

# **Sovereign Debt, Default Risk, and the Liquidity of Government Bonds**

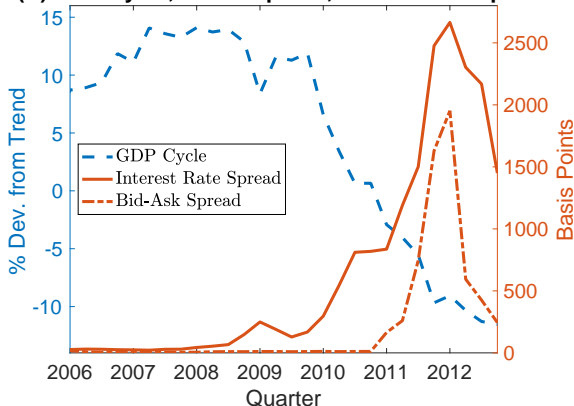
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Gaston Chaumont  
University of Rochester

**Fiscal Policy and Sovereign Debt**  
**IMF-CARF-TCER-Waseda University Conference**  
**June 7, 2024**

# Motivation: Greek GDP Cycle, Credit Spread, and Liquidity

(A) GDP Cycle, Credit Spread, and Bid-Ask Spread



- Credit spreads and bid-ask spreads were very large in the crisis
- Bid-ask spreads arise because bonds are traded in OTC markets
- Liquidity is **endogenous** to macro conditions

# This Paper

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- Transaction cost perspective: resources and time to trade
- Measured mainly through bid-ask spreads

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- How the state of the economy affects sovereign bonds **liquidity**?
- How does **liquidity** affect bond prices and gov't incentives to **default**?

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## Research Questions:

- How the state of the economy affects sovereign bonds **liquidity**?
- How does **liquidity** affect bond prices and gov't incentives to **default**?

## Approach:

- Incorporate trading frictions into a sovereign default model
- Illiquidity due to search frictions in the secondary market
- Liquidity and gross flows traded are endogenous and affects default risk through bond prices

1. Overview of the default model with endogenous illiquidity
2. Some model Implications
3. Conclusions

# Model

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# Model

## Time and markets

- $t = 1, 2, \dots$
- Centralized market: Walrasian, market clearing price for bonds is  $q$
- Decentralized market: subject to search frictions

## Three types of agents:

- Government: Issues long-term bonds only in centralized market
- Dealers: Access centralized market and decentralized market
- Investors: Access only decentralized market
  - High type: Derive utility  $u_h > 0$  from bond holdings ("buyers")
  - Low type: Derive utility  $u_\ell < 0$  from bond holdings ("sellers")
  - Bond holdings are  $a \in \{0, 1\}$
  - Investors distribution:  $\{H_0, H_1, L_1\}$ ,  $\bar{I} = H_0 + H_1 + L_1$

