t) - ln(GDP_{t-1})$). We refer to these variables as y_t^i, where i refers to the country and t to the year. The right-hand-side variable of interest is the log(terms of trade), and we refer to this variable as lp_t^i.

The difference of a variable δ periods ahead with the same variable one period ago is $\Delta y_{t+\delta,t-1}^i = y_{t+\delta}^i - y_{t-1}^i$. The panel regression we run to obtain the response to terms of trade shocks is

$$\Delta y_{t+\delta,t-1}^i = \alpha^\delta + \beta^\delta \Delta lp_{t,t-1}^i + \text{controls}.$$

We run this regression 12 times: for each of the three alternative left-hand-side variables and for $\delta = \{0, 1, 2, 3\}$. The controls consist of two lags of $\Delta y_{t+\delta,t-1}^i$, two lags of $\Delta lp_{t,t-1}^i$, and country fixed effects.

In Figure 9, we plot the coefficients β^δ multiplied by -10 to represent a 10 percent decline in the terms of trade. The standard errors showed by the shaded area in the figure are robust standard errors.

The time period for the regressions in 1980 to 2019 or the latest available observation. The most important exception is the regression for the EMBI spread, which starts in 1997 due to data availability of this variable.

We also conduct this comparison using contemporaneous regressions between these three variables and the terms of trade. The estimated semi-elasticities, similar to those in Drechsel and Tenreyro (2017), are quite similar in the model and the data and resemble the effect at time zero in the analysis presented here.