Productivity and Trade Dynamics in Sudden Stops *

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Abstract

This paper proposes a framework to jointly study productivity and trade dynamics during financial crises. The persistent output loss caused by crises is driven by lower productivity growth, which is determined by changes in product entry and exit margins in domestic and export markets. We calibrate and validate the model using unique data on firms' product portfolios, finding they closely match the behavior of various margins during Chile's 1998 sudden stop. We decompose the sources of the welfare cost of sudden stops, finding that 30% is due to a decline in productivity growth. Lower productivity growth, in turn, is due mostly to slower firm and product entry into the domestic market, while a persistent real exchange rate depreciation induces surviving firms to tilt their product portfolios toward export markets, driving the productivity recovery in the aftermath of the crisis.

Keywords: Endogenous growth, Firm dynamics, Trade dynamics, Sudden Stops.

JEL classification: F10, F41, F43, F44, O33

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1 Introduction

Sudden stops - sharp contractions in capital inflows - often cause severe economic downturns in emerging economies. The persistent effect on output of this type of financial crisis is indicative of a decline in productivity. Studying the slowdown in aggregate productivity, recent work points to distortions affecting firm creation and the expansion of incumbent firms during financial crises. The literature, however, has ignored the role played by exporters in productivity dynamics during sudden stops. Because exporters are central to the trade surpluses observed during crises and are a key driver of productivity growth in open economies, this is potentially an important omission. In fact, exporting firms typically account for the bulk of productivity growth [Bernard and Jensen, 2004], and during sudden stop episodes, the ensuing exchange rate adjustment increases the demand for exports relative to domestic sales. This mechanism gives rise to differential firm and product entry rates into export and domestic markets.

We propose a unified framework to study sudden stops in which the evolution of trade and productivity dynamics shapes the aggregate response of the economy. Our model bridges the endogenous sudden stop literature [Mendoza, 2010] and theories of firm dynamics with endogenous growth [Klette and Kortum, 2004], adding to this framework product-level export dynamics. We use unique data on Chilean manufacturing firms' product portfolios to discipline our quantitative model, and show it matches macro and micro dynamics during sudden stops. With the calibrated model, we show that the slowdown in domestic product entry rates are the key driver of the productivity loss on impact, while export product entry shapes the productivity recovery in the aftermath of the crisis.

We model a small open economy consisting of final and intermediate goods producers. Firms in the intermediate-goods sector innovate to introduce domestic and export product lines. Final-good producers demand intermediate inputs to produce. These firms own a productive asset used as collateral for borrowing to finance working capital. Series of favorable productivity and interest rate shocks cause increases in leverage. Unfavorable shocks arriving at times of high leverage make the collateral constraint binding, raising the effective cost of borrowing during a financial crisis and decreasing the demand of tradable final good producers for intermediate inputs.

Sudden stops have a starkly different impact on exporters and non-exporters. Intermediate goods sold domestically face a lower demand, and therefore lower profits, due to the local crisis. The lower value of domestic product lines reduces product entry into the domestic market by both new and incumbent firms. In contrast, exported products do not rely entirely on local demand, and exports benefit from lower wages while facing a stable foreign demand. This is a classic real exchange rate effect favoring exporters during crises. Therefore, the value of export product lines increases relative to domestic ones, generating incentives for incumbent firms to invest in innovation to introduce exported products. Consistent with the empirical evidence in Alessandria et al. [2014], the extensive margin of exports adjusts gradually and drives a sluggish recovery in productivity.

We discipline the model using novel data on firms' product portfolios. We use unique firm-level data from the Chilean manufacturing census that lists each firm's entire set of products with revenue split between local and foreign sales. The model is calibrated to match the firm-level product portfolio distribution. To validate the model, we document that the model accurately replicates a series of micro and macroeconomic moments. First, the dynamics of firms' product portfolios are central to our framework, and we document the model matches them closely. For instance, the probability that a firm introduces a product in the domestic market is increasing in the firm's number of existing products, and the probability that a firm starts exporting a product previously sold only domestically is increasing in the number of domestic products. Although these moments were not used in the calibration, the model is able to replicate these patterns accurately. Next, and given that our model features endogenous firm dynamics, we further validate it showing that it can accurately replicate the firm size distribution in terms of both employment and number of products sold. In addition, because trade dynamics play a central role in our discussion of productivity, we show the model closely matches exporter premia in terms of revenue, employment and productivity. And because from a macroeconomic perspective our interest is in financial crises, we also validate the model by exploring the response of macroeconomic outcomes to a sudden stop, and contrast it to the evolution of these outcomes during the Chilean 1998 sudden stop.

We then use the calibrated model to illustrate and quantify how the behavior of trade and productivity dynamics shape the response of the economy to sudden stops. Following a sudden stop, as local demand collapses, imports of intermediate goods fall substantially while exports of intermediate goods are more stable, which improves the trade balance. In the model, while imports collapse mostly through an intensive margin adjustment, exports adjust entirely through the extensive margin. This export adjustment is primarily driven by reduced entry into exporting by new firms. During the recovery, the decline in the domestic wage and real exchange rate depreciation increase export profits relative to domestic profits. This induces innovation by incumbent firms aiming to start exporting products that they currently sold only domestically, resulting in exports driving the recovery from sudden stops. To understand the effect of firm-level innovation decisions on aggregate productivity, we decompose productivity growth highlighting the contribution of entrants and incumbents. This decomposition shows that the immediate productivity slowdown during a sudden stop is driven by a decline in product entry to the domestic market. After illustrating these mechanisms, we show that they are supported by the data, as in the aftermath of Chile's 1998 sudden stop, firms tilted their product portfolios toward exported products. In addition, we find that following the sudden stop, for each given firm and product, export revenue falls less relative to revenue from the domestic market especially in industries with a high degree of financial dependence. This is consistent with the financial nature of crises in our model.

We then use our model to study to what extent productivity and trade dynamics account for the welfare loss from a sudden stop. Consumers would be willing to forgo 4.6% of their consumption the period before the crisis to avoid an average sudden stop. We then analyze counterfactuals that keep constant the domestic and/or export innovation rates. We find that about 30% of this welfare loss is due to the endogenous slowdown in productivity growth. Lower productivity growth on impact is explained entirely by lower entry rates for domestic products, while higher entry rates for exported products drive the recovery after the crisis.

We conclude with a brief discussion of the policy implications of our framework, which we analyze at length in the Appendix. Given the pecuniary externality arising from the occasionally binding borrowing constraint, we study a tax on foreign borrowing, which reduces the welfare cost of the crisis but implies a large sacrifice in "normal times". In addition, given the externalities caused by firms' innovation decisions, we analyze an innovation subsidy, which raises growth in normal times but magnifies the cost of crises. We show that a combination of these two policies can improve welfare.

The rest of the paper is organized as follows. Section 1.1 reviews the related literature. Section 2 introduces the model. Section 3 calibrates and validates the model. Section 4 uses the calibrated model to analyze the impact of sudden stops on trade and productitivty dynamics and on welfare. Finally, Section 5 concludes.

1.1 Contribution to the Existing Literature

Our paper contributes to a literature studying the response of the economy to sudden stops. Recent work [Mendoza, 2010, Jeanne and Korinek, 2020, Bianchi, 2011, Bianchi and Mendoza, 2018] models sudden stops as endogenous events using occasionally binding collateral constraints. This approach produces the amplification and asymmetry that these events epitomize, preserving long-run business cycle properties of standard models. Our contribution to this literature is to incorporate productivity and trade dynamics in a heterogeneous-firms framework. This is essential because the slow recovery following these episodes is characterized by slow TFP growth [Meza and Quintin, 2007, Pratap and Urrutia, 2012] and a key role played by the extensive margin of exports [Alessandria et al., 2014]. Recent developments in this literature [Seoane and Yurdagul, 2019, Akıncı and Chahrour, 2018, Flemming et al., 2019] introduce growth-rate trend shocks and news shocks to improve the quantitative performance of these models. Endogenous technological change generates fluctuations in growth rates with similar properties to news and trend shocks. By explicitly modeling endogenous trade and productivity dynamics we provide a measurable microfoundation for these channels, which we validate with microdata.

Our paper is also part of a nascent literature that blends endogenous technological change and international finance with the goal of studying the medium and long-run consequences of large but temporary external shocks. Comin and Gertler [2006] develop a model in which short-run shocks to the economy cause medium-term business cycles using a product-variety-expansion type of endogenous growth framework. A similar framework is used by Queralto [2020] to study Korea's 1997 financial crisis, by Guerron-Quintana and Jinnai [2019] to measure the cost of the U.S. Great Recession, by Gornemann [2014] to explain long-term costs of sovereign crises, by Matsumoto [2021] to study the interplay of FDI and reserve accumulation in emerging countries, and by Ma [2020] to study macroprudential policies. Closer to our paper Ates and Saffie [2021] bridge a version of the Schumpeterian growth model of Klette and Kortum [2004] and the business cycle model of Neumeyer and Perri [2005] and Uribe and Yue [2006] to show that sudden stops have a persistent effect on growth through the composition of entering firms. A key contribution of our model to this literature is incorporating trade dynamics, which are essential to the understanding of sudden stops in emerging markets. In addition, we contribute to this literature by contrasting the model with microdata on firms' domestic and export product portfolios.

These trade dynamics are important, as the literature studying the adjustment of exporters to crises or large devaluations has shown. In this regard, Alessandria et al. [2014] find that the sluggish response of exports to large devaluations is driven by the extensive margin, which adjusts slowly given its forward-looking nature. Alfaro et al. [2018] document that exporting firms' productivity and innovation rise in response to depreciations. Blaum [2019] shows that the response of imports to devaluations is determined in part by the fact that large exporters – which expand during a devaluation – import inputs to produce.

Whereas our model is related to the trade dynamics literature, there are also important differences. Influential models in this literature build on Melitz [2003], in which a firm's productivity is exogenous. In contrast, due to our focus on growth, we allow for endogenous firm productivity, even within each product line. In this regard, our framework is related to models of trade and technological upgrading, such as Costantini and Melitz [2009], Bustos [2011], and Lileeva and Trefler [2010] and to models of multiproduct firms [Bernard et al., 2011, Chatterjee et al., 2013], but with a very different approach which draws from the creative destruction and endogenous technological change literature.

Regarding the connection of our paper to the literature on endogenous technical change, recent work has studied the interaction between trade and productivity dynamics (e.g. [Perla et al., 2021, Buera and Oberfield, 2020, Sampson, 2015, Bloom et al., 2013]). Closest to our paper, Akcigit et al. [2018] also model competition between intermediate goods producers across countries. The link between trade and productivity in our model is more stylized, allowing us to go beyond transitional dynamics and studying aggregate risk with occasionally binding financial constraints. On the empirical front we also make a key contribution to the endogenous technical change literature. In fact, the quantitative literature that builds on Klette and Kortum [2004] has relied on patent- and plant-level data to estimate the parameters governing the expansions and contractions of products [Akcigit and Kerr, 2018, Acemoglu et al., 2018, Lentz and Mortensen, 2008, Cao et al., 2020]. In contrast, we observe the portfolio of domestic and exported products at the plant level. Thus, this is the first paper in the Klette and Kortum [2004] framework that uses product-level data for calibration and validation, and we show that the data indeed validates this class of models.

2 Model

The model consists of an infinite-horizon small open economy (Home). A representative firm produces a tradable final good. This firm borrows working capital within each period and faces an endogenous collateral constraint. Shocks to aggregate productivity and the real interest rate can occasionally make this constraint binding and generate sudden stops.

A set of firms produce differentiated intermediate goods used to assemble the final good in the domestic market or abroad. This intermediate sector is modeled as a version of the Schumpeterian growth model developed by Ates and Saffie [2021], which is a discrete time version of Klette and Kortum [2004] incorporating aggregate risk. These intermediate goods producers innovate to introduce new product lines, competing among them and with foreign firms to become the lowest cost producer under Bertrand competition. These firms also innovate to be able to export these product lines. This setting in the intermediate sector generates endogenous productivity dynamics. In addition it gives rise to trade dynamics at the intensive and extensive margins. An overview of this environment is presented in Figure 1.

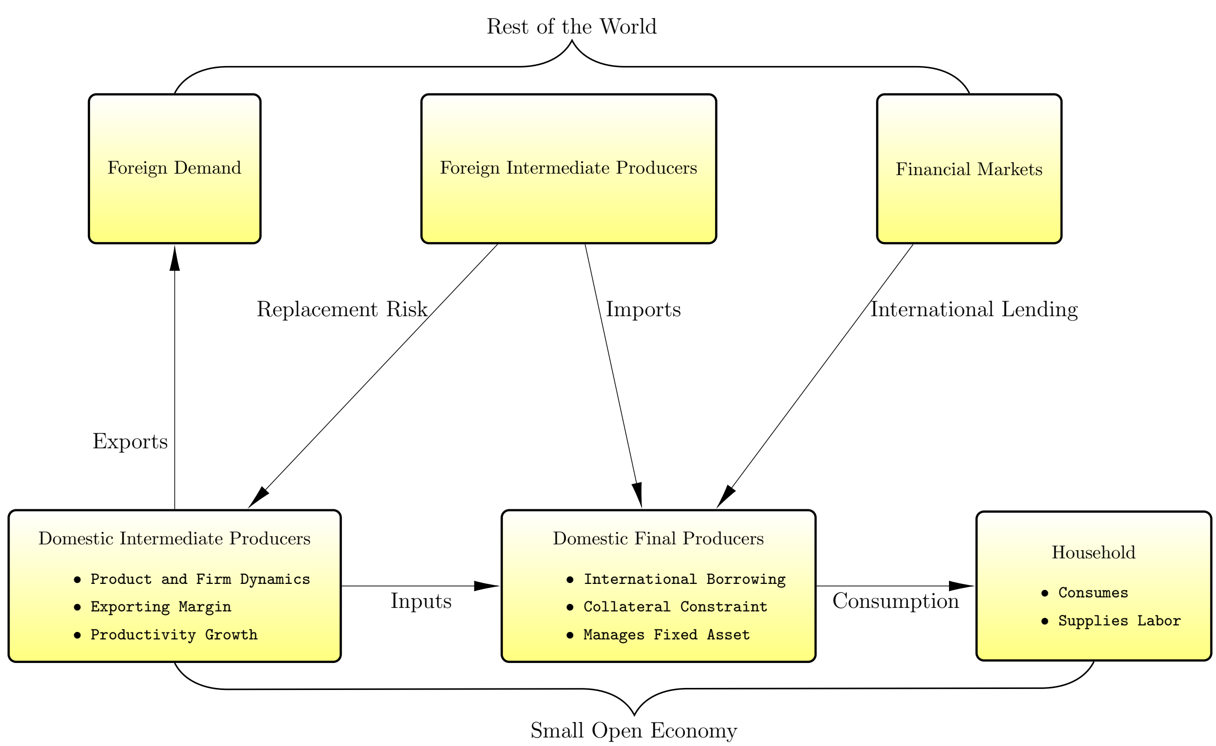


Figure 1: The Model Economy

2.1 Tradable Final Good

A representative firm produces a tradable final good using a set of differentiated intermediate goods $\{y_t(i)\}_{i=0}^1$ according to the production function:

$$Y_t = \exp(\varepsilon_t^A) \exp\left[\int_0^1 \ln y_t(i) di\right] \ , \tag{1}$$

in which ε_t^A is a stochastic productivity shock.

We assume the firm must pay in advance a fixed fraction ϕ of the cost of intermediate inputs. This working capital payment is financed by within-period borrowing from abroad without any interest. In addition, the firm borrows from abroad using a one-period non-contingent bond denoted by B_t. The firm is subject to the following collateral constraint, which states that total borrowing must not be larger than a fixed fraction κ of the value of a productive asset owned by the firm and used as collateral:

$$-B_t + \phi\left[\int_0^1 p_t(i) y_t(i) di\right] \leq \kappa Q_t L_{t-1} \, . \tag{2}$$

where $p_t(i)$ is the price of intermediate good i, Q_t denotes the price of the asset, and L_{t-1} denotes the amount owned by the firm. The firm rents this productive asset at a rate R_t^L to firms in the intermediate good sector, which require it to produce.[1] This asset exists in fixed supply at L. Each period, the firm chooses amounts of each intermediate good $\{y_t(i)\}_{i=0}^1$, the amount of the productive asset L_t to hold, and foreign bond holdings B_t to maximize the discounted value of current and future profits:

$$\max_{\{\{y_t(i)\}_{i=0}^1, L_t, B_t\}_{t=0}^\infty} E_0 \sum\nolimits_{t=0}^\infty \beta^t \lambda_t \Pi_t \, ,$$

subject to the collateral constraint (eq. (2)). λ_t is the marginal utility of tradable goods consumption by households, and future profits are discounted with the same discount rate used by the representative household. Firm profits are:

$$\Pi_t = Y_t - \int_0^1 p_t(i) y_t(i) di - B_t + \exp(\varepsilon_{t-1}^R) R B_{t-1} - Q_t L_t + (Q_t + R_t^L) L_{t-1} \, , \tag{3}$$

[1] The assumption that the firm can own an asset and can borrow from abroad instead of households makes the problem more tractable. A model in which households own the asset and borrow from abroad would be equivalent but less tractable.

where $\exp(\varepsilon_{t-1}^R)R$ is a stochastic gross interest rate on the foreign bond.

The first-order condition with respect to $y_t(i)$ gives demand for each intermediate good:

$$p_t(i)\left(1+\phi\frac{\mu_t}{\lambda_t}\right)=\frac{Y_t}{y_t(i)}\,. \tag{4}$$

In this expression, μ_t stands for the Lagrange multiplier on the borrowing constraint (eq. (2)). When the borrowing constraint is slack, $\mu_t = 0$ and the demand function for intermediate goods (eq. (4)) is standard, equating price and marginal product. When the borrowing constraint binds, a strictly positive μ_t appears as the external financing premium on working capital payments, which increases the effective cost of inputs.

Finally, the first-order condition with respect to L_t determines the asset price:

$$Q_t=\frac{\beta E_t\left[\lambda_{t+1}(Q_{t+1}+R_{t+1}^L)+\mu_{t+1}\kappa Q_{t+1}\right]}{\lambda_t} \tag{5}$$

When the borrowing constraint (eq. (2)) binds, the firm cannot borrow enough and households reduce consumption. Lower consumption implies higher marginal utility λ_t and reduces the asset price Q_t through (5), leading to the Fisherian debt-deflation mechanism as in [Mendoza, 2010].

2.2 Intermediate Goods

There is a continuum of differentiated tradable intermediate goods indexed by $i \in [0,1]$ used to assemble the final good. We refer to these as product lines. These intermediate goods can be produced by Home or Foreign firms. Each product line is produced by a single firm - the lowest cost producer - in a context of Bertrand competition, and each firm may produce multiple products.

These intermediate goods producers have heterogeneous productivity levels $a_t(i)$. They produce using the productive asset $\ell_t(i)$ and labor $h_t(i)$ according to the following production function:[2]

$$y_t(i)=a_t(i)\left(\ell_t(i)\right)^{\alpha}\left(h_t(i)\right)^{1-\alpha}\,. \tag{6}$$

Firms innovate to introduce new product lines by becoming the lowest cost producers.[3] When a

[2]Both factors of production - the productive asset and labor - are internationally immobile.

[3]When a firm introduces a *new* product line, this product is new to the firm, but was previously produced by another firm, given that the continuum of products in the economy is constant.

firm carries on a successful domestic innovation, it obtains a productivity lead equal to $(1+\sigma^D)$ times the previous leading technology, which becomes available to all firms. Firms also innovate to export existing domestic product lines. When a firm carries out a successful export innovation it obtains a larger productivity lead equal to $(1+\sigma^X) > (1+\sigma^D)$ times the previous existing technology. For each product line, only one successful innovation occurs at a time.[4] The probability of successful innovations and the investment firms need to innovate are discussed in the next subsection.

Product lines can be classified into domestic lines (D) (in which the lowest cost producer is a domestic firm), export lines (X) (in which a domestic firm has innovated to be able to sell the product both domestically and abroad) and import lines (M) (in which the lowest cost producer is a foreign firm and the final tradable good producer imports the product). We describe the profits for each of these three types below.

Under Bertrand competition, the firm with the leading technology sets a price equal to the marginal cost of its competitors which have the second-best (i.e. the previous leading) technology. Firms' marginal cost depends on factor prices, trade costs, and their productivity. Home firms trying to sell in the Foreign market face an iceberg trade cost such that shipping $1+\xi$ units is required to sell 1 unit. Foreign firms trying to sell in the Home market face the same iceberg trade cost.

In the case of domestic lines (D), the second lowest marginal cost belongs to domestic firms. Because all domestic firms face the same factor prices, namely the asset rental rate R_t^L and wage W_t, differences in cost between the leading firm and its competitors are due only to differences in productivity. Let $a_t(i)$ denote the productivity level of the lowest cost producer (i.e. the leader) for line i. The price set is equal to the second lowest marginal cost:

$$p_t^D(i) = \widetilde{MC}_t^D(i) = \frac{1}{a_t(i)/(1+\sigma^D)}\overline{\alpha}\left(R_t^L\right)^{\alpha}(W_t)^{1-\alpha}\,, \tag{7}$$

where $\overline{\alpha} = \alpha^{-\alpha}(1-\alpha)^{-(1-\alpha)}$. Profits obtained from this line are:

$$\pi_t^D(i) = p_t(i)y_t(i) - R_t^L\ell_t(i) - W_t h_t(i)\,.$$

[4] This is due to the fact that there is a continuum of products, so the probability that two successful innovations occur for the same product is zero.

Replacing in this expression the demand for intermediate goods by the final tradable good producer (4), profits can be written as:

$$\pi_t^D = Y_t \frac{1}{1+\phi\mu_t/\lambda_t} \frac{\sigma^D}{1+\sigma^D} \,. \tag{8}$$

The following points are worth mentioning. First, profits are independent of the productivity level $a_t(i)$ of the lowest cost producer.[5] Second, profits are a decreasing function of the Lagrange multiplier on the borrowing constraint μ_t. This is because when the borrowing constraint binds, the final tradable good producer lowers its demand for intermediate goods, which translates into lower profits for intermediate goods producers. Third, profits are independent of factor prices. Factor prices impact both the cost and the price (which is equal to the cost of the second-best firm), cancelling out.

Next, we discuss export lines (X), which are owned by domestic firms and sold both domestically and abroad. In the domestic market, prices and profits are identical to those discussed above for domestic lines, with the only difference that the productivity lead is $(1+\sigma^X)$:

$$p_t^X(i) = \widetilde{MC}_t^X(i) = \frac{1}{a_t(i)/(1+\sigma^X)} \overline{\alpha} \left(R_t^L\right)^{\alpha} \left(W_t\right)^{1-\alpha} \tag{9}$$

$$\pi_t^X = Y_t \frac{1}{1+\phi\mu_t/\lambda_t} \frac{\sigma^X}{1+\sigma^X} \,. \tag{10}$$

Due to the larger productivity lead, profits from domestic sales are larger than those of domestic lines. It is worth noting that even for export lines, profits in the domestic market are negatively affected by the borrowing constraint through the Lagrange multiplier μ_t.

In the Foreign market, a representative final tradable good producer demands intermediate goods according to the following production function:

$$Y_t^* = \exp\left[\int_0^1 \ln y_t^*(i) di\right] \,.$$

Foreign production of the final tradable good is not subject to shocks, and Y_t^* grows at a constant

[5] In Appendix A.1 we show that asset and labor inputs for each product line are also independent of productivity. This property enables us to study the aggregate dynamics of the economy without keeping track of heterogeneous productivity levels across product lines.

rate. The demand of this foreign final good producer for each intermediate good is:[6]

$$p_t^*(i) = \frac{Y_t^*}{y_t^*(i)}\,. \tag{11}$$

In the case of export lines, the second lowest marginal cost belongs to foreign intermediate good producers.[7] The price set for exports in the foreign market is equal to this second lowest marginal cost:

$$p_t^*(i) = \widetilde{MC}_t^*(i) = \frac{1}{a_t(i)/(1+\sigma^X)}\overline{\alpha}\left(R_t^{L*}\right)^{\alpha}\left(W_t^*\right)^{1-\alpha} \tag{12}$$

where R_t^{L*} and W_t^* are the asset rental rate and wage in foreign countries respectively.[8] Using the expression for the foreign demand for intermediate goods (eq. (11)), profits from export lines' sales abroad are:

$$\pi_t^* = Y_t^*\left(1 - \frac{1+\xi}{1+\sigma^X}\underbrace{\frac{\left(R_t^L\right)^{\alpha}\left(W_t\right)^{1-\alpha}}{\left(R_t^{L*}\right)^{\alpha}\left(W_t^*\right)^{1-\alpha}}}_{\text{Real exchange rate}}\right) \tag{13}$$

where ξ is the iceberg cost of exporting. Profits from export sales differ from profits from domestic sales of export lines in that they do depend (negatively) on factor prices. Lower domestic factor prices make domestic production cheaper, while the export price is determined by foreign factor prices. Note that the real exchange rate in this economy is defined as the ratio between the domestic and foreign cost as shown in equation (13).

Finally, we discuss import lines (M). In this case the lowest cost producer is a Foreign firm and the domestic final good producer imports the intermediate good. As the demand for intermediate goods by this producer (eq.(4)) has a unit elasticity, the total payment to foreign firms is independent of the price charged:

$$p_t(i)y_t(i) = \frac{Y_t}{1+\phi\mu_t/\lambda_t} \tag{14}$$

The price is equal to the second lowest marginal cost, which in this case belongs to a Home

[6]Note that the revenue obtained by a given firm from an exported product is equal to foreign demand Y_t^*, which grows at a constant rate. Therefore, export fluctuations over time occur entirely through the extensive margin.

[7]In general, the second lowest marginal cost could potentially correspond to a Home producer. However, we verify in the quantitative analysis in the next section that under the calibrated parameters, this is never the case, because the trade cost is large enough to overcome differences in the relative marginal cost.

[8]Foreign output Y_t^* and foreign factor prices R_t^{L*} and W_t^* are assumed to grow at the same constant rate as the long-run growth rate of this economy. We define foreign productivity A_t^* that grows at this rate, and calibrate the three foreign variables divided by A_t^* as model parameters y^*, r^{L*}, and w^*.

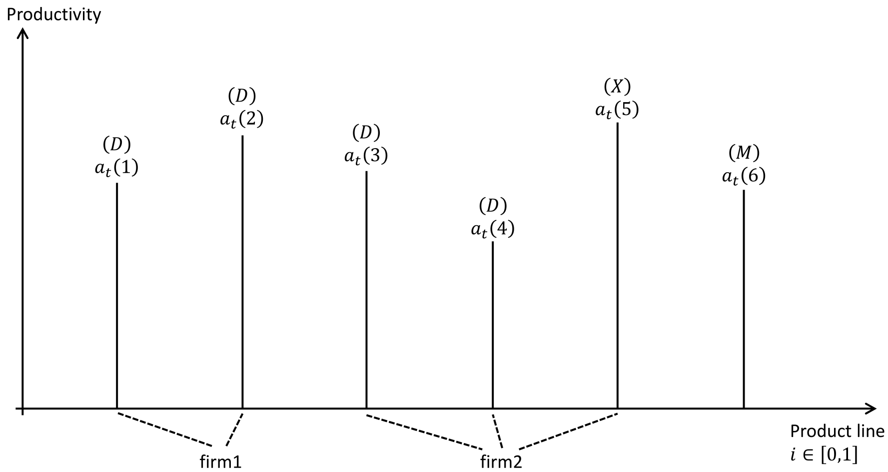
firm. The productivity lead by foreign firms is the same as that of Home exporting firms, $1+\sigma^X$. Consequently the price is the same as that of export lines in equation (9). Note that in equation (14), the right hand side is independent of a product line i, so output $y_t(i)$ is also the same as for export lines.