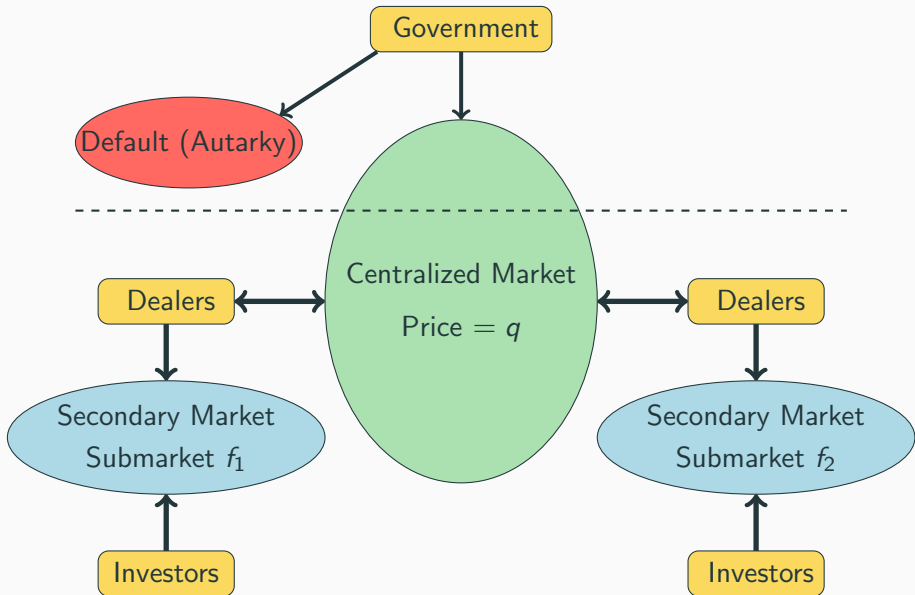


# Graphical Description of the Model

◀ Environment

◀ Dealers

◀ MktClearing



Government's option value of default is

$$V(y, B) = \max_{\delta \in \{0,1\}} (1 - \delta) V^R(y, B) + \delta V^D(y),$$

$$V^D(y) = U(h(y)) + \beta \mathbb{E}_{y'|y} V^D(y')$$

$$V^R(y, B) = \max_{B'} \{ U(c) + \beta \mathbb{E}_{\cdot|y} V(y', B') \}$$

with

$$c = y + q(y, B, B') [B' - (1 - \lambda)B] - \lambda B$$

- $B$  captures potential demand and supply in secondary market:
  - Potential sellers are  $L_1 = \zeta B$  and potential buyers are  $H_0 = \bar{I} - B$
  - Liquidity enters into Gov't problem through  $V^R(\cdot)$
  - In particular, through  $q(y, B, B')$ , the price in centralized market

# Centralized Market Clearing

- Supply of bonds is upward slopping and given by

$$\underbrace{B' - (1 - \lambda) B}_{\text{Government's supply}} + \underbrace{\alpha \left( f_{\ell}^1(s; \overset{+}{q}) \right) (1 - \lambda) \times [\text{Mass of sellers}(\overset{+}{L}_1)]}_{\text{Dealers' supply}}$$

- Demand for bonds is downward slopping and given by

$$\underbrace{\alpha \left( f_h^0(s; \overset{-}{q}) \right) \times [\text{Mass of buyers}(\overset{-}{H}_0)]}_{\text{Dealers' demand}}$$

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- Trading probabilities derived from expected gains from trade:

$$\text{Buyers} : \max_f \alpha(f) \left[ \underbrace{-(f + q(s))}_{\text{ask price}} + u_h + \frac{\mathbb{E}_{y'|y} [1 - \delta(y', B')] [I^1(s') - I^0(s')]}{1 + r} \right]$$

$$\text{Sellers} : \max_f \alpha(f) \left[ \underbrace{(q(s) - f)}_{\text{bid price}} - u_{\ell} - \frac{\mathbb{E}_{y'|y} [1 - \delta(y', B')] I^1(s')}{1 + r} \right]$$

# Quantitative Analysis

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## Model Calibration: New Parameters

Parameter	Value	Target/Source
$u_h$	0.001	$\mathbb{E} u = 0.$
$u_\ell$	-0.160	$\text{mean}(S^{B-A})$
$\zeta$	0.315	Quarterly turnover rate 0.78
$\gamma$	0.00025	Minimum $S^{B-A} \approx 5$ bpts
$\bar{I}$	5.000	Large. Never binding

## Quantitative importance of secondary market frictions

Key Parameters	Baseline	Frictionless	Longer holding	Hold to maturity
Low type probability: $\zeta$	0.315	—	0.1575	0.000
Dealer's entry cost: $\gamma \times 100$	0.025	0.000	0.025	0.025
Moments	Baseline	Frictionless	Longer holding	Hold to maturity
Mean bond spread (%)	3.42	2.67	3.03	2.00
Std. dev. bond spread (%)	2.18	2.67	2.81	1.83
Debt to output (%)	124	139	134	137
Mean bid-ask spread (bps.)	76	—	74	71
Bonds turnover rate (%)	78	—	42	6

1. Trading frictions significantly tighten the borrowing constraint of the government
2. If investors hold bonds for longer horizon spreads are lower and government borrows more

