CARF Working Paper

CARF-F-574

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CARF is presently supported by Nomura Holdings, Inc., Mitsubishi UFJ Financial Group, Inc., Sumitomo Mitsui Banking Corporation., The Norinchukin Bank, The University of Tokyo Edge Capital Partners Co., Ltd., Ernst & Young ShinNihon LLC, The Dai-ichi Life Insurance Company, Limited., and All Nippon Asset Management Co., Ltd.. This financial support enables us to issue CARF Working Papers.

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Abstract

The U.S. economy since 1980 has experienced the growth of finance, manifested by the increases in the value-added of financial services and the value of financial assets. The growth of finance has been associated with the increase in the mutual fund share in the financial assets and the relatively stable unit cost of finance. This paper constructs an incomplete market dynamic general equilibrium model with the islands structure, which has both idiosyncratic and island-level shocks on the firm's productivity. Financial intermediaries trade shares of individual firms and risk-free debts, as well as mutual funds which diversify away idiosyncratic shocks but can not diversify island-level shocks. This model, together with the declining transaction costs on mutual funds and personal and corporate income tax rates calibrated from data, can quantitatively account for these facts.
Keywords: Financial development; mutual funds; unit cost of finance
JEL classification code: E2, G2

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1 Introduction

Since the 1970s, the financial sector has grown significantly in the U.S. The financial sector's value-added excluding the insurance sector has increased from 3.0% of the U.S. GDP in 1980 to 5.6% in 2006. The value of financial assets relative to the U.S. GDP has grown from 1.03 in 1980 to 2.90 in 2007 (for details, see Table 3 in Section 5). Greenwood and Scharfstein (2013) refer to the phenomenon as the growth of finance.

The growth of finance coincided with innovations in the financial sector. Bernstein (1993) and Fox (2009) argue that the development of financial economics has transformed the sector since the 1970s. Many newly developed technologies are related to managing and hedging risks, consequently facilitating the diversification of risks. One notable example is the invention and development of index funds, mutual funds that track certain market indices, such as Standard & Poor's 500. The growth of index funds has provided investors with the means of diversifying risks at lower costs than other mutual funds such as active funds (French, 2008). Accordingly, the share of open-end mutual funds has increased from 4.6% in 1980 to 32.4% in 2007 (French, 2008).

There is also a countervailing view of the growth of finance. Philippon (2015) finds that the unit cost of finance, measured as the ratio of the income of financial intermediaries to the quantities of intermediated assets, has been relatively stable and has not declined. The fact is seemingly puzzling because if financial innovations offer services at lower costs, the unit cost of finance would decline. Philippon (2015) argues that a potential explanation is an increase in market power in the U.S. financial sector.

This paper quantitatively examines these facts in an incomplete market dynamic general equilibrium model. In our model, a household can hold the portfolio from (i) a risky individual stock, (ii) mutual funds that bundle individual stocks, and (iii) risk-free assets, which include corporate and government bonds as well as human assets whose value consists of future wage incomes. Individual stock shares bear the risk of firms' productivity shocks. To tractably introduce mutual funds, we use islands structure in our model. As in Heathcote et al. (2014), there are two types of productivity shocks: idiosyncratic shocks and island shocks. Different from Heathcote et al. (2014), households and physical capital can freely migrate between islands, which results in equated wage rates and returns from physical capital across islands. This setup enables us to focus on the effect of these shocks on the financial aspects with which we are concerned.

We assume that financial intermediaries can construct a mutual fund by forming a portfolio of stock shares of all firms on an island. The mutual fund diversifies away idiosyncratic shocks but cannot diversify island shocks. Transaction costs incurred for constructing the portfolio make mutual funds middle-risk, middle-return assets, while individual stocks are high-risk, high-return assets.

As for the risk-free bonds provided by firms, Merton (1975) shows that in the continuous-time model, the returns from physical capital are "known with certainty" and thus risk-free. We employ this property and assume in the model that firms sell the right to obtain returns from physical capital as risk-free bonds. The corporate provision of risk-free assets through capital income from physical capital also ensures the relatively stable debt-equity ratio, which is consistent with empirical facts.

Using the declining costs for mutual funds reported in French (2008) and the decline in personal and corporate income tax rates since 1980 measured in McGrattan and Prescott (2005) as exogenous variables, we show that the model quantitatively accounts for the following stylized facts on the growth of finance between 1980 and 2006:

1. an increase in the value of financial assets relative to GDP.
2. an increase in the share of mutual funds in total assets.
3. the relatively stable unit cost of finance.
4. an increase in the value-added share of the financial sector.

We follow McGrattan and Prescott (2005) for the first property. In their model, the decrease in taxes on dividends increases the value of equities. In a similar vein, the decline in transaction costs helps increase asset values by depressing the interest rate.

The second property is qualitatively straightforward because, as the transaction costs for mutual funds decline, households choose a higher share of mutual funds. The decline in personal and corporate income tax rates also increases the mutual fund share. The firm value increases with the decrease in tax rates, as explained above. The share of individual stocks is determined by the difference in returns between the individual stock and the mutual fund as well as the volatility of idiosyncratic shocks. However, taxes affect neither the difference in returns nor idiosyncratic shocks. Thus, the increase in the share of mutual funds absorbs the increase in firm value caused by the tax cuts.

The third property is mainly accounted for by the fact that the transaction costs for mutual funds are higher than those for individual stocks and risk-free debts. Low transaction costs for mutual funds cause two opposing effects on the total transaction costs: it reduces the cost per mutual fund transaction, while it increases the share of mutual funds which are relatively expensive. Due to these effects, whether the unit cost of finance, defined by total transaction costs divided by total asset value, increases or decreases is ambiguous.*[1] We show that the unit cost of finance is quantitatively stable.

As for the fourth property, the value-added share of the financial sector is defined as the total transaction costs relative to GDP, which is equal to the unit costs of finance multiplied by the value of financial assets relative to output in our model. As the unit cost of finance is relatively stable from the third property, and the value of financial assets relative to output increases from the first property, the financial sector's value-added share increases.

Our interpretation of the growth of finance that the decline in the fees on a financial transaction causes the portfolio shift toward relatively expensive financial assets follows Gennaioli et al. (2014). Our contribution relative to theirs is to develop a tractable quantitative model and quantify the extent to which the declines in the fees measured in French (2008) account for several stylized facts on the growth of finance. We also incorporate a quantitative property of McGrattan and Prescott (2005) that the decline in the tax rate on dividends accounts for the increase in the firm value relative to GDP in the U.S. since the 1980s. Our model also offers a novel interpretation that the reduction in tax rates also causes the portfolio shift from individual stocks to mutual funds.

Methodologically, we apply the islands structure of Heathcote et al. (2014) to the asset pricing problem. Usually, to analyze incomplete-market models with aggregate shocks, numerical approximation methods such as Krusell and Smith (1998) need to be employed. Gomes and Michaelides (2007) adopt this strategy to explain the equity risk-premium puzzle. A drawback of this approach is that the model becomes analytically intractable. Dindo et al. (2022), extending Di Tella (2017), develop a tractable DGE model with idiosyncratic and aggregate shocks where costly risk pooling is provided by financial intermediaries to analyze how the intermediation costs affect economic growth and business cycles. Their model is tractable because they adopt the AK technology. However, the AK production function is incompatible with the standard Cobb-Douglas production function widely used in DGE models. Heathcote et al. (2014) set up an incomplete-market general equilibrium model, which allows two types of shocks on wages, insurable idiosyncratic shocks and uninsurable island-level shocks. Using the model, they analyze the extent to which households smooth and share growing labor income risks in the U.S. since the 1970s. As in Heathcote et al. (2014), our model incorporates insurable idiosyncratic shocks and uninsurable island-level shocks. In addition, we incorporate aggregate shocks in our quantitative exercise as a robustness check, and obtain results similar to our benchmark case without aggregate shocks.

*[1] The role of declining tax rates is relatively minor because the tax cuts increase total transaction costs and total asset value in the same way.

The plan of the paper is as follows. Section 2 constructs the incomplete market dynamic general equilibrium model with the islands structure. Section 3 solves the model. We discuss how to solve the model and how the model qualitatively explains the stylized facts. Section 4 explains the calibration procedure. Section 5 reports quantitative predictions of the model. Section 6 concludes.

2 Model

2.1 Islands structure

The economy consists of a continuum of "islands" with a measure of 1. A unit mass of firms reside permanently on each island. Each firm incurs both firm-specific and island-specific productivity shocks. There is no common shock across islands.

Each household supplies a unit of labor to a firm. We assume free mobility of goods, capital, assets except for equities, and households across islands. This assumption entails that the rates of return to capital and wage equate across islands and that households face no labor income risks. However, we will later impose a financial structure in which households are barred from diversifying island-level shocks. This structure allows us to tractably analyze the growth of mutual funds such as index funds, without introducing aggregate uncertainty in the economy as a whole as in Heathcote et al. (2014).

2.2 Production

Time is continuous and indexed by t. Firms are monopolistic competitors. There is a unit mass of firms in each of islands with measure 1. A firm is indexed by $e \in E = [0, 1]^2$. The production function of a firm that produces a differentiated intermediate good $y_{e,t}$ is

$$y_{e,t} = \pi_{e,t} k_{e,t}^{\alpha} \ell_{e,t}^{1-\alpha},$$

where $\pi_{e,t}$ is the productivity, $k_{e,t}$ is the physical capital, and $\ell_{e,t}$ is the labor input. We assume that the physical capital is constructed from aggregate goods. The aggregate good Y_t is produced competitively using $y_{e,t}$ from the Dixit-Stiglitz (1977) aggregator:

$$Y_t = \left(\int_E y_{e,t}^{\frac{\phi-1}{\phi}} de \right)^{\frac{\phi}{\phi-1}}, \ \phi > 1.$$

The firm-level productivity evolves according to $\pi_{e,t} = e^{\mu_z t + z_{e,t}}$, where μ_z is the trend growth and $z_{e,t}$ follows a Brownian motion

$$dz_{e,t} = \sigma_{ze} dW_{e,t} + \sigma_{zI} dW_{I,t}.$$

$dW_{e,t}$ and $dW_{I,t}$ are the Wiener processes representing the firm-level and island-level productivity shocks, respectively. Firm-level productivity shocks are idiosyncratic, that is, $dW_{e,t}$ is orthogonal to $dW_{e',t}$ for $e' \neq e$. In contrast, the island shock is common for all the firms on the island.

The firm issues both debts and equities. In the continuous-time model, the return from physical capital is not stochastic and therefore is risk-free (see, for example, Merton, 1975, p. 378). We assume that for all of the investment in its physical capital, firms issue debts $b_{e,t}$ to borrow from households at a risk-free rate $r_{f,t}$. We also assume that when the return from the physical capital is distributed to households, the transaction cost ι_f is incurred for each unit of investment. After the transaction cost is incurred, each household is collected personal income tax τ_{inc}.

