Robust Bounds on Optimal Tax Progressivity

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How should we tax incomes?

Large literature on optimal tax design

- theory: Ramsey (1927), Mirrlees (1971)
- applications: Diamond and Saez (2011), Golosov et al. (2016), Heathcote et al. (2017)

Key predictions depend on hard to measure objects

- distribution of earning potentials (labor productivity)
- distribution of preferences (labor supply elasticity)

Optimal tax design acknowledging uncertainty about distribution of individual characteristics

- build on decision theory under ambiguity to model welfare consequences of statistical uncertainty about type distributions with Mirrlees (1971)
- quantify uncertainty using information from administrative and survey data

Key source of uncertainty

- tails of the productivity distribution with scarce information relative to their welfare implications

Main finding

- concerns for uncertainty call for substantially **lower** tax progressivity for high incomes

Framework

A continuum of households indexed with productivity type $z \sim F(z)$.

Households choose labor supply $n(z)$ subject to an income tax function $T(y)$ where $y(z) = zn(z)$.

A utilitarian government with Pareto weight function $\psi(z)$ chooses $T(y)$ to maximize social welfare.

- trades off redistributive motives and efficiency
- faces uncertainty about the type distribution $F(z)$ and considers a set of distributions statistically close to F

Given a labor income tax function $T(y)$, household of type z solves

$$\max_{c,n} U(c, n; z)$$

subject to the budget constraint

$$c = zn - T(zn)$$

Indirect utility function $\mathcal{U}(z; T)$ and decision rules $\mathcal{C}(z; T)$, $\mathcal{N}(z; T)$, $\mathcal{Y}(z; T)$.

Without uncertainty concerns, the utilitarian government solves

$$\max_{T} \int \psi(z)\mathcal{U}(z;T)dF(z) + V(G)$$

subject to the government budget constraint

$$\int T(\mathcal{Y}(z;T))dF(z) = G.$$

Uncertainty about household characteristics

The government is concerned that distribution $F(z)$ may be misspecified.

- it considers alternative distributions $\widetilde{F}(z)$ that are statistically close to $F(z)$

A measure of statistical closeness is the relative entropy (Kullback–Leibler divergence)

$$\mathcal{E}(F, \widetilde{F}) = \int m(z) \log m(z) \, dF(z)$$

- $m(z) = \frac{d\widetilde{F}(z)}{dF(z)}$ be the Radon–Nikodým derivative of $\widetilde{F}$ with respect to F

For a given benchmark F and entropy bound κ, the set of statistically close distributions is

$$\mathcal{F}(F, \kappa) = \left\{ \widetilde{F} : \mathcal{E}(F, \widetilde{F}) \leq \kappa \right\}$$

- the set $\mathcal{F}(F, \kappa)$ is large and the government does not put a prior on that set

A robust utilitarian government solves the max-min problem

$$\max_{T} \min_{\widetilde{F} \in \mathcal{F}} \int \psi(z) \mathcal{U}(z;T) d\widetilde{F}(z) + V(G)$$

subject to

$$\int T(\mathcal{Y}(z;T)) d\widetilde{F}(z) = G.$$

A robust utilitarian government solves the max-min problem

$$\max_{T} \min_{m:\widetilde{F}\in\mathcal{F}} \int \psi(z)\,\mathcal{U}(z;T)\,m(z)\,dF(z) + V(G)$$

subject to

$$\int T(\mathcal{Y}(z;T))\,m(z)\,dF(z) = G.$$

- utilitarian concern: low weight $m(z)$ on households with high contribution to welfare
- budgetary concern: low weight $m(z)$ on households with high contribution to the budget

Theoretical analysis

The optimal tax problem can be cast as a mechanism design problem (Mirrlees (1971))

- revelation principle allows to focus on direct mechanisms
- workers provide a report z' of their type z
- government offers a menu of allocations $(c(z'), y(z'))$ that incentivizes truthtelling, $z' = z$
- implied tax function $T(y(z)) = y(z) - c(z)$

The robust government solves

$$\max_{c,y} \min_{m:\widetilde{F}\in\mathcal{F}} \int \psi(z)\, U\left(c(z), \frac{y(z)}{z}\right) m(z)\, dF(z) + V(G)$$

subject to incentive compatibility constraints

$$U\left(c(z), \frac{y(z)}{z}\right) \geq U\left(c(z'), \frac{y(z')}{z}\right) \qquad \forall z, z'$$

and the government budget constraint

$$\int (y(z) - c(z))\, m(z)\, dF(z) = G.$$

The robust government solves

$$\min_{m:\widetilde{F}\in\mathcal{F}} \max_{c,y} \int \psi(z)\, U\left(c(z), \frac{y(z)}{z}\right) m(z)\, dF(z) + V(G)$$

subject to incentive compatibility constraints

$$U\left(c(z), \frac{y(z)}{z}\right) \geq U\left(c(z'), \frac{y(z')}{z}\right) \qquad \forall z, z'$$

and the government budget constraint

$$\int (y(z) - c(z))\, m(z)\, dF(z) = G.$$

Fixing $m(z)$ (fixing a distribution $\widetilde{F}(z)$), the problem is as in Mirrlees (1971), now under $\widetilde{F}(z)$.

- ex-post Bayesian interpretation of $\widetilde{F}(z)$ (**min** and **max** can be interchanged)

Incentive-compatibility constraints are type-by-type, do not depend on the distribution.

- misspecification concerns do not alter incentive compatibility

Optimal allocation and the minimizing 'worst-case' distribution determined jointly.

The worst-case distribution is given by $\widetilde{f}(z) = m(z) f(z)$ with

$$m(z) = \bar{m} \exp\left(-\frac{1}{\theta(\kappa)}\left[\psi(z)\mathcal{U}(z) + \mu T(y(z))\right]\right)$$

- utilitarian concern: lower weight on households with high welfare contribution $\psi(z)\mathcal{U}(z)$
- budgetary concern: lower weight on households who generate high tax revenue $T(y(z))$

We first focus on the theoretical characterization of top marginal tax rates.