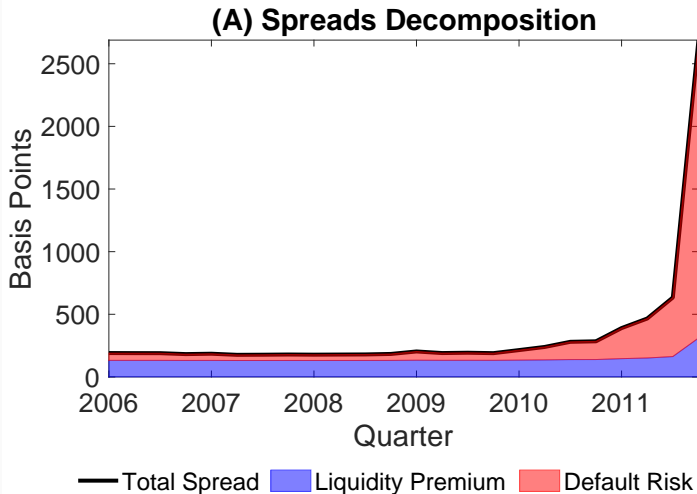
## **Additional model implications**

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## Implication 1: Trading frictions are important for spreads

1. Trading frictions and secondary market flows matter for bond prices



## Implication 2: Risk and liquidity interact in equilibrium

- 2 Changes in default risk affect size of flows and change the liquidity component in bond prices

$$\begin{aligned} \text{Buyers} &: \max_f \alpha(f) \left[ \underbrace{-(f + q(s))}_{\text{ask price}} + \frac{\mathbb{E}_{y'|y} [1 - \delta(y', B')] [l_h^1(s') - l_h^0(s')]}{1 + r} \right] \\ \text{Sellers} &: \max_f \alpha(f) \left[ \underbrace{(q(s) - f)}_{\text{bid price}} - \frac{\mathbb{E}_{y'|y} [1 - \delta(y', B')] l_\ell^1(s')}{1 + r} \right] \end{aligned}$$

## Implication 3: The distribution of bond holders matters

### 3 Changes in the distribution of bond holders affect bond prices

- Supply of bonds is upward slopping and given by

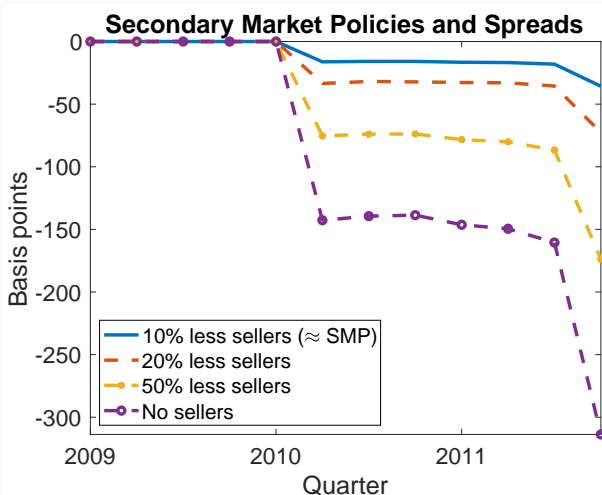
$$\underbrace{B' - (1 - \lambda) B}_{\text{Government's supply}} + \underbrace{\alpha \left( f_{\ell}^1(s; \overset{+}{q}) \right) (1 - \lambda) \times [\text{Mass of sellers}(L_1)]}_{\text{Dealers' supply}}$$

- Demand for bonds is downward slopping and given by

$$\underbrace{\alpha \left( f_h^0(s; \overset{-}{q}) \right) \times [\text{Mass of buyers}(H_0)]}_{\text{Dealers' demand}}$$

## Implication 4: Policy can affect liquidity and bond prices

- 4 Secondary market purchases of bonds can increase bond prices by increasing average holding horizon and reducing sell volumes



## Conclusions

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# Conclusions

I proposed a model of sovereign default with endogenous liquidity in the secondary market

Implications:

1. Bond prices depend on trading frictions and secondary market flows
2. Changes in default risk affect size of flows and change the liquidity component in bond prices
3. Changes in the distribution of bond holders affect bond demand and supply flows, liquidity and prices
4. QE policy can affect liquidity and bond prices by changing the average holding horizon of bonds and reducing sell volumes

