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Loan Screening When Banks Have Superior Information Technology

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ABSTRACT

We analyze the loan market under symmetrically imperfect information: the project quality is unknown to both a bank and a firm, but it is revealed with noise to the bank with cost. We show that there are three equilibria, that is, (i) a screening and separating equilibrium, (ii) a non-screening and pooling equilibrium, and (iii) a non-screening, cheap-information based separating equilibrium. They emerge depending on parameter values and are all socially optimal. In particular, the screening and separating equilibrium emerges when the average project quality is low, or when the international interest rate is high. Policies, such as credit easing, business subsidy, and public loan guarantees, make banks reduce or even stop screening. Then, more capital is allocated to unviable firms, resulting in a smaller national income and lower social welfare.

Key words: imperfect information, costly screening, bank lending standard.

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I INTRODUCTION

We propose a theory of banks' screening of firms based on symmetrically imperfect information. Should banks screen before lending? What is the optimal level of bank screening cost? Is it socially optimal to introduce policies, such as credit easing or public loan guarantees?

Our approach is to view the bank screening process as an optimal mechanism when both a creditor and a borrower do not know a borrower's type. However, before extending a loan, a bank can obtain an imperfect signal with a cost on a loan applicant's real type, which the loan applicant itself does not know. We also assume that the larger the bank paying the screening cost, the more accurate signal the bank can obtain.

Our assumption on information is somewhat different from the conventional assumption of imperfect information, that is, asymmetric information, with which borrowers know their types but lenders do not. We assume that borrowers themselves do not have information on their own types until the production takes place. This informational setup, we think, better captures the real world loan market for small- and medium-sized enterprises (SMEs), especially if SMEs are uncertain whether they can survive when they apply for loans.

We first analyze a baseline model without a government. It is a one-period model, with five stages. Firms' types are predetermined, but they behave identically *ex ante* in the loan market, because they are unaware of their types. Banks are identical *ex ante* and offer loan contracts with contingencies. After receiving loan applications, banks screen, that is, obtain signals of loan applicants with screening cost, and then accept or reject loan applications following the contingencies specified in the loan contract offer. Finally, the firms that receive loans either produce and repay the loans or go bankrupt.

The baseline model shows that the economy has three types of possible Nash equilibria: a screening and separating equilibrium (SS), a non-screening and pooling equilibrium (NP), and a non-screening, cheap-information based separating equilibrium (NS).[2] It is optimal for banks not to

[2]Both banks and firms are usually indifferent between NS equilibrium and NP equilibrium, because banks do not pay to screen in both equilibria. An exception emerges with some government policies, which we discuss later. Otherwise, without loss of generality, we focus on NP equilibrium, refering as non-screening equilibrium. Similarly, we refer SS equilibrium as screening equilibrium hereinafter.

screen under some conditions. When the efficiency of screening is low, the economy ends up with a non-screening equilibrium because the screening cost is simply higher than the loss associated with the higher default without screening. When the average project quality is high, like in booms, the economy ends up in the non-screening equilibrium because the probability of default is sufficiently low even with a random selection of loan candidates. In addition, a non-screening equilibrium is selected (i) when the international interest rate is low; (ii) when a good firm's output is high; or (iii) when a bad firm's output is high (e.g., the cost of default is low, or the recovery upon default is high).

We prove that all types of Nash equilibria are constrained social optimal. We also calculate the gross domestic output and the national income of the baseline model. In the case where the screening equilibrium prevails, the national income is larger, (i) when the efficiency of screening increases, (ii) when the average project quality increases, (iii) when the international interest rate increases, (iv) when a good firm's output increases, or (v) when a bad firm's output increases. In the case where the non-screening equilibrium prevails, the national income is larger, under the mostly same conditions as above. However, the efficiency of screening and the international interest rate do not affect the national income in the non-screening equilibrium because there is no screening and no divergent to foreign investment exist in the non-screening equilibrium.

Governments may support bank loans to small- and medium-sized enterprises (SMEs), for example, by credit easing, or by providing loan guarantees, which are available in major countries. This is because SMEs are often regarded as facing financial constraints, as they do not possess sufficient collaterals and are not qualified to enter the stock market. SMEs' supporting programs were introduced or reinforced amid the Covid-19 pandemic, for example, the Financial Stability Board (2021). Empirical analyses are made for Japan, as in Hoshi, Kawaguchi and Ueda (2022), for Germany, as in Dörr, Murmann and Licht (2021), and for Europe wide, as in Bighelli, Lalinsky and Vanhala (2022).

As extensions of our theory, we examine credit easing, business subsidies, and public loan guarantees. With credit easing, a non-screening equilibrium is more likely to emerge than in the baseline. The credit easing brings welfare loss, since banks loosen the screening standard, allocate excess funds to the domestic market, and extend loans to unviable firms. With direct subsidies to business, a non-screening equilibrium is more likely to emerge than in the baseline, resulting in welfare loss due to the same reason as in the case with credit easing. With public loan guarantees, there exists only one Nash equilibrium, that is, a non-screening, cheap-information based separating equilibrium. The public loan guarantees bring welfare loss again because banks stop screening, invest all to the domestic market, and extend loans to unviable firms. In other words, banks do not make efforts to screen firms but provide loans with low rate to firms with bad signals so that those firms survive perfectly. This mechanism seems to explain the emergence of zombie firms under public loan guarantees.

Lastly, in Appendix I, we drop the assumption of fixed-size loan contracts, and endogenize the loan size under the assumption of a constant-returns-to-scale production function. We find our main conclusions still hold.

Related Literature

Our approach is similar to the theoretical literature on the adverse selection in credit markets. The most closely related paper is Figueroa and Leukhina (2015) who develop the DSGE model under adverse selection. They define the contract loan size as the lending standard which banks use to achieve self-selection. They show that in a boom period, banks tend to relax constraint on loan size and have a pooling equilibrium. In a boom, better economic conditions raise overall returns to investment, so larger loans are made to a worse mix of borrowers. Dell'Ariccia and Marquez (2005) model the collateral requirements related to the lending standard. In a boom period, when collateral liquidation cost is bigger than the loss from lending to bad firms, banks offer pooling contracts. Although the two papers have slightly different definitions of lending standards, the main intuition is similar in that during a boom period, the average project return increases, the costs associated with financing bad borrowers becomes lower than the costs associated with a tight lending standard, thus, lending to bad firms becomes optimal.

However, while the two papers rely on the adverse selection mechanism, Bisin and Gottardi (2006) theoretically prove that the adverse selection in a general equilibrium with more general contracts selects separating equilibrium, not pooling equilibrium. In contrast, our model is not based on the adverse selection, and, therefore, switching to a pooling equilibrium happens with very general contracts. Indeed, empirically, Dvorkin and Shell (2017) show that in a bad recession, loan demand decreases and loan standard is tightened, and the loan market shuts down. Our model can explain these empirical tendencies.

Another closely related paper is by Gorton and Ordonez (2014). In their paper, lenders make collateral loans to borrowers. The final value of collateral is random. Lenders investigate the final value of collateral but with costs. The information about collateral is independent to project quality and, thus, costly acquiring the information is a waste of resources. They find two equilibria, a separating equilibrium with information sensitive loan contracts, and a pooling equilibrium with information insensitive loan contracts. Our model also assumes imperfect information *ex ante*. However, different from Gorton and Ordonez (2014), we assume banks screen to acquire information about the project itself, which can improve social welfare, as in the case with Greenwood and Jovanovic (1990) and Townsend and Ueda (2006). In addition, Gorton and Ordonez (2014) assumes the information is always accurate. However, we allow continuous degrees of screening cost and associated signal accuracy.

Lester et al. (2019) also argue that additional information reduces welfare. They incorporate imperfect competition into a standard adverse selection model, and show that a small increase in information (introduction of a free but noisy signal) reduce welfare when the market competition is strong and that additional information has no effect on welfare under perfect competition. In contrast, we assume banks engage meaningful and costly screening efforts.

Regarding costly acquisition of information, Townsend (1979) is a seminar paper, in which he introduced the costly state verification, which assumes lenders can verify asset quality *ex post* with costs. Bernanke, Gertler and Gilchrist (1999) applied it to a macroecnomic model on financial accelerator. Wang and Williamson (1998) build a model in which lenders can use costly screening *ex ante* to perfectly observe borrowers' types which borrowers themselves know. The loan contract includes repayment and probability of screening, contingent on borrower types. They concluded that there is only separating equilibrium with self selection. Good borrowers strictly prefer a separating contract because there is no cross-subsidy to bad borrowers. In contrast, we assume firms do not know their own types, just like banks have no information, and it needs the costly screening that reveals true types though imperfectly. As a result, in our model, there exists a pooling equilibrium in which banks ease the lending standard and lend to all firms.

In terms of information processing, Van Nieuwerburgh and Veldkamp (2010) use the reduction in entropy to measure screening technology, which is fairly complicated. Our model assumes a simpler screening technology. The closest paper to our screening technology is Ferrante (2018), who defines banks' costly screening as selecting good projects with information accuracy increasing linearly in screening efforts. Their cost function of screening efforts is assumed to be quadratic, in order to get a closed-form solution. Also, Freixas and Rochet (2008), in Chapter 2, introduce a simplified version of Bayesian learning for a moral hazard problem using binary signals. We combine the latter two approaches and assume binary signals and simple Bayesian laws for banks to update the information of types after the costly screening.

Regarding the policy implications of zombie firms, our results are similar to Caballero, Hoshi and Kashyap (2008). Caballero et al. (2008) focus on the effects of zombie firms on the economy, such as productivity, investment, and employment, and classify firms as zombies based on whether they receive loans lower than market rates. Zombie firms are considered to be produced by an evergreening lending strategy by distressed banks. Banks roll over loans to insolvent borrowers, and are unwilling to call in nonperforming loans for the risk of breaking the capital adequacy ratio. Rather, we focus on the economic environment, such as a low interest rate and return differences among firms. Moreover, we show how zombie firms are produced by policies, such as public loan guarantees, credit easing, and direct subsidy to business. We show under those policies banks are demotivated to screen loan applicants, in other words, have a non-screening equilibrium and lend to unviable firms, thus, generating zombie firms.

The rest of the paper is structured as follows. Section 2 describes the model setup. Section 3 solves the optimal decisions of the baseline. Section 4 solves the social planner's problem and discusses the characterization of equilibrium. Section 5 discusses the policy implications of government subsidies. Section 6 concludes.

II The Model Setup under Imperfect Information

A Environment

There are n banks, m firms, and the same m depositors/investors in the market. Each firm has a project that requires 1 unit of investment. Each depositor/investor has 1 unit of funds to save in banks or invest abroad with a fixed return R. Overall, there are m units of fund supply in this economy. Banks, as financial intermediaries, borrow from depositors and lend to firms. Banks can also invest abroad, with fixed return R.[3] Firms are heterogeneous. We assume that firm types are unknown to both the firms themselves and banks before lending. Banks screen firms based on loan applications but with a cost and receive imperfect information on the project returns.

Firms

Firms are risk neutral. Each firm has no initial wealth, borrows funds to invest in one unit of project, and produce the same final goods. Each firm is either of two types: good or bad type, which the firm cannot know *ex ante*. The probability of being a good firm is f, and the probability of being a bad firm is $(1 - f)$. Hence, in the market, there are fm good firms and $(1 - f)m$ bad firms. This type distribution is public information *ex ante*, although the specific type of each firm is not observed by anyone. A good firm produce G amount of goods from 1 unit of investment, while a bad firm produce B amount. We assume that output G and B are fixed with $G > R > B$, where R is an exogenous international interest rate.

Every firm relies on a loan to finance the project. If a firm does not obtain a loan, the firm exits. A firm's type is perfectly revealed only at the production stage after the production. The firm chooses to repay the loan in full or default. For the sake of simplicity, we assume that the firm has limited liability and that, if default, all the firm's output will be taken by the lender bank. Note that, without loss of generality, we consider only debt-type contracts. However, "default" can be considered as a contingent return like an equity-type contract.

[3]Later, we show that due to competition among banks, the depositor's return from bank deposit becomes R, the same as the return from investing abroad.

Banks

Banks are risk neutral. Banks are identical and issue loan contracts to firms. Each loan contract has a fixed loan size of one.[4] Banks can also invest abroad and receive a gross return R. Due to competition, banks end up paying the depositors R, and banks' zero-profit constraint is binding.

A bank's strategy consists of offering a loan contract to a firm with contingencies and screening the firm, that is, collecting information with screening cost C to observe the signal of the firm's type. The signal is not public, observed only by the bank and the firm involved in the screening. The signals of firms' types are not perfect, that is, not necessarily the same as firms' real types. The signals of firms' types are denoted as $\{\hat{G}, \hat{B}\}$, while the real types of firms are denoted as $\{G, B\}$.

The *accuracy* of a signal is assumed to be related to the screening cost C that a bank pays and is expressed as a function of screening cost C, that is, $Pr(G|\hat{G}, C)$ or $Pr(B|\hat{B}, C)$. On the other hand, the *correctness* of a signal is expressed as $Pr(\hat{G}|G, C)$ or $Pr(\hat{B}|B, C)$. Note that the accuracy $Pr(G|\hat{G}, C)$ refers to the probability of a signal $\hat{G}$ is accurately assigned to a G firm by bank under screening cost C, while the correctness $Pr(\hat{G}|G, C)$ refers to the probability of a bank receiving the correct signal $\hat{G}$ on a good firm G under screening cost C.

Bank's screening process in this model represents the process of credit scoring in reality. Firstly, based on the screening budget C, a bank makes a list of screening related actions, for example, a site visit and an equipment quality check. Secondly, the bank screens each loan applicant based on the screening check list and obtains a credit score for each applicant. Third, if a bank wants to decide whether to give a loan or not, the bank should decide a threshold score based on the true probability of good firms f. Note that the bank knows that there exist f good firms among all loan applicants by the Law of Large Numbers. Lastly, in this screening case, the bank assigns $\hat{G}$ signals to firms whose credit score is above the threshold, and $\hat{B}$ signals otherwise.

Theoretically, each bank chooses one from four possible lending strategies: a "lend to $\hat{G}$ only" strategy, a "lend to $\hat{B}$ only" strategy, a "lend to both $\hat{G}$ and $\hat{B}$ with no cross subsidy" strategy and a "lend to both $\hat{G}$ and $\hat{B}$ with a cross subsidy" strategy. Here, "cross subsidy" means that B-type firm can repay conditional on the $\hat{B}$ signal, while no subsidy means B-type firm cannot repay regardless

[4] We also endogenize the loan size in Appendix I and obtain similar results.

of signals. Given the lending strategy, the bank decides the screening strategy, that is, the amount of screening cost C. Based on the lending strategy and the screening strategy, the required repayment of a firm becomes contingent on the observed signal. A loan contract offered by bank i to firm j includes three parts: a screening strategy, a lending strategy (contingent on screening strategy), and loan repayments (contingent on production realization). We assume that banks commit to loan contract specifications throughout all stages, including contingent plans, when offering loan contracts in stage 1.

Information Properties

1. $Pr(G)$ is equal to f, which is an unconditional probability and public information. When banks obtain signals, the aggregate portion of firms that have $\hat{G}$ signals, $Pr(\hat{G})$, is naturally assumed as being equal to f. Hence, $Pr(\hat{G}) = Pr(G)$ and $Pr(\hat{B}) = Pr(B)$.

2. The correctness $Pr(\hat{G}|G, C)$ and $Pr(\hat{B}|B, C)$ are denoted as $f_G(C)$ and $f_B(C)$, respectively. By Bayes' Law, the correctness and the accuracy share the same probabilities (see Table 1).

$$f_G(C) = Pr(G|\hat{G}, C) = \frac{Pr(\hat{G}|G, C)Pr(G)}{Pr(\hat{G})} = Pr(\hat{G}|G, C), \text{ and}$$

$$f_B(C) = Pr(B|\hat{B}, C) = \frac{Pr(\hat{B}|B, C)Pr(B)}{Pr(\hat{B})} = Pr(\hat{B}|B, C).$$

Table 1 summarizes the key probabilities regarding signals.

True type	For G firm	For B firm
Unconditional prob.	$Pr(G) = f$	$Pr(B) = 1 - f$
Accuracy	$Pr(G\|\hat{G}, C) = f_G(C)$	$Pr(B\|\hat{B}, C) = f_B(C)$
Inaccuracy	$Pr(G\|\hat{B}, C) = 1 - f_B(C)$	$Pr(B\|\hat{G}, C) = 1 - f_G(C)$
Signal	For $\hat{G}$ firm	For $\hat{B}$ firm
Unconditional prob.	$Pr(\hat{G}) = f$	$Pr(\hat{B}) = 1 - f$
Correctness	$Pr(\hat{G}\|G, C) = f_G(C)$	$Pr(\hat{B}\|B, C) = f_B(C)$
Incorrectness	$Pr(\hat{G}\|B, C) = 1 - f_B(C)$	$Pr(\hat{B}\|G, C) = 1 - f_G(C)$

Table 1: Key Probabilities regarding Signals

3. The correctness is assumed to naturally have the following properties:

   3.1 The correctness $f_G(\cdot)$ and $f_B(\cdot)$ are increasing, strictly concave, twice continuously differentiable, in terms of C, that is,
   $\frac{\partial f_G(C)}{\partial C} > 0, \frac{\partial^2 f_G(C)}{\partial C^2} < 0, \frac{\partial f_B(C)}{\partial C} > 0,$ and $\frac{\partial^2 f_B(C)}{\partial C^2} < 0.$
   When a bank has more screening budget C, then the bank can afford more screening related actions and has a better understanding of the applicant, hence, the probability of signal accuracy increases. However, additional screening related actions are considered to have diminishing returns as C increases.

   3.2 The correctness $f_G(\cdot)$ and $f_B(\cdot)$ are increasing in two parameters: f, the portion of good firms in the market; and $\alpha \in (0, 1)$, the efficiency of screening technology; that is,
   $\frac{\partial f_G(C)}{\partial f} > 0, \frac{\partial f_G(C)}{\partial \alpha} > 0, \frac{\partial f_B(C)}{\partial f} > 0,$ and $\frac{\partial f_B(C)}{\partial \alpha} > 0.$

4. On the accuracy, the following properties are naturally given:

   4.1 Given the screening cost C, a higher α implies a higher accuracy of a signal $Pr(G|\hat{G}, C)$. If the screening efficiency α increases, for example, by introducing digital transaction records or online meetings, the average cost for one screening related action decreases. Then, the bank can conduct more screening related actions under the same budget C, thus, the probability of signal accuracy increases. By Property 2 above, this means $\frac{\partial f_G(C)}{\partial \alpha} > 0$, the same as Property 3.2. Similarly, $\frac{\partial f_B(C)}{\partial \alpha} > 0$.

   4.2 Given the screening cost C, a higher f implies a higher accuracy of a signal $Pr(G|\hat{G}, C)$. The lower bound of the accuracy is $Pr(G|\hat{G}, C = 0) = f$.[5] As f increases, this lower bound increases. By Property 2 above, this means $\frac{\partial f_G(C)}{\partial f} > 0$, the same as Property 3.2. Similarly, $\frac{\partial f_B(C)}{\partial f} > 0$.

5. Extreme levels of screening costs are naturally defined in terms of the signal correctness:
   $f_G(C = 0) = f$, $f_B(C = 0) = 1 - f, \lim_{C\to\infty} f_G(C) = 1$, and $\lim_{C\to\infty} f_B(C) = 1$.

[5] When $C = 0$, not only does $Pr(G|\hat{G}, C = 0) = f$, but also $Pr(G|\hat{B}, C = 0) = f$. Intuitively, since $C = 0$, signals have no information and $\hat{G}$ and $\hat{B}$ signals are randomly assigned to firms. So, within $\hat{G}$ group and within $\hat{B}$ group, the portion of G are the same, that is f.

$C = 0$ means that the bank does not screen and just receives random signals. On the other extreme, unlimited cost $C \to \infty$ is assumed to give a bank the perfectly accurate signal.

6. For the sake of simplicity, we define a parametric screening function with cost C as follows. Here, e is the Euler number, $f \in (0, 1)$ is the market share of good firm, $\alpha \in (0, 1)$ is the screening efficiency, and $C \in (0, \infty)$ is the screening cost.[6]

$$Pr(\hat{G}|G, C) = Pr(G|\hat{G}, C) = f_G(C) \equiv 1 - (1 - f)e^{-\alpha C}.$$
$$Pr(\hat{B}|B, C) = Pr(B|\hat{B}, C) = f_B(C) \equiv 1 - fe^{-\alpha C}.$$

B Time Line

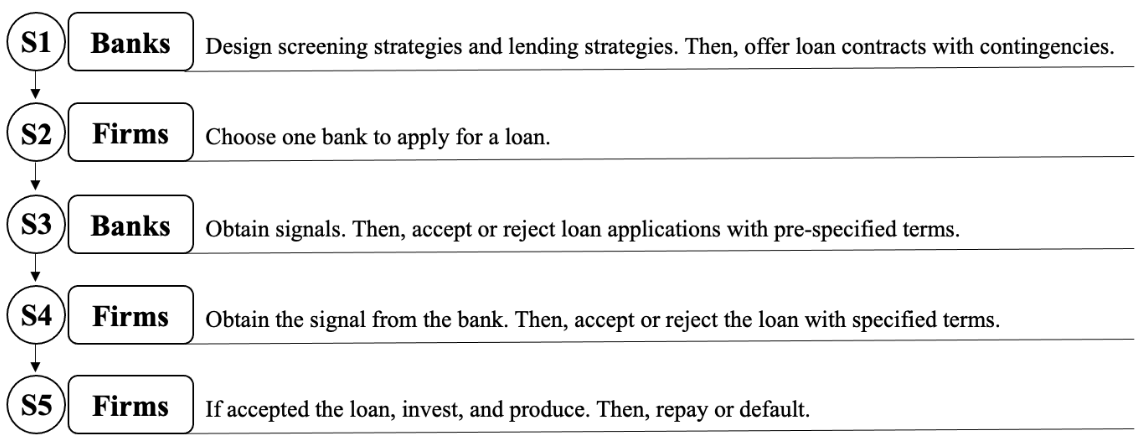

Figure 1: Time line

The model has 5 stages as shown in Figure 1.

- In stage 1, bank i chooses and designs screening strategies and lending strategies. Bank i offers its loan contract, the same one for all potential loan applicants. The loan contract terms are contingent on the signal extracted from the loan applicant in the later stage. Bank i commits the contingencies, which are known to all potential loan applicants.

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[6]The forms of screening functions follow existing literature: Ferrante (2018) and Freixas and Rochet (2008). Ferrante (2018) assumes that information accuracy increases linearly in screening effort, and the cost function of screening effort is convex (quardratic). Freixas and Rochet (2008), in Chapter 2, introduce a simplified version of Bayesian learning for moral hazard using binary signals. We assume binary signals and directly assume that information accuracy is a concave function of screening cost, satisfying the five properties above. To obtain a closed-form solution, we assume an exponential function.

- In stage 2, firm j selects a bank and applies for a loan. In stage 1 and 2, neither bank i nor firm j know firm j's real type.

- In stage 3, bank i obtains the signal by investigating applicant firm j with cost C, and bank i then specifies the loan contract terms contingent on firm j's signal, as described already in the loan offer.

- In stage 4, firm j updates the belief of its real type by the signal, and then chooses to accept the loan offer or to reject the offer (and exit the market).[7] In stage 3 and 4, bank i and firm j know firm j's signal, but not yet the real type.

- In stage 5, conditional on obtaining a loan, firm j produces goods. Naturally, the real type is revealed to the public. Then, the firm j chooses to repay the loan or to default.

For the sake of simplicity, we focus on a pure strategy Nash equilibrium, in which banks are identical, adopt the same screening strategy, and offer the same contracts.[8]

C Notations

For bank i and firm j, bank i's offer to firm j is defined as x_{ij},

$$x_{ij} = \left(Pr(G|\hat{G}, C), (R_{\hat{G}}, L_{\hat{G}}), (R_{\hat{B}}, L_{\hat{B}}) \right),$$

where $Pr(G|\hat{G}, C)$ represents the level of signal accuracy or screening intensity. $R_{\hat{G}}$ and $R_{\hat{B}}$ are the required repayments for $\hat{G}$ and $\hat{B}$ firms, respectively. $L_{\hat{G}}$ and $L_{\hat{B}}$ are the loan sizes for $\hat{G}$ and $\hat{B}$ firms, respectively. We adopt a rather strong assumption about the loan size: either zero or one, that is, $L_{\hat{G}} \in \{0, 1\}$ and $L_{\hat{B}} \in \{0, 1\}$. Also, without loss of generality, $R_{\hat{G}} \in [0, G]$ and $R_{\hat{B}} \in [0, G]$.

In terms of the bank's lending strategy, denote $\hat{\mu}$ as the allocation ratio of funds to $\hat{G}$ firms. Depending on lending strategies, the domain of contract x_{ij} is constrained. There are four cases as

[7] The screening information is private to the bank and the firm, so that the firm cannot apply loans with the screening information. For the sake of simplicity, we assume firms do not have opportunities to apply for loans to other banks at this stage.

[8] This is quasi-symmetric equilibrium, in which the number of client firms can be different and also firms could take different strategies (but firms do take the same strategies as will be explained below). Note that banks can take different strategies in off-equilibrium path.

below.

Case (1): The lending strategy is "lend to $\hat{G}$ only," that is, $\hat{\mu} = 1$. $\mathbf{S_1}$ is denoted as the constrained strategy set of x_{ij} under the "lend to $\hat{G}$ only" strategy. $Pr(G|\hat{G}, C)$ is an increasing function of C. When $C = 0$, $Pr(G|\hat{G}, C) = f$; when $C > 0$, $Pr(G|\hat{G}, C) > f$ with upper bound $Pr(G|\hat{G}, C) \leq 1$. It is easy to show $R_{\hat{G}} \geq R$. Otherwise, the bank participation constraint is not satisfied. Hence, the constrained strategy can be written as

$$x_{ij} = \Big(Pr(G|\hat{G}, C), (R_{\hat{G}}, L_{\hat{G}}), (R_{\hat{B}}, L_{\hat{B}})\Big) \in [f, 1) \times [R, G] \times \{1\} \times [0, G] \times \{0\} \equiv \mathbf{S_1}.$$

Case (2): The lending strategy is "lend to $\hat{B}$ only," that is, $\hat{\mu} = 0$. $\mathbf{S_2}$ is denoted as the constrained strategy set of x_{ij} under the "lend to $\hat{B}$ only" strategy. It is easy to show $R_{\hat{B}} \geq R$. Otherwise, the bank participation constraint is not satisfied. Hence, the constrained strategy can be written as

$$x_{ij} = \Big(Pr(G|\hat{G}, C), (R_{\hat{G}}, L_{\hat{G}}), (R_{\hat{B}}, L_{\hat{B}})\Big) \in [f, 1) \times [0, G] \times \{0\} \times [R, G] \times \{1\} \equiv \mathbf{S_2}.$$

Case (3): The lending strategy is "lend to both $\hat{G}$ and $\hat{B}$ with no cross subsidy," that is, $\hat{\mu} \in (0, 1)$. If a firm with a bad signal ($\hat{B}$) is indeed a bad firm (B), the B firm defaults without cross subsidy. In other words, the required repayment is higher than the output, $R_{\hat{B}} > B$. Hence, the constrained strategy can be written as

$$x_{ij} = \Big(Pr(G|\hat{G}, C), (R_{\hat{G}}, L_{\hat{G}}), (R_{\hat{B}}, L_{\hat{B}})\Big) \in [f, 1) \times [R, G] \times \{1\} \times (B, G] \times \{1\} \equiv \mathbf{S_3}.$$

Case (4): The lending strategy is "lend to both $\hat{G}$ and $\hat{B}$ with a cross subsidy," that is, $\hat{\mu} \in (0, 1)$. Particularly, we only consider the case that good firms subsidize bad firms implicitly, because bad firms have no room to subsidize good firms. Here, we define a cross subsidy in terms of *ex post* true nature of firms (i.e., G and B), instead of the observed terms (i.e., $\hat{G}$ and $\hat{B}$). If a bad signal firm ($\hat{B}$) is indeed a bad firm (B), B firm does not default, in other words, the required repayment is lower than or equal to the output, $R_{\hat{B}} \leq B$. Hence, the constrained strategy can be written as

$$x_{ij} = \Big(Pr(G|\hat{G}, C), (R_{\hat{G}}, L_{\hat{G}}), (R_{\hat{B}}, L_{\hat{B}})\Big) \in [f, 1) \times [R, G] \times \{1\} \times [0, B] \times \{1\} \equiv \mathbf{S_4}.$$

The overall strategy set for x_{ij} is defined naturally as $\mathbf{S} \equiv \mathbf{S_1} \cup \mathbf{S_2} \cup \mathbf{S_3} \cup \mathbf{S_4}$.

There are n banks and m firms in the market. A set of bank i's offers for all firms is denoted as $x_i = \{x_{ij}\}_{j=1}^{m} \in \mathbf{S}^m$, and other banks' offers as $x_{-i} = \Pi_{h \neq i} x_h$. We assume bank i's x_{ij} is the same for all firms. Firm j chooses one bank (or one loan contract offer) from $x_j = \{x_{ij}\}_{i=1}^{n} \in \mathbf{S}^n$. All loan offers are denoted as $x = \{x_1, ..., x_m\} \in \mathbf{S}^{mn}$. $E\left[\pi^f\left(x_{ij}\right)\right]$ represents firm j's expected profit in stage 2 when firm j faces bank i's offer. $E\left[\pi^b(x_{ij}|x_{-i})\right]$ represents bank i's expected profit from bank i's offer to firm j in stage 1, conditional on any other banks' offers.

III Optimal Decisions under Imperfect Information

Under imperfect information, we solve the model backward, starting from stage 5.

A Stage 5

In stage 5, the active firms are those that have accepted the loan contracts in the previous stage 4. Firms produce, unveil the real types, and then decide whether to repay loans in full or to default and declare bankruptcy.[9]

Similar to the perfect information case (see Appendix A), bank's profit is $\pi^b = \hat{\mu}\pi^b_{\hat{G}} + (1-\hat{\mu})\pi^b_{\hat{B}}$. Note that the allocation across true types is not necessarily the same as signal-based allocation $\hat{\mu}$. If firm j repays the loan, $R_{\hat{G}}$ or $R_{\hat{B}}$, the repayment to bank i is contingent on the realization of outputs, G or B. If firm j declares bankrupt, bank i will take over all firm j's output.

A firm's repayment obligations are contingent on signals and loan contracts obtained in the previous stage. Hence, the active firms in stage 5 can be divided into four cases, where $\hat{\mu}$ is bank's allocation ratio of funds to $\hat{G}$ firms. If a firm can repay the loan contract, it will repay. Otherwise, the firm has to declare bankruptcy, and it will get zero return.

(1) $\hat{G}$ firms under a "lend to $\hat{G}$ only" strategy ($\hat{\mu} = 1$) and screening strategy C, with repayment $R_{\hat{G}} \in [0, G]$;

[9] As long as firms can repay loans, they will repay in full. Otherwise, if they do not repay and declare bankruptcy, they will only receive zero output.

(2) $\hat{B}$ firms under a "lend to $\hat{B}$ only" strategy ($\hat{\mu} = 0$) and screening strategy C, with repayment $R_{\hat{B}} \in [0, G]$;

(3) $\hat{G}$ firms under a "lend to both" strategy ($\hat{\mu} \in (0, 1)$)[10] and screening strategy C, with repayment $R_{\hat{G}} \in [0, G]$;

(4) $\hat{B}$ firms under a "lend to both" strategy ($\hat{\mu} \in (0, 1)$) and screening strategy C, with repayment $R_{\hat{B}} \in [0, G]$.

B Stage 4

In stage 4, the active firms are those that have received the loan contract offers contingent on signals in the previous stage 3. Each firm is assumed to have applied for one loan contract to one bank in stage 2, but not all applicants are accepted by banks at this stage. The firm with an accepted application may have an updated belief of its own real type, based on the signal obtained by the bank and shared with the applicant firm. The firm with a loan offer has two options, either accepts the loan contract and produces later in stage 5, or rejects the offer and exits the market in stage 4. Using the updated belief of its type, the threshold of accepting the loan offer and producing goods becomes contingent on the signal and the loan contract offer, as follows.

(1) $\hat{G}$ firms under a "lend to $\hat{G}$ only" strategy ($\hat{\mu} = 1$) and screening strategy C, firms accept the loan offers when $Pr(G|\hat{G}, C)(G - R_{\hat{G}}) + Pr(B|\hat{G}, C) \max\{0, B - R_{\hat{G}}\} \geq 0$;

(2) $\hat{B}$ firms under a "lend to $\hat{B}$ only" strategy ($\hat{\mu} = 0$) and screening strategy C, firms accept the loan offers when $Pr(G|\hat{B}, C)(G - R_{\hat{B}}) + Pr(B|\hat{B}, C) \max\{0, B - R_{\hat{B}}\} \geq 0$;

(3) $\hat{G}$ firms under a "lend to both" strategy ($\hat{\mu} \in (0, 1)$) and screening strategy C, firms accept the loan offers when $Pr(G|\hat{G}, C)(G - R_{\hat{G}}) + Pr(B|\hat{G}, C) \max\{0, B - R_{\hat{G}}\} \geq 0$;

(4) $\hat{B}$ firms under a "lend to both" strategy ($\hat{\mu} \in (0, 1)$) and screening strategy C, firms accept the loan offers when $Pr(G|\hat{B}, C)(G - R_{\hat{B}}) + Pr(B|\hat{B}, C) \max\{0, B - R_{\hat{B}}\} \geq 0$.

[10] Here, the "lend to both" strategy include the "lend to both $\hat{G}$ and $\hat{B}$ with no cross subsidy" strategy and the "lend to both $\hat{G}$ and $\hat{B}$ with a cross subsidy" strategy, since both share the same notations.

C Stage 3

With cost $C > 0$, bank i screens all the loan applications of firms that have chosen bank i. Then, it offers loan contracts specifying repayments $R_{\hat{G}}$ and $R_{\hat{B}}$ to the selected firms, following contingencies in the offered loan terms in stage 1. Note that the screening strategy (C) and the lending strategy ($\hat{\mu}$) are already decided and committed to in stage 1.

D Stage 2: Firm's Optimal Problem

Firm j selects one bank, and applies for one loan contract, as it maximizes its expected profit by comparing loan offers.

Specifically, in stage 2, given all banks' loan contract offers x_j, firm j is assumed to choose one offer, x_{ij}, to maximize firm j's expected profit as in (1).

$$\max_{x_{ij} \in x_j} E\left[\pi^f(x_{ij})\right], \tag{1}$$

which can be expressed as, for $\forall h \neq i, \forall \tilde{x}_{hj} \in x_j$,

$$E\left[\pi^f(x_{ij})\right] \geq E\left[\pi^f(\tilde{x}_{hj})\right], \tag{2}$$

for a specific x_{ij} to be the maximizer of (1). This becomes the firm's incentive compatibility constraint from the viewpoints of banks in stage 1. That is, the offer that firm j chooses should give the highest expected profit among all the offers of banks, that is, x_j.

Depending on lending strategies, the left-hand-side (LHS) of (2) can be further specified into four cases below.

Stage 2 Case (1): Suppose that "lend to $\hat{G}$ only" ($\hat{\mu} = 1$) is the equilibrium lending strategy. Firm j chooses the offer with a "lend to $\hat{G}$ only" lending strategy. If firm j receives a $\hat{G}$ signal, it will receive a loan in later stage. If firm j receives a $\hat{B}$ signal, it will not receive loans in stage 4. Moreover, since $R_{\hat{G}} \geq R$ (by bank participation constraint), B firms *ex post* will default with zero profits in stage 5. Hence, firm's expected profit does not contain the part involving $Pr(B)$. Therefore, firm expected profit, the LHS of condition (2), is specified as below: For $\forall h \neq i, \forall \tilde{x}_{hj} \in x_j$,

$$E\left[\pi^f(x_{ij})\right] = Pr(G)Pr(\hat{G}|G,C)(G - R_{\hat{G}}) \geq E\left[\pi^f(\tilde{x}_{hj})\right]. \tag{3}$$

Stage 2 Case (2): Suppose that "lend to $\hat{B}$ only" is the equilibrium lending strategy ($\hat{\mu} = 0$). By the similar argument as above, condition (2) is specified as below: for $\forall h \neq i, \forall \tilde{x}_{hj} \in x_j$,

$$E\left[\pi^f(x_{ij})\right] = Pr(G)Pr(\hat{B}|G,C)(G - R_{\hat{B}}) \geq E\left[\pi^f(\tilde{x}_{hj})\right]. \tag{4}$$

Stage 2 Case (3): Suppose that "lend to both with no cross subsidy" is the equilibrium lending strategy ($\hat{\mu} \in (0,1)$). By assumption, $R_{\hat{B}} > B$. A bad firm *ex post* will default with zero profit in stage 5. Hence, the firm expected profit does not contain the part with $Pr(B)$. Those with bad signal $\hat{B}$, but turn out to be good G *ex post*, receive positive profits in stage 5. In stage 2, this probability is $Pr(\hat{B}|G,C)$. The condition (2) is then specified as below: for $\forall h \neq i, \forall \tilde{x}_{hj} \in x_j$,

$$E\left[\pi^f(x_{ij})\right] = Pr(G)[Pr(\hat{G}|G,C)(G - R_{\hat{G}}) + Pr(\hat{B}|G,C)(G - R_{\hat{B}})] \geq E\left[\pi^f(\tilde{x}_{hj})\right]. \tag{5}$$

Stage 2 Case (4): Suppose that "lend to both with a cross subsidy" is the equilibrium lending strategy ($\hat{\mu} \in (0,1)$). By assumption $R_{\hat{B}} \leq B$, a bad firm *ex post* (B) with a bad signal ($\hat{B}$) will not default and have positive profits *ex post* in stage 5. However, a bad firm *ex post* (B) with a good signal ($\hat{G}$) will default because $R_{\hat{G}} \geq R$. The condition (2) is then specified as below: for $\forall h \neq i, \forall \tilde{x}_{hj} \in x_j$,

$$\begin{aligned} E\left[\pi^f(x_{ij})\right] =& Pr(G)[Pr(\hat{G}|G,C)(G - R_{\hat{G}}) + Pr(\hat{B}|G,C)(G - R_{\hat{B}})] \\ &+ Pr(B)[Pr(\hat{B}|B,C)(B - R_{\hat{B}})] \\ \geq& E\left[\pi^f(\tilde{x}_{hj})\right]. \end{aligned} \tag{6}$$

E Stage 1: Overall Bank's Optimal Problem

Bank i offers a specific loan contract with contingencies to all firms. The loan terms include the screening strategy (C), the lending strategy ($\hat{\mu}$), and the required loan repayments ($R_{\hat{G}}, R_{\hat{B}}$), contingent on the signals obtained in stage 3.

Bank i has four possible lending strategies: a "lend to $\hat{G}$ only" strategy ($\hat{\mu} = 1$), a "lend to $\hat{B}$ only" strategy ($\hat{\mu} = 0$), a "lend to both $\hat{G}$ and $\hat{B}$ with no cross subsidy" strategy ($\hat{\mu} \in (0, 1)$) and a "lend to both $\hat{G}$ and $\hat{B}$ with a cross subsidy" strategy ($\hat{\mu} \in (0, 1)$).

Specifically, in stage 1, bank i designs contracts for all firms $\{x_{ij}\}_{j=1}^{m}$, by maximizing the bank i's expected profit, conditional on the firm's participation constraint (PC), the firm's incentive compatible constraint (IC), and the bank's participation constraint (PC) in equilibrium. Given other banks' strategies x_{-i}, if bank i's offer is not selected, $E\left[\pi^b(x_{ij}|x_{-i})\right] = 0$; otherwise, bank i's offer is selected and $E\left[\pi^b(x_{ij}|x_{-i})\right]$ becomes a function of x_{ij}. Here, the expectation is taken in stage 1 over the probability of the realization of other banks' strategies, x_{-i}.

$$\max_{\{x_{ij}\}_{j=1}^{m} \in \mathbf{S}^m} \sum_{j=1}^{m} E\left[\pi^b(x_{ij}|x_{-i})\right] \tag{7}$$

$s.t.$

$$E\left[\pi^f\left(x_{ij}\right)\right] \geq 0 \qquad \text{[Firm PC]} \tag{8}$$

$$E\left[\pi^f\left(x_{ij}\right)\right] \geq E\left[\pi^f(\tilde{x}_{hj})\right], \text{ for } \forall j, h \neq i, \forall \tilde{x}_{hj} \in x_j \qquad \text{[Firm IC]} \tag{9}$$

$$\sum_{j=1}^{m} E\left[\pi^b(x_{ij}|x_{-i})\right] \geq 0 \qquad \text{[Bank PC]} \tag{10}$$

Note that constraint (9), the incentive compatible (IC) constraint of firms, is derived from stage 2, that is, condition (2).

Note that firms are identical *ex ante*, so bank i offers the same contract to all firms in stage 1. Each firm chooses only one bank, that is, one loan contract. If more than one bank offers the same contract, we assume that the market share of such a contract is the same among those banks offering the same contract.