- here, we for simplicity assume quasilinear utility

$$U(c,n) = c - \frac{n^{1+\gamma}}{1+\gamma}$$

- insights carry over to general separable preferences

We then provide a quantitative evaluation.

- concave utility, type distribution calibrated to data
- discipline the amount of uncertainty the planner faces

Optimal marginal tax schedule is given by the Diamond (1998)–Saez (2001) 'ABC' formula

$$\frac{T'(y(z))}{1-T'(y(z))} = \underbrace{(1+\gamma)}_{(A)} \underbrace{\frac{\widetilde{\Psi}(z)-\widetilde{F}(z)}{1-\widetilde{F}(z)}}_{(B)} \underbrace{\frac{1-\widetilde{F}(z)}{z\widetilde{f}(z)}}_{(C)}.$$

- (*A*): adverse effect of taxes on labor supply via labor supply elasticity
- (*B*): desire to redistribute

$$\widetilde{\Psi}(z) = \int^{z} \frac{\psi(\zeta)\widetilde{f}(\zeta)}{\int \psi(\xi)\widetilde{f}(\xi)\,d\xi} d\zeta$$

- (*C*): tradeoff between labor supply distortion at z and revenue from taxing types above z

Prototypical example

Assume planner puts zero welfare weight on top households

- $\psi(z) = 0$ for $z \geq \hat{z}$ where $\hat{z}$ is some threshold

Benchmark distribution in the tail above $\hat{z}$ is Pareto with shape parameter α

Without misspecification concerns, the tax formula for $z \geq \hat{z}$ simplifies to

$$\frac{T'(y(z))}{1 - T'(y(z))} = \frac{1+\gamma}{\alpha}.$$

- with a fat-tailed type distribution, taxes at the top are nonzero and quantitatively possibly large (Diamond–Saez)
- intuition: the tax revenue from types above z outweighs the labor supply distortion at z

With misspecification concerns, the tax schedule and distribution $\widetilde{F}(z)$ are determined jointly.

- Distribution $\widetilde{F}(z)$ pins down tax schedule by

$$\frac{T'(y(z))}{1-T'(y(z))} = (1+\gamma)\frac{1-\widetilde{F}(z)}{z\widetilde{f}(z)}$$

- Tax schedule determines distribution $\widetilde{F}(z)$ by

$$\begin{aligned} T(y(z)) &= T(y(\underline{z})) + \int_{y(\underline{z})}^{y(z)} T'(\eta)d\eta \\ m(z) &= \bar{m}\exp\left(-\frac{\mu}{\theta}T(y(z))\right) \end{aligned}$$

The optimal tax schedule is a fixed point of this argument.

Theorem 1.1

Assume preferences are quasilinear and $\kappa > 0$. Then the marginal tax rate vanishes to zero at the top:

$$\lim_{y \to \infty} T'(y) = 0. \tag{1.1}$$

Moreover, if the right tail of z is Pareto distributed with shape parameter α, then

$$\lim_{y \to \infty} \frac{d \log T'(y)}{d \log y} = -\frac{1}{2}. \tag{1.2}$$

Top rate level and the speed of convergence are independent of

- the magnitude of misspecification concerns κ
- shape of the Pareto tail α of the benchmark distribution
- labor supply elasticity γ

Results carry over to

- general (isoelastic) separable utility

$$U(c,n) = \frac{c^{1-\rho}}{1-\rho} - \chi \frac{n^{1+\gamma}}{1+\gamma}$$

- general welfare weights
- other statistical discrepancy functions

Quantitative application

Preferences and technology

- isoelastic preferences: $U(c,n) = \frac{c^{1-\rho}}{1-\rho} - \upsilon \frac{n^{1+\gamma}}{1+\gamma}$ with $\rho = 1$, $\upsilon = 1$, $\gamma = 2$
- government spending $V(G) = \bar{v}G$

Benchmark distribution F

- $\log z$ has exponentially modified Gaussian (EGM) distribution (Heathcote and Tsujiyama (2021))
- left tail of z distribution is lognormal (parameters μ, σ)
- right tail approximately Pareto (parameter α)

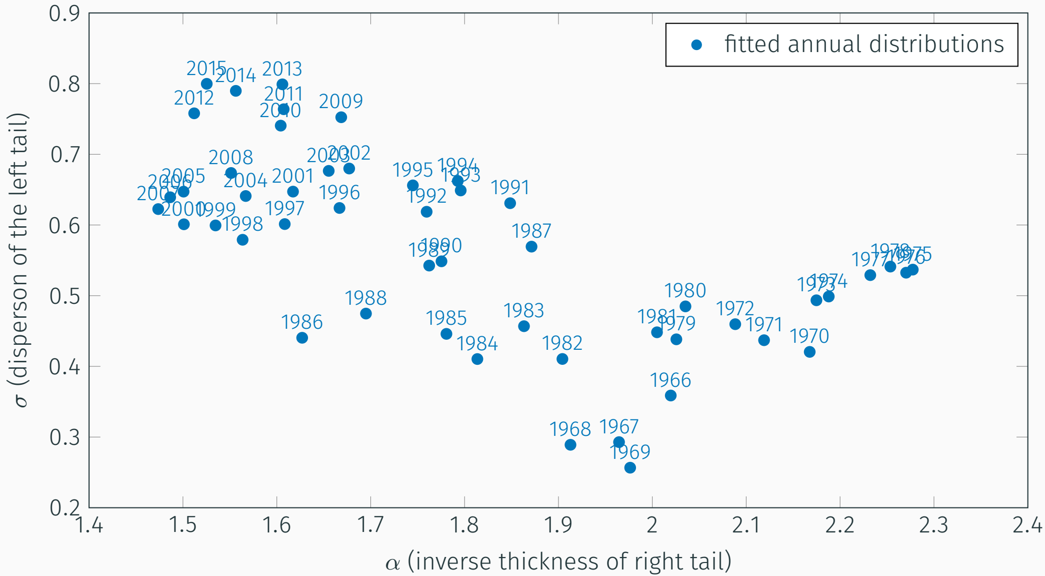
Entropy bound κ

- use time-series variation in observed income distributions (World Income Database)

1. For each year t, we fit the EGM distribution to obtain parameters $(\mu_t, \sigma_t, \alpha_t)$.
2. For each 5-year window $\{t, \ldots, t+4\}$, we construct $\mathcal{F}(F_t, \kappa_t)$ as the set that
   - includes all fitted EGM distributions from years $\{t, \ldots, t+4\}$
   - has the smallest entropy radius κ_t
3. Baseline calibration uses the median of $\{\kappa_t\}$.