The shareholders obtain the firms' residual profits as dividends. Since firms are monopolistic competitors, the residual profits are positive. Dividend $d_{e,t}$ of a firm is

$$(1 - \tau_{\text{corp}}) d_{e,t} = (1 - \tau_{\text{corp}}) \big(p_{e,t} y_{e,t} - w_t \ell_{e,t} - (r_{f,t} + \delta + \iota_f) k_{e,t} \big),$$

where τ_{corp} is the corporate income tax, $p_{e,t}$ is the price of the good the firm produces, w_t is the real wage rate, δ is the depreciation rate of capital. Changes in capital and borrowing, $dk_{e,t}$ and $db_{e,t}$, cancel out by our assumption, $k_{e,t} = b_{e,t}$, that is, the capital investment is always financed by bond issuance.[*2] Then, the firm's dynamic maximization problem is reduced to the static maximization problem, whose first-order conditions are:

$$\frac{\partial p_{e,t} y_{e,t}}{\partial k_{e,t}} = \mathsf{MPK}_t \equiv r_{f,t} + \delta + \iota_f, \tag{1}$$

$$\frac{\partial p_{e,t} y_{e,t}}{\partial \ell_{e,t}} = w_t. \tag{2}$$

The return received by a household that directly purchases a unit of the firm's shares at q_{et}

[*2] This paper does not incorporate the investment tax credit because the existence of both the transaction costs and the investment tax credit complicates the firm's decision problem.

is

$$((1-\tau_{\text{inc}})(1-\tau_{\text{corp}})d_{e,t}dt + dq_{e,t})/q_{e,t} - \iota_q dt = \mu_{q,t}dt + \sigma_{qe,t}dW_{e,t} + \sigma_{qI,t}dW_{I,t}, \quad (3)$$

where τ_{inc} is the personal income tax and ι_q is the transaction cost of holding the share and where $\mu_{q,t}$, $\sigma_{qe,t}$, and $\sigma_{qI,t}$ are endogenous parameters, which turn out to be independent of the characteristics of firm e. When we conduct quantitative exercises, we assume the transaction costs of holding bonds ι_f is the same as the cost of holding individual stocks ι_q.

2.3 Mutual Funds

We assume that financial intermediaries can provide the mutual fund by constructing a market portfolio of equities of firms located on each island. Due to the law of large numbers, the island-specific market portfolio diversifies away idiosyncratic firm-level shocks, while it does not diversify island shocks. The technology for constructing the market portfolio requires transaction costs ι_m per share. We assume that the portfolio management of financial intermediaries is costly and the transaction cost of mutual funds is higher than the costs of holding other assets. Therefore, $\iota_m > \iota_f, \iota_q$.

Under this setting, the return from investing in the mutual funds is

$$\int_{E_I} \left\{\left((1-\tau_{\text{inc}})(1-\tau_{\text{corp}})d_{e,t}dt + dq_{e,t}\right)/q_{e,t} - \iota_m dt\right\}de = \mu_m dt + \sigma_{qI,t}dW_{I,t}, \quad (4)$$

where $E_I \subset E$ denotes the set of firm e residing on island I. Note that firm-level productivity shocks $dW_{e,t}$ are diversified away in the above equation.

2.4 Government

We assume that the government purchases G_t and imposes taxes, and issues the risk-free government debt to pay for the government expenditure, as in the standard dynamic general equilibrium models.

2.5 Households

There is a unit mass of infinitely-lived households indexed by $i \in [0, 1]$. Households supply one unit of labor inelastically and receive real wage w_t. Each household i can hold three kinds of assets, (i) the share of the firm that employs the household i, (ii) the mutual fund specific to the island where the household's employer resides, and (iii) the risk-free bonds $b_{i,t}$ and human asset $h_{i,t}$. The risk-free bonds $b_{i,t}$ are provided by any firms and the government, whereas the human asset is the discounted value of household future wage income w_t minus the lump-sum tax (or transfer) ψ_t. The returns of (i, ii, iii) are given by (3), (4), and the after-tax risk-free rate $\mu_{f,t} \equiv (1-\tau_{\text{inc}})r_{f,t}$, respectively.

Let $a_{i,t}$ be the total asset of household i who currently works at firm $e \in E_I$. Let also $\boldsymbol{\theta}_{i,t} \equiv (\theta^q_{i,t}, \theta^m_{i,t}, \theta^f_{i,t})$ be the shares of the assets (i), (ii), and (iii) in the total asset $a_{i,t}$. Note that $\theta^q_{i,t} + \theta^m_{i,t} + \theta^f_{i,t} = 1$. We assume that $\boldsymbol{\theta}_{i,t} \geq 0$, i.e., households cannot short-sell the assets. Note that $h_{i,t}$ is deterministic since the wage rate and lump-sum tax are deterministic. Then,

$$\mu_{f,t} h_{i,t} = (w_t - \psi_t)dt + dh_{i,t}.$$

Household budget constraint can be written as

$$da_{i,t}/a_{i,t} = \mu_{a,t}dt + \theta^q_{i,t}\sigma_{qe,t}dW_{e,t} + \left(\theta^q_{i,t} + \theta^m_{i,t}\right)\sigma_{qI,t}dW_{I,t},$$

where

$$\mu_{a,t} \equiv \theta^q_{i,t}\mu_{q,t} + \theta^m_{i,t}\mu_{m,t} + \theta^f_{i,t}\mu_{f,t} - c_{i,t}/a_{i,t},$$

and where $c_{i,t}$ is consumption.

Given prices, households solve the following dynamic programming problem. The household utility is the continuous-time formulation of the Epstein and Zin (1989) developed by Duffie and Epstein (1992). We assume that the elasticity of intertemporal substitution (EIS) is one. The value function becomes

$$V_{i,t} \equiv V(a_{i,t}) = \max_{c_{i,t},\boldsymbol{\theta}} \mathbb{E}_t\left[\int_t^\infty f(c_{i,s}, V(a_{i,s}))ds\right],$$

where

$$f(c,V) = \rho(1-\gamma)V\left[\ln(c) - \frac{1}{1-\gamma}\ln((1-\gamma)V)\right].$$

Note that $\rho > 0$ is the rate of time preference, and γ is the coefficient of relative risk aversion.

The Hamilton-Jacobi-Bellman equation of the household problem is written as follows:

$$\rho V_{i,t} = \max_{c_{i,t},\theta^q_{i,t},\theta^m_{i,t}} f(c_{i,t}, V_{i,t}) + \frac{\partial V_{i,t}}{\partial a_{i,t}} \mu_{a,t} a_{i,t}$$
$$+ \frac{1}{2} \frac{\partial^2 V_{i,t}}{\partial a^2_{i,t}} \left((\theta^q_{i,t})^2 \sigma^2_{qe,t} + (\theta^q_{i,t} + \theta^m_{i,t})^2 \sigma^2_{qI,t} \right) a^2_{i,t}. \tag{5}$$

Due to the unit EIS assumption in the household dynamic programming problem (27), the consumption rate $c_{i,t}/a_{i,t}$ and portfolio weights $\boldsymbol{\theta}_{i,t}$ are independent of total asset $a_{i,t}$ (see the appendix for the derivations in this section):

$$\frac{c_{i,t}}{a_{i,t}} = \rho, \tag{6}$$
$$\theta^q_{i,t} = \frac{\mu_{q,t} - \mu_{m,t}}{\gamma \sigma^2_{qe,t}}, \tag{7}$$
$$\theta^q_{i,t} + \theta^m_{i,t} = \frac{\mu_{m,t} - \mu_{f,t}}{\gamma \sigma^2_{qI,t}}. \tag{8}$$

2.6 Market-clearing conditions

Since goods, capital, and assets except for equities are freely mobile across islands, the market-clearing conditions equate the demands and supplies aggregated across islands. Aggregate output Y_t is produced using the Dixit-Stiglitz aggregator defined previously. The price of Y_t is normalized to one. The other aggregate variables are simply summed across islands. For example, the aggregate consumption C_t is defined as $C_t = \int_0^1 c_{i,t} di$. Henceforth, we suppress the subscript i in the household variables such as θ^q_t and θ^m_t, because these variables are identical across households, as shown below.

The market-clearing condition for the aggregate good is

$$C_t + \frac{dK_t}{dt} + G_t + \left(\theta^q_t \iota_q + \theta^m_t \iota_m\right) A_t + \iota_f \left(K_t + B_{G,t}\right) = Y_t - \delta K_t, \tag{9}$$

where A_t, Q_t, and D_t denote the aggregate total assets, the aggregate equity value of firms, and the aggregate dividends. In (9),

$$\left(\theta^q_t \iota_q + \theta^m_t \iota_m\right) A_t + \iota_f \left(K_t + B_{G,t}\right) \tag{10}$$

is the aggregate total transaction costs. In what follows, we interpret the aggregate total transaction costs as the model's counterpart of the value-added of the financial services,[*3] and GDP in the model as the sum of Y_t and the aggregate total transaction costs.

The labor market clearing condition is

$$\int_E \ell_{e,t} de = 1. \tag{11}$$

The shares of each firm e are held either as individual stocks or as a part of mutual funds. At the aggregate level, the demand for the equities is equal to the supply:

$$\left(\theta_t^q + \theta_t^m\right) A_t = Q_t. \tag{12}$$

The market clearing condition for risk-free deposits and debt is

$$\int_0^1 b_{i,t} di = K_t + B_{G,t}, \tag{13}$$

where $B_{G,t}$ is government debts.

The government is indebted with $B_{G,0}$ at $t = 0$. The government debt accumulates as

$$\frac{dB_{G,t}}{dt} = (r_{f,t} + \iota_f) B_{G,t} + G_t - \left(1 - (1 - \tau_{\text{inc}})(1 - \tau_{\text{corp}})\right) D_t - \tau_{\text{inc}} r_{f,t} \left(K_t + B_{G,t}\right) - \psi_t, \tag{14}$$

where G_t denotes the government purchase of goods at t.

2.7 Equilibrium

An equilibrium of the model, given the law of motion of firms' productivities, the initial capital for each firm, the initial individual stocks and mutual funds held by households, and taxes, is a set of households' choice variables, firms' choice variables, price variables, and aggregate variables, such that (i) each firm maximizes profit according to (4), (ii) the value and policy functions solve the household dynamic programming problem (27), (iii) markets clear according to (9)–(13), and (iv) the government debt accumulates according to (14). In the following sections, we focus on the stationary equilibrium.

[*3] That is, a financial sector has a production technology that transforms goods to financial services one-to-one, and is competitive. The payment to the financial intermediary is withheld from dividend payments.