2.3 Innovation and Firm Dynamics

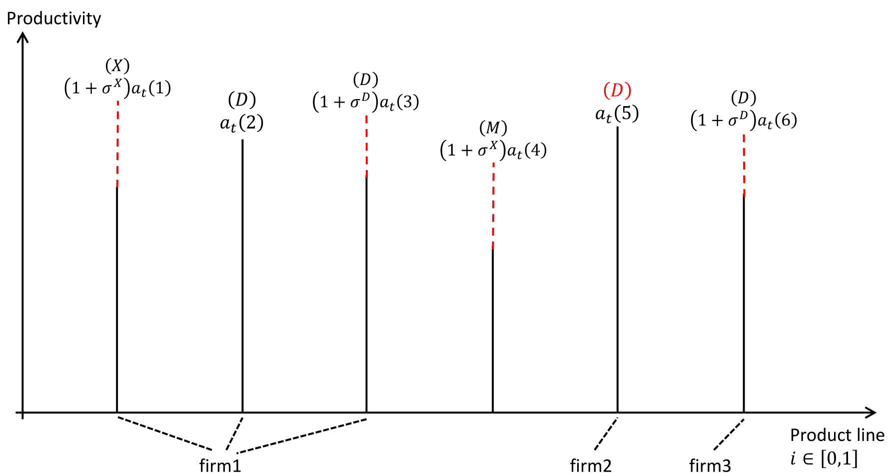
Firm dynamics are shaped by firm entry, innovation by incumbent Home firms, and innovation by Foreign firms. The productivity of each product line evolves with each technological improvement generated by successful innovations. A successful domestic innovation increases the existing productivity of a product line by a factor $1+\sigma^D$. This type of innovation can be done either by an incumbent Home firm adding a new product or by direct entry of a new Home firm. A successful export innovation increases the existing productivity of a product line by a factor $1+\sigma^X$. This class of innovation can be achieved by an incumbent Home firm starting to export a product previously sold domestically, or by direct entry of a new Home firm to the domestic and export markets simultaneously.

Due to entry and innovation, aggregate productivity in the intermediate sector increases over time. Firm dynamics change the status of each product line over time and endogenously determine the extensive margins of imports and exports. Below we explain in detail firms' innovation decisions.

Given that this is the most novel aspect of our model, we start with a simple example to build intuition, and later proceed to analyze firms' innovation decisions systematically. Figure 2 illustrates an example of the evolution of firms' product lines from a period t to $t+1$. First, recall that there is a *constant* set of product lines in the economy and that the number of domestic and export product lines sold by each firm evolves over time. In this example, the top panel shows Home firm 1 producing two domestic product lines, denoted by (D). Home firm 2 produces two domestic and one export line (denoted by (X)). There is also one import line, denoted by (M). In period $t+1$ (bottom panel) Home firm 1 succeeds in an export innovation for product line 1, which becomes an export line. Home firm 1 also succeeds in a domestic innovation and acquires domestic product line 3. Foreign innovation occurs in product lines 4 and 5 owned by Home firm 2. Home firm 2 loses product line 4 and this line becomes an import line. For product line 5, Home firm 2 exits from the Foreign market and this product line becomes a domestic line with no productivity change. Finally, firm entry in Home occurs on product line 6.



a) Period t



b) Period $t+1$

Figure 2: Firm Dynamics

NOTE: This figure provides an example of firms' product dynamics between an initial period t (top panel) and a subsequent period $t+1$ (bottom panel).

2.3.1 Incumbent Firms' Innovation Decisions

Having provided some intuition with this example, we now discuss firms' optimal innovation decisions.

Consider first domestic innovation. A firm owning n^D domestic lines and n^X export lines has $n^D + n^X$ domestic innovation opportunities.[9] For each innovation opportunity, a firm chooses to

[9]The underlying assumption is that a domestic innovation is a spin-off from existing technologies.

invest an amount Z_t^D.[10] The probability of success of a domestic innovation i_t^D is proportional to the amount invested:

$$i_t^D = \eta^D \left(\frac{Z_t^D}{A_t} \right)^{1/\rho} \tag{15}$$

This probability is inversely proportional to the average productivity of intermediate firms A_t (including foreign firms). The functional form is consistent with that used by Ates and Saffie [2021] and Klette and Kortum [2004]. We assume that domestic innovation can take place only on domestic lines and import lines, and does not happen on export lines.

In the case of export innovation, first note that a firm owning n^D domestic product lines has n^D export innovation opportunities. For each innovation opportunity, a firm chooses to invest an amount Z_t^X. The probability of success of an export innovation i_t^X is:

$$i_t^X = \eta^X \left(\frac{Z_t^X}{A_t} \right)^{1/\rho} \tag{16}$$

When a firm's export innovation is successful, a product sold domestically can also be exported and the domestic line becomes an export line.

In the case of foreign firms, there are two types of innovation. Because the home country is a small open economy, we assume that both types of innovations occur with exogenous probabilities. The first type is domestic innovation by Foreign firms (so "domestic" here means the domestic country for Foreign firms). In this case, an export product line owned by a Home firm is forced to exit from the Foreign market, and goes back to being a domestic line. This happens with an exogenous probability i^{FX} for each export product line. The second type of innovation by Foreign firms is an export innovation. In this case, a domestic product line is forced to exit from the Home market, and this product line becomes an import line. This happens with an exogenous probability i^{FD}.

Having described domestic and export innovation opportunities for domestic firms, we can now characterize the optimal innovation decisions for domestic firms which are the result of forward-looking optimizing behavior.

As is common in Schumpeterian growth models, innovation is undirected in the sense that innovation is equally likely to apply to any product line. This feature is preserved in this model

[10]This investment is measured in units of the tradable final good.

because operating profits are independent of a firm's productivity level, so firms with a given productivity are indifferent among any product lines that can be introduced. Undirected innovations carried on by a continuum of firms imply that each product line faces the same replacement probability d_t. The probability of i successes in n trials for a binomial process with success probability p is:

$$P(i,n,p) = \binom{n}{i} p^i (1-p)^{1-i} .$$

The value of a Home firm with n^D domestic lines and n^X export lines can be written in a recursive form as follows:

$$\begin{aligned} V_t(n^D, n^X) = & \max_{Z_t^D, Z_t^X} \{ n^D \pi_t^D + n^X (\pi_t^X + \pi_t^*) - (n^D + n^X) Z_t^D - n^D Z_t^X \\ & + \sum_{i=0}^{n^D+n^X} P(i, n^D + n^X, i_t^D) \sum_{j=0}^{n^D} P(j, n^D, d_t) \sum_{k=0}^{n^D-j} P(k, n^D - j, (1-d_t) i_t^X) \sum_{m=0}^{n^X} P(m, n^X, i^{FD}) \\ & E_t \left[\Lambda_{t,t+1} V_{t+1}(n^D + i - j - k + m, n^X + k - m) \right] \} . \end{aligned}$$

The first line represents operating profits minus innovation investment costs. The second and third line add up the expected value of a firm across all the possible combinations of innovations and replacement on n^D domestic lines and n^X export lines in the next period. The first summation adds up across all the possibilities for domestic innovations from 0 to $n^D + n^X$ successes. The second summation adds up over the number of domestic lines being replaced from 0 to n^D. The third summation adds up over the number of successful export innovations. It is worth noting at this point that given that there is a continuum of product lines and innovation decisions are simultaneous, the probability that two or more innovations occur at the same time for a same product is zero. Thus the effective success probability is given by $(1 - d_t) i_t^X$. The last summation adds up over the number of export lines being replaced from 0 to n^X. $\Lambda_{t,t+1}$ in the last line is the stochastic discount factor by households.

We use a guess-and-verify method to show that the value of a firm with n^D domestic lines and n^X export lines is equal to the sum of n^D times the value of a single domestic line and n^X times the value of a single export line:

$$V_t(n^D, n^X) = n^D V_t(1,0) + n^X V_t(0,1) .$$

The proof is shown in Appendix A.3. This linear relation enables us to aggregate firm dynamics in a tractable way and study how firm dynamics affect endogenous growth and the extensive margins of imports and exports. It enables us to do so without having to keep track of the firm size distribution. The value of a single domestic line is given by:

$$\begin{aligned} V_t(1,0) \quad = \quad & \max_{Z_t^D, Z_t^X} \left\{ \pi_t^D - Z_t^D - Z_t^X \right. \\ & \left. + \left(i_t^D + (1-d_t)(1-i_t^X) \right) E_t \left[\Lambda_{t,t+1} V_{t+1}(1,0) \right] + (1-d_t) i_t^X E_t \left[\Lambda_{t,t+1} V_{t+1}(0,1) \right] \right\} . \end{aligned} \tag{17}$$

The first-order condition with respect to Z_t^D pins down the optimal investment for domestic innovation opportunities:

$$\eta^D \frac{1}{\rho} \left(\frac{Z_t^D}{A_t} \right)^{1/\rho - 1} \frac{1}{A_t} E_t \left[\Lambda_{t,t+1} V_{t+1}(1,0) \right] = 1 \,. \tag{18}$$

The first-order condition with respect to Z_t^X pins down the optimal investment for export innovation opportunities:

$$(1-d_t)\eta^X \frac{1}{\rho} \left(\frac{Z_t^X}{A_t} \right)^{1/\rho - 1} \frac{1}{A_t} \left(E_t \left[\Lambda_{t,t+1} V_{t+1}(0,1) \right] - E_t \left[\Lambda_{t,t+1} V_{t+1}(1,0) \right] \right) = 1 \,. \tag{19}$$

Note that investment is forward-looking in the sense that as the expected value of a product line increases, firms increase their investment.

The value of a single export product line is:

$$V_t(0,1) = \max_{Z_t^D} \left\{ \pi_t^X + \pi_t^* - Z_t^D + (i_t^D + i^{FX}) E_t \left[\Lambda_{t,t+1} V_{t+1}(1,0) \right] + (1 - i^{FX}) E_t \left[\Lambda_{t,t+1} V_{t+1}(0,1) \right] \right\} , \tag{20}$$

and the first-order condition with respect to Z_t^D is identical to the case of domestic product lines in equation (18).

2.3.2 Entry by Domestic Firms

Firm entry results from innovation by households. Households invest in two different types of innovation to start firms with domestic or export product lines. In both cases, new firms poach a product line from incumbent firms and start with a single product line. Households invest an amount Z_t^{ED} to create new firms with a domestic line, and Z_t^{EX} to create new firms with an export line.[11] The number of firms created from Z_t^{ED} and Z_t^{EX} units of investment is:

$$e_t^D = \eta^{ED} \left(\frac{Z_t^{ED}}{A_t} \right)^{1/\rho} \tag{21}$$

and

$$e_t^X = \eta^{EX} \left(\frac{Z_t^{EX}}{A_t} \right)^{1/\rho} , \tag{22}$$

respectively. In both cases, the optimal investment (Z_t^{ED} or Z_t^{EX}) is such that the marginal benefit and marginal cost of investment are equal:

$$\eta^{ED} \frac{1}{\rho} \left(\frac{Z_t^{ED}}{A_t} \right)^{1/\rho - 1} \frac{1}{A_t} E_t \left[\Lambda_{t,t+1} V_{t+1}(1,0) \right] = 1 \tag{23}$$

$$\eta^{EX} \frac{1}{\rho} \left(\frac{Z_t^{EX}}{A_t} \right)^{1/\rho - 1} \frac{1}{A_t} E_t \left[\Lambda_{t,t+1} V_{t+1}(0,1) \right] = 1 \, . \tag{24}$$

2.3.3 Productivity Growth and The Extensive Margins of Trade

We can now characterize how firm dynamics translate into aggregate productivity growth and into the extensive margins of exports and imports. We denote the share of domestic lines by θ_t^D, and the share of export lines by θ_t^X. The share of imported product lines is then $1 - \theta_t^D - \theta_t^X$. The rate at which domestic product lines are replaced (d_t) is the sum of the probability that a product line is replaced due to domestic entry, domestic innovation, or foreign innovation:

$$d_t = \left(e_t^D + e_t^X + (\theta_{t-1}^D + \theta_{t-1}^X) i_t^D \right) \frac{1}{1 - \theta_{t-1}^X} + i^{FD} \, . \tag{25}$$

In this expression, the firm entry rate and the domestic innovation rate are divided by the share of domestic and import lines. This is because these innovations affect only domestic and import lines, and thus the probability that each domestic product line is replaced due to these innovations is the number of product lines facing these innovations divided by the total number of product lines that could potentially receive these innovations. Note also that the domestic innovation rate is equal to the probability that a domestic innovation by an incumbent firm is successful (i_t^D) times the share of domestically-owned product lines (which is the sum of domestic

[11] This investment is measured in units of the tradable final good.

and export lines). The law of motion for the share of domestic lines is:

$$\theta_t^D = \theta_{t-1}^D + (e_t^D + (\theta_{t-1}^D + \theta_{t-1}^X)i_t^D)\frac{1-\theta_{t-1}^D-\theta_{t-1}^X}{1-\theta_{t-1}^X} + \theta_{t-1}^X i^{FX} \\ - \theta_{t-1}^D(1-d_t)i_t^X - \theta_{t-1}^D i^{FD} - e_t^X\frac{\theta_{t-1}^D}{1-\theta_{t-1}^X}\,. \quad (26)$$

This share increases due to i) entry of domestic lines and domestic innovation by incumbent firms that occur on import lines, and ii) innovation by Foreign firms that pushes export lines back to being domestic lines. It decreases due to i) export innovation, ii) foreign innovation that forces domestic lines to exit, and iii) entry of export lines that occurs on domestic lines.

The law of motion for the share of export lines θ_t^X is:

$$\theta_t^X = \theta_{t-1}^X + \theta_{t-1}^D(1-d_t)i_t^X + e_t^X - \theta_{t-1}^X i^{FX}\,. \quad (27)$$

This share increases due to export innovations by incumbent firms and entry of export lines, and decreases due to foreign innovations that turn export lines back into domestic lines. The share of import lines is consequently $1-\theta_t^D-\theta_t^X$. Note that the extensive margin of imports is determined by endogenous changes in the share of import product lines. The extensive margin of exports is determined by endogenous changes in the share of export lines.

Finally, aggregate production of the tradable final good is:

$$Y_t = \exp(\varepsilon_t^A)\exp\left[\int_0^1 \ln y_t(i)di\right] = \\ \exp(\varepsilon_t^A)A_t\left[\left(\ell_t^D\right)^\alpha\left(h_t^D\right)^{1-\alpha}\right]^{\theta_{t-1}^D}\left[\left(\ell_t^X\right)^\alpha\left(h_t^X\right)^{1-\alpha}\right]^{\theta_{t-1}^X}\left[\frac{1}{1+\xi}\left(\ell_t^M\right)^\alpha\left(h_t^M\right)^{1-\alpha}\right]^{1-\theta_{t-1}^D-\theta_{t-1}^X}, \quad (28)$$

where ℓ_t^D,ℓ_t^X and ℓ_t^M are the amounts of the productive asset used by each product line, and h_t^D, h_t^X and h_t^M are the amounts of labor hired by each product line. Note that ℓ_t^M and h_t^M are factors employed abroad. The average productivity of intermediate firms (A_t) is:

$$A_t = \exp\left[\int_0^1 \ln a_t(i)di\right]\,. \quad (29)$$

A_t grows as the productivity of each product line $a_t(i)$ improves through domestic firm entry, innovation by incumbent Home firms, and foreign innovation.[12] The growth rate of A_t is:

$$\frac{A_{t+1}}{A_t} = 1 + g_t = (1+\sigma^D)^{e_t^D + (\theta_{t-1}^D + \theta_{t-1}^X) i_t^D} (1+\sigma^X)^{e_t^X + \theta_{t-1}^D (1-d_t) i_t^X} (1+\sigma^X)^{i^{FD}} . \tag{30}$$

Note that the three terms in the right hand side correspond to the sum of domestic firm entry and domestic innovations, exporting firm entry and export innovations, and foreign innovations respectively. Note also that although the average productivity A_t grows endogenously, the long-run trend growth is exogenous because the foreign economy grows at an exogenous rate.[13]

2.4 Households

The representative household consumes final tradable goods and supplies labor elastically. In addition it invests Z_t^{ED} and Z_t^{EX} units of the tradable good in domestic and export entry. It receives wage $W_t H_t$, profits from the tradable good producers, and profits from domestic intermediate good producers. The representative household's optimization problem is then to maximize:

$$\max_{\{C_t, H_t, Z_t^{ED}, Z_t^{EX}\}_{t=0}^{\infty}} E_0 \sum\nolimits_{t=0}^{\infty} \beta^t \left[\ln \left(C_t - A_t \frac{(H_t)^\omega}{\omega} \right) \right] ,$$

subject to the budget constraint:

$$C_t + Z_t^{ED} + Z_t^{EX} = W_t H_t + \Pi_t + \theta_{t-1}^D \left(\pi_t^D - Z_t^D - Z_t^X \right) + \theta_{t-1}^X \left(\pi_t^X + \pi_t^* - Z_t^D \right) . \tag{31}$$

Optimal investment in domestic entry Z_t^{ED} and Z_t^{EX} are determined by equations (23) and (24). The marginal utility of consumption λ_t is then given by:

$$\lambda_t = \frac{1}{C_t - A_t \frac{(H_t)^\omega}{\omega}},$$

and the stochastic discount factor is given by $\Lambda_{t,t+1} = \beta \lambda_{t+1} / \lambda_t$.

[12] Note that A_t is not necessarily the productivity level of this economy, because A_t includes productivity of Foreign firms. But the long-run growth rate of this economy is determined by growth in A_t.

[13] This economy features semi-endogenous growth consistent with the evidence in Bloom et al. [2020]. In this class of models, productivity exhibits persistent endogenous deviations from an exogenous long-run trend. In our model, foreign productivity A_t^*, which grows at a constant rate, acts as a long-run trend, and domestic aggregate productivity A_t endogenously fluctuates around it.

Finally, the trade balance is:

$$\begin{aligned} TB_t &= \underbrace{Y_t - C_t - Z_t^{ED} - Z_t^{EX} - \theta_{t-1}^D \left(Z_t^D + Z_t^X\right) - \theta_{t-1}^X Z_t^D}_{\text{final tradable output - absorption}} \\ &+ \underbrace{\theta_{t-1}^X Y_t^*}_{\text{exports of intermediate goods}} - \underbrace{\left(1 - \theta_{t-1}^D - \theta_{t-1}^X\right) \frac{Y_t}{1 + \phi \mu_t / \lambda_t}}_{\text{imports of intermediate goods}}. \end{aligned}$$

In Appendix A.1 we define the equilibrium of the economy and describe the stationarized equilibrium conditions that we use to solve the model numerically.

3 Calibration and Validation

In this section, we show that a calibrated version of the model can replicate the micro and macroeconomic dynamics of the Chilean sudden stop. After validating the model, we use it to analyze the determinants of trade and productivity dynamics following sudden stops. Finally, we quantify the implications of sudden stops for long-run productivity, welfare, and policy.

3.1 Data: Firms' Domestic and Export Product Portfolios

We use unique data on firms' product portfolios of Chilean manufacturing firms. Our firm-product level data comes from the Chilean Annual Survey of Manufactures, which contains data on the universe of manufacturing plants with 10 or more employees. While the standard information on plant-level outcomes of this Census has been used extensively in the literature, we have access to an additional form that records each product produced by each firm. These data report separately domestic and export sales of each of these products. We use annual firm-level product data for 1996-1999. We are able to aggregate the plant level data to the firm level. These data have been used by Navarro [2012] and Garcia-Marin and Voigtländer [2019]. To the best of our knowledge, the Chilean Census of Manufactures is the only firm-level database reporting firm-product level data and distinguishing between domestic and export markets.

3.2 Calibration

The model is calibrated at an annual frequency. There are 18 parameters to be determined in the model. We take conventional values from the literature, and calibrate others to target the Chilean economy. In addition we use the firm-product level data described above. Table 1 shows the values of 9 externally-calibrated parameters. The discount factor $\beta = 0.96$ and the interest rate on foreign bonds $R = 1.05$ are commonly used values for annual models. The parameter for the labor supply elasticity $\omega = 1.455$ is set following Mendoza [1991]. Regarding the production parameters, the asset's share in tradable production $\alpha = 0.08$ is set to target a capital to output ratio equal to 2, consistent with the Chilean economy. The iceberg trade cost $\xi = 0.21$ follows the estimation by Anderson and Van Wincoop [2004]. The fraction of the input cost subject to the working capital requirement ϕ varies widely depending on how it is estimated. We set its value to 0.2 so that the mean total credit-to-GDP ratio in a long-run simulation matches the data.[14] The coefficient on the borrowing constraint κ is set to 0.2 based on Mendoza [2010]. The amount of productive asset $L = 0.6$ is set to target the cross-country frequency of sudden stops of 7.7% [Eichengreen et al., 2008, Jeanne and Rancière, 2011]. The concavity parameter governing productivity-enhancing investment ρ is set to 1.5, which is within the range reported in the literature [Comin and Gertler, 2006, Akcigit and Kerr, 2018].

Table 1: Externally-Determined Parameters

Variable		Value	Source
β	Discount factor	0.96	Standard
R	Foreign bond interest rate	1.05	Standard
ω	Frisch elasticity $1/(\omega - 1)$	1.455	Mendoza [1991]
α	Asset share in production	0.08	Targets Capital to Output ratio (Chile)
ξ	Iceberg trade cost	0.21	Anderson and Van Wincoop [2004]
ϕ	Fraction of input subject to working cap. req.	0.2	Targets Total credit to GDP ratio (Chile)
κ	Coefficient on borrowing constraint	0.2	Mendoza [2010]
L	Amount of productive asset	0.6	Targets Frequency of Sudden Stops
ρ	Concavity of innovation investment	1.5	Median value from literature

Seven parameters related to firm dynamics and growth (σ^D,σ^X, η^D, η^{ED}, η^{EX}, η^X, and y^*) are jointly calibrated to match seven moments in a long-run simulation of the model and the associated

[14] According to the External Wealth of Nations Database by Lane and Milesi-Ferretti [2007], the average net foreign asset position in Chile in 1990-2005 is 38.2%. We target this value and set $\phi = 0.2$ accordingly.

stationary firm distribution with the Chilean data on firms' product portfolios described earlier.[15] The seven targeted moments are (1) the long-run aggregate growth rate, (2) the relative profit of non-exporting firms to exporting firms, (3) the share of single-product non-exporting firms, (4) the share of exporting firms among single-product firms, (5) the average number of products owned by non-exporting firms, (6) the average number of exported products owned by exporting firms, and (7) the share of exports in total revenue for exporting firms. Because foreign innovation on export lines corresponds to domestic innovation by foreign firms, its rate i^{FX} is set equal to the domestic innovation rate by domestic firms. Similarly, foreign innovation on domestic lines corresponds to export innovation by foreign firms, and thus its rate i^{FD} is set equal to the export innovation rate by domestic firms. Therefore, by setting the domestic parameter values, the exogenous foreign innovation rate on domestic lines and export lines are also determined. The values of these parameters and the corresponding targeted moments are listed in Table 2.

Table 2: Jointly-Determined Parameters

Variable		Value	Target	Model	Data
σ^D	Domestic innovation step size	0.06	Aggregate growth rate	2.5%	2.5%
σ^X	Export innovation step size	0.31	Relative profit of nonexporters to exporters	27.8%	26.2%
η^{ED}	Nonexporter entry coefficient	1.46	Share of single-product nonexporters	37.1%	38.3%
η^{EX}	Exporter entry coefficient	0.31	Share of exporters in single-product firms	20.9%	21%
η^D	Domestic innovation coefficient	2.97	Average number of products by nonexporters	2.24	2.56
η^X	Export innovation coefficient	0.53	Average number of exported products by exporters	1.05	1.7
y^*	Foreign demand	0.74	Export revenue share for exporters	30.5%	35.9%
i^{FX}	Foreign innovation rate on X lines	0.23	Domestic innovation rate by domestic firms		
i^{FD}	Foreign innovation rate on D lines	0.02	Export innovation rate by domestic firms		

The two aggregate shocks to the economy are the productivity shock to the final tradable sector ε_t^A and the interest rate shock to the return of the foreign bond ε_t^R. We take the stochastic process for these shocks from Mendoza [2010], in which ε_t^A and ε_t^R follow a joint discrete Markov process with two realizations for each variable. In particular, ε_t^A takes the values ± 0.0134 and ε_t^R takes the values ± 0.0196 with the same autocorrelation 0.59 and a negative correlation -0.67 between

[15] We derive the stationary firm distribution using the mean entry and innovation rates in a long-run model simulation. The detailed steps are explained in Appendix A.4.

ε_t^A and ε_t^R. Finally, the foreign factor prices divided by foreign productivity, r^{L*} and w^*, are set equal to the long-run simulation mean of corresponding domestic values R_t^L and W_t divided by A_t. We solve the model numerically using a global method to deal with the occasionally binding constraint, as we explain in detail in Appendix A.2.

3.3 Model Validation

Having calibrated the model we assess its quantitative success by comparing non-targeted micro and macro moments. In particular, the model matches the firm-level product portfolio dynamics in normal times, the product and size distributions, and the differences between exporters and non-exporters. From an aggregate perspective the calibrated model is consistent with the macroeconomic time series during the Chilean sudden stop. Note that these are moments that are not targeted in the calibration.

3.3.1 Microeconomic Moments

First, we assess the ability of the model to match patterns in terms of firms' product portfolios. We study how a firm's existing product portfolio shapes the addition or removal of products from the domestic and export markets.

We study these transitions based on a balanced panel for 1996-1997 (i.e. prior to the sudden stop) with 3503 firms out of which 825 (23.5%) are exporters in 1996 and 870 (24.8%) are exporters in 1997.[16] We focus on 4 transitions: i) adding new products to the domestic market, ii) adding domestic products to export markets, iii) dropping products from the domestic market, and iv) dropping export products turning them to solely domestic products. To this end, we estimate the following equation:

$$Y_f = \beta_1 \cdot X_f + \phi_s + \epsilon_f, \tag{32}$$

where Y_f is a dummy variable taking a value of one if a transition takes place and zero otherwise, and X_f captures the existing number of products in a firm's portfolio. The regression includes industry (s) fixed effects (at the four-digit level) in order to compare across firms within a given industry.[17]

[16] Appendix Table A.3 documents the raw frequency of different type of firms' product portfolios' transitions.

[17] Given the inclusion of fixed effects, we estimate this equation using a linear probability model. The results are robust when a probit model is used for estimation.