The set $\mathcal{F}(F_t, \kappa_t)$ is rich:

- it contains all distributions that are close to F_t
- not only the parameterized EGM family


fitted annual distributions
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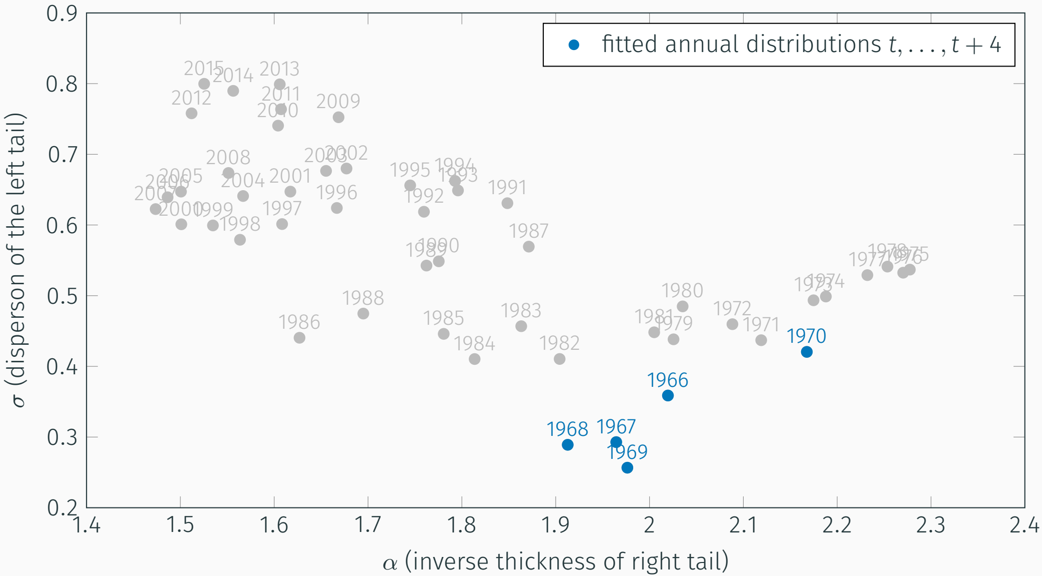
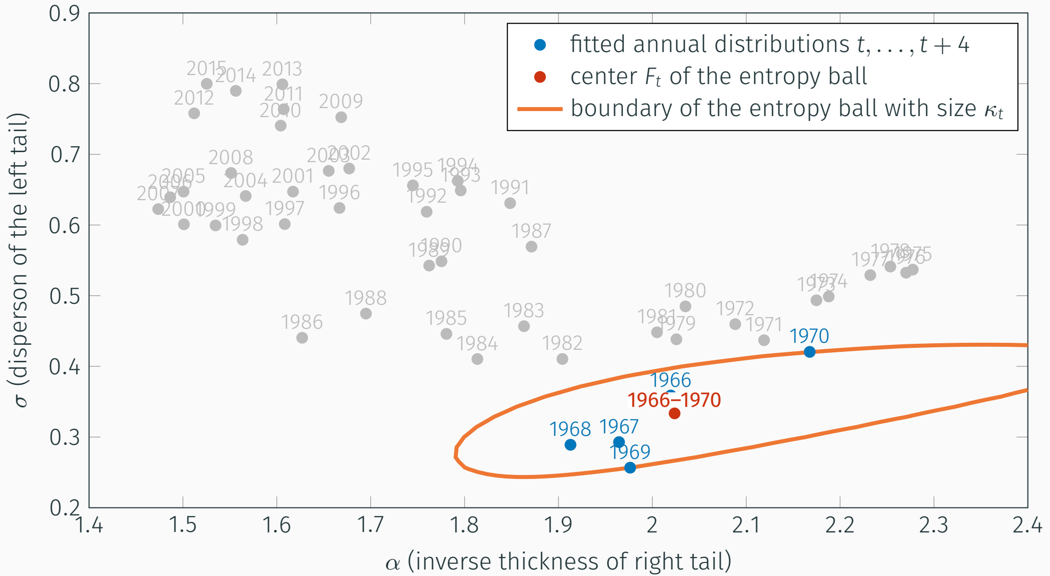
Quantifying uncertainty in income distributions