3 Solving the Model

3.1 Aggregate Dynamics

In this section, we consider the aggregate dynamics of the model economy. Let the relative productivity of firm e be $\tilde{\pi}_{e,t} \equiv \pi_{e,t}^{\phi-1}/\overline{\mathbb{E}}\left\{\pi_{e,t}^{\phi-1}\right\}$ and define $\Pi_t \equiv \overline{\mathbb{E}}\left\{\pi_{e,t}^{\phi-1}\right\}^{\frac{1}{\phi-1}}$, where $\overline{\mathbb{E}}\left\{\pi_{e,t}^{\phi-1}\right\}$ denotes the cross-sectional average (average across islands) of $\pi_{e,t}^{\phi-1}$. We can show that the steady state growth rate of aggregate variables, g, coincides with the growth rate of $\Pi_t^{\frac{1}{1-\alpha}}$ in the steady state as

$$g = \left\{\mu_z + (\phi - 1)\frac{\sigma_{ze}^2 + \sigma_{zI}^2}{2}\right\}/(1-\alpha).$$

We suppress the time subscript of variables in the steady state. Then, firm-side variables can be derived as follows:

$$\ell_{e,t} = \frac{p_{e,t}y_{e,t}}{Y_t} = \frac{k_{e,t}}{K_t} = \frac{d_{e,t}}{D_t} = \frac{q_{e,t}}{Q_t} = \tilde{\pi}_{e,t}, \tag{15}$$

where

$$Y_t \equiv \left(\frac{\alpha(1-1/\phi)}{\mathsf{MPK}_t}\right)^{\frac{\alpha}{1-\alpha}} \Pi_t^{\frac{1}{1-\alpha}}, \tag{16}$$

$$K_t \equiv \left(\frac{\alpha(1-1/\phi)}{\mathsf{MPK}_t}\right)^{\frac{1}{1-\alpha}} \Pi_t^{\frac{1}{1-\alpha}}, \tag{17}$$

$$D_t \equiv (1/\phi)\cdot \overline{py}_t, \tag{18}$$

$$Q_t \equiv \int_t^\infty (1-\tau_{\text{inc}})(1-\tau_{\text{corp}})D_s\,\mathbb{E}\left(\frac{\Lambda_{e,s}}{\Lambda_{e,t}}\tilde{\pi}_{e,t,s}\right)ds, \tag{19}$$

$\Lambda_{e,s}$ is the stochastic discount factor of a household that directly purchases the stocks of firm e, and $\tilde{\pi}_{e,t,s} \equiv \exp\Big(-(1/2)(\phi-1)^2\big(\sigma_{ze}^2+\sigma_{zI}^2\big)(s-t) + (\phi-1)\sigma_{ze}\big(W_{e,s}-W_{e,t}\big) + (\phi-1)\sigma_{zI}\big(W_{e,s}-W_{e,t}\big)\Big)$, which does not depend on the productivity level of firm e. Dividing these aggregate variables by $\Pi_t^{\frac{1}{1-\alpha}}$, we obtain the detrended variables, $\widetilde{Y}_t$, $\widetilde{K}_t$, $\widetilde{D}_t$, and $\widetilde{Q}_t$. At the stationary equilibrium, these detrended variables become constant.

Using the aggregate variables, portfolio variables are written as follows:

$$\mu_{q,t} = (1-\tau_{\text{inc}})(1-\tau_{\text{corp}})\frac{\widetilde{D}_t}{\widetilde{Q}_t} + \left(\frac{d\widetilde{Q}_t \Big/ dt}{\widetilde{Q}_t} + g\right) - \iota_q, \tag{20}$$

$$\mu_{m,t} = (1-\tau_{\text{inc}})(1-\tau_{\text{corp}})\frac{\widetilde{D}_t}{\widetilde{Q}_t} + \left(\frac{d\widetilde{Q}_t \Big/ dt}{\widetilde{Q}_t} + g\right) - \iota_m, \tag{21}$$

$$\sigma_{qe,t} = (\phi - 1)\sigma_{ze,t}, \tag{22}$$

$$\sigma_{qI,t} = (\phi - 1)\sigma_{zI,t}. \tag{23}$$

Given the exogenous variables including $\left\{\widetilde{G_t}, \widetilde{\psi}_t\right\}_{t=0}^{\infty}$, the steady state of the economy is the fixed point of the system of the following differential equations:

$$\frac{d\widetilde{\mathbf{S}}_t}{dt} = \mu_{\widetilde{\mathbf{S}}}(\widetilde{\mathbf{S}}_t), \quad \text{where } \widetilde{\mathbf{S}}_t \equiv \left(\widetilde{K}_t, \widetilde{H}_t, \widetilde{Q}_t, \widetilde{B}_{G,t}\right).$$

Given $\widetilde{\mathbf{S}}_t$, other variables at t can be computed as follows:

1. Compute $\widetilde{A}_t = \widetilde{K}_t + \widetilde{H}_t + \widetilde{Q}_t + \widetilde{B}_{G,t}$.
2. Given $\widetilde{K}_t$, MPK_t, $r_{f,t}$ and $\mu_{f,t}$ are computed.
3. Given MPK_t, $\widetilde{Y}_t$ and $\widetilde{D}_t$ are obtained.
4. θ_t^q, $\mu_{q,t}$, $\mu_{m,t}$, and $d\widetilde{Q}_t \Big/ dt$ are jointly determined by (7), (20), (21), and the following equation:

$$\frac{d\widetilde{Q}_t}{dt} = -(1-\tau_{\text{inc}})(1-\tau_{\text{corp}})\widetilde{D}_t + \left(\mu_{f,t} - g + \iota_q + \gamma\frac{\widetilde{Q}}{\widetilde{A}}\sigma_{qI}^2 + \gamma\theta_t^q\sigma_{qe}^2\right)\widetilde{Q}_t. \tag{24}$$

   θ_t^m is computed from (8).
5. Compute $d\widetilde{H}_t \Big/ dt$ by

$$\frac{d\widetilde{H}_t}{dt} = -\left((1-\alpha)(1-1/\phi)\widetilde{Y}_t - \widetilde{\psi}_t\right) + \left(\mu_{f,t} - g\right)\widetilde{H}_t.$$

6. Using the variables obtained as above and $\widetilde{C}_t = \rho\widetilde{A}_t$, compute dK/dt from the resource

constraint (9):

$$\frac{d\widetilde{K}_t}{dt} = \widetilde{Y}_t - (\delta + g)\widetilde{K}_t - \widetilde{C}_t - \widetilde{G}_t - \left(\theta^q \iota_q + \theta^m \iota_m\right)\widetilde{A}_t - \iota_f\left(\widetilde{K}_t + \widetilde{B}_{G,t}\right).$$

7. Compute $d\widetilde{B}_{G,t} \Big/ dt$ by

$$\begin{aligned}\frac{d\widetilde{B}_{G,t}}{dt} =& (r_{f,t} + \iota_f - g)\widetilde{B}_{G,t} + \widetilde{G}_t - \left(1 - (1 - \tau_{\text{inc}})(1 - \tau_{\text{corp}})\right)\widetilde{D}_t \\ & - \tau_{\text{inc}} r_{f,t}\left(\widetilde{K}_t + \widetilde{B}_{G,t}\right) - \widetilde{\psi}_t.\end{aligned}$$

3.2 Qualitative Predictions of the Model

This section shows how the four stylized facts are qualitatively explained in the model.

Result 1. An increase in the value of financial assets relative to GDP The personal and corporate income taxes have decreased since the 1980s. As in McGrattan and Prescott (2005), the decline of the taxes, exogenous in the model, drives up the firm value Q_t (see (24) at the stationary equilibrium with $d\widetilde{Q}_t \Big/ dt = 0$), while it has little effect on GDP. Similarly, (24) shows that the decline in the transaction cost ι_q also contributes to the increase in Q_t.

Result 2. An increase in the share of mutual funds in total assets First, since 1980, the transaction costs of mutual funds ι_m have decreased more than those of individual stocks ι_q. In the model, this effect decreases the share of individual stocks:

$$\theta^q = \frac{\iota_m - \iota_q}{\gamma \sigma_{qe}^2}. \tag{25}$$

Second, from Result 1 and (12), the cut in corporate and individual income taxes, by increasing the firm value share in total assets Q_t/A_t, increases $\theta^q + \theta^m$. These two effects increase the mutual fund share θ^m.

Result 3. Relatively stable unit cost of finance In the model, we define the unit cost of finance (uc) as aggregate total transaction costs (aggregate tc) given in (10) divided by financial assets that equal the sum of the firm values, physical capitals, and government debts, that is, $F \equiv Q + K + B_G$. Then,

$$\text{uc} \equiv \frac{\text{aggregate tc}}{F} = \frac{\theta^q A}{F}\iota_q + \frac{\theta^m A}{F}\iota_m + \frac{K + B_G}{F}\iota_f.$$

Two countervailing effects by the decrease in ι_m operate on the unit cost of finance. First, as a direct effect, the second term of the RHS of the above equation decreases by the decline in ι_m, which further decreases the unit cost of finance. Second, Result 2 shows that the decline in ι_m decreases θ^q and increases θ^m. Since $\iota_m > \iota_q$, this indirect substitution effect increases the unit cost of finance. These two effects cancel each other out, resulting in a relatively stable unit cost of finance.[*4]

Result 4. An increase in the value-added share of the financial sector We measure the model's value-added of the financial sector by aggregate total transaction costs. Then, the value-added share of the financial sector is equal to

$$\frac{\text{aggregate tc}}{F}\frac{F}{\text{GDP}} = \text{uc} \times \frac{F}{\text{GDP}}.$$

By Result 1, F/GDP increases. By Result 3, the unit cost of finance is relatively stable. Therefore, the value-added share of the financial sector increases with the declines in tax rates and transaction costs of mutual funds.

4 Calibration

We calibrate parameters to match the U.S. data. Using the calibrated model, in the next section, we evaluate the quantitative properties of the model and compare them with the U.S. data.

4.1 Constant parameters

Table 1 Constant parameters

g	Steady-state growth rate	2%
ρ	Discount rate	4%
α	Capital share	1/3
γ	Relative risk aversion	1.6
δ	Depreciation rate	10%
ϕ	Elasticity of substitution	5.0

Constant parameters are calibrated as in Table 1. The unit of time is annual. The first three

[*4] This implies that our result of stable unit costs is *quantitative*: the model can generate mildly increasing or decreasing unit costs depending on parameter values. Thus, our model conforms to the literature that finds varying trends of the unit cost of finance across economies as in Bazot (2018) and Gunji et al. (2021).

parameter values are standard in the growth models. We set the coefficient of relative risk aversion γ to 1.6 so that the model's portfolio shares at 1980 roughly match those in data. We set the depreciation rate δ to 10% as in Gomes and Michaelides (2007). The elasticity of substitution in the CES aggregate production function is set to 5.0, which implies that 20% of a firm's sales is rent and is in the range of standard values.

4.2 Non-constant parameters

Table 2 Non-constant parameters

		1980	2006
τ_{inc}	Individual income tax	40.0%	17.3%
τ_{corp}	Corporate income tax	35.8%	34.9%
$(\phi-1)\sigma_{zI}$	Island-level volatility	2%	1%
$(\phi-1)\sigma_{ze}$	Firm-level idiosyncratic volatility	45%	45%
G/GDP	Government expenditure-to-GDP ratio	0.22	0.22
B/GDP	Debt-to-GDP ratio	0.31	0.61
ι_q	Costs on individual stocks	1.00% + 0.55%	1.00% + 0.21%
ι_m	Costs on mutual funds	1.00% + 2.63%	1.00% + 1.16%
ι_f	Costs on the risk-free debts	1.00% + 0.55%	1.00% + 0.21%

Notes: Tax rates on assets are taken from McGrattan and Prescott (2005). The value of island-level volatility is taken from "Real Aggregate Sales" in Figure 6 in Comin and Mulani (2006) and the value of the firm-level volatility from "Total Economy" in Figure 2.6 in Davis et al. (2007). The government expenditure-to-GDP and the debt-to-GDP ratio are taken from National Economic Accounts and Treasury Bulletin, retrieved from FRED (for details of data used in the paper, see also the appendix). The value of (transaction) costs on assets are taken from Table 6 in French (2008). 1.00% is added for the unit cost of finance in 1980 to match with data in 1980. Without the addition of 1.00%, the model's predictions of the unit cost of finance become lower than the data.

We list non-constant calibrated parameters in Table 2. The first two rows show the tax rates on individual and corporate incomes, which are taken from McGrattan and Prescott (2005). We set the island-level and firm-level productivity shocks, σ_{zI} and σ_{ze}, to match the aggregate and idiosyncratic volatilities. We set $(\phi-1)\sigma_{zI}$, which corresponds to the volatility of firm-level variables such as output, firm's value, and employment by island-level shocks to the aggregate volatility ("Real Aggregate Sales" in Figure 6 in Comin and Mulani, 2006). We set the idiosyncratic volatility $(\phi-1)\sigma_{ze}$ to match the firm-level volatility in data (taken from "Total Economy" in Figure 2.6 in Davis et al., 2007). We set the government expenditure-to-GDP and debt-to-GDP ratios to match data.