Panel A in Table 3 summarizes the estimation results in the data. The structure of the model assumes that the total number of product determines the probability of adding a new domestic product to the firm's portfolio. Therefore, column 1 studies how the total number of products in a firms' portfolio affects the probability of introducing a new domestic product. We estimate that one additional product produced by a firm in period t is associated to a 0.042 standard deviation increase in the probability that a firm introduces one or more new domestic products between periods t and $t+1$. This coefficient is relatively large, as the unconditional probability of adding new domestic product (shown in the first row of Table A.3) is 0.15 and this is equivalent to a 1.5 percentage point increase. Because in the model a firm innovates to transform to start exporting a domestic product it already produces, it is the number of domestic products the key determinants of entering the foreign market at the product level. Column 2 indicates that one additional domestic product sold by a firm in period t leads to a 0.017 standard deviation increase in the probability that a firm introduces a new export product between periods t and $t+1$. Similarly, column 3 shows that one additional domestic product sold by a firm in period t is associated to a 0.181 standard deviation increase in the probability that a firm drops a domestic product between periods t and $t+1$. Because exporter products become domestic after being innovated upon by foreigners, column 4 controls for the number of exported products of firms that export in t. The estimated equation shows that one additional exported product sold by a firm in period t is associated to a 0.047 standard deviations increase in the probability that a firm drops an exported product (subsequently sold only domestically) between periods t and $t+1$.

Panel B in Table 3 contrasts these results with model generated firm-product data. In particular, we simulate 5000 firms and estimate a regression equivalent to (32) on the model-based data for the 4478 firms present in two consecutive periods without any aggregate shock and starting from the stationary firm distribution. This equation excludes of course industry fixed effects, as the model corresponds to a single industry.

$$Y_f = \beta_0 + \beta_1 \cdot X_f + \epsilon_f. \tag{33}$$

All the elasticities have the same sign and significance in model and data. Moreover, while the estimated coefficients are larger in the simulated data, the model effectively replicates the ranking between the coefficients. In fact, the lowest coefficient is in both cases the one governing the transition to exports and the highest one is the elasticity related to dropping a domestic product.

These novel product level facts have not been documented in the literature before and our tractable framework, while not explicitly targeting them, is consistent with these patterns.

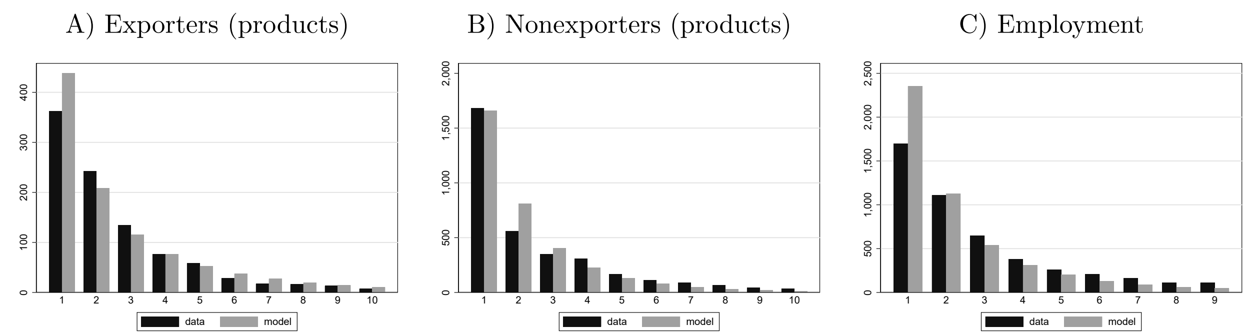
Table 3: Transitions and Firm Characteristics

	(1) Not Produced to Domestic	(2) Domestic to Exported	(3) Domestic to Not Produced	(4) Exported to Domestic
Panel A: Data				
Number of Products	0.042***			
	(0.006)			
Number of Domestic Products		0.017***	0.181***	
		(0.004)	(0.006)	
Number of Exported Products				0.047***
				(0.011)
Industry Fixed Effects	Yes	Yes	Yes	Yes
Observations	3996	3996	3996	870
Panel B: Model				
Number of Products	0.175***			
	(0.007)			
Number of Domestic Products		0.022***	0.266***	
		(0.003)	(0.006)	
Number of Exported Products				0.027**
				(0.013)
Observations	4478	4478	4478	1120

NOTE: Panel A reports the results of the estimation of equation (32). Panel B reports the results of the estimation of equation (33) using simulated data. The dependent variable in each column is (1) the probability that a firm introduces to the domestic market a product not sold previously, (2) the probability that a firm introduces to the export market a product previously sold domestically, (3) the probability that a firm withdraws a product from the domestic market, subsequently not selling it, and (4) the probability that a firm withdraws a product from the export market, subsequently selling it domestically.

Firm dynamics are key to the predictions of the model regarding the evolution of productivity and trade. Therefore, we further validate the model using the empirical firm-size distribution. In the model, each firm is characterized by the number of domestic and export lines it owns, (n^D, n^X). Let $\delta_t(n^D, n^X)$ denote the measure of firms that own n^D domestic lines and n^X export lines at period t. We derive the stationary firm-size distribution $\delta(n^D, n^X)$ using the long-run simulation mean of entry and innovation rates.[18] Note that while the shares of exporters and non-exporters selling a single product are targeted in the model calibration, the rest of the distribution is nontargeted. We also evaluate the firm-size distribution in terms of employment. Because model and data can be in different scales, we evaluate the size distribution relative to the smallest firm.

Figure 3: Model vs. Data: Size Distribution



NOTE: Panel A shows the distribution of the total number of products sold by exporting firms in the data and in the simulated model. Panel B does the same for non-exporting firms. Panel C shows the distribution of total employment in the data and in the simulated model. Employment is reported in multiples of the employment of the smallest firm.

Figure 3 compares model and data in terms of firm size. Panel A focuses on firms that export at least one of their products, showing the total number of products sold by these firms. Panel B does the same exercise for non-exporting firms. The model closely tracks the fact that most firms sell a small number of products, and the number of firms declines monotonically with firm size. Panel C shows the total firm-size distribution in terms of employment. Once again model and data show a similar pattern, with most firms concentrated in lower size bins.

Note also that the model displays a tight connection between firm-level employment and the number of products sold. We test the validity of this assumption by estimating the following regression:

$$\log \mathrm{E}_f = \beta_1 \cdot \mathrm{N}_f + \phi_s + \epsilon_f \, , \tag{34}$$

where E_f denotes total firm-level employment, N_f denotes the total number of products sold, and ϕ_s represents industry fixed effects. Table 4 shows the result of this regression for both the model and data. The results confirm the implicit model assumption that firms with more products have

[18]The detailed steps are explained in Appendix A.4.

more workers. Given the parsimony of the model and the non-targeted nature of this moment, it is not surprising that the magnitudes of the coefficients are relatively different.

Table 4: Firm employment and product portfolios

	(1) Data	(2) Model
	(log) Employment	(log) Employment
Total number of products	0.109***	0.880***
	(0.014)	(0.007)
Industry Fixed Effects	Yes	
Observations	5185	4937

NOTE: This table reports the results of the estimation of equation (34). Column 2, corresponding to the model simulation, excludes industry fixed effects because the model corresponds to a single industry. All variables are standardized to have mean zero and standard deviation one. ***, **, and * denote statistical significance at a 1, 5 and 10 percent confidence level.

The quantitative trade literature has systematically documented and validated its models by comparing exporting and non-exporting firms [Bernard and Jensen, 1999]. Because of the central role of trade dynamics in our framework, we further validate the calibrated model based on exporter premia in terms of revenue, employment and productivity. For that purpose, we estimate the following firm-level regression:

$$\log \mathrm{Y}_f = \beta_1 \cdot \mathrm{Exporter}_f + \phi_s + \epsilon_f\,, \tag{35}$$

where Y_f is each of these outcomes, $\mathrm{Exporter}_f$ is a dummy that indicates if a firm exports at least one product, and ϕ_s represents industry fixed effects. Table 5 shows that both the model and data generate strong exporter premia in terms of revenue, employment, and estimated firm-level revenue productivity. Moreover, the quantitative fit between model and data in terms of employment and revenue is surprisingly accurate.[19]

3.3.2 Macroeconomic Moments

Having shown the close fit between model and data regarding non-targeted microeconomic moments, we now assess the response of macroeconomic outcomes in the model to a sudden stop,

[19] As can be expected when using estimated revenue productivity, the fit between model and data in this case is mostly qualitative.

Table 5: Exporter premia

	(1)	(2)	(3)	(4)	(5)	63)
	Data			Model		
	Revenue	Employment	Productivity	Revenue	Employment	Productivity
Exporter	1.07***	1.12***	0.61***	1.02***	0.92***	2.19***
	(0.03)	(0.03)	(0.03)	(0.03)	(0.03)	(0.142)
Industry Fixed Effects	Yes	Yes	Yes			
Observations	5185	5185	4923	4937	4937	4937

NOTE: This table reports the results of the estimation of a regression of log revenue, log employment or productivity on a dummy variable equal to one for exporting firms and industry fixed effects. Note that industry fixed effects are not included in columns 4 through 6 given that the model corresponds to a single industry. Firm log revenue, log employment, and productivity are standardized to have mean zero and standard deviation one. The estimation of firm productivity is described in Appendix G.3. ***, **, and * denote statistical significance at a 1, 5 and 10 percent confidence level.

and compare it to Chile's 1998 sudden stop. In 1998, Chile faced a severe sudden stop common to several other emerging markets as a consequence of the Russian default that year and the Asian financial crisis that had started in mid 1997. This event was both unanticipated and exogenous to the Chilean economy, which had few direct ties with the countries in which this crisis originated. The sudden stop sharply decreased capital inflows and GDP growth. Calvo and Talvi [2005] discuss this episode in detail.[20]

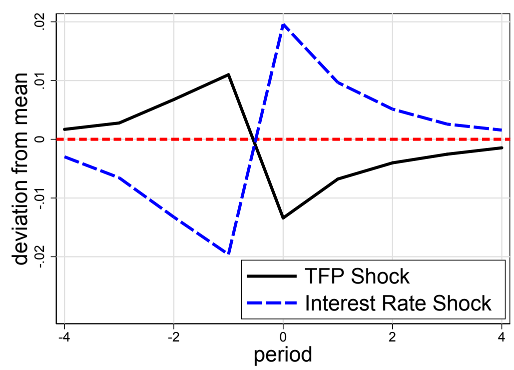
To obtain macroeconomic time series from the model, we simulate the model for 10,000 periods with stochastic shocks. Figure 4 illustrates the average path of productivity shocks and interest rate shocks, which cause the sudden stop.[21] Before a sudden stop occurs, the aggregate productivity shock and the interest rate shock are favorable, implying that the country is facing low interest rates and high exogenous productivity. When these favorable shocks reverse to bad shocks of low productivity and a high interest rate, the asset price declines and forces the borrowing constraint to bind. Households are then forced to cut consumption, which reduces the asset price further,

[20] Note that in 1998, Chile removed its capital controls, which had been in place since 1991. Andreasen et al. [2019] and Andreasen et al. [2021] study the effect of capital controls on firm dynamics and welfare. This structural change might have facilitated the outflow of capitals during the crisis.

[21] Following Bianchi and Mendoza [2018], sudden stops are identified as events in which the current account adjusted for its trend is at least two standard deviations above its mean. Under this definition, the unconditional probability of sudden stops in the model is 7.7%, which is in line with empirical estimations in Eichengreen et al. [2008] and Jeanne and Rancière [2011].

and the amplification mechanism is set in motion. These developments of exogenous shocks and the subsequent endogenous dynamics are all consistent with Mendoza [2010].

Figure 4: Sudden Stops Dynamics: TFP and interest rate shocks



Note: This figure shows the path of the productivity shock and the interest rate shock (in deviation from the mean) around sudden stop episodes.

Figure 5 show the average dynamics of manufacturing output, the asset price, the trade balance to GDP ratio and the current account to GDP ratio around sudden stops in the model and in the data during the Chilean crisis. In all cases, the simulated model (dashed blue lines) follows a similar path compared to the actual data (solid black lines). Recalling that we calibrate our stochastic processes following Mendoza [2010], not surprisingly, the macro time series in our model are consistent with the literature. Although Mendoza [2010] did not target the Chilean sudden stop, the average sudden stop in the calibrated model shows a reasonable fit to the macro time series. For instance, manufacturing output falls by 9.2% in the model and 7.7% in the data, while the asset price, the trade balance-to-GDP and current account to GDP ratio both track their empirical counterparts during the sudden stop.[22,23]

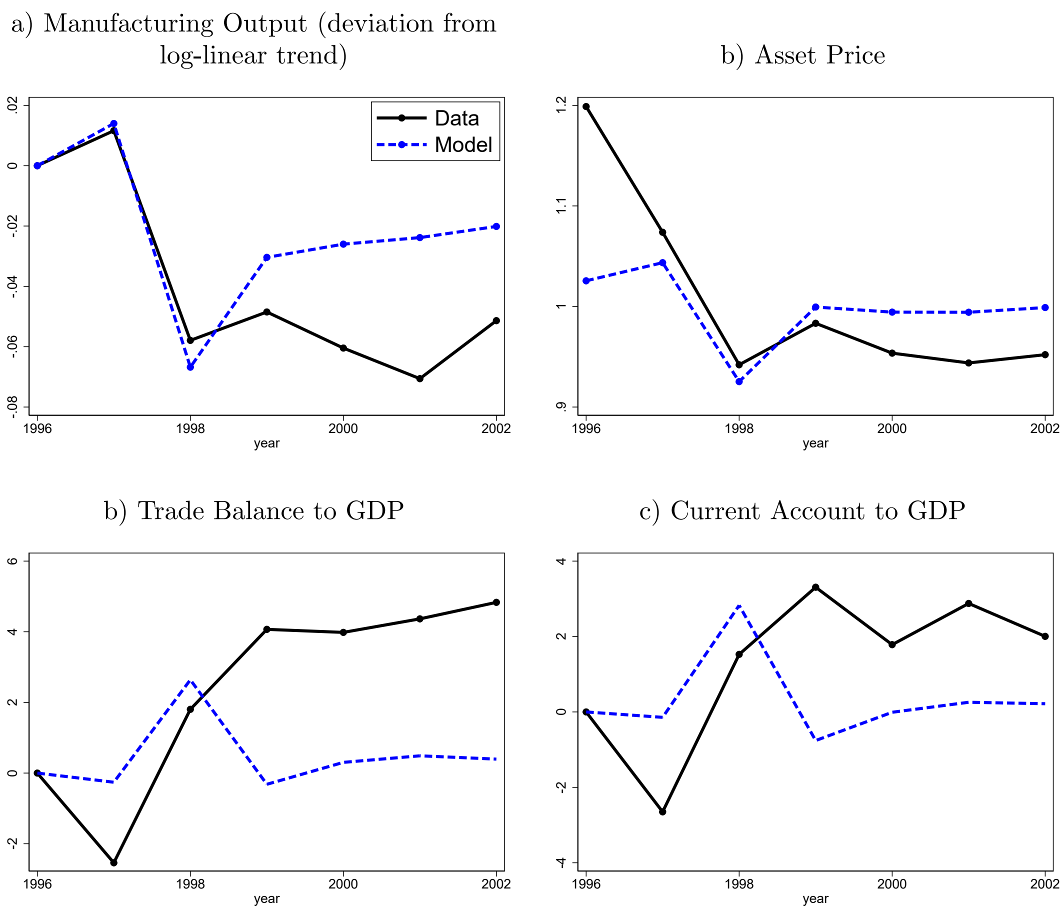
4 Quantitative Analysis

Having validated the model, we proceed to study how sudden stops affect the dynamics of firm's product portfolios, trade, and productivity. We first use the model to illustrate the key novel

[22]Following Mendoza [2010], we compute Tobin's Q as the empirical counterpart of the asset price.

[23]The rather quick rebound from the crisis is a common feature in this class of models as the constraint only binds for one period.

Figure 5: Macroeconomic Outcomes



NOTE: This figure shows the path of manufacturing output (in deviation from log-linear trend), the asset price, the trade balance to GDP ratio (in levels) and the current account to GDP ratio (in levels) around sudden stop episodes. All series are normalized to zero in the initial period. Data on manufacturing output, the trade balance to GDP ratio and the current account to GDP ratio are obtained from the Central Bank of Chile. In the data, we use Tobin's Q for the asset price, computed following Mendoza [2010] as the ratio between the market value of equity and debt outstanding over the book value of equity. It is computed using data on listed firms from Worldscope, as the median across firms.

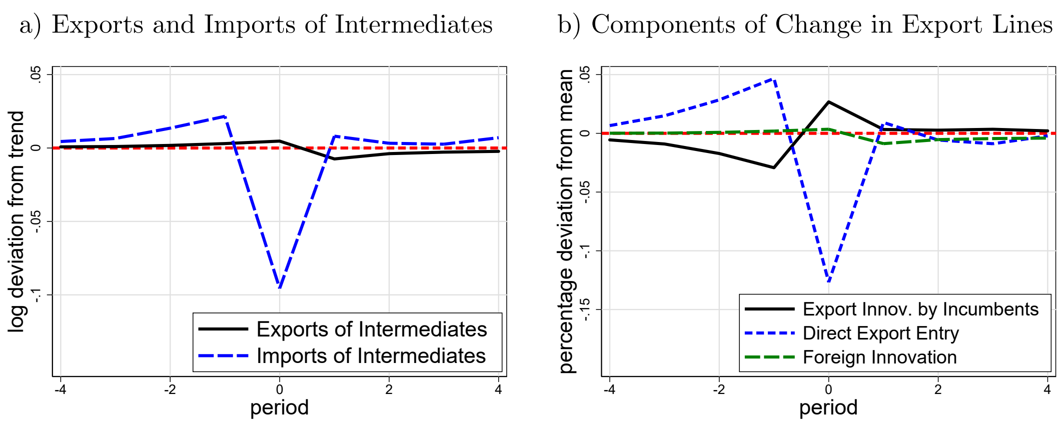
mechanisms of our framework, and later use counterfactual simulations to understand the impact of these mechanisms on welfare.

4.1 Trade and Productivity Dynamics During Sudden Stops

Because the crisis has a differential effect on domestic demand and it generates an exchange rate depreciation, we expect that incumbent firms will tilt their product portfolio toward exported products.

Figure 6 studies the dynamics of intermediate imports and exports. Panel 6a shows that, during a sudden stop, imports fall substantially while exports are relatively unaffected. To better understand these dynamics, it is useful to distinguish between the intensive and the extensive trade margins.[24] The key determinant of imports of intermediate goods is local demand, which is given by equation (4). Because the local demand collapses during the crisis, every domestic product and every imported product sees its demand fall. Thus, most of the sharp decrease in imports is an intensive margin adjustment. There is a sharp contrast with exports in this regards. Equation (11) shows that as long as foreign demand is unaffected by the crisis, the value of exports per product (the intensive margin) grows at a constant rate.[25] Thus, the adjustment of exports occurs at the extensive margin.[26]

Figure 6: Sudden Stop Dynamics: Imports and Exports



NOTE: This figure shows the path of exports and imports of intermediate goods (in log deviation from trend) and the components of the change in export lines (in percentage deviation from the mean) around sudden stop episodes.

To understand the drivers of the mild decrease on the extensive margin of exports, we need to revisit the drivers of the share of exported products. Following equation (27), the change in

[24]Trade growth at the intensive margin refers to changes in trade volumes of continuing firm-products pairs, while trade growth at the extensive margin refers to firms' addition or withdrawal of traded products as well as firm entry and exit.

[25]Several of Chile's trading partners also experienced the consequences of the Russian default, therefore, it can be argued that the demand for Chilean exports also decreased. For the sake of simplicity we abstract from shocks to Y_t^* as they will increase the dimensionality of the problem without bringing further insights.

[26]In Appendix Section G.6 we show that this is consistent with the behavior of exports and imports during Chile's 1998 sudden stop.

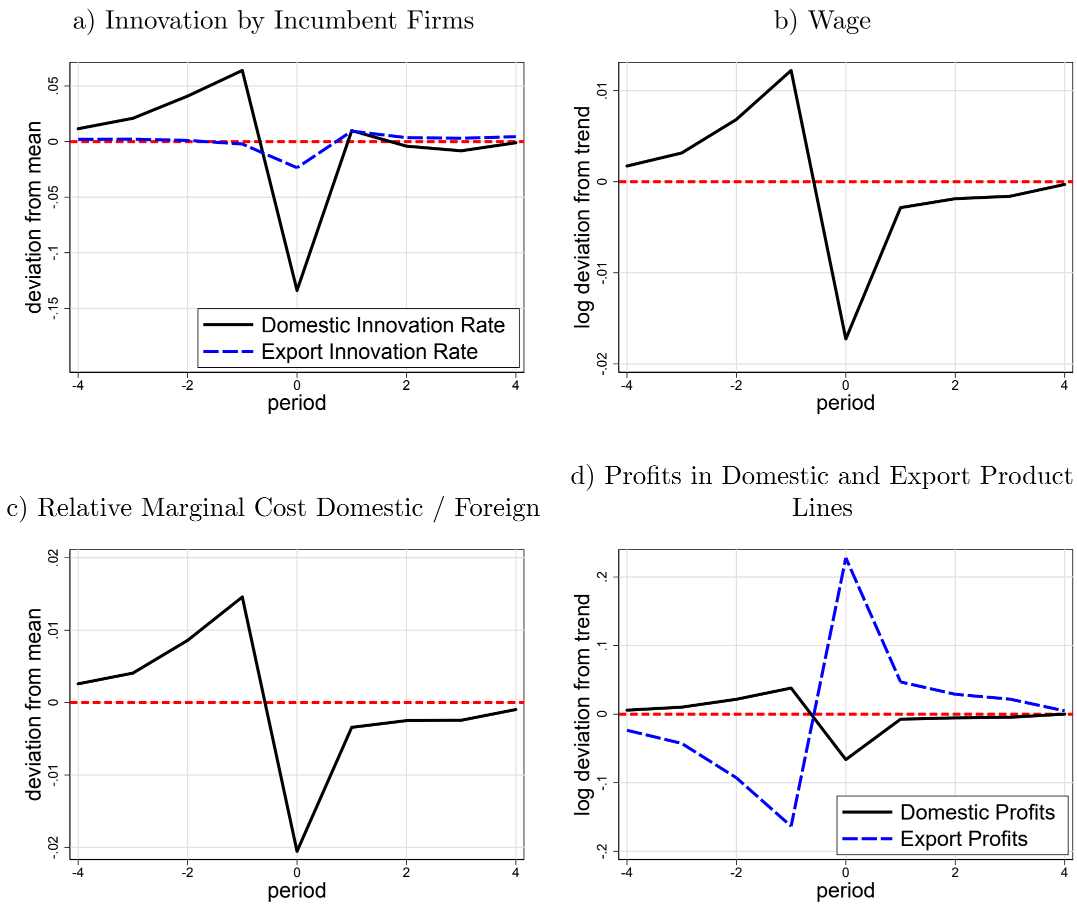
the share of exported products can be decomposed into three terms: the addition of export lines by incumbent firms, the addition of export lines by new firms that start exporting immediately, and the negative pressure of foreign innovation. Figure 6b indicates that the decline in the share of export lines is due to a large decline in direct export entry (i.e. entry into exporting by new firms) while the innovation by incumbent firms is rather stable. The reason for this difference between new export lines by incumbents versus direct export entry becomes clear from comparing equations (19) and (24). The first one, that determines the introduction of new export lines by incumbents, depends on the difference in value between export and domestic lines, which increases during a sudden stop. In contrast, the latter depends only on the value of export lines, which faces a sharp drop in a sudden stop.[27] Another difference comes from the replacement rate d_t in equation (27). Incumbent export innovation can materialize only when the domestic line is not replaced by other innovations, thus a drop in the replacement rate during a sudden stop increases the effective success probability of incumbent export innovation.

Figure 7 uncovers another interesting asymmetry, this time between exporting and domestic innovation by incumbent firms. Panel 7a shows that the export innovation effort of incumbent firms decreases by 3% while the domestic innovation effort decreases by 20%.[28] To shed light on this differential behavior, we need to study the dynamics of the value of export product lines. Although exporters are exposed to the local market only in their domestic sales, all their production is done locally. Therefore, the 3% decline in the domestic wage depicted in panel 7b decreases their production cost. Moreover, the asset rental rate also falls in the local economy, triggering a real exchange rate depreciation of 3.5% in panel 7c. The insulation from the local demand and the depreciation of the exchange rate imply radically different dynamics for domestic and export profits. Panel 7d shows that export profits increase by 23% above trend, while domestic profits fall by 7%. As the domestic demand recovers and wages rebound, most of the gap closes within a year. However, even three years after the episode, we see relatively higher profits from the foreign market. Because the value of export product lines depends on future discounted profits, this future gap should be driving the differential export innovation effort of incumbent firms. Note that the interest rate increases and consumption drops during the crisis, decreasing the present discounted value of profits for both domestic and exported products. This negative effect dominates, and

[27]Note that, consistent with the model, in Appendix Table A.4 we document a larger decline in export entry during Chile's sudden stop among entrants (firms not active in the previous year) than among incumbent firms.

[28]Although the export innovation rate i_t^X drops by 3%, the effective innovation rate $(1 - d_t)i_t^X$ increases due to a drop in the replacement d_t, as shown in Figure 6b.

Figure 7: Sudden Stop Dynamics: Differential Innovation by Incumbents



NOTE: This figure shows the domestic and export innovation rates (in deviation from the mean), the domestic wage (in log deviation from trend), the ratio of domestic to foreign marginal cost (in deviation from the mean), domestic and export profits (in log deviation from trend).

thus even the exporting effort decreases. Nevertheless, the relative decrease is six times larger for domestic innovation, and we expect incumbents to tilt their product portfolios towards exported products. This prediction is at the core of our mechanism and the next section uses Chilean product data to test it.

The overall decrease in innovation effort by entrants and incumbents affects the dynamics of the productivity index. Using a log approximation on equation (30) that defines the growth of the productivity index, we get the following decomposition:

$$g_t \approx e_t^D \sigma^D + e_t^X \sigma^X + (\theta_{t-1}^D + \theta_{t-1}^X) i_t^D \sigma^D + \theta_{t-1}^D (1 - d_t) i_t^X \sigma^X + i^{FD} \sigma^X \,, \tag{36}$$

where the first two components represent innovation by new firms entering with either their first purely domestic or exported product, the third and fourth components represent domestic and export innovation by incumbent firms, and the last component is the foreign innovation effort. Figure 8 shows the effect of the sudden stop on each of these components and their effect on the medium-run dynamics of domestic GDP. Panel 8a decomposes the total growth rate of the productivity index on each of its components. First, innovation by incumbent firms accounts for 64% of the growth rate on the productivity index (50% is domestic, 14% is export innovation), innovation by entrants accounts for 20% (14% export and 6% domestic), while the remainder 16% is due to innovation by foreign firms. Second, the sudden stop decreases all the components of the productivity index, but the lion's share of the productivity slowdown is due to incumbent's domestic innovation. Third, the boom that precedes the crisis fuels domestic innovation and generates an increase in endogenous productivity that further stimulates the economy. Akıncı and Chahrour [2018] show that news shocks have the potential to generate booms that precede sudden stops. And as discussed by Gornemann et al. [2020], endogenous growth affects the expectations of future productivity triggering wealth effects that act similarly to news shocks. Panel 8b compares the log GDP dynamics with its 10 years pre-sudden stop trends. Note first that the log GDP is above its 10 years trend before the crisis. By fueling the boom, endogenous innovation is likely to worsen the over-borrowing externality that leads to a sudden stop, therefore affecting macroprudential policy design. Because the sudden stop affects the *growth* of the productivity index, it persistently decreases the *level* of the productivity index, generating hysteresis in every growing variable. In fact, panel 8b shows that five years after the crisis GDP remains 1.5% below its pre-crisis trend.[29] These patterns are consistent with the empirical cross-country experience [Cerra and Saxena, 2008, Reinhart and Rogoff, 2009]. In Section 4.3 we study the welfare consequences of this persistent productivity loss.