The loan market is perfectly competitive, implying that banks have zero profit and any rents go to firms.

Lemma 1. *A bank's expected profit of each contract offer to a firm is always zero in equilibrium: for* $\forall i, j, E\left[\pi^b(x_{ij}|x_{-i})\right] = 0$.

Proof. See Appendix B. *Q.E.D.*

Note that this lemma does not exclude the possibility of cross subsidy *ex post* by a bank between G and B firms. This lemma is about a bank's expected profit before even observing signals $\hat{G}$ and $\hat{B}$.

Bank i's maximization problem can be simplified as below, (11) to (14). Since bank i's expected profit from firm j by contract x_{ij} is non-negative, we can focus on bank i's maximization problem of a single contract offer to firm j like (11), as if it is a representative firm. Moreover, by Lemma 1, we can replace bank's participation constraint (10) by bank's zero profit constraint (14).

$$\max_{x_{ij} \in \mathbf{S}} E\left[\pi^b(x_{ij}|x_{-i})\right] \qquad \text{[Bank profit]} \qquad (11)$$

$$s.t.$$

$$E\left[\pi^f\left(x_{ij}\right)\right] \geq 0 \qquad \text{[Firm PC]} \qquad (12)$$

$$E\left[\pi^f\left(x_{ij}\right)\right] \geq E\left[\pi^f(\tilde{x}_{hj})\right], \text{ for } \forall h \neq i, \forall \tilde{x}_{hj} \in x_j \qquad \text{[Firm IC]} \qquad (13)$$

$$E\left[\pi^b(x_{ij}|x_{-i})\right] = 0 \qquad \text{[Bank ZPC]} \qquad (14)$$

Note that because banks get zero profit by Lemma 1, a firm would get positive profits and its IR constraint (12) is not binding, under most cases.[11]

For stage 1 as a whole, we have the following proposition, regarding the existence of an equilibrium. For one of the case below, we use a bit new equilibrium concept, that is, the *cheap information equilibrium*. It is similar to the cheap talk equilibrium in the sense that signal is uninformative, but unlike the cheap talk equilibrium, signal emission is not decided strategically, but signal extraction is. Proof is given in Appendix C.

Proposition 1. *There are three types of pure strategy Nash equilibria, depending on the parameter values: (1) the screening separating equilibrium (SS) with the screening cost* $C > 0$*; (2) the non-screening pooling equilibrium (NP) with the screening cost* $C = 0$ *and banks offering the same contract regardless of signals; (3) the non-screening, cheap-information based separating equilibrium (NS) with the screening cost* $C = 0$ *and banks offering different contracts contingent on uninformative signals.*

[11] If a firm's expected profit $E\left[\pi^f\left(x_{ij}\right)\right] < 0$, the firm will not enter the loan market in stage 1, hence, there will be no loan contracts in the market, as described in Appendix E.

Define [12]

$$\Delta \equiv f\left[G - R - \frac{1}{\alpha f}\ln[\alpha f(1-f)(G-B)] - \frac{1}{\alpha f}\right] - [fG + (1-f)B - R].$$

The equilibria are described as below.

Case (1): *When $\alpha f(1-f)(G-B) > 1$, $\Delta \geq 0$, and $G - R - \frac{1}{\alpha f}\ln[\alpha f(1-f)(G-B)] - \frac{1}{\alpha f} \geq 0$, there is a screening separating equilibrium (SS), in which the "lend to $\hat{G}$ only" strategy is the equilibrium strategy. In the equilibrium contract x^*_{ij}, the equilibrium screening cost C^* and the equilibrium repayment of a $\hat{G}$ firm $R^*_{\hat{G}}$ are as below*

$$C^* = \frac{1}{\alpha}\ln[\alpha f(1-f)(G-B)] > 0,$$
$$R^*_{\hat{G}} = \frac{R + \frac{C^*}{f} - \frac{B}{\alpha f(G-B)}}{1 - \frac{1}{\alpha f(G-B)}}.$$

In stage 1, bank i uses the "lend to $\hat{G}$ only" strategy and offers the contract x^*_{ij}, and bank i's expected profit from firm j before screening is

$$E\left[\pi^b(x^*_{ij})\right] = Pr(\hat{G})[Pr(G|\hat{G}, C^*)R^*_{\hat{G}} + Pr(B|\hat{G}, C^*)B - R] - C^* = 0.$$

In stage 2, firm j's expected profit before screening is

$$E\left[\pi^f\left(x^*_{ij}\right)\right] = Pr(G)Pr(\hat{G}|G, C^*)(G - R^*_{\hat{G}}) = f\left[G - R - \frac{1}{\alpha f}\ln[\alpha f(1-f)(G-B)] - \frac{1}{\alpha f}\right] > 0.$$

In stage 3, bank i's expected profit from firm j after screening is contingent on the signal of firm j,

$$E\left[\pi^b(x^*_{ij})|\hat{G}\right] = Pr(G|\hat{G}, C^*)R^*_{\hat{G}} + Pr(B|\hat{G}, C^*)B - R - C^*/Pr(\hat{G}) = 0.$$
$$E\left[\pi^b(x^*_{ij})|\hat{B}\right] = 0.$$

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[12] Here, we interpret Δ as the difference of firm's expected profits in stage 2 under optimal contracts of a "lend to $\hat{G}$ only" strategy and of a "lend to both" strategy.

In stage 4, firm j's expected profit after screening is contingent on signals,

$$E\left[\pi^f\left(x_{ij}^*\right)|\hat{G}\right] = Pr(G|\hat{G},C^*)(G-R_{\hat{G}}^*) = G-R-\frac{1}{\alpha f}\ln[\alpha f(1-f)(G-B)]-\frac{1}{\alpha f}.$$

$$E\left[\pi^f\left(x_{ij}^*\right)|\hat{B}\right] = 0.$$

In stage 5, firm j's profit after production is contingent on signals and real types,

$$\pi^f\left(x_{ij}^*|\hat{G},G\right) = G-R_{\hat{G}}^* = G-\frac{R+\frac{C^*}{f}-\frac{B}{\alpha f(G-B)}}{1-\frac{1}{\alpha f(G-B)}}.$$

$$\pi^f\left(x_{ij}^*|\hat{G},B\right) = 0.$$

In stage 5, bank i's profit from firm j after production is contingent on signals and real types,

$$\pi^b\left(x_{ij}^*|\hat{G},G\right) = R_{\hat{G}}^*-R-\frac{C^*}{f} = \frac{R+\frac{C^*}{f}-\frac{B}{\alpha f(G-B)}}{1-\frac{1}{\alpha f(G-B)}}-R-\frac{1}{\alpha f^2}\ln[\alpha f(1-f)(G-B)].$$

$$\pi^b\left(x_{ij}^*|\hat{G},B\right) = B-R-\frac{C^*}{f} = B-R-\frac{1}{\alpha f^2}\ln[\alpha f(1-f)(G-B)].$$

$$\pi^b\left(x_{ij}^*|\hat{B},B\right) = 0.$$

$$\pi^b\left(x_{ij}^*|\hat{B},G\right) = 0.$$

Case (2): *When $\alpha f(1-f)(G-B)>1$, $\Delta<0$,and $fG+(1-f)B\geq R$, or when $\alpha f(1-f)(G-B)\leq 1$ and $fG+(1-f)B\geq R$, there are non-screening equilibria, including non-screening pooling equilibrium (NP) and non-screening cheap-information based separating equilibrium (NS), as below.*[13]

Subcase (2.1): *There is a non-screening pooling equilibrium (NP), in which the "lend to both with no cross subsidy" strategy is the equilibrium strategy. In the equilibrium contract x_{ij}^*, C^* is the*

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[13] When $\alpha f(1-f)(G-B)>1$, $\Delta=0$, both screening and non-screening equilibria exist. Under this case, there could exist a mixed strategy equilibrium. Since we focus on pure strategy equilibrium, we do not go deep into this case.

equilibrium screening cost, and R_p^* is the equilibrium repayment of a firm, regardless of the signal.

$$\begin{aligned}
&C^* = 0.\\
&R_p^* = R_{\hat{G}}^* = R_{\hat{B}}^* = \frac{R-(1-f)B}{f}.
\end{aligned}$$

In stage 1, bank i uses the "lend to both with no cross subsidy" strategy and offers the contract x_{ij}^*, and bank i's expected profit from firm j before getting any signals is

$$\begin{aligned}
E\left[\pi^b(x_{ij}^*)\right] =& Pr(\hat{G})[Pr(G|\hat{G},C^*)R_p^* + Pr(B|\hat{G},C^*)B] + Pr(\hat{B})[Pr(G|\hat{B},C^*)R_p^* + Pr(B|\hat{B},C^*)B]\\
&- R - C^* = 0.
\end{aligned}$$

In stage 2, firm j's expected profit before getting any signals is

$$E\left[\pi^f(x_{ij}^*)\right] = Pr(G)\left[Pr(\hat{G}|G,C^*)(G-R_p^*) + Pr(\hat{B}|G,C^*)(G-R_p^*)\right] = fG + (1-f)B - R.$$

In stage 3, bank i's expected profit from firm j after getting any signals is not contingent on signals,

$$\begin{aligned}
E\left[\pi^b(x_{ij}^*)|\hat{G}\right] &= Pr(G|\hat{G},C^*)R_p^* + Pr(B|\hat{G},C^*)B - R - C^* = 0.\\
E\left[\pi^b(x_{ij}^*)|\hat{B}\right] &= Pr(G|\hat{B},C^*)R_p^* + Pr(B|\hat{B},C^*)B - R - C^* = 0.
\end{aligned}$$

In stage 4, firm j's expected profit after getting any signals is not contingent on signals,

$$\begin{aligned}
E\left[\pi^f(x_{ij}^*)|\hat{G}\right] &= Pr(G|\hat{G},C^*)(G-R_p^*) = fG + (1-f)B - R.\\
E\left[\pi^f(x_{ij}^*)|\hat{B}\right] &= Pr(G|\hat{B},C^*)(G-R_p^*) = fG + (1-f)B - R.
\end{aligned}$$

In stage 5, firm j's profit after production is contingent on real types (not on signals),

$$\begin{aligned}
&\pi^f\left(x_{ij}^*|\hat{G},G\right) = \pi^f\left(x_{ij}^*|\hat{B},G\right) = G - R_p^* = G - \frac{R-(1-f)B}{f}.\\
&\pi^f\left(x_{ij}^*|\hat{G},B\right) = \pi^f\left(x_{ij}^*|\hat{B},B\right) = 0.
\end{aligned}$$

In stage 5, bank i's profit from firm j after production is contingent on real types (not on signals),

$$\pi^b\left(x_{ij}^*|\hat{G},G\right) = \pi^b\left(x_{ij}^*|\hat{B},G\right) = R_p^* - R - C^* = \frac{R-(1-f)B}{f} - R > 0.$$
$$\pi^b\left(x_{ij}^*|\hat{G},B\right) = \pi^b\left(x_{ij}^*|\hat{B},B\right) = B - R < 0.$$

Subcase (2.2): *There are a set of non-screening, cheap-information based separating equilibria (NS), in which the "lend to both with no cross subsidy" strategy is the equilibrium strategy. In the equilibrium contract x_{ij}^*, C^* is the equilibrium screening cost, and $R_{\hat{G}}^*$ and $R_{\hat{B}}^*$ are the equilibrium repayments of a $\hat{G}$ firm and a $\hat{B}$ firm.*

$$C^* = 0.$$
$$\left(R_{\hat{G}}^*, R_{\hat{B}}^*\right) \in \left\{R_{\hat{G}}^* \in [R,G], R_{\hat{B}}^* \in (B,G] \mid fR_{\hat{G}}^* + (1-f)R_{\hat{B}}^* = \frac{R-(1-f)B}{f}\right\}.$$

In stage 1, bank i uses the "lend to both with no cross subsidy" strategy and offers the contract x_{ij}^*, and bank i's expected profit from firm j before separating firms with cheap information is

$$E\left[\pi^b(x_{ij}^*)\right] = Pr(\hat{G})[Pr(G|\hat{G},C^*)R_{\hat{G}}^* + Pr(B|\hat{G},C^*)B] + Pr(\hat{B})[Pr(G|\hat{B},C^*)R_{\hat{B}}^* + Pr(B|\hat{B},C^*)B] - R - C^* = 0.$$

In stage 2, firm j's expected profit before separating firms with cheap information is

$$E\left[\pi^f(x_{ij}^*)\right] = Pr(G)\left[Pr(\hat{G}|G,C^*)(G-R_{\hat{G}}^*) + Pr(\hat{B}|G,C^*)(G-R_{\hat{B}}^*)\right] = fG + (1-f)B - R.$$

In stage 3, bank i's expected profit from firm j after separating firms with cheap information is contingent on signals,

$$E\left[\pi^b(x_{ij}^*)|\hat{G}\right] = Pr(G|\hat{G},C^*)R_{\hat{G}}^* + Pr(B|\hat{G},C^*)B - R - C^* = fR_{\hat{G}}^* + (1-f)B - R.$$
$$E\left[\pi^b(x_{ij}^*)|\hat{B}\right] = Pr(G|\hat{B},C^*)R_{\hat{B}}^* + Pr(B|\hat{B},C^*)B - R - C^* = fR_{\hat{B}}^* + (1-f)B - R.$$

In stage 4, firm j's expected profit after separating firms with cheap information is contingent on *signals,*

$$E\left[\pi^f(x^*_{ij})|\hat{G}\right] = Pr(G|\hat{G},C^*)(G-R^*_{\hat{G}}) = f(G-R^*_{\hat{G}})$$
$$E\left[\pi^f(x^*_{ij})|\hat{B}\right] = Pr(G|\hat{B},C^*)(G-R^*_{\hat{B}}) = f(G-R^*_{\hat{B}})$$

In stage 5, firm j's profit after production is contingent on signals and real types,

$$\pi^f\left(x^*_{ij}|\hat{G},G\right) = G-R^*_{\hat{G}}.$$
$$\pi^f\left(x^*_{ij}|\hat{B},G\right) = G-R^*_{\hat{B}}.$$
$$\pi^f\left(x^*_{ij}|\hat{G},B\right) = \pi^f\left(x^*_{ij}|\hat{B},B\right) = 0.$$

In stage 5, bank i's profit from firm j after production is contingent on signals and real types,

$$\pi^b\left(x^*_{ij}|\hat{G},G\right) = R^*_{\hat{G}} - R - C^* = R^*_{\hat{G}} - R.$$
$$\pi^b\left(x^*_{ij}|\hat{B},G\right) = R^*_{\hat{B}} - R - C^* = R^*_{\hat{B}} - R.$$
$$\pi^b\left(x^*_{ij}|\hat{G},B\right) = \pi^b\left(x^*_{ij}|\hat{B},B\right) = B-R.$$

Subcase (2.3): *There are a set of non-screening, cheap-information based separating equilibria, in which the "lend to both with a cross subsidy" strategy is the equilibrium strategy. In the equilibrium contract x^*_{ij}, C^* is the equilibrium screening cost, and $R^*_{\hat{G}}$ and $R^*_{\hat{B}}$ are the equilibrium repayments of a $\hat{G}$ firm and a $\hat{B}$ firm.*

$$C^* = 0.$$
$$\left(R^*_{\hat{G}}, R^*_{\hat{B}}\right) \in \left\{R^*_{\hat{G}} \in [R,G], R^*_{\hat{B}} \in [0,B] \mid fR^*_{\hat{G}} + \frac{1-f}{f}R^*_{\hat{B}} = \frac{R-(1-f)fB}{f}\right\}.$$

In stage 1, bank i uses the "lend to both with a cross subsidy" strategy and offers the contract x^*_{ij}, and bank i's expected profit from firm j before separating firms with cheap information is

$$E\left[\pi^b(x^*_{ij})\right] = Pr(\hat{G})\left[Pr(G|\hat{G},C^*)R^*_{\hat{G}} + Pr(B|\hat{G},C^*)B\right] + Pr(\hat{B})R^*_{\hat{B}} - R - C^* = 0.$$

In stage 2, firm j's expected profit before separating firms with cheap information is

$$\begin{aligned}E\left[\pi^f(x^*_{ij})\right] =& Pr(G)\left[Pr(\hat{G}|G,C^*)(G-R^*_{\hat{G}})+Pr(\hat{B}|G,C^*)(G-R^*_{\hat{B}})\right]\\ &+Pr(B)\left[Pr(\hat{B}|B,C^*)(B-R^*_{\hat{B}})\right]\\ =& f[G-fR^*_{\hat{G}}-(1-f)R^*_{\hat{B}}]+(1-f)(1-f)(B-R^*_{\hat{B}})\\ =& fG+(1-f)^2B-f^2R^*_{\hat{G}}-(1-f)R^*_{\hat{B}}\\ =& fG+(1-f)B-R.\end{aligned}$$

In stage 3, bank i's expected profit from firm j after separating firms with cheap information is contingent on signals,

$$\begin{aligned}E\left[\pi^b(x^*_{ij})|\hat{G}\right] &= Pr(G|\hat{G},C^*)R^*_{\hat{G}}+Pr(B|\hat{G},C^*)B-R-C^* = fR^*_{\hat{G}}+(1-f)B-R.\\ E\left[\pi^b(x^*_{ij})|\hat{B}\right] &= R^*_{\hat{B}}-R-C^* = R^*_{\hat{B}}-R.\end{aligned}$$

In stage 4, firm j's expected profit after separating firms with cheap information is contingent on signals,

$$\begin{aligned}E\left[\pi^f(x^*_{ij})|\hat{G}\right] &= Pr(G|\hat{G},C^*)(G-R^*_{\hat{G}}) = f(G-R^*_{\hat{G}})\\ E\left[\pi^f(x^*_{ij})|\hat{B}\right] &= Pr(G|\hat{B},C^*)(G-R^*_{\hat{B}})+Pr(B|\hat{B},C^*)(B-R^*_{\hat{B}}) = fG+(1-f)B-R^*_{\hat{B}}\end{aligned}$$

In stage 5, firm j's profit after production is contingent on signals and real types,

$$\begin{aligned}\pi^f\left(x^*_{ij}|\hat{G},G\right) &= G-R^*_{\hat{G}}.\\ \pi^f\left(x^*_{ij}|\hat{B},G\right) &= G-R^*_{\hat{B}}.\\ \pi^f\left(x^*_{ij}|\hat{G},B\right) &= 0.\\ \pi^f\left(x^*_{ij}|\hat{B},B\right) &= B-R^*_{\hat{B}}.\end{aligned}$$

In stage 5, bank i's profit from firm j after production is contingent on signals and real types,

$$\pi^b\left(x^*_{ij}|\hat{G},G\right) = R^*_{\hat{G}} - R - C^* = R^*_{\hat{G}} - R.$$
$$\pi^b\left(x^*_{ij}|\hat{B},G\right) = R^*_{\hat{B}} - R - C^* = R^*_{\hat{B}} - R.$$
$$\pi^b\left(x^*_{ij}|\hat{G},B\right) = B - R.$$
$$\pi^b\left(x^*_{ij}|\hat{B},B\right) = R^*_{\hat{B}} - R.$$

Note that firms are indifferent between the non-screening, cheap-information based separating equilibria (NS) and the non-screening pooling equilibrium (NP), because firms receive the same expected profit in stage 2 in all those equilibria. Intuitively, in non-screening equilibria, the optimal screening cost is zero, in other words, signals are randomly assigned to firms. Within the $\hat{G}$ group and the $\hat{B}$ group, the share of G type firms are the same, that is, f. Hence, the allocation of contract repayment, $R_{\hat{G}}$ and $R_{\hat{B}}$, does not matter in the equilibrium. Without loss of generality, we focus on the NP equilibrium hereinafter when NP and NS equilibria are indifferent.

IV SOCIAL PLANNER'S PROBLEM AND CHARACTERIZATION OF EQUILIBRIUM

First, we briefly describe the benchmark case of a social planner's problem under perfect information. In this case, firms' types are public information. Social planner will lend to G-type firms only. There are fm amount of G-type firms in the market, so fm amount of funding will fund domestic firms. $(1-f)m$ amount of funding will go to foreign market with return R per unit of funding. The national income $NI = [fG + (1-f)R]m$. This is the same allocation as in the decentralized equilibrium. Proof is simple and omitted.

Next, we consider a constrained social planner's problem under imperfect information, as described in the previous section. In other words, neither the social planner nor firms know firms' types until production. The social planner needs to use screening cost C per firm to observe the signal of firm's type. The social planner is benevolent, thus, has zero profit. Consider the two lending strategies: a screening strategy and a non-screening strategy. The social planner chooses z_j for all firm: $\forall j \in \{1, 2, ..., m\}$. $z_j = (Pr(G|\hat{G}, C_j), L_{\hat{G},j}, L_{\hat{B},j}) \in [f, 1) \times \{0, 1\} \times \{0, 1\}$. Because there is no externalities, it is natural to have the following proposition.

Proposition 2. *Both the screening and the non-screening Nash equilibria achieve the constrained social optima.*

Proof. See Appendix D. *Q.E.D.*

National income (NI) consists of the income of banks, firms, and depositors. Gross domestic product (GDP) consists of the output of firms. Recall that we assume there are n banks and m firms, and there are total m units of funding in the market, which are owned by depositors/investors. Investors can save money in banks, or invest abroad with a fixed return R. Banks absorb the savings and lend to firms or invest abroad with a fixed return R. As shown already, in any equilibrium, the depositor's or investor's return of a unit funding is R. The expected NI and GDP in "SS" and "NP" equilibria are expressed as below.[14]

$$E(GDP_{SS}) = \left[fG - \frac{1}{\alpha} - \frac{1}{\alpha}ln(\alpha f(1-f)(G-B))\right]m \tag{15}$$

$$E(NI_{SS}) = \left[fG - \frac{1}{\alpha} + (1-f)R - \frac{1}{\alpha}ln(\alpha f(1-f)(G-B))\right]m \tag{16}$$

$$E(GDP_{NP}) = [fG + (1-f)B]m \tag{17}$$

$$E(NI_{NP}) = [fG + (1-f)B]m \tag{18}$$

In Appendix E, we show how the equilibrium changes for different values of five parameters: f, R, B, G, or α, *ceteris paribus*. They are shown in detail. Moreover, we also show in Appendix E how GDP and NI change in five parameters: α, f, R, G, or B, *ceteris paribus*. Below is the summary of findings.

A non-screening equilibrium is selected (i) when the efficiency of screening is low (the screening technology is under-developed); (ii) when the average project quality is high; (iii) when the international interest rate is low; (iv) when good firm's output is high; or (v) when bad firm's output is high (e.g., the cost of default is low, or the recovery upon default is high).

Moreover, we calculate the national income (NI) and the gross domestic output (GDP) under different parameterizations. In the screening equilibrium, the national income is larger, (i) when the

[14]The expected NI and GDP in "NS" equilibrium are same as "NP" equilibrium, thus, omitted.

efficiency of screening increases, (ii) when the average project quality increases, (iii) when the international interest rate increases, (iv) when good firm's output increases, or (v) when bad firm's output increases. In the non-screening equilibrium, the national income is larger, under mostly same conditions. However, the efficiency of screening and the international interest rate do not affect the national income in the non-screening equilibrium because there is no screening and no divergent to foreign investment exist in the non-screening equilibrium.

We discuss two aspects of policy implications. First, the development of bank screening technology will improve the social welfare. The improvement of screening technology reduces screening cost, motivates banks to use screening, and improves social welfare (See Appendix E). Second, the government subsidies lower banks' screening incentive and cause welfare loss as in the section below.

V Policy Implications

In this section, we discuss three policies. The first policy is credit easing, which means that the central bank lowers the funding cost of banks if banks lend domestically. The second policy is a business subsidy, which means that the government gives away a fixed amount of money to firms. The third policy is a public loan guarantee, which means that the government covers the loss of banks if banks' loans to firms are defaulted.

A Credit Easing

Credit easing was adopted by some central banks, such as the Bank of Japan since 2009 and the Bank of England since 2016.[15] The central bank offers banks a lower funding cost than the international interest rate R, if banks lend to domestic borrowers. Let τ denote as the level of the subsidy and let $\tilde{R}$ denote as domestic interest rate. Bank's zero profit condition means arbitrage free with international interest rate, that is, $\tilde{R} = (1-\tau)R$. Thus, the banks' funding cost is $(1-\tau)R$ per unit of loan, and the central bank provides a subsidy of τR per unit of loan. The cost of credit easing

[15]For Japan, the Comprehensive Monetary Easing (CME) was introduced in late 2009, in which the Bank of Japan expanded its policy toolkit to include outright purchases of corporate bonds and commercial papers. See Fasano-Filho, Wang and Berkmen (2012). For UK, the Corporate Bond Purchase Scheme (CBPS) conducted by the Bank of England was introduced in 2016. See D'Amico and Kaminska (2019).

is financed by the lump sum tax (inflation tax) on consumers. With the introduction of credit easing policy, we have the following proposition, regarding the existence of an equilibrium. Proof is given in Appendix F.

Proposition 3. *Under the credit easing policy, there exist pure strategy Nash Equilibria, which are the screening separating equilibrium (SS) and the non-screening pooling equilibrium (NP), depending on the parameter values.*[16]

Define

$$\Delta_\tau \equiv f\left[G-(1-\tau)R-\frac{1}{\alpha f}\ln[\alpha f(1-f)(G-B)]-\frac{1}{\alpha f}\right]-[fG+(1-f)B-(1-\tau)R].$$

The equilibria are described as below.

(i) When $\alpha f(1-f)(G-B)>1$, $\Delta_\tau \geq 0$, *and* $G-(1-\tau)R-\frac{1}{\alpha f}\ln[\alpha f(1-f)(G-B)]-\frac{1}{\alpha f}\geq 0$, *there is a screening separating equilibrium (SS), in which the "lend to* $\hat{G}$ *only" strategy is the equilibrium strategy. Following,* C^*_τ *is the equilibrium screening cost, and* $R^*_{\hat{G}\tau}$ *is the equilibrium repayment of a* $\hat{G}$ *firm.*

$$C^*_\tau = \frac{1}{\alpha}\ln[\alpha f(1-f)(G-B)].$$

$$R^*_{\hat{G}\tau} = \frac{(1-\tau)R+\frac{C^*}{f}-\frac{B}{\alpha f(G-B)}}{1-\frac{1}{\alpha f(G-B)}}.$$

(ii) When $\alpha f(1-f)(G-B)>1$, $\Delta_\tau<0$,[17] *and* $fG+(1-f)B\geq(1-\tau)R$, *or when* $\alpha f(1-f)(G-B)\leq 1$ *and* $fG+(1-f)B\geq(1-\tau)R$, *there is a non-screening pooling equilibrium (NP), in which the "lend to both with no cross subsidy" strategy is is the equilibrium strategy.*[18] *Following,* C^*_τ *is the equilibrium screening cost, and* $R^*_{p\tau}$ *is the equilibrium repayment of*

[16] Additionally, there exists a non-screening, cheap-information based separating equilibrium (NS), indifferent from non-screening pooling equilibrium (NP), in terms of firm's and bank's expected profits. Here, the discussion of NS equilibrium is omitted.

[17] When $\alpha f(1-f)(G-B)>1$, $\Delta_\tau=0$, both screening and non-screening equilibria exist. Under this case, there could exist a mixed strategy equilibrium. Since we focus on pure strategy equilibrium, we do not go deep into this case.

[18] Here, we skip the discussion of the non-screening, cheap-information based separating equilibria, in which everyone is indifferent to the non-screening pooling equilibrium, following the same logic as in the baseline.

a firm, regardless of the signal.

$$C_\tau^* = 0.$$
$$R_{p\tau}^* = \frac{(1-\tau)R - (1-f)B}{f}.$$

We omit the full description of firm's and bank's expected profits but compare firm's expected profit in stage 2 in the credit easing case to the baseline (as in Proposition 1).

In the screening separating equilibrium, the equilibrium screening cost is the same as the baseline. $\hat{G}$ firm's required repayment is smaller than the baseline, but firm's expected profit in stage 2, denoted as $E\left[\pi^f_{SS\tau}(x^*_{ij})\right]$, is bigger than the baseline by $f\tau R$.

$$E\left[\pi^f_{SS\tau}(x^*_{ij})\right] = f\left[G - (1-\tau)R - \frac{1}{\alpha f}\ln[\alpha f(1-f)(G-B)] - \frac{1}{\alpha f}\right]$$

In the non-screening pooling equilibrium, firm's required repayment is smaller than the baseline, but firm's expected profit in stage 2, denoted as $E\left[\pi^f_{NP\tau}(x^*_{ij})\right]$, is bigger than the baseline by τR.

$$E\left[\pi^f_{NP\tau}(x^*_{ij})\right] = fG + (1-f)B - (1-\tau)R.$$

Next, we calculate the NI and the GDP of the screening equilibrium and the non-screening equilibrium, as below. In the screening equilibrium, NI becomes lower than the baseline level, while GDP remains the same as the baseline level. In the non-screening equilibrium, NI and GDP remain the same as the baseline levels.

$$E(NI_{SS\tau}) = \left[fG - \frac{1}{\alpha} + (1-f)(1-\tau)R - \frac{1}{\alpha}ln(\alpha f(1-f)(G-B))\right]m \tag{19}$$

$$E(GDP_{SS\tau}) = \left[fG - \frac{1}{\alpha} - \frac{1}{\alpha}ln(\alpha f(1-f)(G-B))\right]m \tag{20}$$

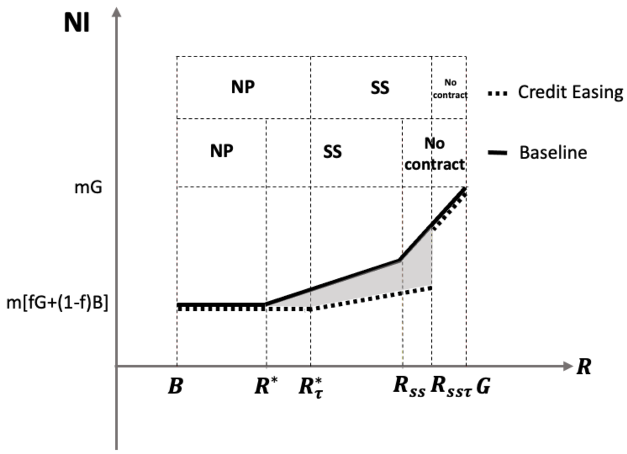
$$E(NI_{NP\tau}) = [fG + (1-f)B]m \tag{21}$$

$$E(GDP_{NP\tau}) = [fG + (1-f)B]m \tag{22}$$

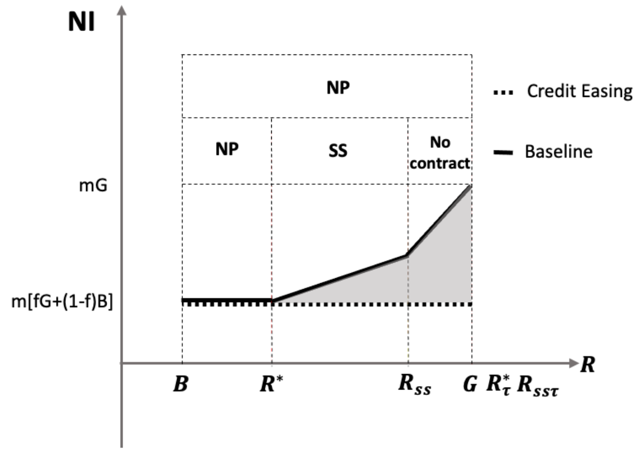
Let us look at the effects of changes in R. Credit easing affects the switching thresholds of equilibria. Let R_τ^* denote the interest rate that satisfies $\Delta_\tau(R_\tau^*) = 0$ under credit easing. Compared to the baseline, $(1-\tau)R_\tau^* = R^*$. In addition, let $R_{SS\tau}$ denote the interest rate that satisfies $E\left[\pi_{SS\tau}^f(R_{SS\tau})\right] = 0$, that is, $(1-\tau)R_{SS\tau} = R_{SS}$. Also let $R_{NP\tau}$ denote the interest rate that satisfies $E\left[\pi_{NP\tau}^f(R_{NP\tau})\right] = 0$, that is, $(1-\tau)R_{NP\tau} = R_{NP}$.

In Figure 2a, the upper row shows the case that screening equilibrium exists with $R \in (B, G)$ in the baseline (see Appendix Figure 10, panel (a)). The lower row shows the case that $\forall R \in (B, G)$, screening equilibrium does not exist in the baseline (see Appendix Figure 10, panel (c)). For columns, we discuss two examples of parameter settings. The left panels (a) and (c) show the case that τ is small enough that satisfies $R_{SS\tau} < G$ or $R_{NP\tau} < G$, while the right panels (b) and (d) show the case that τ is big enough that satisfies $R_{SS\tau} \geq G$ or $R_{NP\tau} \geq G$.[19] Figure 2b shows the case of GDP instead of NI.

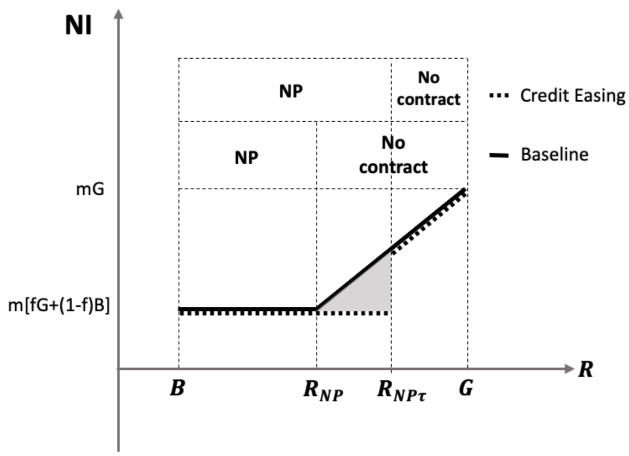
[19]Here, we omit the case $R_\tau^* < G < R_{SS\tau}$, in which $\forall R \in [B, R_\tau^*]$, there is a non-screening equilibrium while $\forall R \in (R_\tau^*, G]$, there is a screening equilibrium. The key message is the same as the two examples in Figure 2a.



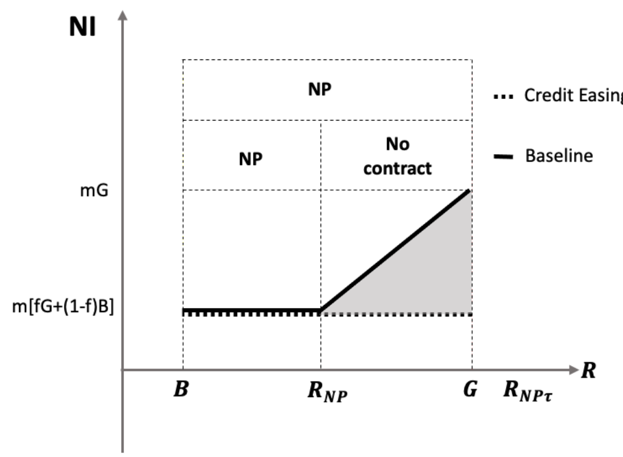
(a) $R_{SS\tau} < G$



(b) $R^*_\tau \geq G$

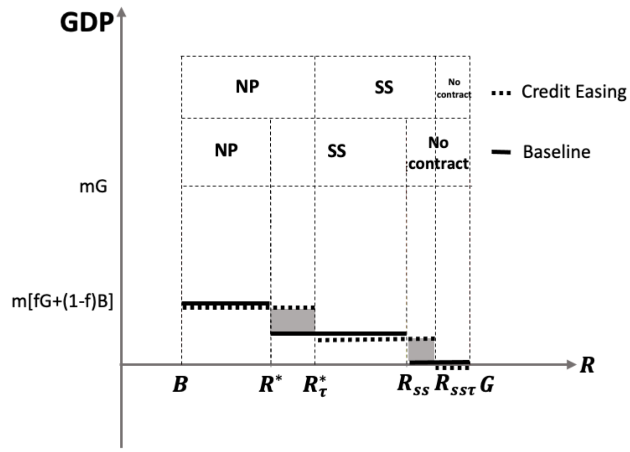


(c) $R_{NP\tau} < G$

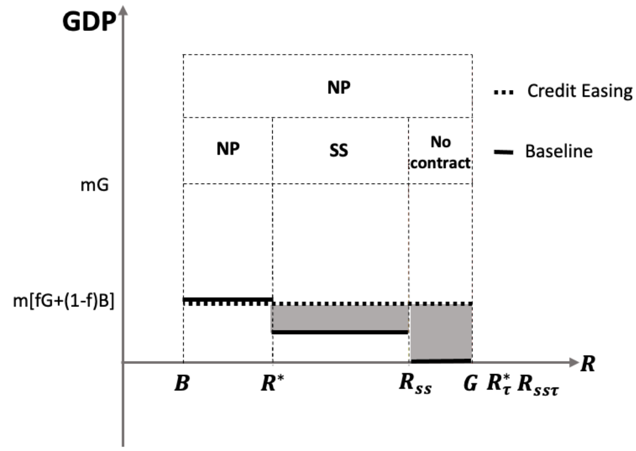


(d) $R_{NP\tau} \geq G$

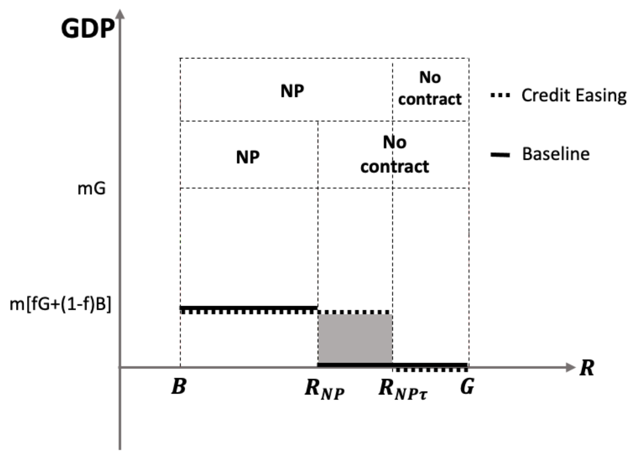
Figure 2a: Changes in NI under credit easing



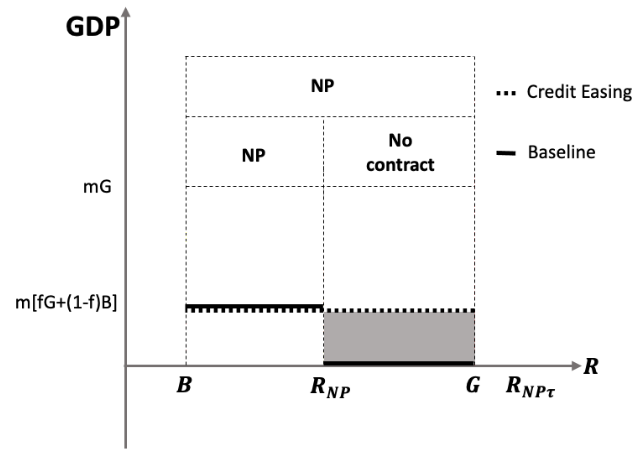
(e) $R_{SS\tau} < G$



(f) $R^*_\tau \geq G$



(g) $R_{NP\tau} < G$



(h) $R_{NP\tau} \geq G$

Figure 2b: Changes in GDP under credit easing

The shaded areas are the changes of NI and GDP. With credit easing, NI decreases in screening equilibrium and remains the same in non-screening equilibrium, while GDP remains the same for both screening and non-screening equilibria. However, all thresholds move rightward. With such a change, as seen from the figures, NI decreases for some values of R, while GDP increases for some values of R.

Intuitively, credit easing encourages banks to offer excess loans and allocate excess fund to the domestic market rather than investing abroad. Hence, firms are more likely to get funds, but together with a lower funding cost, and some of them are nonviable firms under the international interest rate. As a result, the average asset quality deteriorates and NI decreases compared to the baseline. However, given that more firms get loans to produce, GDP increases under some parameter values compared to the baseline. Still, NI, not GDP, captures social welfare, which are shown to decline with credit easing, compared to the screening equilibrium under the baseline.

B Business Subsidy

During the COVID-19 pandemic, many countries introduced subsidies to business, especially SMEs, (e.g., Blanchard, Philippon, Pisani-Ferry et al. (2020) for Europe and the US case and Hoshi et al. (2022) for Japan's case). We model this policy as government subsidy S to all firms. The subsidy can be considered as a temporary increase of firm's output from G and B to $(G+S)$ and $(B+S)$. For the sake of simplicity, we assume that the size of the increase is exogenous.[20] With the introduction of this business subsidy policy, we have the following proposition, regarding the existence of an equilibrium. Proof is given in Appendix G.

Proposition 4. *With business subsidy, there are two types of pure strategy Nash Equilibria, the screening separating equilibrium (SS) and the non-screening pooling equilibrium (NP), depending on the parameter values.*[21] *Let us define*

$$\Delta_s \equiv f\left[G+S-R-\frac{1}{\alpha f}\ln[\alpha f(1-f)(G-B)]-\frac{1}{\alpha f}\right]-[fG+(1-f)B+S-R].$$

The equilibria are described as below.