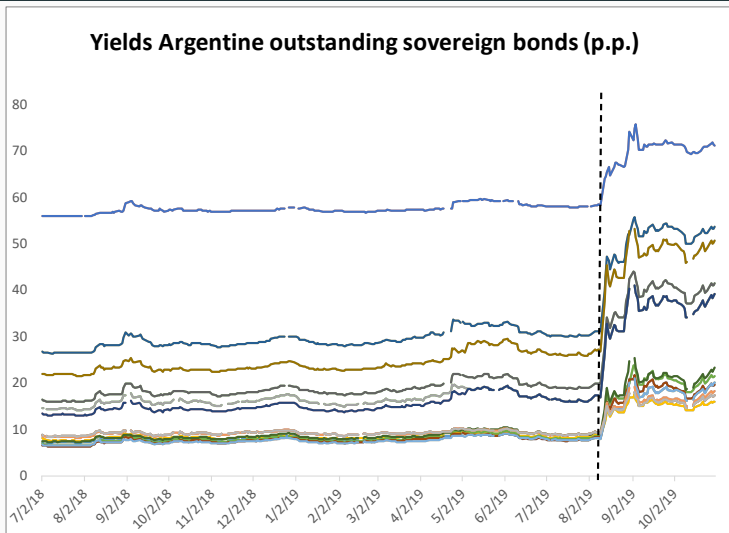
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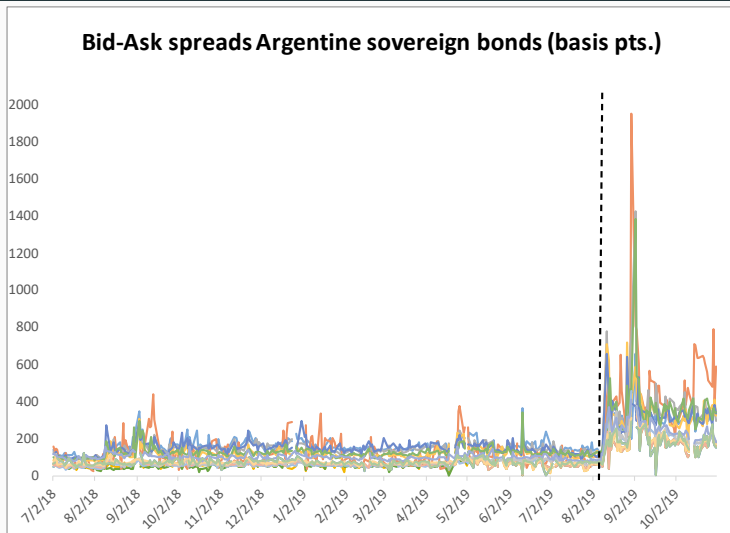
# Default “shock” and yields in Argentina



- Updated belief of elections upcoming election outcomes lead to large increase in bond yields