fitted annual distributions $t, \dots, t+4$
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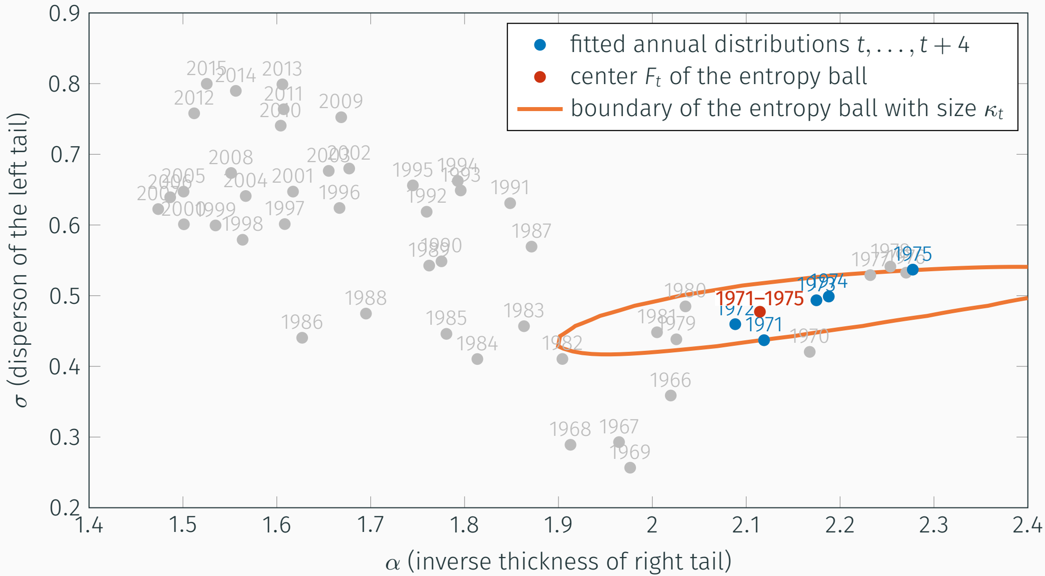
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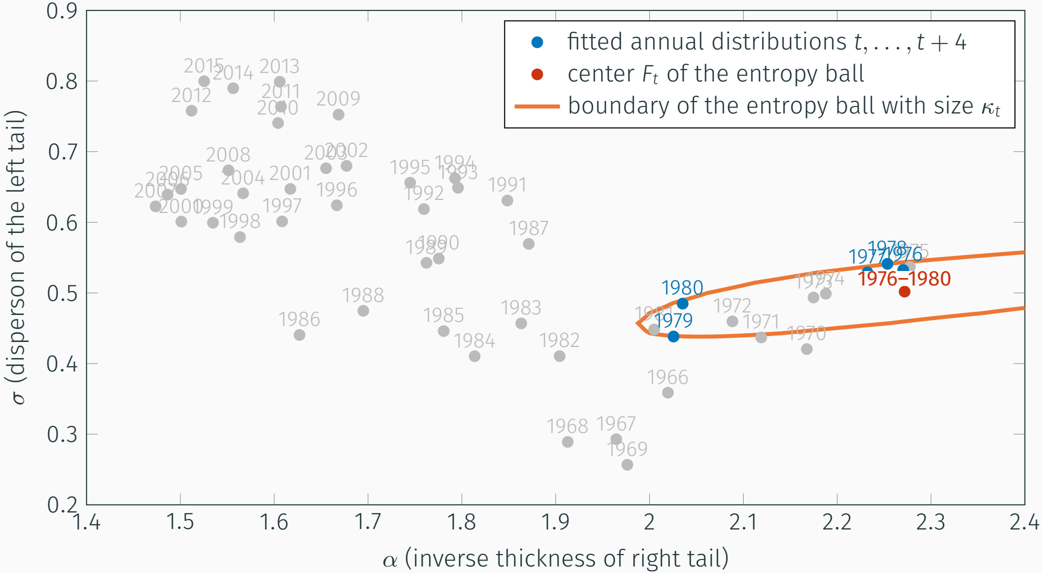
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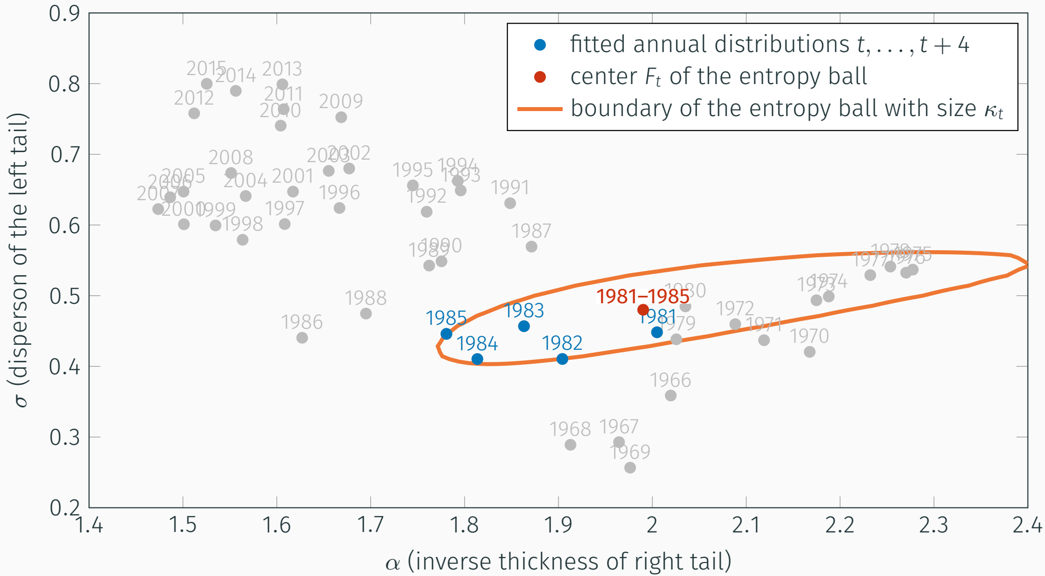