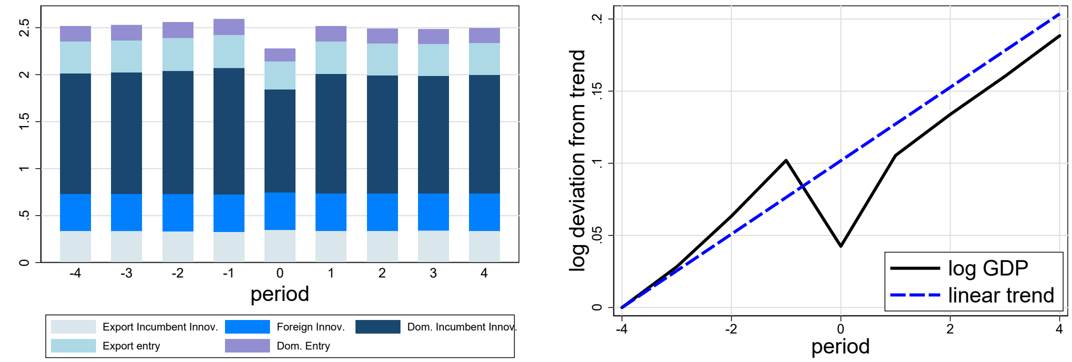
We perform further analyses in appendices. Because of the central role of the real exchange rate dynamics during crises, Appendix C decomposes the decline in the relative marginal cost into cyclical changes in factor prices and a decline in productivity. We find that, because of its persistence, a decline in productivity is the main driver of product dynamics after crises. Appendix D examines how the possibility of future sudden stops affects firms' innovation decisions ex-ante and shows that it discourages domestic innovation but slightly encourages export innovation

[29] Because the foreign country grows at an exogenous rate, the model features semi-endogenous growth. Therefore, log GDP will eventually reach its long-run trend and the hysteresis in the limit is not permanent.

Figure 8: Sudden Stops, Productivity and GDP

a) Decomposition of the Growth Rate of the Productivity Index A_t

b) Log GDP and its trend



Note: Panel a) shows the components of a decomposition of the growth rate of the productivity index, based on equation (36). Panel b) shows the path of GDP (in log deviation from trend) and its trend.

through the expectation of high export profits during crises. Further, Appendix E solves an alternative model without a working capital requirement ($\phi = 0$) and shows that a smaller drop in the relative marginal cost in this model leads to a muted response of export innovation during and after a crisis. Finally, Appendix F shows a version of the model without a borrowing constraint has even more muted crisis dynamics, and the boom-bust cycle around sudden stops becomes symmetric, which is inconsistent with the empirical regularities of sudden stops.

4.2 The Dynamics of Firm Product Portfolios during the Sudden Stop

The source of the productivity cost of a sudden stop is the disruption it triggers on the dynamics of firms' product portfolios. Before quantifying the welfare effects of the productivity slowdown, we test the transmission channel by studying the behavior of Chilean manufacturing firms. First, at the extensive margin, the larger decrease in incumbents' domestic innovation rate with respect to their export innovation rate predicts that incumbents should tilt their portfolios toward exported products during the sudden stop. Second, because of the exchange rate depreciation and the relative decline of the domestic demand with respect to the foreign demand, the model predicts that the profits and revenue of non-exporting firms falls relative to exporters. Third, because the crisis is financial in nature, we should expect that the relative revenue advantage from exporting a product is magnified in industries that rely more on external finance.

Our empirical analysis is based on the dataset of firms' product portfolios described earlier in Section 3.1. We focus on a panel of firms between 1996 and 1999, with the last two years being considered as crisis years. We observe the revenue of each product sold by each firm, and can distinguish between revenue from domestic or export sales for each individual product. In addition, we see total firm revenue, profits, and we estimate firm-level productivity.[30] We compare the empirical results with model-based data from a simulated panel of firms. We simulate two pre-crisis and two crisis periods to be consistent with the horizon of the Chilean data. We start with a panel of 5000 firms at the stationary firm distribution.

We first document that, during the crisis, incumbents decrease relatively more the rate at which they introduce domestic products when compared to the rate at which they start exporting a product that they previously sold domestically. To this end, we estimate the following regression:

$$Y_{ft} = \beta_1 \times \mathbf{1}[\text{Sudden Stop}_t] + \phi_f + \epsilon_{ft}, \tag{37}$$

in which Y_{ft} is a dummy variable taking a value of one if a given transition takes place between years $t-1$ and t, and zero otherwise. This regression includes firm fixed effects and a dummy variable indicating the sudden stop period. The results are shown in the first two columns in Table 6. We find that during the sudden stop, the probability that firms introduce domestic products falls relative to the previous period. In contrast, the probability of exporting products previously sold only domestically, falls to a much lesser extent. Specifically, column 1 indicates that the probability that a firm introduces one (or more) new domestic products during the sudden stop is 8.3 percentage points lower (0.22 standard deviations lower) than in the previous years. Column 2 shows that the probability that a firm introduces one or more new exported products previously sold domestically is 1.8 percentage points lower (0.10 standard deviations lower) during the sudden stop than before. Columns 3 and 4 replicate the analysis with model simulated data. Note that the probability of adding a domestic product decreases by a similar magnitude in model and data while the probability of starting to export is not significantly affected by the sudden stop in the model. Thus, the data confirms one of the key drivers of the productivity loss in the model, the sharp decrease of domestic innovation by incumbent firms.

Second, we compare the empirical firm-level dynamics of revenue, profits and productivity with

[30] The estimation of firm productivity is described in Appendix G.3.

Table 6: Probability of Adding Products during the Sudden Stop

	(1)	(2)	(3)	(4)
	Data		Model	
	Not Sold to Domestic	Domestic to Exported	Not Sold to Domestic	Domestic to Exported
1[Sudden Stop$_t$]	-0.083***	-0.018***	-0.091***	-0.003
	(0.006)	(0.003)	(0.008)	(0.004)
Firm Fixed Effects	Yes	Yes	Yes	Yes
Observations	15523	15523	12533	12533

NOTE: This table reports the results of the estimation of equation (37). The transitions in each column are (1) a firm introduces to the domestic market a product not sold previously, (2) a firm introduces to the export market a product previously sold domestically. ***, **, and * denote statistical significance at a 1, 5 and 10 percent confidence level.

their model counterpart during the sudden stop. In particular, we estimate firm-level regressions with these outcomes as functions of a dummy variable indicating the sudden stop period interacted with an exporter status dummy variable. We include firm and year fixed effects.

$$\log\left(\mathrm{Y}_{ft}\right) = \beta_1 \times \mathbf{1}[\text{Sudden Stop}_t] \times \mathbf{1}[\text{Exporter}_{ft}] + \phi_f + \delta_t + \epsilon_{ft}. \quad (38)$$

The first three columns in Table 7 show the empirical results while the last three columns report their model-based counterpart. We find a relative increase in revenue (column 1), profits (column 2) and productivity (column 3) for exporting firms relative to non-exporting firms during the sudden stop period. Specifically, exporters experience a 0.04 standard deviations increase in revenue, a 0.063 standard deviations increase in profits, and a 0.093 standard deviations increase in productivity during the sudden stop relative to the change experienced by non-exporters. The magnitude and significance of these coefficients are closely replicated in columns 4 through 6 by their model counterpart. Therefore, the data also provide support for the exchange rate and demand channels opening a performance wedge between exporters and non-exporters during the crisis.

Table 7: Revenue, Profits and Productivity of Exporters vs. Nonexporters

	(1)	(2) Data	(3)	(4)	(5) Model	(6)
	Revenue	Profits	Productivity	Revenue	Profits	Productivity
$\mathbf{1}$[Sudden Stop$_t$] $\times$ $\mathbf{1}$[Exporter$_{ft}$]	0.040***	0.063***	0.093***	0.115***	0.120***	0.091***
	(0.006)	(0.010)	(0.016)	(0.016)	(0.013)	(0.009)
Year Fixed Effects	Yes	Yes	Yes	Yes	Yes	Yes
Firm Fixed Effects	Yes	Yes	Yes	Yes	Yes	Yes
Observations	21213	20797	19547	18711	18711	18711

NOTE: This table reports the results of the estimation of equation (38). In columns 1 and 4 the dependent variable is log revenue. In columns 2 and 5 the dependent variable is log profits. In columns 3 and 6, the dependent variable is productivity. Firm log revenue, log profits and productivity are standardized to have mean zero and standard deviation one. The estimation of firm productivity is described in Appendix G.3. ***, **, and * denote statistical significance at a 1, 5 and 10 percent confidence level.

Third, we use cross-industry variation to assess the financial nature of the transmission channel. In the model economy, the relative gap between exporters and non-exporters during sudden stops is partially driven by the working capital constraint. Recall that the representative final good producer pays in advance a fraction ϕ of the cost of intermediate inputs (2). In normal times when the Lagrangian multiplier μ_t is zero, profits from domestic sales (equation (8) for non-exporters and equation (10) for exporters) do not depend on ϕ. When a sudden stop occurs and the multiplier μ_t is strictly positive, profits from domestic sales depend negatively on the extent of financial dependence ϕ, while profits from export sales (eq.(13)) are not affected by ϕ. Because working capital is more important in more financially dependent industries, a sudden stop should generate a larger decline in domestic relative to export sales in industries with a larger degree of financial dependence.

We use our data at the most granular level, with each observation corresponding to a firm $\times$ product $\times$ market (domestic or export) $\times$ period combination. We compile data on industries' external finance dependence from Rajan and Zingales [1996]. We estimate the following equation of revenue as a function of the interaction between a dummy for exported products and a dummy for the sudden stop years. In addition, we include the triple interaction between year dummies, the exported product dummy, and the financial dependence measures. We include firm and product-

Table 8: Sudden Stops and External Finance Dependence

$\mathbf{1}[\text{Sudden Stop}_t] \times \mathbf{1}[\text{Exported}_m] \times \text{Fin. Dep.}_p$	0.270***
	(0.086)
Firm Fixed Effects	Yes
Product-Market Fixed Effects	Yes
Product-Year Fixed Effects	Yes
Market-Year Fixed Effects	Yes
Observations	41804

NOTE: This table reports the results of the estimation of equation (39). Log revenue and financial dependence are standardized to have a mean zero and standard deviation one. ***, **, and * denote statistical significance at a 1, 5 and 10 percent confidence level.

market, product-year and market-year fixed effects.

$$\log\left(\text{Revenue}_{fpmt}\right) = \beta_1 \times \mathbf{1}[\text{Sudden Stop}_t] \times \mathbf{1}[\text{Exported}_m] \times \text{Fin. Dep.}_p + \phi_f + \rho_{pm} + \delta_{pt} + \nu_{mt} + \epsilon_{fpmt}. \tag{39}$$

This regression compares the gap in revenue between domestic and export sales caused by sudden stops in industries with different degree of financial dependence. The results are reported in Table 8. The positive term on the triple interaction shows that the gap in revenue from exporting increases relatively more in more financially dependent industries. This suggests that domestic demand for intermediate goods decreases more relatively to foreign demand in more financially dependent industries.[31] Therefore, the data supports a financial channel driving the gap between the domestic and foreign demand for intermediate goods.

Summing up all the previous results, we have described in detail for the first time in the literature the differential dynamics of revenue and product entry rates between export and domestic markets during a sudden stop episode. These findings are consistent with the model's predictions and further validate the calibrated model which we will use in the next section for counterfactuals.

[31] This difference is economically significant. Consider the total elasticity of revenue to the interaction between the sudden stop dummy and the exported dummy (i.e. the additional revenue obtained from export sales relative to domestic sales during the sudden stop relative to the previous years). This elasticity is -0.06 at the 10th percentile of financial dependence and 0.05 at the 90th percentile.

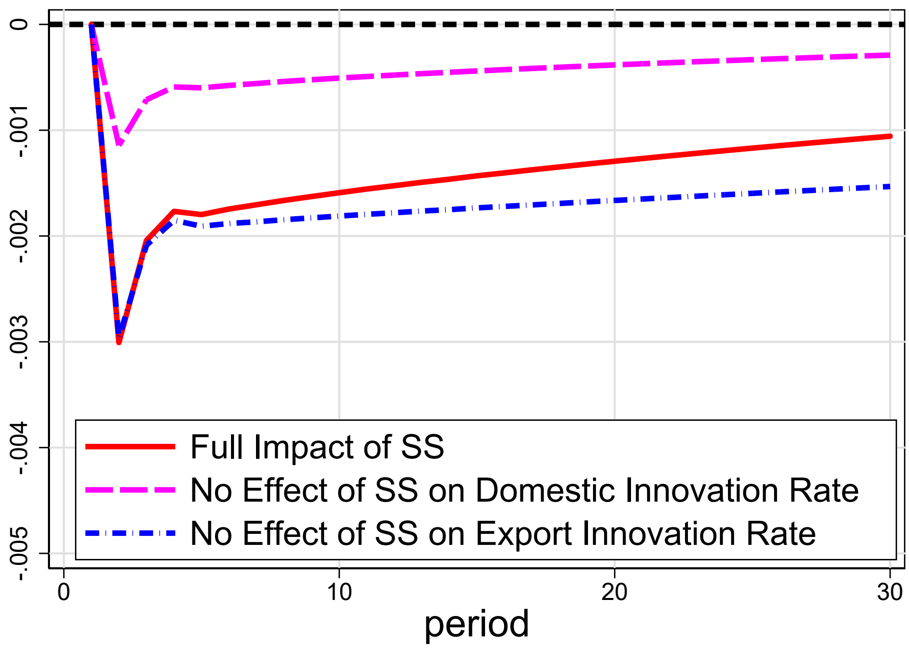
4.3 Sudden Stops, Welfare and Productivity

Finally, we measure to what extent the growth and trade dynamics account for the welfare loss caused by sudden stops. We conduct the following simulations: we set the initial state of the economy as the average state when sudden stops take place in the previous simulation. Then we create two economies facing different shocks: the first economy receives a good shock of high TFP and a low interest rate in period 1. The second economy receives a bad shock of low TFP and a high interest rate in period 1. Only this second economy faces a sudden stop, triggered by the bad shock. For the subsequent periods, both economies face the same random shocks. Henceforth, we refer to the first economy as the *no-SS economy*, and the second economy as the *SS economy*. We simulate these two economies a thousand times with stochastic shocks from period 2 onward, and compare the average productivity paths and expected welfare. This analysis allows us to compute the average productivity and welfare losses due to a sudden stop.

We also conduct the following counterfactual analyses to disentangle the effects of the product and trade dynamics on productivity and welfare. In the first counterfactual, we take the path of the productivity growth rate g_t from the no-SS economy and feed it into the SS economy. In this economy, a sudden stop happens in period 1, but it does not affect productivity at all. In the second counterfactual exercise, we take the path of the domestic innovation rate i_t^D from the no-SS economy and feed it into the SS economy to assess how much the domestic innovation rate accounts for the productivity loss and the welfare loss. Finally, in the third counterfactual we take the path of the export innovation rate i_t^x from the no-SS economy and feed it into the SS economy.

Figure 9 plots the productivity loss in percentage terms in each counterfactual simulation. At each period after the sudden stop, we calculate the percentage difference of the counterfactual productivity index relative to the no-SS level. A negative number reflects that the counterfactual economy has a lower productivity index at that horizon after a sudden stop. Because the model features semi-endogenous growth, every counterfactual has no productivity loss in the limit but the model does generate persistent losses for at least 30 years. The solid red line shows that productivity falls by 0.3% on impact following a sudden stop compared to the no-SS economy, and its recovery is slow. The dashed purple line corresponds to the productivity loss when the path of the domestic innovation rate is replaced by its equivalent in the no-SS case. In this case, the productivity loss is about a third of the full effect. This means that about two-thirds of the productivity loss from sudden stops come from a lower domestic product innovation rate.

Figure 9: Productivity Loss after a Sudden Stops



NOTE: This figure shows, for different counterfactual experiments, the deviations of the productivity index from the no-SS level after a sudden stops,

Finally, the dot-dashed blue line corresponds to the productivity loss when the path of the export innovation rate is replaced by its equivalent in the no-SS case. In this case, the productivity decline on impact is not different from the baseline case, but the recovery is slower. This means that boosted export product innovation after a sudden stop drives the recovery of productivity.

Table 9: Welfare Loss from Sudden Stops

Economy	Welfare loss (%, relative to baseline)
Total Cost of Sudden Stop	4.62%
No Effect of SS on Productivity Growth g_t	3.24% (70.1%)
No Effect of SS on Domestic Innovation Rate i_t^D	3.49% (75.5%)
No Effect of SS on Export Innovation Rate i_t^X	5.04% (109.1%)

Table 9 shows the welfare loss in each counterfactual simulation compared to the case of no sudden stop. Following Durdu and Mendoza [2006], we translate the welfare loss into the compensating variation in period-1 consumption that equates the expected lifetime utility of each counterfactual to the no-SS case. Specifically, let V_1^{noss} denote the expected lifetime utility in the no-SS economy, and $E_1[V_2^{cf}]$ denote the expected utility in a counterfactual economy from period 2 onward. The compensating variation v^{cf} satisfies the following equation:

$$V_1^{noss} = \ln\left((1+v^{cf})c_1 - A_1\frac{H_1^{\omega}}{\omega}\right) + E_1[V_2^{cf}] \ , \tag{40}$$

The result in Table 9 shows that the welfare loss from a sudden stop corresponds to 4.62% of consumption in period 1.[32] If the path of productivity growth is replaced with the path from the no sudden stop case, the welfare loss would reduce to 3.24%, which is 70.1% of the total welfare loss from a sudden stop. This implies that about 30% of the welfare loss from a sudden stop comes from the endogenous distortions to productivity. Next, if the path of the domestic innovation rate is replaced with the path in the no sudden stop case, the welfare loss would reduce to 3.49%, which is close to the case in which the productivity growth rate is replaced. This is in line with the earlier result that a large fraction of the productivity loss comes from the lower domestic product innovation rate. In contrast, if the path of export innovation rate is replaced with the path in the no sudden stop case, the welfare loss would increase to 5.04%, which is 9.1% higher than the total welfare loss caused by a sudden stop. This implies that boosted export product innovation after the sudden stop helps productivity to converge faster, thereby reducing the welfare loss from the sudden stop.

Given the pecuniary externality arising from the collateral borrowing constraint in this class of models, the literature has studied the capacity of a tax on foreign borrowing to ameliorate the welfare cost of sudden stops [Bianchi and Mendoza, 2018]. Allowing for product dynamics enriches this analysis in several dimensions. First, endogenous product dynamics feed the boom that precedes a financial crisis potentially, augmenting its severity. Second, macroprudential policies typically impose borrowing taxes in normal times, thereby decreasing the present discounted value of profits and discouraging innovation in normal times. Third, a model with endogenous entry dynamics features other externalities. Even without a financial crisis, the level of innovation can

[32] 4.62% may look small compared to the decline in consumption when a sudden stop takes place, which is roughly 10%. This is because labor disutility also falls during a sudden stop, which reduces the compensating variation measured in terms of consumption only. If we compute the compensating variation in terms of consumption minus labor disutility, then it would be 12.24% instead of 4.62%. The decomposition analysis would not be affected by this alternative measure.

be inefficient as agents do not fully internalize neither the effects of their innovation on aggregate productivity nor its effects on the aggregate rate of creative destruction [Aghion et al., 2014]. In the case of underinvestment in innovation, the literature typically proposes innovation subsidies [Acemoglu et al., 2018]. However, these subsidies could increase systemic risk by affecting the severity of a sudden stop. Appendix B studies the effect of simple macroprudential policies and innovation subsidies in the context of our model. We find that a borrowing tax indeed reduces the welfare cost of sudden stops, but a negative welfare effect through discouraging innovation more than offsets a welfare gain by the stabilization effect.[33] We also study a flat innovation subsidy. This policy amplifies the over-borrowing externality because it boosts pre-crisis growth. However, a welfare gain through encouraging innovation dominates and this policy improves expected welfare. Given that the tax on foreign borrowing helps mitigate the cost of sudden stops and the innovation subsidy helps in normal times, we also analyze the effect of the joint policy and find that it can improve the expected welfare slightly more than the subsidy alone. There is space for future research to study a richer set of policies and lever the interactions between firm-level subsidies and macroprudential policy.

5 Conclusions

In this paper we introduce a new model of endogenous trade and productivity to study the impact of sudden stops on firm dynamics and economic growth. This theory stems from two key empirical facts. First, financial crises have persistent output effects that can be explained by distortions on firm dynamics. Second, exports are not only an essential adjustment margin during sudden stops episodes, but also a key contributor to productivity growth. Our model captures these two facts by extending the framework of leverage-driven financial crisis [Jeanne and Korinek, 2020, Bianchi and Mendoza, 2018] to include endogenous product portfolio dynamics at the firm level [Klette and Kortum, 2004] including exporting decisions at the product level.

We discipline the model using unique data on firms' products portfolios in domestic and export markets for Chile around the sudden stop episode of 1998. The calibrated model captures key product-level dynamics from the data in tranquil and crises times. Therefore, we show for the first time that the Klette and Kortum [2004] model is successful when contrasted to this type of

[33] In Ma [2020], a welfare gain through stabilization effects dominates a welfare loss through growth slowdowns, and thus a borrowing tax alone brings a positive expected welfare gain.

microdata. Moreover, our framework provides a path for dynamic stochastic general equilibrium models to use granular data to discipline their performance and test their effects.

We show that the lion's share of the productivity distortion during a financial crisis is due to the decrease in the development of local products, while the speed of the recovery is determined by the entry into exporting in the aftermath of the crisis. The calibrated model suggests that macroprudential policies [Benigno et al., 2013, Bianchi and Mendoza, 2018] should be combined with firm-level policies that palliate the negative effects of the former on firms' innovation efforts during normal times. Therefore, a fruitful direction for future research is to study optimal macroprudential policy coupled with firm-level subsidies that discriminate between exporters and non-exporters. The macroprudential policy analysis could further be enriched by studying the effect on productivity dynamics of the exchange rate adjustment in a context in which firms hold debt in foreign currency [Céspedes et al., 2004] .

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Online Appendix

A Theory Appendix

A.1 Equilibrium and Stationarized Equilibrium

This section defines the equilibrium of the economy and describes the stationarized equilibrium.

A.1.1 Factor Allocation

Before defining the equilibrium, we derive the expressions for asset and labor allocations. First we show that the total cost of production $R_t^L \ell_t(i) + W_t h_t(i)$ is equal to production $y_t(i)$ times the marginal cost. The latter can be written as:

$$y_t(i) \times MC_t(i) = a_t(i)(\ell_t(i))^{\alpha}(h_t(i))^{1-\alpha} \times \frac{1}{a_t(i)}\overline{\alpha}(R_t^L)^{\alpha}(W_t)^{1-\alpha} = \overline{\alpha}(R_t^L \ell_t(i))^{\alpha}(W_t h_t(i))^{1-\alpha} \quad \text{(A.1)}$$

Using the cost minimization condition $R_t^L \ell_t(i)/W_t h_t(i) = \alpha/(1-\alpha)$,

$$y_t(i) \times MC_t(i) = \frac{1}{\alpha} R_t^L \ell_t(i) = \frac{1}{1-\alpha} W_t h_t(i) = R_t^L \ell_t(i) + W_t h_t(i) \quad \text{(A.2)}$$

This shows that production times the marginal cost is equal to the total cost. Next, profit for a product line can be written as follows:

$$\pi_t(i) = p_t(i) y_t(i) - (R_t^L \ell_t(i) + W_t h_t(i)) = (p_t(i) - MC_t(i)) y_t(i) \quad \text{(A.3)}$$

Recall that the optimal price is equal to the marginal cost for the second-best rival firm. Whether the rival firm is a domestic firm or a foreign firm depends on whether the product is sold domestically or exported. We first consider the case of domestic sales by domestic and export lines, for which the second-best rival is a domestic firm. The case of exports and imports is examined next.

For domestic sales, given that the rival firm is also a domestic firm, the rival's marginal cost is $1 + \sigma^s$ times the marginal cost for the leader, where $s = D, X$ depending on the type of the product line. Therefore,

$$\pi_t(i) = \sigma^s MC_t(i) y_t(i) \quad \text{(A.4)}$$

Using (A.2),

$$\pi_t(i) = \sigma^s \frac{1}{\alpha} R_t^L \ell_t(i) = \sigma^s \frac{1}{1-\alpha} W_t h_t(i) \tag{A.5}$$

In the main text, we derived another expression for profits in equation (8). Thus we have:

$$\sigma^s \frac{1}{\alpha} R_t^L \ell_t(i) = \sigma^s \frac{1}{1-\alpha} W_t h_t(i) = Y_t \frac{1}{1+\phi\mu_t/\lambda_t} \frac{\sigma^s}{1+\sigma^s} \tag{A.6}$$

This equation shows that the asset and labor input $\ell_t(i)$ and $h_t(i)$ are independent of productivity level $a_t(i)$, and depend only on the type of product lines, $s = D, X$. Combining this equation with $s = D$ and $s = X$, we obtain the relative factor input between domestic lines and export lines:

$$\frac{\ell_t^D}{\ell_t^X} = \frac{h_t^D}{h_t^X} = \frac{1+\sigma^X}{1+\sigma^D} \tag{A.7}$$

Next, for exports by export lines, demand is exogenously given by Y_t^*. Using the demand equation for each type of intermediate good,

$$\begin{aligned} Y_t^* &= p_t(i) y_t(i) = MC_t^*(i) \times \frac{1}{1+\xi} a_t(i) \ell_t(i)^\alpha h_t(i)^{1-\alpha} \\ &= \frac{1}{a_t^*(i)} \overline{\alpha} (R_t^{L*})^\alpha (W_t^*)^{1-\alpha} \times \frac{1}{1+\xi} a_t(i) \ell_t(i)^\alpha h_t(i)^{1-\alpha} \end{aligned} \tag{A.8}$$

Note that the amount that exporting firms can actually sell to the foreign demand is their output minus the loss due to an iceberg cost. Using $a_t(i) = (1+\sigma^X) a_t^*(i)$,

$$Y_t^* = \overline{\alpha} (R_t^{L*})^\alpha (W_t^*)^{1-\alpha} \frac{1+\sigma^X}{1+\xi} \ell_t(i)^\alpha h_t(i)^{1-\alpha} \tag{A.9}$$

Cost minimization implies:

$$\frac{R_t^L \ell_t(i)}{W_t h_t(i)} = \frac{\alpha}{1-\alpha} \tag{A.10}$$

(A.9) and (A.10) imply that the factor input for export lines is also independent of the productivity level. It follows that the relation between aggregate factor inputs L_t^*, H_t^* and individual factor inputs ℓ_t^*, h_t^* satisfy $\ell_t^* = L_t^*/\theta_{t-1}^X$ and $h_t^* = H_t^*/\theta_{t-1}^X$. Then equation (A.9) can be written as

$$Y_t^* = \overline{\alpha} (R_t^{L*})^\alpha (W_t^*)^{1-\alpha} \frac{1+\sigma^X}{1+\xi} \frac{1}{\theta_{t-1}^X} (L_t^*)^\alpha (H_t^*)^{1-\alpha} \tag{A.11}$$

We also utilize the fact that Y_t^*, R_t^{L*}, W_t^* all grow at the same exogenous rate $\overline{g}$. This implies that

$(R_t^{L*})^\alpha (W_t^*)^{1-\alpha}/Y_t^*$ is a constant. Let ω^* denote this constant. Then this equation can be written as follows:

$$1 = \overline{\alpha}\omega^* \frac{1+\sigma^X}{1+\xi}\frac{1}{\theta_{t-1}^X}(L_t^*)^\alpha (H_t^*)^{1-\alpha} \tag{A.12}$$

This equation pins down the factor inputs for exports.

