A Behavioral Explanation for the Puzzling Persistence of the Aggregate Real Exchange Rate

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The Law-of-One-Price and Purchasing Power Parity

- LOP

$$\frac{S_t P^*_{\text{beer},t}}{P_{\text{beer},t}} = 1$$

where S_t is the nominal exchange rate, $*$ denotes "foreign"

- PPP

$$\frac{S_t P^*_t}{P_t} = 1$$

where P_t and P^*_t are CPIs of home and foreign countries

The Law-of-One-Price and Purchasing Power Parity

- LOP deviations (or good-level RER)

$$\frac{S_t P^*_{\text{beer},t}}{P_{\text{beer},t}} = q_{it} \neq 1$$

where S_t is the nominal exchange rate, $*$ denotes "foreign"

- PPP deviations (aggregate RER)

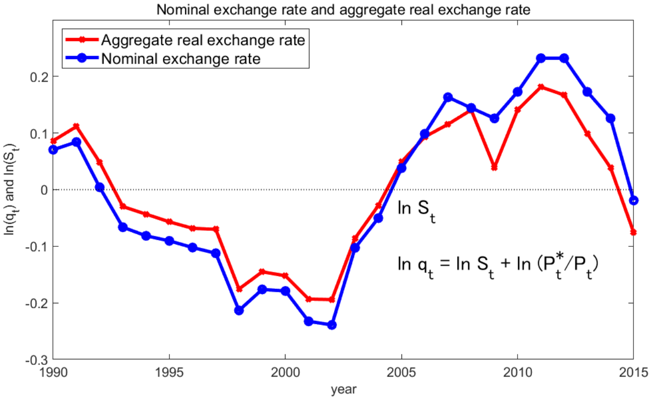
$$\frac{S_t P^*_t}{P_t} = q_t \neq 1$$

where P_t and P^*_t are CPIs of home and foreign countries

- In reality, LOP & PPP do not hold

PPP puzzle 1

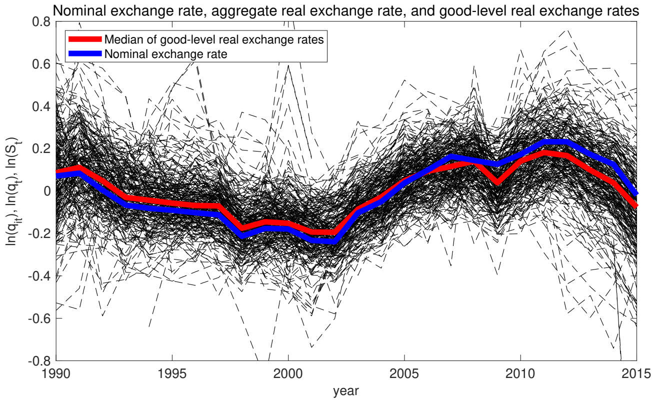
- PPP deviations are extremely persistent



  - too persistent to be explained by reasonable degree of nominal price rigidities (Rogoff 1996)
  - Half-life = 3-5 years

PPP puzzle 2

- LOP deviations are much less persistent than PPP deviations



- Half-life ($\simeq 1.2$ year) is lower than that of q_t (3-5 years)
- Imbs, Mumtaz, Ravn and Rey (2005), Crucini and Shintani (2008), Carvalho and Nechio (2011)

Research questions

Q1: Does the behavioral inattention model help to solve the PPP puzzle?

- We consider a behavioral model of inattention ("sparse-based model") by Gabaix (2014, 2020)
- Gabaix (2014, 2020) discusses models with attention parameter m:

$$
\begin{aligned}
m &= 1 \qquad \text{(if agents are fully attentive)} \\
m &< 1 \qquad \text{(if agents are inattentive)}
\end{aligned}
$$

- We introduce m into the model of LOP deviations used in Crucini, Shintani and Tsuruga (2010a, 2010b, 2013, 2015)

Q2: Do micro price data support the behavioral inattention?