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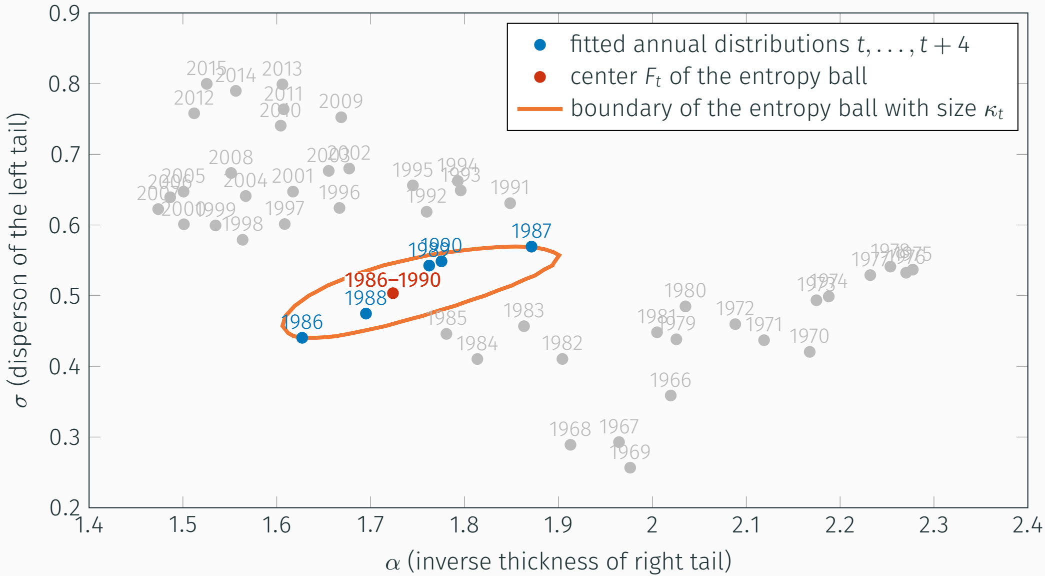
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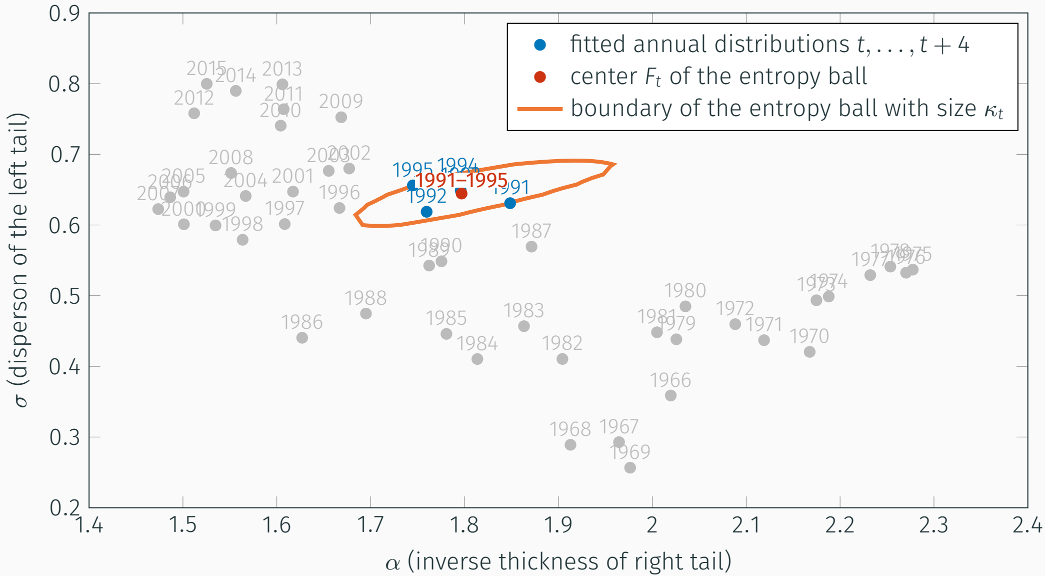
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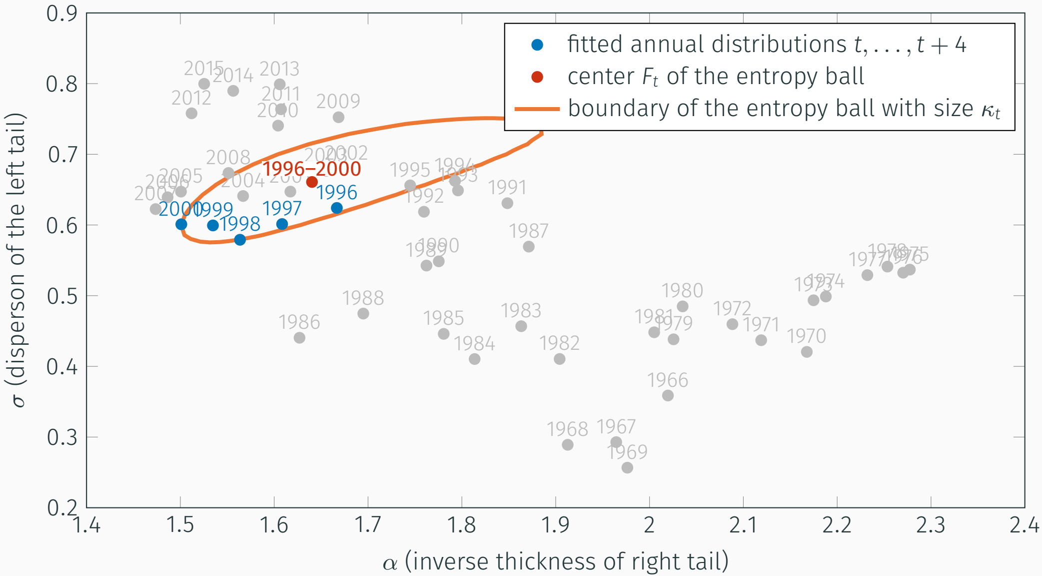
fitted annual distributions $t, \ldots, t+4$
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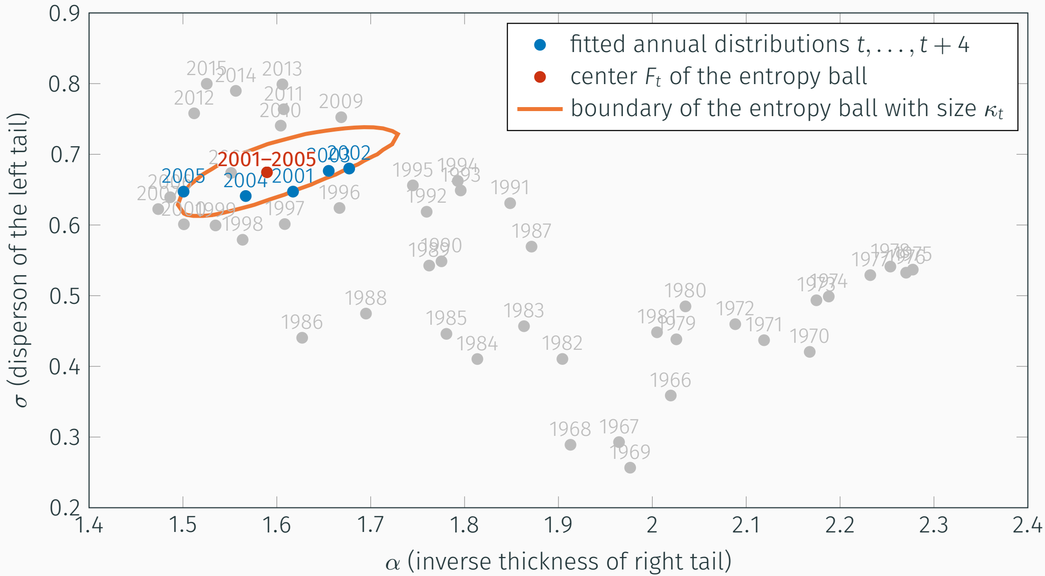
