

# Online Appendix to The Demand for Money at the Zero Interest Rate Bound

Tsutomu Watanabe\*      Tomoyoshi Yabu†

December 2, 2022

## A Structural Break Tests

### A.1 Gregory-Hansen Tests

Gregory, Nason, and Watt (1996) and Gregory and Hansen (1996) point out that if a simple cointegration test is used when some variables are cointegrated with structural breaks, the result will be biased toward accepting the null of no cointegration. To investigate this issue in more detail, we employ tests proposed by Gregory and Hansen (1996) that can detect a cointegrating relationship allowing a single structural break in the intercept and the slope.

Following the procedure proposed by Gregory and Hansen (1996), we (1) consider the middle 70 percent of the sample as candidates for the timing of a structural break, which is denoted by  $T_B$ ; (2) obtain the residual from a regression of  $\ln m_t$  on a constant,  $D_t$ ,  $r_t$  (or  $\ln r_t$ ), and  $D_t \times r_t$  (or  $D_t \times \ln r_t$ ), where  $D_t$  is 1 for  $t > T_B$  and zero otherwise; (3) conduct unit root tests for the residual and obtain test statistics  $\text{ADF}(T_B)$ ,  $Z_t(T_B)$ , and  $Z_\alpha(T_B)$ ; and (4) look for the minimal values of these test statistics over all possible break points. The test statistics thus obtained are denoted by  $\text{Inf-ADF}$ ,  $\text{Inf-}Z_t$ , and  $\text{Inf-}Z_\alpha$ .

Table A1 shows the values of  $\text{Inf-ADF}$ ,  $\text{Inf-}Z_t$ , and  $\text{Inf-}Z_\alpha$  for the semi-log and log-log

---

\*Graduate School of Economics, University of Tokyo. E-mail: watanabe@e.u-tokyo.ac.jp, Website: <https://sites.google.com/site/twatanabelab/>.

†Faculty of Business and Commerce, Keio University. E-mail: tyabu@fbc.keio.ac.jp.

Table A1: Gregory-Hansen Tests

	Inf-ADF	Inf- $Z_t$	Inf- $Z_\alpha$
semi-log specification	-4.521	-3.295	-21.222
log-log specification	-4.875*	-2.678	-16.109

Note: The three test statistics (Inf-ADF, Inf- $Z_t$ , and Inf- $Z_\alpha$ ) are computed using the procedure proposed by Gregory and Hansen (1996). \* indicates that the null hypothesis of no cointegration is rejected at the 10% level. For the ADF test, the lag length is chosen based on the AIC with the maximum lag length set to 8. For the PP tests, the long-run variance is estimated using a pre-whitened quadratic spectral kernel based on Andrews (1991) and Andrews and Monahan (1992).

specifications. For the semi-log specification, the test statistics are much larger in absolute value than in Table 2 in the main text, suggesting the possibility of structural breaks. However, we still cannot reject the null hypothesis, implying that our test fails to detect a cointegration relationship even when allowing for the possibility of a structural break.<sup>1</sup> Turning to the log-log specification, we see that the test statistics do not change that much from those in Table 2 in the main text, implying that the likelihood of a structural break is negligible. The result from the ADF test indicates that the null hypothesis of no cointegration is rejected at the 10 percent significance level.<sup>2</sup>

## A.2 Kejriwal-Perron Tests

Gregory-Hansen tests aim to detect a cointegrating relationship even when there is a structural break but do *not* aim to examine whether there actually are structural breaks. In this subsection, we employ the test proposed by Kejriwal and Perron (2010) to check whether there are any structural changes in the intercept and the slope coefficient of the money demand equation.

<sup>1</sup>The date associated with the minimal value of the test statistics is 2008:Q4, when the three month TB rate fell below 1 percent (1.5% in 2008:Q3 to 0.3% in 2008:Q4).

<sup>2</sup>Note that the significance level associated with the ADF test is lower than that presented in Table 2 in the main text. This may be due to a loss of statistical power of cointegration tests when the Gregory-Hansen test is employed in cases where there exists a cointegration relationship with no breaks.