Theoretical finding

Q1: Does the behavioral inattention model help to solve the PPP puzzle?

A1: Yes

- We derive the relationship between LOP deviations and PPP deviations
- If $m = 1$

$$\ln q_{it} = \lambda \ln q_{it-1} + e_t + e_{it}$$

(λ: the degree of price stickiness, e: iid shocks)

- If $m < 1$

$$\ln q_{it} = \lambda \ln q_{it-1} + (1 - m)(1 - \lambda) \ln q_t + e_t + e_{it}$$

  - **Complementarity** generates persistence of q_t and q_{it}

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Complementarity

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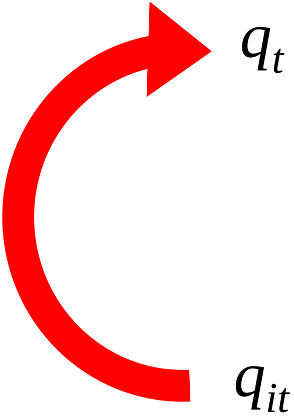
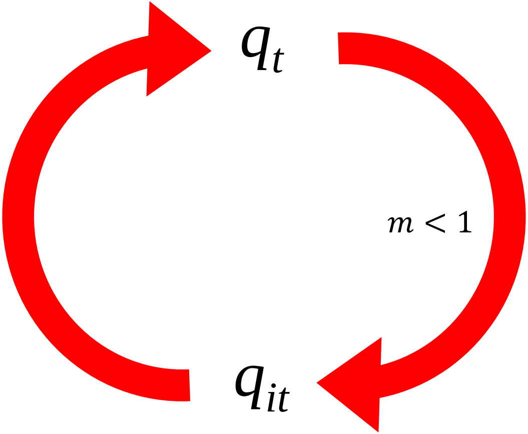


$m = 1$

- No complementarities if $m = 1$

Complementarity

$$\ln q_{it} = \lambda \ln q_{it-1} + (1-m)(1-\lambda)\ln q_t + e_t + e_{it}$$



- Compelementarity generates slow aggregate real exchange rate if $m < 1$

Empirical findings

Q2: Do micro price data support the behavioral inattention?

A2: Yes

- We test the model of LOP deviations with behavioral inattention
  - Competing Hypotheses

$$\begin{aligned} H_0 &: \quad m = 1 \text{ (fully attentive)} \\ H_1 &: \quad m < 1 \text{ (inattentive)} \end{aligned}$$

  - H_0: $m = 1$ is strongly rejected by the data
- Our estimates of m are $m = 0.11 - 0.25$
- Under the estimated m, the model explains the PPP puzzle

The persistence of the PPP and LOP deviations

- With $m = 1$ (**full attention**)

	Model ($m = 1$)	Model ($m < 1$)	Data
Aggregate real exchange rates (PPP deviations)			
Half-life	**0.6**		**2.4-4.9**
Good-level real exchange rates (LOP deviations)			
Half-life	0.6		1.2-1.6

- Note: The unit of a period is year

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The persistence of the PPP and LOP deviations

- With $m = 0.1$ (**behavioral inattention**)

	Model ($m = 1$)	Model ($m < 1$)	Data
Aggregate real exchange rates (PPP deviations)			
Half-life	0.6	**2.5-3.7**	2.4-4.9
Good-level real exchange rates (LOP deviations)			
Half-life	0.6	**1.0-1.2**	1.2-1.6

- Note: The unit of a period is year

A simple model of inattention

- 2 steps
  1. Rational firm's profit function (log-approximated to the second order)

$$\max_{\hat{p}} \pi(\hat{p}, \hat{p}^{opt}) = \max_{\hat{p}} \left[-\frac{c}{2} \left(\hat{p} - \hat{p}^{opt} \right)^2 \right]$$