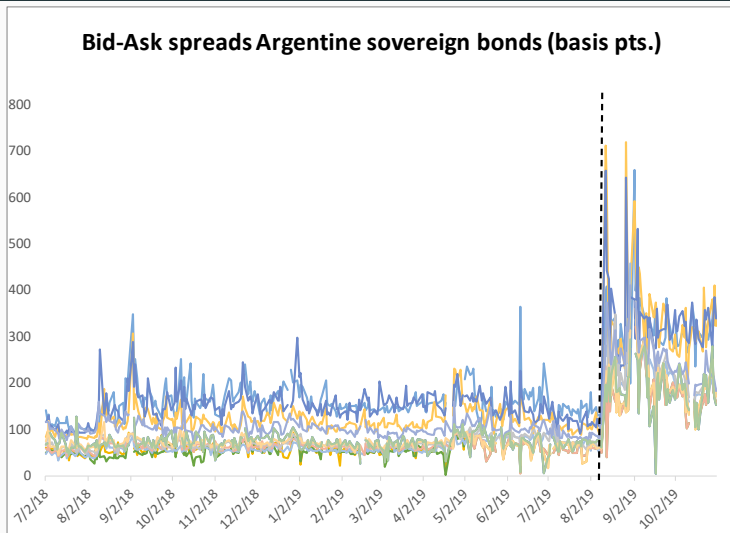


# Default “shock” and Bid-Ask Spreads in Argentina



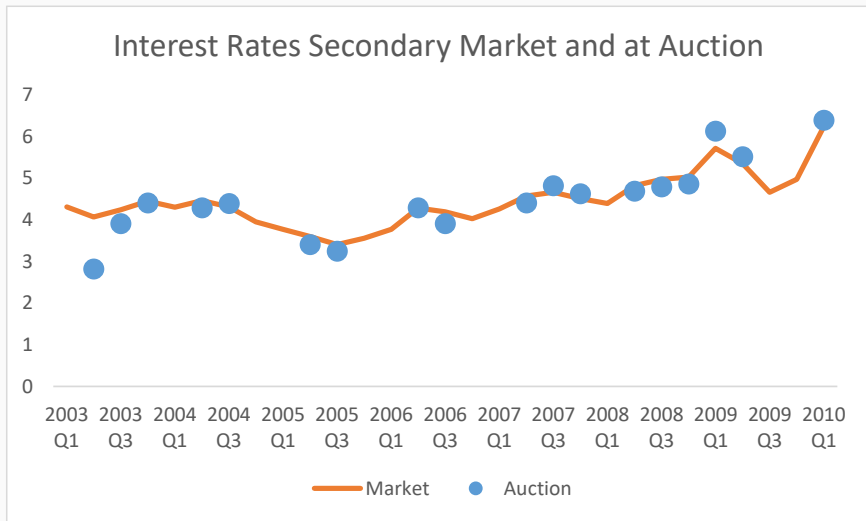
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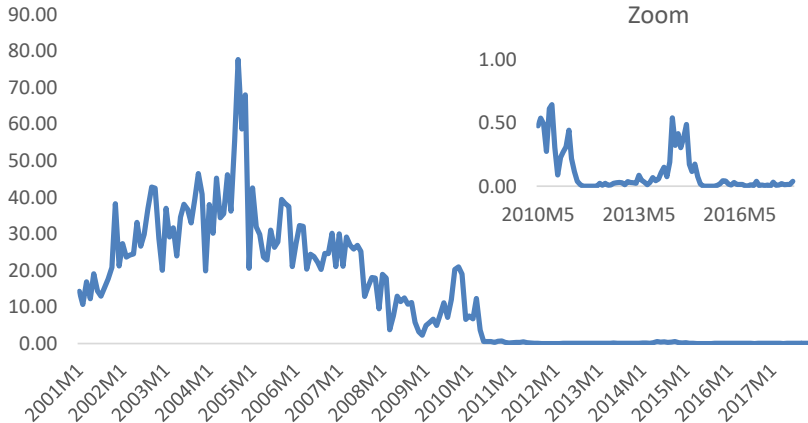


- Updated belief of elections upcoming election outcomes lead to large increase in bid-ask spreads

# Greece: Interest Rates in the Market and at Issuance

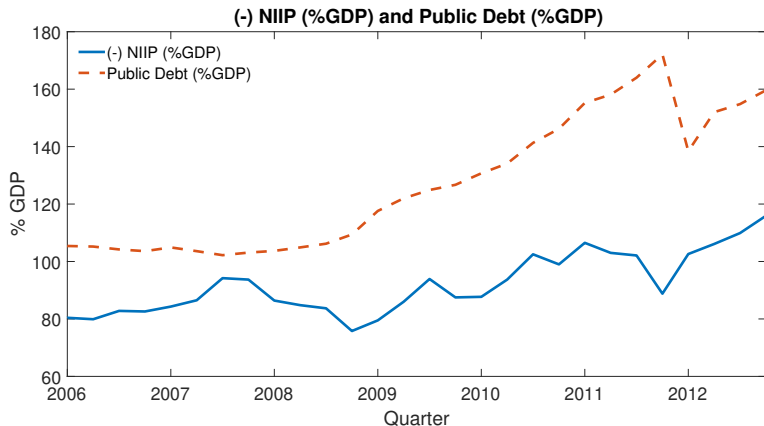
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## Volumes: Monthly Turnover Rate HDAT



$$\text{Turnover} = \frac{\text{Transactions}}{\text{Outstanding Bonds}} \times 100.$$

# Greece: Debt and International Investment Position

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- Only a few banks can directly purchase bonds from gov't [▶ Dealers](#)
- All other investors trade bonds in the secondary market:
  - Participants: banks, institutional investors, private investors, etc.
  - Volumes in secondary market **12 times** larger than primary market
- Secondary markets are OTC where transactions are
  - Decentralized
  - Bilateral
  - Costly
  - Time consuming
- A standard liquidity measure is the bid-ask spreads