Specifically, we consider the following money demand equation with  $k$  breaks (that is, there are  $k + 1$  different regimes):

$$\ln m_t = \alpha_i + \beta_i z_t + \sum_{j=-\nu_T}^{\nu_T} \delta_j \Delta z_{t-j} + u_t \quad \text{for } T_{i-1} < t \leq T_i \quad (\text{A.1})$$

where  $i$  represents the regime ( $i = 1, \dots, k + 1$ ). By convention,  $T_0 = 0$  and  $T_{k+1} = T$ , where  $T$  represents the sample size. The explanatory variable  $z_t$  is either  $r_t$  in the case of the semi-log form or  $\ln r_t$  in the case of the log-log form. We add leads and lags of  $\Delta z_t$  as auxiliary variables to correct for potential endogeneity between  $z_t$  and  $u_t$ . The number of leads and lags is set to 2 (i.e.,  $\nu_T = 2$ ).

Let  $\lambda = \{\lambda_1, \dots, \lambda_k\}$  denote the vector of break fractions with  $\lambda_i = T_i/T$ . Note that  $\lambda$  is an element of the set  $\Lambda_\epsilon = \{\lambda : |\lambda_{i+1} - \lambda_i| \geq \epsilon, \lambda_1 \geq \epsilon, \lambda_k \leq 1 - \epsilon\}$  for some  $\epsilon > 0$ . Therefore, each regime contains at least as many observations as  $[\epsilon T]$ , where  $[\cdot]$  denotes the greatest integer that is less than or equal to its argument. The trimming parameter  $\epsilon$  is set to 0.15.

Kejriwal and Perron (2010) employ the sup Wald statistic to test the null of no break against the alternative of  $k$  breaks in cointegrated models that allow for both  $I(1)$  and  $I(0)$  regressors. The test statistic is defined as follows:

$$\sup F_T^*(k) = \sup_{\lambda \in \Lambda_\epsilon} \frac{\text{SSR}_0 - \text{SSR}_k}{\hat{\sigma}^2} \quad (\text{A.2})$$

where  $\text{SSR}_0$  and  $\text{SSR}_k$  are the sum of squared residuals under the null of no break and the alternative of  $k$  breaks, and  $\hat{\sigma}^2$  is the long-run variance computed using the residuals from the model estimated under the null of no break. Based on the sup Wald test shown above, Kejriwal and Perron (2010) propose a double-maximum test in which the alternative hypothesis contains an unknown number of breaks between 1 and the upper bound  $M$ :

$$\text{UDmax}(M) = \max_{1 \leq k \leq M} \sup F_T^*(k) \quad (\text{A.3})$$

This is known as the most useful test to determine if there are any structural changes. We set the upper bound to 5 ( $M = 5$ ).

The other test proposed by Kejriwal and Perron (2010) is a test of the null hypothesis of  $k$  breaks against the alternative of  $k + 1$  breaks. This test makes it possible to identify the number of breaks. We use the following sequential test procedure: We start by testing if there are any structural breaks by applying either the  $\sup F_T^*(1)$  test or the UDmax(5) test. If we reject the null hypothesis of no break, we then test for one versus two breaks. We continue this process until we fail to reject the null. The number of breaks estimated in this way, which is equal to the number of rejections, is a consistent estimate of the true number of breaks.

Table A2: Kejriwal-Perron Tests

	Semi-log form $\ln m = \alpha + \beta r$	Log-log form $\ln m = \alpha + \beta \ln r$
$\sup F_T^*(1)$	15311.634***	7.537
$\sup F_T^*(2)$	4.850	4.481
$\sup F_T^*(3)$	3.416	3.348
$\sup F_T^*(4)$	2.626	2.613
$\sup F_T^*(5)$	2.155	2.102
UDmax(5)	15311.634***	7.537
No. of breaks	1	0
Break dates	2007:Q4	No breaks

Note: The break dates are estimated by minimizing the sum of squared residuals based on eq. (A.1). \*\*\* indicates that the null hypothesis of no cointegration is rejected at the 1% level.

Table A2 presents the test statistics  $\sup F_T^*(k)$  and UDmax(5) for the semi-log and log-log specifications. The table shows the number of breaks we detect as well as the break dates estimated by minimizing the sum of squared residuals based on eq. (A.1). For the semi-log specification, the null of no breaks is rejected for  $\sup F_T^*(1)$  and UDmax(5), suggesting that there is a single break. We also conduct the sequential procedure to confirm that the number of breaks is one. The date of the single structural break estimated based on the minimization of the sum of squared residuals is the fourth quarter of 2007, implying that a new regime starts

in 2008:Q1. On the other hand, for the log-log specification, none of the test statistics indicate that there are structural breaks.