     $\hat{p}$: firm's actual price, $\hat{p}^{opt}$: the optimal price (in terms of log-deviations), c: constant
     - The rational agent's optimal price

$$\hat{p} = \hat{p}^{opt}$$

  2. Behavioral agents replaces $\pi\left(\hat{p}, \hat{p}^{opt}\right)$ with "attention augmented profit function"

$$\max_{\hat{p}} \widetilde{\pi}(\hat{p}, \hat{p}^{opt}, m) = \max_{\hat{p}} \left[-\frac{c}{2} \left(\hat{p} - m\hat{p}^{opt} \right)^2 \right]$$

     - In a special case with $m = 1$, $\widetilde{\pi}\left(\hat{p}, \hat{p}^{opt}, 1\right) = \pi\left(\hat{p}, \hat{p}^{opt}\right)$
     - The action becomes choice of m

$$\hat{p}(m) = m\hat{p}^{opt}$$

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     - The action becomes ▸ choice of m

$$\hat{p}(m) = m\hat{p}^{opt}$$

Choice of attention

- How do firms determine m?
- The degree of attention can be endogeneized by solving

$$\begin{aligned} & \max_{m \in [0,1]} \mathbb{E}\left[\pi(\hat{p}(m), \hat{p}^{opt}) - \mathcal{C}(m)\right] \\ = & \max_{m \in [0,1]} \mathbb{E}\left[-\frac{c}{2}(\hat{p}(m) - \hat{p}^{opt})^2 - \mathcal{C}(m)\right] \end{aligned}$$

  where $\mathcal{C}(m)$ is a cost of paying attention
- Trade-off
  - Benefit: correction of distorted action
  - Cost: paying attention
- back

Overview of the model

- Follows Kehoe and Midrigan (2007), Crucini, Shintani and Tsuruga (2010, 2013)
- Households
  - $U(c_t, n_t) = \ln c_t - \chi n_t$ max. problem
- Firms
  - Set prices in monopolistically competitive market (Home and Foreign, local currency pricing) CES
  - Calvo pricing with its parameter λ
  - Use technology: $y_{it}(z) = a_{it} n_{it}(z)$
  - Must pay trade cost to send goods from a country to the other resources
  - We introduce Gabaix's behavioral inattention to price setting
- Governments
  - Control money supply

Rational firms' pricing: step 1

- Home firm's pricing under **full attention**

$$\hat{p}_{Hit} = (1-\lambda\delta)\mathbb{E}_t \sum_{k=0}^{\infty} (\lambda\delta)^k (\widehat{mc}_{Hit+k})$$

$$\hat{p}^*_{Hit} = (1-\lambda\delta)\mathbb{E}_t \sum_{k=0}^{\infty} (\lambda\delta)^k (\widehat{mc}^*_{Hit+k})$$

where all variables are the log-deviations, p_{Hit}: relative price of good i, a_{it}: productivity, δ: discount factor,

- $\hat{p}^*_{Fit}$ and $\hat{p}_{Fit}$ are analogously derived

Behavioral firms' pricing: step 2

- Home firm's pricing under **behavioral inattention**

$$\hat{p}_{Hit}(m_H) = (1-\lambda\delta)\mathbb{E}_t\sum_{k=0}^{\infty}(\lambda\delta)^k(m_H\widehat{mc}_{Hit+k})$$

$$\hat{p}^*_{Hit}(m^*_H) = (1-\lambda\delta)\mathbb{E}_t\sum_{k=0}^{\infty}(\lambda\delta)^k(m^*_H\widehat{mc}^*_{Hit+k})$$

- $\hat{p}^*_{Fit}(m^*_F)$ and $\hat{p}_{Fit}(m_F)$ are analogously derived
- The optimal (relative) prices are insensitive to the aggregate shocks price index