$$S^{B-A} \equiv \frac{q^A - q^B}{(q^A + q^B)/2} \times 10,000$$

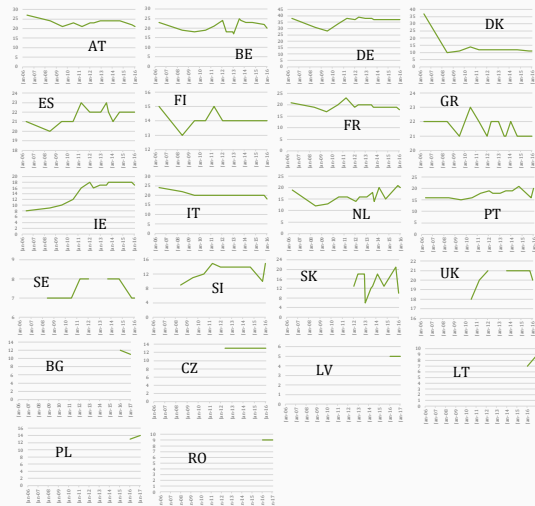
# Greece: List of Primary Dealers [◀ Back](#)

## A. List of Primary Dealers<sup>1</sup>

FIRM	Greece (GR)	Firm's location
Alpha Bank	X	Athens
Banca IMI	X	Milan
Bank of America Merrill Lynch	X	London
Barclays	X	London
BNP Paribas	X	London
Citigroup	X	London
Commerzbank	X	Frankfurt
Crédit Suisse	X	London
Deutsche Bank	X	Frankfurt
EFG Eurobank-Ergasias	X	Athens
Goldman Sachs	X	London
HSBC	X	Athens
J.P. Morgan	X	London
Morgan Stanley	X	London
National Bank of Greece	X	Athens
NatWest Markets	X	London
Nomura	X	London
Piraeus Bank	X	Athens
Société Générale	X	Paris
UBS	X	London
Unicredit	X	Munich
<b>TOTAL</b>	<b>21</b>	

Source: AFME and Greek Central Bank

# Europe: Number of Primary Dealers 2006-2017

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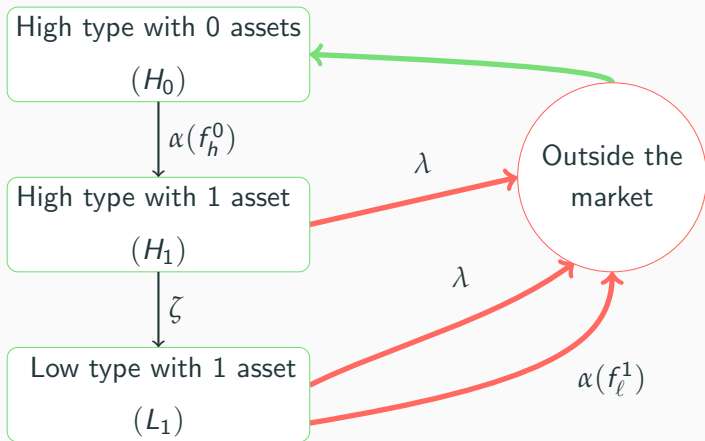
Source: Association for Financial Markets in Europe



## **Appendix to the Model**

# Investors Types and Transitions

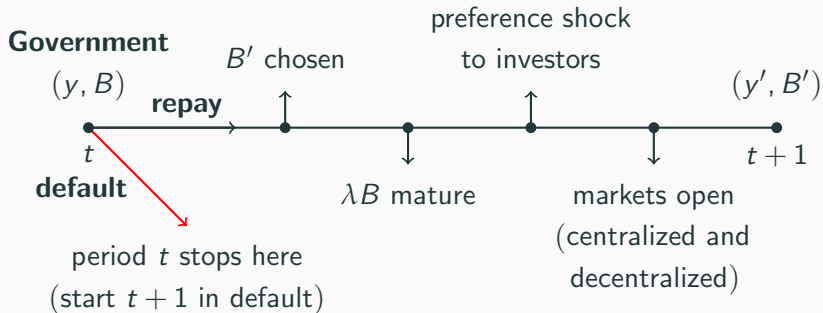
**Investors:**  $\bar{I} = H_0 + H_1 + L_1$



- $\alpha(\cdot)$  is trade prob.,  $\lambda$  mature prob.,  $\zeta$  prob. type switches


# Timing of Actions

Environment



- Maximizes representative household utility:  $\sum_t \beta^t U(c_t)$
- Income  $y \in Y \equiv \{y_1, y_2, \dots, y_N\}$  with Markov transitions  $\pi_{i,j}$
- Issues debt in the centralized market
  - Each bond matures randomly with probability  $\lambda \in [0, 1]$
- Can default on their debt obligations
  - There is an output loss  $h(y) \leq y$
  - Government is excluded from debt markets
  - Regains market access with prob.  $\phi \in (0, 1)$ , and  $B = 0$
- Takes as given a pricing schedule  $q(\cdot)$  and policies of investors and dealers