Parameters on transaction costs of assets are calibrated using data in French (2008). The transaction costs of individual stocks and risk-free debts, ι_q and ι_f, are calibrated from the trading cost of equities in Table 6 in French (2008). The transaction costs on mutual funds are calibrated from the trading cost plus the mutual fund costs, both in Table 6 in French (2008). We add the base costs of 1.00% to these transaction costs because, without the base costs, the unit cost of finance in the model becomes lower than that in the data (1.00% in Table 2 is the base cost).

5 Quantitative Predictions of the Model

This section analyzes the quantitative predictions of the model and evaluates the extent to which the model can account for the growth of finance since the 1980s in the U.S.

5.1 Benchmark results

Table 3 Predicted and actual values on the growth of finance

	Predicted		Actual	
	1980	2006	1980	2006
Individual stock share	33%	9%	29%	17%
Mutual fund share	3%	31%	3%	25%
Corporate debt share	44%	33%	37%	29%
Government debt share	20%	27%	30%	29%
Financial assets-GDP ratio	1.56	2.24	1.03	2.09
Firm value-value added ratio	2.28	3.20	1.30	2.90
Financial value added share	4.1%	5.6%	3.0%	5.6%
Unit cost of finance	1.63%	1.55%	1.96%	1.95%

Notes: Financial asset shares, such as Individual stock share, are the share of each asset value in the sum of the financial asset values. Financial assets-GDP ratio is the sum of the financial asset values divided by GDP. Firm value-value added ratio is the firm value that consists of corporate equities and debts divided by the value added of firms. Firm value and the value added of firms in the data are those of the nonfinancial corporate sector. Actual these values are constructed by authors from the Financial Accounts of the United States, the National Economic Accounts, and Treasury Bulletin; some are retrieved from FRED. The actual financial value added shares are the value added of the financial services excluding insurance sector divided by GDP, which are taken from Appendix Table 1 in Greenwood and Scharfstein (2013). The actual values of the unit cost of finance are taken from Philippon (2015). For the derivations of predicted values, see the main text and the appendix of the paper.

This section compares the model's predictions with U.S. data (for the data used in the paper, see also the appendix). We first explain the adjustments necessary for the comparison. In the model, GDP, except for the value-added of financial services, consists of the value added of the nonfinancial corporate sector. In contrast, in the U.S. data, the share of the nonfinancial corporate sector, of which we compute the corporate equity and debt values, amounts to only about 50% of U.S. GDP and is roughly constant over time. Most of the remaining consist of the noncorporate sector, whose asset values are difficult to measure appropriately. To bridge the gap, in the following comparison, we make the following adjustments to the results of the model: we multiply the model's asset values of the corporate sector by the actual value-added share of the nonfinancial corporate sector, and divide it by the model's value-added share of the nonfinancial corporate sector.[*5]

Table 3 summarizes the main results of the paper: the model's predictions and the corresponding actual values on the growth of finance. The first four rows in Table 3 are the share of individual stock value, mutual fund value, corporate debt value, and government debt value in the sum of the four financial asset values. The model's predictions on these shares in 1980 are roughly similar to those in the data in 1980, although the model's prediction of the government debt share is somewhat lower than the data. The lower government debt share is caused by the higher firm value-value added ratio in the model, which we discuss below. Our model can account for the change in portfolio shares from 1980 to 2006; for example, it can account for the portfolio shift from individual stocks to mutual funds, though the model somewhat overpredicts the shift.

The fourth and fifth rows in Table 3 are related to the financial asset value relative to GDP. The model quantitatively accounts for the increase in the financial assets-GDP ratio, while the model's predictions on the level are somewhat higher than that of the data. The discrepancy between the model and data on the level is caused by a higher firm value in the model. The fifth row reports the ratio of the firm value and the value added of the nonfinancial corporate sector. We confirm that the prediction of the model is higher than that of the data. For 1980, a possible cause is that the model does not take into account that the arrival of new technologies in the 1970s, such as microelectronics and information and communication technologies, destructed the value of existing physical capitals, as argued by Laitner and Stolyarov (2003). For 2006, a lower firm value-GDP ratio in the data might be accidental: in the 2010s, the actual value of the firm value-GDP ratio on average exceeds 3.50.

The sixth row in Table 3 reports the financial value added share. As we explained above, we

[*5] For example, the model's prediction on the value of individual stocks after adjustment is calculated by

$$\theta_t^q A_t \times \left(\frac{Y_{\text{corp}}}{\text{GDP}}\right)_{\text{Data}} \Bigg/ \left(\frac{Y_{\text{corp}}}{\text{GDP}}\right)_{\text{Model}},$$

where $(Y_{\text{corp}}/\text{GDP})_{\text{Data}}$ and $(Y_{\text{corp}}/\text{GDP})_{\text{Model}}$ are the actual and model's value-added shares of the nonfinancial corporate sector.

interpret the total transaction costs in the model as the value added of the financial services. The results deriving from this formulation match data and capture the increase in the value-added share of the financial services.

Finally, the last row in Table 3 reports the unit cost of finance, which is defined as the ratio of the value added of the financial services and the value of financial assets. Though the level of the unit cost in our model is slightly lower than that in data, the results in the model can capture the salient property in data that the unit cost of finance is constant over time. Moreover, the level of the measured unit cost excluding insurance (see Figure 17 in Philippon, 2014) is around 1.5%, which is close to the model's prediction.

5.2 Counterfactual exercises

In order to further investigate the causes of the changes between 1980 and 2006, we conduct counterfactual exercises, in which either only transaction costs or taxes decrease from the 1980 level to the 2006 level while the other exogenous variables, including tax rates, are unchanged at the 1980 level.

The second column in Table 4 shows the counterfactual results for 2006 in which only transaction costs change from the 1980 level. The counterfactual results for the transaction costs-only case quantitatively capture the increase in the mutual fund share and the decrease in individual stocks in the benchmark result. It is caused by the fact that the transaction costs of mutual funds decrease much more than those of other assets (see Table 2).

The results for another counterfactual exercise in which only tax rates change from the 1980 level are shown in the last column in Table 4. Although quantitatively smaller than the transaction costs-only case, the mutual fund share increased from the 1980 economy. The result may appear puzzling at first because the decrease in the personal and corporate tax rates applies to all of the returns from financial assets. This is explained as follows. Due to the decrease in tax rates, the equity value as well as the share of equities increased. In the meanwhile, the individual stock share does not increase, because as shown in (25), it is the increased difference in the returns between individual stocks and mutual funds that increases the individual stocks share. These two effects increase the mutual fund share.

As is consistent with the implication of McGrattan and Prescott (2005), the effect of taxes on the increase in the firm value. The decrease in transaction costs also contributes to the rise in firm value (see Firm value-value added shares in Table 4). The increase in firm value caused by both of these factors mainly accounts for the increase in the financial assets-GDP ratio. The increase in the debt-to-GDP ratio, which is kept constant in the counterfactual exercises, also contributes to the increase in the financial asset value.

Table 4 Counterfactual exercises

	Benchmark 2006	Transaction costs only 2006	Taxes only 2006
Individual stock share	9%	14%	25%
Mutual fund share	31%	27%	17%
Corporate debt share	33%	41%	42%
Government debt share	27%	18%	16%
Financial assets-GDP ratio	2.24	1.69	1.89
Firm value-value added ratio	3.20	2.52	2.87
Financial value added share	5.6%	4.1%	5.8%
Unit cost of finance	1.55%	1.49%	1.93%

Notes: "Benchark 2006" is the benchmark results for 2006 in Table 3. "Transaction costs only 2006" results from the counterfactual exercise where only transaction costs change to the 2006 level. "Taxes only 2006" results from the counterfactual exercise where only tax rates change to the 2006 level. See notes in Table 3 for the definitions of variables.

5.3 Impacts on household wealth distribution

With incorporating a dissipation shock as in Moll et al. (2022), our model generates a stationary Pareto distribution of household wealth. The model parameter is modified so that ρ is now interpreted as a sum of the rate of time preference ϱ and a dissipation shock p in which household enters a state with an infinitely impatient time preference. Once entered the dissipation state, the household consumes all its asset a_t instantly and derives zero utility (see Appendix C in Moll et al., 2022). In aggregation across households, the dissipation functions similarly to the depreciation of capital. Hence, the aggregate capital depreciates at rate $\delta + p$, which is the only alteration from the benchmark model given ρ is unchanged. The aggregate variables in the stationary equilibrium exhibit almost no changes from the benchmark. In our model, aggregate variables in the stationary equilibrium are determined independently from the distribution of wealth. The wealth distribution is then determined given the prices and policies as in Aoki and Nirei (2017).

In our model, the individual household i accumulates wealth as follows. We suppress the index i henceforth.

$$da_t/a_t = \mu_a dt + \theta^q \sigma_{qe} dW_e + (\theta^q + \theta^m)\sigma_{qI} dW_I$$

where $\mu_a = \theta^q \mu_q + \theta^m \mu_m + \theta^f \mu_f - \rho$. Now we define $\tilde{a}_t$ as household wealth after detrended by growth factor g. The household wealth is reset to the steady-state value of detrended human asset, $\tilde{h}$. Denoting a diffusion parameter of the wealth accumulation as $\sigma_a := \sqrt{(\theta^q \sigma_{eq})^2 + ((\theta^q + \theta^m)\sigma_{qI})^2}$ and also $\zeta := \sqrt{2p\sigma_a^2 + \mu_a^2}$, we obtain a Pareto exponent of the right tail of the household wealth distribution, when $p > 0$, as

$$\lambda_1 = \frac{\zeta - \mu_a}{\sigma_a^2}.$$

The entire distribution follows a double-Pareto distribution

$$f(\log \tilde{a}) = \begin{cases} (p/\zeta)e^{-\lambda_1(\log \tilde{a} - \log \tilde{h})} & \text{if } \tilde{a} \geq \tilde{h}, \\ (p/\zeta)e^{\lambda_2(\log \tilde{a} - \log \tilde{h})} & \text{otherwise} \end{cases} \tag{26}$$

where $\lambda_2 \equiv (\zeta + \mu_a)/\sigma_a^2$.

The double-Pareto distribution emerges as a result of idiosyncratic multiplicative shocks (Reed, 2001; Toda, 2011). Note that, by applying the change of variable to (26), the density function of $\tilde{a}$ follows a power function with exponent $\lambda_1 + 1$ in the right tail. Hence, λ_1 is the Pareto exponent defined as the tail index of the cumulative distribution of $\tilde{a}$, and determines the inequality in the right tail of the wealth distribution. It is immediately seen that the Pareto exponent is negatively related to trend μ_a and diffusion σ_a^2 while it is positively related to p, i.e., $\partial\lambda_1/\partial\mu_a < 0$, $\partial\lambda_1/\partial\sigma_a^2 < 0$, and $\partial\lambda_1/\partial p > 0$, ceteris paribus.