(i) When $\alpha f(1-f)(G-B)>1$, $\Delta_s \geq 0$, and $G+S-R-\frac{1}{\alpha f}\ln[\alpha f(1-f)(G-B)]-\frac{1}{\alpha f}\geq 0$, there exists a screening separating equilibrium, in which the "lend to $\hat{G}$ only" strategy is the equilibrium strategy. Following, C_s^* is the equilibrium screening cost, and $R_{\hat{G}s}^*$ is the equilibrium

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[20]There is another type of subsidy that the government subsidizes a part of required repayment, that is $\phi R_{\hat{G}}, \phi R_{\hat{B}}$. Such subsidy can be considered as a temporary increase of firm's output from G and B to $G+\phi R_{\hat{G}}$ and $B+\phi R_{\hat{G}}$. The calculation of equilibrium is a bit complicated in this case.

[21]There also exists a non-screening, cheap-information based separating equilibrium (NS), indifferent from non-screening pooling equilibrium (NP), in terms of firm's and bank's expected profits under the same parameter values. Here, the discussion of NS equilibrium is omitted.

repayment of a $\hat{G}$ firm.

$$C_s^* = \frac{1}{\alpha}\ln[\alpha f(1-f)(G-B)].$$
$$R_{\hat{G}s}^* = \frac{R + \frac{C^*}{f} - \frac{B+S}{\alpha f(G-B)}}{1 - \frac{1}{\alpha f(G-B)}}.$$

(ii) When $\alpha f(1-f)(G-B) > 1$, $\Delta_s < 0$, and $fG + (1-f)B + S - R \geq 0$, or when $\alpha f(1-f)(G-B) \leq 1$ and $fG + (1-f)B + S - R \geq 0$, there exists a non-screening pooling equilibrium (NP), in which the "lend to both with no cross subsidy" strategy is the equilibrium strategy. Following, C_s^* is the equilibrium screening cost, and R_{ps}^* is the equilibrium repayment of a firm, regardless of the signal.

$$C_s^* = 0.$$
$$R_{ps}^* = \frac{R - (1-f)(B+S)}{f}.$$

We omit the full description of firm's and bank's expected profits but compare firm's expected profit in stage 2 in the business subsidy case to the baseline (as in Proposition 1).

In the screening equilibrium, the equilibrium screening cost is the same as the baseline. $\hat{G}$ firm's required repayment is smaller than the baseline, but firm's expected profit in stage 2, denoted as $E\left[\pi_{SSs}^f(x_{ij}^*)\right]$, is bigger than the baseline, due to the subsidy.

$$E\left[\pi_{SSs}^f(x_{ij}^*)\right] = f\left[G + S - R - \frac{1}{\alpha f}\ln[\alpha f(1-f)(G-B)] - \frac{1}{\alpha f}\right].$$

In the non-screening equilibrium, firm's required repayment is smaller than the baseline, but firm's expected profit in stage 2, denoted as $E\left[\pi_{NPs}^f(x_{ij}^*)\right]$, is bigger than the baseline, due to the subsidy.

$$E\left[\pi_{NPs}^f(x_{ij}^*)\right] = fG + (1-f)B + S - R.$$

We can calculate the NI and the GDP of the screening equilibrium and the non-screening equilibrium as below. In both screening and non-screening equilibria, both NI and GDP are the same as in the baseline.

$$E(NI_{SSs}) = \left[fG - \frac{1}{\alpha} + (1-f)R - \frac{1}{\alpha} ln(\alpha f(1-f)(G-B)) \right] m \tag{23}$$

$$E(GDP_{SSs}) = \left[fG - \frac{1}{\alpha} - \frac{1}{\alpha} ln(\alpha f(1-f)(G-B)) \right] m \tag{24}$$

$$E(NI_{NPs}) = [fG + (1-f)B]m \tag{25}$$

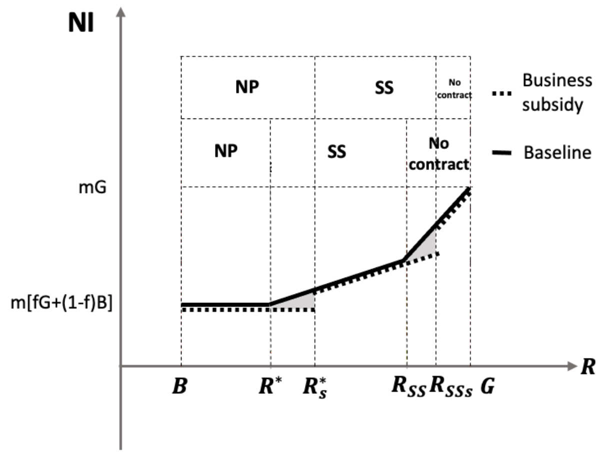
$$E(GDP_{NPs}) = [fG + (1-f)B]m \tag{26}$$

$$\tag{27}$$

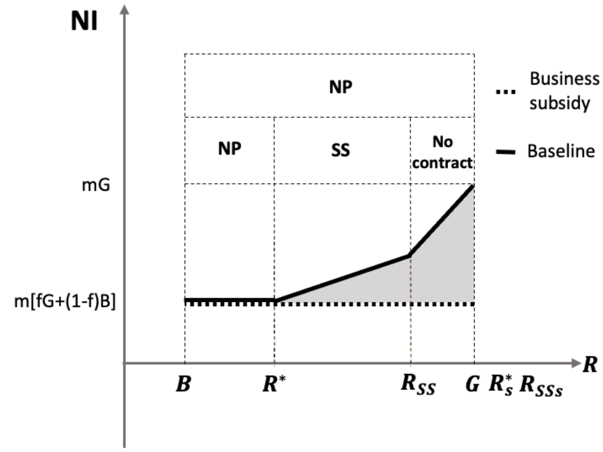
Let us look at the effects of changes in R. Business subsidies affect the switching thresholds of screening and non-screening equilibria. Let R_s^* define as $\Delta_s(R_s^*) = 0$. Compared to the baseline, $\Delta_s > \Delta$, $R_s^* > R^*$. Let R_{SSs} define as $E\left[\pi^f_{SSs}(R_{SSs})\right] = 0$. We can show that $R_{SSs} > R_{SS}$. Let R_{NPs} define as $E\left[\pi^f_{NPs}(R_{NPs})\right] = 0$. We can show that $R_{NPs} > R_{NP}$.

In figure 3a, the upper row shows the case that the screening equilibrium exists with $R \in (B, G)$ in the baseline. The lower row shows the case that screening equilibrium does not exist for any $R \in (B, G)$ in the baseline. The left column panels (a) and (c) show the case that the subsidy S is small enough that satisfies $R_{SSs} < G$ or $R_{NPs} < G$, while the right column panels (b) and (d) show the case that the subsidy S is big enough that satisfies $R_{SSs} \geq G$ or $R_{NPs} \geq G$.[22] Figure 3b shows the case of GDP instead of NI.

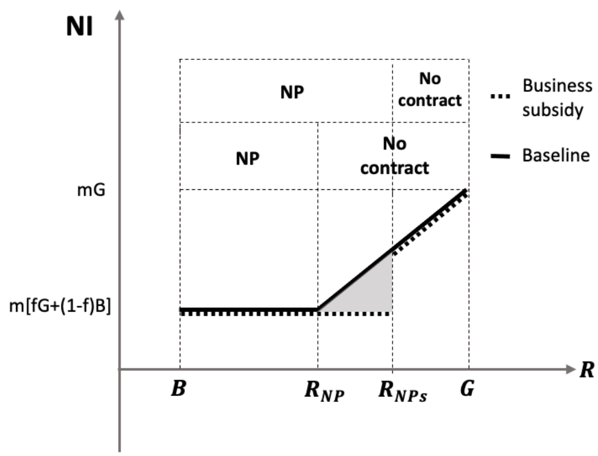
[22]Here, we omit the case $R_s^* < G < R_{SSs}$, in which $\forall R \in [B, R_s^*]$, there is a non-screening equilibrium; $\forall R \in (R_s^*, G]$, there is a screening equilibrium. The key outcomes are the same as the two examples in Figure 3a.



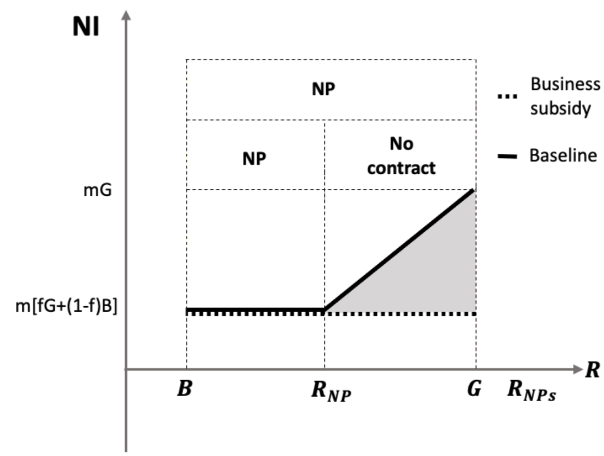
(a) $R_{SSs} < G$



(b) $R_s^* \geq G$

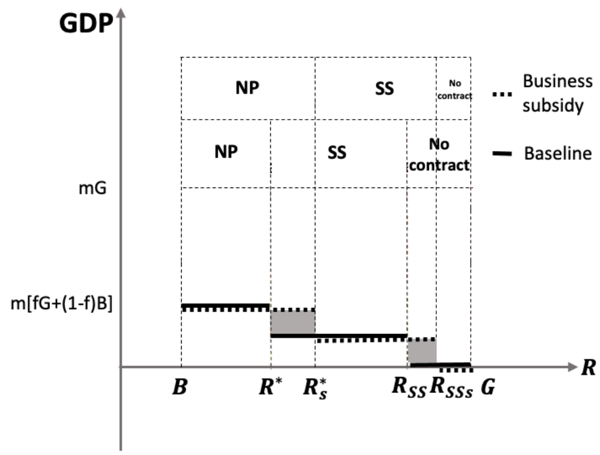


(c) $R_{NPs} < G$

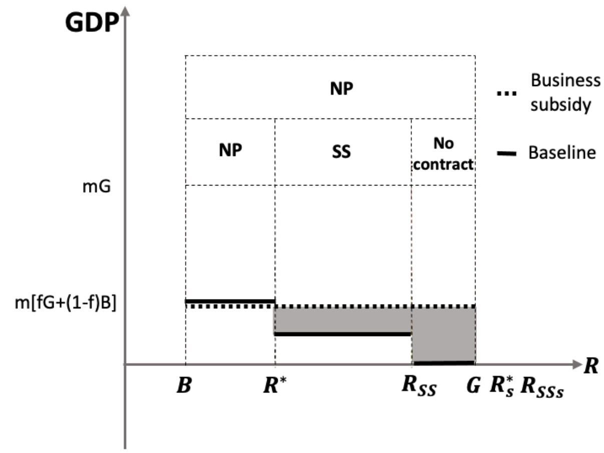


(d) $R_{NPs} \geq G$

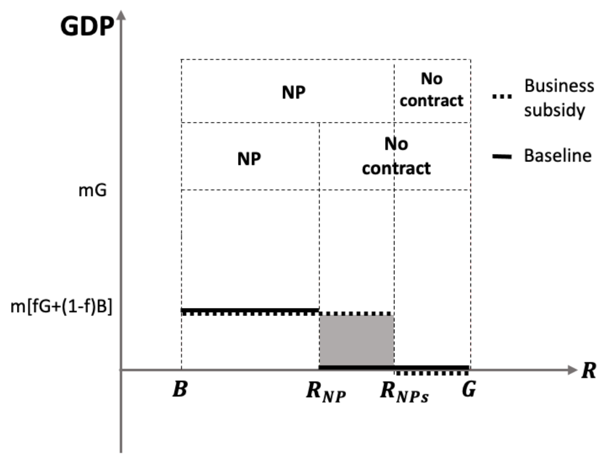
Figure 3a: Changes in NI under business subsidy



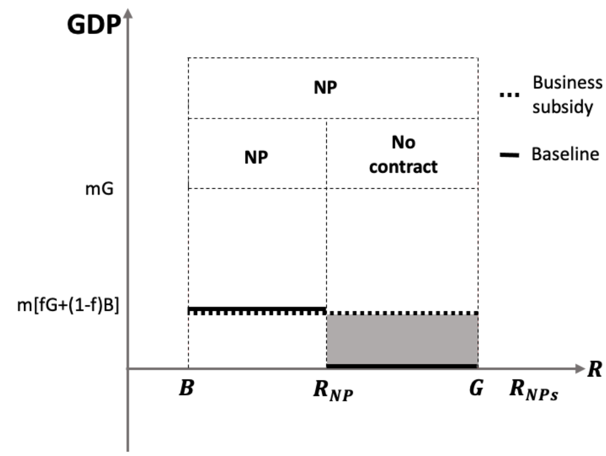
(e) $R_{SSs} < G$



(f) $R_s^* \geq G$



(g) $R_{NPs} < G$



(h) $R_{NPs} \geq G$

Figure 3b: Changes in GDP under business subsidy

The shaded areas show the changes in expected NI and expected GDP. With direct business subsidy, NI and GDP keep the same levels in screening and non-screening equilibria as in the baseline. However, all thresholds move rightward. As shown in Figure 4, NI decreases for some values of R, while GDP increases for some values of R.

Intuitively, the business subsidy increases firm's output temporarily, which encourages banks to offer excess loans and allocate excess fund to the domestic market. Direct business subsidy reallocates resources between depositors and firms with distortion, and income loss (NI) occurs as banks allocate capital excessively in the domestic market,which includes an increase in overall output (GDP) but reduce overall welfare.

C Public Loan Guarantees

Public loan guarantees have been used by some countries. In particular, those used in Japan are quite generous, often 100 percent in the past decade or so, and strengthened during the COVID-19 pandemic (Hoshi et al., 2022). We model a simple version of public loan guarantees, as follows. The government provides a complete loan guarantee plan *ex ante* with the required loan rate contingent on the signal, $R_{\hat{G}}$ or $R_{\hat{B}}$. The government expenditure on loan guarantees comes from the lump sum tax, paid by consumers. For a given loan contract, if a borrower repays, the borrower will pay the required loan rate in the contract; if a borrower defaults, the lender will acquire the borrower's output, and the government will pay the rest of the repayment to achieve the full repayment at $R_{\hat{G}}$ or $R_{\hat{B}}$.

Because of the competitive banking industry, banks' zero-profit constraint is still binding. With the introduction of public loan guarantees, we have the following proposition regarding the existence of an equilibrium. Proof is given in Appendix H.

Proposition 5. *Under public loan guarantees, there always exists a non-screening, cheap-information based separating Nash equilibrium (NS), in which the "lend to both with a cross subsidy" strategy is is the equilibrium strategy.*[23] *Following,* C^* *is the equilibrium screening cost, and* $R^*_{\hat{G}}$ *and* $R^*_{\hat{B}}$ *are the equilibrium repayments of a* $\hat{G}$ *firm and a* $\hat{B}$ *firm.*

$$C^* = 0.$$
$$R^*_{\hat{B}} = 0.$$
$$R^*_{\hat{G}} = \frac{R}{f}.$$

We omit the full description of firm's and bank's expected profits but show firm's expected profit in stage 2, $E\left[\pi^f_{NS}(x^*_{ij})\right]$ as below.

$$E\left[\pi^f_{NS}(x^*_{ij})\right] = \begin{cases} f(G-R) + (1-f)^2 B, & \text{if} \quad fG \geq R, \\ (1-f)[fG + (1-f)B], & \text{if} \quad fG < R. \end{cases} \tag{28}$$

[23] Under public loan guarantees, non-screening pooling strategy is dominated by non-screening, cheap-information based separating strategy. Also, screening separating equilibrium vanishes.

Note that if $R^*_{\hat{G}} \leq G$, that is, $fG \geq R$, the defaulted firms are B firms with $\hat{G}$ signals. If $fG < R$, the defaulted firms are all G and B firms with $\hat{G}$ signals. In addition, a depositor's expected return are denoted as $E\left(\pi^d_{NS}\right)$, as below.

$$E\left(\pi^d_{NS}\right) = \begin{cases} fR + f(1-f)B, & \text{if} \quad fG \geq R, \\ f[fG + (1-f)B], & \text{if} \quad fG < R. \end{cases} \tag{29}$$

NI and the GDP are expressed as below,

$$E(NI_{NS}) = [fG + (1-f)B]m \tag{30}$$

$$E(GDP_{NS}) = [fG + (1-f)B]m \tag{31}$$

Let us look at the effects of changes in R. In Figure 4a, the left panel implies the case that screening equilibrium exists with some $R \in (B, G)$ in the baseline and the right panel shows the case that screening equilibrium does not exist for any $R \in (B, G)$ in the baseline. On the other hand, with public loan guarantees, there always exists non-screening, cheap-information based separating equilibrium, for $\forall R \in (B, G)$. Figure 4b shows the case of GDP instead of NI for the same setup of parameter values as in Figure 4a.

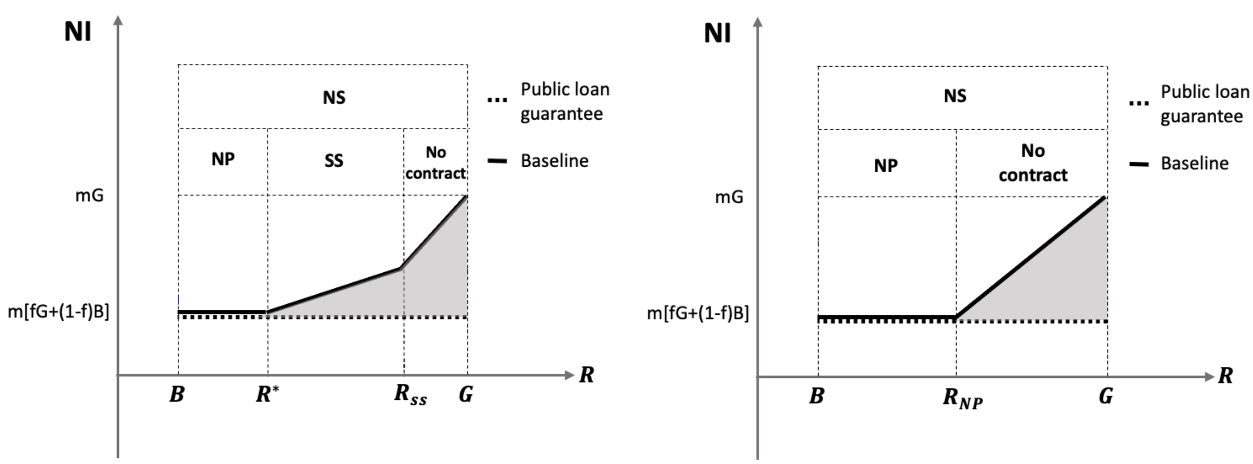


Figure 4a: Changes in NI under public loan guarantees

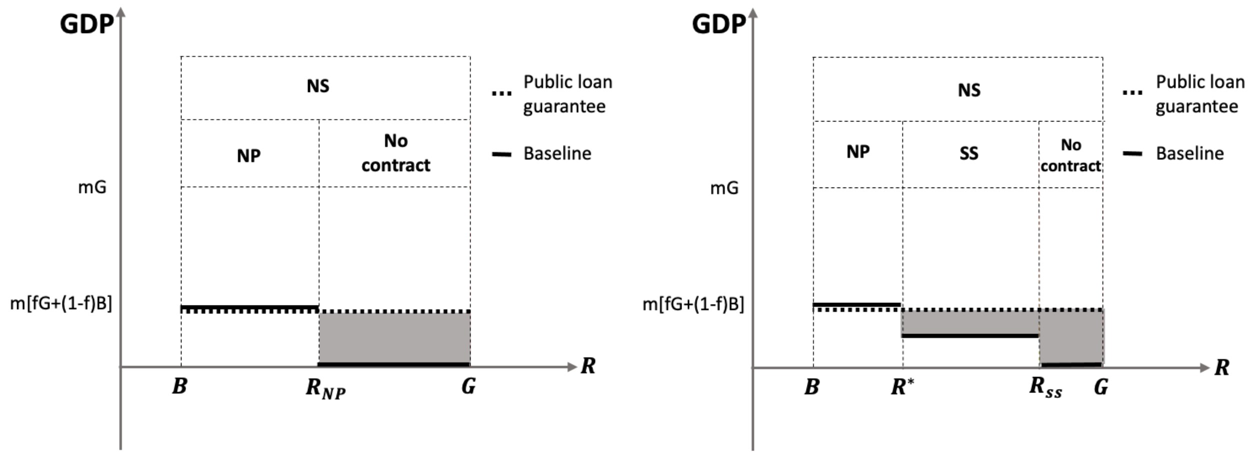


Figure 4b: Changes in GDP under public loan guarantees

The shaded areas show the changes in expected NI and expected GDP. Under public loan guarantees, there always exists non-screening, cheap-information based separating equilibrium (NS). The levels of expected NI and GDP in the NS equilibrium in the public loan guarantee case are the same as in the NP equilibrium in the baseline. The expected NI in NS equilibrium is lower than in the SS equilibrium, while the expected GDP in NS equilibrium is higher than in the SS equilibrium. This is because in the SS equilibrium, banks invest f portion of capital in the domestic market and the rest in the foreign market with return R, while in the NS equilibrium, banks lose the incentive of investing abroad, and invest all capital to domestic borrowers.

When public loan guarantees are provided, banks have no loss when lending to bad firms, so banks have no incentive to screen firms nor invest abroad. Here, non-screening strategy is optimal for banks, thus, more bad firms are financed in the market. Since banks facilitate as many firms as possible to produce, GDP increases under some values of R, for which the screening equilibrium and no contract cases emerge in the baseline.

In addition, under public loan guarantees and non-screening strategy, banks can choose $R^*_{\hat{G}}$ and $R^*_{\hat{B}}$ freely, as long as they satisfy $fR^*_{\hat{G}} + (1-f)R^*_{\hat{B}} = R$, irrelevant to firms' default rate.[24] Banks then choose them to maximize firm's expected profits in the competitive loan market. Here, firms prefer the "lend to both with a cross subsidy" strategy to the "lend to both without cross subsidy" strategy, because the former strategy has a lower default rate and provides firms with a higher

[24]This is derived from bank's zero profit constraint, because the government will guarantee that banks will receive the contract repayments.

expected profit.[25] This brings a non-screening, cheap-information based separating Nash equilibrium under public loan guarantees. Note that this optimal loan contract offers a bad firm (with $\hat{B}$ signals) a lower loan rate.

Here, banks do not make efforts to screen firms but provide loans with low rate to firms with bad signals so that those firms survive perfectly. This mechanism seems to explain the emergence of zombie firms under public loan guarantees. Intuitively, public loan guarantees generate a moral hazard problem in the sense that, for a sizable range of parameter values, banks might prefer screening *ex ante* in the baseline but stop screening completely after introducing public loan guarantees. Interestingly, 100 percent public loan guarantees generate a bigger welfare loss than the credit easing in our model. Recall that the credit easing does not eliminate screening equilibrium when the subsidy level τ is small. However, when τ is big, the credit easing is as bad as public loan guarantees, in terms of welfare loss.

VI Conclusion

We studied banks' optimal screening effort of loan contracts under symmetrically imperfect information. Unlike the usual setting of adverse selection with asymmetric information, we describe a case that neither firms nor banks are aware of firms' types *ex ante*. We think that this setup is realistic: Many small- and medium-sized entrepreneurs are uncertain whether they will default when borrowing loans from banks. We then allow banks to choose their screening efforts, which is represented by a costly and imperfect signal extraction technology. The more a bank spends in the screening process, the more accurate a signal of the firm's type a bank can obtain.

For the sake of simplicity, we adopt three assumptions: First, every loan contract has a fixed size of one.[26] Second, banks commit to the loan contract and each firm can only choose one bank to apply. Third, we only consider pure strategy Nash equilibrium.[27]

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[25]By definition, under the "lend to both without cross subsidy" strategy, B type firms default with 100 percent, while under the "lend to both with a cross subsidy" strategy, B type firms with $\hat{B}$ signals do not default. For detailed calculations, please refer to Appendix H.

[26]We obtain similar results with endogenous loan size in Appendix I and obtain similar results, where we allow more productive firms to receive more funding under the screening euqilibrium.

[27]There is a possibility of a mixed equilibrium when screening equilibrium and non-screening equilibrium are indifferent, but the mixed equilibrium should not make much difference to the conclusion.

In the baseline model, we have found three types of pure strategy Nash equilibria: the screening separating equilibrium (SS), the non-screening pooling equilibrium (NP), and the non-screening cheap-information based separating equilibrium (NS). However, the outcomes of the latter two are the same in terms of the ex-ante expected profits for banks and firms, thus, we focus only on the first two equilibria, SS and NP.

We prove that the three equilibria emerge depending on the parameter values and they are all socially optimal. Obviously, any policy interventions are detrimental to the economy. We then analyze the effects of the three polices: credit easing, business subsidies, and public loan guarantees. Under all three policies, banks reduce screening efforts, or even stop them, and, thereby, extend loans to unviable firms. This also means that banks allocate excess capital to the domestic market. This, in turn, increases GDP, but reduces national income (NI), thus, generates welfare loss. In Appendix, we show this declines of NI and welfare loss appears together with the declines of GDP, if we assume investment size can be flexible (instead of fixed at one as in the baseline). This is because less productive firms receive more funding than the case without these policies under the screening equilibrium.

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Appendices

A Hypothetical Perfect Information Case

Here, let us assume a hypothetical case where there is no information friction, that is, firms' types are always public information. There are no stages 3 and 4, which deal with signals. The sequence of events is as follows: First, a bank provides a menu of available loan contracts contingent on a firm's real type. A contract includes the lending strategy and the required repayment conditional on firm's type $\{G, B\}$. Second, a firm chooses a loan contract from the menu. Third (i.e., stage 5, without stage 3 and 4), a firm produces and repays.

In the last stage, if a firm can repay the loan contract, it will repay. Otherwise, the firm has to declare bankruptcy and it will get zero return. Formally, we can state the firms' maximization problems as below. R_G is the repayment of a G-type firm specified by the bank, R_B of a B-type firm. π_G^f is the return of a G-type firm, π_B^f of a B-type firm. By limited liability, firms' profits are non-negative. G-type firm's decision is to pay in full ($d_G = 1$), or not ($d_G = 0$). When $d_G = 0$, banks will take over the business and a G-type firm has zero profit.

For a G-type firm, given R_G,

$$\max_{d_G} \pi_G^f = d_G(G - R_G) + (1 - d_G)0 = d_G(G - R_G) \tag{32}$$

The optimal decision is $d_G = 1$ if $R_G \leq G$, $d_G = 0$ if $R_G > G$.

Similarly, for a B-type firm, given R_B,

$$\max_{d_B} \pi_B^f = d_B(B - R_B) + (1 - d_B)0 = d_B(B - R_B) \tag{33}$$

In stage 2, firms choose the best loan rates for them, given their types. This just creates zero profit condition for banks. The optimal decision is $d_B = 1$ if $R_B \leq B$, $d_B = 0$ if $R_B > B$.

In stage 1, bank's maximization problem is to choose loan rate R_G for G-type firms, R_B for

B-type firms, and μ the allocation ratio of funds to G-type firms.

$$\begin{aligned}
\max_{\mu, R_G, R_B} \quad & \pi^b = \mu\pi^b_G + (1-\mu)\pi^b_B \\
\text{s.t.} \quad & \pi^b_G = \min\{R_G, G\} - D \\
& \pi^b_B = \min\{R_B, B\} - D \\
& \pi^b = 0 \\
& D \geq R
\end{aligned} \tag{34}$$

Here, π^b is a bank's profit from a general loan contract. π^b_G is a bank's profit from a loan contract with a G-type firm, π^b_B with a B-type firm. D is the repayment to a depositor. There are three constraints: the firm's limited liability constraint, the bank's zero profit constraint, and the depositor's participation constraint.

Given a firm's optimal decision under the limited liability, bank's profit becomes $\pi^b_G = \min\{R_G, G\} - D$, $\pi^b_B = \min\{R_B, B\} - D$. In addition, the banking sector has perfect competition, which means that banks have zero profit and offer R as interest payment per unit of deposits, that is, $D = R$ and $\pi^b = 0$.

An equilibrium consists of the bank loan allocation μ, the loan contract (R_G, R_B), and the firms' repayment strategy (d_G, d_B), with which each agent behaves optimally, and both deposit market and credit market clear.[28]

Proposition 6. *Under perfect information, there is a unique screening separating equilibrium. In the equilibrium, the bank lending strategy is $\mu = 1$, that is, "lend to G only," the G-type firms' repayment $R_G = R$, and the B-type firms' strategy is indeterminant, i.e., R_B can be random values. G-type firms have $d_G = 1$, which means that G-type firms enter the market, apply for loans and produce. B-type firms are indifferent between $d_B = 0$ and 1, because banks will not offer loan contracts to B-type firms.*

Proof is omitted. Intuitively speaking, if a bank lends to all firms, B-type firms only repay up to B, then the bank needs to acquire a repayment higher than R from G-type firms (cross subsidization). On the other hand, if a bank lends to G-type firms only, the bank charges G-type firms R as loan repayment. Then, when firms choose the loan contracts, G-type firms prefer banks

[28] Loan allocation μ is the aggregate of pure strategy loan allocation to each firm. It could be viewed as mixed strategy, but we do not allow mixed strategy for each bank-firm pair.

that only lend to G-type firms at R. Note that the remaining funds from depositors, which amounts to $(1-f)m$, are invested abroad and receive return R per unit of fund.

B Proof of Lemma 1

First, we prove a bank's expected profit from a firm cannot be positive in equilibrium by contradiction. Suppose firm j chooses bank i, bank i obtains a positive profit from firm j in equilibrium, denoted as $E\left[\pi^b(x_{ij}|x_{-i})\right] = 2\epsilon > 0$, and firm j's expected profit as $E\left[\pi^f\left(x_{ij}\right)\right]$. Then, there exists a profitable deviation by another bank h which offers almost the same contract as bank j, but gives ϵ to firm j. Then, bank h's profit becomes $E[\pi^b(x_{hj}|x_{-i})] = \epsilon$ and firm j's profit becomes $E[\pi^f(x_{hj})] = E\left[\pi^f\left(x_{ij}\right)\right] + \epsilon$ by choosing bank h. Then, bank i loses the client firm j, and $E\left[\pi^b(x_{ij}|x_{-i})\right] = 0$. This contradicts the assumption.

Second, we prove a bank's expected profit from a firm cannot be negative in equilibrium by contradiction. Suppose bank i obtains a negative profit from firm j in equilibrium, $E\left[\pi^b(x_{ij}|x_{-i})\right] < 0$. By bank participation constraint (10), there must be some firm k which gives positive profit to bank i, $E\left[\pi^b(x_{ik}|x_{-i})\right] > 0$. However, bank profit cannot be positive from firm k as shown above, which is a contradiction. Hence, it must be the case that $E\left[\pi^b(x_{ik}|x_{-i})\right] = 0$.

C Calculation of Bank's Optimal Problem in Stage 1

Depending on lending strategies, a bank's expected profit $E\left[\pi^b(x_{ij}|x_{-i})\right]$ and a firm's expected profit $E\left[\pi^f\left(x_{ij}\right)\right]$ can be simplified differently. Specific expressions of the constrained profit maximization problem for bank i, (11) - (14), can be described in four cases, as below, similar to stage 2.

1 Stage 1 Case (1)

Suppose the "lend to $\hat{G}$ only" ($\hat{\mu} = 1$) is the equilibrium lending strategy in stage 1, before observing signals. Bank i offers the "lend to $\hat{G}$ only" strategy, then receives loan applicants, and screens the applicant firms. If bank i obtains $\hat{G}$ signal for applicant firm j, then bank i would provide the loan to firm j. With $\hat{B}$ signal, bank i would not provide the loan to firm j. The bank's optimal problem, with respect to a firm in stage 1, is as below.

$$\max_{x_{ij}\in\mathbf{S_1}} E\left[\pi^b(x_{ij}|x_{-i})\right] = Pr(\hat{G})\left[Pr(G|\hat{G},C)R_{\hat{G}} + Pr(B|\hat{G},C)B - R - \frac{C}{f}\right] \quad \text{[Bank profit]} \tag{35}$$

$s.t.$

$$E\left[\pi^f\left(x_{ij}\right)\right] = Pr(G)Pr(\hat{G}|G,C)(G - R_{\hat{G}}) \geq 0 \quad \text{[Firm PC]} \tag{36}$$

$$E\left[\pi^f\left(x_{ij}\right)\right] = Pr(G)Pr(\hat{G}|G,C)(G - R_{\hat{G}}) \geq E\left[\pi^f(\tilde{x}_{hj})\right], \text{ for } \forall h \neq i, \forall \tilde{x}_{hj} \in x_j \quad \text{[Firm IC]} \tag{37}$$

$$E\left[\pi^b(x_{ij}|x_{-i})\right] = Pr(\hat{G})\left[Pr(G|\hat{G},C)R_{\hat{G}} + Pr(B|\hat{G},C)B - R - \frac{C}{f}\right] = 0 \quad \text{[Bank ZPC]} \tag{38}$$

Here, the probability of getting signal $\hat{G}$, $Pr(\hat{G})$ is entered in equation (35) and (38) with signal accuracy $Pr(G|\hat{G},C)$, because the bank's expected profit maximizing problem is formulated in stage 1, before observing any signals. On the other hand, for firm's expected profit in (36) and (37), the probability of true type $Pr(G)$ enters with signal correctness $Pr(\hat{G}|G,C)$.

Regarding the screening cost, we assume in case (1) the overall screening cost mC is evenly charged to all $\hat{G}$ firms. All applicant firms are screened, and assigned signals $\hat{G}$ and $\hat{B}$, where the ratios of $\hat{G}$ and $\hat{B}$ firms in the market $Pr(\hat{G}) = f$ and $Pr(\hat{B}) = 1 - f$, which are consistent with the distribution of the true types. Only $\hat{G}$ firms are offered with contracts, and charged with the screening cost. So, in each contract to a $\hat{G}$ firm, screening cost $C/Pr(\hat{G}) = C/f$ is subtracted.

1.1 Subcases of Case (1) IC constraint

We further simplify the firm's incentive compatible (IC) constraint (37). The right-hand-side (RHS) represents any possible deviations that firm j takes an offer from bank h other than bank i. Instead of solving the RHS part, that is, $E\left[\pi^f(\tilde{x}_{hj})\right]$, we use an equivalent condition that describes the highest possible expected profit of firms by taking bank h's offer, as (39) below. Obviously bank h is deviating from the equilibrium strategy to lure firm j. Deviant bank h's offer can be chosen from the strategy set S but it should deliver strictly positive profits for individually rational bank h. We define such a restricted strategy set for deviant bank h as $\mathbf{X}_j = \{\tilde{x}_{hj} \in \mathbf{S} | E\left[\pi^b(\tilde{x}_{hj}|x_{-h})\right] > 0\}$.

Conditional on the lending strategies of deviant bank h, the sufficient condition for firm's IC constraint can be divided into four maximization problems. Here, the restricted strategy set for bank $h \neq i$, $\mathbf{X}_j$ is divided into $\mathbf{X}_{1j}$, $\mathbf{X}_{2j}$, $\mathbf{X}_{3j}$, and $\mathbf{X}_{4j}$, defined as
$\mathbf{X}_{1j} = \{\tilde{x}_{hj} \in \mathbf{S_1} | E\left[\pi^b(\tilde{x}_{hj}|x_{-h})\right] > 0\}$, $\mathbf{X}_{2j} = \{\tilde{x}_{hj} \in \mathbf{S_2} | E\left[\pi^b(\tilde{x}_{hj}|x_{-h})\right] > 0\}$,
$\mathbf{X}_{3j} = \{\tilde{x}_{hj} \in \mathbf{S_3} | E\left[\pi^b(\tilde{x}_{hj}|x_{-h})\right] > 0\}$, and $\mathbf{X}_{4j} = \{\tilde{x}_{hj} \in \mathbf{S_4} | E\left[\pi^b(\tilde{x}_{hj}|x_{-h})\right] > 0\}$. Recall that $\mathbf{S_1}, \mathbf{S_2}, \mathbf{S_3}$, and $\mathbf{S_4}$ represent the set of bank's loan contract offers under lending strategies "lend to $\hat{G}$ only," "lend to $\hat{B}$ only," "lend to both $\hat{G}$ and $\hat{B}$ with no cross subsidy," and "lend to both $\hat{G}$ and $\hat{B}$ with a cross subsidy." Using these, the right hand side of the IC constraint (37) can be expressed as below.

$$
\begin{aligned}
&\max\Bigg\{\max_{\tilde{x}_{hj}\in\mathbf{X}_{1j}} Pr(G)Pr(\hat{G}|G,\tilde{C})(G-\tilde{R}_{\hat{G}}),\\
&\max_{\tilde{x}_{hj}\in\mathbf{X}_{2j}} Pr(G)Pr(\hat{B}|G,\tilde{C})(G-\tilde{R}_{\hat{B}}),\\
&\max_{\tilde{x}_{hj}\in\mathbf{X}_{3j}} Pr(G)[Pr(\hat{G}|G,\tilde{C})(G-\tilde{R}_{\hat{G}})+Pr(\hat{B}|G,\tilde{C})(G-\tilde{R}_{\hat{B}})],\\
&\max_{\tilde{x}_{hj}\in\mathbf{X}_{4j}} Pr(G)[Pr(\hat{G}|G,\tilde{C})(G-\tilde{R}_{\hat{G}})+Pr(\hat{B}|G,\tilde{C})(G-\tilde{R}_{\hat{B}})]+Pr(B)[Pr(\hat{B}|B,\tilde{C})(B-\tilde{R}_{\hat{B}})].\Bigg\}
\end{aligned}
\tag{39}
$$

To further simplify (39), we calculate the four maximization problems of deviant bank h under the restricted strategy sets $\mathbf{X}_{1j}$, $\mathbf{X}_{2j}$, $\mathbf{X}_{3j}$, and $\mathbf{X}_{4j}$, and denote as RHS-X1, RHS-X2, RHS-X3, and RHS-X4.

RHS-X1: If deviant bank h chooses the "lend to $\hat{G}$ only" strategy, firm j's highest possible expected profit by choosing deviant bank h's offer can be assessed as (40) below.

$$\max_{\tilde{x}_{hj}\in\mathbf{X}_{1j}} E\left[\pi^f(\tilde{x}_{hj})\right] = Pr(G)Pr(\hat{G}|G,\tilde{C})(G-\tilde{R}_{\hat{G}}) \tag{40}$$

Because $\tilde{x}_{hj} \in \mathbf{X}_{1j}$, the loan contract of deviant bank h should give deviant bank h a strictly positive expected profit.

$$E\left[\pi^b(\tilde{x}_{hj}|x_{-h})\right] = Pr(\hat{G})\left[Pr(G|\hat{G},\tilde{C})\tilde{R}_{\hat{G}} + Pr(B|\hat{G},\tilde{C})B - R - \frac{\tilde{C}}{f}\right] = \epsilon > 0. \tag{41}$$

Here, using $Pr(\hat{G}|G,C) = 1-(1-f)e^{-\alpha C}$, and $Pr(G) = Pr(\hat{G}) = f$, (40) and (41) become

$$E\left[\pi^f(\tilde{x}_{hj})\right] = f\left[1-(1-f)e^{-\alpha\tilde{C}}\right](G-\tilde{R}_{\hat{G}}). \tag{42}$$

$$E\left[\pi^b(\tilde{x}_{hj}|x_{-h})\right] = f\left[\left[1-(1-f)e^{-\alpha\tilde{C}}\right]\tilde{R}_{\hat{G}} + (1-f)e^{-\alpha\tilde{C}}B - R - \frac{\tilde{C}}{f}\right] = \epsilon. \tag{43}$$

By substituting (43) to (42) to eliminate $\tilde{R}_{\hat{G}}$, we obtain

$$E\left[\pi^f(\tilde{x}_{hj})\right] = f\left[G-(1-f)e^{-\alpha\tilde{C}}(G-B) - R - \frac{\tilde{C}}{f} - \frac{\epsilon}{f}\right]. \tag{44}$$

Here, firm j's expected profit becomes a function of bank's screening cost $\tilde{C}$. Then, we can find the loan contract, specifically $\tilde{C}$, by deviant bank h that offers firm j the highest expected profit. To see it, we take the first derivative of the firm's profit (44) with respect to $\tilde{C}$,

$$\frac{\partial E\left[\pi^f(\tilde{x}_{hj})\right]}{\partial\tilde{C}} = f\left[\alpha(1-f)(G-B)e^{-\alpha\tilde{C}} - \frac{1}{f}\right]. \tag{45}$$

It is easy to calculate that the second order condition is always negative. Because $e^{-\alpha\tilde{C}}|_{C=0} = 1$ and decreases with C, there are two cases regarding the firms' profit.