# Dealers

- Risk-neutral and there is competitive entry at cost  $\gamma > 0$
- No Inventories: permanent access to centralized market 
- Dealers are static and profits are  $\Pi(f) = -\gamma + \rho(f)f$
- By free entry:

$$\rho(f) = \frac{\gamma}{f}$$

- Matching function implies  $\alpha(f)$  increasing and concave

## Dealer's Problem and Market Tightness [◀ Back](#)

Dealers choose which submarket  $f$  to visit to maximize profits given by

$$\max_f \Pi(f) = \max_f \{-\gamma + \rho(\theta(f)) [f + q(s) - q(s)]\}$$

Then, free entry implies that

$$\Pi(f) \leq 0 \text{ and } \theta(f) \geq 0,$$

with complementary slackness.

Market tightness in each submarket is given by

$$\theta(f) = \begin{cases} \rho^{-1}\left(\frac{\gamma}{f}\right) & \text{if } \Pi(f) = 0, \\ 0 & \text{otherwise.} \end{cases}$$

# How are Bonds Priced? An Example

Assume investors hold bonds until maturity and a Telephone-Line matching function

$$\mathcal{M}(d, n) = \frac{d \times n}{d + n}$$

Then, market clearing price is

$$q(s) = \underbrace{\frac{1}{1+r} \mathbb{E}_{y'|y} \{ [I_h^1(s') - I_h^0(s')] }_{\text{Value of holding bond}} \underbrace{[1 - \delta(y', B')]_{\text{Default Risk}}} - \underbrace{\gamma \left[ \frac{1}{1 - \frac{\Delta B}{H_0}} \right]^2}_{\text{Liquidity Component}}$$

Liquidity Components:

- Intermediation frictions  $\gamma$ : more efficient matching reduce liquidity penalty
- Relative supply  $\Delta B / H_0 = [B' - (1 - \lambda) B] / [\bar{I} - (1 - \lambda) B]$ : the larger relative supply the higher liquidity penalty
- Bond maturity  $\lambda$ : longer maturity implies less liquidity penalty ( $\bar{I} > B'$ )

Implied interest rate,  $r_g(s)$ :

$$q(s) = \frac{\lambda + (1 - \lambda)z}{\lambda + r_g(s)}$$

Interest Rate Spread,  $S^R(s)$ :

$$\begin{aligned} S^R(s) &\equiv (1 + r_g(s))^4 - (1 + r)^4 \\ &= \left[ 1 + \frac{\lambda + (1 - \lambda)z}{q(s)} - \lambda \right]^4 - (1 + r)^4. \end{aligned}$$

Bid-Ask Spread

$$S^{B-A}(s) \equiv \frac{q^A(s) - q^B(s)}{q(s)}$$

$$q^B(s) \equiv q(s) - f^B(s)$$

$$q^A(s) \equiv q(s) + f^A(s)$$



Government utility function

$$U(c) = \frac{c^{1-\sigma}}{1-\sigma}$$

Output under default

$$h(y) = y - \max\{0, d_0 y + d_1 y^2\}$$

Matching function

$$\mathcal{M}(d, n) = \frac{d \times n}{d + n}$$

Parameter	Value	Target/Source
$\sigma$	2.000	Literature
$\rho_y$	0.953	AR(1) for GDP Cycle
$\eta_y$	0.020	AR(1) for GDP Cycle
$\phi$	0.050	5 years average exclusion
$\beta$	0.976	Match Default Probability of 0.68%
$d_0$	-0.522	average $r(s) - r$
$d_1$	0.650	standard deviation of $r(s) - r$
$\lambda$	0.039	6.5 year average time to maturity
$z$	0.011	So price in PM is at par-value
$r$	0.010	~4% annual

# Non-Targeted Moments

Moment	Model	Data
$\sigma_c / \sigma_y$	1.07	0.98
$\rho_{S^R, tb/y}$	0.58	0.71
$\rho_{S^R, c}$	-0.77	-0.45
$\rho_{S^R, y}$	-0.75	-0.56
$\rho_{y, tb/y}$	-0.43	-0.59