The Pareto exponent can be also rewritten as $\lambda_1 = \sqrt{2p/\sigma_a^2 + (\mu_a/\sigma_a^2)^2} - \mu_a/\sigma_a^2$. λ_1 is determined by the dissipation rate p, the diffusion of household wealth σ_a^2, and the trend-diffusion ratio μ_a/σ_a^2. A lower dissipation rate leads to a smaller Pareto exponent and, thus, a greater inequality in household wealth. This holds because households have a higher chance of drawing many periods of successful risky investments. Dissipation shock p can be alternatively interpreted as the birth rate of new households with no financial wealth. Thus, a greater influx at the mode of the distribution $\tilde{a} = \tilde{h}$ results in greater equality in the stationary wealth distribution. The effect of the birth rate is offset by the effect of diffusion σ_a^2, which reduces λ_1 and increases inequality. If μ_a is zero, which holds approximately under our calibrated parameter values, we obtain $\lambda_1 = \sqrt{2p}/\sigma_a$. Thus, the Pareto exponent is determined by the balance between the influx effect p and the diffusion effect σ_a^2 (Nirei and Souma, 2007; Nirei and Aoki, 2016).

The influx-diffusion balance is also adjusted by term μ_a/σ_a^2 in the above expression. With other terms fixed, the effect of μ_a/σ_a^2 on λ_1 is negative. Hence, the direct negative effect of σ_a^2 on λ_1 via p/σ_a^2 is mitigated by the effect via μ_a/σ_a^2. Also, a greater trend μ_a leads to a smaller λ_1. This generates ambiguity in the effect of financial transaction costs on the Pareto exponent through general equilibrium. As we have seen, a reduction in transaction costs enhances capital accumulation, thereby reducing the steady-state risk-free rate. Therefore, there can be a downward effect on wealth trend growth, leading to a decrease in wealth inequality.

To quantify these effects, we compute the Pareto exponent λ_1 for our stationary equilibria calibrated for 1980 and 2006. The dissipation rate p is set to 0.0008, so that the λ_1 matches an estimate for the U.S. households in 1980. Then, λ_1 in our stationary equilibrium is 1.86 for the 1980 calibration and 3.02 for the 2006 calibration. The U.S. Pareto exponents are 1.87 in 1980 and 1.47 in 2006.[*6] Hence, the effect of lowered costs of financial transactions in our model does not explain the rise in income concentration observed in the U.S. in the same period. If anything, the predicted impact is strongly equalizing.

This is intuitive: the lowered cost of mutual funds contributes to the lowered volatility of household wealth portfolio, resulting in less dispersion in the stationary distribution of household wealth. In fact, σ_a declines from 0.0289 in 1980 to 0.0133 in 2006 in our model, while μ_a increases only from -0.0003 to $-7e^{-7}$. Given that μ_a is almost nil, λ_1 is dominated by $\sqrt{2p}/\sigma_a$. Hence, the decrease of σ_a more than half has a strong upward effect on the Pareto exponent. The equalizing effect of the widespread use of mutual funds identified by our model implies that the reason for the decreased Pareto exponent of the U.S. household wealth has to be found elsewhere. Our preferred hypothesis is that entrepreneurial risk-taking behavior induced by lowered tax rates contributed to the decreased Pareto exponent, as argued in Aoki and Nirei (2017) using a similar model to this paper.

5.4 Aggregate shocks

The benchmark model uses island structure in order to incorporate undiversified shocks. Although the island structure is handy for our analysis, questions remain as to what extent our quantitative results are attributable to the particular structure. For a robustness check, this section analyzes an alternative economy in which there are aggregate shocks instead of island-level shocks.

The firm-level productivity evolves according to $\pi_{e,t} = e^{\mu_z t + z_{e,t} + Z_t}$. μ_z is the trend growth. $z_{e,t}$ follows a Brownian motion

$$dz_{e,t} = \sigma_{ze} dW_{e,t},$$

which now only contains firm-level productivity shocks. Z_t contains aggregate shocks and

[*6] We inferred the U.S. Pareto exponent from World Inequality Database (WID, Alvaredo et al., 2022). WID reports Pareto coefficients $\lambda_1/(\lambda_1 - 1)$ for the pre-tax income of top 1% income earners.

follows an Ornstein-Uhlenbeck process

$$dZ_t = -\mu_Z Z_t dt + \sigma_Z dW_Z,$$

where dW_Z is the Wiener process representing aggregate productivity shocks. As in the benchmark exercise, the government expenditure G is a constant fraction of GDP. In order to reduce an unnecessary state variable, we set the detrended lump-sum taxation/transfers $\widetilde{\psi}$ so that the detrended government debts $\widetilde{B}_G$ is constant. Then,

$$\widetilde{\psi} = (r_f - g)\widetilde{B}_G + \widetilde{G} - \big(1 - (1 - \tau_{\text{inc}})(1 - \tau_{\text{corp}})\big)\widetilde{D} - \tau_{\text{inc}} r_f \left(\widetilde{K} + \widetilde{B}_G\right).$$

We set the ratio of G and GDP and the average ratio of B_G in GDP to match the data.

A difference from the benchmark case is the property that human assets become risky because wage, the dividend of human assets, fluctuates over time. We solve the partial differential equations which describe the dynamics of aggregate asset values, $\widetilde{Q}$ and $\widetilde{H}$, as well as the value function, given the state variables $\widetilde{K}$ and Z, to obtain quantitative results of the economy. The details of the solution method are provided in the Appendix.

For quantitative exercises, we set μ_Z to 0.25×4, the value used in Ahn et al. (2017) (we adjust their quarterly value to an annual one). We set σ_Z to 0.028 for the volatility of output to roughly match the data. The results of the economy with aggregate shocks are shown in Table 5. The differences from the benchmark case results are small.

Table 5 Predicted values under aggregate shocks

	Predicted	
	1980	2006
Individual stock share	31%	9%
Mutual fund share	7%	32%
Corporate debt share	42%	32%
Government debt share	19%	27%
Financial assets-GDP ratio	1.62	2.29
Firm value-value added ratio	2.39	3.29
Financial value added share	4.4%	5.8%
Unit cost of finance	1.72%	1.56%

Notes: Variables such as portfolio shares change depending on the state variables, aggregate physical capital and aggregate productivity. The values reported in the table are those when the state variables take median values so that aggregate productivity level Z_t is equal to zero (the mean value) and the growth rate of aggregate physical capital is equal to trend growth rate g.

6 Conclusion

This paper constructs an incomplete market dynamic general equilibrium model and quantitatively investigates the mechanism underlying the facts on the growth of finance, which has occurred in the U.S. since the 1980s: (i) an increase in the financial asset value relative to GDP, (ii) an increase in the mutual fund share, (iii) stable unit cost of finance, (iv) an increase in the value added share of the financial sector. Our model accounts for these facts by declining transaction costs for financial assets and declining individual and corporate tax rates. Our interpretation of the growth of finance is neoclassical in that the unit cost of finance does not decline despite technological progress in the financial sector, not because market power or rent changes, but because the decrease in transaction costs attracts demands for financial services previously avoided due to the high transaction costs. The island structure of the model enables us to calibrate parameters from aggregate and firm-level volatilities and quantitatively analyze the substitutable nature of assets between individual stocks and mutual funds in a tractable way.

There are some caveats in our analysis. First, this paper does not analyze transitional dynamics. In order for the transitional dynamics of the model to be consistent with data, some uncertainties are necessary for future transaction costs and tax rates. Without these uncertainties, the value of equities, the unit cost of finance, and the value added of the financial services immediately jump up after the revelation of information, that is inconsistent with the data. Second, this paper does not analyze the increase in mortgages and mortgage-backed securities, another pillar of the growth or finance (Greenwood and Scharfstein, 2013). Individual stocks and mutual funds in our model can also be interpreted as holding and self-financing a house and issuing mortgage-backed securities. However, in order to appropriately account for the role of mortgages on the growth of finance, it is necessary to model the housing problem and mortgage-backed securities realistically. We leave them for future research.

Appendix

A Derivations in Section 2

A.1 Hosehold's problem

We reproduce the HJB equation in the text here:

$$\rho V = \max_{c,\theta^q,\theta^m} f(c,V) + V_a\mu_a a + \frac{1}{2}V_{aa}\left((\theta^q)^2\sigma_{qe}^2 + (\theta^q+\theta^m)^2\sigma_{qI}^2\right)a^2. \tag{27}$$

Differentiating (27) by θ^q, we obtain

$$V_a(\mu_q-\mu_f)a + V_{aa}\left[\theta^q\sigma_{qe}^2 + (\theta^q+\theta^m)\sigma_{qI}^2\right]a^2 = 0.$$

By rearranging the equation, we obtain

$$\theta^q = -\frac{V_a}{aV_{aa}}\frac{\mu_q-\mu_f}{\sigma_{qe}^2+\sigma_{qI}^2} - \theta^m\frac{\sigma_{qI}^2}{\sigma_{qe}^2+\sigma_{qI}^2}.$$

Differentiating (27) by θ^m, we obtain

$$V_a(\mu_m-\mu_f)a + (\theta^q+\theta^m)\sigma_{qI}^2V_{aa}a^2 = 0.$$

By rearranging this equation, we obtain

$$\theta^q+\theta^m = -\frac{V_a}{aV_{aa}}\frac{\mu_m-\mu_f}{\sigma_{qI}^2}.$$

Using $-V_a/(aV_{aa}) = 1/\gamma$, we obtain equation (8) in the text. Using the equations derived above, we obtain equation (7) in the text.

B Derivations in Section 3

B.1 Growth rate

To compute g, we need to compute the time derivative of $\overline{\mathbb{E}}\left\{\pi_e^{\phi-1}\right\}$,

$$\frac{d\overline{\mathbb{E}}\left\{\pi_e^{\phi-1}\right\}}{dt} = \frac{\mathbb{E}\left[d\pi_e^{\phi-1}\right]}{dt},$$

and thus $d\pi_e^{\phi-1}$.

Using Ito's lemma and defining $f(z_e,t) = \pi_{e,t}^{\phi-1} = e^{(\phi-1)(\mu_z t + z_e)}$, we obtain

$$\begin{aligned} df(z_e,t) =& \left((\phi-1)\mu_z\pi_e^{\phi-1} + \frac{\sigma_{ze}^2+\sigma_{zI}^2}{2}(\phi-1)^2\pi_e^{\phi-1}\right)dt \\ &+ (\phi-1)\sigma_{ze}\pi_e^{\phi-1}dW_e + (\phi-1)\sigma_{zI}\pi_e^{\phi-1}dW_I. \end{aligned} \tag{28}$$

Thus,

$$\mathbb{E}\left[d\pi_e^{\phi-1}\right] = \left((\phi-1)\mu_z + \frac{\sigma_{ze}^2+\sigma_{zI}^2}{2}(\phi-1)^2\right)\pi_e^{\phi-1}dt.$$

Using the above results, we obtain

$$\begin{aligned} g &= \frac{d\overline{\mathbb{E}}\left\{\pi_e^{\phi-1}\right\}^{\frac{1}{(\phi-1)(1-\alpha)}}\Big/dt}{\overline{\mathbb{E}}\left\{\pi_e^{\phi-1}\right\}^{\frac{1}{(\phi-1)(1-\alpha)}}} \\ &= \frac{1}{(\phi-1)(1-\alpha)}\frac{d\overline{\mathbb{E}}\left\{\pi_e^{\phi-1}\right\}\Big/dt}{\overline{\mathbb{E}}\left\{\pi_e^{\phi-1}\right\}} \\ &= \frac{\mu_z + (\phi-1)\frac{\sigma_{ze}^2+\sigma_{zI}^2}{2}}{1-\alpha}. \end{aligned}$$