boundary of the entropy ball with size κ_t
1996–2000
σ (disperson of the left tail)
α (inverse thickness of right tail)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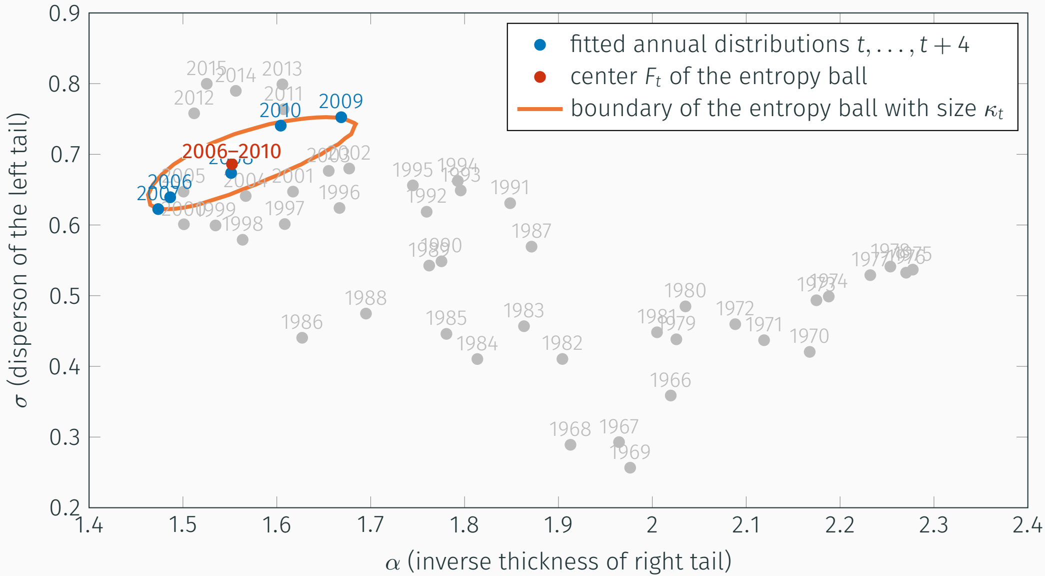
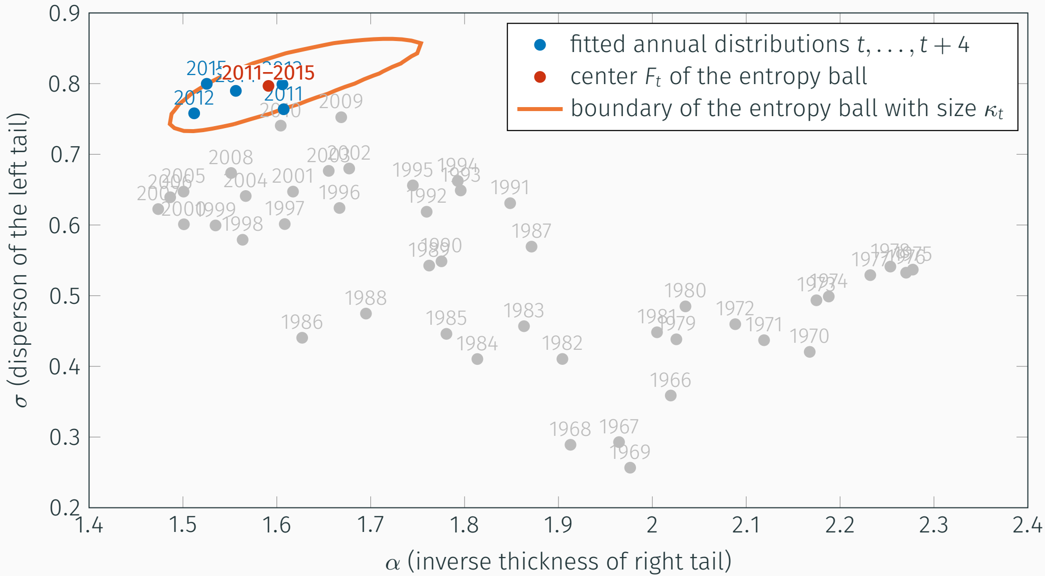
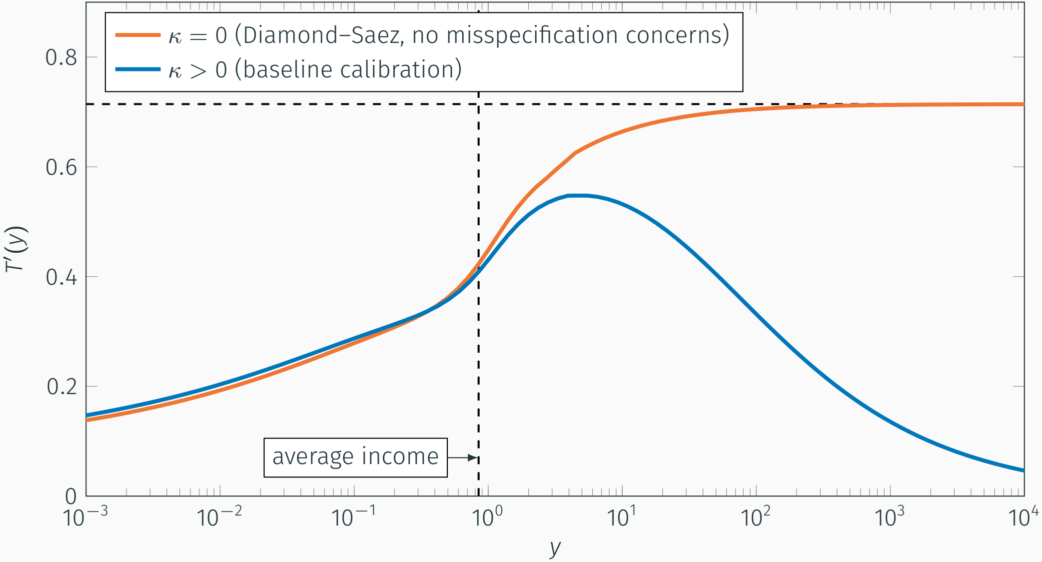
QUANTIFYING UNCERTAINTY IN INCOME DISTRIBUTIONS