First, when $\alpha f(1-f)(G-B) > 1$, we have $\frac{\partial E\left[\pi^f(\tilde{x}_{hj})\right]}{\partial\tilde{C}}|_{\tilde{C}=0} > 0$ and $\frac{\partial E\left[\pi^f(\tilde{x}_{hj})\right]}{\partial\tilde{C}}|_{\tilde{C}\to\infty} < 0$, that is, $E\left[\pi^f(\tilde{x}_{hj})\right]$ is an inverted-U shape function of $\tilde{C}$, hitting zero at some point. In this case, there is an internal solution $\tilde{C}^* > 0$, at which $\frac{\partial E\left[\pi^f(\tilde{x}_{hj})\right]}{\partial\tilde{C}} = 0$, that is,

$$e^{-\alpha\tilde{C}^*} = \frac{1}{\alpha f(1-f)(G-B)} \tag{46}$$

Second, when $\alpha f(1-f)(G-B) \leq 1$, it is always the case that $\frac{\partial E\left[\pi^f(\tilde{x}_{hj})\right]}{\partial\tilde{C}}|_{\tilde{C}=0} \leq 0$. In this case, $E\left[\pi^f(\tilde{x}_{hj})\right]$ is a monotonically decreasing function of $\tilde{C}$, so a corner solution $\tilde{C}^* = 0$ should be chosen, given the screening cost is non-negative.

The optimal screening cost is, thus, characterized as below.

$$\tilde{C}_1^* = \begin{cases} \frac{1}{\alpha}\ln[\alpha f(1-f)(G-B)], & \text{if} \quad \alpha f(1-f)(G-B) > 1, \\ 0, & \text{otherwise.} \end{cases} \tag{47}$$

Zero screening cost implies that the bank makes no screening efforts. Let $\tilde{x}_{hj}^{1*} \in \mathbf{X}_{1j}$ denote the bank h's loan contract that uses screening cost $\tilde{C}_1^*$. By substituting (46) and (47) into (44), firm j's highest expected profit from deviant bank h's offer is as below.

$$E\left[\pi^f(\tilde{x}_{hj}^{1*})\right] = \begin{cases} f\left[G - R - \dfrac{1}{\alpha f}\ln[\alpha f(1-f)(G-B)] - \dfrac{1}{\alpha f} - \dfrac{\epsilon}{f}\right], & \text{if} \quad \alpha f(1-f)(G-B) > 1, \\ f\left[fG + (1-f)B - R - \dfrac{\epsilon}{f}\right], & \text{otherwise.} \end{cases}$$

Since $E\left[\pi^f(\tilde{x}_{hj}^{1*})\right]$ is a decreasing function of ϵ in either case, firm j's expected profit will be higher if deviant bank h's expected profit ϵ is smaller. So, the highest expected profit for firm j would be realized if deviant bank h takes ϵ to the limit of zero.

$$\sup E\left[\pi^f(\tilde{x}_{hj}^{1*})\right] = \begin{cases} \lim\limits_{\epsilon\to 0} E\left[\pi^f(\tilde{x}_{hj}^{1*})\right] = f\left[G - R - \dfrac{1}{\alpha f}\ln[\alpha f(1-f)(G-B)] - \dfrac{1}{\alpha f}\right], & \text{if} \quad \alpha f(1-f)(G-B) > 1, \\ \lim\limits_{\epsilon\to 0} E\left[\pi^f(\tilde{x}_{hj}^{1*})\right] = f[fG + (1-f)B - R], & \text{otherwise.} \end{cases} \tag{48}$$

RHS-X2: If deviant bank h chooses the "lend to $\hat{B}$ only" strategy, firm j's highest possible expected profit by choosing deviant bank h's offer can be assessed as (49) below.

$$\max_{\tilde{x}_{hj}\in \mathbf{X}_{2j}} E\left[\pi^f(\tilde{x}_{hj})\right] = Pr(G)Pr(\hat{B}|G,\tilde{C})(G - \tilde{R}_{\hat{B}}) \tag{49}$$

Because $\tilde{x}_{hj} \in \mathbf{X}_{2j}$, the loan contract of deviant bank h should give deviant bank h a strictly positive expected profit.

$$E\left[\pi^b(\tilde{x}_{hj}|x_{-h})\right] = Pr(\hat{B})\left[Pr(G|\hat{B},\tilde{C})\tilde{R}_{\hat{B}} + Pr(B|\hat{B},\tilde{C})B - R - \frac{\tilde{C}}{1-f}\right] = \epsilon > 0. \tag{50}$$

Here, using $Pr(\hat{B}|G,\tilde{C}) = (1-f)e^{-\alpha\tilde{C}}$, (49) becomes

$$E\left[\pi^f(\tilde{x}_{hj})\right] = f(1-f)e^{-\alpha\tilde{C}}(G - \tilde{R}_{\hat{B}}). \tag{51}$$

Using $Pr(G|\hat{B},\tilde{C}) = fe^{-\alpha\tilde{C}}$, (50) becomes

$$E\left[\pi^b(\tilde{x}_{hj}|x_{-h})\right] = (1-f)\left[fe^{-\alpha\tilde{C}}\tilde{R}_{\hat{B}} + \left(1 - fe^{-\alpha\tilde{C}}\right)B - R - \frac{\tilde{C}}{1-f}\right] = \epsilon. \tag{52}$$

By substituting (52) to (51) to eliminate $\tilde{R}_{\hat{B}}$, we obtain

$$E\left[\pi^f(\tilde{x}_{hj})\right] = f(1-f)e^{-\alpha\tilde{C}}G + (1-f)\left[\left(1 - fe^{-\alpha\tilde{C}}\right)B - R - \frac{\tilde{C}}{1-f} - \frac{\epsilon}{1-f}\right]. \qquad (53)$$

Then, we can find the loan contract, the screening cost $\tilde{C}$, by deviant bank h that offers firm j the highest expected profit. The first derivative is calculated as in (54), which is always negative. Hence, $E\left[\pi^f(\tilde{x}_{hj})\right]$ is a decreasing function of $\tilde{C}$, so the corner solution $\tilde{C}_2^* = 0$ should be chosen.

$$\frac{\partial E\left[\pi^f(\tilde{x}_{hj})\right]}{\partial \tilde{C}} = -\alpha f(1-f)(G-B)e^{-\alpha\tilde{C}} - 1 < 0 \qquad (54)$$

$$\tilde{C}_2^* = 0. \qquad (55)$$

Let $\tilde{x}_{hj}^{2*} \in \mathbf{X}_{2j}$ denote the bank h's loan contract which uses screening cost $\tilde{C}_2^* = 0$. By substituting (55) into (53), firm j's highest expected profit from deviant bank h's offer is as below.

$$E\left[\pi^f\left(\tilde{x}_{hj}^{2*}\right)\right] = (1-f)\left[fG + (1-f)B - R - \frac{\epsilon}{1-f}\right].$$

Since $E\left[\pi^f(\tilde{x}_{hj}^{2*})\right]$ is a decreasing function of ϵ, firm j's expected profit will be higher if deviant bank h's expected profit ϵ is smaller. So, the highest expected profit for firm j would be realized if deviant bank h takes ϵ to the limit of zero.

$$\sup E\left[\pi^f\left(\tilde{x}_{hj}^{2*}\right)\right] = \lim_{\epsilon\to 0} E\left[\pi^f(\tilde{x}_{hj}^{2*})\right] = (1-f)[fG + (1-f)B - R].$$

RHS-X3: If deviant bank h chooses the "lend to both with no cross subsidy" strategy, firm j's highest possible expected profit by choosing deviant bank h's offer can be assessed by (56) below.

$$\max_{\tilde{x}_{hj}\in\mathbf{X}_{3j}} E\left[\pi^f(\tilde{x}_{hj})\right] = Pr(G)\left[Pr(\hat{G}|G,\tilde{C})(G - \tilde{R}_{\hat{G}}) + Pr(\hat{B}|G,\tilde{C})(G - \tilde{R}_{\hat{B}})\right] \qquad (56)$$

Because $\tilde{x}_{hj} \in \mathbf{X}_{3j}$, the loan contract of deviant bank h should give deviant bank h a strictly positive expected profit.

$$E\left[\pi^b(\tilde{x}_{hj}|x_{-h})\right] = Pr(\hat{G})\left[Pr(G|\hat{G},\tilde{C})\tilde{R}_{\hat{G}} + Pr(B|\hat{G},\tilde{C})B\right] +$$
$$Pr(\hat{B})\left[Pr(G|\hat{B},\tilde{C})\tilde{R}_{\hat{B}} + Pr(B|\hat{B},\tilde{C})B\right] - R - \tilde{C} = \epsilon > 0. \qquad (57)$$

Here, using $Pr(\hat{G}|G,C) = Pr(G|\hat{G},C) = 1-(1-f)e^{-\alpha C}$ and
$Pr(\hat{B}|B,C) = Pr(B|\hat{B},C) = 1 - fe^{-\alpha C}$, (56) and (57) become as below.

$$E\left[\pi^f(\tilde{x}_{hj})\right] = f\left[\left(1-(1-f)e^{-\alpha\tilde{C}}\right)(G-\tilde{R}_{\hat{G}}) + (1-f)e^{-\alpha\tilde{C}}(G-\tilde{R}_{\hat{B}})\right]. \tag{58}$$

$$E\left[\pi^b(\tilde{x}_{hj}|x_{-h})\right] = f\left[\left(1-(1-f)e^{-\alpha\tilde{C}}\right)\tilde{R}_{\hat{G}} + (1-f)e^{-\alpha\tilde{C}}B\right] +$$
$$(1-f)\left[fe^{-\alpha\tilde{C}}\tilde{R}_{\hat{B}} + \left(1-fe^{-\alpha\tilde{C}}\right)B\right] - R - \tilde{C} = \epsilon. \tag{59}$$

By substituting (59) to (58) to eliminate $\tilde{R}_{\hat{G}}$ and $\tilde{R}_{\hat{B}}$, we obtain

$$\begin{aligned} E\left[\pi^f(\tilde{x}_{hj})\right] &= f\left[G - \left(1-(1-f)e^{-\alpha\tilde{C}}\right)\tilde{R}_{\hat{G}} - (1-f)e^{-\alpha\tilde{C}}\tilde{R}_{\hat{B}}\right] \\ &= fG + f(1-f)e^{-\alpha\tilde{C}}B + (1-f)\left(1-fe^{-\alpha\tilde{C}}\right)B - R - \tilde{C} - \epsilon \\ &= fG + (1-f)B - R - \tilde{C} - \epsilon. \end{aligned} \tag{60}$$

Then, we can find the loan contract, specifically the screening cost $\tilde{C}$, by deviant bank h that offers firm j the highest expected profit. The first derivative is calculated as in (61), which is always negative. $E\left[\pi^f(\tilde{x}_{hj})\right]$ is a decreasing function of $\tilde{C}$, so the corner solution $\tilde{C}_3^* = 0$ should be chosen.

$$\frac{\partial E\left[\pi^f(\tilde{x}_{hj})\right]}{\partial\tilde{C}} = -1 < 0. \tag{61}$$

$$\tilde{C}_3^* = 0. \tag{62}$$

Let $\tilde{x}_{hj}^{3*} \in \mathbf{X}_{3j}$ denote the bank h's loan contract which uses screening cost $\tilde{C}_3^* = 0$. By substituting (62) into (60), firm j's highest expected profit from deviant bank h's offer is as below.

$$E\left[\pi^f\left(\tilde{x}_{hj}^{3*}\right)\right] = fG + (1-f)B - R - \epsilon.$$

Since $E\left[\pi^f(\tilde{x}_{hj}^{3*})\right]$ is a decreasing function of ϵ, firm j's expected profit will be higher if deviant bank h's expected profit ϵ is smaller. So, the highest expected profit for firm j would be realized if deviant bank h takes ϵ to the limit of zero.

$$\sup E\left[\pi^f\left(\tilde{x}_{hj}^{3*}\right)\right] = \lim_{\epsilon\to 0} E\left[\pi^f(\tilde{x}_{hj}^{3*})\right] = fG + (1-f)B - R.$$

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RHS-X4: If deviant bank h chooses the "lend to both with a cross subsidy" strategy, firm j's highest possible expected profit by choosing deviant bank h's offer can be assessed by (63) below.

$$\max_{\tilde{x}_{hj}\in\mathbf{X}_{4j}} E\left[\pi^f(\tilde{x}_{hj})\right] = Pr(G)\left[Pr(\hat{G}|G,\tilde{C})(G-\tilde{R}_{\hat{G}}) + Pr(\hat{B}|G,\tilde{C})(G-\tilde{R}_{\hat{B}})\right] + Pr(B)\left[Pr(\hat{B}|B,\tilde{C})(B-\tilde{R}_{\hat{B}})\right] \tag{63}$$

Note that unlike (56) in case (IC-3), cross-subsidy implies $R_B < B$, so that a firm can repay in full R_B even with the bad realization of the firm's type. Because $\tilde{x}_{hj}\in\mathbf{X}_{4j}$, the loan contract of deviant bank h should give deviant bank h a strictly positive expected profit.

$$E\left[\pi^b(\tilde{x}_{hj}|x_{-h})\right] = Pr(\hat{G})[Pr(G|\hat{G},\tilde{C})\tilde{R}_{\hat{G}} + Pr(B|\hat{G},\tilde{C})B] + \Pr(\hat{B})\tilde{R}_{\hat{B}} - R - \tilde{C} = \epsilon > 0. \tag{64}$$

Here, using $Pr(\hat{G}|G,C) = Pr(G|\hat{G},C) = 1-(1-f)e^{-\alpha C}$ and
$Pr(\hat{B}|B,C) = Pr(B|\hat{B},C) = 1-fe^{-\alpha C}$, (63) and (64) become as below.

$$E\left[\pi^f(\tilde{x}_{hj})\right] = f\left[\left(1-(1-f)e^{-\alpha C}\right)(G-\tilde{R}_{\hat{G}}) + (1-f)e^{-\alpha C}(G-\tilde{R}_{\hat{B}})\right] + (1-f)\left[\left(1-fe^{-\alpha C}\right)(B-\tilde{R}_{\hat{B}})\right]. \tag{65}$$

$$E\left[\pi^b(\tilde{x}_{hj}|x_{-h})\right] = f\left[\left(1-(1-f)e^{-\alpha C}\right)\tilde{R}_{\hat{G}} + (1-f)e^{-\alpha C}B\right] + (1-f)\tilde{R}_{\hat{B}} - R - \tilde{C} = \epsilon. \tag{66}$$

By substituting (66) to (65), to eliminate $\tilde{R}_{\hat{G}}$ and $\tilde{R}_{\hat{B}}$, we obtain

$$\begin{aligned} E\left[\pi^f(\tilde{x}_{hj})\right] =& fG + (1-f)\left(1-fe^{-\alpha C}\right)B - f\left[\left(1-(1-f)e^{-\alpha C}\right)\tilde{R}_{\hat{G}} + (1-f)e^{-\alpha C}\tilde{R}_{\hat{B}}\right] \\ & - (1-f)\left(1-fe^{-\alpha C}\right)\tilde{R}_{\hat{B}} \\ =& fG + (1-f)\left(1-fe^{-\alpha C}\right)B - f\left(1-(1-f)e^{-\alpha C}\right)\tilde{R}_{\hat{G}} - (1-f)\tilde{R}_{\hat{B}} \\ =& fG + (1-f)B - R - \tilde{C} - \epsilon. \end{aligned} \tag{67}$$

Then, we can find the loan contract, specifically the screening cost $\tilde{C}$, by deviant bank h that offers firm j the highest expected profit. The first derivative is calculated as in (68), which is always negative. $E\left[\pi^f(\tilde{x}_{hj})\right]$ is a decreasing function of $\tilde{C}$, so the corner solution $\tilde{C}_4^* = 0$ should be chosen.

$$\frac{\partial E\left[\pi^f(\tilde{x}_{hj})\right]}{\partial \tilde{C}} = -1 < 0. \tag{68}$$

$$\tilde{C}_4^* = 0. \tag{69}$$

Let $\tilde{x}_{hj}^{4*} \in \mathbf{X}_{4j}$ denote the bank h's loan contract which uses screening cost $\tilde{C}_4^* = 0$. By substituting (69) into (67), firm j's highest expected profit from deviant bank h's offer is as below.

$$E\left[\pi^f\left(\tilde{x}_{hj}^{4*}\right)\right] = fG + (1-f)B - R - \epsilon.$$

Since $E\left[\pi^f(\tilde{x}_{hj}^{4*})\right]$ is a decreasing function of ϵ, firm j's expected profit will be higher if deviant bank h's expected profit ϵ is smaller. So, the highest expected profit for firm j would be realized if deviant bank h takes ϵ to the limit of zero.

$$\sup E\left[\pi^f\left(\tilde{x}_{hj}^{4*}\right)\right] = \lim_{\epsilon \to 0} E\left[\pi^f(\tilde{x}_{hj}^{4*})\right] = fG + (1-f)B - R.$$

Next, by summarizing RHS-X1, RHS-X2, RHS-X3, and RHS-X4, the right hand side of the incentive compatible constraint (39) can be expressed as below.

$$\begin{aligned} \max \Bigg\{ & \Bigg\{ f\left[G - R - \frac{1}{\alpha f}\ln[\alpha f(1-f)(G-B)] - \frac{1}{\alpha f}\right]\Big|_{\alpha f(1-f)(G-B)>1}, \\ & f[fG + (1-f)B - R]|_{\alpha f(1-f)(G-B)\le 1} \Bigg\}, \\ & (1-f)[fG + (1-f)B - R], \\ & fG + (1-f)B - R. \Bigg\} \end{aligned} \tag{70}$$

Here, the first two lines represent RHS-X1, the third line represents RHS-X2. The fourth line represents RHS-X3 and RHS-X4. The third line is smaller than the fourth line, since the third line is equal to the fourth line multiplied by $1 - f < 1$. Therefore, we will not consider RHS-X2. Then, we can simplify (70) as below.

$$\max\left\{\left\{f\left[G-R-\frac{1}{\alpha f}\ln[\alpha f(1-f)(G-B)]-\frac{1}{\alpha f}\right]|_{\alpha f(1-f)(G-B)>1},\right.\right.$$
$$\left.f[fG+(1-f)B-R]|_{\alpha f(1-f)(G-B)\leq 1}\right\}, \tag{71}$$
$$\left.fG+(1-f)B-R.\right\}$$

When $\alpha f(1-f)(G-B)>1$, define Δ as the first line RHS-X1 minus the third line RHS-X3 (or RHS-X4) in (71), as below.

$$\Delta \equiv f\left[G-R-\frac{1}{\alpha f}\ln[\alpha f(1-f)(G-B)]-\frac{1}{\alpha f}\right]-[fG+(1-f)B-R]. \tag{72}$$

Here, if $\Delta>0$, then the "lend to $\hat{G}$ only" strategy should be chosen by a deviant bank over the "lend to both" strategy. Firm j's IC constraint (71), originally (37), in which "lend to $\hat{G}$ only" is the equilibrium strategy, can be further classified into four subcases as below.

Subcase (1-a): IC constraint: When $\alpha f(1-f)(G-B)>1$, and $\Delta>0$, the "lend to $\hat{G}$ only" contract provides firm j the highest expected profit compared to contracts under other lending strategies. Here, while bank i chooses the "lend to $\hat{G}$ only" strategy, deviant bank h may also choose the "lend to $\hat{G}$ only" strategy, and the firm j's IC constraint (37) becomes

$$Pr(G)Pr(\hat{G}|G,C)(G-R_{\hat{G}})\geq f\left[G-R-\frac{1}{\alpha f}\ln[\alpha f(1-f)(G-B)]-\frac{1}{\alpha f}\right]. \tag{73a}$$

which is

$$ff_G(C)(G-R_{\hat{G}})\geq f\left[G-R-\frac{1}{\alpha f}\ln[\alpha f(1-f)(G-B)]-\frac{1}{\alpha f}\right]. \tag{73b}$$

Subcase (1-b): IC constraint: When $\alpha f(1-f)(G-B)>1$, and $\Delta<0$, the "lend to both" contract provides firm j the highest expected profit compared to contracts under other lending strategies. "Lend to both" strategies include case (IC-3) "lend to both with no cross subsidy" and case (IC-4) "lend to both with a cross subsidy." Here, while the bank i chooses the "lend to $\hat{G}$ only" strategy, the deviant bank h may choose the "lend to both" strategy, and the firm j's IC constraint (37) becomes

$$Pr(G)Pr(\hat{G}|G,C)(G - R_{\hat{G}}) \geq fG + (1-f)B - R. \tag{74}$$

Subcase (1-c): IC constraint: When $\alpha f(1-f)(G-B) > 1$, and $\Delta = 0$, the "lend to $\hat{G}$ only" contract and the "lend to both" contract can provide the same highest firm's expected profit. Under this case, there could appear a mixed strategy equilibrium.[29] We do not go deep into this case, and we assume all banks choose the same pure strategy. Without loss of generality, case (1-c) is combined with case (1-a). Hereinafter, we do not consider this case separately.

Subcase (1-d): IC constraint: When $\alpha(1-f)(G-B) \leq 1$, the "lend to both" contract provides firm j the highest expected profit compared to contracts under other lending strategies, because in (71) the second line is f multiplying the third line, and f is smaller than 1. Here, while the bank i chooses the "lend to $\hat{G}$ only" strategy, deviant bank h may choose the "lend to both" strategy, and the firm j's IC constraint (37) becomes

$$Pr(G)Pr(\hat{G}|G,C)(G - R_{\hat{G}}) \geq fG + (1-f)B - R. \tag{75}$$

Next, we incorporate subcases of IC constraint, (1-a), (1-b), and (1-d) into the bank's optimal problem.

[29]This mixed strategy equilibrium can be achieved by asymmetric pure strategies or by lottery by each bank.

1.2 Subcases of Case (1) Bank's Problem

1.2.1 Subcase (1-a) Bank's Problem

In subcase (1-a), $\alpha f(1-f)(G-B) > 1$ and $\Delta \geq 0$, the deviant bank h adopts the "lend to $\hat{G}$ only" strategy. Firm's IC constraint becomes (73b). Bank i's optimal problem is expressed as below.

$$\max_{x_{ij}\in \mathbf{S_1}} f\left[f_G(C)R_{\hat{G}} + (1-f_G(C))B - R - \frac{C}{f}\right] \qquad \text{[Bank profit]}$$

$$s.t.$$

$$f_G(C)(G-R_{\hat{G}}) \geq 0 \qquad \text{[Firm IR]}$$

$$f_G(C)(G-R_{\hat{G}}) \geq G - R - \frac{1}{\alpha f}\ln[\alpha f(1-f)(G-B)] - \frac{1}{\alpha f} \qquad \text{[Firm IC]}$$

$$f\left[f_G(C)R_{\hat{G}} + (1-f_G(C))B - R - \frac{C}{f}\right] = 0 \qquad \text{[Bank ZPC]}$$

Note that if the RHS of the firm's IC constraint is less than zero, only the firm's IR constraint binds, and vice versa. Here, we analyze the case with the firm's IC constraint, that is, the RHS of the firm's IC constraint is positive without firm's IR constraint.

By banks' zero-profit constraint,

$$R_{\hat{G}} = \frac{R + C/f - (1-f_G(C))B}{f_G(C)} \equiv k(C).$$

We substitute $R_{\hat{G}}$ to the firm's IC constraint as below.

$$f_G(C)(G-k(C)) \geq G - R - \frac{1}{\alpha f}ln(\alpha f(1-f)(G-B)) - \frac{1}{\alpha f}.$$

Then, we form the Lagrangian as below.

$$\begin{aligned}\mathcal{L}(C) =& f\left[f_G(C)k(C) + (1-f_G(C))B - R - \frac{C}{f}\right] + \lambda\Big\{f_G(C)(G-k(C)) - \Big[G - R - \\ & \frac{1}{\alpha f}ln(\alpha f(1-f)(G-B)) - \frac{1}{\alpha f}\Big]\Big\}\end{aligned} \tag{76}$$

A bank maximizes this Lagrangian by choosing $C \geq 0$. The first order condition is written as below.

$$0 = \frac{\partial \mathcal{L}}{\partial C} = f\left[f'_G(C)k(C) + f_G(C)k'(C) - f'_G(C)B - \frac{1}{f}\right] + \lambda[f[f'_G(C)(G-k(C)) - f_G(C)k'(C)]]. \tag{77}$$

In the Lagrangian (76), the firm's IC constraint should be binding, and $\lambda > 0$. Recall that in stage 2, firm j's maximization problem (1) is derived from the first order condition with respect to C to maximize firm j's expected profit by (deviant) bank as (45), that is,

$$\frac{\partial E\left[\pi^f(x_{ij})\right]}{\partial C} = f[f'_G(C)(G - k(C)) - f_G(C)k'(C)] = 0. \tag{78}$$

By (78), we have $f'_G(C)G = f'_G(C)k(C) + f_G(C)k'(C)$. By substituting (78) to (77), the first order condition becomes

$$0 = \frac{\partial \mathcal{L}}{\partial C} = f\left[f'_G(C)k(C) + f_G(C)k'(C) - f'_G(C)B - \frac{1}{f}\right].$$
$$0 = f\left[f'_G(C)(G - B) - \frac{1}{f}\right]. \tag{79}$$

Here, $f'_G(C) = \alpha(1-f)e^{-\alpha C} > 0$, $f'_G(C)|_{C=0} = \alpha(1-f)$, and the second derivative $f''_G(C) < 0$ is negative. We can calculate $\frac{\partial \mathcal{L}}{\partial C}|_{C=0} = \alpha(1-f)(G-B) - \frac{1}{f} > 0$, $\frac{\partial^2 \mathcal{L}}{\partial C^2} < 0$. Hence, the Lagrangian (76) is an increasing and concave function of C, and there is an internal solution of C^*.

In summary, when $\alpha f(1-f)(G-B) > 1$, $\Delta \geq 0$, and $G - R - \frac{1}{\alpha f}\ln[\alpha f(1-f)(G-B)] - \frac{1}{\alpha f} \geq 0$,[30] the "lend to $\hat{G}$ only" strategy by banks is an equilibrium strategy. The equilibrium loan contract uses the "lend to $\hat{G}$ only" lending strategy, and the screening strategy and the required repayment is characterized in the loan contract x^*_{ij} , where

$$C^* = \frac{1}{\alpha}\ln[\alpha f(1-f)(G-B)].$$
$$R^*_{\hat{G}} = \frac{R + \frac{C^*}{f} - \frac{B}{\alpha f(G-B)}}{1 - \frac{1}{\alpha f(G-B)}}.$$

Both are derived from (79). $R^*_{\hat{B}}$ can be arbitrary because banks do not lend to $\hat{B}$ firms.

Lemma 2. *When $\alpha f(1-f)(G-B) > 1$, $\Delta \geq 0$, and $G - R - \frac{1}{\alpha f}\ln[\alpha f(1-f)(G-B)] - \frac{1}{\alpha f} \geq 0$, the "lend to $\hat{G}$ only" strategy by banks is an equilibrium strategy.*

[30]This condition is from firm's PC constraint. $E\left[\pi^f\left(x_{ij}\right)\right]$ represents firm's expected profit before screening in stage 2, which is non-negative by firm's participation constraint. Otherwise, firms will not enter the market in the first place. $E\left[\pi^f\left(x^*_{ij}\right)\right] = Pr(G)Pr(\hat{G}|G,C)(G - R_{\hat{G}}) = f\left[G - R - \frac{1}{\alpha f}\ln[\alpha f(1-f)(G-B)] - \frac{1}{\alpha f}\right] \geq 0.$

1.2.2 Subcases (1-b) and (1-d) Bank's Problem

In subcase (1-b), $\alpha f(1-f)(G-B) > 1$, and $\Delta < 0$, bank i offers the "lend to $\hat{G}$ only" strategy in equilibrium. Deviant bank h offers the "lend to both" strategy to give firms the highest expected profit in a deviation among all possible deviation offers. In this case, we have the IC constraint (74).

However, under the "lend to $\hat{G}$ only" strategy, firms at most receive the following expected profit, derived as the same way as for the first line of (71), which is

$$f\left[G - R - \frac{1}{\alpha f}\ln[\alpha f(1-f)(G-B)] - \frac{1}{\alpha f}\right].$$

Under the "lend to both" strategy, firms receive the third line of (71). By our current assumption here $\Delta < 0$,

$$f\left[G - R - \frac{1}{\alpha f}\ln[\alpha f(1-f)(G-B)] - \frac{1}{\alpha f}\right] < fG + (1-f)B - R.$$

Therefore, in this parametric values case, bank i does not take the "lend to $\hat{G}$ only" strategy in an equilibrium. In other words, this case cannot be taken in an equilibrium.

In subcase (1-d), $\alpha f(1-f)(G-B) \leq 1$, bank i offers the "lend to $\hat{G}$ only" strategy in equilibrium. Deviant bank h offers the "lend to both" strategy to give firms the highest expected profit in a deviation among all possible deviation offers. In this case, we have the IC constraint (75).

However, under the "lend to $\hat{G}$ only" strategy, firms at most receive the following expected profit, derived as the same way as for the second line of (71), which is

$$f[fG + (1-f)B - R].$$

Under the "lend to both" strategy, firms receive the third line of (71). We have

$$f[fG + (1-f)B - R] < fG + (1-f)B - R.$$

Therefore, in this parametric values case, bank i does not take the "lend to $\hat{G}$ only" strategy in an equilibrium. In other words, this case cannot be taken in an equilibrium.

In summary, for stage 1 case (1), we have following results.

Lemma 3. *When $\alpha f(1-f)(G-B) > 1$ and $\Delta < 0$, or when $\alpha f(1-f)(G-B) \leq 1$, the "lend to $\hat{G}$ only" strategy cannot be an equilibrium strategy.*

To summarize stage 1 case (1), the "lend to $\hat{G}$ only" strategy can be an equilibrium strategy, when $\alpha f(1-f)(G-B) > 1$, $\Delta \geq 0$, and $G - R - \frac{1}{\alpha f}\ln[\alpha f(1-f)(G-B)] - \frac{1}{\alpha f} \geq 0$.

2 Stage 1 Case (2)

Suppose the "lend to $\hat{B}$ only" ($\hat{\mu} = 0$) is the equilibrium lending strategy in stage 1 before observing signals. Bank i offers the "lend to $\hat{B}$ only" strategy, then receive loan applicants, and screens the applicant firms. If bank i obtains $\hat{B}$ signal for firm j, then bank i would provide the loan to firm j. With the $\hat{G}$ signal, bank i would not provide the loan to firm j.

The bank optimal problem per firm in stage 1 is defined as below.

$$\max_{x_{ij}\in \mathbf{S_2}} E\left[\pi^b(x_{ij}|x_{-i})\right] = Pr(\hat{B})\left[Pr(G|\hat{B},C)R_{\hat{B}} + Pr(B|\hat{B},C)B - R - \frac{C}{1-f}\right] \quad \text{[Bank profit]} \tag{80}$$

$$s.t. \tag{81}$$

$$E\left[\pi^f\left(x_{ij}\right)\right] = Pr(G)Pr(\hat{B}|G,C)(G - R_{\hat{B}}) \geq 0 \quad \text{[Firm PC]} \tag{82}$$

$$E\left[\pi^f\left(x_{ij}\right)\right] = Pr(G)Pr(\hat{B}|G,C)(G - R_{\hat{B}}) \geq E\left[\pi^f(\tilde{x}_{hj})\right], \text{ for } \forall h \neq i, \forall \tilde{x}_{hj} \in x_j \quad \text{[Firm IC]} \tag{83}$$

$$E\left[\pi^b(x_{ij}|x_{-i})\right] = Pr(\hat{B})\left[Pr(G|\hat{B},C)R_{\hat{B}} + Pr(B|\hat{B},C)B - R - \frac{C}{1-f}\right] = 0 \quad \text{[Bank ZPC]} \tag{84}$$

Now, we prove that this case cannot be taken in an equilibrium.

Under the "lend to $\hat{B}$ only" strategy, firms can receive at most the following expected profit, derived as the same way as for the third line of (70), which is

$$f[fG + (1-f)B - R].$$

Suppose deviant bank h offers the "lend to both" strategy, then firms can receive at most the fourth line of (70). We have

$$f[fG + (1-f)B - R] < fG + (1-f)B - R.$$

Therefore, in this parametric values case, bank i does not take the "lend to $\hat{B}$ only" strategy in an equilibrium. In other words, this case cannot be taken in an equilibrium. We have the following lemma.

In summary, for stage 1 case (2), we have following results.

Lemma 4. *The "lend to $\hat{B}$ only" strategy cannot be an equilibrium strategy, under any parameter values.*

3 Stage 1 Case (3)

Suppose the "lend to both with no cross subsidy" ($\hat{\mu} \in (0,1)$, $R_{\hat{B}} > B$), is the equilibrium lending strategy in stage 1 before observing signals. Bank i offers the lending strategy "lend to both with no cross subsidy", then receives loan applicants, and screens the applicant firms. If bank i obtains $\hat{G}$ signal for firm j, then bank i would provide the loan rate $R_{\hat{G}}$ to firm j. With $\hat{B}$ signal, bank i would provide the loan rate $R_{\hat{B}}$ to firm j.

The bank optimal problem per firm is defined as below.

$$\max_{x_{ij}\in\mathbf{S_3}} E\left[\pi^b(x_{ij}|x_{-i})\right] = Pr(\hat{G})\left[Pr(G|\hat{G},C)R_{\hat{G}} + Pr(B|\hat{G},C)B\right] + Pr(\hat{B})\left[Pr(G|\hat{B},C)R_{\hat{B}} + Pr(B|\hat{B},C)B\right] - R - C \quad \text{[Bank profit]} \tag{85}$$

$s.t.$

$$E\left[\pi^f(x_{ij})\right] = Pr(G)\left[Pr(\hat{G}|G,C)(G-R_{\hat{G}}) + Pr(\hat{B}|G,C)(G-R_{\hat{B}})\right] \geq 0 \quad \text{[Firm PC]} \tag{86}$$

$$E\left[\pi^f(x_{ij})\right] = Pr(G)\left[Pr(\hat{G}|G,C)(G-R_{\hat{G}}) + Pr(\hat{B}|G,C)(G-R_{\hat{B}})\right] \geq E\left[\pi^f(\tilde{x}_{hj})\right], \text{ for } \forall h \neq i, \forall \tilde{x}_{hj} \in x_j \quad \text{[Firm IC]} \tag{87}$$

$$E\left[\pi^b(x_{ij}|x_{-i})\right] = Pr(\hat{G})\left[Pr(G|\hat{G},C)R_{\hat{G}} + Pr(B|\hat{G},C)B\right] + Pr(\hat{B})\left[Pr(G|\hat{B},C)R_{\hat{B}} + Pr(B|\hat{B},C)B\right] - R - C = 0 \quad \text{[Bank ZPC]} \tag{88}$$

3.1 Subcases of Case (3) IC constraint

By the same argument to derive (71), firm j's IC constraint (87) can be expressed as below:

$$\begin{aligned} &Pr(G)\left[Pr(\hat{G}|G,C)(G-R_{\hat{G}}) + Pr(\hat{B}|G,C)(G-R_{\hat{B}})\right] \\ &\geq \max\Bigg\{\Bigg\{ f\left[G - R - \frac{1}{\alpha f}\ln[\alpha f(1-f)(G-B)] - \frac{1}{\alpha f}\right]|_{\alpha f(1-f)(G-B)>1}, \\ &f[fG+(1-f)B-R]|_{\alpha f(1-f)(G-B)\leq 1}\Bigg\}, \\ &fG+(1-f)B-R.\Bigg\}. \end{aligned} \tag{89}$$

Then, firm j's IC constraint (89) can be simplified into four cases as below.

Subcase (3-a): IC constraint: When $\alpha f(1-f)(G-B) > 1$, and $\Delta > 0$, the "lend to $\hat{G}$ only" contract provides firm j the highest expected profit, compared to contracts under other lending strategies. Hence, while bank i chooses the "lend to both with no cross subsidy" strategy, deviant bank h may choose the "lend to $\hat{G}$ only" strategy, and firm j's IC constraint (89) becomes

$$Pr(G)\left[Pr(\hat{G}|G,C)(G-R_{\hat{G}})+Pr(\hat{B}|G,C)(G-R_{\hat{B}})\right]\geq f\left[G-R-\frac{1}{\alpha}ln[\alpha(1-f)(G-B)]-\frac{1}{\alpha}\right]. \tag{90}$$

Subcase (3-b): IC constraint: When $\alpha f(1-f)(G-B)>1$, and $\Delta<0$, the "lend to both" contract provides firm j the highest expected profit, compared to contracts under other lending strategies. Firm j's IC constraint (89) becomes

$$Pr(G)\left[Pr(\hat{G}|G,C)(G-R_{\hat{G}})+Pr(\hat{B}|G,C)(G-R_{\hat{B}})\right]\geq fG+(1-f)B-R. \tag{91}$$

Subcase (3-c): IC constraint: When $\alpha f(1-f)(G-B)>1$, and $\Delta=0$, the "lend to $\hat{G}$ only" contract and the "lend to both" contract can provide the same highest firm's expected profit. Under this case, there could exist a mixed strategy equilibrium. Since we focus on pure strategy equilibrium, we do not go deep into this case, and we assume all banks choose the same pure strategy. Without loss of generality, case (3-c) is combined with case (3-b). Hereinafter, we do not construct this case separately.

Subcase (3-d): IC constraint: When $\alpha(1-f)(G-B)\leq 1$, the "lend to both" contract provides firm j the highest expected profit compared to contracts under other lending strategies. Firm j's IC constraint (89) becomes

$$Pr(G)\left[Pr(\hat{G}|G,C)(G-R_{\hat{G}})+Pr(\hat{B}|G,C)(G-R_{\hat{B}})\right]\geq fG+(1-f)B-R. \tag{92}$$

This is the same constraint as in case (3-b), (91). Next, we incorporate subcases of IC constraint, (3-a), (3-b), and (3-d) into bank's optimal problem.

3.2 Subcases of Case (3) Bank's Problem

3.2.1 Subcase (3-a) Bank's Problem

In case (3-a), $\alpha f(1-f)(G-B) > 1$ and $\Delta > 0$, bank i offers the "lend to both with no cross subsidy" strategy in equilibrium. Deviant bank h offers the "lend to $\hat{G}$ only" strategy to give firms the highest expected profit in a deviation among all possible deviation offers. In this case, we have the IC constraint (90).

However, under the "lend to both with no cross subsidy" strategy, firms at most receive the following expected profit, derived as the same way as for the third line of (71), which is

$$fG + (1-f)B - R.$$

Under the "lend to $\hat{G}$ only" strategy, firms receive the first line of (71). By our current assumption here $\Delta > 0$,

$$f\left[G - R - \frac{1}{\alpha f}\ln[\alpha f(1-f)(G-B)] - \frac{1}{\alpha f}\right] > fG + (1-f)B - R.$$

Therefore, in this parametric values case, bank i does not take the "lend to both with no cross subsidy" strategy in an equilibrium. In other words, this case cannot be taken in an equilibrium.

3.2.2 Subcases (3-b) and (3-d) Bank's Problem

In subcases (3-b) and (3-d), firm's IC constraint is (101). Here, the bank i's optimal problem is simplified as below:

$$\max_{x_{ij}\in \mathbf{S_3}} f[f_G(C)R_{\hat{G}} + (1-f_G(C))B] + (1-f)[f_B(C)B + (1-f_B(C))R_{\hat{B}}] - R - C \quad \text{[Bank profit]}$$

$s.t.$

$$f\left[f_G(C)\left(G - R_{\hat{G}}\right) + (1-f_G(C))\left(G - R_{\hat{B}}\right)\right] \geq 0 \quad \text{[Firm PC]}$$

$$f\left[f_G(C)\left(G - R_{\hat{G}}\right) + (1-f_G(C))\left(G - R_{\hat{B}}\right)\right] \geq fG + (1-f)B - R \quad \text{[Firm IC]}$$

$$f[f_G(C)R_{\hat{G}} + (1-f_G(C))B] + (1-f)[f_B(C)B + (1-f_B(C))R_{\hat{B}}] - R - C = 0 \quad \text{[Bank ZPC]}$$

Recall that $Pr(\hat{G}|G,C) = Pr(G|\hat{G},C) = f_G(C) = 1-(1-f)e^{-\alpha C}$,
$Pr(\hat{B}|B,C) = Pr(B|\hat{B},C) = f_B(C) = 1 - fe^{-\alpha C}$. Then, we have $f_B(C) = 1 - f\frac{1-f_G(C)}{1-f}$. By

banks' zero-profit constraint, $R_{\hat{G}}, R_{\hat{B}}$ are functions of C, as following:

$$R_{\hat{G}} \equiv g(C) = \frac{R + C - (1-f)B}{f f_G(C)} - \frac{1 - f_G(C)}{f_G(C)} b(C).$$
$$R_{\hat{B}} \equiv b(C) = \frac{R + C - (1-f)B}{f(1 - f_G(C))} - \frac{f_G(C)}{1 - f_G(C)} g(C).$$

We substitute $R_{\hat{G}}, R_{\hat{B}}$ to the firm's IC constraint as below:

$$fG + (1-f)B - R - C \geq fG + (1-f)B - R,$$

which can be further simplified as

$$C \leq 0.$$

There are two subcases. First, the RHS of firm's IC constraint is less than zero. We can show that firm's IR constraint is violated, as in case (1-a). No firms enter in stage 1, and hence there is no equilibrium in this case. Second, the RHS of firm's IC constraint is non-negative, which indicates that $fG + (1-f)B \geq R$. Given that the screening cost is non-negative, we have the optimal screening cost $C^* = 0$.