### A.3 Rolling Regressions

Next, we conduct a rolling regression to see how the interest rate elasticity changed over time. Figure A1 shows the result for the semi-log specification with the window length set to 20 quarters (i.e., we use observations from  $t$  to  $t - 19$ ). The semi-elasticity estimated looks stable up until 2009 but exhibits a substantial change afterward: it starts to decline from 2010 onward and reaches -81 in 2013:Q4. The observed instability of the estimated coefficient reflects the fact that the money-income ratio increased substantially after the third quarter of 2010, but the semi-log form failed to track it. Turning to the log-log specification, Figure A2 shows that the interest rate elasticity of money demand appears to be stable over the entire sample period. In particular, it is noteworthy that in the log-log specification the interest rate elasticity of money demand has remained very stable since 2010, which is in sharp contrast to the case of the semi-log specification. This is consistent with the result from the Kejriwal-Perron test that the null of no structural breaks is not rejected.

### A.4 Extrapolation Tests

Finally, we conduct extrapolation tests to check the stability of the estimated parameters from a slightly different perspective. Specifically, we first split the sample period into the period up to 2009 (from the first quarter of 1980 to the fourth quarter of 2009) and the period after 2010 (from the first quarter of 2010 to the fourth quarter of 2013).<sup>3</sup> Next, we use the data for the first half of the sample period to estimate a money demand equation in semi-log and log-log

---

<sup>3</sup>In December 2008, the Fed lowered the federal funds target rate from 1.25% to 0-0.25% and then maintained the target rate at near zero until December 2015, when it began to raise interest rates. Thus, in the second half of the sample period, the federal funds target rate remained unchanged near zero throughout. However, the market interest rate used by Ireland (2009) and our study (namely, the six-month commercial paper rate for 1980 to 1997 and the three-month AA nonfinancial commercial paper rate from 1998 onward) did not remain stuck at zero but showed some, albeit small, fluctuations.