fitted annual distributions $t, \ldots, t+4$
center F_t of the entropy ball
boundary of the entropy ball with size κ_t
2011–2015
σ (disperson of the left tail)
α (inverse thickness of right tail)


$\kappa = 0$ (Diamond–Saez, no misspecification concerns)
$\kappa > 0$ (baseline calibration)
average income
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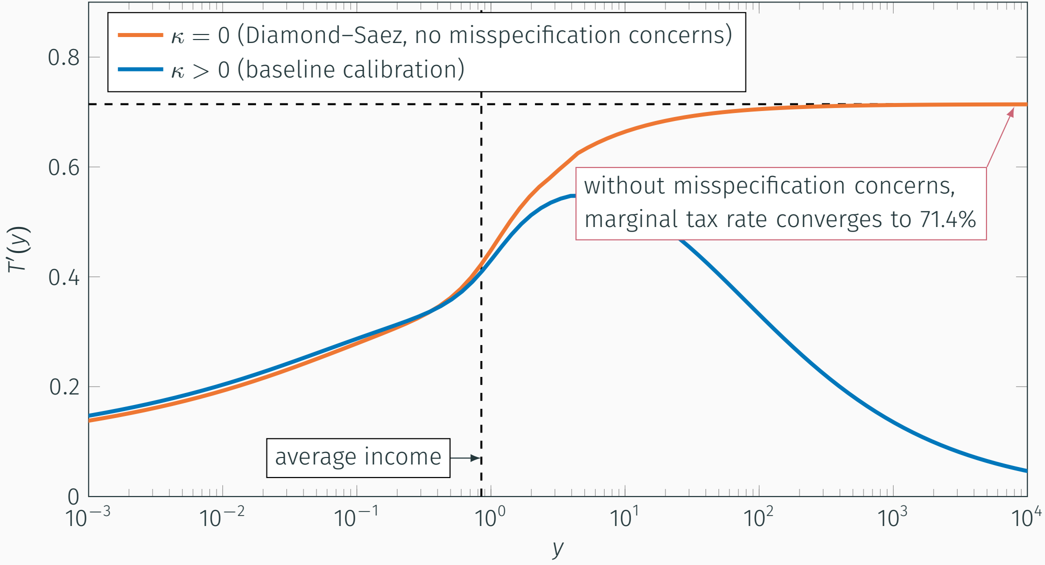
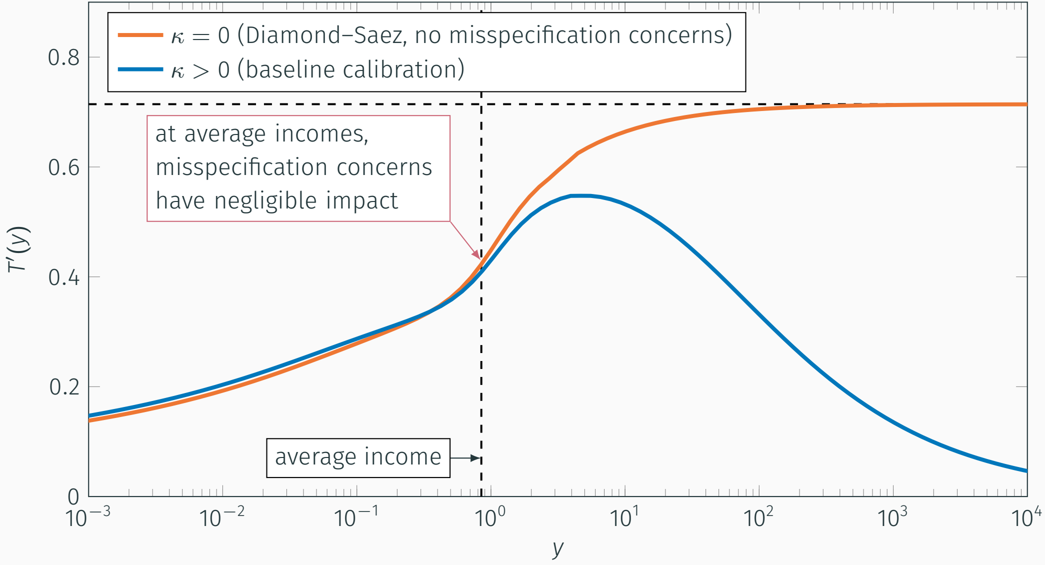
Optimal marginal tax schedules