In the equilibrium, $C^* = 0$. Bank ZPC means that

$$fR^*_{\hat{G}} + (1-f)R^*_{\hat{B}} = \frac{R - (1-f)B}{f} \in [R, G]. \tag{93}$$

Note that a firm chooses one bank in stage 2, and cannot switch to other banks (loan contracts) in later stages, implying that a firm only values the expected profit *ex ante*, that is, the expected repayment in stage 2. Here, $R^*_{\hat{G}}$ and $R^*_{\hat{B}}$ may be different, but it does not affect firm's strategy in stage 2 as long as satisfying (93), because firms do not know whether they will get $\hat{G}$ or $\hat{B}$ signals. The firm's expected profit in stage 2 as below.

$$\begin{aligned}
E\left[\pi^f(x^*_{ij})\right] &= Pr(G)\left[Pr(\hat{G}|G,C)(G - R^*_{\hat{G}}) + Pr(\hat{B}|G,C)(G - R^*_{\hat{B}})\right] \\
&= f\left[f(G - R^*_{\hat{G}}) + (1-f)(G - R^*_{\hat{B}})\right] \\
&= f\left[G - \left(fR^*_{\hat{G}} + (1-f)R^*_{\hat{B}}\right)\right] \qquad (94)
\end{aligned}$$

Here, the remaining question is whether banks want to offer different $R^*_{\hat{G}}$ and $R^*_{\hat{B}}$. Remember that under $C^* = 0$, $R^*_{\hat{G}}$ and $R^*_{\hat{B}}$ do not convey any information, and firms are indifferent between $R^*_{\hat{G}}$ and $R^*_{\hat{B}}$ as long as (93) is satisfied. This means banks are also indifferent between offering symmetric strategy and offering asymmetric strategies.

In summary, when $\alpha f(1-f)(G-B) > 1$, $\Delta \leq 0$, and $fG + (1-f)B \geq R$; or when $\alpha f(1-f)(G-B) \leq 1$ and $fG + (1-f)B \geq R$,[31] the "lend to both with no cross subsidy" strategy by banks is the equilibrium strategy of a set of non-screening, cheap-information based equilibria. The equilibrium contract uses the "lend to both with no cross subsidy" strategy, and the screening strategy and the required repayment is characterized in the loan contract x^*_{ij} , where

$$C^* = 0.$$

$$fR^*_{\hat{G}} + (1-f)R^*_{\hat{B}} = \frac{R-(1-f)B}{f}.$$

In particular, there is a non-screening pooling equilibrium contract, in which

$$R^*_{\hat{G}} = R^*_{\hat{B}} \equiv R^*_p = \frac{R-(1-f)B}{f} > R.$$

Because $R^*_p \geq R \geq B$, the assumption of no cross subsidy holds.

In summary, for stage 1 case (3), we have following results.

Lemma 5. *When $\alpha(1-f)(G-B) > 1$, $\Delta < 0$, and $fG + (1-f)B \geq R$, or when $\alpha(1-f)(G-B) \leq 1$ and $fG + (1-f)B \geq R$, the "lend to both with no cross subsidy" strategy by banks is an equilibrium strategy.*

4 Stage 1 Case (4)

Suppose the "lend to both with a cross subsidy" ($\hat{\mu} \in (0,1)$, $R_{\hat{B}} \leq B$), is the equilibrium lending strategy in stage 1 before observing signals. Bank i provides the "lend to both with a cross subsidy" strategy, then receives loan applicants and screens the applicant firms. If bank i obtains $\hat{G}$ signal for firm j, then bank i would provide the loan rate $R_{\hat{G}}$ to firm j. With $\hat{B}$ signal, bank i would provide the loan rate $R_{\hat{B}}$ to firm j.

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[31] This condition is from firm's IR constraint. $E\left[\pi^f\left(x_{ij}\right)\right]$ represents firm's expected profit before screening in stage 2, which is non-negative by firm's participation constraint. Otherwise, firms will not enter the market in the first place. By (93) and (94), $E\left[\pi^f\left(x^*_{ij}\right)\right] = fG + (1-f)B - R \geq 0$.

The bank optimal problem per firm is defined as below.

$$\max_{x_{ij}\in\mathbf{S_4}} E\left[\pi^b(x_{ij}|x_{-i})\right] = Pr(\hat{G})\left[Pr(G|\hat{G},C)R_{\hat{G}} + Pr(B|\hat{G},C)B\right] + Pr(\hat{B})R_{\hat{B}} - R - C \quad \text{[Bank profit]} \tag{95}$$

$s.t.$

$$E\left[\pi^f(x_{ij})\right] = Pr(G)\left[Pr(\hat{G}|G,C)(G-R_{\hat{G}}) + Pr(\hat{B}|G,C)(G-R_{\hat{B}})\right] + Pr(B)\left[Pr(\hat{B}|B,C)(B-R_{\hat{B}})\right] \geq 0 \quad \text{[Firm PC]} \tag{96}$$

$$E\left[\pi^f(x_{ij})\right] = Pr(G)\left[Pr(\hat{G}|G,C)(G-R_{\hat{G}}) + Pr(\hat{B}|G,C)(G-R_{\hat{B}})\right] + Pr(B)\left[Pr(\hat{B}|B,C)(B-R_{\hat{B}}))\right] \geq E\left[\pi^f(\tilde{x}_{hj})\right], \text{ for } \forall h\neq i, \forall \tilde{x}_{hj}\in x_j \quad \text{[Firm IC]} \tag{97}$$

$$E\left[\pi^b(x_{ij}|x_{-i})\right] = Pr(\hat{G})\left[Pr(G|\hat{G},C)R_{\hat{G}} + Pr(B|\hat{G},C)B\right] + \Pr(\hat{B})R_{\hat{B}} - R - C = 0 \quad \text{[Bank ZPC]} \tag{98}$$

4.1 Subcases of Case (4) IC constraint

By the same argument to derive (71), firm's IC constraint (97) can be expressed as below:

$$\begin{aligned} &Pr(G)[Pr(\hat{G}|G,C)(G-R_{\hat{G}}) + Pr(\hat{B}|G,C)(G-R_{\hat{B}})] + Pr(B)[Pr(\hat{B}|B,C)(B-R_{\hat{B}})] \\ &\geq \max\left\{\left\{f\left[G-R-\frac{1}{\alpha f}\ln[\alpha f(1-f)(G-B)] - \frac{1}{\alpha f}\right]|_{\alpha f(1-f)(G-B)>1},\right.\right. \\ &\left. f[fG+(1-f)B-R]|_{\alpha f(1-f)(G-B)\leq 1}\right\}, \\ &\left. fG+(1-f)B-R.\right\}. \end{aligned} \tag{99}$$

Then, firm j's IC constraint (99) can be simplified into four cases as below.

Subcase (4-a): IC constraint: When $\alpha f(1-f)(G-B) > 1$ and $\Delta > 0$, the "lend to $\hat{G}$ only" contract provides firm j the highest expected profit, compared to contracts under other lending strategies. Hence, while bank i chooses the "lend to both with a cross subsidy" strategy, deviant bank h may choose the "lend to $\hat{G}$ only" strategy, and the firm j's IC constraint (99) becomes

$$\begin{aligned} &Pr(G)\left[Pr(\hat{G}|G,C)(G-R_{\hat{G}})+Pr(\hat{B}|G,C)(G-R_{\hat{B}})\right] \\ &+Pr(B)\left[Pr(\hat{B}|B,C)(B-R_{\hat{B}})\right] \geq f\left[G-R-\frac{1}{\alpha}ln[\alpha(1-f)(G-B)]-\frac{1}{\alpha}\right]. \end{aligned} \tag{100}$$

Subcase (4-b): IC constraint: When $\alpha f(1-f)(G-B)>1$ and $\Delta<0$, the "lend to both" contract provides firm j the highest expected profit compared to contracts under other lending strategies, and firm j's IC constraint (99) becomes

$$\begin{aligned} &Pr(G)\left[Pr(\hat{G}|G,C)(G-R_{\hat{G}})+Pr(\hat{B}|G,C)(G-R_{\hat{B}})\right] \\ &+Pr(B)\left[Pr(\hat{B}|B,C)(B-R_{\hat{B}})\right] \geq fG+(1-f)B-R. \end{aligned} \tag{101}$$

Subcase (4-c): IC constraint: When $\alpha f(1-f)(G-B)>1$ and $\Delta=0$, the "lend to $\hat{G}$ only" contract and the "lend to both" contract can provide the same highest firm's expected profit. Under this case, there could be a mixed strategy equilibrium appears. Since we focus on pure strategy equilibrium, we do not go deep into this case, and we assume all banks choose the same pure strategy. Without loss of generality, case (4-c) is combined with case (4-b). Hereinafter, we do not construct this case separately.

Subcase (4-d): IC constraint: When $\alpha(1-f)(G-B)\leq 1$, the "lend to both" contract provides firm j the highest expected profit compared to contracts under other lending strategies, and the firm j's IC constraint (99) becomes

$$\begin{aligned} &Pr(G)\left[Pr(\hat{G}|G,C)(G-R_{\hat{G}})+Pr(\hat{B}|G,C)(G-R_{\hat{B}})\right] \\ &+Pr(B)\left[Pr(\hat{B}|B,C)(B-R_{\hat{B}})\right] \geq fG+(1-f)B-R. \end{aligned} \tag{102}$$

This is the same constraint as in case (4-b) (101).

Next, we incorporate subcases of IC constraint, (4-a), (4-b), and (4-d) into the bank's optimal problem.

4.2 Subcases of Case (4) Bank's Problem

4.2.1 Subcase (4-a) Bank's Problem

In subcase (4-a), $\alpha f(1-f)(G-B) > 1$ and $\Delta > 0$, bank i offers the "lend to both with a cross subsidy" in equilibrium. Deviant bank h offers the "lend to $\hat{G}$ only" strategy to give firms the highest expected profit in a deviation among all possible deviation offers. In this case, we have the IC constraint (100).

However, under the "lend to both with a cross subsidy" strategy, firms at most receive the following expected profit, derived as the same way as for the third line of (71), which is

$$fG + (1-f)B - R.$$

Under the "lend to $\hat{G}$ only" strategy, firms receive the first line of (71). By our current assumption here $\Delta > 0$,

$$f\left[G - R - \frac{1}{\alpha f}\ln[\alpha f(1-f)(G-B)] - \frac{1}{\alpha f}\right] > fG + (1-f)B - R.$$

Therefore, in this parametric values case, bank i does not take the "lend to both with a cross subsidy" strategy in an equilibrium. In other words, this case cannot be taken in an equilibrium.

4.2.2 Subcases (4-b) and (4-d) Bank's Problem

In subcases (4-b) and (4-d), firm's IC constraint is (101). Here, the bank i's optimal problem is simplified as below:

$$\max_{x_{ij}\in \mathbf{S_4}} f[f_G(C)R_{\hat{G}} + (1-f_G(C))B] + (1-f)R_{\hat{B}} - R - C \qquad \text{[Bank profit]}$$

$s.t.$

$$f[G - f_G(C)R_{\hat{G}} - (1-f_G(C))R_{\hat{B}}] + (1-f)f_B(C)(B - R_{\hat{B}}) \geq 0 \qquad \text{[Firm PC]}$$

$$f[G - f_G(C)R_{\hat{G}} - (1-f_G(C))R_{\hat{B}}] + (1-f)f_B(C)(B - R_{\hat{B}}) \geq fG + (1-f)B - R \qquad \text{[Firm IC]}$$

$$f[f_G(C)R_{\hat{G}} + (1-f_G(C))B] + (1-f)R_{\hat{B}} - R - C = 0 \qquad \text{[Bank ZPC]}$$

By banks' zero-profit constraint, $R_{\hat{G}}, R_{\hat{B}}$ are functions of C as following:

$$R_{\hat{G}} \equiv h(C) = \frac{R + C - f(1 - f_G(C))B}{f f_G(C)} - \frac{1-f}{f f_G(C)} l(C),$$
$$R_{\hat{B}} \equiv l(C) = \frac{R + C - f(1 - f_G(C))B}{1-f} - \frac{f f_G(C)}{1-f} h(C).$$

Recall that $Pr(\hat{G}|G, C) = Pr(G|\hat{G}, C) = f_G(C) = 1 - (1-f)e^{-\alpha C}$,
$Pr(\hat{B}|B, C) = Pr(B|\hat{B}, C) = f_B(C) = 1 - fe^{-\alpha C}$. Then, we have $f_B(C) = 1 - f\frac{1-f_G(C)}{1-f}$. We substitute $R_{\hat{G}}, R_{\hat{B}}, f_B(C)$ to the IC constraint and it becomes

$$fG + (1-f)B - R - C \geq fG + (1-f)B - R,$$

which can be further simplified as

$$C \leq 0.$$

There are two subcases. First, the RHS of firm's IC constraint is less than zero. We can show that firm's IR constraint is violated, as in case (1-a). No firms enter in stage 1, hence, there is no equilibrium in this case. Second, the RHS of firm's IC constraint is non-negative, which indicates that $fG + (1-f)B \geq R$. Given that the screening cost is non-negative, we have the optimal screening cost $C^* = 0$.

In the equilibrium, $C^* = 0$. Bank ZPC means that

$$f^2 R^*_{\hat{G}} + (1-f)R^*_{\hat{B}} = R - (1-f)fB \tag{103}$$

Note that a firm chooses one bank in stage 2, and cannot switch to other banks (loan contracts) in later stages, implying that a firm only values the expected profit *ex ante*, that is, the expected repayment in stage 2. Here, $R^*_{\hat{G}}$ and $R^*_{\hat{B}}$ may be different, but it does not affect firm's strategy in stage 2 as long as satisfying (103), because firms do not know whether they will get $\hat{G}$ or $\hat{B}$ signals.

The firm's equilibrium expected profit in stage 2 as below.

$$\begin{aligned}E\left[\pi^f(x^*_{ij})\right] =& Pr(G)\left[Pr(\hat{G}|G,C)(G-R^*_{\hat{G}})+Pr(\hat{B}|G,C)(G-R^*_{\hat{B}})\right]\\ &+Pr(B)\left[Pr(\hat{B}|B,C)(B-R^*_{\hat{B}})\right]\\ =& f\left[f(G-R^*_{\hat{G}})+(1-f)(G-R^*_{\hat{B}})\right]+(1-f)^2(B-R^*_{\hat{B}})\\ =& fG+(1-f)^2B-[f^2R^*_{\hat{G}}+(1-f)R^*_{\hat{B}}] \end{aligned} \tag{104}$$

Here, the remaining question is whether banks want to offer different $R^*_{\hat{G}}$ and $R^*_{\hat{B}}$. Remember that under $C^*=0$, $R^*_{\hat{G}}$ and $R^*_{\hat{B}}$ do not convey any information, and firms are indifferent between $R^*_{\hat{G}}$ and $R^*_{\hat{B}}$, as long as (103) is satisfied. This means banks are also indifferent between offering pooling strategy and offering cheap information strategies.

In summary, when $\alpha f(1-f)(G-B)>1$, $\Delta\le 0$, and $fG+(1-f)B\ge R$, or when $\alpha f(1-f)(G-B)\le 1$ and $fG+(1-f)B\ge R$, [32] the "lend to both with a cross subsidy" strategy by banks is the equilibrium strategy of a set of cheap information non-screening equilibria. The equilibrium contract uses the "lend to both with a cross subsidy" strategy, and the screening strategy and the required repayment is characterized in the loan contract x^*_{ij} , where

$$\begin{aligned}&C^*=0.\\ &fR^*_{\hat{G}}+\frac{1-f}{f}R^*_{\hat{B}}=\frac{R-(1-f)fB}{f}.\end{aligned}$$

Let $R^*_{\hat{G}}=R^*_{\hat{B}}\equiv R^*_p$, by (103), we have $R^*_{\hat{G}}=R^*_{\hat{B}}\equiv R^*_p=\frac{R-(1-f)fB}{1-(1-f)f}>\frac{B-(1-f)fB}{1-(1-f)f}=B$. Since $R^*_p\ge B$, the assumption of cross subsidy does not hold. Under "lend only to both with a cross subsidy" strategy, there is no pooling Nash equilibrium.

In summary, for stage 1 case (4), we have following results.

Lemma 6. *When $\alpha(1-f)(G-B)>1$, $\Delta<0$, and $fG+(1-f)B\ge R$, or when $\alpha(1-f)(G-B)\le 1$ and $fG+(1-f)B\ge R$, the "lend to both with a cross subsidy" strategy by banks is an equilibrium strategy.*

[32] This condition is from firm's PC constraint. $E\left[\pi^f\left(x_{ij}\right)\right]$ represents firm's expected profit before screening in stage 2, which is non-negative by firm's participation constraint. Otherwise, firms will not enter the market in the first place. By (103) and (104), $E\left[\pi^f\left(x^*_{ij}\right)\right]=fG+(1-f)B-R\ge 0$.

D PROOF OF PROPOSITION 2

For the case of screening strategy, $z_j = (Pr_j(G|\hat{G}, C_j), L_{\hat{G},j}, L_{\hat{B},j}) \in [f, 1) \times \{1\} \times \{0\}$. We consider the "lend to $\hat{G}$ only" strategy. Denote economy-wide net output as W_s.

$$\max_{\{z_j\}_{j=1}^m} \quad W_s \equiv \sum_{j=1}^{m} Pr(\hat{G})[Pr(G|\hat{G}, C_j)G + Pr(B|\hat{G}, C_j)B - R] - C_j$$

$$s.t. \qquad Pr(\hat{G})[Pr(\hat{G})[Pr(G|\hat{G}, C_j)G + Pr(B|\hat{G}, C_j)B - R] - C_j \geq 0, \text{ for } \forall j \in \{1, 2, ..., m\}$$

The Lagrangian is as below.

$$\mathcal{L} = \sum_{j=1}^{m} \Big\{ Pr(\hat{G})[Pr(G|\hat{G}, C_j)G + Pr(B|\hat{G}, C_j)B - R] - C_j \Big\} - \sum_{j=1}^{m} \lambda_j \Big\{ Pr(\hat{G})[Pr(\hat{G})[Pr(G|\hat{G}, C_j)G + Pr(B|\hat{G}, C_j)B - R] - C_j \Big\}$$

By $Pr(G|\hat{G}, C_j) = f_G(C_j)$, the Lagrangian can be rewritten as below.

$$\mathcal{L} = \sum_{j=1}^{m} \{f[f_G(C_j)G + (1 - f_G(C_j))B - R] - C_j\} - \sum_{j=1}^{m} \lambda_j \{f[f_G(C_j)G + (1 - f_G(C_j))B - R] - C_j\}$$

The first order conditions are for $\forall j \in \{1, 2, ..., m\}$,

$$\frac{\partial \mathcal{L}}{\partial C_j} = (1 + \lambda_j)(f f'_G(C_j)(G - B) - 1) = 0.$$

Firms are identical, thus, $C_1 = C_2 = ... = C_m \equiv C$. The first order conditions imply $f f'_G(C^*)(G - B) - 1 = 0$. Recall that in the decentralized baseline, the first order condition in the screening equilibrium is $f'_G(C^*)(G - B) - 1/f = 0$. Therefore, the optimal screening cost of the screening strategy in the social planner's problem is the same as the optimal screening cost in the screening equilibrium in the decentralised economy. The optimal overall output is

$$W_s^* = \Big[fG - \frac{1}{\alpha} - \frac{1}{\alpha} ln(\alpha f(1 - f)(G - B)) \Big] m.$$

For the case of non-screening strategy, $z_j = (Pr_j(G|\hat{G}, C), L_{\hat{G},j}, L_{\hat{B},j}) \in [f, 1) \times \{1\} \times \{1\}$.

Denote economy-wide net output as W_p.

$$\max_{\{z_j\}_{j=1}^m} \quad W_p \equiv \sum_{j=1}^{m} Pr(\hat{G})[Pr(G|\hat{G},C_j)G + Pr(B|\hat{G},C_j)B] + Pr(\hat{B})[Pr(G|\hat{B},C_j)G + Pr(B|\hat{B},C_j)B] - R - C_j$$

$$s.t. \quad Pr(\hat{G})[Pr(G|\hat{G},C_j)G + Pr(B|\hat{G},C_j)B] + Pr(\hat{B})[Pr(G|\hat{B},C_j)G + Pr(B|\hat{B},C_j)B] - R - C_j \geq 0, \ \text{for } \forall j \in \{1,2,...,m\}$$

The Lagrangian is as below.

$$\mathcal{L} = \sum_{j=1}^{m} \Big\{ Pr(\hat{G})[Pr(G|\hat{G},C_j)G + Pr(B|\hat{G},C_j)B] + Pr(\hat{B})[Pr(G|\hat{B},C_j)G + Pr(B|\hat{B},C_j)B] - R - C_j \Big\} - \sum_{j=1}^{m} \lambda_j \Big\{ Pr(\hat{G})[Pr(G|\hat{G},C_j)G + Pr(B|\hat{G},C_j)B] + Pr(\hat{B})[Pr(G|\hat{B},C_j)G + Pr(B|\hat{B},C_j)B] - R - C_j \Big\}$$

where $Pr(\hat{B})Pr(G|\hat{B},C_j) = Pr(\hat{B})\frac{Pr(\hat{B}|G,C_j)Pr(G)}{Pr(\hat{B})} = Pr(\hat{B}|G,C_j)Pr(G) = Pr(\hat{G})Pr(B|\hat{G},C_j)$. By Simplifying the Lagrangian, we have

$$\mathcal{L} = \sum_{j=1}^{m} [fG + (1-f)B - R - C_j] - \sum_{j=1}^{m} \lambda_j [fG + (1-f)B - R - C_j]$$

The first order conditions are for $\forall j \in \{1,2,...,m\}$,

$$\frac{\partial \mathcal{L}}{\partial C_j} = -(1+\lambda_j) < 0.$$

Firms are identical. $C_1 = C_2 = ... = C_m \equiv C$. The first order conditions imply $C^* = 0$, So the optimal screening cost of the non-screening strategy in the social planner's problem is the same as the optimal screening cost in the non-screening equilibrium in the decentralised economy. The optimal overall output is

$$W_p^* = [fG + (1-f)B - R]m.$$

The timing of switching between screening equilibrium and non-screening equilibrium depends on Δ_w, is consistent with Δ in the decentralized baseline, as shown below. When $\Delta_w > 0$, the optimal solution of the social planner's problem is the same as the screening equilibrium. When $\Delta_w \leq 0$, the optimal solution of the social planner's problem is the same as the non-screening equilibrium.

$$\begin{aligned}\Delta_w =& W_s - W_p \\ =& \left[fG - \frac{1}{\alpha} - \frac{1}{\alpha} ln(\alpha f(1-f)(G-B)) \right] m \\ & - [fG + (1-f)B - R]m \\ =& m\Delta\end{aligned}$$

E Characterization of Equilibrium and Comparative Analysis on NI and GDP

We investigate the effects of changes in five parameters: α, f, R, G, and B, *ceteris paribus*. For each parameter value, how equilibrium changes and how NI and GDP change are shown below. Without loss of generality, we focus on the screening separating equilibrium (also written as SS equilibrium or screening equilibrium) and the non-screening pooling equilibrium (also written as NP equilibrium or non-screening equilibrium) in this section.

According to proposition 1, Δ is an important threshold to determine screening or non-screening equilibrium. Δ can be simplified as below.

$$\begin{aligned}\Delta &= f\left[G - R - \frac{1}{\alpha f} \ln[\alpha f(1-f)(G-B)] - \frac{1}{\alpha f} \right] - [fG + (1-f)B - R] \\ &= (1-f)(R-B) - \frac{1}{\alpha} \ln[\alpha f(1-f)(G-B)] - \frac{1}{\alpha} \end{aligned} \tag{105}$$

A Changes in α

1 Characterization of Equilibrium

Depending on the existence of Δ, there are two cases: (1) $f(1-f)(G-B) > 1$, which implies that Δ exists for $\forall \alpha \in [\underline{\alpha}, 1)$, where $\underline{\alpha} = \frac{1}{f(1-f)(G-B)}$; and (2) $f(1-f)(G-B) \leq 1$, which implies that Δ does not exist for $\forall \alpha \in (0,1)$. Note that firm's expected profit *ex ante* is non-negative, and we introduce two key values of α: α_{SS} for screening equilibrium case and α_{NP} for non-screening equilibrium case. At α_{SS}, firm's expected profit in screening strategy is $E[\pi^f_{SS}(\alpha_{SS})] = 0$. At α_{NP}, firm's expected profit in non-screening strategy is $E[\pi^f_{NP}(\alpha_{NP})] = 0$. $E[\pi^f_{SS}]$ is an increasing function of α. $E[\pi^f_{NP}]$ is irrelevant to α.

Subcase (α-1): $f(1-f)(G-B) > 1$, then Δ exists for $\forall\alpha \in [\underline{\alpha}, 1)$. The first derivative $\frac{\partial\Delta}{\partial\alpha} = \frac{1}{\alpha^2} ln\alpha f(1-f)(G-B) > 0$, Δ is an increasing function of α. The signs of $\Delta(\alpha = \underline{\alpha})$ and $\Delta(\alpha = 1)$ are uncertain. $E[\pi^f_{NP}] = fG + (1-f)B - R$, which is not related to α. If $E[\pi^f_{NP}] > 0$, $\Delta(\alpha = \underline{\alpha}) = -(1-f)E[\pi^f_{NP}] < 0$, and vice versa. Hence, there are three possibilities: (1) $\Delta(\alpha = \underline{\alpha}) < 0$, $\Delta(\alpha = 1) < 0$; (2) $\Delta(\alpha = \underline{\alpha}) < 0$, $\Delta(\alpha = 1) > 0$; (3) $\Delta(\alpha = \underline{\alpha}) > 0$, $\Delta(\alpha = 1) > 0$.

Sub-subcase (α-1.1): $\Delta(\alpha = \underline{\alpha}) < 0$ (that is, $E[\pi^f_{NP}] > 0$), $\Delta(\alpha = 1) < 0$, as in Figure 5, panel (a). For $\forall\alpha \in (0, 1)$, there is a non-screening equilibrium.

Sub-subcase (α-1.2): $\Delta(\alpha = \underline{\alpha}) < 0$ (that is, $E[\pi^f_{NP}] > 0$), $\Delta(\alpha = 1) > 0$, as in Figure 5, panel (b). Denote α^* satisfies $\Delta(\alpha^*) = 0$. At α^*, $E[\pi^f_{SS}] = E[\pi^f_{NP}] > 0$, thus, $\alpha_{SS} > \alpha^*$. When $\alpha \in (0, \alpha^*)$, there is a non-screening equilibrium; when $\alpha \in [\alpha^*, 1)$, there is a screening equilibrium.

Sub-subcase (α-1.3): $\Delta(\alpha = \underline{\alpha}) > 0$ (that is, $E[\pi^f_{NP}] < 0$), $\Delta(\alpha = 1) > 0$, as in Figure 5, panel (c). At $\underline{\alpha}$, $C^* = 0$, $E[\pi^f_{SS}] = f(fG + (1-f)B - R) < 0$, thus, $\alpha_{SS} > \underline{\alpha}$. When $\alpha \in (0, \alpha_{SS})$, there is no investment domestically; when $\alpha \in [\alpha_{SS}, 1)$, there is a screening equilibrium.[33]

Subcase (α-2): $f(1-f)(G-B) \leq 1$, then Δ does not exist for $\forall\alpha \in (0, 1)$. If $E[\pi^f_{NP}] > 0$, there is a non-screening equilibrium for $\forall\alpha \in (0, 1)$. If $E[\pi^f_{NP}] \leq 0$, there is no investment domestically for $\forall\alpha \in (0, 1)$.

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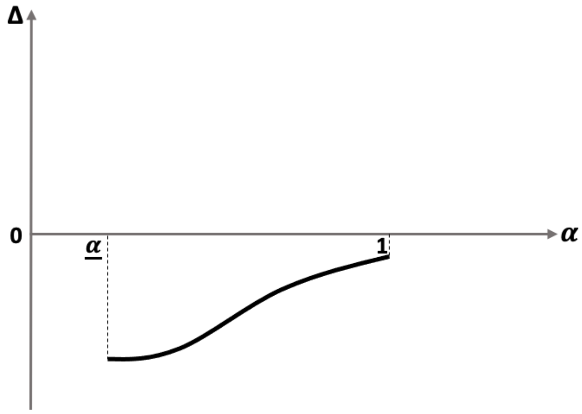
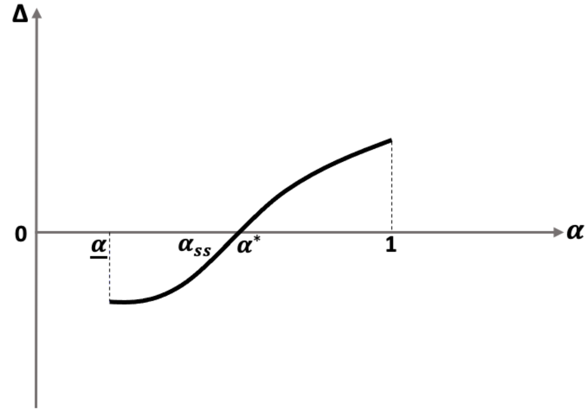
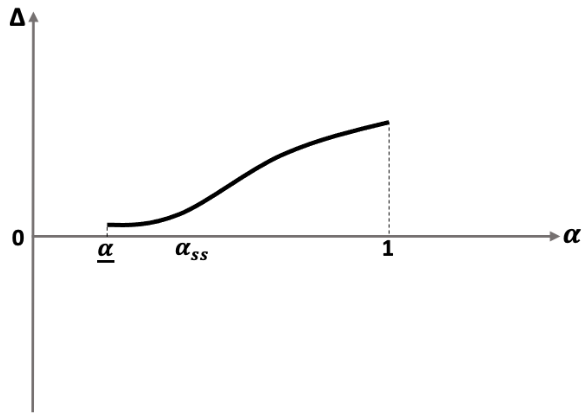
[33] For the case $\Delta(\alpha = \underline{\alpha}) = 0$, $\alpha_{SS} = \alpha_{NP} = \underline{\alpha}$, the characterization of equilibrium is consistent with subcase (α-1.3).



(e) $\Delta(\alpha = \underline{\alpha}) < 0, \Delta(\alpha = 1) < 0$



(f) $\Delta(\alpha = \underline{\alpha}) < 0, \Delta(\alpha = 1) > 0$



(g) $\Delta(\alpha = \underline{\alpha}) > 0, \Delta(\alpha = 1) > 0$

Figure 5: The plot of Δ on α

2 Comparative Analysis on NI and GDP

As in Figure 6, NI and GDP vary as α changes. The figures are plotted according to the equations (15) to (18). For the upper row, the two figures (a) and (b) characterize the case non-screening equilibrium exists for some $\alpha \in (0, 1)$ (subcase (α-1.2) above). When $\alpha \in (0, \alpha^*)$, there is a non-screening equilibrium, and NI and GDP do not depend on α. When $\alpha \in [\alpha^*, 1)$, there is a screening equilibrium, and NI and GDP are increasing and concave functions of α. For the lower row, the two figures (c) and (d) characterize the case that non-screening equilibrium does not exist for any $\alpha \in (0, 1)$ (subcase (α-1.3) above). When $\alpha \in (0, \alpha_{SS})$, no loan contracts are offered in the domestic market, and all investments go abroad, thus, $NI = mR, GDP = 0$. When $\alpha \in [\alpha_{SS}, 1)$, there is a screening equilibrium. Regarding α case, there are other conditions depending on other parameters, that is, subcase (α-1.1) and subcase (α-2). In subcase (α-1.1), for all $\alpha \in (0, 1)$, there is a non-screening equilibrium. In subcase (α-2), for all $\alpha \in (0,1)$, there is either a non-screening equilibrium or no loan contracts offered in the domestic market. We skip the details of those two cases here.

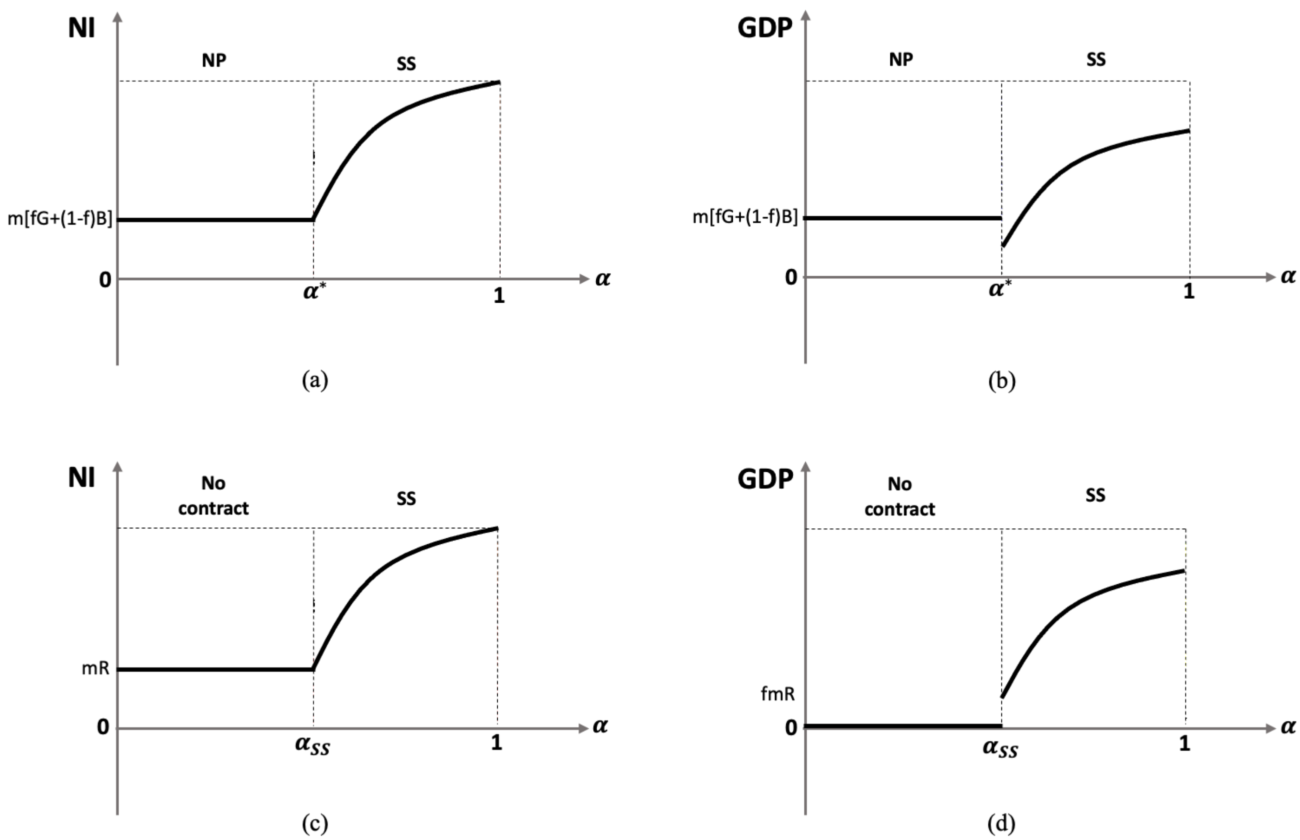


Figure 6: Comparative analysis: α case.

3 Implications of Advancement in Screening Technology

After the pandemic, digitization is one of the new trends of financial institutions. Using new tools such as cloud computing, big data analysis, and mobile payments, banks should be able to improve the accuracy of screening, thus, reduce the screening cost.

In our model, the signal accuracy is defined as $Pr(G|\hat{G}, C) = f_G(C) \equiv 1 - (1-f)e^{-\alpha C}$. The improvement of screening technology implies the increase of α. As in Figure 5, as α becomes bigger, the baseline model is more likely to have a screening equilibrium where banks conduct screening and only lend to $\hat{G}$. As in Figure 6, the NI in the screening equilibrium is bigger than the NI in the non-screening equilibrium and the NI in the case of full foreign investment. As α increases, the NI in the screening equilibrium increases. Note that if the initial screening technology is underdeveloped (α is very low), a big investment of the screening technology is necessary to generate a screening equilibrium; otherwise, a small investment is just a waste of money in the sense that the non-screening equilibrium (or foreign investment) still prevails.

Overall, the improvements in the screening technology will motivate banks to use screening, and make the economy to have the screening equilibrium with higher NI.

B Changes in f

1 Characterization of Equilibrium

Depending on the existence of Δ, there are two cases: (1) $\alpha(G-B) \leq 4$, which implies that $\forall f \in (0,1), \alpha f(1-f)(G-B) \leq 1$; and (2) $\alpha(G-B) > 4$, which means that $\exists f \in (0,1), \alpha f(1-f)(G-B) > 1$. Note that firm's expected profit *ex ante* is non-negative, and we introduce two key values of f: f_{SS} for screening equilibrium case and f_{NP} for non-screening equilibrium case. When $f = f_{SS}$, firm's expected profit in screening strategy is $E[\pi^f_{SS}(f_{SS})] = f_{SS}(G-R) - \frac{1}{\alpha} ln[\alpha f_{SS}(1-f_{SS})(G-B)] - \frac{1}{\alpha} = 0$. When $f = f_{NP}$, firm's expected profit in non-screening strategy is $E[\pi^f_{NP}(f_{NP})] = f_{NP}G + (1-f_{NP})B - R = 0$.

Subcase (f-1): $\alpha(G-B) \leq 4$, then for $\forall f \in (0,1), \alpha f(1-f)(G-B) \leq 1$. screening equilibrium does not exist, and by (105), Δ does not exist. When $f \in (0, f_{NP})$, there is no investment domestically; when $f \in [f_{NP}, 1)$, there is a non-screening equilibrium.

Subcase (f-2): $\alpha(G-B) > 4$, then $\exists f \in (0,1), \alpha f(1-f)(G-B) > 1$. Denote $\underline{f}$ and $\bar{f}$ that satisfies $\alpha f(1-f)(G-B) = 1$, that is, $C^*_{SS} = 0$. Note that $\underline{f} < \frac{1}{2} < \bar{f}$. Δ exists for $f \in [\underline{f}, \bar{f}]$. The first derivative $\frac{\partial \Delta}{\partial f} = -(R-B) - \frac{1}{\alpha}(\frac{1}{f} - \frac{1}{1-f})$, and the second order condition is always positive. At $\underline{f}$ and $\bar{f}$, $C^*_{SS} = 0$, $\Delta(f) = -(1-f)(fG + (1-f)B - R) = -(1-f)E[\pi^f_{NP}]$, and $E[\pi^f_{SS}] = fE[\pi^f_{NP}]$. The signs of $\Delta(\underline{f})$ and $\Delta(\bar{f})$ are uncertain. $\frac{\partial \Delta}{\partial f}|_{\underline{f}} < 0$, but the sign of $\frac{\partial \Delta}{\partial f}|_{\bar{f}}$ is uncertain. Then, we further divide this case based on the signs of $\Delta(\underline{f})$ and $\Delta(\bar{f})$.