Figure A1: Interest Rate Semi-elasticity of Money Demand

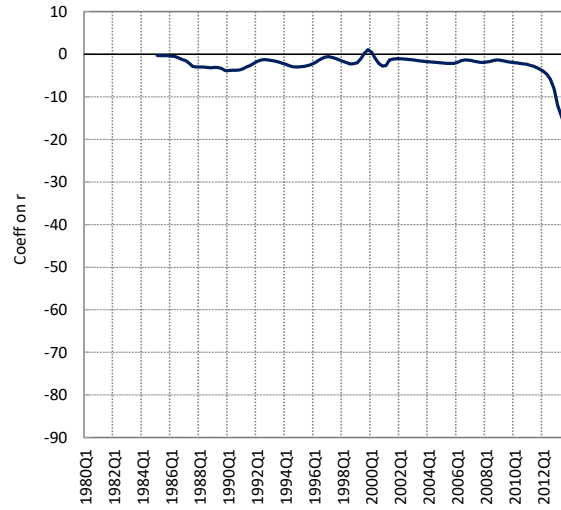
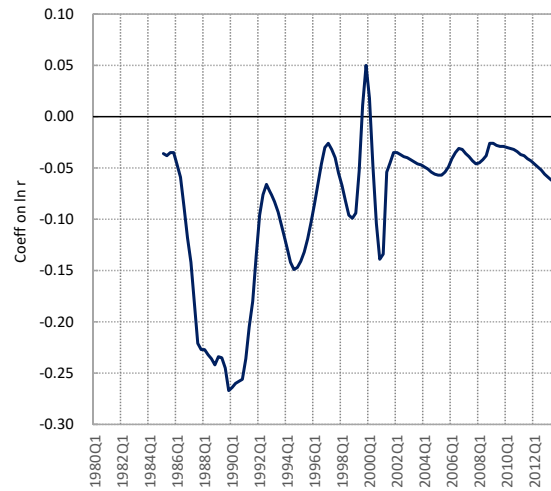
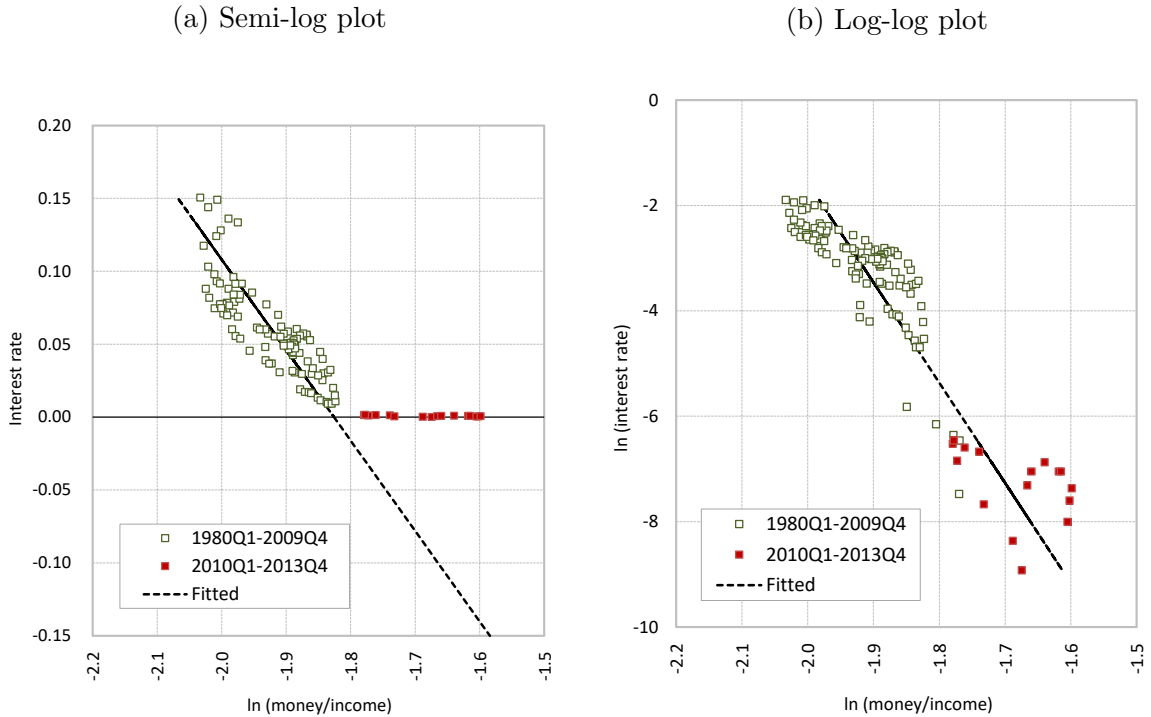


Figure A2: Interest Rate Elasticity of Money Demand



form with the money-income ratio as the dependent variable and the market interest rate as the independent variable. We then calculate the money-income ratio for the second half of the

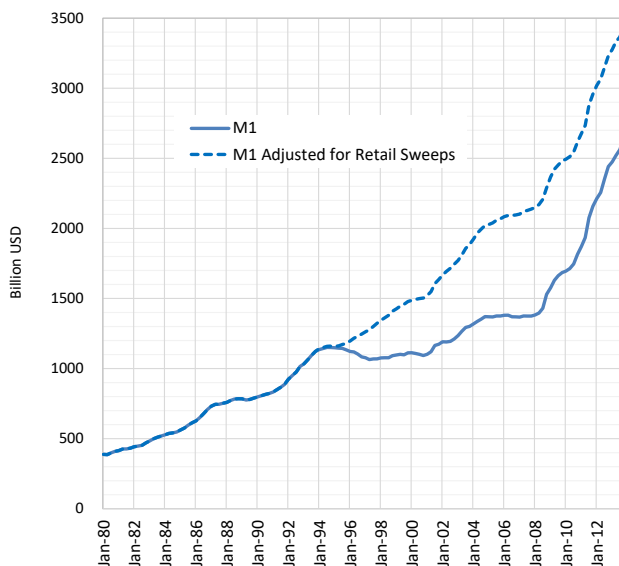
Figure A3: Extrapolation



sample period by substituting the actual values of the interest rate during the corresponding period into the estimated equation.