Proposition 1

Under the preferences given by $U(c,n) = \ln c - \chi n$, the CIA constraints, the stochastic processes of money supply, the stochastic processes of the labor productivity, and the Calvo pricing with the degree of price stickiness $\lambda \in (0,1)$, the stochastic process of the good-level real exchange rate is given by:

$$\ln q_{it} = \lambda \ln q_{it-1} + (1-m)(1-\lambda)\ln q_t + \lambda \varepsilon_t^n + \psi \varepsilon_{it}^r$$

where

- $m \in (0,1]$: the degree of attention, $m = \omega m_H + (1-\omega) m_F$
- ψ: param. for real frictions (> 0 with trade cost)
- ε_t^n: nominal shock, $\varepsilon_t^n = \Delta \ln S_t \sim i.i.d.(0, \sigma_n^2)$
- ε_{it}^r: real shock, $\varepsilon_{it}^r = (\varepsilon_{it} - \varepsilon_{it}^*) \sim i.i.d.(0, \sigma_r^2)$

Takeaway: Direct relationship between $\ln q_{it}$ and $\ln q_t$

Testable implication of behavioral LOP

- We define modified LOP deviations $\widetilde{q}_{it}$ and PPP deviations $\widetilde{q}_t$

$$\underbrace{\ln q_{it} - \lambda \ln q_{it-1} - \lambda \varepsilon_t^n}_{\ln \widetilde{q}_{it}} = (1-m)\underbrace{(1-\lambda)\ln q_t}_{\ln \widetilde{q}_t} + \underbrace{\psi \varepsilon_{it}^r}_{u_{it}}$$

  where we replace ε_t^n by $\Delta \ln S_t$

- We can use this regression as a test for full attention

$$\ln \widetilde{q}_{it} = \alpha + \beta \ln \widetilde{q}_t + u_{it},$$

- The null hypothesis of full attention $(H_0 : m = 1 \text{ or } \beta = 0)$ is significantly rejected in favor of behavioral inattention

Data

- We use the annual micro price data of US-Canadian city pairs and UK-Euro city pairs
  - The *Worldwide Cost of Living Survey* by Economic Intelligence Unit
  - Our regression has variations in three dimensions

$$\ln q_{ijt}$$

    - 274 goods (i)
    - 17 US cities and 4 Canadian cities = 68 city pairs (j)
    - (or 19 Euro cities and 2 UK cities = 38 city pairs (j))
    - 26 years from 1990 to 2015 (t)

Test for behavioral inattention (US–Canada)

- Modified RER with common λ

$$\ln \widetilde{q}_{ijt} = \alpha_{ij} + \beta \ln \widetilde{q}_t + \gamma' X_{ijt} + u_{ijt}$$

	(1)	(2)	(3)	(4)
$\ln \widetilde{q}_t$	0.844***	0.802***	0.812***	0.806***
	(0.030)	(0.028)	(0.029)	(0.029)
# of Obs.	389,500	389,500	389,500	389,500
city-pairs FE	N	Y	N	Y
Control for productivity	N	N	Y	Y
$\hat{m}$	0.156	0.198	0.188	0.194

Note: *10% significance level, **5% significance level, ***1% significance level. For λ, the median value of the degrees of price stickiness ($\lambda = 0.34$) is used. All specifications include the fixed effect at the good level. The standard errors are clustered by goods.

Test for behavioral inattention (UK–Euro)

- Modified RER with common λ

$$\ln \widetilde{q}_{ijt} = \alpha_{ij} + \beta \ln \widetilde{q}_t + \gamma' X_{ijt} + u_{ijt}$$

	(1)	(2)	(3)	(4)
$\ln \widetilde{q}_t$	0.856***	0.851***	0.853***	0.868***
	(0.042)	(0.043)	(0.042)	(0.042)
# of Obs.	214,115	214,115	213,064	213,064
city-pairs FE	N	Y	N	Y
Control for productivity	N	N	Y	Y
$\hat{m}$	0.144	0.149	0.147	0.132

Note: *10% significance level, **5% significance level, ***1% significance level. For λ, the median value of the degrees of price stickiness ($\lambda = 0.34$) is used. All specifications include the fixed effect at the good level. The standard errors are clustered by goods.