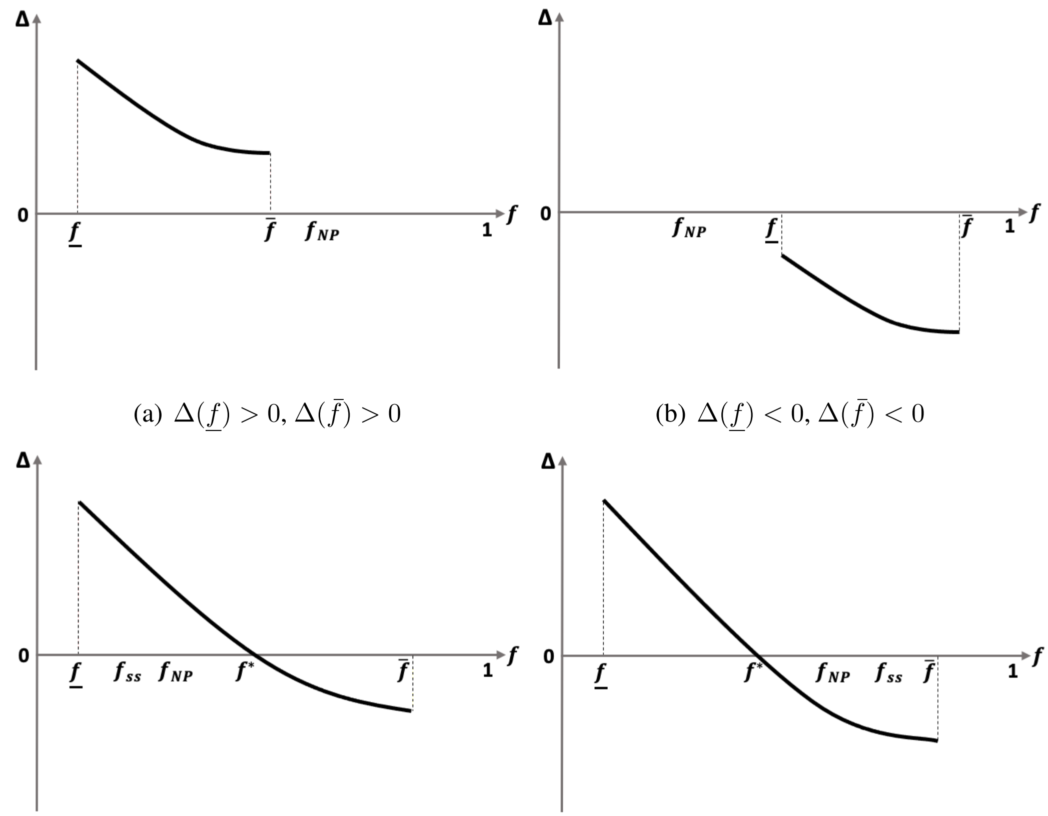
Sub-subcase (f-2.1): $\Delta(\underline{f}) > 0$ and $\Delta(\bar{f}) > 0$, as in Figure 7, panel (a). Since $\Delta(\bar{f}) > 0$, $E[\pi^f_{NP}(\bar{f})] < 0$. $E[\pi^f_{NP}]$ is an increasing function of f, thus, $f_{NP} > \bar{f}$. Here, the value of f_{SS} is important. Note that $\frac{\partial E[\pi^f_{SS}]}{\partial f} = G - R - \frac{1}{\alpha}(\frac{1}{f} - \frac{1}{1-f})$, $\frac{\partial E[\pi^f_{SS}]}{\partial f}|_{\bar{f}} > 0$, but the sign of $\frac{\partial E[\pi^f_{SS}]}{\partial f}|_{\underline{f}}$ is uncertain. $E[\pi^f_{SS}(\underline{f})] < 0$ and $E[\pi^f_{SS}(\bar{f})] < 0$. So, $f_{SS} \notin [\underline{f}, \bar{f}]$, which implies f_{SS} does not matter for the characterization of equilibrium. When $f \in (0, f_{NP})$, there is no investment domestically; when $f \in [f_{NP}, 1)$, there is a non-screening equilibrium.

Sub-subcase (f-2.2): $\Delta(\underline{f}) < 0$ and $\Delta(\bar{f}) < 0$, as in Figure 7, panel (b).[34] Δ is always negative, so f_{SS} does not matter for the characterization of equilibrium. Since $\Delta(\underline{f}) < 0$, $E[\pi^f_{NP}(\underline{f})] > 0$, and $f_{NP} < \underline{f}$. When $f \in (0, f_{NP})$, there is no investment domestically; when $f \in [f_{NP}, 1)$, there is a non-screening equilibrium.

Sub-subcase (f-2.3): $\Delta(\underline{f}) > 0$ and $\Delta(\bar{f}) < 0$. This means $E[\pi^f_{SS}(\underline{f})] < 0$ and $E[\pi^f_{SS}(\bar{f})] > 0$, $f_{SS} \in [\underline{f}, \bar{f}]$. Moreover, $E[\pi^f_{NP}(\underline{f})] < 0$ and $E[\pi^f_{NP}(\bar{f})] > 0$, so $f_{NP} \in [\underline{f}, \bar{f}]$. Denote f^* satisfies $\Delta(f^*) = 0$. Then, we analyze the relative sizes of f_{SS}, f_{NP}, and f^*, which can be divided to two possibilities. (1) If $f_{NP} < f^*$, then $\Delta(f_{NP}) > 0$. Since $E[\pi^f_{NP}(f_{NP})] = 0$, $E[\pi^f_{SS}(f_{NP})] > 0$, thus, $f_{SS} < f_{NP}$. (2) If $f_{NP} > f^*$, then $\Delta(f_{NP}) < 0$, $E[\pi^f_{SS}(f_{NP})] < 0$, thus, $f_{SS} > f_{NP}$. For the possibility of $f_{NP} = f^*$, it can be combined with either case above. In subcase (f-2.3), we analyze the former possibility, $f_{SS} < f_{NP} < f^*$, as in Figure 7, panel (c). When $f \in (0, f_{SS})$, there is no investment domestically; when $f \in [f_{SS}, f^*)$, there is a screening equilibrium; when $f \in [f^*, 1)$, there is a non-screening equilibrium.

Sub-subcase (f-2.4): $\Delta(\underline{f}) > 0$ and $\Delta(\bar{f}) < 0$. We analyze the latter possibility, $f^* < f_{NP} < f_{SS}$, as in Figure 7, panel (d). When $f \in (0, f_{NP})$, there is no investment domestically; when $f \in [f_{NP}, 1)$, there is a non-screening equilibrium.

[34] For the case when $\Delta(\underline{f}) = 0$ and $\Delta(\bar{f}) = 0$, $\underline{f} = f_{NP}, \bar{f} = 1$, and the result is the same as subcase (f-2.2).



(a) $\Delta(\underline{f}) > 0, \Delta(\bar{f}) > 0$

(b) $\Delta(\underline{f}) < 0, \Delta(\bar{f}) < 0$

(c) $\Delta(\underline{f}) > 0, \Delta(\bar{f}) < 0, f_{SS} < f_{NP} < f^*$

(d) $\Delta(\underline{f}) > 0, \Delta(\bar{f}) < 0, f^* < f_{NP} < f_{SS}$

Figure 7: The plot of Δ on f

2 Comparative Analysis on NI and GDP

As in Figure 8, NI and GDP vary as f changes. For the upper row, the two figures (a) and (b) characterize the case that screening equilibrium exists for some $f \in (0, 1)$ (subcase (f-2.3) above). More specifically, when $f \in (0, f_{SS})$, no loan contracts are offered in the domestic market, all investments go abroad, thus, $GDP = 0$ with $NI = mR$. When $f \in [f_{SS}, f^*)$, there is a screening equilibrium, banks allocate f share of funding to domestic market and use the rest to invest abroad, so NI is an increasing convex function of f, and so does GDP. When $f \in [f^*, 1)$, there is a non-screening equilibrium, banks allocate all fundings to the domestic market, so NI is an increasing linear function of f, and so does GDP. Here, f^* is the threshold from screening equilibrium to non-screening equilibrium, and it is easy to identify as $NI_{SS} = NI_{NP}$ and $GDP_{SS} < GDP_{NP}$. The reason of the discontinuity of the GDP at f^* is that banks stop foreign investments and increase lending in the domestic market.[35]

For the lower row, the two figures (c) and (d) characterize the case that screening equilibrium does not exist for all $f \in (0,1)$ (all subcases other than (f-2.3)). When $f \in (0, f_{NP})$, there is no loans offered in the domestic market, thus, $GDP = 0$ with $NI = mR$. When $f \in [f_{NP}, 1)$, there is a non-screening equilibrium.

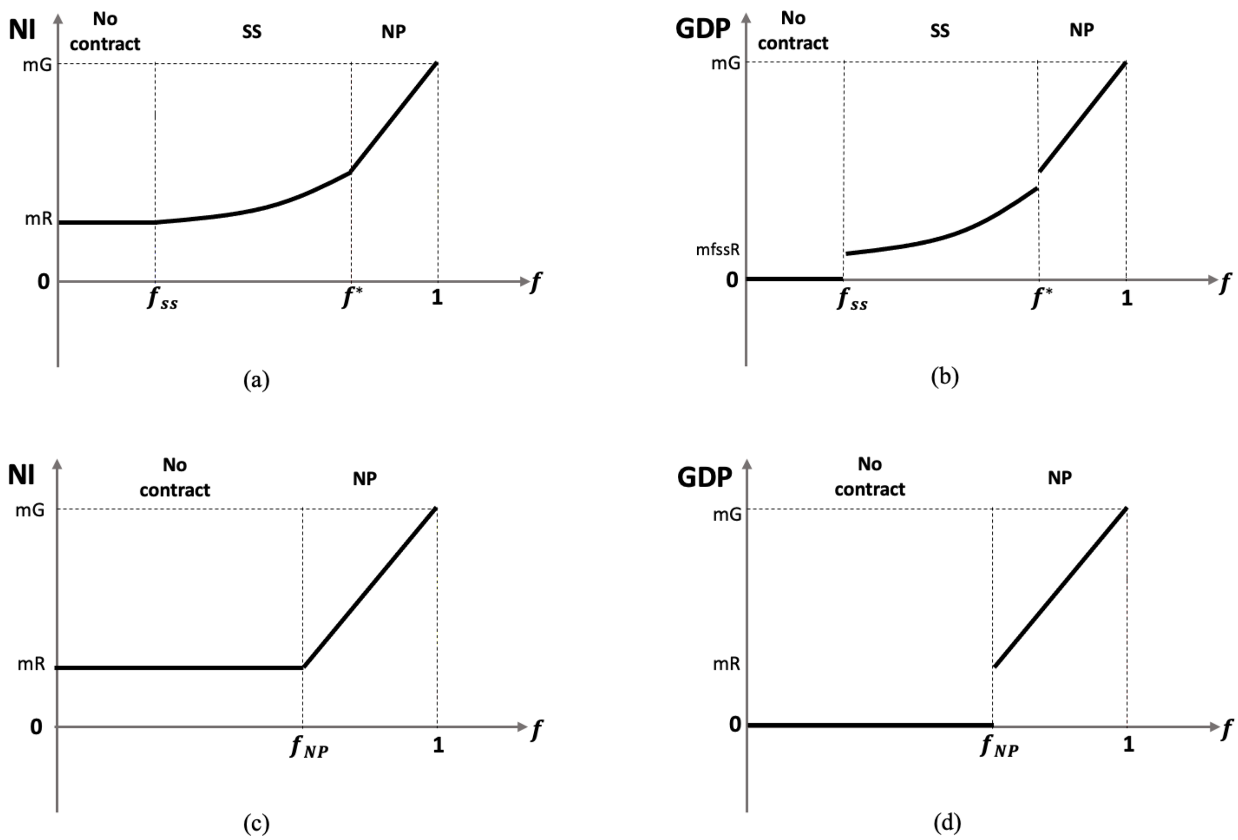


Figure 8: Comparative analysis: f case

C Changes in R

1 Characterization of Equilibrium

Depending on the existence of Δ, there are two cases: (1) $\alpha f(1-f)(G-B) > 1$, which implies that Δ exists for $\forall R \in (B,G)$; and (2) $\alpha f(1-f)(G-B) \leq 1$, which implies that Δ does not exist for $\forall R \in (B,G)$. Note that firm's expected profit *ex ante* is non-negative, and we introduce two key values of R: R_{SS} for screening equilibrium case and R_{NP} for non-screening equilibrium case. At

[35]The baseline model uses fixed-size loan contracts. When the equilibrium switches between no contract, screening equilibrium, and non-screening equilibrium, the foreign investments will flow into (or out of) the domestic market and cause a jump of the domestic output. The discontinuity disappears when we endogenize the loan size and use constant-returns-to-scale production technology.

R_{SS}, firm's expected profit in screening strategy is $E[\pi^f_{SS}(R_{SS})] = 0$. At R_{NP}, firm's expected profit in non-screening strategy is $E[\pi^f_{NP}(R_{NP})] = 0$.

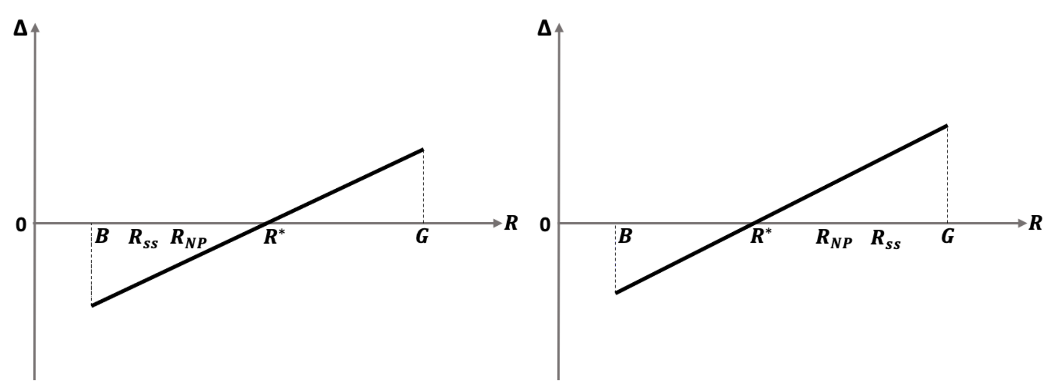
Subcase (R-1): $\alpha f(1-f)(G-B) > 1$, then Δ exists for $\forall R \in (B, G)$. Δ is an increasing linear function of R. $\Delta(R = B) < 0$,

$\Delta(R = G) > \frac{1}{\alpha}[\alpha f(1-f)(G-B) - 1 - ln[\alpha f(1-f)(G-B)]] \geq 0$. Denote R^* satisfies $\Delta(R^*) = 0$. Both $E[\pi^f_{SS}]$ and $E[\pi^f_{NP}]$ are decreasing linear functions of R, with $E[\pi^f_{SS}(R = B)] > 0$, $E[\pi^f_{NP}(R = B)] > 0$, $E[\pi^f_{SS}(R = G)] < 0$, and $E[\pi^f_{NP}(R = G)] < 0$. Thus, $R_{SS} \in (B, G)$ and $R_{NP} \in (B, G)$. The relative sizes of $R*$, R_{SS}, and R_{NP} can be divided into two possibilities: (1) $R_{SS} < R_{NP} < G^*$. (2) $R_{SS} > R_{NP} > G^*$.

Sub-subcase (R-1.1): $R_{SS} < R_{NP} < G^*$, as in Figure 9, panel (a). When $R \in (B, R_{NP})$, there is a non-screening equilibrium; when $R \in [R_{NP}, G)$, there is no investment domestically.

Sub-subcase (R-1.2): $R_{SS} > R_{NP} > G^*$, as in Figure 9, panel (b). When $R \in (B, R^*)$, there is a non-screening equilibrium; when $R \in [R^*, R_{SS})$, there is a screening equilibrium; when $R \in [R_{SS}, G)$, there is no investment domestically.

Subcase (R-2): $\alpha f(1-f)(G-B) \leq 1$, then Δ does not exist for $\forall R \in (B, G)$. When $R \in (B, R_{NP})$, there is a non-screening equilibrium; when $R \in [R_{NP}, G)$, there is no investment domestically.



(a) $R_{SS} < R_{NP} < G^*$ (b) $R_{SS} > R_{NP} > G^*$

Figure 9: The plot of Δ on R

2 Comparative Analysis on NI and GDP

As in Figure 10, NI and GDP vary as R changes. For the upper row, the two figures (a) and (b) characterize the case that screening equilibrium exists for some $R \in (B, G)$, (subcase (R-1.2) above). More specifically, when $R \in (B, R^*)$, there is a non-screening equilibrium. The levels of NI and GDP in pooling equilibrium do not depend on R. When $R \in [R^*, R_{SS})$, there is a screening equilibrium. NI is a linear increasing function of R, with a slope of $(1-f)m$. GDP does not depend on R. When $R \in [R_{SS}, G)$, no loan contracts are offered in the domestic market, all investments go abroad, thus, NI is a linear increasing function of R with a slope of m, and GDP is zero given that there is no domestic production. For the lower row, the two figures (c) and (d) characterize the case that screening equilibrium does not exist for any $R \in (B, G)$ (subcase (R-1.1, R-2) above). When $R \in (B, R_{NP})$, there is a non-screening equilibrium. The level of NI and GDP in pooling equilibrium do not depend on R. When $R \in [R_{NP}, G)$, no loan contracts are offered in the domestic market, all investments go abroad, thus, NI is a linear increasing function of R, with a slope of m, and GDP is zero.

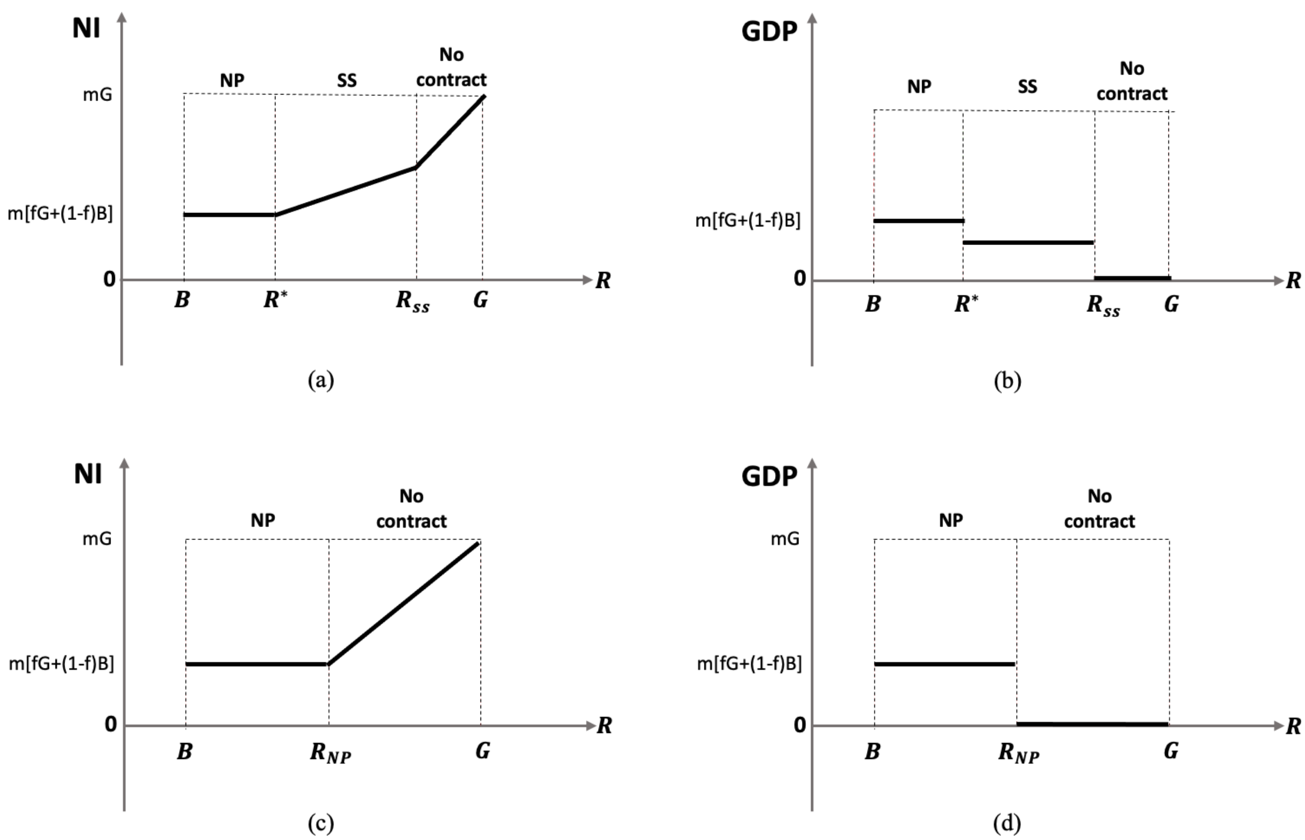
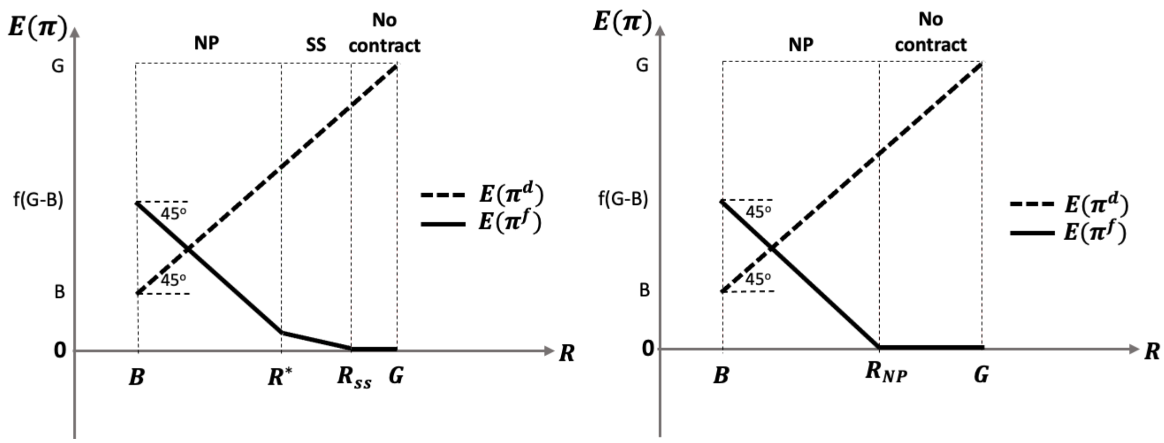


Figure 10: Comparative analysis: R case

R affects the return of depositors (or investors), Figure 11 shows the expected returns of depositors and firms. The expected returns of depositors or investors ($E(\pi^d)$) are always R per unit of money under all occasions. The expected returns of firms ($E(\pi^f)$) depend on the equilibrium. The

LHS figure (a) shows the case that screening equilibrium exists for some $R \in (B, G)$. When $R \in (B, R^*)$, there is a non-screening equilibrium, and the firm's expected profit is a linear decreasing function of R, with a slope of 1. When $R \in [R^*, R_{SS})$, there is a screening equilibrium, and the firm's expected profit is a linear decreasing function of R, with a slope of $(1 - f)$. When $R \in [R_{SS}, G)$, no loan contracts are offered in the domestic market, thus, firm's expected profit is zero. The RHS figure (b) shows the case that screening equilibrium does not exist for any $R \in (B, G)$. When $R \in (B, R_{NP})$, there is a non-screening equilibrium, and the firm's expected profit is a linear decreasing function of R, with a slope of 1. When $R \in [R_{NP}, G)$, no loan contracts are offered in the domestic market, thus, firm's expected profit is zero.



(a) (b)

Figure 11: R case: firm's expected profit $E(\pi^f)$ and depositor's expected profit $E(\pi^d)$

When R is high, the model shows a tendency for capital outflow. The financial sector might be overheated and investors have a big expected return, while the real sector stays at a constrained status. R can be interpreted as the return of foreign investment, With a big R, banks tend to invest abroad, instead of financing the domestic project. As R increases, the NI increases, while the GDP remains at a lower level.

D Changes in G

1 Characterization of Equilibrium

Depending on the existence of Δ, there are two cases: (1) $B + \frac{1}{\alpha f(1-f)} \leq R$, which means that for $\forall G \in (R, \infty), \alpha f(1-f)(G - B) > 1$; and (2) $B + \frac{1}{\alpha f(1-f)} > R$, which means that

$\forall G \in (\underline{G}, \infty), \alpha f(1-f)(G-B) > 1$, where $\underline{G} = B + \frac{1}{\alpha f(1-f)}$. Note that firm's expected profit *ex ante* is non-negative, and we introduce two key values of G: G_{SS} for screening equilibrium case and G_{NP} for non-screening equilibrium case. At G_{SS}, firm's expected profit in screening strategy is $E[\pi^f_{SS}(G_{SS})] = 0$. At G_{NP}, firm's expected profit in non-screening strategy is $E[\pi^f_{NP}(G_{NP})] = 0$. $E[\pi^f_{SS}]$ is an increasing and convex function of G, and $E[\pi^f_{NP}]$ is an increasing linear function of G, and $G_{NP} > R$, $G_{SS} > R$.

Subcase (G-1): $B + \frac{1}{\alpha f(1-f)} \leq R$, then for $\forall G \in (R, \infty), \alpha f(1-f)(G-B) > 1$. We can plot Δ on $G \in (R, \infty)$. The first derivative $\frac{\partial \Delta}{\partial G} = -\frac{1}{\alpha(G-B)} < 0$, and the second order condition $\frac{\partial^2 \Delta}{\partial G^2} > 0$. The sign of $\Delta(G = R)$ is uncertain. Then, we further divide this case based on the sign.

Sub-subcase (G-1.1): $\Delta(G = R) < 0$, as in Figure 12, panel (a). At $G = R$, $E[\pi^f_{NP}(G = R)] < 0$, and $E[\pi^f_{NP}]$ is an increasing linear function of G, thus, $G_{NP} < R$. When $G \in (R, G_{NP})$, there is no investment domestically; when $G \in [G_{NP}, \infty)$, there is a non-screening equilibrium.[36]

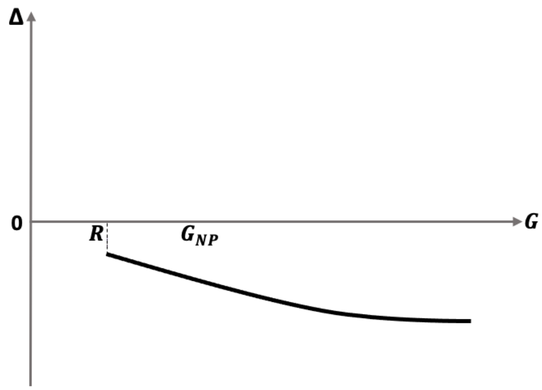
Sub-subcase (G-1.2): $\Delta(G = R) > 0$. Denote G^* satisfies $\Delta(G^*) = 0$. Regarding the relative values of G_{SS} and G_{NP}, if $G_{NP} < G^*$, then $G_{SS} < G_{NP} < G^*$; and if $G_{NP} > G^*$, $G_{SS} > G_{NP} > G^*$. The relative sizes of $G*$, G_{SS}, and G_{NP} can be divided into two possibilities: (1) $G_{SS} < G_{NP} < G^*$; and (2) $G_{SS} > G_{NP} > G^*$. In subcase (G-1.2), we show the first possibility, as in Figure 12, panel (b). When $G \in (R, G_{SS})$, there is no investment domestically; when $G \in [G_{SS}, G^*)$, there is a screening equilibrium; when $G \in [G^*, \infty)$, there is a non-screening equilibrium.

Sub-subcase (G-1.3): $\Delta(G = R) > 0$, $G_{SS} > G_{NP} > G^*$, as in Figure 12, panel (c). When $G \in (R, G_{NP})$, there is no investment domestically; when $G \in [G_{NP}, \infty)$, there is a non-screening equilibrium.

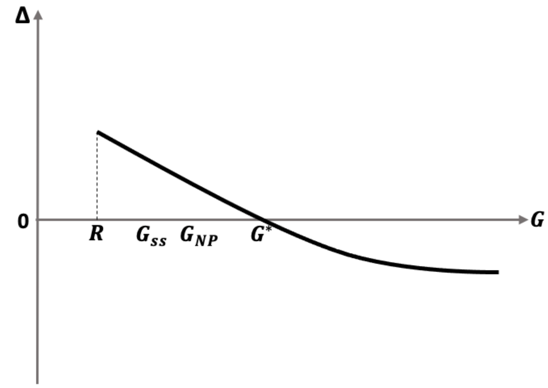
Subcase (G-2): $B + \frac{1}{\alpha f(1-f)} > R$, then for $\forall G \in (\underline{G}, \infty), \alpha f(1-f)(G-B) > 1$. The plot of Δ and the characterization of equilibrium is similar to subcases of (G-1). We skip the plot and divide the equilibrium characterization into four possibilities. (1) $\Delta(\underline{G}) < 0$. When $G \in (R, G_{NP})$, there is no investment domestically; when $G \in [G_{NP}, \infty)$, there is a non-screening equilibrium. (2) $\Delta(\underline{G}) > 0$, $G_{SS} > G_{NP} > G^* > \underline{G}$. When $G \in (R, G_{NP})$, there is no investment domestically; when $G \in [G_{NP}, \infty)$, there is a non-screening equilibrium. (3) $\Delta(\underline{G}) > 0$, $G_{SS} < G_{NP} < \underline{G} < G^*$

[36] For the case $\Delta(G = R) = 0$, $G_{NP} = R$, for $\forall G \in (R, \infty)$, there is a non-screening equilibrium.

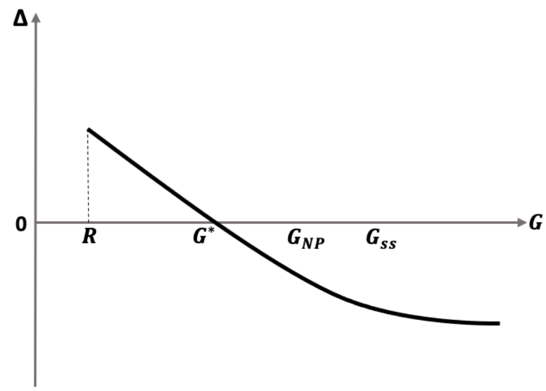
or $G_{SS} < \underline{G} < G_{NP} < G^*$. When $G \in (R, \underline{G})$, there is no investment domestically; when $G \in [\underline{G}, G^*)$, there is a screening equilibrium; when $G \in [G^*, \infty)$, there is a non-screening equilibrium. (4) $\Delta(\underline{G}) > 0$, $\underline{G} < G_{SS} < G_{NP} < G^*$. When $G \in (R, G_{SS})$, there is no investment domestically; when $G \in [G_{SS}, G^*)$, there is a screening equilibrium; when $G \in [G^*, \infty)$, there is a non-screening equilibrium.



(a) $\underline{G} \leq R, \Delta(G = R) < 0$



(b) $\underline{G} \leq R, \Delta(G = R) > 0, G_{SS} < G_{NP} < G^*$



(c) $\underline{G} \leq R, \Delta(G = R) > 0, G_{SS} > G_{NP} > G^*$

Figure 12: The plot of Δ on G

2 Comparative Analysis on NI and GDP

As in Figure 13, NI and GDP vary as G changes. For the upper row, the two figures (a) and (b) characterize the case that screening equilibrium exists for some $G \in (R, \infty)$ (subcase (G-1.2) above). When $G \in (R, G_{SS})$, no loan contracts are offered in the domestic market, and all investments go abroad, thus, $NI = mR, GDP = 0$. When $G \in [G_{SS}, G^*)$, there is a screening equilibrium, and NI and GDP are increasing and convex functions of G. When $G \in [G^*, \infty)$, there is a non-screening equilibrium, and NI and GDP are increasing and linear functions of G. For the lower row, the two figures (c) and (d) characterize the case that screening equilibrium does not exist for any $G \in (R, \infty)$ (subcase (G-1.1, 1.3) above). When $G \in (R, G_{NP})$, no loan contracts are offered in the domestic market. When $G \in [G_{NP}, \infty)$, there is a non-screening equilibrium.

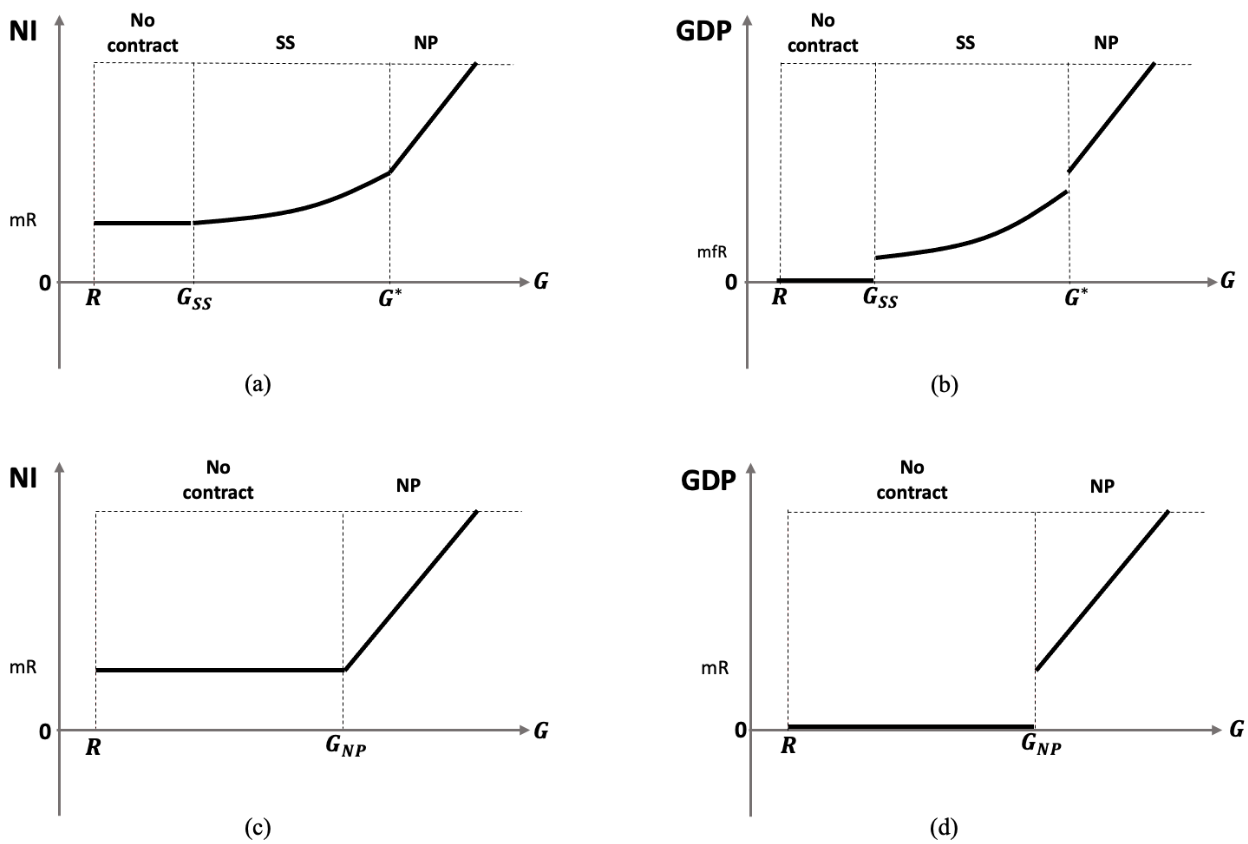


Figure 13: Comparative analysis: G case

E Changes in B

1 Characterization of Equilibrium

Depending on the existence of Δ, there are two cases: (1) $G - \frac{1}{\alpha f(1-f)} > R$, which means that for $\forall B \in [0, R), \alpha f(1-f)(G-B) > 1$; and (2) $G - \frac{1}{\alpha f(1-f)} \leq R$, which means that $\forall B \in [0, \bar{B}), \alpha f(1-f)(G-B) > 1$, where $\bar{B} = G - \frac{1}{\alpha f(1-f)}$. Note that firm's expected profit *ex ante* is non-negative, and we introduce two key values of B: B_{SS} for screening equilibrium case and B_{NP} for non-screening equilibrium case. At B_{SS}, firm's expected profit in screening strategy is $E[\pi^f_{SS}(B_{SS})] = 0$. At B_{NP}, firm's expected profit in non-screening strategy is $E[\pi^f_{NP}(B_{NP})] = 0$. $E[\pi^f_{SS}]$ is an increasing and concave function of B, and $E[\pi^f_{NP}]$ is an increasing linear function of B.

Subcase (B-1): $\bar{B} = G - \frac{1}{\alpha f(1-f)} > R$, then for $\forall B \in [0, R), \alpha f(1-f)(G-B) > 1$. We can plot

Δ on $B \in [0, R)$. The first derivative $\frac{\partial \Delta}{\partial B} = -(1-f) + \frac{1}{\alpha(G-B)} < -(1-f) + f(1-f) < 0$, and the second order condition $\frac{\partial^2 \Delta}{\partial B^2} = \frac{1}{\alpha(G-B)^2} > 0$. Note that
$\Delta(B = R) = -\frac{1}{\alpha} ln[\alpha f(1-f)(G-R)] - \frac{1}{\alpha} < 0$. Δ is negative when $B = R$.
$\Delta(B = 0) = (1-f)R - \frac{1}{\alpha} ln[\alpha f(1-f)G] - \frac{1}{\alpha}$. The sign of $\Delta(B = 0)$ is uncertain, so is the sign of $E[\pi^f_{NP}(B = 0)]$. Then, we further divide this case based on the signs.

Sub-subcase (B-1.1): $\Delta(B = 0) \leq 0$ and $B_{NP} \leq 0$, as in Figure 14, panel (a). For any $B \in [0, R)$, there is a non-screening equilibrium.

Sub-subcase (B-1.2): $\Delta(B = 0) \leq 0$ and $B_{NP} > 0$, as in Figure 14, panel (b). When $B \in [0, B_{NP})$, there is no investment domestically; when $B \in [B_{NP}, R)$, there is a non-screening equilibrium.

Sub-subcase (B-1.3): $\Delta(B = 0) > 0$. Denote B^* satisfies $\Delta(B^*) = 0$. Regarding the relative values of B_{SS} and B_{NP}, if $B_{NP} > B^*$, then $E[\pi^f_{NP}(B^*)] > 0$, and $E[\pi^f_{SS}(B^*)] > 0$, implying that $B_{SS} > B^*$, thus, $B_{SS} > B_{NP} > B^*$. Similarly, If $B_{NP} < B^*$, then $B_{SS} < B_{NP} < B^*$. The relative sizes of B^*, B_{SS}, and B_{NP} can be divided into three possibilities: (1) $B_{SS} > B_{NP} > B^*$. (2) $0 < B_{SS} < B_{NP} < B^*$. (3) $B_{SS} < 0 < B^*$, where B_{NP} does not matter. In subcase (B-1.3), we show the first possibility in Figure 14, panel (c)[37]. Here, when $B \in [0, B_{NP})$, there is no investment domestically; when $B \in [B_{NP}, R)$, there is a non-screening equilibrium.

Sub-subcase (B-1.4): $\Delta(B = 0) > 0$ and $0 < B_{SS} < B_{NP} < B^*$, as in Figure 14, panel (d). When $B \in [0, B_{SS})$, there is no investment domestically; when $B \in [B_{SS}, B^*)$, there is a screening equilibrium; when $B \in [B^*, R)$, there is a non-screening equilibrium.

Sub-subcase (B-1.5): $\Delta(B = 0) > 0$ and $B_{SS} < 0 < B^*$, as in Figure 14, panel (e). When $B \in [0, B^*]$, there is a screening equilibrium; when $B \in [B^*, R)$, there is a non-screening equilibrium.

Subcase (B-2): $G - \frac{1}{\alpha f(1-f)} \leq R$. $\bar{B} = G - \frac{1}{\alpha f(1-f)}$. At $\bar{B}$, $C^* = 0$,
$\Delta(\bar{B}) = -(1-f)(fG + (1-f)\bar{B} - R)$. If $\Delta(\bar{B}) \leq 0$,$B_{NP} \leq \bar{B}$. The plots of Δ on B remain almost the same as subcases of (B-1), except replacing the upper limit R with $\bar{B}$. The characterization of equilibrium is the same as subcases of (B-1). If $\Delta(\bar{B}) > 0$, as in Figure 14, panel (f). Hence, when $B \in [0, B_{NP})$, there is no investment domestically; when $B \in [B_{NP}, R)$, there is a non-screening equilibrium.

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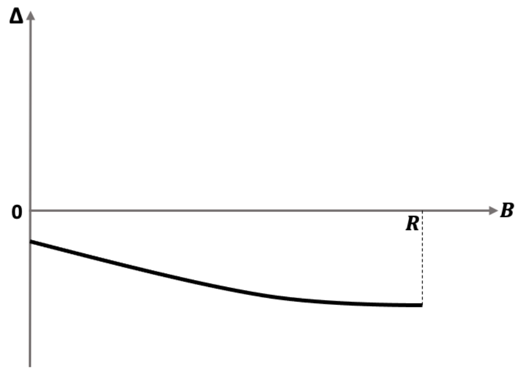
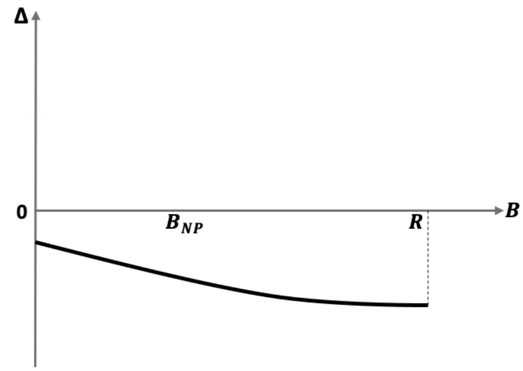
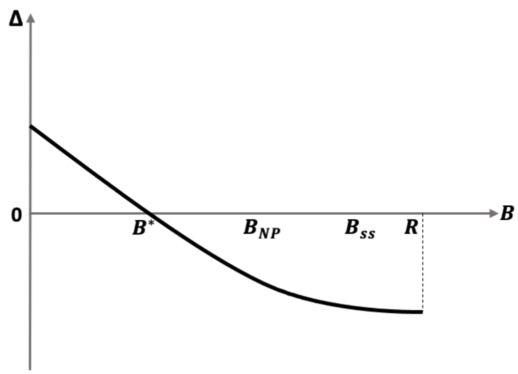
[37]The case $B_{SS} = B_{NP} = B^*$ can be combined with subcase (B-1.3).



