Comments on “Robust bounds on optimal tax progressivity”

Tomoyuki Nakajima

June 7, 2024

University of Tokyo

- Very interesting and important paper!
- Mirrleesian optimal taxation problem, where the planner is uncertain about the distribution of productivity of workers.
- robust control approach
- Such uncertainty reduces the optimal tax rate at the top (to zero).
- Quantitative results illustrate how the tax function is affected.

Mirrleesian taxation problem

Optimal taxation problem

- a continuum of workers indexed by their productivity $z \in \mathbb{R}_+$
- $f : \mathbb{R}_+ \to \mathbb{R}_+$ is the probability density function of z.
- Worker's utility maximization problem:

$$\max_{c,n} \mathcal{U}(c, n) \quad \text{s.t.} \quad c = zn - T(zn)$$

  - Solution: $c = c(z)$, $n = n(z)$, $y(z) = zn(z)$.

- Optimal taxation problem:

$$\max_{T(\cdot)} \int_0^\infty \psi(z) U\left(c(z), n(z)\right) f(z)\, dz$$
$$\text{s.t.} \int_0^\infty \big(y(z) - c(z)\big) f(z)\, dz \geq B.$$

where $\psi(z)$ is the Pareto weights with

$$\int_0^\infty \psi(z) f(z)\, dz = 1$$

ABC formula

- quasi-linear utility:

$$\mathcal{U}(c,n) = c - \frac{n^{1+\gamma}}{1+\gamma}$$

- ABC formula:

$$\frac{T'(y(z))}{1-T'(y(z))} = (1+\gamma)\frac{\Psi(z)-F(z)}{1-F(z)}\frac{1-F(z)}{zf(z)}$$

where

$$F(z) = \int_0^z f(\zeta)\,d\zeta$$
$$\Psi(z) = \int_0^z \psi(\zeta)f(\zeta)\,d\zeta.$$

Marginal tax rate at the top

- If the productivity distribution is bounded, then the optimal marginal tax rate at the top is zero

$$T'(y(\bar{z})) = 0$$

  This remains true as long as the distribution is thin-tailed (e.g. normal, log-normal, etc),

$$\lim_{z \to \infty} T'(y(z)) \to 0$$

- This result is overturned if the distribution is fat-tailed. For instance, with a Pareto with parameter α,

$$\frac{1 - F(z)}{zf(z)} = \frac{1}{\alpha}$$

  Thus, if $\Phi(z) = 1$ for large z,

$$\frac{T'(y(z))}{1 - T'(y(z))} = (1 + \gamma)\frac{1}{\alpha}$$

  According to Diamond and Saez (2011), $\gamma = 4$ and $\alpha = 1.875$, and

$$\lim_{z \to \infty} T'(y(z)) \approx 73\%$$

Robust taxation

Robust taxation problem

- The government is uncertain about the distribution of productivities.
- It designs the tax function so as to perform well under the "worst-case distribution."
  - The worst-case distribution is chosen endogenously based on a relative entropy penalization.
- Robust taxation problem:

$$\max_{T(\cdot)} \min_{m(\cdot)} \int_0^\infty \psi(z) U\left(c(z), n(z)\right) m(z) f(z)\, dz + \theta \int_0^\infty m(z) \log m(z) f(z)\, dz$$

$$\text{s.t.} \quad \int_0^\infty \big(y(z) - c(z)\big) f(z)\, dz \geq B,$$

$$\int_0^\infty m(z) f(z)\, dz = 1.$$

  - This reduces to the standard Mirrleesian program if $\theta = \infty$.
- The worst-case distortion:

$$m(z) = \frac{\exp\left(-\frac{1}{\theta}\left[\psi(z)\mathcal{U}(z) + \mu T(y(z))\right]\right)}{\int_{\underline{z}}^{\overline{z}} \exp\left(-\frac{1}{\theta}\left[\psi(z)\mathcal{U}(z) + \mu T(y(z))\right]\right) f(\zeta)\, d\zeta'}$$

ABC formula

- Modified ABC formula:

$$\frac{T'(y(z))}{1-T'(y(z))} = (1+\gamma)\frac{\widetilde{\Psi}(z)-\widetilde{F}(z)}{1-\widetilde{F}(z)}\frac{1-\widetilde{F}(z)}{z\widetilde{f}(z)}$$

where

$$\widetilde{F}(z) = \int_0^z \tilde{f}(\zeta)\,d\zeta = \int_0^z m(\zeta)f(\zeta)\,d\zeta$$

$$\widetilde{\Psi}(z) = \int_0^z \frac{\psi(\zeta)\tilde{f}(\zeta)}{\int_0^\infty \psi(\xi)\widetilde{f}(\xi)\,d\xi}\,d\zeta.$$

- If $\psi(z) = 0$ for large z, then

$$\frac{T'(y(z))}{1-T'(y(z))} = (1+\gamma)\frac{1-\widetilde{F}(z)}{z\widetilde{f}(z)}$$

Marginal tax rate at the top

- Theorem 3.1: If $\theta < \infty$, then

$$\lim_{z \to \infty} T'(y(z)) = 0$$

- The worst-case distribution $\tilde{f}$ puts lower weights on high types.
  - Even though the objective distribution f is Pareto, the worst case distribution $\tilde{f}$ is thin-tailed, and thus the marginal tax rate at the top is zero.
- Extensions:
  - concave separable preferences
  - welfare concerns at the top
  - power divergence functions

Quantitative result

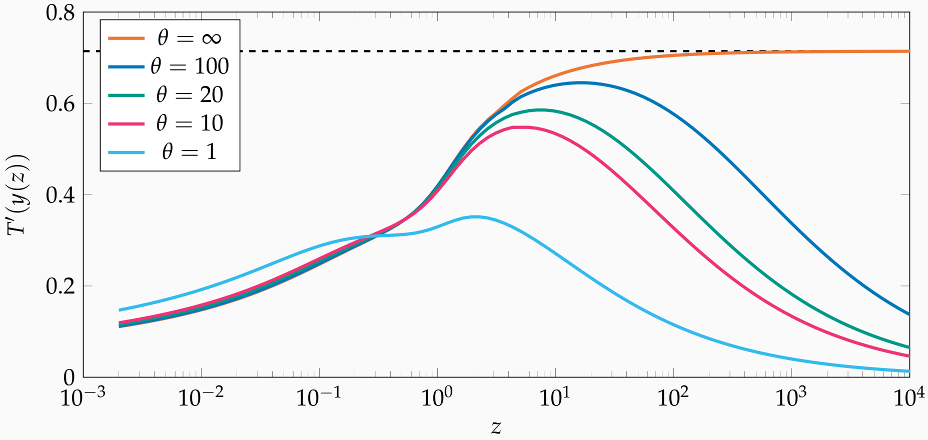

Figure 3: Optimal marginal tax schedules for alternative levels of misspecification concerns. The dashed line corresponds to the limiting marginal tax rate for the rational case.

Comments

Comment 1

- What is the gain of adopting a robust policy in this problem?
- How much welfare would be lost when the productivity distribution is indeed given by the worst-case distribution?

Comment 2

- Other kinds of uncertainty or fear of misspecification might also be relevant.
- Indeed, some of those might lead to an increase in the progressiveness of optimal taxation.
- misspecification regarding the social welfare function
    - inequality aversion/relative income concerns
- misspecification regarding the wage determination
    - rent seeking activity
- Is there a way to choose the kinds of misspecification the government should focus on?