Finally, we show that factor inputs by foreign firms for import lines are linear in the factor inputs by exporting firms for domestic sales. Demand for imported intermediate goods by the final producer is given by the same equation as other product lines:

$$p_t(i)y_t(i) = Y_t \frac{1}{1+\phi\mu_t/\lambda_t} \tag{A.13}$$

The optimal price is the marginal cost for the closest rival firm, and in this case it is a domestic firm. Therefore,

$$\frac{1+\sigma^X}{a_t(i)}\overline{\alpha}(R_t^L)^\alpha (W_t)^{1-\alpha} \times \frac{1}{1+\xi} a_t(i)(\ell_t^F)^\alpha (h_t^F)^{1-\alpha} = Y_t \frac{1}{1+\phi\mu_t/\lambda_t} \tag{A.14}$$

$$\overline{\alpha}(R_t^L)^\alpha (W_t)^{1-\alpha}\frac{1+\sigma^X}{1+\xi}(\ell_t^F)^\alpha (h_t^F)^{1-\alpha} = Y_t \frac{1}{1+\phi\mu_t/\lambda_t} \tag{A.15}$$

Note that foreign exporters are subject to an iceberg cost, and thus they can sell only a fraction $1/(1+\xi)$ of their output. The same equation for export lines' domestic sales is:

$$\overline{\alpha}(R_t^L)^\alpha (W_t)^{1-\alpha}(1+\sigma^X)(\ell_t^X)^\alpha (h_t^X)^{1-\alpha} = Y_t \frac{1}{1+\phi\mu_t/\lambda_t} \tag{A.16}$$

Comparing these two equations, foreign exporters' factor input is given by the following equation:

$$\frac{1}{1+\xi}(\ell_t^F)^\alpha (h_t^F)^{1-\alpha} = (\ell_t^X)^\alpha (h_t^X)^{1-\alpha} \tag{A.17}$$

This means that foreign exporters use more inputs than domestic exporting firms, but the effective inputs that contribute to production are the same due to an iceberg cost.

Using this result, final goods production can be written as follows:

$$
\begin{aligned}
Y_t &= \exp(\varepsilon_t^A)\exp\left[\int_0^1 \ln y_t(i)di\right] = \exp(\varepsilon_t^A)\exp\left[\int_0^1 \ln a_t(i)di\right]\times\exp\left[\int_0^1 \ln\left(\ell_t(i)^\alpha h_t(i)^{1-\alpha}\right)\right] \\
&= \exp(\varepsilon_t^A)A_t\exp\left[\theta_{t-1}^D \ln\left\{\left(\frac{L_t^D}{\theta_{t-1}^D}\right)^\alpha\left(\frac{H_t^D}{\theta_{t-1}^D}\right)^{1-\alpha}\right\}\times(1-\theta_{t-1}^D)\ln\left\{\left(\frac{L_t^X}{\theta_{t-1}^X}\right)^\alpha\left(\frac{H_t^X}{\theta_{t-1}^X}\right)^{1-\alpha}\right\}\right] \\
&= \exp(\varepsilon_t^A)A_t\left\{\left(\frac{L_t^D}{\theta_{t-1}^D}\right)^\alpha\left(\frac{H_t^D}{\theta_{t-1}^D}\right)^{1-\alpha}\right\}^{\theta_{t-1}^D}\left\{\left(\frac{L_t^X}{\theta_{t-1}^X}\right)^\alpha\left(\frac{H_t^X}{\theta_{t-1}^X}\right)^{1-\alpha}\right\}^{1-\theta_{t-1}^D} \\
&= \exp(\varepsilon_t^A)A_t\frac{\left(L_t^D\right)^\alpha\left(H_t^D\right)^{1-\alpha}}{\theta_{t-1}^D}\left(\frac{1+\sigma^D}{1+\sigma^X}\right)^{1-\theta_{t-1}^D}
\end{aligned}
\tag{A.18}
$$

A.1.2 Other First-Order Conditions

The first-order condition for the foreign bond by the final tradable producer, and the complementary slackness condition for the borrowing constraint are given as follows:

$$u_c(t) - \mu_t = \beta R\exp(\varepsilon_t^R)E_t\left[(u_c(t+1)\right] \tag{A.19}$$

$$\mu_t\left[-B_t + \phi Y_t\frac{1}{1+\phi\mu_t/u_c(t)} - \kappa Q_t L\right] = 0, \mu_t \geq 0 \tag{A.20}$$

First-order condition for labor supply by households is given as follows:

$$A_t H_t^{\omega-1} = W_t \tag{A.21}$$

Market clearing conditions for labor and asset are given as follows:

$$H_t = \theta_{t-1}^D h_t^D + \theta_{t-1}^X h_t^X + \theta_{t-1}^X h_t^* \tag{A.22}$$

$$L = \theta_{t-1}^D \ell_t^D + \theta_{t-1}^X \ell_t^X + \theta_{t-1}^X \ell_t^* \tag{A.23}$$

A.1.3 Equilibrium

Definition: *The equilibrium of the model economy is defined by the initial states* $R\exp(\varepsilon_{-1}^R)B_{-1}$, θ_{-1}^D, θ_{-1}^X, *initial foreign productivity and factor prices* A_0^*, R_0^{L*}, W_0^*, *initial productivity level for each product line* $\{a_0(i)\}_{i=0}^1$, *stochastic process* $\{\varepsilon_t^A, \varepsilon_t^R\}_{t=0}^\infty$, *and the following:*

1. *Tradable goods producer optimally chooses intermediate inputs, foreign bond, and asset holdings* $\left\{\{y_t(i)\}_{t=0}^{1}, B_t, L_t\right\}$*, given intermediate goods price* $\{p_t(i)\}_{i=0}^{1}$*, asset price* Q_t*, and asset return* R_t^L*, satisfying (1), (3), (4), (5), (A.19), and (A.20).*

2. *Intermediate goods producing firms:*

   - *For domestic and exporting product lines, optimally choose price and production inputs* $\left\{\{p_t(i)\}_{i\in\theta_{t-1}^D,\theta_{t-1}^X}, \ell_t^D, h_t^D, \ell_t^X, h_t^X\right\}$*, given individual productivity* $a_t(i)$*, factor prices* W_t *and* R_t^L*, and domestic demand (4), satisfying (6), (7), (9), and (A.6).*
   - *For exporting product lines, optimally choose price and inputs* $\left\{\{p_t(i)\}_{i\in\theta_{t-1}^X}, \ell_t^*, h_t^*\right\}$*, given individual productivity* $a_t(i)$*, factor prices* W_t *and* R_t^L*, and foreign demand (11), satisfying (6), (12), and (A.10).*
   - *Optimally choose investment* $\left\{Z_t^D, Z_t^X\right\}$ *given the expected value of product lines in (17) and (20), satisfying (15), (16), (18), and (19).*

3. *Households optimally choose consumption, labor supply, and investment* $\left\{C_t, H_t, Z_t^{ED}, Z_t^{EX}\right\}$ *subject to the budget constraint (31), given real wage* W_t *and the expected value of product lines in (17) and (20), satisfying (21), (22), (23), (24), and (A.21).*

4. *Aggregate variables* $A_t, \theta_t^D, \theta_t^X, d_t$ *follow (25), (26), (27), (30).*

5. *Foreign productivity* A_t^* *and factor prices* R_t^{L*}, W_t^* *grow at the exogenous rate* $\bar{g}$*.*

6. *Labor and asset markets clear, satisfying (A.22), (A.23).*

Definition: *The balanced growth path of the model economy is a non-stochastic* ($\varepsilon_t^A = \varepsilon_t^R = 0$) *equilibrium in which* C_t, Y_t, B_t, Q_t, A_t, R_t^L, W_t, π_t^D, π_t^X, π_t^*, V_t^D, V_t^X, Z_t^D, Z_t^X, Z_t^{ED}, Z_t^{EX} *grow at the same rate* $\bar{g}$ *along with the exogenous foreign variables* A_t^*, R_t^{L*}, W_t^**, the borrowing constraint is binding with* λ_t *and* $\mu_t > 0$ *growing at the same rate, and* θ_t^D, θ_t^X, e_t^D, e_t^X, i_t^D, i_t^X, d_t, ℓ_t^D, ℓ_t^X, ℓ_t^*, H_t, h_t^D, h_t^X, h_t^* *are constant.*

A.1.4 Stationarized Equilibrium

To stationarize the model, we divide the equilibrium conditions by aggregate productivity A_t. We denote stationarized variables using lower-case letters, and use g_t to denote the productivity growth rate $A_{t+1}/A_t - 1$. We also make some arrangements and reduce the number of equations.

The following is the complete list of equations to characterize the stationarized equilibrium of the model:

■ Final goods producer

$$y_t = \exp(\varepsilon_t^A)\frac{(L_t^D)^\alpha (H_t^D)^{1-\alpha}}{\theta_{t-1}^D}\left(\frac{1+\sigma^D}{1+\sigma^X}\right)^{1-\theta_{t-1}^D} \tag{A.24}$$

$$\lambda_t - \mu_t = \beta R \exp(\varepsilon_t^R)\frac{1}{1+g_t}E_t(\lambda_{t+1}) \tag{A.25}$$

$$\lambda_t q_t = \beta E_t\left[\lambda_{t+1}(q_{t+1} + r_{t+1}^L) + \mu_{t+1}\kappa q_{t+1}\right] \tag{A.26}$$

$$\mu_t\left[-b_t + \phi y_t^T \frac{1}{1+\phi\mu_t/\lambda_t} - \kappa q_t L\right] = 0 \tag{A.27}$$

■ Intermediate goods producing firms

$$r_t^L = \frac{1}{1+\sigma^D}\alpha y_t \frac{\theta_{t-1}^D}{L_t^D}\frac{1}{1+\phi\mu_t/\lambda_t} \tag{A.28}$$

$$w_t = \frac{1}{1+\sigma^D}(1-\alpha) y_t \frac{\theta_{t-1}^D}{H_t^D}\frac{1}{1+\phi\mu_t/\lambda_t} \tag{A.29}$$

$$L_t^X = L_t^D \frac{\theta_{t-1}^X}{\theta_{t-1}^D}\frac{1+\sigma^D}{1+\sigma^X} \tag{A.30}$$

$$H_t^X = H_t^D \frac{\theta_{t-1}^X}{\theta_{t-1}^D}\frac{1+\sigma^D}{1+\sigma^X} \tag{A.31}$$

$$\pi_t^D = \frac{\sigma_t^D}{1+\sigma_t^D} y_t \frac{1}{1+\phi\mu_t/\lambda_t} \tag{A.32}$$

$$\pi_t^X = \frac{\sigma_t^X}{1+\sigma_t^X} y_t \frac{1}{1+\phi\mu_t/\lambda_t} \tag{A.33}$$

$$\pi_t^* = a_t^* y^* - \frac{1+\xi}{1+\sigma^X}\frac{1}{\omega^*}(r_t^L)^{\alpha}(w_t)^{1-\alpha} \tag{A.34}$$

$$1 = \frac{1+\sigma^X}{1+\xi}\overline{\alpha}\omega^*\frac{1}{\theta_{t-1}^X}(L_t^*)^{\alpha}(H_t^*)^{1-\alpha} \tag{A.35}$$

$$\frac{r_t^L L_t^*}{w_t H_t^*} = \frac{\alpha}{1-\alpha} \tag{A.36}$$

$$\begin{aligned} v_t(1,0) = \pi_t^D - z_t^D - z_t^X + \left[i_t^D + (1-d_t)(1-i_t^X)\right] E_t(\Lambda_{t,t+1}v_{t+1}(1,0)) \\ + (1-d_t)i_t^X E_t(\Lambda_{t,t+1}v_{t+1}(0,1)) \end{aligned} \tag{A.37}$$

$$\begin{aligned} v_t(0,1) = \pi_t^X + \pi_t^* - z_t^D + (i_t^D + i^{FX})E_t(\Lambda_{t,t+1}v_{t+1}(1,0)) \\ + (1-i^{FX})E_t(\Lambda_{t,t+1}v_{t+1}(0,1)) \end{aligned} \tag{A.38}$$

$$i_t^D = \eta^D (z_t^D)^{1/\rho} \tag{A.39}$$

$$\eta^D \frac{1}{\rho}(z_t^D)^{1/\rho-1}E_t(\Lambda_{t,t+1}v_{t+1}(1,0)) = 1 \tag{A.40}$$

$$i_t^X = \eta^X (z_t^X)^{1/\rho} \tag{A.41}$$

$$(1-d_t)\eta^X \frac{1}{\rho}(z_t^X)^{1/\rho-1}\left[E_t(\Lambda_{t,t+1}v_{t+1}(0,1)) - E_t(\Lambda_{t,t+1}v_{t+1}(1,0))\right] = 1 \tag{A.42}$$

■Aggregate variables

$$d_t = i^{FD} + (e_t^D + e_t^X + (\theta_{t-1}^D + \theta_{t-1}^X)i_t^D)\frac{1}{1-\theta_{t-1}^X} \tag{A.43}$$

$$\theta_t^D = \theta_{t-1}^D + (e_t^D + (\theta_{t-1}^D + \theta_{t-1}^X) i_t^D) \frac{1 - \theta_{t-1}^D - \theta_{t-1}^X}{1 - \theta_{t-1}^X} + \theta_{t-1}^X i^{FX}$$
$$- \theta_{t-1}^D (1 - d_t) i_t^X - \theta_{t-1}^D i^{FD} - e_t^X \frac{\theta_{t-1}^D}{1 - \theta_{t-1}^X} \tag{A.44}$$

$$\theta_t^X = \theta_{t-1}^X + \theta_{t-1}^D (1 - d_t) i_t^X + e_t^X - \theta_{t-1}^X i^{FX} \tag{A.45}$$

$$\frac{A_{t+1}}{A_t} = 1 + g_t = (1 + \sigma^D)^{e_t^D + (\theta_{t-1}^D + \theta_{t-1}^X) i_t^D} (1 + \sigma^X)^{e_t^X + \theta_{t-1}^D (1 - d_t) i_t^X} (1 + \sigma^X)^{i^{FD}} \tag{A.46}$$

$$a_{t+1}^* = \frac{1 + \overline{g}}{1 + g_t} a_t^* \tag{A.47}$$

■**Households**

$$c_t + z_t^{ED} + z_t^{EX} = y_t - \theta_{t-1}^D (z_t^D + z_t^X) - \theta_{t-1}^X z_t^D + \theta_{t-1}^X a_t^* y^*$$
$$- (1 - \theta_{t-1}^D - \theta_{t-1}^X) \frac{y_t}{1 + \phi \mu_t / \lambda_t} - b_t + R \exp(\varepsilon_{t-1}^R) \frac{b_{t-1}}{1 + g_{t-1}} \tag{A.48}$$

$$H_t^{\omega - 1} = w_t \tag{A.49}$$

$$\lambda_t = \frac{1}{c_t - H_t^{\omega} / \omega} \tag{A.50}$$

$$e_t^D = \eta^{ED} (z_t^{ED})^{1/\rho} \tag{A.51}$$

$$\eta^{ED} \frac{1}{\rho} (z_t^{ED})^{1/\rho - 1} E_t (\Lambda_{t,t+1} v_{t+1}(1, 0)) = 1 \tag{A.52}$$

$$e_t^X = \eta^{EX} (z_t^{EX})^{1/\rho} \tag{A.53}$$

$$\eta^{EX} \frac{1}{\rho} (z_t^{EX})^{1/\rho - 1} E_t (\Lambda_{t,t+1} v_{t+1}(0, 1)) = 1 \tag{A.54}$$

■**Market clearing**

$$H_t = H_t^D + H_t^X + H_t^* \tag{A.55}$$

$$L = L_t^D + L_t^X + L_t^* \tag{A.56}$$

A.2 Numerical Solution

In this section we sketch the numerical solution method. The solution method is a version of the policy function iteration, modified to deal with the occasionally binding constraint. Below is the procedure to obtain the numerical solution.

1. We set the equally-spaced grid points for the endogenous state variables, foreign debt $R\exp(\varepsilon_{t-1}^R)b_{t-1}/(1+g_{t-1})$, share of domestic product lines θ_{t-1}^D, share of export product lines θ_{t-1}^X, relative productivity of foreign countries over the domestic country $a_t^* = A_t^*/A_t$. There are also 2 states for stochastic shocks ε_t^A and ε_t^R respectively.

2. For each grid point, we set the initial guess for five variables: L_t^D, b_t, q_t, $E_t(\Lambda_{t,t+1}v_{t+1}(1,0))$, and $E_t(\Lambda_{t,t+1}v_{t+1}(0,1))$.

3. For each grid point, we do the following:

   - We leave the five variables we have made guesses for as unknown variables. We express all the other endogenous variables as functions of the state variables and the five unknowns. In this process, we first assume that the borrowing constraint is not binding and proceed. Later we check if the constraint is satisfied. If it is not satisfied, we recalculate all the variables using the binding borrowing constraint. The other endogenous variables, which include next-period state variables, are now functions of the five variables.

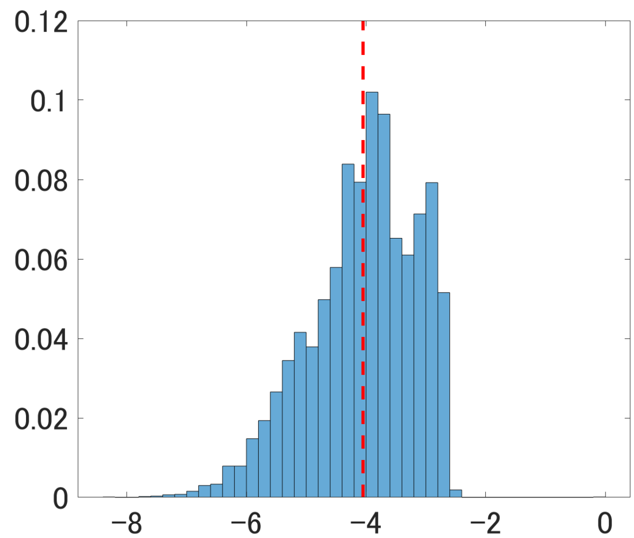
   - Using four-dimensional linear interpolation over the next-period state variables and the guess for the five variables L_t^D, b_t, q_t, $E_t(\Lambda_{t,t+1}v_{t+1}(1,0))$, and $E_t(\Lambda_{t,t+1}v_{t+1}(0,1))$, we compute all the endogenous variables next period. We then calculate all the forward-looking expectation terms, such as the right hand side of the Euler equations and the value functions.

- All the equilibrium conditions are now the functions of the initial five unknowns. There are five equations we did not use in step (a), thus five equations in total. We solve for the five unknowns using non-linear solver.

4. We check the gap between the guess and the newly-obtained values for the five variables. If they are close enough, we stop. If not, we update the guess by the newly-obtained values, and go back to step 3. We repeat this process until the gap becomes sufficiently small.

We check the accuracy of the numerical solution using the Euler equation error. We simulate the model for 100,000 periods with stochastic shocks and compute the Euler equation error for each period. Figure A.1 plots the distribution of the Euler equation errors obtained in this way. The average error is -4 and the maximum error is -2.5, which is reasonably small compared to the literature.

Figure A.1: Distribution of Euler Equation Errors



A.3 Proof of Linear Relations in Value Functions

This section shows the detailed procedure of the guess-and-verify method to prove the linear relation in value functions for intermediate goods producers. We guess the linear relation $V_t(n^D, n^X) = n^D V_t(1, 0) + n^X V_t(0, 1)$ and prove it. We first work on the value of a firm with a single domestic product line:

$$
\begin{aligned}
V_t(1,0) &= \max_{Z_t^D, Z_t^X} \left\{ \pi_t^D - Z_t^D - Z_t^X \right. \\
&+ \left. \left[\sum_{i=0}^{1} P(i,1,i_t^D) \left\{ \sum_{j=0}^{1} P(j,1,d_t) \left(\sum_{k=0}^{1-j} P\left(k,1-j,i_t^X\right) E_t\left[\Lambda_{t,t+1} V_{t+1}(1+i-j-k,k)\right] \right) \right\} \right] \right\}
\end{aligned} \tag{A.57}
$$

Using the linear relation, the summations in the second line can be written as follows:

$$
\begin{aligned}
& \sum_{i=0}^{1} P(i,1,i_t^D) \left\{ \sum_{j=0}^{1} P(j,1,d_t) \left(\sum_{k=0}^{1-j} P\left(k,1-j,i_t^X\right) E_t\left[\Lambda_{t,t+1} V_{t+1}(1+i-j-k,k)\right] \right) \right\} \\
=\ & \sum_{i=0}^{1} P(i,1,i_t^D) \left\{ \sum_{j=0}^{1} P(j,1,d_t) \left(\sum_{k=0}^{1-j} P\left(k,1-j,i_t^X\right) E_t\left[\Lambda_{t,t+1}[(1+i-j-k)V_{t+1}(1,0) + kV_{t+1}(0,1)]\right] \right) \right\} \\
=\ & E_t\left[\Lambda_{t,t+1} V_{t+1}(1,0)\right] \sum_{i=0}^{1} P(i,1,i_t^D) \sum_{j=0}^{1} P(j,1,d_t) \sum_{k=0}^{1-j} P\left(k,1-j,i_t^X\right)(1+i-j-k) \\
& + E_t\left[\Lambda_{t,t+1} V_{t+1}(0,1)\right] \sum_{i=0}^{1} P(i,1,i_t^D) \sum_{j=0}^{1} P(j,1,d_t) \sum_{k=0}^{1-j} P\left(k,1-j,i_t^X\right)(k) \\
=\ & (i_t^D + (1-d_t)(1-i_t^X)) E_t\left[\Lambda_{t,t+1} V_{t+1}(1,0)\right] + (1-d_t) i_t^X E_t\left[\Lambda_{t,t+1} V_{t+1}(0,1)\right]
\end{aligned} \tag{A.58}
$$

Therefore, we have:

$$
\begin{aligned}
V_t(1,0) &= \max_{Z_t^D, Z_t^X} \left\{ \pi_t^D - Z_t^D - Z_t^X \right. \\
&\left. + \left(i_t^D + (1-d_t)(1-i_t^X)\right) E_t\left[\Lambda_{t,t+1} V_{t+1}(1,0)\right] + (1-d_t) i_t^X E_t\left[\Lambda_{t,t+1} V_{t+1}(0,1)\right] \right\}
\end{aligned} \tag{A.59}
$$

Similarly, we can show that the value of a firm with a single export line is given as follows:

$$
V_t(0,1) = \max_{Z_t^D} \left\{ \pi_t^X + \pi_t^* - Z_t^D + (i_t^D + i^{FX}) E_t\left[\Lambda_{t,t+1} V_{t+1}(1,0)\right] + (1 - i^{FX}) E_t\left[\Lambda_{t,t+1} V_{t+1}(0,1)\right] \right\} \tag{A.60}
$$

which corresponds to equations (17) and (20) in the main text. Next we work on the value of a firm with general n^D domestic lines and n^X export lines:

$$
\begin{aligned}
V_t(n^D, n^X) = \max_{Z_t^D, Z_t^X} & \left\{ n^D \pi_t^D + n^X(\pi_t^X + \pi_t^*) - (n^D + n^X) Z_t^D - n^D Z_t^X \right. \\
& + \sum_{i=0}^{n^D+n^X} P(i, n^D + n^X, i_t^D) \sum_{j=0}^{n^D} P(j, n^D, d_t) \sum_{k=0}^{n^D-j} P(k, n^D - j, i_t^X) \sum_{m=0}^{n^X} P(m, n^X, i^{FX}) \\
& \left. E_t\left[\Lambda_{t,t+1} V_{t+1}(n^D + i - j - k, n^X + k - m)\right] \right\}
\end{aligned} \tag{A.61}
$$

Using the linear relation in the value function,

$$
\begin{aligned}
V_t(n^D,n^X) = \max_{Z_t^D,Z_t^X} \{ & n^D\pi_t^D + n^X(\pi_t^X+\pi_t^*) - (n^D+n^X)Z_t^D - n^D Z_t^X \\
& + E_t[\Lambda_{t,t+1}V_{t+1}(1,0)] \sum_{i=0}^{n^D+n^X} P(i,n^D+n^X,i_t^D) \sum_{j=0}^{n^D} P(j,n^D,d_t) \sum_{k=0}^{n^D-j} P(k,n^D-j,i_t^X) \sum_{m=0}^{n^X} P(m,n^X,i^{FX})(n^D+i-j-k+m) \\
& + E_t[\Lambda_{t,t+1}V_{t+1}(0,1)] \sum_{i=0}^{n^D+n^X} P(i,n^D+n^X,i_t^D) \sum_{j=0}^{n^D} P(j,n^D,d_t) \sum_{k=0}^{n^D-j} P(k,n^D-j,i_t^X) \sum_{m=0}^{n^X} P(m,n^X,i^{FX})(n^X+k-m)
\end{aligned}
\tag{A.62}
$$

The second line can be written as follows:

$$
E_t[\Lambda_{t,t+1}V_{t+1}(1,0)](n^D + (n^D+n^X)i_t^D - n^D d_t - n^D(1-d_t)i_t^X + n^X * i^{FX} \tag{A.63}
$$

The third line can be written as follows:

$$
E_t[\Lambda_{t,t+1}V_{t+1}(0,1)](n^X + n^D(1-d_t)i_t^X - n^X i^{FX} \tag{A.64}
$$

Therefore $V_t(n^D,n^X)$ can be written as follows::

$$
\begin{aligned}
V_t(n^D,n^X) &= \max_{Z_t^D,Z_t^X} \{ n^D\pi_t^D + n^X(\pi_t^X+\pi_t^*) - (n^D+n^X)Z_t^D - n^D Z_t^X \\
&\quad + E_t[\Lambda_{t,t+1}V_{t+1}(1,0)](n^D + (n^D+n^X)i_t^D - n^D d_t - n^D(1-d_t)i_t^X + n^X i^{FX} \\
&\quad + E_t[\Lambda_{t,t+1}V_{t+1}(0,1)](n^X + n^D(1-d_t)i_t^X - n^X i^{FX})\} \\
&= \max_{Z_t^D,Z_t^X} \{ n^D\pi_t^D + n^X(\pi_t^X+\pi_t^*) - (n^D+n^X)Z_t^D - n^D Z_t^X \\
&\quad + n^D \left\{(i_t^D + (1-d_t)(1-i_t^X))E_t[\Lambda_{t,t+1}V_{t+1}(1,0)] + (1-d_t)i_t^X E_t[\Lambda_{t,t+1}V_{t+1}(0,1)]\right\} \\
&\quad + n^X \left\{(i_t^D + i^{FX})E_t[\Lambda_{t,t+1}V_{t+1}(1,0)] + (1-i^{FX})\right\} E_t[\Lambda_{t,t+1}V_{t+1}(0,1)]\} \\
&= n^D V_t(1,0) + n^X V_t(0,1)
\end{aligned}
\tag{A.65}
$$

This verifies that the initial guess $V_t(n^D,n^X) = n^D V_t(1,0) + n^X V_t(0,1)$ is correct.