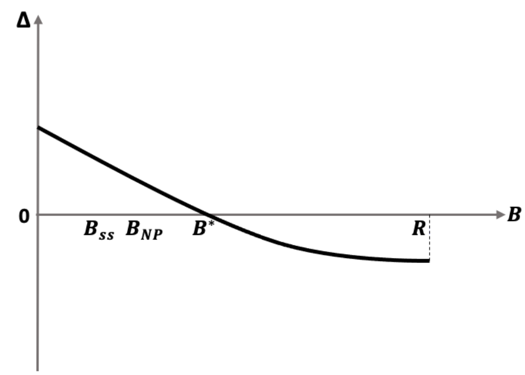
(a) $\bar{B} > R$, $\Delta(B=0) < 0$, $B_{NP} < 0$



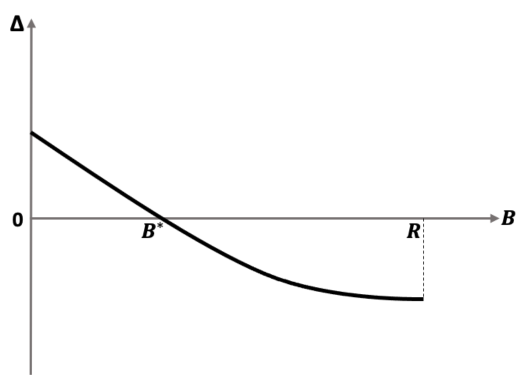
(b) $\bar{B} > R$, $\Delta(B=0) < 0$, $B_{NP} \geq 0$



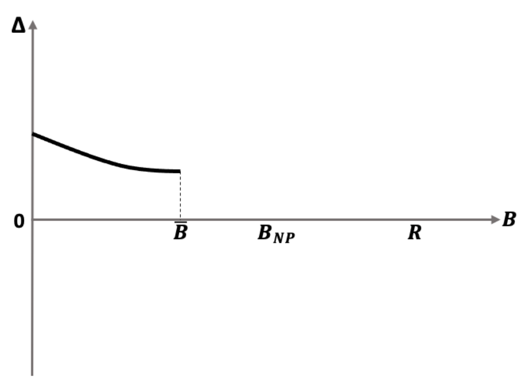
(c) $\bar{B} > R$, $\Delta(B=0) > 0$, $B_{SS} > B_{NP} > B^*$



(d) $\bar{B} > R$, $\Delta(B=0) > 0$, $0 < B_{SS} < B_{NP} < B^*$



(e) $\bar{B} > R$, $\Delta(B=0) > 0$, $B_{SS} < 0 < B^*$



(f) $\bar{B} \leq R$, $\Delta(\bar{B}) > 0$

Figure 14: The plot of Δ on B

2 Comparative Analysis on NI and GDP

As in Figure 15, NI and GDP vary as B changes. For the first row, the two figures (a) and (b) characterize the case that screening equilibrium exists for some B (subcase (B-1.4) above). When $B \in [0, B_{SS})$, no loan contracts are offered in the domestic market, and all investments go abroad. When $B \in [B_{SS}, B^*)$, there is a screening equilibrium. When $B \in [B^*, R)$, there is a non-screening equilibrium. For the second row, the two figures (c) and (d) characterize the case that screening equilibrium does not exist for any $B \in [0, R)$ (subcase (B-1.2, 1.3) above). When $B \in [0, B_{NP})$, no loan contracts are offered in the domestic market, and all investments go abroad. When $B \in [B_{NP}, R)$, there is a non-screening equilibrium. Regarding B case, there are other conditions, that is, subcase (B-1.1) and subcase (B-1.5) above. In subcase (B-1.1), there is a non-screening equilibrium for any $B \in [0, R)$. In subcase (B-1.5), when $B \in [0, B^*]$, there is a screening equilibrium; when $B \in [B^*, R)$, there is a non-screening equilibrium. We skip the details of those two cases here.

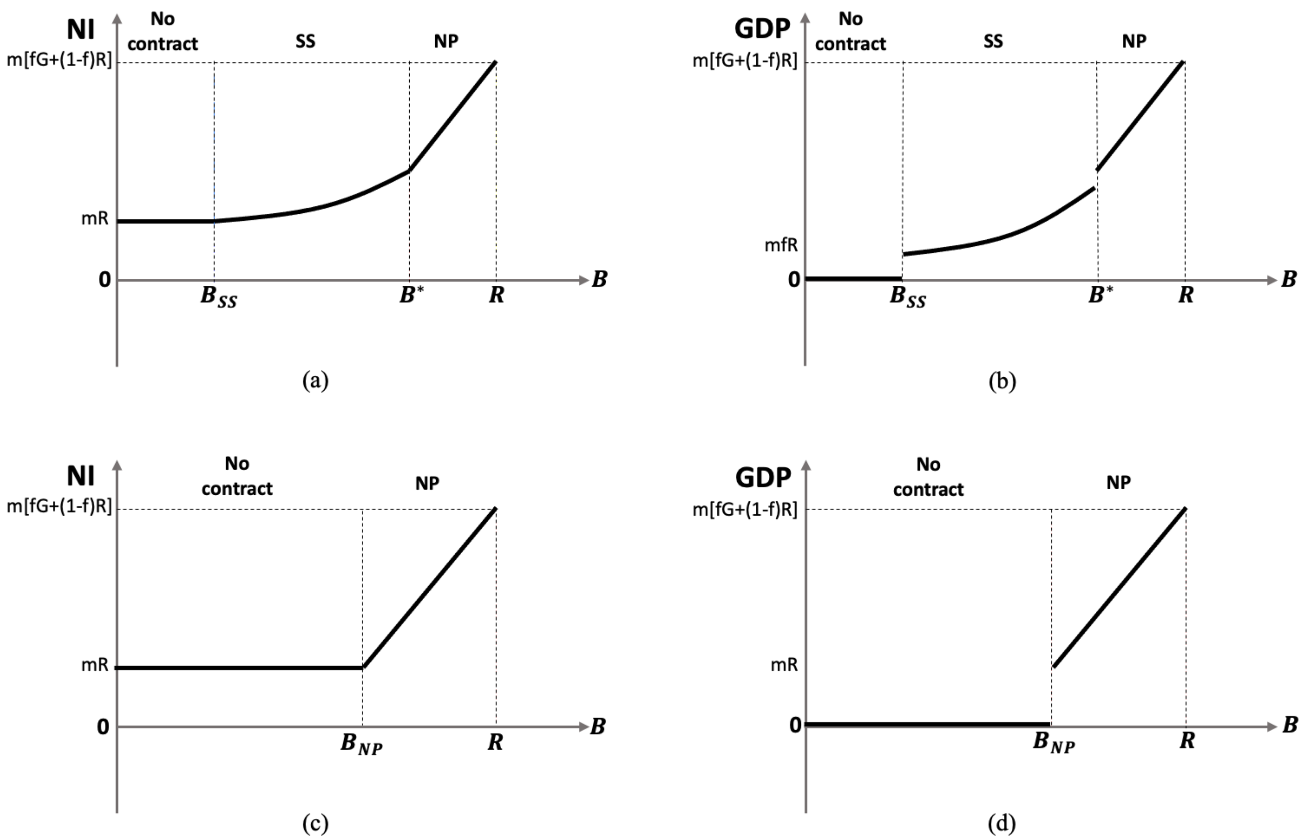


Figure 15: Comparative analysis: B case

In summary, the equilibrium varies with parameter values as follows.

(1) When the efficiency of screening is low (α is low), a non-screening equilibrium is more likely to emerge. Note that α can be interpreted as a multiplier of the screening cost. When the financial market or the screening technologies are well developed, α should be high. In other words, to achieve the same level of signal accuracy, an economy with lower α needs to spend a higher level of screening cost. When the screening cost is more expensive than the loss of identification mistakes,

the optimal decision will be to give up screening, so there is a non-screening equilibrium.

(2) When the average project quality is high (f is high), the economy tends to have a non-screening equilibrium. A high average project quality indicates that banks are likely to pick good firms under a blind selection, and the possibility of lending to bad firms, that is, the possibility of default is small, so banks are unwilling to spend money on screening, given a small loss in non-screening strategy.

(3) When the international interest rate R is low, the economy tends to have a non-screening equilibrium. Banks' funding cost per unit of loan is R. Given G, B constant, when R becomes lower, bank's loss of an identification mistake is lower. Since the screening cost is simply higher than the loss associated with the higher default without screening, banks are less likely to screen.

(4) When the good firm's output G is high relative to R, the economy tends to have a non-screening equilibrium. Since we assume a fixed size of loan contract, only f share of funding is allocated for domestic market in the screening strategy, while all funding is used for domestic market in the non-screening strategy. If the good firm's output G is high, it is optimal to bear the loss of default to fund every good firm.[38]

(5) When the bad firm's output B is high relative to R, the economy tends to have a non-screening equilibrium. Given R constant, a higher B implies a smaller loss of default. Given the cost of conducting screening, banks are less likely to screen.

In addition, under certain extreme conditions,[39] there is no loan contract in equilibrium, as firm's expected profit *ex ante* are negative and will not go to banks to borrow loans. Hence, banks collect deposits and invest abroad to receive the international interest rate R. In other words, the domestic credit market collapses. This is consistent with Dvorkin and Shell (2017) who have shown empirically there is no market when market asset quality is extremely bad, loan demand decreases, and loan standard is tightened.

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[38]In the later section, we relax the assumption of the fixed-size contract, and use the constant-return-to-scale production technology, and we find the characterization of equilibrium in G case is slightly different, while the other four cases remain the same.

[39]Such extreme conditions include a very low α, a very low f, a very low G, a very low B, or a very high R.

F PROOF OF PROPOSITION 3

Under a credit easing policy, the central bank offers banks a lower funding cost than the international interest rate R, if banks lend to domestic borrowers. Denote τ as the level of the subsidization. Let us denote the domestic interest rate by $\tilde{R}$. Bank's zero profit condition means arbitrage with international interest rate, that is, $\tilde{R} = (1-\tau)R$. Thus, bank's funding cost is $(1-\tau)R$ per unit of loan, and the central bank provides a subsidy of τR per unit of loan. The expenditure on credit easing is financed by the lump sum tax (inflation tax) on households (depositors).

We follow the setup of baseline, and adjust the funding cost of banks to $(1-\tau)R$ for loans. Under the "lend to $\hat{G}$ only" strategy, firm's expected profit (36) remains the same, while bank's expected profit (35) becomes[40]

$$E\left[\pi^b_\tau(x_{ij}|x_{-i})\right] = Pr(\hat{G})\left[Pr(G|\hat{G},C)R_{\hat{G}} + Pr(B|\hat{G},C)B - (1-\tau)R - \frac{C}{f}\right] \tag{106}$$

Under the "lend to both with no cross subsidy" strategy, firm's expected profit (86) remains the same, while bank's expected profit (85) becomes

$$\begin{aligned} E\left[\pi^b_\tau(x_{ij}|x_{-i})\right] =& Pr(\hat{G})[Pr(G|\hat{G},C)R_{\hat{G}} + Pr(B|\hat{G},C)B] + Pr(\hat{B})[Pr(G|\hat{B},C)R_{\hat{B}} + \\ & Pr(B|\hat{B},C)B] - (1-\tau)R - C \end{aligned} \tag{107}$$

Under the "lend to both with a cross subsidy" strategy, firm's expected profit (96) remains the same, while bank's expected profit (95) becomes

$$E\left[\pi^b_\tau(x_{ij}|x_{-i})\right] = Pr(\hat{G})\left[Pr(G|\hat{G},C)R_{\hat{G}} + Pr(B|\hat{G},C)B\right] + Pr(\hat{B})R_{\hat{B}} - (1-\tau)R - C \tag{108}$$

As shown above, the formulas of firm's expected profit remains the same as the baseline, we only replace $E\left[\pi^b(x_{ij}|x_{-i})\right]$ with $E\left[\pi^b_\tau(x_{ij}|x_{-i})\right]$ for the bank's optimal problem. Here, we find two types of pure strategy Nash equilibria, that is, the screening equilibrium and the non-screening

[40] The "lend to $\hat{B}$ only" strategy is omitted given that it cannot be an equilibrium strategy, as showed in Lemma 4.

equilibrium. We denote Δ_τ as below

$$\begin{aligned}\Delta_\tau &\equiv E\left(\pi^f_{SS\tau}\right) - E\left(\pi^f_{NP\tau}\right)\\ &= f\left[G-(1-\tau)R-\frac{1}{\alpha f}\ln[\alpha f(1-f)(G-B)]-\frac{1}{\alpha f}\right]-[fG+(1-f)B-(1-\tau)R].\end{aligned}$$

The rest of the calculation is the same as the baseline.

G PROOF OF PROPOSITION 4

Under business subsidy policy, the government provides a subsidy S to all firms. The subsidy can be considered as a temporary increase of firm's output from G and B to $(G+S)$ and $(B+S)$. Note the size of the increase is exogenous.

If $B+S \geq R$, we can consider this case as $f=1$. Because how we define bad firms is their output smaller than the international interest rate. Under $f=1$, there is always a non-screening equilibrium.

If $B+S<R$, then $f<1$, and all firms' outputs increase S. We follow the setup of baseline, and adjust firms' outputs. [41]

Under the "lend to $\hat{G}$ only" strategy, bank's expected profit becomes

$$E\left[\pi^b(x_{ij}|x_{-i})\right] = Pr(\hat{G})\left[Pr(G|\hat{G},C)R_{\hat{G}} + Pr(B|\hat{G},C)(B+S) - R - \frac{C}{f}\right]. \tag{109}$$

Firm's expected profit becomes

$$E\left[\pi^f(x_{ij}|x_{-i})\right] = Pr(\hat{G})\left[Pr(G|\hat{G},C)(G+S-R_{\hat{G}})\right]. \tag{110}$$

Under the "lend to both with no cross subsidy" strategy, bank's expected profit becomes

$$\begin{aligned}E\left[\pi^b(x_{ij}|x_{-i})\right] =&Pr(\hat{G})[Pr(G|\hat{G},C)R_{\hat{G}} + Pr(B|\hat{G},C)(B+S)] + Pr(\hat{B})[Pr(G|\hat{B},C)R_{\hat{B}}+\\ &Pr(B|\hat{B},C)(B+S)] - R - C.\end{aligned} \tag{111}$$

[41] The "lend to $\hat{B}$ only" strategy is omitted given that it cannot be an equilibrium strategy, as showed in Lemma 4.

Firm's expected profit becomes

$$E\left[\pi^f(x_{ij}|x_{-i})\right] = Pr(G)\left[Pr(\hat{G}|G,C)(G+S-R_{\hat{G}}) + Pr(\hat{B}|G,C)(G+S-R_{\hat{B}})\right]. \tag{112}$$

Under the "lend to both with a cross subsidy" strategy, bank's expected profit becomes

$$E\left[\pi^b(x_{ij}|x_{-i})\right] = Pr(\hat{G})\left[Pr(G|\hat{G},C)R_{\hat{G}} + Pr(B|\hat{G},C)(B+S)\right] + Pr(\hat{B})R_{\hat{B}} - R - C. \tag{113}$$

Firm's expected profit becomes

$$\begin{aligned}E\left[\pi^f(x_{ij}|x_{-i})\right] =& Pr(G)\left[Pr(\hat{G}|G,C)(G+S-R_{\hat{G}}) + Pr(\hat{B}|G,C)(G+S-R_{\hat{B}})\right]\\ &+ Pr(B)\left[Pr(\hat{B}|B,C)(B+S-R_{\hat{B}})\right]\end{aligned} \tag{114}$$

We replace firm's and bank's expected profits into the setup of baseline, and find two types of pure strategy Nash equilibria: the screening equilibrium and the non-screening equilibrium.

Denote Δ_s as below.

$$\begin{aligned}\Delta_s &\equiv E\left(\pi^f_{SSs}\right) - E\left(\pi^f_{NPs}\right)\\ &= f\left[G+S-R-\frac{1}{\alpha f}\ln[\alpha f(1-f)(G-B)] - \frac{1}{\alpha f}\right] - [fG+(1-f)B+S-R].\end{aligned}$$

The rest of the calculation is the same as the baseline.

H PROOF OF PROPOSITION 5

Under public loan guarantees, the government provides a complete loan guarantee plan *ex ante* with the required loan rate contingent on the signal, $R_{\hat{G}}$ or $R_{\hat{B}}$. The government expenditure on loan guarantees is from the lump sum tax, paid by consumers. For a given loan contract, if a borrower repays, the borrower will pay the required loan rate in the contract; if a borrower defaults, the lender will acquire the borrower's output, and the government will pay the rest of the repayment to achieve the overall repayment at $R_{\hat{G}}$ or $R_{\hat{B}}$. Due to the competitive banking industry, banks' zero-profit constraint is still binding.

We follow the setup of baseline and adjust the expressions of bank's expected profit. Under the "lend to $\hat{G}$ only" strategy, bank's expected profit becomes

$$E\left[\pi^b(x_{ij}|x_{-i})\right] = Pr(\hat{G})\left(R_{\hat{G}} - R - \frac{C}{f}\right) = fR_{\hat{G}} + (1-f)R - R - C. \quad (115)$$

Firm's expected profit remains

$$E\left[\pi^f(x_{ij}|x_{-i})\right] = Pr(\hat{G})\left[Pr(G|\hat{G},C)(G - R_{\hat{G}})\right] = ff_G(G - R_{\hat{G}}). \quad (116)$$

Under the "lend to both with no cross subsidy" strategy, bank's expected profit becomes

$$E\left[\pi^b(x_{ij}|x_{-i})\right] = Pr(\hat{G})R_{\hat{G}} + Pr(\hat{B})R_{\hat{B}} - R - C = fR_{\hat{G}} + (1-f)R_{\hat{B}} - R - C. \quad (117)$$

Firm's expected profit remains

$$\begin{aligned} E\left[\pi^f(x_{ij}|x_{-i})\right] &= Pr(G)\left[Pr(\hat{G}|G,C)(G - R_{\hat{G}}) + Pr(\hat{B}|G,C)(G - R_{\hat{B}})\right] \\ &= f\left[G - f_G R_{\hat{G}} - (1-f_G)R_{\hat{B}}\right] = ff_G(G - R_{\hat{G}}) + f(1-f_G)(G - R_{\hat{B}}). \quad (118) \end{aligned}$$

We can show that the "lend to $\hat{G}$ only" strategy is weakly dominated by the "lend to both with no cross subsidy" strategy. For any loan contract under the "lend to $\hat{G}$ only" strategy, $R_{\hat{G}}$ and C satisfies (115) equals to zero (bank's zero profit constraint). There is always a contract under the "lend to both with no cross subsidy" strategy, where $R_{\hat{G}}$ and C are the same as the former contract, and let $R_{\hat{B}} = R$. Bank's zero profit constraint in (117) is satisfied, and firm's expected profit (118) is bigger than in (116), since $R_{\hat{B}} = R < G$.

Under the "lend to both with no cross subsidy" strategy, following the similar calculation as in the baseline model, we find that the optimal screening cost and the required repayments satisfy

$$C^* = 0 \quad (119)$$

$$fR^*_{\hat{G}} + (1-f)R^*_{\hat{B}} = R \quad (120)$$

Firm's expected profit (in stage 2), bank's expected profit (in stage 1), and depositor's expected return are denoted as $E\left[\pi^f(x^*_{ij})\right]$, $E\left[\pi^b(x^*_{ij})\right]$, and $E(\pi^d)$, as below, which are irrelevant to the allocation between $R^*_{\hat{G}}$ and $R^*_{\hat{B}}$.[42]

$$E\left[\pi^f(x^*_{ij})\right] = f(G-R) + (1-f)(B-B) = f(G-R). \tag{121}$$

$$E\left[\pi^b(x^*_{ij})\right] = 0. \tag{122}$$

$$E(\pi^d) = R - (1-f)(R-B) = fR + (1-f)B. \tag{123}$$

Next, we discuss the "lend to both with a cross subsidy" strategy, and divide into two subcases: (1) $fG \geq R$; (2) $fG < R$.

Subcase (1): When $fG \geq R$, assume G-type firms with $\hat{G}$ signal repay, bank's expected profit becomes

$$E\left[\pi^b(x_{ij}|x_{-i})\right] = Pr(\hat{G})R_{\hat{G}} + Pr(\hat{B})R_{\hat{B}} - R - C = fR_{\hat{G}} + (1-f)R_{\hat{B}} - R - C. \tag{124}$$

Firm's expected profit remains

$$\begin{aligned} E\left[\pi^f(x_{ij}|x_{-i})\right] =& Pr(G)\left[Pr(\hat{G}|G,C)(G-R_{\hat{G}}) + Pr(\hat{B}|G,C)(G-R_{\hat{B}})\right] \\ &+ Pr(B)\left[Pr(\hat{B}|B,C)(B-R_{\hat{B}})\right] \\ =& f\left[G - f_G R_{\hat{G}} - (1-f_G)R_{\hat{B}}\right] + (1-f)f_B\left(B - R_{\hat{B}}\right). \end{aligned} \tag{125}$$

Following the similar calculation as in the baseline model, we find that the optimal screening cost

$$C^* = 0.$$

Using bank's zero profit constraint, firm's expected profit is a decreasing function of $R_{\hat{B}}$.

$$\begin{aligned} E\left[\pi^f(x_{ij}|x_{-i})\right] &= f\left[G - fR_{\hat{G}} - (1-f)R_{\hat{B}}\right] + (1-f)^2\left(B - R_{\hat{B}}\right) \\ &= f(G-R) + (1-f)^2\left(B - R_{\hat{B}}\right). \end{aligned}$$

[42] Intuitively, G firms repay either $R^*_{\hat{G}}$ or $R^*_{\hat{B}}$ contingent on signals. Given $C^* = 0$, with probability f, a G firm repay $R^*_{\hat{G}}$, and with probability $(1-f)$, a G firm repay $R^*_{\hat{B}}$. B firms default regardless of signals. Hence, the allocation between $R^*_{\hat{G}}$ and $R^*_{\hat{B}}$ does not matter, as long as they satisfy $fR^*_{\hat{G}} + (1-f)R^*_{\hat{B}} = R$.

So, $E\left[\pi^f(x_{ij}|x_{-i})\right]$ is maximized when $R_{\hat{B}}$ takes the minimal value as 0, hence

$$R^*_{\hat{B}} = 0, R^*_{\hat{G}} = \frac{R}{f}.$$

Since $fG \geq R$, $R^*_{\hat{G}} \leq G$, a G-type firm with $\hat{G}$ signal is able to repay the loan. Firm's expected profit (in stage 2), bank's expected profit (in stage 1), and depositor's expected return are denoted as $E\left[\pi^f(x^*_{ij})\right]$, $E\left[\pi^b(x^*_{ij})\right]$, and $E(\pi^d)$, as below

$$E\left[\pi^f(x^*_{ij})\right] = f(G-R) + (1-f)^2 B,$$
$$E\left[\pi^b(x^*_{ij})\right] = 0,$$
$$E(\pi^d) = fR + f(1-f)B.$$

Intuitively, G firms repay either $R^*_{\hat{G}}$ or $R^*_{\hat{B}}$ contingent on signals. Specifically, given $C^* = 0$, a G firm repay $R^*_{\hat{G}}$ conditional on $\hat{G}$ signals (with probability f), and repay $R^*_{\hat{B}}$ conditional on $\hat{B}$ signals (with probability $(1-f)$). B firms default conditional on $\hat{G}$ signals (with probability f), and repay $R^*_{\hat{B}}$ conditional on $\hat{B}$ signals (with probability $(1-f)$). In stage 2, a firm chooses a bank to maximize firm's expected profit before realizing firms' types. With probability $Pr(G) = f$, the firm is G type, the allocation between $R^*_{\hat{G}}$ and $R^*_{\hat{B}}$ does not matter; with probability $Pr(B) = 1-f$, the firm is B type, the lower the $R^*_{\hat{B}}$, the higher the firm's expected profit in stage 2. Hence, the optimal solution is the corner solution where $R^*_{\hat{B}} = 0$.

Subcase (2): When $fG < R$, assume G-type firms with $\hat{G}$ signal default, and bank's and firm's expected profits become

$$E\left[\pi^b(x_{ij}|x_{-i})\right] = Pr(\hat{G})R_{\hat{G}} + Pr(\hat{B})R_{\hat{B}} - R - C = fR_{\hat{G}} + (1-f)R_{\hat{B}} - R - C. \tag{126}$$

$$\begin{aligned} E\left[\pi^f(x_{ij}|x_{-i})\right] &= Pr(G)Pr(\hat{B}|G,C)(G - R_{\hat{B}}) + Pr(B)Pr(\hat{B}|B,C)(B - R_{\hat{B}}) \\ &= f(1-f_G)(G - R_{\hat{B}}) + (1-f)f_B\left(B - R_{\hat{B}}\right). \end{aligned} \tag{127}$$

Following the similar calculation as in the baseline model, we find that the optimal screening cost and equilibrium repayments satisfy

$$C^* = 0,$$
$$fR^*_{\hat{G}} + (1-f)R^*_{\hat{B}} = R.$$

Using bank's zero profit constraint, firm's expected profit can be simplified as a decreasing function of $R_{\hat{B}}$.

$$\begin{aligned} E\left[\pi^f(x_{ij}|x_{-i})\right] &= f(1-f_G)(G-R_{\hat{B}}) + (1-f)f_B\left(B-R_{\hat{B}}\right) \\ &= (1-f)[(1-f_B)G + f_B B - R_{\hat{B}}] \\ &= (1-f)[fG + (1-f)B - R_{\hat{B}}]. \end{aligned}$$

Hence, the optimal required repayments are

$$R^*_{\hat{B}} = 0, R^*_{\hat{G}} = \frac{R}{f}.$$

Since $fG < R$, a G-type firm with $\hat{G}$ signal is unable to repay the loan. Firm's expected profit (in stage 2), bank's expected profit (in stage 1), and depositor's expected return are denoted as $E\left[\pi^f(x^*_{ij})\right]$, $E\left[\pi^b(x^*_{ij})\right]$, and $E(\pi^d)$, as below.

$$E\left[\pi^f(x^*_{ij})\right] = (1-f)[fG + (1-f)B].$$
$$E\left[\pi^b(x^*_{ij})\right] = 0.$$
$$E(\pi^d) = f[fG + (1-f)B].$$

Intuitively, all firms default conditional on $\hat{G}$ signals and repay conditional on $\hat{B}$ signals. The lower the $R_{\hat{B}}$, the higher the firm's expected profit. Since firms choose the loan contract that maximizes firm's expected profit, the optimal solution is the corner solution where $R^*_{\hat{B}} = 0$.

Next, we check the corner solution, let $R^*_{\hat{G}} = G$, then $R^*_{\hat{B}} = \frac{R-fG}{1-f} > 0$. Since $E(\pi^f)$ is a decreasing function of $R_{\hat{B}}$, $E(\pi^f)|_{R^*_{\hat{B}}=\frac{R-fG}{1-f}} < E(\pi^f)|_{R^*_{\hat{B}}=0}$. Hence, there is no corner solution such that $R^*_{\hat{G}} = G$.

To summarize subcases (1) and (2), under the "lend to both with a cross subsidy" strategy, the optimal screening cost and the required repayments are as below,

$$C^* = 0, \tag{128}$$

$$R^*_{\hat{B}} = 0, R^*_{\hat{G}} = \frac{R}{f}. \tag{129}$$

In stage 1, by bank zero profit constraint, bank's expected profit is denoted as

$$E\left[\pi^b(x^*_{ij})\right] = 0 \tag{130}$$

In stage 2, firm's expected profit is denoted as $E\left[\pi^f(x^*_{ij})\right]$, as below

$$E\left[\pi^f(x^*_{ij})\right] = \begin{cases} f(G-R) + (1-f)^2 B, & \text{if} \quad fG \geq R, \\ (1-f)[fG + (1-f)B], & \text{if} \quad fG < R. \end{cases} \tag{131}$$

In addition, depositor's expected return are denoted as $E(\pi^d)$, as below.

$$E(\pi^d) = \begin{cases} fR + f(1-f)B, & \text{if} \quad fG \geq R, \\ f[fG + (1-f)B], & \text{if} \quad fG < R. \end{cases} \tag{132}$$

By (121) and (28), we show that firm's expected profit under the "lend to both with no cross subsidy" strategy is smaller than firm's expected profit under the "lend to both with a cross subsidy" strategy.[43] Hence, the "lend to both with no cross subsidy" strategy is dominated by the "lend to both with a cross subsidy" strategy.

The rest of the calculation is the same as the baseline.

I Different Production Technology: constant return to scale

In the baseline, we assume the production technology has a fixed scale, that is, each firm has 1 unit of project. In this section, we instead assume the production technology has a constant return to scale, denoted as CRS case: for ψ unit of capital, the production could be either ψG or ψB. We still keep the assumption that there are m unit of funding in the market and no access to foreign

[43] Particularly, when $fG < R$, $f(G-R) < (1-f)R < (1-f)[fG + (1-f)B]$. We assume $fG + (1-f)B > R$, in the sense that $fG + (1-f)B$ is the average market asset quality, and R is the opportunity cost of investing abroad. If $fG + (1-f)B \leq R$, banks (depositors) will not enter domestic market in the first place.

investment for domestic firms.

We assume investors either invest all in domestic market or invest all abroad, since we relax the constraint on loan size. In screening strategy, we assume for each $\hat{G}$ firm, the loan size becomes bigger than one, which is $1/Pr(\hat{G}) = 1/f$; for each $\hat{B}$ firm, the loan size is 0. In non-screening strategy, we assume for all firms, the loan size is 1, the same as in the baseline. Depositor's return of saving in banks is still R for one unit of deposit for any equilibrium.

A Equilibrium

The calculation of the "lend to both $\hat{G}$ and $\hat{B}$" strategy is the same as the baseline, but the calculation of the "lend to $\hat{G}$ only" strategy includes a new part with loan size, $1/Pr(\hat{G})$, compared to the baseline. The bank maximization problem in stage 1 is adjusted as below (consistent to (35), (36), (37), and (38) in the baseline).

$$
\begin{aligned}
\max_{x_{ij}\in\mathbf{S_1}} \quad & E\left[\pi^b(x_{ij}|x_{-i})\right] = \frac{1}{Pr(\hat{G})}Pr(\hat{G})\left[Pr(G|\hat{G},C)R_{\hat{G}} + Pr(B|\hat{G},C)B - R - \frac{C}{f}\right] \\
s.t. \quad & E\left[\pi^f\left(x_{ij}\right)\right] = \frac{1}{Pr(\hat{G})}Pr(G)Pr(\hat{G}|G,C)(G - R_{\hat{G}}) \geq 0 \\
& E\left[\pi^f\left(x_{ij}\right)\right] = \frac{1}{Pr(\hat{G})}Pr(G)Pr(\hat{G}|G,C)(G - R_{\hat{G}}) \geq E\left[\pi^f(\tilde{x}_{hj})\right], \text{ for } \forall h \neq i, \forall \tilde{x}_{hj} \in x_j \\
& E\left[\pi^b(x_{ij}|x_{-i})\right] = \frac{1}{Pr(\hat{G})}Pr(\hat{G})\left[Pr(G|\hat{G},C)R_{\hat{G}} + Pr(B|\hat{G},C)B - R - \frac{C}{f}\right] = 0
\end{aligned}
$$

Following the similar calculation as in the baseline (Section III), we have the following proposition.

Proposition 7. *Under the constant-return-to-scale (CRS) production technology, there are three types of pure strategy Nash Equilibria,the screening separating equilibrium (SS), the non-screening pooling equilibrium (NP), and the non-screening cheap-information based separating equilibrium (NS), depending on the parameter values.*
Define[44]

$$\Delta^c = [G - R - \frac{1}{\alpha}ln[\alpha(1-f)(G-B)] - \frac{1}{\alpha}] - [fG + (1-f)B - R].$$

Case (1): *When* $\alpha(1-f)(G-B) > 1$, $\Delta^c \geq 0$, *and* $G - R - \frac{1}{\alpha}ln[\alpha(1-f)(G-B)] - \frac{1}{\alpha} \geq 0$,

[44]Here, we interpret Δ^c as the difference of firm's expected profits in stage 2 under optimal contracts of the "lend to $\hat{G}$ only" strategy and of the "lend to both" strategy.

there is a screening separating equilibrium (SS), in which the "lend to $\hat{G}$ only" strategy is the equilibrium strategy. In the equilibrium contract x^*_{ij}, the equilibrium screening cost C^* and the equilibrium repayment of a $\hat{G}$ firm $R^*_{\hat{G}}$ are as below.

$$C^* = \frac{1}{\alpha}\ln[\alpha(1-f)(G-B)].$$

$$R^*_{\hat{G}} = \frac{R + C^* - \frac{1}{\alpha(G-B)}B}{1 - \frac{1}{\alpha(G-B)}}.$$

In stage 1, bank i use the "lend to $\hat{G}$ only" strategy and offer the contract x^*_{ij}, and bank i's expected profit from firm j before screening is

$$E[\pi^b(x^*_{ij})] = \frac{1}{Pr(\hat{G})}Pr(\hat{G})\left[Pr(G|\hat{G},C)R_{\hat{G}} + Pr(B|\hat{G},C)B - R - \frac{C}{f}\right] = 0.$$

In stage 2, firm j's expected profit before screening is

$$E\left[\pi^f(x^*_{ij})\right] = \frac{1}{Pr(\hat{G})}Pr(\hat{G})Pr(\hat{G}|G,C)(G - R_{\hat{G}}) = G - R - \frac{1}{\alpha}ln[\alpha(1-f)(G-B)] - \frac{1}{\alpha}.$$

In stage 3, bank i's expected profit from firm j after screening is contingent on the signal of firm j,

$$E\left[\pi^b(x^*_{ij})|\hat{G}\right] = \frac{1}{Pr(\hat{G})}\left[Pr(G|\hat{G},C)R_{\hat{G}} + Pr(B|\hat{G},C)B - R - \frac{C}{f}\right] = 0.$$

$$E\left[\pi^b(x^*_{ij})|\hat{B}\right] = 0.$$

In stage 4, firm j's expected profit after screening is contingent on signals,

$$E\left[\pi^f(x^*_{ij})|\hat{G}\right] = \frac{1}{Pr(\hat{G})}Pr(G|\hat{G},C)(G - R_{\hat{G}}) = \frac{1}{f}\left[G - R - \frac{1}{\alpha}ln[\alpha(1-f)(G-B)] - \frac{1}{\alpha}\right].$$

$$E\left[\pi^f(x^*_{ij})|\hat{B}\right] = 0.$$

In stage 5, firm j's profit after production is contingent on signals and real types,

$$\pi^f\left(x^*_{ij}|\hat{G},G\right) = \frac{1}{Pr(\hat{G})}(G - R_{\hat{G}}) = \frac{1}{f}\left[G - \frac{R + C^* - \frac{1}{\alpha(G-B)}B}{1 - \frac{1}{\alpha(G-B)}}\right].$$

$$\pi^f\left(x^*_{ij}|\hat{G},B\right) = 0.$$

In stage 5, bank i's profit from firm j after production is contingent on signals and real types,

$$\pi^b\left(x_{ij}^*|\hat{G},G\right)=\frac{1}{Pr(\hat{G})}\left(R_{\hat{G}}^*-R-\frac{C^*}{f}\right)=\frac{1}{f}\left[\frac{R+C^*-\frac{1}{\alpha(G-B)}B}{1-\frac{1}{\alpha(G-B)}}-R-\frac{1}{\alpha f}\ln[\alpha(1-f)(G-B)]\right].$$
$$\pi^b\left(x_{ij}^*|\hat{G},B\right)=\frac{1}{Pr(\hat{G})}\left(B-R-\frac{C^*}{f}\right)=\frac{1}{f}\left[B-R-\frac{1}{\alpha f}\ln[\alpha(1-f)(G-B)]\right].$$
$$\pi^b\left(x_{ij}^*|\hat{B},B\right)=0.$$
$$\pi^b\left(x_{ij}^*|\hat{B},G\right)=0.$$

Case (2): *When $\alpha(1-f)(G-B)>1$, $\Delta^c<0$, and $fG+(1-f)B\geq R$, or when $\alpha(1-f)(G-B)\leq 1$ and $fG+(1-f)B\geq R$, there are the non-screening pooling equilibrium (NP), and the non-screening cheap-information based separating equilibrium (NS), and the characterizations of the equilibria are the same as the baseline (2) in Proposition 1.*

B Comparative Analysis on NI and GDP

In CRS case, the expected NI and GDP in "SS" and "NP" equilibria are calculated as below.[45]

$$E(NI_{SS}^c)=E(GDP_{SS}^c)=\left[G-\frac{1}{\alpha}-\frac{1}{\alpha}ln[\alpha(1-f)(G-B)]\right]m. \tag{133}$$

$$E(NI_{NP}^c)=E(GDP_{NP}^c)=[fG+(1-f)B]m. \tag{134}$$

Although the values of NI and GDP of the screening equilibrium in the CRS case are different from the values in the baseline, the values of NI and GDP of the non-screening equilibrium in the CRS case are the same as the values in the baseline. The existence condition of the screening equilibrium in the CRS case are also different from the existence condition in the baseline. In the non-screening equilibrium, banks use all the funds in the domestic market in both the baseline and the CRS case, but in the screening equilibrium, banks use all the funds in the domestic market only in the CRS case.

For the comparative analysis, we discuss changes in five parameter values, consistent with the baseline (Appendix E). The key difference is that at the switch points between SS and NP, there are no kinks in the expected NI curve and no gaps in the expected GDP curves. Details are below.

The α case is showed in Figure 16, similar to the baseline. We first introduce two key values of α:

---

[45] Same as in the baseline, the the non-screening pooling equilibrium (NP) and the non-screening cheap-information based separating equilibrium (NS) are indifferent in terms of the expected NI and GDP. Without loss of generality, we omit NS equilibrium and focus on SS and NP equilibria.

At $\alpha^{c*} = \frac{1}{(1-f)(G-B)}$, $C^{c*} = 0$, we find that $\Delta^c = 0, \partial\Delta^c/\partial\alpha = 0, \partial NI/\partial\alpha|\alpha^{c*}_{-} = \partial NI/\partial\alpha|\alpha^{c*}_{+}$. At α^c_{SS}, firm's expected profit in screening strategy $E[\pi^{fc}_{SS}] = 0$. Note that firm's expected profit in non-screening strategy does not relate to α. The upper row shows the case that $fG + (1-f)B - R \geq 0$. When $\alpha \in (0, \alpha^{c*})$, there is a non-screening equilibrium, in which the NI does not depend on α. When $\alpha \in [\alpha^{c*}, 1)$, there is a screening equilibrium. The lower row shows the case that $fG + (1-f)B - R < 0$. When $\alpha \in (0, \alpha^c_{SS})$, all investment go abroad. When $\alpha \in [\alpha^c_{SS}, 1)$, there is a screening equilibrium.

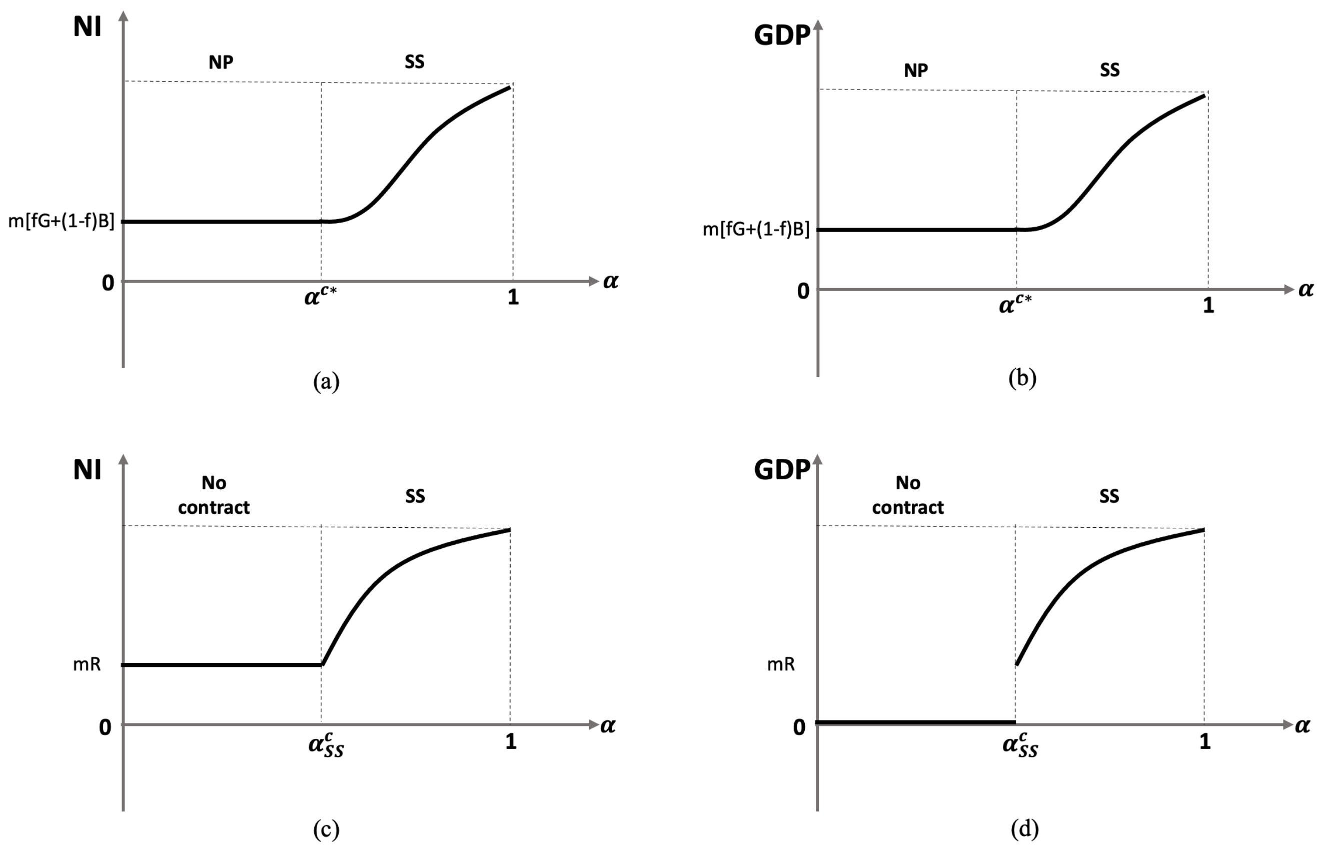


Figure 16: Comparative analysis: α case under CRS

The f case is similar to the baseline, as in Figure 17. We first introduce three key values of f: At $f^{c*} = 1 - \frac{1}{\alpha(G-B)}$, $C^{c*} = 0$, and $\Delta^c = 0$, we find that $\partial\Delta^c/\partial f = 0, \partial NI/\partial f|_{f^{c*}_{-}} = \partial NI/\partial f|_{f^{c*}_{+}}$. At f^c_{SS}, firm's expected profit in screening strategy $E[\pi^{fc}_{SS}] = 0$. At f^c_{NP}, firm's expected profit in non-screening strategy $E[\pi^{fc}_{NP}] = 0$. The upper row shows the cases that screening equilibrium exists for some $f \in (0,1)$.[46] When $f \in (0, f^c_{SS})$, all investments go abroad, $NI = mR, GDP = 0$.