$\kappa = 0$ (Diamond–Saez, no misspecification concerns)
$\kappa > 0$ (baseline calibration)
at average incomes, misspecification concerns have negligible impact
average income
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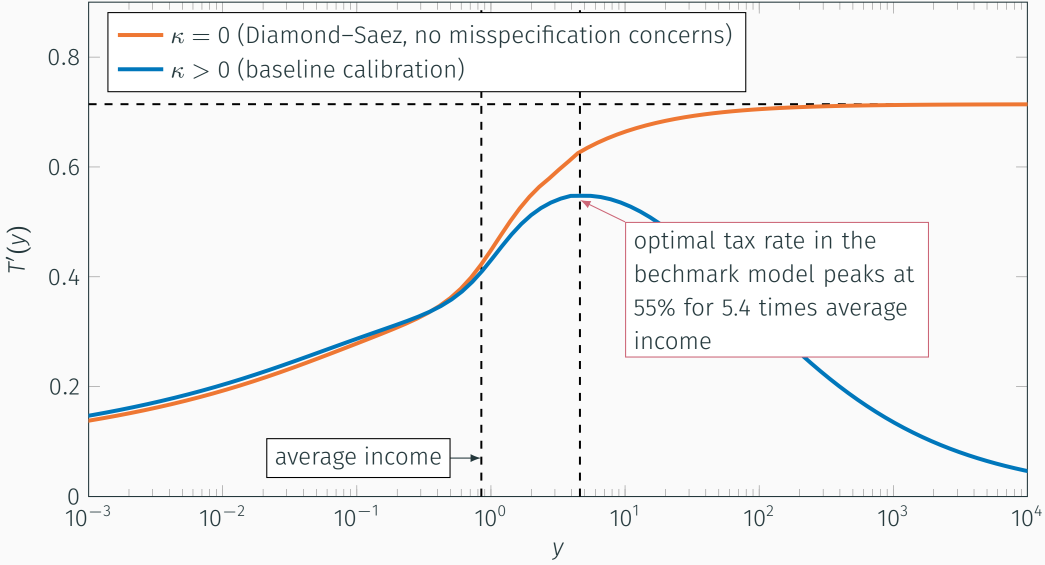
$\kappa = 0$ (Diamond–Saez, no misspecification concerns)
$\kappa > 0$ (baseline calibration)
0.8
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$T'(y)$
optimal tax rate in the bechmark model peaks at 55% for 5.4 times average income
average income
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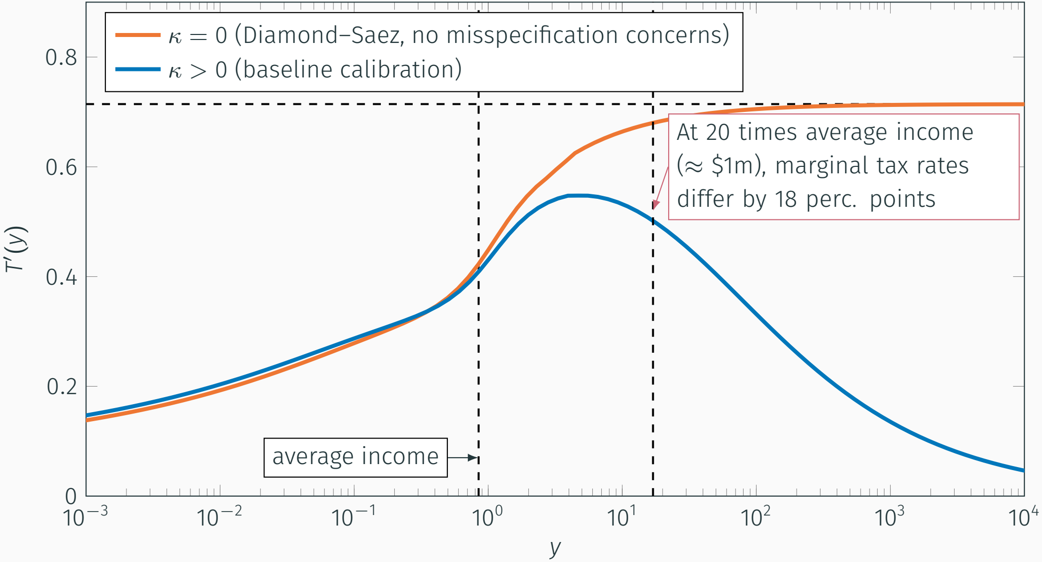
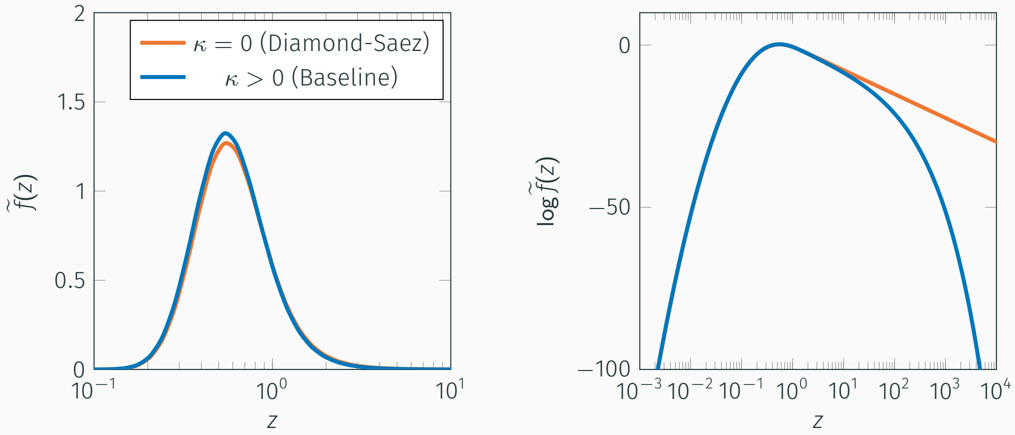
Optimal marginal tax schedules





- Worst-case distribution thins out more quickly than the benchmark distribution
- The optimal marginal tax rates are lower at the right tail due to smaller tax gain above z.

The worst-case density is characterized by the distortion

$$m(z) = \bar{m} \exp\left(-\frac{1}{\theta}\left[\mathcal{U}(z) + \mu T(y(z))\right]\right)$$

Right tail of the type distribution

- dominated by budgetary concerns

Left tail of the type distribution

- without redistribution, we would have $\lim_{z\to 0} \mathcal{U}(z) = -\infty$, and $\lim_{z\to 0} m(z) = \infty$
- but redistributive transfers bound $\mathcal{U}(z)$ from below, and so $m(z)$ is bounded above
- this makes misspecification concerns at the bottom quantitatively small

Conclusion

Acknowledging distributional uncertainty points toward lower progressivity.

- especially at the top, where budgetary concerns (per household) are most severe
- the left tail is well insured, leading to only modest concerns
- insights robust to variation in underlying distributions and preferences

Magnitude of misspecification concerns can be disciplined using

- administrative data: time-series variability in income distributions
- survey evidence: measure distinguishability in finite samples

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