A.4 Firm Size Distribution

This section shows the law of motion for the share of each firm size and how to derive the firm size distribution. Each firm is characterized by the number of domestic and export lines it owns, (n^D,n^X). The law of motion for the firm size (n^D,n^X) is the formula that gives us the measure (number) of firms that own (n^D,n^X) given the firm size distribution in the previous period. Let $\delta_t(n^D,n^X)$ denote the measure of firms that own n^D domestic lines and n^X export lines at period t. Because the total measure of intermediate goods is one and each firm owns at least one product line, the measure of firms is between 0 and 1, i.e. $\delta_t(n^D, n^X) \in [0,1]$ $\forall t, n^D, n^X$. In order for a firm to become a firm with (n^D, n^X) in the next period, there are some conditions to be satisfied. For example, a firm with (i, j) at period $t-1$ can own at most $2i + j$ domestic lines, because this is the case in which all domestic innovations $(i + j)$ are successful, all export innovations fail, and no replacement on domestic lines happens. So, if a firm owns (i, j) that satisfies $2i + j < n^D$, this firm cannot become a firm with (n^D, n^X) in the next period for any n^X. Let (i, j) denote the number of domestic and export lines that a firm owns at period $t-1$. Let (k, ℓ) denote the number of successes in domestic innovation and export innovation respectively. Let q denote the number of export lines that are hit by foreign innovation and turn back to domestic lines. Let r^D denote the number of replacements that happen on domestic lines. Then consider a case in which this firm becomes a firm with (n^D, n^X). The table below lists all the notation:

Table A.1: Notation

Symbol	Description
n^D	domestic lines next period
n^X	export lines next period
i	domestic lines this period
j	export lines this period
k	successful domestic innovation
ℓ	successful export innovation
q	export lines to become domestic
r^D	replacements on domestic lines

These variables satisfy the following equations and inequality:

- $n^D = i + k - \ell + q - r^D$
- $n^X = j + \ell - q$
- $n^D + n^X = i + j + k - r^D$
- $\ell + r^D \leq i$ (ℓ and r^D do not happen on the same domestic line)

Given these restrictions, the conditions that each variable needs to satisfy are given as follows:

i:

- $i \geq 0$

j:

- $j \geq 0$
- $n^D \leq i + k + j \rightarrow j \geq I_+ \left\{ n^D/2 - i \right\}$
- $n^X \leq i + j \rightarrow j \geq n^X - i$
- $n^D + n^X \leq 2i + 2j \rightarrow j \leq I_+ \left\{ (n^D + n^X)/2 - i \right\}$

k:

- $k \geq 0$
- $k \leq i + j$
- $n^D \leq i + k + j \rightarrow k \geq n^D - i - j$
- $n^D + n^X \leq i + j + k \rightarrow k \geq n^D + n^X - i - j$

ℓ:

- $\ell \geq 0$
- $\ell \leq i$
- $n^D \leq i + k - \ell + j \rightarrow \ell \leq i + k + j - n^D$
- $n^X \leq j + \ell \rightarrow \ell \geq n^X - j$

q:

- $\ell + r^D = i + k + q - n^D \leq i \rightarrow q = j + \ell - n^X \leq n^D - k \rightarrow \ell \leq n^D + n^X - j - k$
- $q = j + \ell - n^X \leq j \rightarrow \ell \leq n^X$

r^D:

- $\ell + r^D = i + k + q - n^D \leq i$

where $I_+\{x\}$ is the smallest integer that is equal to or greater than x. Summarizing these conditions, i, j, k, ℓ are subject to the following restrictions:

i:

- $i \geq 0$

j:

- $\max\{0, n^X - i\} \leq j \leq I_+\{(n^D + n^X)/2 - i\}$

k:

- $\max\{0, n^D + n^X - i - j\} \leq k \leq i + j$

ℓ:

- $\max\{0, n^X - j\} \leq \ell \leq \min\{i, i + k + j - n^D, n^D + n^X - j - k, n^X\}$

Using these conditions, the law of motion for $\delta_t(n^D, n^X)$ can be written as follows:

$$\delta_t(n^D, n^X) = \sum_i \sum_j \delta_{t-1}(i,j) \sum_k \sum_\ell P(k, i+j, i^D) P(\ell, i - r^D, i^X) P(q, j, i^{FX}) P(r^D, i, d) \quad \text{(A.66)}$$

subject to the above constraints on i, j, k, ℓ. There are two special cases, for which new firm entry is added: domestic firm entry e^{ED} is added to the case of $(n^D, n^X) = (1, 0)$, and exporting firm entry e^{EX} is added to the case of $(n^D, n^X) = (0, 1)$.

To derive the stationary firm distribution which is used for calibration in the main text, we use the long-run simulation mean of new entry, incumbent innovation, and replacement rate, $e^{ED}, e^{EX}, i^D, i^X, d$, and iterate this law of motion for large enough (n^D, n^X) until the distribution converges for every firm size.

B Policy Analysis

In this section we assess to what extent a tax on foreign borrowing, a subsidy on innovation investment or a combination of these two policies can improve welfare.

B.1 A Tax on Foreign Borrowing

The endogenous asset price Q_t in the borrowing limit introduces a pecuniary externality into the model. As shown in the literature [Bianchi and Mendoza, 2018], this externality induces over-borrowing in normal times and calls for policy interventions. In this subsection, we study whether a tax on foreign borrowing can improve welfare. Because of the rich structure of the model, we perform a quantitative study focusing on a Taylor-rule style tax as in Bianchi and Mendoza [2018] and Jeanne and Korinek [2020], which is given by the following equation:

$$\tau_t = \max\left\{\left(\frac{b_t}{\bar{b}}\right)^{\iota} - 1, 0\right\} ,$$

and characterized by the parameters $\bar{b} < 0$ and $\iota > 0$. This means that the tax rate τ_t is 0 when foreign debt divided by aggregate productivity b_t is smaller than $\bar{b}$ in absolute value, and becomes positive as b_t exceeds $\bar{b}$. The Euler equation with respect to foreign borrowing is now:

$$u_c(C_t, H_t) = \beta R \exp\left(\varepsilon_t^R\right)(1+\tau_t) E_t\left[u_c(C_{t+1}, H_{t+1})\right].$$

The tax collected is rebated to the final tradable producer as a lump-sum transfer. Following Bianchi and Mendoza [2018], the welfare impact of the policy given the initial state of the economy is measured as the permanent additional consumption that makes households indifferent between having or not having this policy. Given the set of initial conditions $\boldsymbol{S}$, the welfare impact of the policy, denoted by γ, is defined by:

$$E_0 \sum_{t=0}^{\infty} \beta^t \ln\left(\left(1+\gamma(\boldsymbol{S})\right) C_t - A_t \frac{(H_t)^{\omega}}{\omega}\right) = E_0 \sum_{t=0}^{\infty} \beta^t \ln\left(C_t^p - A_t^p \frac{(H_t^p)^{\omega}}{\omega}\right)$$

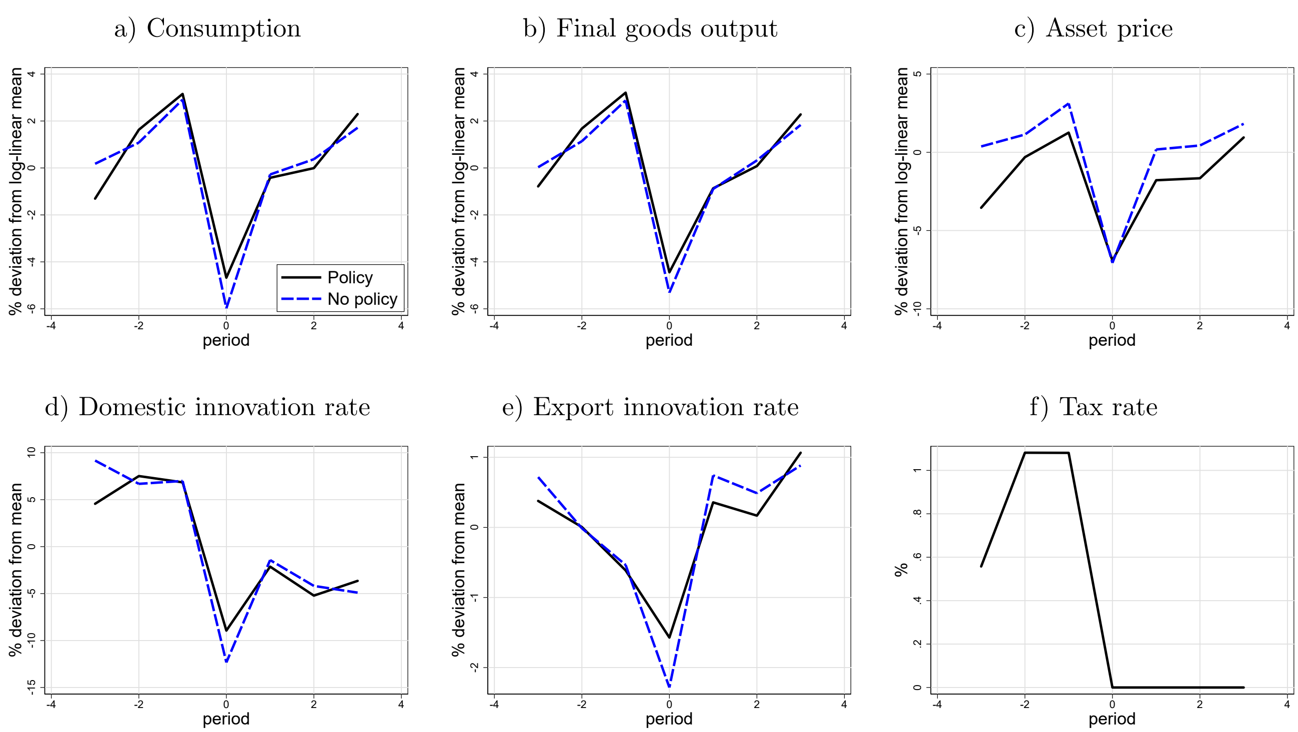
where C_t, H_t, A_t in the left hand side are consumption, labor supply, and aggregate productivity in the model without the policy.[34] Variables with a superscript p in the right hand side correspond to an economy with the policy. Finally, note that the ergodic distribution of $\boldsymbol{S}$ is used to summarize

[34] Welfare gains are calculated for each initial condition $\boldsymbol{S} \equiv R\exp(\varepsilon_{t-1}^R)B_{t-1}, \theta_{t-1}^D, \theta_{t-1}^X, a_t^*, \{\varepsilon_t^A, \varepsilon_t^R\}$.

the consumption equivalent welfare gains of a given policy.

We use numerical maximization for $\bar{b}$ and ι to find the Taylor-rule tax configuration that gives the highest expected welfare. This rule is characterized by $\bar{b} = -0.1225$ and $\iota = 0.6$. To illustrate the effects of the policy, Figure A.2 compares the sudden stop dynamics without policy and under the optimal Taylor-rule tax. We set the initial state of the economy as the average $\boldsymbol{S}$ three periods before the average sudden stop depicted in Section 4 in the main text. To this economy, we feed high values of the productivity shock ε_t^A and low values of the interest rate shock ε_t^R for the first 3 periods (from $t = -3$ to -1), and then low values of ε_t^A and high values of ε_t^R for the following 4 periods (from $t = 0$ to 3).

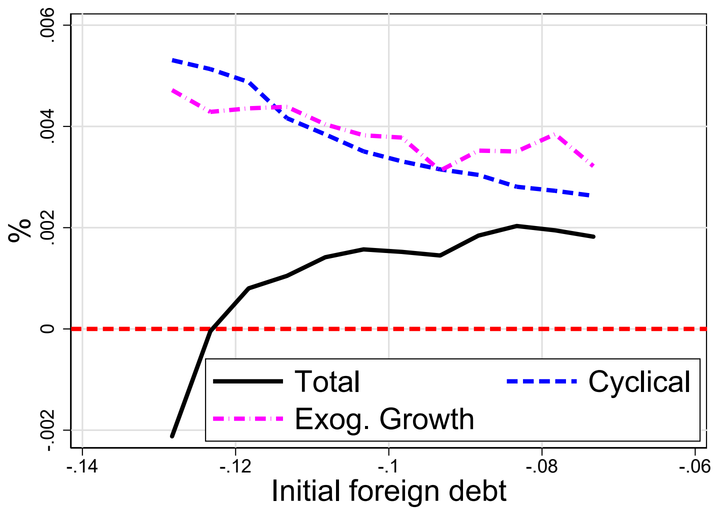
Figure A.2: Sudden stop dynamics under Taylor-rule tax



Note: Consumption, final goods output, and asset price are expressed in percentage deviations from the log-linear mean under no policy. Domestic and export innovation rates are percentage deviations from the mean under no policy. The tax rate is a raw value.

The tax on foreign borrowing reduces foreign borrowing and thus also reduces consumption in the initial period. At the cost of this initial lower level of consumption, the tax mitigates the negative impact of a sudden stop on the economy. As shown in panels a) through c), consumption, final output, and the asset price fall by less under the policy. Similar effects are observed for

Figure A.3: Welfare gain/loss by tax on foreign borrowing



NOTE: This figure plots the permanent consumption gain/loss under the optimal Taylor-rule tax on foreign borrowing relative to the case without policy. The horizontal axis is the initial amount of foreign debt. $\theta^D_{t-1}, \theta^X_{t-1}, a^*_t$ are set to the long-run simulation mean, and the initial business cycle shocks are high productivity and low interest rate.

the innovation rates in panels d) and e). The tax reduces innovation investment and thus the innovation rates at the initial periods, but mitigates the drop in innovation rates during the sudden stop. The last panel shows that the tax rate before a sudden stop materializes is about 1.1%.

Figure A.3 plots the value of γ (i.e. the measure of the welfare impact of the crisis) as a function of the initial foreign debt. The solid black line ("Total") shows that the optimal Taylor-rule tax achieves small welfare gains in some states but it also delivers welfare losses in other states. Evaluated by the ergodic distribution of the state variables under no policy, the expected welfare impact of this optimal Taylor-rule tax is -0.0007%. Therefore, a simple Taylor style rule reduces welfare in the calibrated model. To understand the reason behind these welfare losses, we follow Ma [2020] and decompose the total welfare impact of the policy into a cyclical component and a trend component. We can build a stationary version of the model by dividing all the nonstationary variables by the aggregate productivity index A_t. Thus consumption C_t can be written as $A_t \times c_t$, where c_t is the stationary consumption level. Using c_t, the cyclical component of the welfare impact given the initial states can be measured by $\gamma^c(\boldsymbol{S})$ that satisfies the following:

$$E_0 \sum_{t=0}^{\infty} \beta^t \ln \left((1+\gamma^c(\boldsymbol{S}))c_t - \frac{(H_t)^\omega}{\omega} \right) = E_0 \sum_{t=0}^{\infty} \beta^t \ln \left(c_t^p - \frac{(H_t^p)^\omega}{\omega} \right) ,$$

where c_t is stationary consumption in the model without policy, and c_t^p is stationary consumption under the policy. The cyclical component of the welfare impact γ^c captures the welfare gain through mitigating crises and smoothing consumption, putting aside the effect of the policy on growth. Because most models in the literature only feature exogenous growth, this cyclical component corresponds to the common welfare measure in the literature. The dashed blue line ("Cyclical") in Figure A.3 plots the value of γ^c as a function of the state foreign debt. It shows that the cyclical component of the welfare impact is always positive and higher than the total impact. Evaluated by the ergodic distribution of the state variables, the cyclical component of the welfare impact is 0.005%. This magnitude is comparable to Bianchi and Mendoza [2010], in which the expected welfare gain by the optimal policy under a log-utility function is 0.001%.[35]

This result also implies that the trend component of the welfare impact, $\gamma - \gamma^c$, is always negative. The result of this decomposition sheds light on the underlying forces in the model. On the one hand, the tax on foreign borrowing corrects a pecuniary externality and thereby stabilizes the economy contributing to a welfare gain. On the other hand, this policy is associated with a welfare loss through its negative impact on growth. The negative growth effects of the policy dominate the welfare gains of stabilization, and therefore the total expected welfare impact is slightly negative. It is important to note that our model features semi-endogenous growth. This implies that the long-run growth rate of the model is exogenous. Therefore, policies cannot affect the long-run growth level; they only affect how productivity growth fluctuates.[36]

For comparison purposes, Figure A.3 also shows the welfare gains of the optimal policy under exogenous growth. In this alternative model, all the variables related to growth such as innovation rates, shares of domestic and export product lines, and the aggregate productivity growth rate, are set to the long-run simulation mean from the baseline model. The optimal Taylor-rule tax for this economy is given by $\bar{b} = -0.1175$ and $\iota = 0.4$. As shown in Figure A.3, the welfare gain

[35]We use 15 grid points for debt, 5 grid for θ^D, θ^X, a^*, and 4 grid points for business cycle shocks. These are 7500 potential initial conditions. For each we do 500-period stochastic simulations 1,000 times with and without policy and calculate a set of 7500 values of γ. Then we use the ergodic distribution to build the consumption equivalent welfare gain from the policy.

[36]Under a fully endogenous growth model welfare losses of structural policies could be even larger due to permanent effects in growth rates.

under this policy is almost identical to the cyclical component of the welfare gain in the baseline model. The expected welfare gain evaluated using the ergodic distribution is 0.003%, which is also consistent with the expected welfare impact of the cyclical component.

B.2 Subsidy on Innovation Investment

Having documented that a borrowing tax can trigger welfare losses because of its effects on productivity growth dynamics, it is natural to study if a subsidy to innovation investment can potentially correct the non-cyclical welfare losses. There are several externalities associated with innovation investment in the model. First, because future innovations build on the current knowledge stock, innovations create a positive spillover to future innovators. Second, an increase in the share of domestic product lines θ_t^D implies that this country has more opportunities for export innovation investment. These positive externalities cause socially under-investment. There are also negative externalities due to creative destruction. In fact, a higher innovation rate implies a higher replacement rate d_t, which would discourage innovation investment. Besides, higher domestic productivity growth increases the cost of domestic production and reduces export profits. These negative externalities can lead to over-investment in innovation. The optimal investment is determined by the total effect of these positive and negative externalities.

Because the main welfare losses of a borrowing tax are a-cyclical, we consider a fixed-rate subsidy on incumbent firms' investment in both domestic and export innovation, Z_t^D and Z_t^X. We assume that these subsidies are financed by a lump-sum tax on firms that receive these subsidies. Denoting the fixed subsidy rate by τ^s, the first-order conditions with respect to Z_t^D and Z_t^X are given as follows:

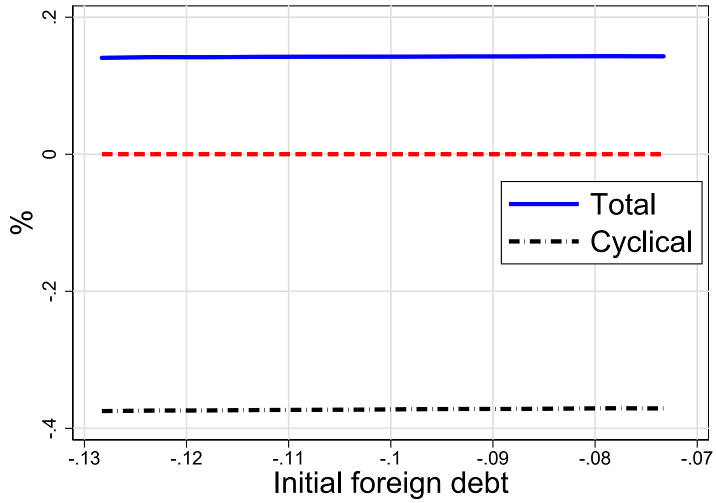
$$\eta^D \frac{1}{\rho}\left(\frac{Z_t^D}{A_t}\right)^{1/\rho-1}\frac{1}{A_t}E_t\left[\Lambda_{t,t+1}V_{t+1}^D\right] = 1-\tau^s$$

$$(1-d_t)\eta^X \frac{1}{\rho}\left(\frac{Z_t^X}{A_t}\right)^{1/\rho-1}\frac{1}{A_t}E_t\left[\Lambda_{t,t+1}\left(V_{t+1}^X - V_{t+1}^D\right)\right] = 1-\tau^s$$

The marginal cost of investment is now given by $1-\tau^s$ instead of 1. All the other equilibrium conditions remain the same.

The welfare maximizing investment subsidy is 18% ($\tau^s = 0.18$). Figure A.4 plots the permanent consumption gain as a function of the initial foreign debt. The blue line ("Total") indicates that

Figure A.4: Welfare gain/loss by subsidy on incumbent investment

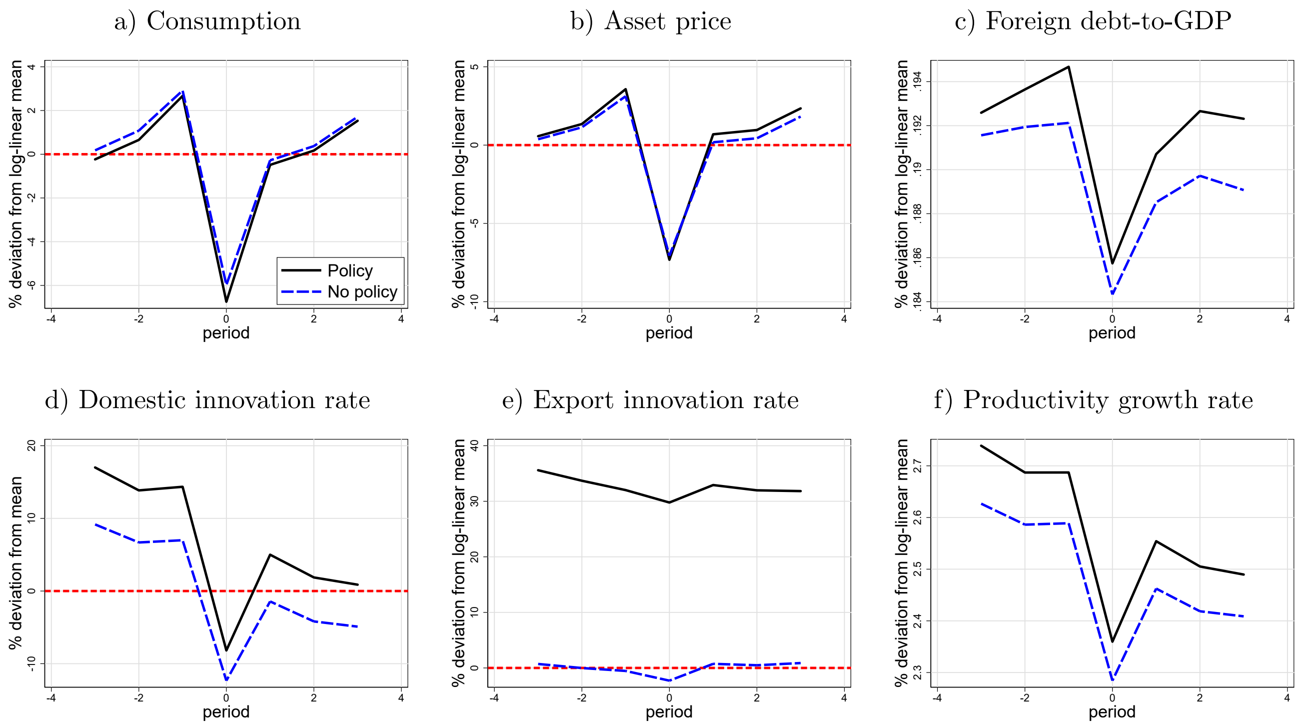


Note: This figure plots the permanent consumption gain/loss under 18% investment subsidy relative to the case without policy. The horizontal axis is the initial amount of foreign debt. $\theta^D_{t-1}, \theta^X_{t-1}, a^*_t$ are set to the long-run simulation mean, and the initial business cycle shocks are high productivity and low interest rate.

the welfare gain by the 18% subsidy on incumbent investment is stable at 0.14% over the initial foreign debt. This welfare gain suggests that there is an important net-positive growth externality in the calibrated model. We decompose this welfare gain into a cyclical and trend component. The red dashed line in Figure A.4 indicates that the cyclical component is -0.37%. This means that the trend component is 0.51% of permanent consumption. Evaluated by the ergodic distribution of the state variables, the expected welfare gain from this subsidy is 0.13% of permanent consumption, whereas the expected welfare impact of the cyclical component is -0.40%.

To understand this result, Figure A.5 compares the sudden stop dynamics under no policy and the optimal investment subsidy. This figure is created using the same procedure as Figure A.2. In panels a) through c) we observe that consumption is initially lower and foreign debt is larger under the investment subsidy. This is because private agents borrow more to increase investment in innovation. When a sudden stop happens, consumption and the asset price fall by more under the subsidy because the pre-crisis debt is larger. In this way, the subsidy on innovation investment induces larger foreign borrowing and exacerbates sudden stops, thereby lowering the cyclical welfare gains. This is reflected by a large negative cyclical component of the welfare impact in Figure A.4. At the cost of this short-run negative impact on the economy, the subsidy induces

Figure A.5: Sudden stop dynamics under investment subsidy

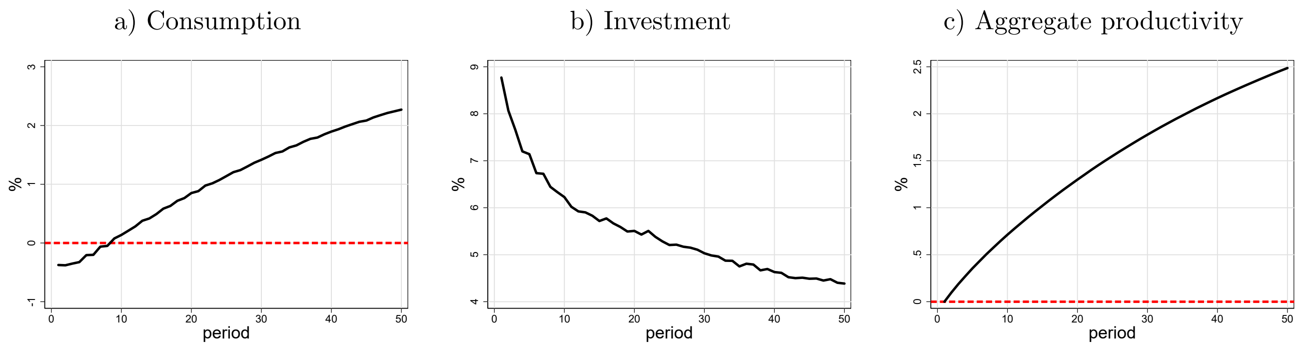


Note: Consumption and asset price are expressed in percentage deviations from the log-linear mean under no policy. Domestic and export innovation rates are percentage deviations from the mean under no policy. Debt-to-GDP ratio and productivity growth rate are raw values.

higher investment and promotes productivity growth. Panels d) through f) in Figure A.5 show that both domestic and export innovation rates are substantially higher under the subsidy, and aggregate productivity grows faster at least in the short run.