[46]Here, we only show the case that $f^c_{SS} \geq 0$. $E[\pi^{fc}_{SS}]$ is an increasing function of f, so $E[\pi^{fc}_{SS}] \geq 0$ when $f \geq f^c_{SS}$.

When $f \in [f^c_{SS}, f^{c*})$, there is a screening equilibrium, and NI is a convex and increasing function of f. When $f \in [f^{c*}, 1)$, there is a non-screening equilibrium, and NI is a linear and increasing function of f. [47] The lower row shows the cases that screening equilibrium does not exist for all $f \in (0, 1)$. When $f \in (0, f^c_{NP})$, there is no equilibrium. When $f \in [f^c_{NP}, 1)$, there is a non-screening equilibrium.

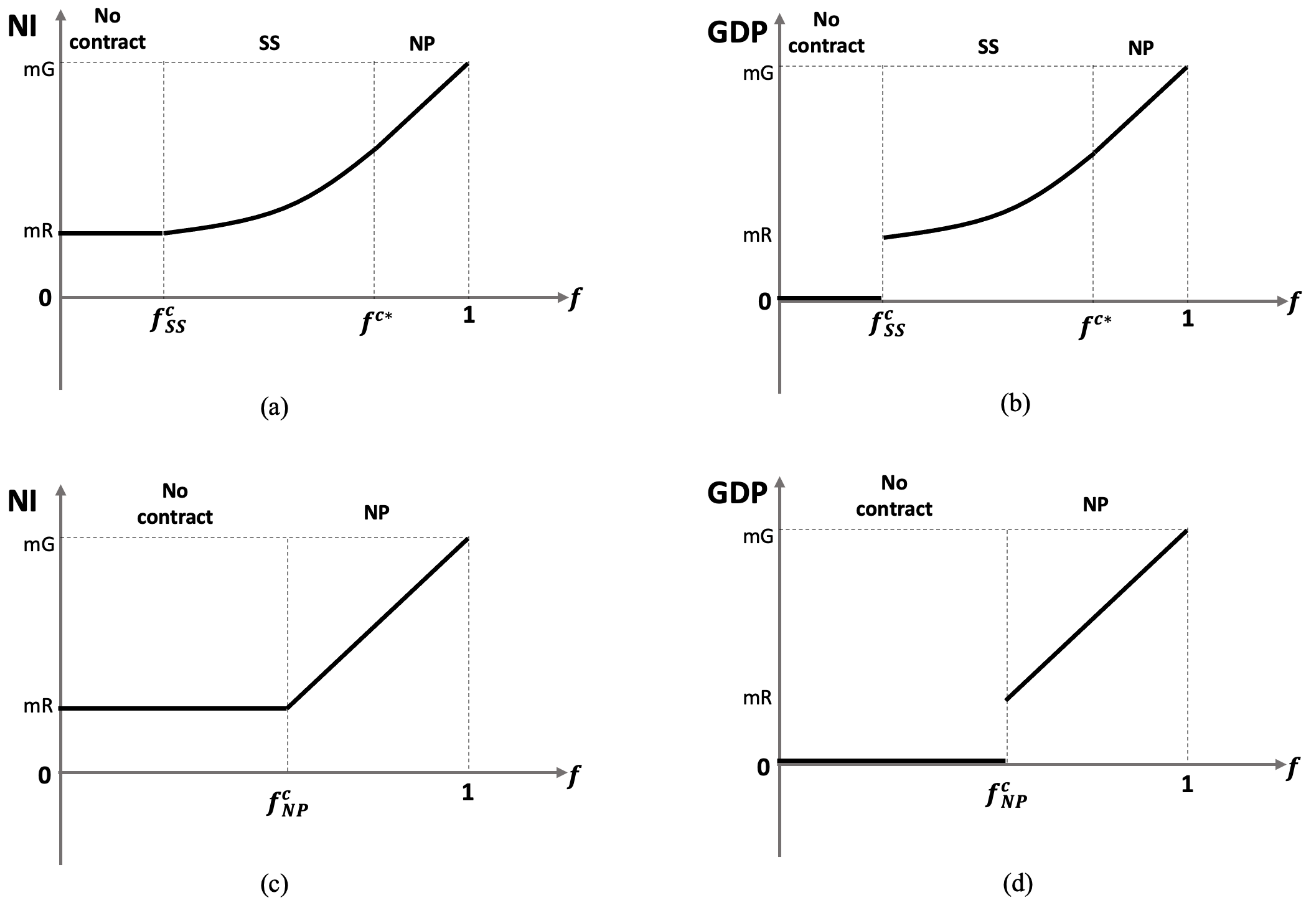


Figure 17: Comparative analysis: f case under CRS

The R case is showed in Figure 18. Different from the baseline, the switching point between screening and non-screening equilibria does not depend on R. We introduce two key values of R. At R^c_{SS}, firm's expected profit in screening strategy $E[\pi^{fc}_{SS}] = 0$. At R^c_{NP}, firm's expected profit in non-screening strategy $E[\pi^{fc}_{NP}] = 0$. The upper row shows the case $G - \frac{1}{\alpha} - \frac{1}{\alpha} ln[\alpha(1-f)(G-B)] > fG + (1-f)B$. When $R \in (B, R^c_{SS})$, there is a screening

When $f^c_{SS} < 0$, the "no contract" area disappears, but the key message is the same.

[47] A main difference from the baseline is that in the CRS case, NI is differentiable at f^{c*} and there is no jump of GDP. The intuition is that banks either invest all capital in the domestic market or invest all in foreign markets.

equilibrium. When $R \in [R^c_{SS}, G)$, all investments go abroad. The lower row shows the case $G - \frac{1}{\alpha} - \frac{1}{\alpha} ln[\alpha(1-f)(G-B)] \leq fG + (1-f)B$. When $R \in (B, R^c_{NP})$, there is a non-screening equilibrium. When $R \in [R^c_{NP}, G)$, all investments go abroad.

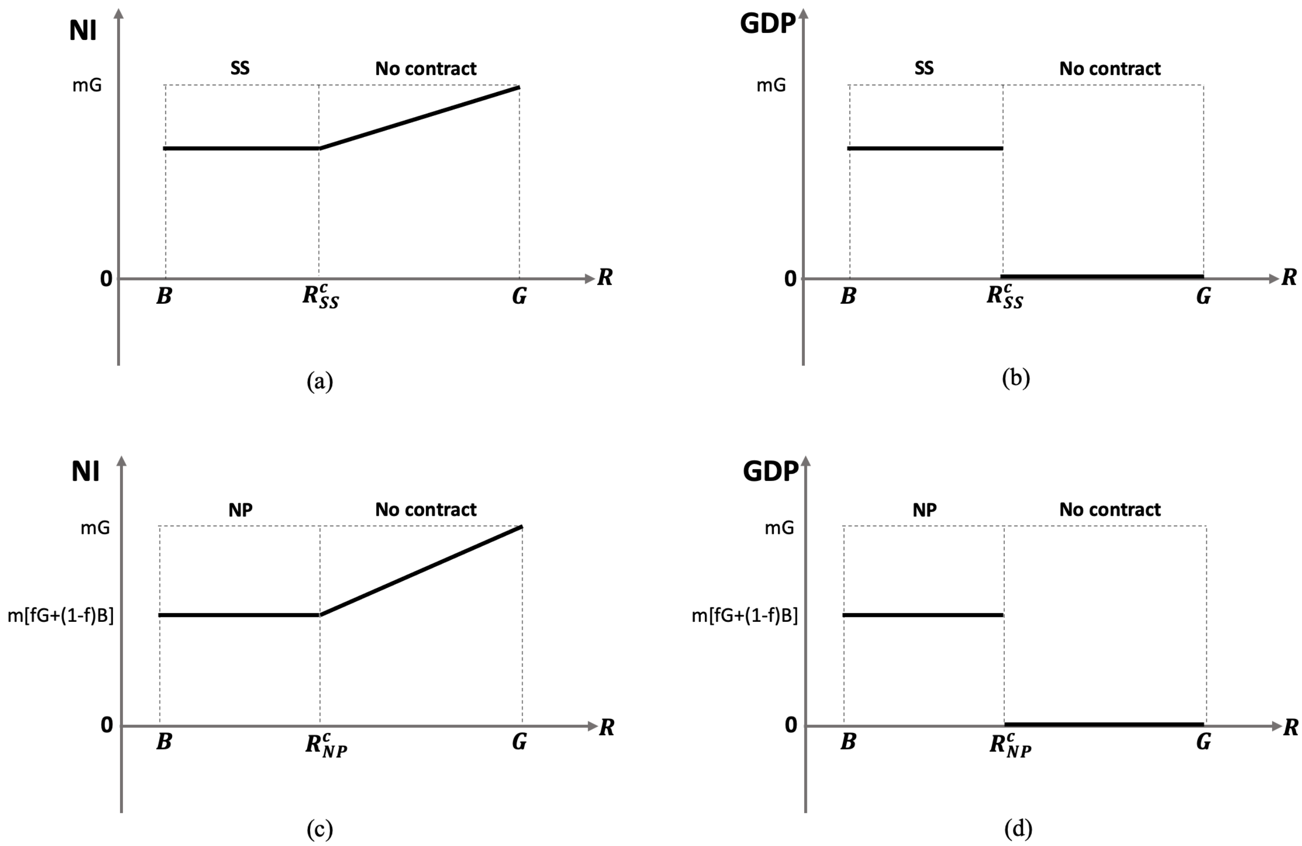


Figure 18: Comparative analysis: R case under CRS

The G case is showed in Figure 19. We first introduce three key values of G: At G^c_{SS}, firm's expected profit in screening strategy $E[\pi^{fc}_{SS}] = 0$. At G^c_{NP}, firm's expected profit in non-screening strategy $E[\pi^{fc}_{NP}] = 0$. At $G^{c*}, C^{c*} = 0$, and $\Delta^c = 0$, we find that $\partial \Delta^c / \partial G = 0, \partial NI/\partial G|_{G^{c*}_-} = \partial NI/\partial G|_{G^{c*}_+}$. In Figure 19, the upper row characterizes the case that non-screening equilibrium exists for some $G \in (R, \infty)$. The lower row characterizes the case that non-screening equilibrium does not exist for all $G \in (R, \infty)$. The CRS results are fairly different from the baseline. When G is high, the equilibrium in the baseline tends to be a non-screening equilibrium, while the equilibrium in the CRS case tends to be a screening equilibrium. Intuitively, when good firms have a high level of production,that is, G is high, banks will lend to good firms as much as possible. The baseline assumes a fixed-size loan contract: When G is high, banks would like to given money to as many good firms as possible, even at the expense of making mistakes of lending to bad firms, thus, there is a non-screening equilibrium. The CRS case assumes that the production technology is constant-return-to-scale: The aggregate loan amount in the market is the same for screening and non-screening strategies. hence the loan size is endogenous, and the loan size in screening strategy is bigger than in the non-screening strategy. G-type firms gets more money in screening strategy than in non-screening strategy, hence there is a screening equilibrium.

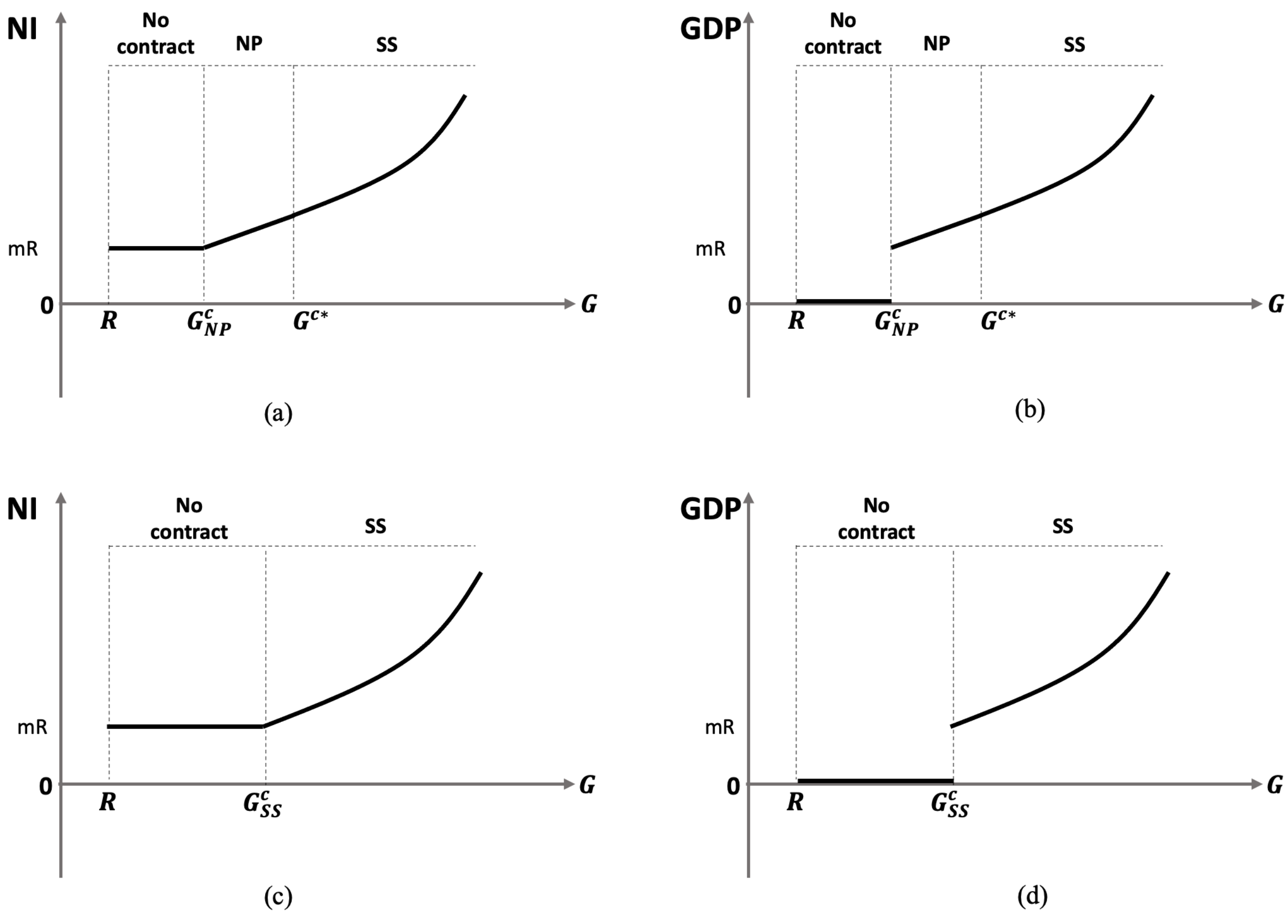


Figure 19: Comparative analysis: G case under CRS

The B case is showed in Figure 20, similar to the baseline. We first introduce three key values of B: At B^c_{SS}, firm's expected profit in screening strategy $E[\pi^{fc}_{SS}] = 0$. At B^c_{NP}, firm's expected profit in non-screening strategy$E[\pi^{fc}_{NP}] = 0$. At $B^{c*}, C^{c*} = 0$, and $\Delta^c = 0$, we find that $\partial \Delta^c / \partial B = 0, \partial NI/\partial B|_{B^{c*}_-} = \partial NI/\partial B|_{B^{c*}_+}$. The upper row characterizes the cases that screening equilibrium exists for some $B \in [0, R)$. The lower row characterizes the cases that screening equilibrium does not exist for all $B \in [0, R)$.

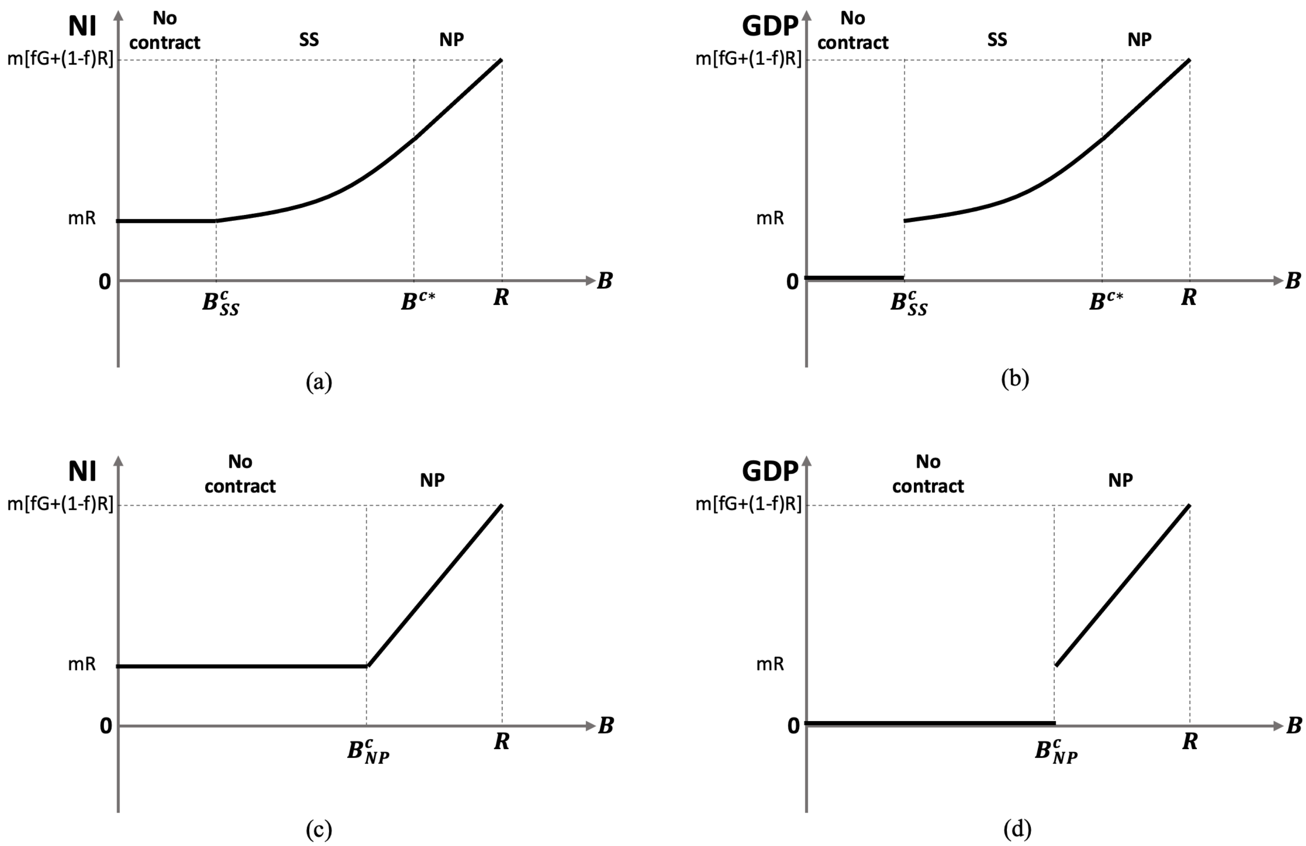


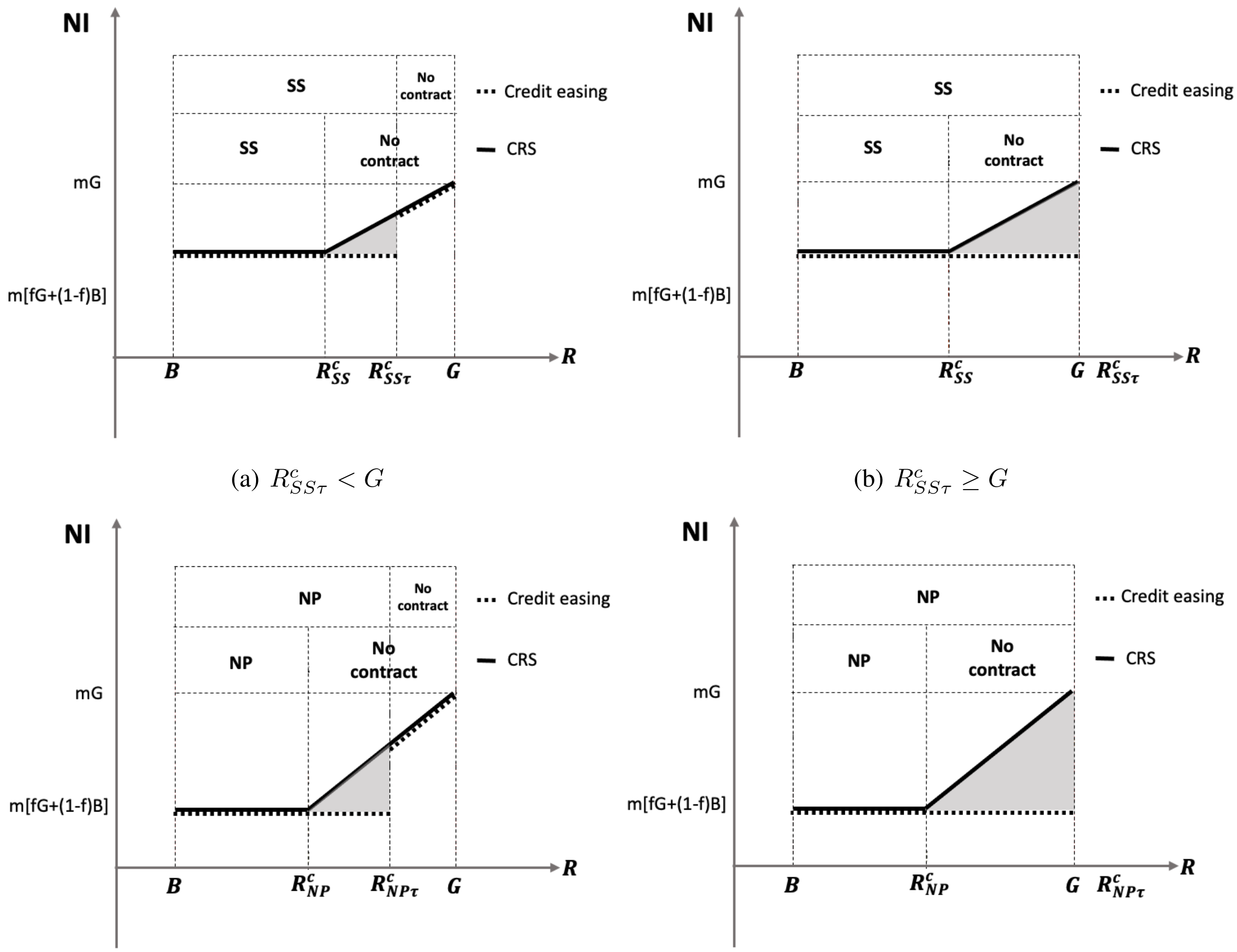
Figure 20: Comparative analysis: B case under CRS

C Policy Implications

In this subsection, we discuss the same policies as in Section V (credit easing, business subsidies, and public loan guarantees), under the CRS settings. We show the effects in various R values.

1 Credit Easing

Denote $R^c_{SS\tau}$ satisfying $(1-\tau)R^c_{SS\tau} = R^c_{SS}$, and $R^c_{NP\tau}$ satisfying $(1-\tau)R^c_{NP\tau} = R^c_{NP}$. Figure 21a shows the level of NI under all kinds of parameters range. The upper row shows the case that $\exists R \in (B, G)$, screening separating equilibrium exists in the baseline. The lower row shows the case that for all $R \in (B, G)$, screening separating equilibrium does not exist in the baseline. With credit easing, all thresholds move rightward. Figure 21b shows the changes of GDP under the same setup of parameter values as in Figure 21a. The shaded areas are the changes of aggregate values. As seen from the figures, NI decreases for some values of R, while GDP increases for some values of R. The intuition is the same as the baseline.



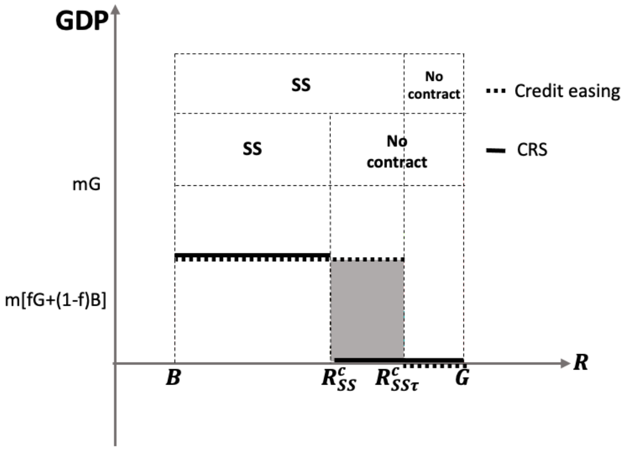
(a) $R^c_{SS\tau} < G$

(b) $R^c_{SS\tau} \geq G$

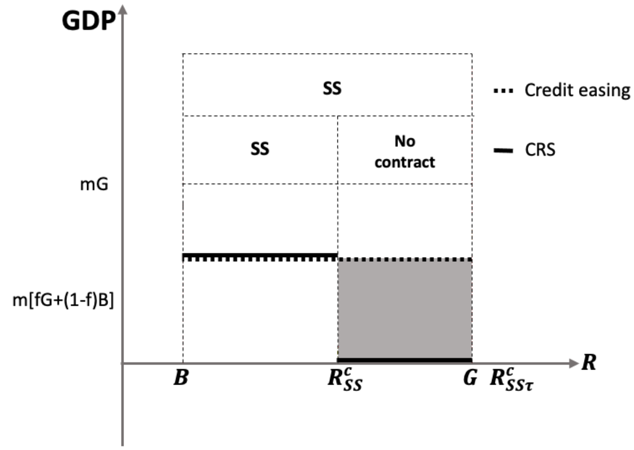
(c) $R^c_{NP\tau} < G$

(d) $R^c_{NP\tau} \geq G$

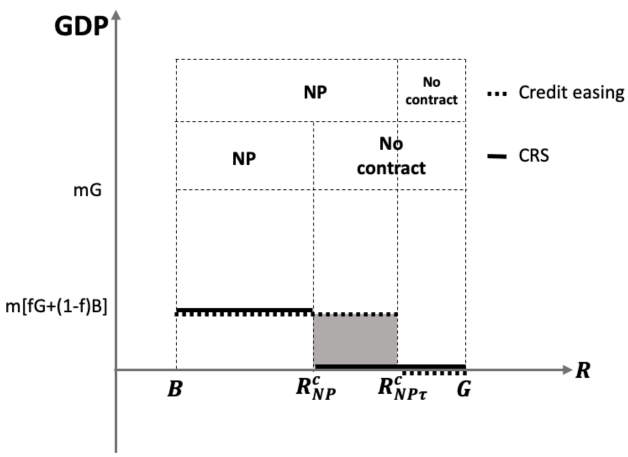
Figure 21a: Changes in NI under credit easing (CRS case)



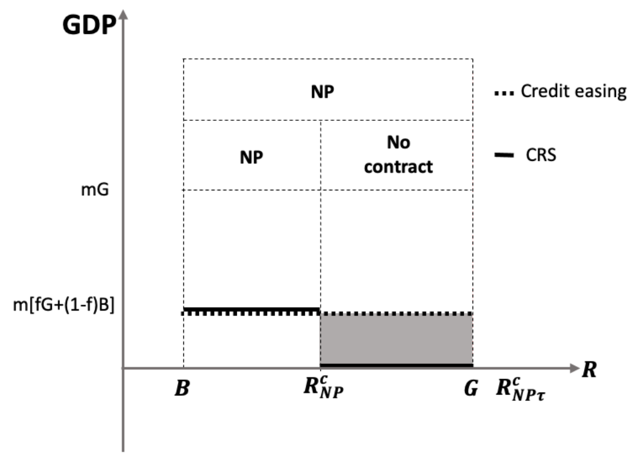
(e) $R^c_{SS\tau} < G$



(f) $R^c_{SS\tau} \geq G$



(g) $R^c_{NP\tau} < G$

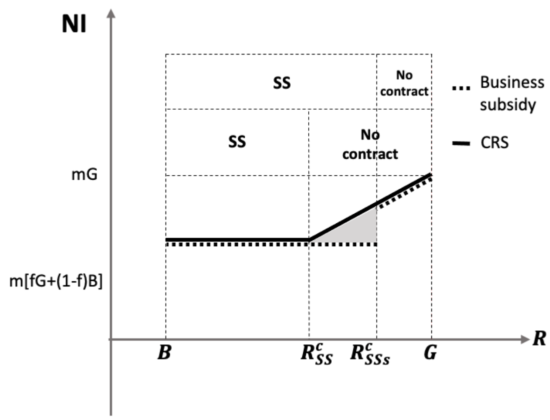


(h) $R^c_{NP\tau} \geq G$

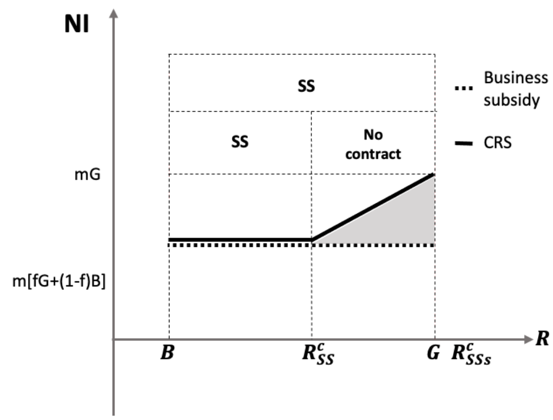
Figure 21b: Changes in GDP under credit easing (CRS case)

2 Business Subsidy

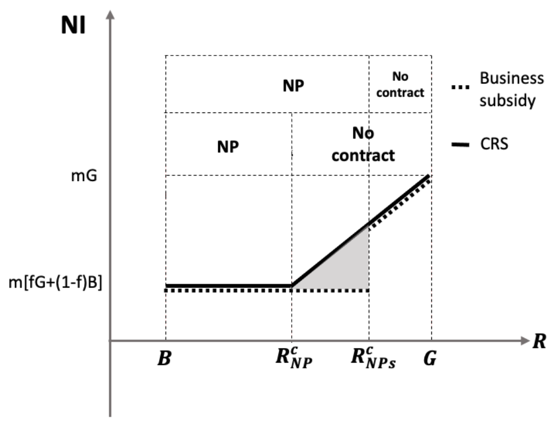
Denote R^c_{SSs} that satisfies $E\left[\pi^{fc}_{SSs}(R_{SSs})\right] = 0$, and R^c_{NPs} that satisfies $E\left[\pi^{fc}_{NPs}(R_{NPs})\right] = 0$. Figure 22a shows the level of NI under all kinds of parameters range. The upper row shows the case that $\exists R \in (B, G)$, screening separating equilibrium exists. The lower row shows the case that for all $R \in (B, G)$, screening separating equilibrium does not exist. With business subsidy, all thresholds move rightward. Figure 22b shows the changes of GDP under the same setup of parameter values as in Figure 22a. The shaded areas are the changes of aggregate values. As seen from the figures, NI decreases for some values of R, while GDP increases for some values of R. The intuition is the same as the baseline.



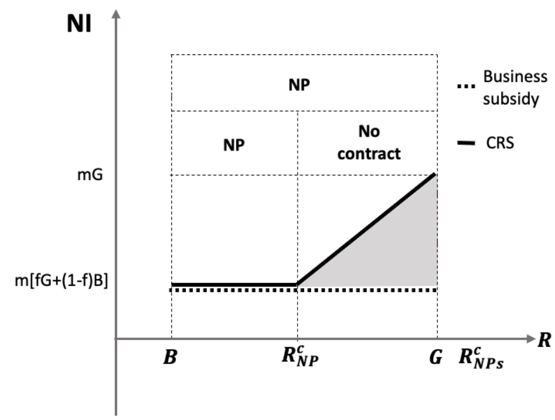
(a) $R_{SSs}^c < G$



(b) $R_{SSs}^c \geq G$

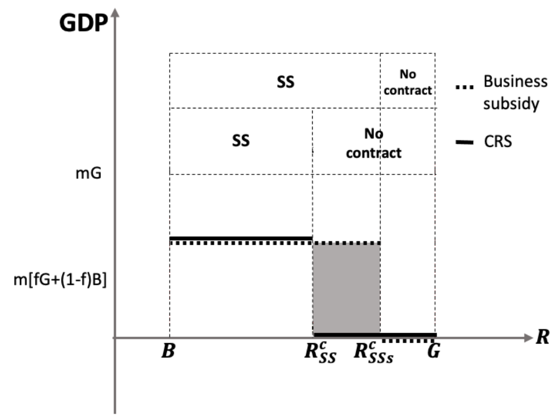


(c) $R_{NPs}^c < G$

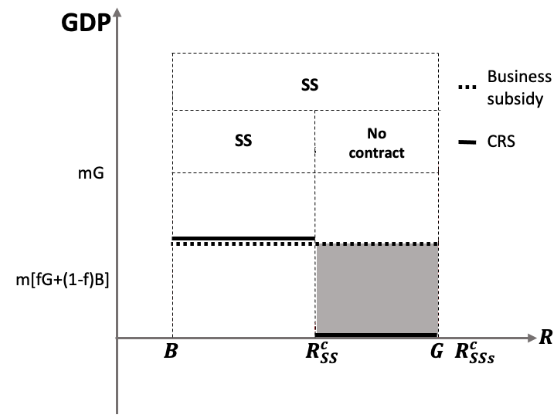


(d) $R_{NPs}^c \geq G$

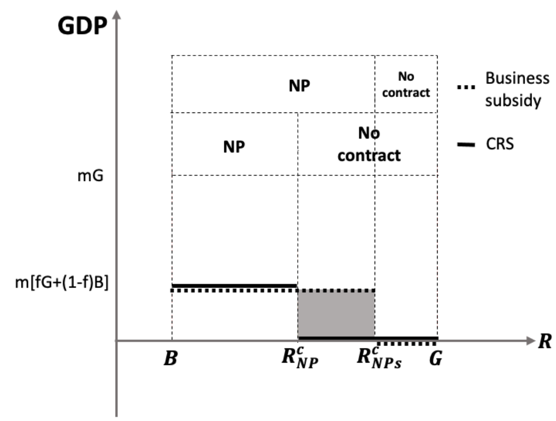
Figure 22a: Changes in NI under business subsidy (CRS case)



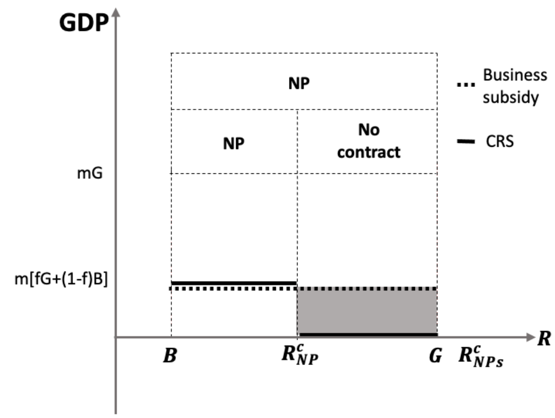
(e) $R^c_{SSs} < G$



(f) $R^c_{SSs} \geq G$



(g) $R^c_{NPs} < G$



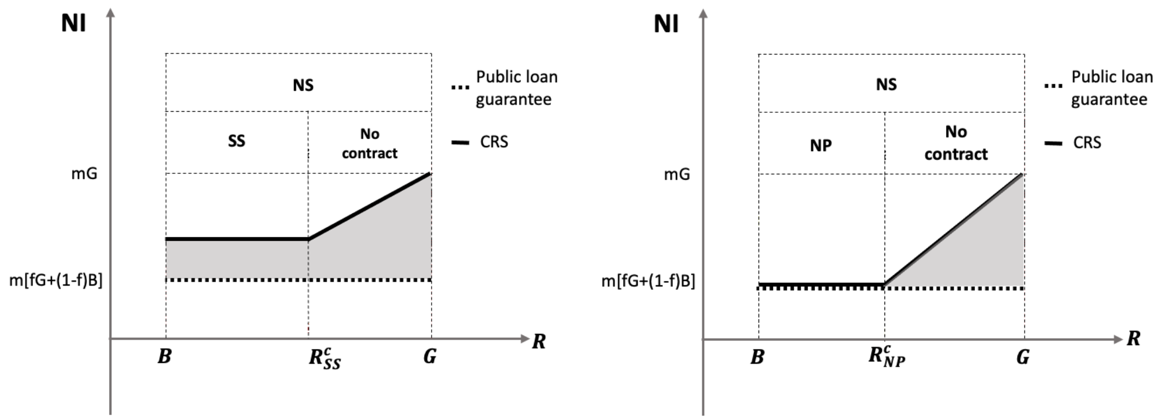
(h) $R^c_{NPs} \geq G$

Figure 22b: Changes in GDP under business subsidy (CRS case)

3 Public Loan Guarantees

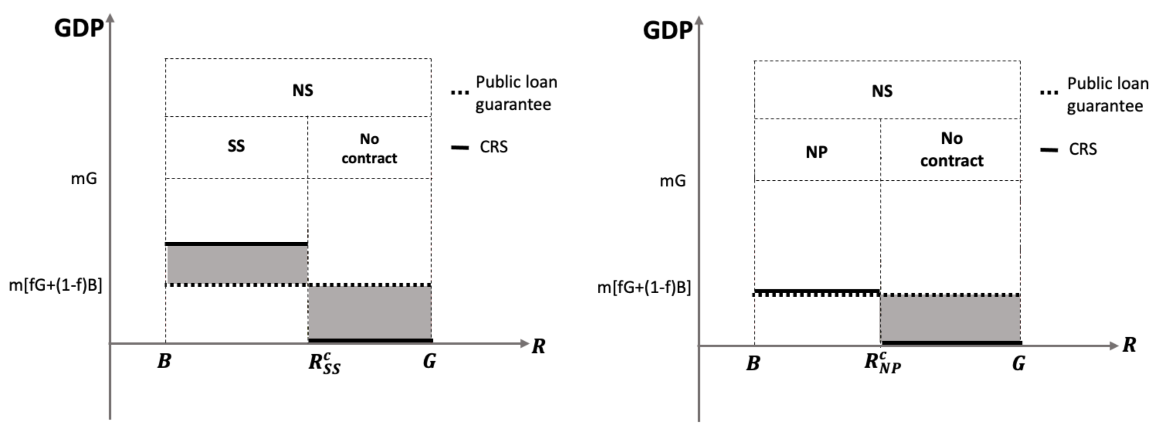
Figure 23a shows the level of NI under all kinds of parameters range. The left panel shows the case that $\exists R \in (B, G)$, screening separating equilibrium exists in the baseline. The right panel shows the case that for all $R \in (B, G)$, screening separating equilibrium does not exist in the baseline. With public loan guarantees, there is always a non-screening, cheap information based separating equilibrium. Figure 23b shows the changes of GDP under the same setup of parameter values as in Figure 23a. The shaded areas are the changes of aggregate values. As seen from the figures, NI decreases for some values of R, while GDP either increases or decreases for some values of R. The intuition for the increase of GDP is the same as the baseline. [48]

[48] When $R \in [B, R^c_{SS})$, GDP decreases after introducing the public loan guarantee, because both SS equilibrium (baseline) and NS equilibrium (public loan guarantee case) invest all fundings into domestic market under CRS assumption. In the NS equilibrium, the equilibrium screening cost is zero, and hence more loans are lent to unviable firms.



(a) (b)

Figure 23a: Changes in NI under public loan guarantees (CRS case)



(c) (d)

Figure 23b: Changes in GDP under public loan guarantees (CRS case)