Figure A3 shows the results for the semi-log specification (left) and the log-log specification (right). Starting with the right panel, this shows that the observations since 2010 (depicted by the red squares) are around the regression line estimated using data up to 2009, suggesting that the money demand function in log-log form in the second half of the sample period is essentially unchanged from that in the first half. Turning to the left panel, the observations after 2010 deviate substantially from the regression line, indicating that the large increase in the money-income ratio since 2010 is not captured well by the semi-log specification. An important implication of this result is that, given the regression line estimated using data

Figure B1: M1 vs. M1 Adjusted for Retail Sweeps



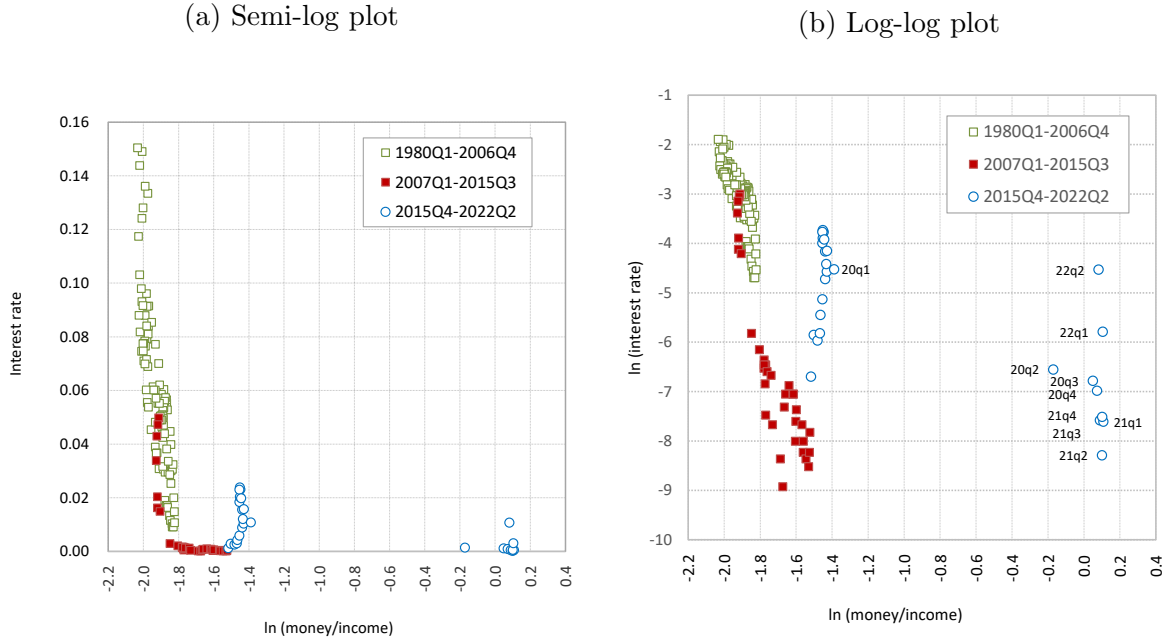
through 2009, interest rates would have had to fall to -15% to create the observed increase in the money-income ratio.

## B Further Extension of the Sample Period

Ireland (2009) estimated the money demand function using data from the first quarter of 1980 to the fourth quarter of 2006, while in our analysis we changed the endpoint of Ireland’s sample period to the fourth quarter of 2013. This allowed us to include more observations from periods with low interest rates. Specifically, the sample period in our analysis includes 24 quarters with interest rates below 1% compared to only three quarters in Ireland (2009). However, our sample period ends in the fourth quarter of 2013 and does not include more recent data with a lot more fluctuations in interest rates and the money-income ratio. The reason why we chose the fourth quarter of 2013 as our endpoint is that the monetary aggregate we employed (“M1 adjusted for retail sweeps”), which is the same as the one used by Ireland, is available only until that



Figure B2: Extension of Sample Period



quarter. The series has not been extended since then. In this subsection, we extend the sample period to the second quarter of 2022 by connecting the series to the M1 series.<sup>4/5</sup> Series other than the monetary aggregate (i.e., interest rates and nominal GDP) are the same as those used by Ireland (2009) and in the main text, but extended to the second quarter of 2022.

Figure B2 shows the semi-log plot (left) and the log-log plot (right) for the extended sample period. We see, first, that the data for Ireland’s (2009) sample period (1980:Q1 to 2006:Q4)

<sup>4</sup>Since January 1994, the Federal Reserve has allowed depository institutions to sweep retail customer transaction deposits, which are subject to statutory reserve requirement ratios as high as 10%, into savings deposits that have a zero percent reserve requirement ratio. Given this regulatory reform, commercial banks have started using deposit-sweeping software to dynamically reclassify the balances in checking accounts above a certain level as money market deposit accounts (MMDAs) and to reclassify them back when the balances on the checking accounts are too low. This software effectively creates a shadow MMDA for every checking account, and the shadow MMDA is included in M2, but not in M1. Figure B1 shows “M1” and “M1 adjusted for retail sweeps.” The difference between the two represents the total amount of sweeps of transaction deposits into the shadow MMDAs, which is not trivial (more than 30% of M1) and fluctuates substantially over time.