B.2 Dividends $d_{e,t}$

Due to the characteristics of the CES function, under the profit maximization, we have

$$\begin{aligned}\mathsf{MPK}_t &= \frac{\partial p_{e,t} y_{e,t}}{\partial k_{e,t}} = (1-1/\phi)\alpha \frac{p_{e,t} y_{e,t}}{k_{e,t}},\\ w_t &= \frac{\partial p_{e,t} y_{e,t}}{\partial \ell_{e,t}} = (1-1/\phi)(1-\alpha)\frac{p_{e,t} y_{e,t}}{\ell_{e,t}}.\end{aligned}$$

Therefore,

$$\begin{aligned}d_{e,t} dt &= \max_{k_{e,t}, \ell_{e,t}} \left(p_{e,t} y_{e,t} - w_t \ell_{e,t} - \mathsf{MPK}_t k_{e,t}\right) dt\\ &= (1/\phi) p_{e,t} y_{e,t} dt.\end{aligned}$$

B.3 Aggregate variables

1. Firms in the aggregate goods sector maximize profit as follows:

$$\max_{y_e} \left(\int y_e^{\frac{\phi-1}{\phi}} de\right)^{\frac{\phi}{\phi-1}} - \int p_e y_e de.$$

   The FOC is

$$p_e y_e = X^{\frac{1}{\phi-1}} y_e^{\frac{\phi-1}{\phi}}, \tag{29}$$

   where

$$X \equiv \int y_e^{\frac{\phi-1}{\phi}} de. \tag{30}$$

2. By substituting (29) into the FOC of the profit maximization problem for labor, we obtain

$$(1-\alpha)(1-1/\phi) p_e y_e / \ell_e = w.$$

Substituting this equation into the market condition of labor,

$$\int \ell_e de = L,$$

we obtain

$$w = (1-\alpha)(1-1/\phi)\frac{\overline{p_e y_e}}{L},$$

where

$$\overline{p_e y_e} \equiv \int p_e y_e de.$$

Substituting this equation into the FOC for labor, we obtain

$$\ell_e = \frac{p_e y_e}{\overline{p_e y_e}} L = \frac{p_e y_e}{\overline{p_e y_e}}. \tag{31}$$

Similarly,

$$k_e = \frac{p_e y_e}{\overline{p_e y_e}} K, \tag{32}$$

where K is the aggregate stock of capital.

3. Using (29), we obtain

$$\overline{p_e y_e} = \int p_e y_e de = X^{\frac{1}{\phi-1}} \int y_e^{\frac{\phi-1}{\phi}} = X^{\frac{\phi}{\phi-1}}. \tag{33}$$

Using (29) and the above equation,

$$\frac{p_e y_e}{\overline{p_e y_e}} = \frac{y_e^{\frac{\phi-1}{\phi}}}{X}. \tag{34}$$

4. Substituting (34) into (31) and (32), and further substituting the results into $y_e = \pi_e k_e^{\alpha} \ell^{1-\alpha}$,

$$y_e^{\frac{\phi-1}{\phi}} = \left(\pi_e \frac{K^{\alpha} L^{1-\alpha}}{X}\right)^{\phi-1}. \tag{35}$$

Substituting (35) into (30) and rearranging,

$$X^{\frac{\phi}{\phi-1}} = K^{\alpha} L^{1-\alpha} \overline{\mathbb{E}} \left\{ \pi_e^{\phi-1} \right\}^{1/(\phi-1)}. \tag{36}$$

Substituting (35) and (36) into (29),

$$p_e y_e = X^{\frac{\phi}{\phi-1}} \tilde{\pi}_e. \tag{37}$$

5. Using (37),

$$\overline{p_e y_e} = \int p_e y_e de = X^{\frac{\phi}{\phi-1}}, \tag{38}$$

and thus

$$\frac{p_e y_e}{\overline{p_e y_e}} = \tilde{\pi}_e.$$

Substituting the equation into (31),

$$\ell_e = \tilde{\pi}_e.$$

6. Summing up the FOC for capital over firms, we obtain

$$K = \alpha(1 - 1/\phi) \frac{\overline{p_e y_e}}{\mathsf{MPK}}. \tag{39}$$

Substituting (38) into (39),

$$K = \left(\frac{\alpha(1 - 1/\phi)}{\mathsf{MPK}} \right)^{\frac{1}{1-\alpha}} \overline{\mathbb{E}} \left\{ \pi_e^{\phi-1} \right\}^{\frac{1}{(\phi-1)(1-\alpha)}}. \tag{40}$$

Substituting (36) and (40) into (33), we obtain

$$\overline{p_e y_e} = \left(\frac{\alpha(1 - 1/\phi)}{\mathsf{MPK}} \right)^{\frac{\alpha}{1-\alpha}} \overline{\mathbb{E}} \left\{ \pi_e^{\phi-1} \right\}^{\frac{1}{(\phi-1)(1-\alpha)}}.$$

7. Let $\Lambda_{I,t}$ be the stochastic discount factor. Then,

$$
\begin{aligned}
q_{e,t} =& \mathbb{E} \int \frac{\Lambda_{I,s}}{\Lambda_{I,t}} d_{e,s} ds \\
=& \mathbb{E} \int \frac{\Lambda_{I,s}}{\Lambda_{I,t}} \frac{1}{\phi} p_{e,s} y_{e,s} ds \\
=& \frac{1}{\phi} \mathbb{E} \int \frac{\Lambda_{I,s}}{\Lambda_{I,t}} \overline{py}_s \tilde{\pi}_{e,s} ds.
\end{aligned} \tag{41}
$$

As shown below, $\tilde{\pi}_e$ has the following property:

$$
d\tilde{\pi}_{e,s} = (\phi - 1)\sigma_{ze}\tilde{\pi}_{e,s} dW_{e,s} + (\phi - 1)\sigma_{zI}\tilde{\pi}_{e,s} dW_{I,s}.
$$

Using Ito's lemma,

$$
d \ln \tilde{\pi}_{e,s} = -\frac{(\phi - 1)^2 \left(\sigma_{ze}^2 + \sigma_{zI}^2\right)}{2} dt + (\phi - 1)\sigma_{ze} dW_{e,s} + (\phi - 1)\sigma_{zI} dW_{I,s}.
$$

Therefore,

$$
\tilde{\pi}_{e,s} = \tilde{\pi}_{e,t} \cdot \tilde{\pi}_{e,t,s},
$$

where

$$
\begin{aligned}
\tilde{\pi}_{e,t,s} \equiv \exp \Bigg(& -\frac{(\phi - 1)^2 \left(\sigma_{ze}^2 + \sigma_{zI}^2\right)}{2} (s - t) + (\phi - 1)\sigma_{ze} \left(W_{e,s} - W_{e,t}\right) \\
& + (\phi - 1)\sigma_{zI} \left(W_{e,s} - W_{e,t}\right) \Bigg).
\end{aligned}
$$

Substituting the result into (41), we obtain

$$
q_{e,t} = \overline{q}_t \cdot \tilde{\pi}_{e,t},
$$

where

$$
\overline{q}_t \equiv \int_t^\infty (1 - \tau_{\text{inc}})(1 - \tau_{\text{corp}}) \overline{d}_s \, \mathbb{E} \frac{\Lambda_{e,s}}{\Lambda_{e,t}} \tilde{\pi}_{e,t,s} ds,
$$

and $\Lambda_{e,s}$ is the stochastic discount factor of a household that directly purchases the stocks of firm e.

B.4 Returns

As defined in Section 2.1,

$$\mu_{q,t}dt + \sigma_{qe,t}dW_{e,t} + \sigma_{qI,t}dW_{I,t} = \frac{(1-\tau_{\text{inc}})(1-\tau_{\text{corp}})d_{e,t}dt}{q_{e,t}} + \frac{dq_{e,t}}{q_{e,t}} - \iota_q dt.$$

The first term in the RHS of the above equation is rewritten as

$$\frac{(1-\tau_{\text{inc}})(1-\tau_{\text{corp}})d_{e,t}}{q_{e,t}}dt = (1-\tau_{\text{inc}})(1-\tau_{\text{corp}})\frac{\bar{d}_t}{\bar{q}_t}dt = (1-\tau_{\text{inc}})(1-\tau_{\text{corp}})\frac{\widetilde{D}_t}{\widetilde{Q}_t}dt.$$

The second term is, by substituting $q_{e,t} = \bar{q}_t\tilde{\pi}_{e,t}$,

$$\begin{aligned}\frac{dq_{e,t}}{q_{e,t}} &= \frac{(d\bar{q}_t)\tilde{\pi}_{e,t} + \bar{q}_t(d\tilde{\pi}_{e,t})}{q_{e,t}} \\ &= \frac{d\bar{q}_t/dt}{\bar{q}_t}dt + \frac{\bar{q}_t(d\tilde{\pi}_{e,t})}{q_{e,t}}.\end{aligned}$$

Note that

$$\frac{d\bar{q}_t/dt}{\bar{q}_t} = g + \frac{d\widetilde{Q}_t\Big/dt}{\widetilde{Q}_t}.$$

Note also that using (28),

$$d\tilde{\pi}_e = (\phi-1)\sigma_{ze}\tilde{\pi}_e dW_e + (\phi-1)\sigma_{zI}\tilde{\pi}_e dW_I.$$

Thus,

$$\frac{\bar{q}_t(d\tilde{\pi}_{e,t})}{q_{e,t}} = (\phi-1)\sigma_{ze}dW_{e,t} + (\phi-1)\sigma_{zI}dW_{I,t}.$$

Taking the above results into account, we derive the following equations.

$$
\begin{aligned}
\mu_{q,t} =&(1-\tau_{\text{inc}})(1-\tau_{\text{corp}})\frac{\widetilde{D}_t}{\widetilde{Q}_t}+g+\frac{d\widetilde{Q}_t\Big/dt}{\widetilde{Q}_t}-\iota_q,\\
\sigma_{qe} =&(\phi-1)\sigma_{ze},\\
\sigma_{qI} =&(\phi-1)\sigma_{zI}.
\end{aligned}
$$

Similarly, the return of the mutual funds is expressed as

$$
\mu_m dt+\sigma_{m,t}dW_{I,t}=\int\left\{\left((1-\tau_{\text{inc}})(1-\tau_{\text{corp}})d_{e,t}dt+dq_{e,t}\right)/q_{e,t}-\iota_m\right\}de.
$$

Since idiosyncratic shocks are diversified away, only island shocks remain. The first term in the RHS of the above equation is, by equation (14) in the text,

$$
\int\frac{(1-\tau_{\text{inc}})(1-\tau_{\text{corp}})d_{e,t}dt}{q_{e,t}}de=(1-\tau_{\text{inc}})(1-\tau_{\text{corp}})\frac{\bar{d}_t}{\bar{q}_t}dt=(1-\tau_{\text{inc}})(1-\tau_{\text{corp}})\frac{\widetilde{D}_t}{\widetilde{Q}_t}dt.
$$

The second term is, by substituting $q_{e,t}=\bar{q}_t\tilde{\pi}_{e,t}$,

$$
\begin{aligned}
\int\frac{dq_{e,t}}{q_{e,t}}de=&\int\frac{(d\bar{q}_t)\tilde{\pi}_{e,t}+\bar{q}_t(d\tilde{\pi}_{e,t})}{q_{e,t}}de\\
=&\frac{d\bar{q}_t/dt}{\bar{q}_t}dt+\int\frac{\bar{q}_t(d\tilde{\pi}_{e,t})}{q_{e,t}}de\\
=&g dt+\frac{d\widetilde{Q}_t\Big/dt}{\widetilde{Q}_t}dt+(\phi-1)\sigma_{zI}dW_{I,t}.
\end{aligned}
$$

Using these results, we can obtain the following results:

$$
\begin{aligned}
\mu_{m,t} =&(1-\tau_{\text{inc}})(1-\tau_{\text{corp}})\frac{\widetilde{D}_t}{\widetilde{Q}_t}+g+\frac{d\widetilde{Q}_t\Big/dt}{\widetilde{Q}_t}-\iota_m,\\
\sigma_m =&(\phi-1)\sigma_{zI}.
\end{aligned}
$$

C Data used in Section 5

Here, we explain how the U.S. asset allocation data are constructed. We construct asset allocation data of households from the production side data, not household wealth data. There are two reasons for doing so. First, households indirectly hold equities. For example, when banks hold equities, households indirectly hold equities by holding bank deposits. Moreover, some households hold the equities of noncorporate businesses, which may hold equities of public firms, residences, and real estate. Second, the household wealth data includes the real estate. Real estate should be excluded because it is not treated in the model.