Next we examine the trend component of the welfare impact under the investment subsidy. We simulate the model without policy and under investment subsidy for 1,000 times starting from the long-run mean state, and take the mean of log gaps in variables under investment subsidy relative to the case of no policy. The result is presented in Figure A.6. Panel a) shows that consumption is lower under the subsidy for the first 8 periods on average. This lower consumption is due to higher investment shown in the panel b). Total investment, which is a sum of investment in domestic and export entry and domestic and export incumbent innovation, is about 9% higher initially under the subsidy. As the aggregate productivity grows faster under the subsidy, as shown in panel c), consumption also becomes higher under the subsidy. In the long run, consumption is permanently higher by about 4% under the 18% subsidy. This long-run higher consumption accounts for the

Figure A.6: Mean log gaps under investment subsidy and no policy



NOTE: These figures are created by simulating the model without policy and under investment subsidy for 1,000 times starting from the long-run mean state of the model, and taking the mean of log deviations of each variable under investment subsidy relative to the case of no policy.

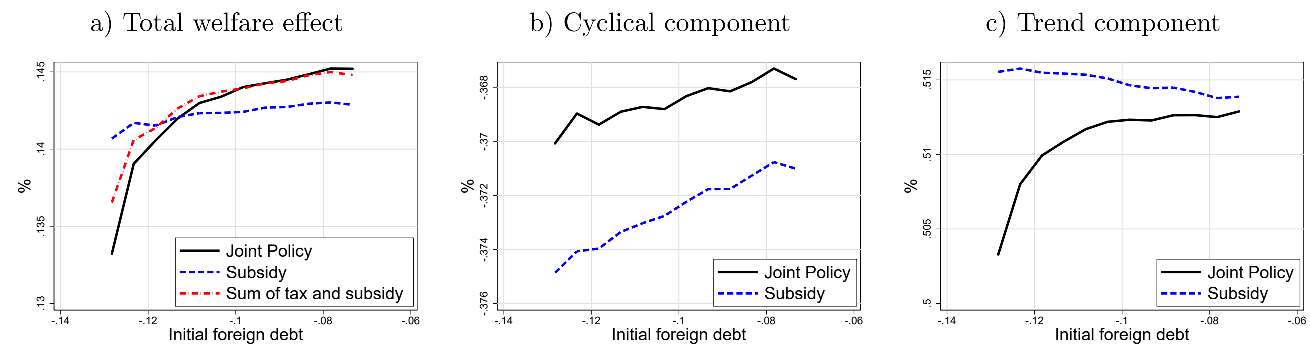
trend component of the welfare impact, which corresponds to 0.51% permanent consumption gain.

B.3 Joint Policy

We have documented that a tax on foreign borrowing can stabilize the economy during crises at the cost of lower investment in innovation. We have also shown that investment subsidies can increase innovation investment at the cost of lower short-run consumption and deeper crises. A natural question is if a joint policy of a borrowing tax and an innovation subsidy can improve welfare more than each policy individually. We answer this question by introducing the tax and the subsidy at the same time, with the policy parameters set to the optimal ones found in the previous subsections ($\bar{b} = -0.1225$ and $\iota = 0.6$ for the tax and $\tau^s = 0.18$ for the subsidy).

Figure A.7 plots the welfare effect of this joint policy, along with the welfare effect of the optimal subsidy for comparison. Panel a) shows that the joint policy (solid black line) brings higher expected welfare than the optimal subsidy (dashed blue line) when the initial debt is relatively small, but lower expected welfare when the initial debt is large. The same panel also shows that the welfare gain from the joint policy is very close to the sum of the welfare gain from the optimal tax and from the optimal subsidy (dot-dashed red line) when these policies are introduced individually. Evaluated using the ergodic distribution of the state variables under no policy, the expected welfare gain from the joint policy is 0.130%, whereas it is 0.127% under the optimal subsidy. Thus, the joint policy brings slightly higher expected welfare. This result is

Figure A.7: Welfare gain/loss by joint policy



NOTE: These panels plot the permanent consumption gain/loss under a joint policy and investment subsidy relative to the case without policy. Panel a) also shows the sum of welfare gain/loss by the optimal Taylor-rule tax and subsidy. Panel b) plots the cyclical component, and panel c) plots the trend component of welfare gain by a joint policy and investment subsidy.

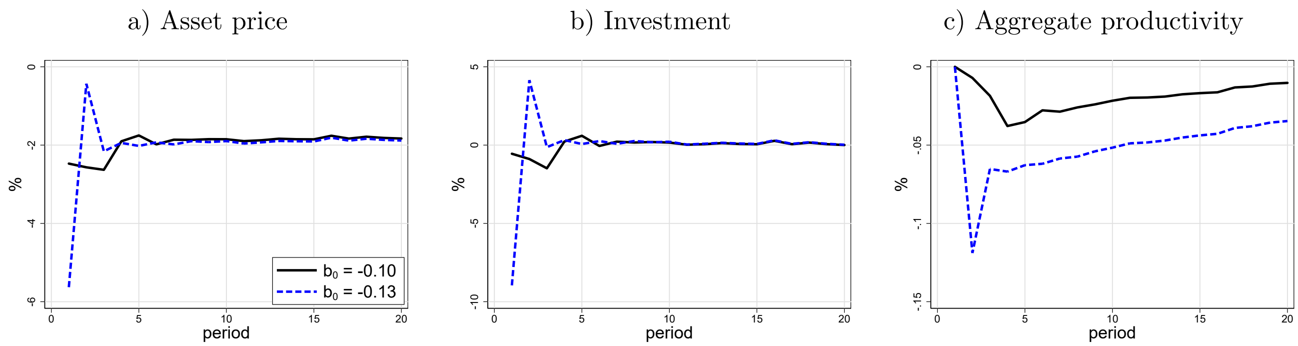
consistent with the moderate welfare effect of the borrowing tax shown in Section B.1.

Panel b) in Figure A.7 plots the cyclical components of the welfare effect of the joint policy and the optimal subsidy. It shows that the cyclical component is always higher under the joint policy than under the subsidy, although it is negative under both policies. A higher cyclical component under the joint policy reflects the stabilization effect of the tax on foreign borrowing. Panel c) shows that the trend component is always higher under the subsidy, and the gap becomes larger as the initial debt becomes larger. In particular, the trend component under the joint policy declines quickly as the initial debt becomes larger. This lower trend component reverses the welfare comparison between the two policies when the initial debt is large.

To understand this reversal at high debt levels, we compare the dynamic path under the joint policy and the optimal subsidy starting from different levels of initial debt. We start the simulations with initial debt levels $b_0 = -0.10$ and $b_0 = -0.13$, with the other state variables set to the mean of a long-run simulation. We simulate the model 1,000 times under the joint policy and the optimal subsidy, and take the mean of these simulations. Figure A.8 plots the percentage gap in the mean dynamics for the first 20 periods of this exercise. The plotted values are the percentage deviations of the values under the joint policy relative to the values under the subsidy.

Abstracting from the volatility of the first periods, the panel a) shows that the asset price under the joint policy is on average 2% lower than under the subsidy. The reason for this lower price is that a borrowing tax reduces the stochastic discount factor of households. Because the

Figure A.8: Initial debt and average dynamic path



NOTE: These panels plot the percentage gap of the values under joint policy relative to the values under the subsidy, taken from the mean dynamics of 1,000 simulations. The solid black lines correspond to simulations starting from $b_0 = -0.10$, and the dashed blue lines are simulations starting from $b_0 = -0.13$.

asset price is determined by the household's forward-looking decision, this force works even when the tax is not imposed today. The possibility of a tax being imposed in future lowers the future expected asset price, which in turn lowers the asset price today.

Under the more indebted initial condition (dashed blue lines), the borrowing constraint binds on the first period, triggering a sudden stop. Interestingly, the asset price falls 6% more under the joint policy (panel a)) on impact. Panel b) shows that due to a sudden stop, total investment drops 9% more under the joint policy. Panel c) shows that the aggregate productivity is lower by 0.12% on impact, and 0.04% lower even after 20 periods. This persistently lower productivity accounts for the low trend component under the joint policy when initial debt is large.

In contrast, when the initial debt is not large (solid black lines), a sudden stop does not happen in period 1, although the asset price is similarly lower under the joint policy. Panels b) and c) show that the tax on foreign borrowing reduces investment and slows down productivity growth, but the impact is substantially limited and short-lived.

In sum, joint innovation subsidies with borrowing taxes have the potential to improve welfare and ameliorate the incidence of crises. Future work should further explore optimal policy jointly allowing for time varying innovations subsidies.

C Relative Productivity and Marginal Cost

This section disentangles the factors that affect firm dynamics after sudden stops. Section 4 in the main text shows that persistently low marginal cost after sudden stops increases export profits and induces export innovations, thereby promoting the recovery of aggregate productivity. We design a counterfactual analysis to study the determinants of the persistently low marginal cost after sudden stops, and how the decline in marginal cost affects the recovery of productivity after a sudden stop.

Equation (13) in the main text shows that export profits π_t^* are determined by the relative marginal cost:

$$\text{Relative marginal cost} = \frac{\left(R_t^L\right)^\alpha \left(W_t\right)^{1-\alpha}}{\left(R_t^{L*}\right)^\alpha \left(W_t^*\right)^{1-\alpha}}$$

Dividing both the denominator and numerator by the domestic productivity index A_t and denoting stationary values with lower-case letters,

$$\text{Relative marginal cost} = \frac{\left(r_t^L\right)^\alpha \left(w_t\right)^{1-\alpha}}{\left(r^{L*}\right)^\alpha \left(w^*\right)^{1-\alpha} \times A_t^*/A_t} = \frac{\left(r_t^L\right)^\alpha \left(w_t\right)^{1-\alpha}}{\left(r^{L*}\right)^\alpha \left(w^*\right)^{1-\alpha}} \frac{1}{a_t^*}, \tag{A.67}$$

where $a_t^* = A_t^*/A_t$ is foreign productivity relative to domestic productivity. Because foreign factor prices R_t^{L*} and W_t^* grow at the exogenous rate $\bar{g}$, foreign factor prices divided by foreign productivity A_t^* are constant, and we denote them with r^{L*} and w^*. Equation (A.67) indicates that the relative marginal cost is determined by two factors: the stationary domestic marginal cost $\left(r_t^L\right)^\alpha \left(w_t\right)^{1-\alpha}$ and relative foreign productivity a_t^*. Thus, export profits are higher when the stationary domestic marginal cost is lower and/or relative foreign productivity is higher.

We design a counterfactual experiment to show that the relative foreign productivity is the key driver of persistently low marginal cost after sudden stops. To do this, we first show that a sudden stop causes persistently higher relative foreign productivity. We take the following steps:

1. We set the initial state of the economy to the average values one period before a sudden stop.

2. We consider two different economies.

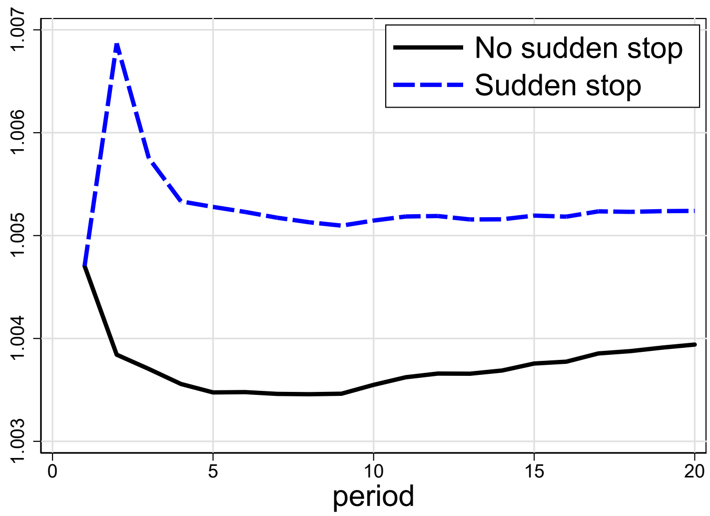
    (a) In the first economy, the initial business cycle shocks are set to high productivity and low interest rate. In this case, a sudden stop is not triggered at the initial period.

(b) In the second economy, initial business cycle shocks are set to low productivity and high interest rate. This shock triggers a sudden stop at the initial period.

3. In the following periods, we feed the same random shocks and simulate the two economies.

4. We repeat this procedure 1,000 times and compute the mean path of relative foreign productivity over 1,000 simulations in these two economies.

Figure A.9: Relative foreign productivity after a sudden stop

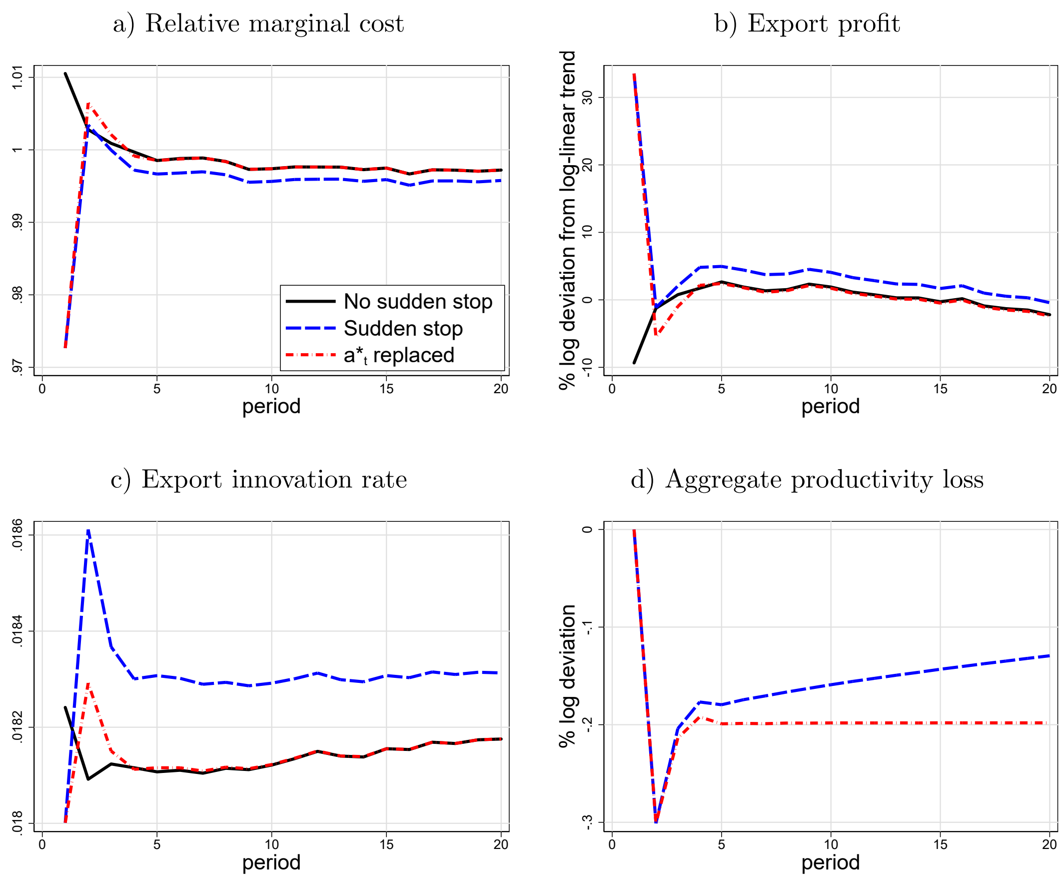


NOTE: This figure plots the mean paths of foreign productivity relative to domestic productivity over 1,000 simulations. The solid black line is the path in the case of no sudden stop at period 1, and the dashed blue line is the path after a sudden stop at period 1.

Figure A.9 plots the mean paths of relative foreign productivity in the economy without a sudden stop (solid black line) and with a sudden stop (dashed blue line) at period 1. Without a sudden stop, the domestic economy experiences a high productivity and a low interest rate shock that stimulates innovation. Naturally, domestic productivity increases persistently causing relative foreign productivity to decline. In contrast, in the case of a sudden stop, innovation investment drops sharply and domestic productivity growth slows down. Relative foreign productivity jumps up by 0.2% on impact and stays higher than in the case of no sudden stop by more than 0.1%. This is another illustration of the inherent endogenous hysteresis of the model.

Having established that a sudden stop causes persistently higher relative foreign productivity, we design a counterfactual analysis that compares three economies in order to disentangle the role of productivity and factor prices. In the first economy, the initial shocks are high productivity and a low interest rate, which do not trigger a sudden stop. In the second economy, the initial shocks are low productivity and a high interest rate, which triggers a sudden stop. The third economy is the counterfactual economy, in which the initial shock is the same as in the second economy and a sudden stop happens, but the path of relative foreign productivity is replaced by the path in the first economy. We simulate these three economies for 1,000 times with the same random shocks after period 2 and compare their dynamics.

Figure A.10: Sudden stop dynamics: microeconomic outcomes



Note: This figure plots the mean paths of selected variables over 1,000 simulations under the three economies. Export profit is log deviations from the log-linear trend in the case of no sudden stop. Aggregate productivity loss is log deviations relative to the path without a sudden stop.

Figure A.10 shows the average path of the relative marginal cost, export profits, exporting innovation rate, and the loss in productivity growth in the three economies. Figure A.10a shows that when a sudden stop occurs (dashed blue line) the relative marginal cost drops by 3.8%

on impact, and stays lower than the path without a sudden stop by 0.2% persistently. Recall that in the counterfactual economy (dot-dashed red line) the sudden stop occurs but relative productivity follows the path of the no-sudden stop economy (solid black line). Interestingly, in the counterfactual economy the relative marginal cost drops on impact, but quickly recovers, essentially joining the path of the economy without a sudden stop. This result indicates that the persistently lower marginal cost after a sudden stop is driven by persistently higher relative foreign productivity.

Figure A.10b shows the evolution of export profits in deviations from the long-run growth rate path. When a sudden stop occurs, the reduction in relative marginal cost increases export profits by more than 30%. After a sudden stop, export profits stay higher than the path without a sudden stop by about 2% in the baseline economy, while, in the counterfactual economy the increase is only temporary and once again we see a lack of persistence after the crisis. In the baseline economy, the sudden stop triggers a persistent increase in export profits, therefore affecting the expected return to innovation. Figure A.10c shows that after a sudden stop, the export innovation rate jumps up and stays persistently higher afterwards. In the counterfactual economy, the initial jump is smaller, and the following path is identical to the case without a sudden stop. Because innovation decisions are forward looking, the future path of marginal cost is more important than the decrease on impact of the marginal cost in affecting firm dynamics.

Figure A.10d shows the paths of aggregate productivity in the economy with a sudden stop (dashed blue line) and in the counterfactual economy (dot-dashed red line) relative to productivity in the economy without a sudden stop. The dashed blue line is the same as the path in Figure 9 in the main text. It shows that aggregate productivity drops by 0.3% on impact in a sudden stop, and the recovery process is slow. The counterfactual path shows that the initial decline is the same as in the case of a sudden stop, but there is no recovery in productivity in the following periods. This result indicates that persistently high relative foreign productivity and associated low relative marginal cost fully account for the productivity recovery after a sudden stop. In fact, this is the force that makes the baseline economy a semi-endogenous growth model with the exogenously-growing foreign economy's productivity acting as a long-run attractor due to the gap in marginal costs.

We compute the welfare loss in this counterfactual economy in the same way as in Section 4.3 in the main text. The welfare loss in terms of a one time consumption loss in the counterfactual economy is 5.32%. This is 15% higher than the welfare cost of a sudden stop in the baseline model. Therefore, the recovery process is an important component of the welfare cost of financial crises.

Summing up, the persistence of a lower marginal cost is the main driver of firms' export innovation. This persistence depends solely on the dynamics of relative productivity. The short-run decrease in stationary factor prices mainly affects the short-run impact of the crisis without directly distorting the dynamics of productivity accumulation.

D Possibility of Sudden Stops and Ex-Ante Innovation

This section shows that as a result of the occasionally binding borrowing constraint and the global solution method, the expectation that sudden stops can take place with a non-zero probability has an impact on firms' innovation decisions in "normal times". To make this point, we construct counterfactual firm profits during a sudden-stop period assuming that the constraint does not bind, and examine how this assumption would affect firm investment before the crisis.

If we remove the borrowing constraint from the model, domestic profits π_t^D, π_t^X and export profits π_t^* can be obtained by solving a static problem. Given the state variables $R\exp(\varepsilon_{t-1}^R)B_{t-1}$, θ_{t-1}^D, θ_{t-1}^X, a_t^*, and $\{\varepsilon_t^A, \varepsilon_t^R\}$, we can solve the simultaneous 13-equation system that consists of (A.24), (A.28), (A.29), (A.30), (A.31), (A.32), (A.33), (A.34), (A.35), (A.36), (A.49), (A.55), and (A.56) and derive 13 endogenous variables: H_t, H_t^D, H_t^X, H_t^*, L_t^D, L_t^X, L_t^*, Y_t, W_t, R_t^k, π_t^D, π_t^X, and π_t^*. We use these counterfactual profits denoted by $\tilde{\pi}_t^D$, $\tilde{\pi}_t^X$, and $\tilde{\pi}_t^*$. We study how these future counterfactual profits would affect firms' investment today. In particular:

1. We set the state variables at their ergodic mean values one period before a sudden stop happens;

2. Given the state variables, we derive firm investment in domestic and export innovation Z_t^D and Z_t^X using the decision rules. These are the actual investment by firms in the baseline model with the possibility of a binding constraint;

3. We derive all the other endogenous variables including the next period state variables, using the corresponding decision rules;

4. There are 4 possible states in the next period, high and low realizations for ε_{t+1}^A and ε_{t+1}^R respectively. For each state, given the state variables obtained in step 3, we compute counterfactual profits $\tilde{\pi}_{t+1}^D$, $\tilde{\pi}_{t+1}^X$, and $\tilde{\pi}_{t+1}^*$ by solving the static problem described above;

5. We compute counterfactual values of domestic and export product lines $\tilde{V}_{t+1}^D$ and $\tilde{V}_{t+1}^X$ for each of 4 states by replacing π_{t+1}^D, π_{t+1}^X, and π_{t+1}^* in equations (17) and (20) by the counterfactual profits obtained in step 4;

6. For the period before the crisis, we compute the counterfactual expected value of a domestic and an export product line $E_t[\tilde{V}_{t+1}^D]$ and $E_t[\tilde{V}_{t+1}^X]$ using the counterfactual value of product lines and the transition matrix;

7. Using the counterfactual expected values of product lines, we derive the counterfactual firm investment $\tilde{Z}_t^D$ and $\tilde{Z}_t^X$ from the first-order conditions (18) and (19), and we compare them with the actual investment Z_t^D and Z_t^X obtained in step 2.

Table A.2 summarizes the result of this analysis. The numbers in the table are percentage deviations of the counterfactual values from the values in the baseline model. The second column shows that the counterfactual domestic profits in the next period are higher by 4.0% and 4.6% when the next period productivity is low. This gap is due to the absence of a borrowing constraint in the counterfactual simulation. When the next period productivity is high, a sudden stop does not happen in the baseline model, and thus there is no gap in profits. In contrast to the domestic profits, export profits in the third column are lower by 4% and 6% under the counterfactual simulation. This is because in the baseline model, a sudden stop increases export profits by reducing the domestic production cost. The fourth and fifth columns show how the expected value of product lines is also affected by the expectation of a binding constraint. Due to a higher domestic profit, the counterfactual expected value of a domestic product line is higher by 0.21%. There are two reasons for this limited gap. First, the probability of low productivity in the next period is only 22%. Second, the expected value of a product line is the sum of next period profits and the future continuation value of a firm, but we manipulate only the former. The counterfactual expected value of an export line has a negative but even smaller gap relative to the baseline model. This is because an export product line has both domestic and export profits, and higher domestic profit and lower export profit offset each other. The last two columns show the counterfactual firm investment in domestic and export innovation. Without the expectation of a binding constraint next period, firms invest 0.62% more in domestic innovation, and 0.10% less in export innovation. This result suggests that stabilization policies such as borrowing taxes can have effects even in normal times. When a capital control is introduced, because it is expected to mitigate the impact of a future sudden stop on firm profits, firms will invest more in domestic innovation but may be discouraged to invest in export innovation. Optimal design of capital controls should therefore take this heterogeneous effect into account.

Table A.2: Counterfactual firm profits, expected value, and investment

$(\varepsilon^A_{t+1}, \varepsilon^R_{t+1})$	$\tilde{\pi}^D_{t+1}, \tilde{\pi}^X_{t+1}$	$\tilde{\pi}^*_{t+1}$	$E_t[\tilde{V}^D_{t+1}]$	$E_t[\tilde{V}^X_{t+1}]$	$\tilde{Z}^D_t$	$\tilde{Z}^X_t$
(high,high)	0%	0%				
(low,high)	4.0%	−4.0%	0.21%	−0.06%	0.62%	−0.10%
(high,low)	0%	0%				
(low,low)	4.6%	−6.0%				

NOTE: Numbers in this table are percentage deviations of the counterfactual values under no suddens stops from the values in the baseline model.

E Model without a Working Capital Requirement

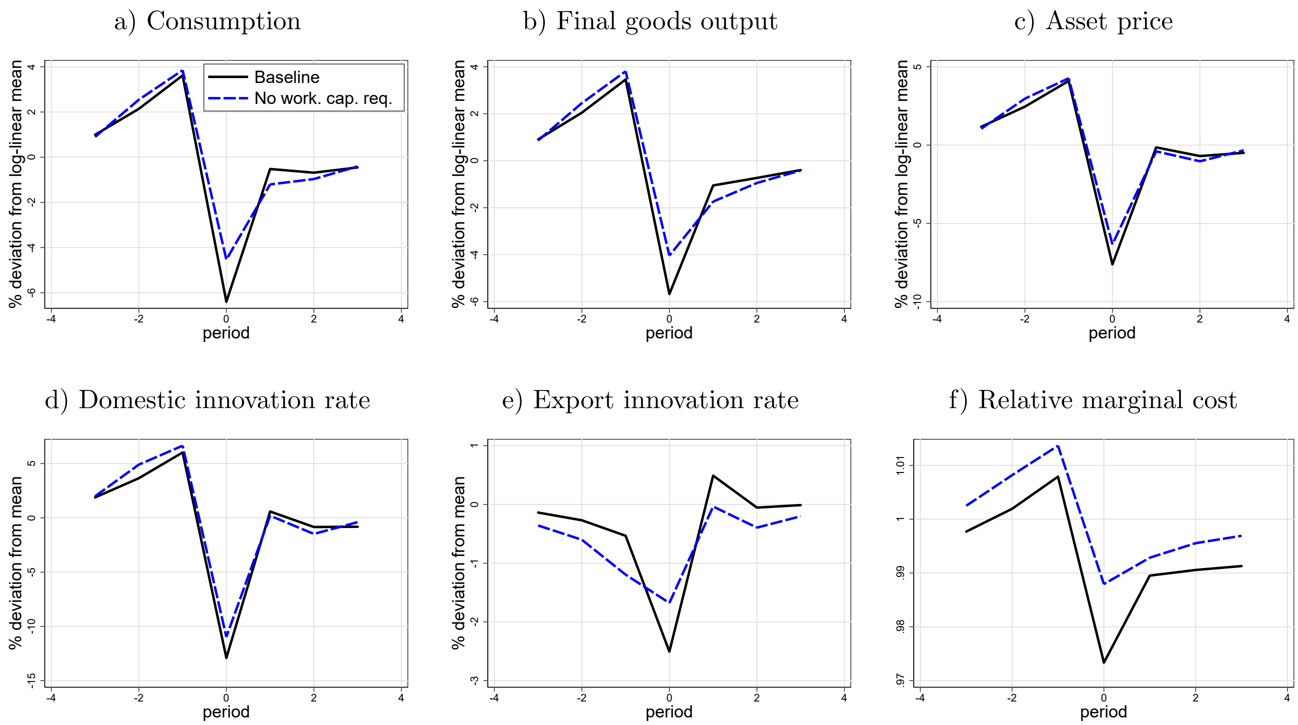
This section studies a version of the baseline model without a working capital requirement to understand how important this channel is for the quantitative behavior of the model. Removing the working capital ($\phi = 0$), the borrowing constraint in equation (2) becomes:

$$-B_t \leq \kappa Q_t L.$$

In this alternative model, a binding constraint does not affect firms' profits in the domestic market, but still affects innovation through lower consumption and the stochastic discount factor. Importantly, without changing the borrowing limit $\kappa Q_t L$ from the baseline model, this alternative economy would be able to borrow substantially more, which makes it difficult to compare the models with and without working capital. Therefore, we set $\kappa = 0.1$ instead of the $\kappa = 0.2$ used in the baseline model, so that the amount of foreign debt is practically the same in both models. All the other parameter values are unchanged. We solve this model numerically, simulate it for 10,000 periods with stochastic shocks, and derive the average sudden stop dynamics in the same way as in the baseline model.