<sup>5</sup>Note that M1 was redefined in May 2020 to include savings as well as checking deposits, so that the money-income ratio we use is disconnected before and after 2020:Q2.

depicted by the green squares are aligned on a straight line in the semi-log plot. Ireland (2009) concludes, based on this fact, that the semi-log form is a better fit.

Second, looking at the data through the third quarter of 2015 (i.e., just before the Fed raised the federal funds target rate from 0-0.25% to 0.25-0.50% in December 2015), we see that the data no longer line up on a single straight line in the semi-log plot. This is consistent with the results of the Kejriwal-Perron test in Section A that the semi-log form does not fit the data well for this period unless a structural change is taken into account. In the log-log plot, however, all points up to the third quarter of 2015 are on a straight line. Based on this fact, we concluded in the main text that the log-log form performs better for this longer sample period.

Third, looking at the period through the second quarter of 2022, neither the semi-log form nor the log-log form fits the data well. While it is probably not surprising that our specifications do not track changes in the money-income ratio during the pandemic, the fit for earlier periods is also not very good. The Fed began raising the federal funds target rate in December 2015 and subsequently raised it nine more times through December 2018. Nonetheless, the money-income ratio rose during this period, albeit slightly, rather than falling. Neither the semi-log nor the log-log specification can account for this positive correlation.

The most recent data show some very interesting patterns. That is, in the log-log plot on the right, the money-income ratio has hardly decreased despite the upward trend in interest rates since the second quarter of 2021. Although it would be hasty to rush to conclusions regarding the most recent period, because many things are happening during the pandemic, it may not be a coincidence that we find that the negative correlation between interest rates and money-income ratio disappeared during the two phases of rising interest rates, 2015:Q3-2019:Q1 and 2021:Q2 onward.<sup>6</sup>

---

<sup>6</sup>In another study (Watanabe and Yabu, 2019) we use Japanese data for the period from 1985 to 2017 to estimate a money demand function. We show that the demand for money did *not* decline in 2006 when the Bank of Japan terminated quantitative easing and started to raise the policy rate, indicating that there was an upward shift in the money demand schedule. We argued that this was similar (but in the opposite direction) to the downward shift in the money demand schedule repeatedly observed in high inflation economies, where the

## C Money Demand Functions in an Economy with Non-negligible Storage Costs of Money

Recent studies on negative interest rate policy, including Rognlie (2016) and Eggertsson et al. (2019), argue that the cost of holding cash is not negligible. In this section, we introduce the storage cost of money into Sidrauski's (1967) model closely following Eggertsson et al. (2019) to examine how the demand for money and the welfare gain of lowering interest rates differ with and without the storage cost of money.

### C.1 Log-log form

Let us start with a version of Sidrauski's (1967) model. The representative household maximizes the present value of the sum of utilities,

$$U_t = \sum_{T=t}^{\infty} \beta^{T-t} U(c_T, z_T)$$

where  $c$  and  $z$  denote consumption and real money balances. Following Lucas (2000), we assume that the current period utility function is given by

$$U(c, z) = \frac{1}{1-\sigma} \left[ c \varphi \left( \frac{z}{c} \right) \right]^{1-\sigma} \quad (\text{C.1})$$

where  $\sigma > 0$  and  $\sigma \neq 1$ , and  $\varphi(\cdot)$  is a strictly increasing and concave function. We will specify  $\varphi(\cdot)$  later.

The household faces the following flow budget constraint:

$$M_t + (1 + r_{t-1})B_{t-1} = M_{t-1} + B_t + P_t y_t - P_t c_t - S(M_{t-1}) \quad (\text{C.2})$$

where  $B_t$ ,  $r_t$ ,  $P_t$ ,  $y_t$ , and  $S(M_{t-1})$  denote a one period risk-free bond held by the household, the nominal interest rate associated with the bond, the price level, income, and the storage cost of

---

demand for money does not increase even after inflation has subsided. We also argued that the upward shift in the money demand schedule in 2006 points to high switching costs between money and interest-bearing assets.

money. Note that  $S(M_{t-1})$  represents nominal storage costs and depends on nominal (rather than real) money balances. The first order conditions for utility maximization imply

$$\frac{U_z}{U_c} = \frac{\varphi'(\frac{z}{c})}{\varphi(\frac{z}{c}) - \frac{z}{c}\varphi'(\frac{z}{c})} = r + S'(M) \quad (\text{C.3})$$

Following Eggertsson et al. (2019), we assume that the marginal storage cost is positive and constant, so that  $S'(M) = \theta > 0$ .