In our construction, a household's financial assets consist of equities and debts. Equities consist of individual stocks and mutual funds. Debts consist of corporate debt and government debt. We explain below how they are constructed.

C.1 Corporate debts and equities

We measure the values of corporate debts and equities as follows. We mainly extract data from the Flow of Funds (Federal Reserve, 2019) "L.223 Corporate Equities, Nonfinancal corporate business."

We first measure the "gross" debts using the nonfinancial corporate sector data in the Flow of Funds Accounts by "Nonfinancial corporate business; total liabilities" − $(3/4)\times$ "Nonfinancial corporate business; foreign direct investment in U.S. (market value); liability." We exclude three-quarters of foreign direct investments since, as pointed out by McGrattan and Prescott (2004), "[a]bout 1/4 of the total is debt." To obtain the value of debts used in our analysis, we subtract from the "gross" debts financial debt assets owned by firms that is, "Nonfinancial corporate business; private foreign deposits; asset" + "Nonfinancial corporate business; checkable deposits and currency; asset" + "Nonfinancial corporate business; total time and savings deposits; asset" + "Nonfinancial corporate business; money market fund shares; asset" + "Nonfinancial corporate business; security repurchase agreements; asset" + "Nonfinancial corporate business; debt securities; asset" + "Nonfinancial corporate business; loans; asset" + "Nonfinancial corporate business; trade receivables; asset" + $(1/10)\times$"Nonfinancial corporate business; U.S. direct investment abroad (market value); asset." We include $1/10$ of U.S. direct investment abroad, following McGrattan and Prescott (2004) in the financial debt assets owned by firms.

We next measure "gross" equities using the nonfinancial corporate sector data in the Flow of Funds Accounts by "Nonfinancial corporate business; corporate equities; liability" + $(3/4)\times$ "Nonfinancial corporate business; foreign direct investment in U.S. (market value); liability." As explained above, it includes three-quarters of foreign direct investments. To obtain the value of equities used in our analysis, we subtract from the "gross" equities financial equity assets owned by firms that is, "Nonfinancial corporate business; corporate equities; asset" + "Nonfinancial corporate business; mutual fund shares; asset" + "Nonfinancial corporate business; equity in Fannie Mae and Farm Credit System; asset“ + “Nonfinancial corporate business; equity investment in finance company subsidiaries; asset” + $(9/10)\times$ “Nonfinancial corporate business; U.S. direct investment abroad (market value); asset.” As explained above, we include $9/10$ of U.S. direct investment abroad in the financial equity assets owned by firms.

Second, we compute the shares of equities and debts from the values of debts and equities obtained above.

Third, we measure the firm value from the sum of debts and equities, both computed above, minus “Nonfinancial corporate business; insurance receivables due from property-casualty insurance companies; asset.”

C.2 Individual stocks and mutual funds

Equities consist of individual stocks and mutual funds. The section explains how the values of individual stocks and mutual funds are measured. The data on the shares of individual stocks and mutual funds are taken from Table 1 of French (2008). In the table, he reports the shares of several kinds of equities in the U.S. equity. We categorize direct holdings, defined contribution (DC) plans, defined benefit (DB) plans, and employee stock ownership plans (ESOPs) as individual stocks, and open-end funds, closed-end funds (CEFs), exchange-traded funds (ETFs), public funds, nonprofits, banks and insurance, and hedge funds as mutual funds.[*7]

Multiplying the firm value by the share of equities and the shares of individual stocks and mutual funds, we can compute the values of individual stocks and mutual funds.

C.3 Government debt

The U.S. government debt is taken from “Total Public Debt [GFDEBTN]”, provided by U.S. Department of the Treasury (2019a), which is retrieved from Federal Reserve Bank of St. Louis (2019).

C.4 Transaction costs

We measure the transaction costs of financial assets using data in Table 6 in French (2008). First, we measure the transaction cost of individual stocks, the risk-free corporate and government debts in the model, ι_q, from the cost of “trading” in Table 6. Second, we measure

[*7] We include DC and DB plans in individual stocks because as French (2008) writes, “[t]o avoid double counting, the allocations to DC and defined benefit (DB) plans in [his] Table 1 do not include the mutual funds they own (p. 1543).”

the transaction costs of mutual funds, ι_m, from the sum of the cost of trading and the cost of "mutual funds" in Table 6.

C.5 Government expenditure

The U.S. government debt is taken from "Federal Government: Current Expenditures [FG-EXPND]", provided by Bureau of Economic Analysis (2019), which is retrieved from Federal Reserve Bank of St. Louis (2019).

D Aggregate shocks in Section 5

In this robustness check, instead of island shocks, we introduce aggregate shocks to the firm-level productivity. Now, the firm-level producticity is $\pi_{e,t} = e^{\mu_z t + z_{e,t} + Z_t}$. μ_z is the trend growth as before. $z_{e,t}$ now only contains firm-level shocks,

$$dz_{e,t} = \sigma_{ze} dW_{e,t}.$$

Z_t contains aggregate shocks and follows an Ornstein-Uhlenbeck process,

$$dZ_t = -\mu_Z Z_t dt + \sigma_Z dW_{Z,t},$$

where dW_Z is the Wiener process representing aggregate productivity shocks. We set σ_Z for the output volatility to roughly match the data.

As in the benchmark exercise, the government expenditure G_t is a constant fraction of GDP. To reduce an unnecessary state variable, we set lump-sum taxation/transfers ψ_t so that the detrended government debts $\widetilde{B}_G$ is constant:

$$\widetilde{\psi} = (r_f - g)\widetilde{B}_G + \widetilde{G} - \big(1 - (1 - \tau_{\text{inc}})(1 - \tau_{\text{corp}})\big)\widetilde{D} - \tau_{\text{inc}} r_f \Big(\widetilde{K} + \widetilde{B}_G\Big).$$

In the calibration, we set the ratio of G_t and GDP and the average ratio of B_G in GDP to match the data.

Notable characteristics of the model with aggregate shocks are as follows. First, the state variables of the economy are the aggregate productivity Z_t and aggregate capital stock K_t. Second, the aggregate human asset H_t becomes a risky asset.

D.1 Derivations in Section 2

D.1.1 Household's problem

Equation (5) is rewritten as follows.

$$\rho V = \max_{c,\theta^q,\theta^m} f(c,V) + V_a\mu_a a + \frac{1}{2}V_{aa}\left((\theta^q\sigma_{qe}a)^2 + (\theta^q\sigma_{qZ} + \theta^m\sigma_m + \theta^h\sigma_h)^2 a^2\right)$$
$$+ V_{\widetilde{K}}\frac{d\widetilde{K}_t}{dt} + V_Z(-\mu_Z Z_t) + \frac{1}{2}V_{ZZ}\sigma_Z^2 + V_{Za}\Big(\theta^q\sigma_{qZ} + \theta^m\sigma_m + \theta^h\sigma_h\Big)a\sigma_Z. \tag{42}$$

Differentiating (42) by θ^q, we obtain

$$V_a(\mu_q - \mu_f)a + V_{aa}\Big[\theta^q\sigma_{qe}^2 + \sigma_{qZ}\Big(\theta^q\sigma_{qZ} + \theta^m\sigma_m + \theta^h\sigma_h\Big)\Big]a^2 + V_{Za}\sigma_{qZ}\sigma_Z a = 0. \tag{43}$$

Differentiating (42) by θ^m, we obtain

$$V_a(\mu_m - \mu_f)a + V_{aa}a^2\sigma_m\Big(\theta^q\sigma_{qZ} + \theta^m\sigma_m + \theta^h\sigma_h\Big) + V_{Za}\sigma_m\sigma_Z a = 0. \tag{44}$$

We conjecture that the value function takes the form

$$V(a,\widetilde{K},Z) = \frac{a^{1-\gamma}}{1-\gamma}I(\widetilde{K},Z). \tag{45}$$

Then, $-V_a/(aV_{aa}) = 1/\gamma$ and $-V_{Za}/(aV_{aa}) = (1/\gamma)I_Z/I$. By comparing (3) and (4) in the main text, we find that $\sigma_{qZ} = \sigma_m$. By rearranging the equations, we again obtain equation (8) in the text,

$$\theta^q = \frac{\mu_q - \mu_m}{\gamma\sigma_{qe}^2}.$$

Differentiating (42) by c, we obtain equation (6) in the text,

$$c/a = \rho.$$

D.2 Derivations in Section 3

D.2.1 Growth rates

1. The relative productivity $\tilde{\pi}_{e,t}$ can be rewritten as follows:

$$\tilde{\pi}_{e,t} \equiv \frac{\pi_{e,t}^{\phi-1}}{\overline{\mathbb{E}}\left\{\pi_{e,t}^{\phi-1}\right\}} = \frac{e^{(\phi-1)(gt+z_{e,t}+Z_t)}}{\overline{\mathbb{E}}\left[e^{(\phi-1)(gt+z_{e,t}+Z_t)}\right]} = e^{(\phi-1)z_{e,t}-\frac{(\phi-1)^2\sigma_{ze}^2}{2}t}.$$

Using the result, the differential of $\tilde{\pi}_{e,t}$ is

$$d\tilde{\pi}_{e,t} = (\phi-1)\sigma_{ze}\tilde{\pi}_{e,t}dW_{e,t}.$$

2. Π_t can be written as follows:

$$\Pi_t \equiv \overline{\mathbb{E}}\left\{\pi_{e,t}^{\phi-1}\right\}^{\frac{1}{(1-\alpha)(\phi-1)}} = e^{\tilde{g}t+\frac{1}{1-\alpha}Z_t}$$

where

$$\tilde{g} \equiv \frac{1}{1-\alpha}\left(g+(\phi-1)\frac{\sigma_{ze}^2}{2}\right).$$

D.2.2 Returns

An individual stock's drift and stochastic terms are computed in the following way. As defined in Section 2.1,

$$\mu_{q,t}dt + \sigma_{qe,t}dW_{e,t} + \sigma_{qZ,t}dW_{Z,t} = \frac{\big((1-\tau_{\text{inc}})(1-\tau_{\text{corp}})d_{e,t}\big)dt}{q_{e,t}} + \frac{dq_{e,t}}{q_{e,t}} - \iota_q dt.$$

The first and the third terms in the RHS of the above equation are, by equation (14) in the text,

$$\frac{\big((1-\tau_{\text{inc}})(1-\tau_{\text{corp}})d_{e,t}\big)}{q_{e,t}} - \iota_q = (1-\tau_{\text{inc}})(1-\tau_{\text{corp}})\frac{\bar{d}_t}{\bar{q}_t} - \iota_q = (1-\tau_{\text{inc}})(1-\tau_{\text{corp}})\frac{\widetilde{D}_t}{\widetilde{Q}_t} - \iota_q.$$

The second term is, by substituting $q_{e,t} = \widetilde{Q}_t e^{\tilde{g}t} \tilde{\pi}_{e,t}$,

$$\begin{aligned}\frac{dq_{e,t}}{q_{e,t}} &= \frac{d\widetilde{Q}_t}{\widetilde{Q}_t} + \tilde{g}dt + \frac{d\tilde{\pi}_{e,t}}{\tilde{\pi}_{e,t}} \\ &= \left\{\mu_{\widetilde{Q},t} + \tilde{g}\right\}dt + (\phi - 1)\sigma_{ze} dW_{e,t} + \sigma_{\widetilde{Q},t} dW_{Z,t}.\end{aligned}$$

Combining these results, we obtain (20), (22), and (23) in the main text. A mutual fund's drift and stochastic terms are computed similarly.