Figure A.11 compares the sudden stop dynamics in the baseline model (solid black lines) and those in the model without working capital (dashed blue lines). Panels a) through c) show the average dynamics of consumption, final good production, and the asset price around the sudden stop, expressed as percentage deviations from the log-linear mean of each variable in each model. The dynamics are overall milder in the model without working capital. Consumption drops to 6.4% below the trend in the baseline model, whereas it drops to 4.5% in the model without working

Figure A.11: Sudden stop dynamics in the model without working capital



Note: Consumption, final goods output, and the asset price are expressed in percentage deviations from the log-linear mean of each variable in each model. Domestic and export innovation rates are percentage deviations from the mean of each variable in each model. The relative marginal cost is the raw value.

capital. Similarly, final good production and the asset price fall to 5.7% and 7.6% below the trend in the baseline model, but only to 4.0% and 6.3% in the model without working capital. Therefore, working capital amplifies the crisis in the order of 40%.

Panels d) through f) show the domestic and export innovation rates, and the relative domestic marginal cost relative to the foreign cost. Consistent with panels a) through c), both the domestic and export innovation rates fall less in the model without working capital. Another difference is that after a sudden stop, the export innovation rate is persistently lower in the model without working capital. This difference comes from the transition of the relative marginal cost. As the innovation rates fall less in the model without working capital, productivity growth does not slow down as much and the relative marginal cost stays relatively high after a sudden stop, as shown in panel f). This implies that export profits stay low, and thus the export innovation rate is persistently lower than in the baseline model.

F Model without a Borrowing Constraint

This section compares the baseline model to an alternative economy that does not face a borrowing constraint. This comparison allows us to study how the borrowing constraint amplifies negative shocks. If the borrowing constraint is removed from the baseline model, the country would keep accumulating foreign debt without limit because the parameters are set such that $\beta R < 1 + \bar{g}$.[37] Therefore, we adjust the interest rate R such that $\beta R = 1 + \bar{g}$. This implies $R = 1.0677$ instead of the $R = 1.05$ value of the baseline model. An issue associated with this parameter change is that the model shows random walk dynamics and is not stationary, as discussed in Schmitt-Grohé and Uribe [2003]. Therefore, we follow Schmitt-Grohé and Uribe [2003] and introduce a debt-elastic interest rate to make the model stationary:

$$R_t = R + \psi_b \left(\exp \left(-\frac{B_t}{A_t} + \bar{b} \right) - 1 \right) \tag{A.68}$$

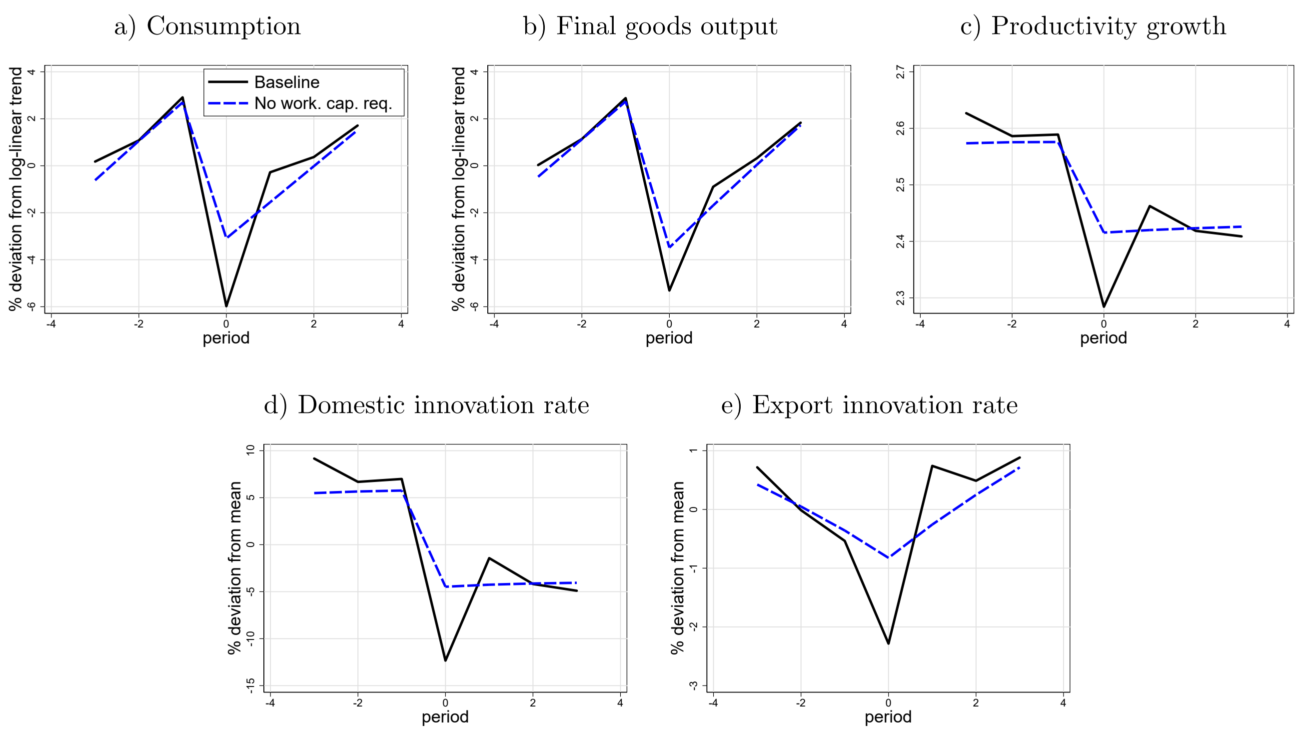
where $\psi_b > 0$ is the coefficient on the debt-elastic interest rate and $\bar{b} < 0$ is foreign debt adjusted by aggregate productivity in the long run. As in Schmitt-Grohé and Uribe [2003], we assume that private agents do not internalize that their choice of foreign bond would affect the interest rate through this equation. We use ψ_b to target the standard deviation of B_t/A_t in the baseline model leading to $\psi_b = 2$, a value consistent with Garcia-Cicco et al. [2010]. Similarly, we set $\bar{b}$ to -0.12, which is the mean of a long-run simulation. We solve this model numerically using the same global method and the same stochastic shock process $\left\{\varepsilon_t^A, \varepsilon_t^R\right\}$ as in the baseline model.

Figure A.12 compares the sudden stop dynamics in the baseline model (solid black lines) and those in the model without a borrowing constraint (dashed blue lines). This figure is created by the same method as Figure A.2 in the policy analysis section (Appendix B). Namely, we set the initial state to the values 3 periods before the average sudden stop dynamics, and feed good shocks for the first 3 periods and feed bad shocks for the latter 4 periods to each economy. Consumption and final goods output are percentage deviations from the log-linear trend in each model. Domestic and export innovation rates are percentage deviations from the mean of each model.

First note that all variables decrease more during the crisis in the baseline model than in the model without a borrowing constraint. This is due to the binding borrowing constraint and the

[37] In models with no growth, such as Bianchi [2011] and Bianchi and Mendoza [2018], $\beta R < 1$ is assumed so that the country always borrows from abroad. In our model, there is growth and thus the condition for the country to be always borrowing is $\beta R < 1 + \bar{g}$ given the log utility.

Figure A.12: Sudden stop dynamics in the model without a borrowing constraint



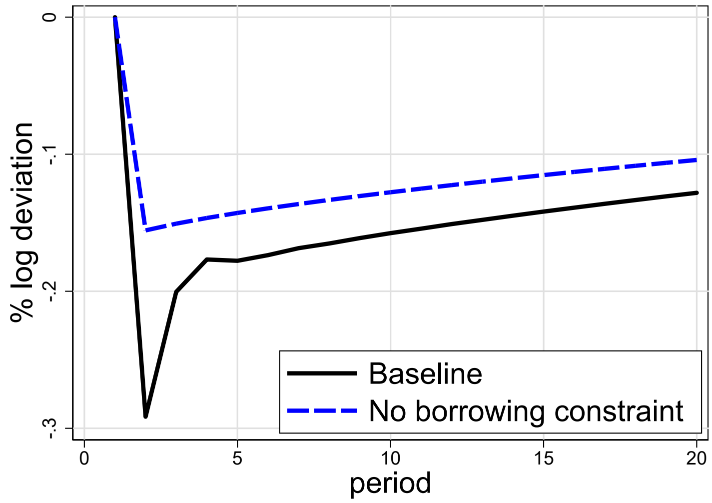
Note: Consumption and final goods output are expressed in percentage deviations from the log-linear trend in each model. Domestic and export innovation rates are percentage deviations from the mean in each model. The productivity growth rate is a raw value.

amplification effect through the debt-deflation dynamics. If the shocks were re-calibrated in the economy without borrowing constraints, then the normal business cycles of the economy would be too extreme. In fact, the appeal of models based on Mendoza [2010] is that the same model can nest normal business cycles and deep financial crises.

Second, crisis dynamics in the model without a borrowing constraint show a symmetric boom and bust around the trend, whereas in the baseline model the size of the bust during a crisis is larger than the size of a boom before a crisis. A boom in consumption before a crisis is in the order of 3% in both models. The decline during the crisis has a similar absolute size in the model without a borrowing constraint, but in the baseline model the absolute change during the crisis practically doubles the size of the preceding boom. Output, the productivity growth rate, and the innovation rates show similar patterns. These asymmetric feature of business cycles are also well documented by Mendoza [2010].

Figure A.13 plots the average path of log deviations in productivity relative to the path without

Figure A.13: Productivity in the baseline model and the model without a borrowing constraint



NOTE: The figure plots log deviations in productivity relative to the path without a sudden stop in each model.

a sudden stop in each model. This figure is created by the same method as Figure 9 in the main text. The initial drop in productivity in the baseline model is roughly twice as large as the fall in the model without a borrowing constraint. The quick recovery of productivity in the next period in the baseline model happens for two reasons. First, foreign debt substantially shrinks on impact of a crisis due to the binding borrowing constraint, and private agents borrow and invest aggressively in the next period. Second, a larger slowdown of productivity growth implies that the relative marginal cost becomes lower, which makes exports more profitable and induces more investment in export innovation, as shown in Figure A.12.

Overall, the occasionally binding constraint with an endogenous collateral asset price amplifies the dynamics and makes the dynamics asymmetric, thereby improving the model's ability to explain the crisis dynamics reported in the literature without sacrificing its performance in normal times.

G Empirical Appendix

G.1 Product Transitions

Here we document the frequency with which firms add or drop products from the domestic and export markets. We document these transitions based on a balanced panel for 1996-1997 (i.e. prior to the sudden stop) with 3512 firms out of which 825 (23.5%) are exporters in 1996 and 870 (24.8%) are exporters in 1997.

We count the number of firms adding or dropping products and group them according to their initial and final status (not sold, sold exclusively domestically, or exported). We define six transitions of interest. The frequency of these transitions is shown in Column 1 in Table A.3. We find that 15.4% of firms add one or more domestic product not produced the previous year. 2.6% of firms add one or more new products simultaneously to the domestic and export markets. We also find that 5.3% of firms add one or more products to the export market sold exclusively in the domestic market the previous year.

We also find that 16.0% of firms drop one or more domestic products. 2.7% of firms drop one or more product both sold domestically and exported. Finally, 4.5% of firms drop one or more products from the export market that transitions to be sold only domestically the next period.

Table A.3 also documents the number of products added or dropped in each transition. In each case, there is a larger probability of adding or dropping a single product, and the probability of each event is decreasing in the number of products added or dropped. In all cases, the decrease in the probability of adding or dropping a single product to adding or dropping more than one is quite steep. For instance, while 10% of firms introduce a new product to the domestic market, only 3% introduce 2 products and 3% introduce three or more; and while 5% introduce a single product previously sold domestically to the export market, only 0.4% introduce two, and 0.4% introduce three or more.

Table A.3: Number of Firms Per Transition

	Number of Products Added or Dropped			
	Any	1	2	3+
Not Produced to Domestic	0.15	0.10	0.03	0.03
Domestic to Exported	0.05	0.05	0.004	0.004
Domestic to Not Produced	0.16	0.10	0.03	0.03
Exported to Domestic	0.05	0.04	0.005	0.003
Not Produced to Domestic + Exported	0.03	0.02	0.003	0.002
Domestic + Exported to Not Produced	0.03	0.02	0.004	0.001

Note: This table reports the frequency of firms' product transitions. The transitions in each row are (1) a firm introduces to the domestic market a product not sold previously, (2) a firm introduces to the export market a product previously sold domestically, (3) a firm withdraws a product from the domestic market, subsequently not selling it, (4) a firm withdraws a product from the export market, subsequently selling it domestically, (5) a firm introduces simultaneously to the domestic and export market a product not sold previously , and (6) a firm simultaneously withdraws a product from the domestic and export markets, subsequently not selling it.

G.2 Extensive Margin: Domestic, Export and Import Lines

Figure 6b in the main text indicates that, in the model, the decline in the share of export lines is due primarily to a decline in direct export entry (i.e. entry into exporting by new firms).

We contrast this with the data, and document in Table A.4 that in fact we see a larger decline in export entry during the sudden stop among entrants (firms not active in the previous year) than among incumbent firms.

G.3 Productivity Estimation

We estimate firm-level productivity using Wooldridge [2009]'s method, which builds on Levinsohn and Petrin [2003]. First, we estimate the following production for each 3-digit industry s: function:

$$logy_{it} = d_t^s + \beta_{sl} logl_{it} + \beta_{sk} logk_{it} + logz_{it} + \epsilon_{it} \tag{A.69}$$

Table A.4: Export Entry by Incumbent and New Firms

	(1) Share of Exported Products by New Firms	(2) Share of Exported Products by Incumbent Firms
1997	0.091	0.087
1998	0.051	0.068
1999	0.061	0.063

NOTE: This table reports, for each year, the share of new products introduced by firms to the export market. In column 1 we count new export products sold by entrants (firms not present in the previous period). In column 2 we count new export products sold by incumbent firms.

In this expression y_{it} is real value added for firm i in year t, d_t^s is a year fixed effect, l_{it} is total employment, and k_{it} is the real stock of capital. The coefficients β_{sl} and β_{sk} are the elasticities of value added with respect to labor and capital respectively, and are industry-specific. Having estimated these elasticities, firm productivity is defined as follows:

$$logz_{it} = logy_{it} - \hat{\beta}_{sl} logl_{it} - \hat{\beta}_{sk} logk_{it} \quad \text{(A.70)}$$

G.4 Data Description

In this section, we further describe the data on firms' product portfolios. Table A.5 provides summary statistics on the number of firms, number of products, number of firm-product pairs and the mean number of products per firm by two-digit industry.

To provide a better sense of what a product consists of in the data, note that there are about 1800 different products in each year. These products are highly disaggregated. Some examples within the textile sector are men's coats, women's coats, children's coats, leather jackets, and wedding dresses. The products are defined according to the Chilean product classification.

G.5 Evidence of a "Replacement Effect"

In the model, under Bertrand competition, the addition of a product by a firm implies that at the same time another firm withdraws a product from the same market. Here we provide evidence that is concordant with such a replacement effect.

First, for each two-digit industry, Figure A.14a shows that the number of firms introducing

Table A.5: Descriptive Statistics

Industry Code	Industry Name	Number of Firms	Number of Different Products	Number of Firm-Product Pairs	Mean Number of Products per Firm
15	Food and beverages	1593	247	3657	2.30
16	Tobacco products	2	2	2	1.00
17	Textiles	319	169	686	2.15
18	Apparel	374	135	1873	5.01
19	Leather products	228	76	442	1.94
20	Wood products	380	86	793	2.09
21	Paper and paper products	115	82	270	2.35
22	Publishing, printing, etc.	191	41	323	1.69
23	Coke, refined petroleum, etc.	5	31	43	8.60
24	Chemicals and chemical products	263	280	887	3.37
25	Rubber and plastic products	293	261	844	2.88
26	Non-metallic mineral products	222	151	600	2.70
27	Basic Metals	74	79	155	2.09
28	Fabricated metal products	403	337	1084	2.69
29	Machinery and equipment	211	225	566	2.68
31	Electrical machinery	69	77	185	2.68
32	Radio, television and comm. equip.	9	9	11	1.22
33	Medical, precision and optical inst.	16	30	34	2.13
34	Motor vehicles	81	91	202	2.49
35	Other transport equipment	40	35	65	1.63
36	Furniture; other mftg.	309	293	1655	5.36

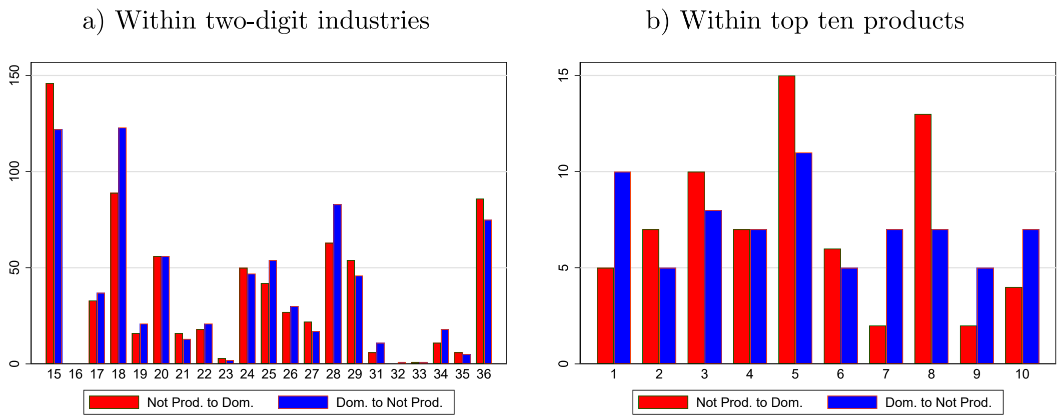
NOTE: This table reports, for each two-digit industry, the number of firms, the number of different products in the industry, the number of firm-product pairs, and the mean number of products per firm. The data correspond to 1996.

one or more new products to the domestic market is similar to the number removing one or more products from the domestic market. Figure A.14b shows the same pattern at the product level, considering the ten products with the largest number of firms.

G.6 Decomposing Export and Import Growth

In the model, export growth occurs entirely through the extensive margin. In this appendix we show that we find a similar pattern in the data when analyzing the change in exports during the sudden stop, between 1997 and 1998. For each two-digit industry, we decompose the change in exports between these two years into an intensive and extensive margin. Equation A.71 shows this standard decomposition, in which v_t stands for total exports in period t and v_{it} corresponds to exports by firm-product i in period t. The intensive margin refers to changes in exports within continuing firm-product pairs (the first term on the right hand side) and the extensive margin refers to exports among entering firm-product pairs minus exports among exiting firm-product

Figure A.14: Product addition or removal: replacement effect



NOTE: Panel a) plots for each two-digit industry, the number of firms introducing one or more new products to the domestic market and the number removing one or more products from the domestic market. Panel b) plots for each product (considering the top ten products with the largest number of producers), the number of firms introducing one or more new products to the domestic market and the number removing one or more products from the domestic market.

pairs (the last two terms on the right hand side).

$$\frac{\Delta v_t}{v_{t-1}} = \sum_{i \in \text{Cont.}} \frac{\Delta v_{i,t}}{v_{t-1}} + \sum_{i \in \text{Enter}} \frac{v_{i,t}}{v_{t-1}} - \sum_{i \in \text{Exit}} \frac{v_{i,t-1}}{v_{t-1}} \tag{A.71}$$

Consistent with the model, we find that most industries see a decline in exports as the sudden stop hits. In addition, we find that this is mostly due to an extensive margin adjustment, as shown in Figure A.15. For each two-digit industry, the figure plots the percent change in exports during 1997-1998 in the horizontal axis, and the component of that change in exports due to the extensive margin adjustment in the vertical axis. Most industries lie close to the 45 degree line, implying that most of the export adjustment occurs is due to the extensive margin. In fact, a linear fit line (weighted by industry exports in the initial period) is very close to the 45 degree line, with a slope of 1.21 (s.e.=0.08).[38] The R-squared is 0.92 implying that the extensive margin explains a very large fraction of the variation in exports.

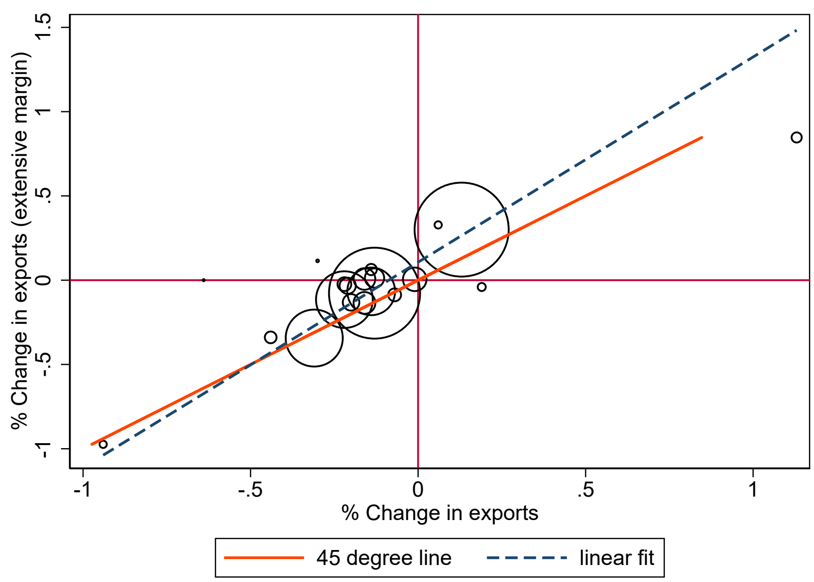
Finally, we compare the role of the extensive margin for exports and imports. To perform a valid comparison, the product definition must be equivalent in both cases. For this reason, we use export and import customs data for 1997-1998, in which each observation corresponds to a firm

[38] An unweighted linear fit is also close to the 45 degree line, with a slope of 0.74 (s.e. = 0.10) and R-squared 0.73. Note that we find very similar results when replicating this exercise for 3-digit industries.

- HS8 product - year. We perform the same decomposition described earlier for each HS 2-digit sector and construct the same plots described earlier using ENIA with this new data source. The results are shown in Figure A.16. As before, we find that most industries are found close to the 45 degree line, such that the export adjustment occurs primarily through the extensive margin. A linear fit line (weighted by industry exports in the initial period) has a slope of 0.70 (s.e.=0.08 and R-squared= 0.49).

In the case of imports, the model predicts that the adjustment should be partly through the intensive margin, given the decline in local demand. Consistent with this prediction, we find that the extensive margin is less relevant in the case of imports compared to exports.[39] A linear fit line (weighted by industry exports in the initial period) has a slope of 0.30 (s.e.=0.05 and R-squared= 0.31).

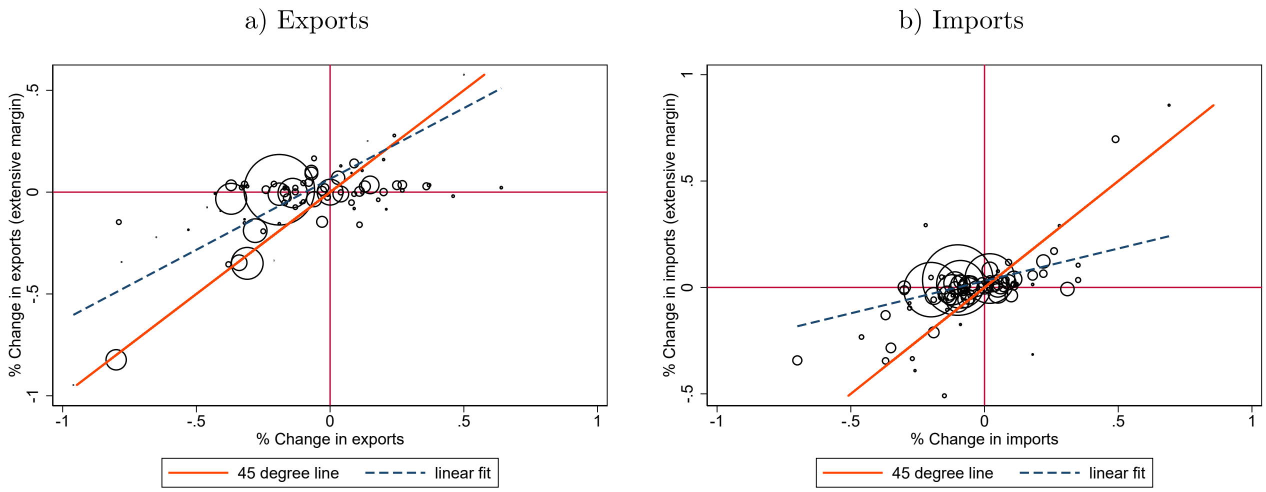
Figure A.15: Extensive Margin of Export Growth



Note: This figure plots the relationship between the change in exports between 1997 and 1998, and the share of that change that corresponds to the extensive margin. Each observation corresponds to a two-digit industry. The size of each circle corresponds to the total exports of the industry in the initial year. See text.

[39]In the case of imports, we do not observe the foreign exporting firm, so the extensive margin is based on the identity of the Chilean importer. This is a good approximation given patterns of exporting-importing matching [Benguria, 2021].

Figure A.16: Extensive Margin of Export and Import Growth based on Customs Data



NOTE: Panel a) plots the relationship between the change in exports between 1997 and 1998, and the share of that change that corresponds to the extensive margin. Each observation corresponds to a HS 2-digit code. The size of each circle corresponds to the total exports of the HS2 code in the initial year. Panel b) is equivalent for imports. See text.