Following Lucas (2000), we consider an endowment economy characterized by a balanced growth equilibrium path, on which the money growth rate is constant, maintained by a constant ratio of transfers to income. In this setting, the money-income ratio, given by  $m = z/y$ , is also constant. Then eq. (C.3) can be rewritten as

$$\frac{\varphi'(m)}{\varphi(m) - m\varphi'(m)} = r + \theta$$

This implies that, if money demand is of log-log form, i.e.,

$$m(r + \theta) = A(r + \theta)^\alpha \quad \text{for } r + \theta > 0 \quad (\text{C.4})$$

with  $A > 0$  and  $\alpha < 0$ , then the function  $\varphi(\cdot)$  solves a differential equation of the form

$$\frac{\varphi'(m)}{\varphi(m)} = \frac{\psi(m)}{1 + m\psi(m)} = \frac{A^{-1/\alpha}m^{1/\alpha}}{1 + mA^{-1/\alpha}m^{1/\alpha}}$$

where  $\psi(\cdot)$  is the inverse money demand function (i.e.,  $(\psi(\cdot) \equiv m^{-1}(\cdot))$ ). The solution to this differential equation is given by

$$\varphi(m) = \left(1 + A^{-\frac{1}{\alpha}}m^{\frac{1+\alpha}{\alpha}}\right)^{\frac{\alpha}{1+\alpha}}$$

Conversely, if the utility function (C.1) is specified as

$$U(c, z) = \frac{1}{1 - \sigma} \left[ c \left( 1 + A^{-\frac{1}{\alpha}} \left( \frac{c}{z} \right)^{\frac{1+\alpha}{\alpha}} \right)^{\frac{\alpha}{1+\alpha}} \right]^{1-\sigma} \quad (\text{C.5})$$

then the money demand function derived from utility maximization is of log-log form.

Starting from the utility function given by (C.5), we end up with a money demand function of the following form:

$$m = A(r + \theta)^\alpha \quad (\text{C.6})$$

$$\text{or} \quad \ln m = \ln A + \alpha \ln(r + \theta)$$

which is close to a standard log-log form but differs from it in that a constant term,  $\theta$ , is added to  $r$  before taking the logarithm. Note that  $m$  takes a finite value even when  $r \leq 0$  as long as  $r > -\theta$ , which is an important difference from the case of no storage costs.

When the money demand function takes a log-log form, Bailey's (1956) measure for the welfare gain achieved by lowering the interest rate (and inflation) from  $r$  to zero is given by

$$\begin{aligned} w(r) &= \int_0^r m(x)dx - rm(r) \\ &= \frac{A}{1 + \alpha} [(r + \theta)^{1+\alpha} - \theta^{1+\alpha}] - rA(r + \theta)^\alpha \end{aligned} \quad (\text{C.7})$$

implying that  $w'(r) > 0$  for  $r > 0$ ,  $w'(r) < 0$  for  $-\theta < r < 0$ , and that  $w'(0) = 0$  and  $w''(0) > 0$ . An important difference from the case without the storage cost of money is that there exists a finite satiation level of money even for the log-log money demand function, at which the marginal utility of money coincides with the marginal storage cost of money, and that the satiation level is achieved by setting  $r = 0$  (i.e., the Friedman rule). Any deviation from the Friedman rule, whether  $r > 0$  or  $r < 0$ , ends up with a suboptimal outcome.