D.3 Computation in Section 4

We numerically solve the partial differential equations of the detrended aggregate firm value, $\widetilde{Q}$, the detrended aggregate human asset, $\widetilde{H}$, and the value function I using the explicit upwind method. To do so, we first compute $\mu_{\widetilde{Q}}$, $\sigma_{\widetilde{Q}}$, $\mu_{\widetilde{H}}$, and $\sigma_{\widetilde{H}}$.

D.3.1 Computation of $\mu_{\widetilde{Q}}$, $\sigma_{\widetilde{Q}}$, $\mu_{\widetilde{H}}$, and $\sigma_{\widetilde{H}}$

Applying Ito's lemma to $\widetilde{Q}(\widetilde{K}, Z)$, we obtain

$$\begin{aligned}d\widetilde{Q}(\widetilde{K}, Z) &= \mu_{\widetilde{Q}} \widetilde{Q} dt + \sigma_{\widetilde{Q}} \widetilde{Q} dW_Z \\ &= \frac{\widetilde{Q}_{\widetilde{K}} \frac{d\widetilde{K}}{dt} + \widetilde{Q}_Z(-\mu_Z Z) + \widetilde{Q}_{ZZ} \frac{\sigma_Z^2}{2}}{\widetilde{Q}} \widetilde{Q} dt + \frac{\widetilde{Q}_Z}{\widetilde{Q}} \sigma_Z \widetilde{Q} dW_Z.\end{aligned}$$

$\widetilde{Q}_{\widetilde{K}}$, $\widetilde{Q}_Z$, and $\widetilde{Q}_{ZZ}$ are the derivatives of $\widetilde{Q}$, and numerically computed by the upwind scheme from $\widetilde{Q}(\widetilde{K}, Z)$. Similarly,

$$\begin{aligned}d\widetilde{H}(\widetilde{K}, Z) =& \mu_{\widetilde{H}} \widetilde{H} dt + \sigma_{\widetilde{H}} \widetilde{H} dW_Z \\ =& \frac{\widetilde{H}_{\widetilde{K}} \frac{d\widetilde{K}}{dt} + \widetilde{H}_Z(-\mu_Z Z) + \widetilde{H}_{ZZ} \frac{\sigma_Z^2}{2}}{\widetilde{H}} \widetilde{H} dt + \frac{\widetilde{H}_Z}{\widetilde{H}} \sigma_Z \widetilde{H} dW_Z.\end{aligned}$$

D.3.2 Computation of $\widetilde{Q}$ and $\widetilde{H}$

1. First, suppose that $\theta^m > 0$. Using the pricing equation, we numerically compute $\widetilde{Q}$ by updating it from n to $n+1$ as follows

$$\frac{\widetilde{Q}_t^{n+1} - \widetilde{Q}_t^n}{\Delta} = (1 - \tau_{\text{inc}})(1 - \tau_{\text{corp}}) \widetilde{D}_t^n - \iota_m \widetilde{Q}_t^n + \mathbb{E}_t \left[\frac{d\left(\Lambda_t^n Q_t^n\right)}{e^{\tilde{g}t} \cdot \Lambda_t^n \cdot \Delta} \right].$$

Note that

$$\frac{d\Lambda_t}{\Lambda_t} = -\mu_f dt - \gamma\sigma_{ae} dW_e - \left(\gamma\sigma_{aZ} - \frac{I_Z}{I}\sigma_Z\right) dW_Z,$$

where σ_{ae} is the stochastic term of idiosyncratic shocks of da_{it}/a_{it}

$$\sigma_{ae} = \theta^q \sigma_{qe},$$

and σ_{aZ} is the stochastic term of aggregate shocks of da_{it}/a_{it}

$$\sigma_{aZ} = (\theta^q + \theta^m)\sigma_{\widetilde{Q}} + \theta^h \sigma_{\widetilde{H}} = \frac{\widetilde{Q}_t}{\widetilde{A}_t}\sigma_{\widetilde{Q}} + \frac{\widetilde{H}_t}{\widetilde{A}_t}\sigma_{\widetilde{H}}.$$

Using the result, the last term is further calculated as

$$\begin{aligned}\mathbb{E}_t\left[\frac{d(\Lambda_t^n Q_t^n)}{e^{\tilde{g}t}\cdot\Lambda_t^n\cdot\Delta}\right] &= \mathbb{E}_t\left[\frac{Q_t^n d\Lambda_t^n + \Lambda_t^n dQ_t^n + d\Lambda_t^n dQ_t^n}{e^{\tilde{g}t}\cdot\Lambda_t^n\cdot\Delta}\right] \\ &= -\mu_f \widetilde{Q}_t^n + \left(\mu_{\widetilde{Q}} + \tilde{g}\right)\widetilde{Q}_t^n - \left(\gamma\sigma_{aZ} - \frac{I_Z}{I}\sigma_Z\right)\sigma_{\widetilde{Q}}\widetilde{Q}_t^n.\end{aligned}$$

2. Next, suppose the case where $\theta^m = 0$, a corner solution case. The pricing equation is

$$\frac{\widetilde{Q}_t^{n+1} - \widetilde{Q}_t^n}{\Delta} = (1-\tau_{\text{inc}})(1-\tau_{\text{corp}})\widetilde{D}_t^n - \iota_q \widetilde{Q}_t^n + \mathbb{E}_t\left[\frac{d\left(\Lambda_t^n q_{e,t}^n\right)}{e^{\tilde{g}t}\cdot\Lambda_t^n\cdot\Delta}\right].$$

The last term becomes

$$\mathbb{E}_t\left[\frac{d\left(\Lambda_t^n q_{e,t}^n\right)}{e^{\tilde{g}t}\cdot\Lambda_t^n\cdot\Delta}\right] = \mathbb{E}_t\left[\frac{d(\Lambda_t^n Q_t^n)}{e^{\tilde{g}t}\cdot\Lambda_t^n\cdot\Delta}\right] - \gamma\theta^q\sigma_{qe}^2\widetilde{Q}_t^n.$$

3. Similarly,

$$\frac{\widetilde{H}_t^{n+1} - \widetilde{H}_t^n}{\Delta} = \widetilde{w}_t^n - \widetilde{\psi}_t^n + \mathbb{E}_t\left[\frac{d(\Lambda_t^n H_t^n)}{e^{\tilde{g}t}\cdot\Lambda_t^n\cdot\Delta}\right].$$

The last term is further calculated as

$$
\begin{aligned}
\mathbb{E}_t\left[\frac{d\left(\Lambda_t^n H_t^n\right)}{e^{\tilde{g}t}\cdot\Lambda_t^n\cdot\Delta}\right] &= \mathbb{E}_t\left[\frac{H_t^n d\Lambda_t^n + \Lambda_t^n dH_t^n + d\Lambda_t^n dH_t^n}{e^{\tilde{g}t}\cdot\Lambda_t^n\cdot\Delta}\right] \\
&= -\mu_f \widetilde{H}_t^n + \left(\mu_{\widetilde{H}} + \tilde{g}\right)\widetilde{H}_t^n - \left(\gamma\sigma_{aZ} - \frac{I_Z}{I}\sigma_Z\right)\sigma_{\widetilde{H}}\widetilde{H}_t^n.
\end{aligned}
$$

D.3.3 Computation of the HJB equation

From (27) and (45), the HJB equation is rewritten as

$$
\begin{aligned}
\rho I =& \frac{f}{\frac{a^{1-\gamma}}{1-\gamma}} + I\cdot(1-\gamma)\left\{\mu_a - \frac{1}{2}\gamma\cdot\left((\theta^q(\phi-1)\sigma_{ze})^2 + \left((\theta^q+\theta^m)\sigma_{\widetilde{Q}} + \theta^h\sigma_{\widetilde{H}}\right)^2\right)\right\} \\
&+ I_{\widetilde{K}}\cdot\frac{d\widetilde{K}}{dt} + I_Z\cdot\left\{-\mu_Z\cdot Z + (1-\gamma)\left((\theta^q+\theta^m)\sigma_{\widetilde{Q}} + \theta^h\sigma_{\widetilde{H}}\right)\sigma_Z\right\} + \frac{1}{2}I_{ZZ}\sigma_Z^2,
\end{aligned}
$$

where

$$
\begin{aligned}
\frac{f}{\frac{a^{1-\gamma}}{1-\gamma}} =& \rho(1-\gamma)I\left(\ln\rho - \frac{\ln I}{1-\gamma}\right), \\
\frac{d\widetilde{K}}{dt} =& \widetilde{Y} - (\delta+\tilde{g})\widetilde{K} - \widetilde{C} - \widetilde{G} - \widetilde{X}, \\
\widetilde{X} =& \left(\iota_m + \frac{\theta\widetilde{A}}{\widetilde{Q}}\left(\iota_q - \iota_m\right)\right)(1-\tau_{\text{inc}})(1-\tau_{\text{corp}})\widetilde{D}_t + \iota_f(1-\tau_{\text{inc}})(\widetilde{K}_t + \widetilde{B}_{Gt}), \\
\widetilde{G} =& \left(1-(1-\tau_{\text{inc}})(1-\tau_{\text{corp}})\right)\widetilde{D} + \tau_{\text{inc}}\left[r_f\left(\widetilde{K}+\widetilde{B}_G\right) + \widetilde{w}\right] - (r_f - \tilde{g})\widetilde{B}_G.
\end{aligned}
$$

Using the equation, we update the part of the value function I as follows:

$$
\begin{aligned}
\frac{I_t^{n+1} - I_t^n}{\Delta} =& -\rho I_t^n + \rho(1-\gamma)I_t^n\left(\ln\rho - \frac{\ln I_t^n}{1-\gamma}\right) \\
&+ I_t^n\cdot(1-\gamma)\left\{\mu_a - \frac{1}{2}\gamma\cdot\left((\theta^q(\phi-1)\sigma_{ze})^2 + \left((\theta^q+\theta^m)\sigma_{\widetilde{Q}} + \theta^h\sigma_{\widetilde{H}}\right)^2\right)\right\} \\
&+ I_{\widetilde{K},t}^n\cdot\frac{d\widetilde{K}}{dt} + I_{Z,t}^n\cdot\left\{-\mu_Z\cdot Z + (1-\gamma)\left((\theta^q+\theta^m)\sigma_{\widetilde{Q}} + \theta^h\sigma_{\widetilde{H}}\right)\sigma_Z\right\} + \frac{1}{2}I_{ZZ,t}^n\sigma_Z^2,
\end{aligned}
$$

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