## C.2 Semi-log form

Following Cysne (2009), we assume the current period utility function by

$$U(c, z) = g[c + \lambda(z)] \quad (\text{C.8})$$

where  $g'(\cdot) > 0$ ,  $g''(\cdot) \leq 0$ ,  $\lambda'(\cdot) > 0$ , and  $\lambda''(\cdot) < 0$ . The first order conditions for utility maximization imply

$$\frac{U_z}{U_c} = \lambda'(z) = r + \theta \quad (\text{C.9})$$

If the money demand function is of semi-log form, i.e.,

$$m(r + \theta) = B \exp[\alpha'(r + \theta)] \quad (\text{C.10})$$

with  $B > 0$  and  $\alpha' < 0$ , then the corresponding differential equation is given by

$$\lambda'(m) = \frac{1}{\alpha'} (\ln m - \ln B)$$

and the solution to this differential equation is

$$\lambda(m) = \frac{m}{\alpha'} \left[ 1 + \ln \left( \frac{B}{m} \right) \right]$$

Conversely, if the utility function is specified as

$$U(c, z) = g \left[ c + \frac{z}{\alpha'} \left( 1 + \ln \left( \frac{B}{z} \right) \right) \right] \quad (\text{C.11})$$

then the money demand function derived from utility maximization is of semi-log form.

If we start from the utility function given by (C.11), we obtain a money demand function of the form

$$\begin{aligned} m &= B \exp[\alpha'(r + \theta)] & (\text{C.12}) \\ \text{or} \quad \ln m &= (\ln B + \alpha'\theta) + \alpha'r \end{aligned}$$

which reduces to a standard semi-log form when  $\theta = 0$ . The welfare gain of lowering the interest rate toward zero is given by

$$\begin{aligned} w(r) &= \int_0^r m(x) dx - rm(r) \\ &= \frac{B}{\alpha'} [(1 - \alpha'r) \exp(\alpha'(r + \theta)) - \exp(\alpha'\theta)] \end{aligned} \quad (\text{C.13})$$

implying that  $w'(r) > 0$  for  $r > 0$ ,  $w'(r) < 0$  for  $r < 0$ , and that  $w'(0) = 0$  and  $w''(0) > 0$ . Again, any deviation from the Friedman rule ( $r = 0$ ), whether  $r > 0$  or  $r < 0$ , leads to a suboptimal outcome.

## References

- [1] Andrews, Donald W. K. 1991. "Heteroskedasticity and Autocorrelation Consistent Covariance Matrix Estimation." *Econometrica*, 59(3): 817-58.
- [2] Andrews, Donald W. K., and J. Christopher Monahan. 1992. "An Improved Heteroskedasticity and Autocorrelation Consistent Covariance Matrix Estimator." *Econometrica*, 60(4): 953-966.
- [3] Cysne, Rubens P. 2009. "Bailey's Measure of the Welfare Costs of Inflation as a General-Equilibrium Measure." *Journal of Money, Credit and Banking*, 41(2-3): 451-59.
- [4] Eggertsson, Gauti B., Ragnar E. Juelsrud, Laurance H. Summers, and Ella Getz Wold. 2019. "Negative Nominal Interest Rates and the Bank Lending Channel." *NBER Working Paper* No. 25416.
- [5] Gregory, Allan W., and Bruce E. Hansen. 1996. "Residual-Based Tests for Cointegration in Models with Regime Shifts." *Journal of Econometrics*, 70(1): 99-126.
- [6] Gregory, Allan W., James M. Nason, and David G. Watt. 1996. "Testing for Structural Breaks in Cointegrated Relationships." *Journal of Econometrics*, 71(1-2): 321-41.
- [7] Ireland, Peter N. 2009. "On the Welfare Cost of Inflation and the Recent Behavior of Money Demand." *American Economic Review*, 99(3): 1040-52.
- [8] Kejriwal, Mohitosh, and Pierre Perron. 2010. "Testing for Multiple Structural Changes in Cointegrated Regression Models." *Journal of Business and Economic Statistics*, 28(4): 503-22.
- [9] Lucas, Robert E., Jr. 2000. "Inflation and Welfare." *Econometrica*, 68(2): 247-74.
- [10] Rognlie, Matthew. 2016. "What Lower Bound? Monetary Policy with Negative Interest Rates." [http://mattrognlie.com/negative\\_rates.pdf](http://mattrognlie.com/negative_rates.pdf) (accessed September 6, 2019).

- [11] Sidrauski, Miguel. 1967. "Rational Choice and Patterns of Growth in a Monetary Economy." *American Economic Review*, 57(2): 534-544.
- [12] Watanabe, Tsutomu, and Tomoyoshi Yabu. 2019. "How Large is the Demand for Money at the ZLB? Evidence from Japan?" *CARF Working Paper Series* CARF-F-465, September 2019.