Test for behavioral inattention (US–Canada)

- Modified RER with good-specific λ_i

$$\ln \widetilde{q}_{ijt} = \alpha_{ij} + \beta \ln \widetilde{q}_{it} + \gamma' X_{ijt} + u_{ijt}$$

	(1)	(2)	(3)	(4)
$\ln \widetilde{q}_{it}$	0.894***	0.862***	0.883***	0.880***
	(0.029)	(0.028)	(0.032)	(0.033)
# of Obs.	389,500	389,500	389,500	389,500
city-pairs FE	N	Y	N	Y
Control for productivity	N	N	Y	Y
$\hat{m}$	0.106	0.138	0.117	0.120

Note: *10% significance level, **5% significance level, ***1% significance level. For λ, the good-specific values of the degrees of price stickiness (λ_i) are used. All specifications include the fixed effect at the good level. The standard errors are clustered by goods.

Test for behavioral inattention (UK–Euro)

- Modified RER with good-specific λ_i

$$\ln \widetilde{q}_{ijt} = \alpha_{ij} + \beta \ln \widetilde{q}_{it} + \gamma' X_{ijt} + u_{ijt}$$

	(1)	(2)	(3)	(4)
$\ln \widetilde{q}_{it}$	0.866***	0.834***	0.864***	0.840***
	(0.047)	(0.049)	(0.048)	(0.049)
# of Obs.	171,606	171,606	170,750	170,750
city-pairs FE	N	Y	N	Y
Control for productivity	N	N	Y	Y
$\hat{m}$	0.134	0.166	0.136	0.160

Note: *10% significance level, **5% significance level, ***1% significance level. For λ, the good-specific values of the degrees of price stickiness (λ_i) are used. All specifications include the fixed effect at the good level. The standard errors are clustered by goods.

Test for behavioral inattention: Summary

- The null hypothesis of $m = 1$ is significantly rejected and robust to various specifications
- The estimated degree of inattention ranges between 0.11–0.25
  - Baseline estimate $m = 0.106$ for US-Canada
  - Baseline estimate $m = 0.134$ for UK-Euro
- What are the implications for PPP puzzle?

Propositions 2 & 3

Under the same assumptions in Proposition 1,

$$HL_q > HL_{q|m=1} \text{ (PPP puzzle 1)}$$

$$HL_q > HL_{qi} \text{ (PPP puzzle 2)}$$

provided $m \in (0,1)$, $\lambda \in (0,1)$, $\tau \in (0,\infty)$, *and* $\sigma_r/\sigma_n \in (0,\infty)$

- Eq Prop 2
- Eq Prop 3

The persistence of the PPP and LOP deviations

- For US-Canada with $m = 0.106$

	Model ($m = 1$)	Model ($m < 1$)	95% CI	Data
Aggregate real exchange rates (PPP deviations)				
Half-life	0.6	3.7	[2.5, 7.6]	4.9
Good-level real exchange rates (LOP deviations)				
Half-life	0.6	1.2	[1.0, 2.1]	1.6

- Note: The unit of a period is year

The persistence of the PPP and LOP deviations

- For UK-Euro with $m = 0.134$

	Model ($m = 1$)	Model ($m < 1$)	95% CI	Data
Aggregate real exchange rates (PPP deviations)				
Half-life	0.6	2.5	[1.7, 4.9]	2.4
Good-level real exchange rates (LOP deviations)				
Half-life	0.6	1.0	[0.8, 1.5]	1.2

- Note: The unit of a period is year

Conclusion

- Two puzzles on PPP and LOP deviations
  1. The persistence of the aggregate RER is too high to be explained by reasonable degree of nominal price rigidities
  2. The good-level RER is less persistent than the aggregate RER
- The behavioral model by Gabaix (2014, 2020) could explain these puzzles

	Model ($m = 1$)	Model ($m < 1$)	Data
Aggregate real exchange rates (PPP deviations)			
Half-life	0.6	2.5-3.7	2.4-4.9
Good-level real exchange rates (LOP deviations)			
Half-life	0.6	1.0-1.2	1.2-1.6

The way forward

1. Roundabout production

   - It may also generate the feedback effect from the aggregate RER to the good-level RER
   - The model with roundabout production function generates complementarity

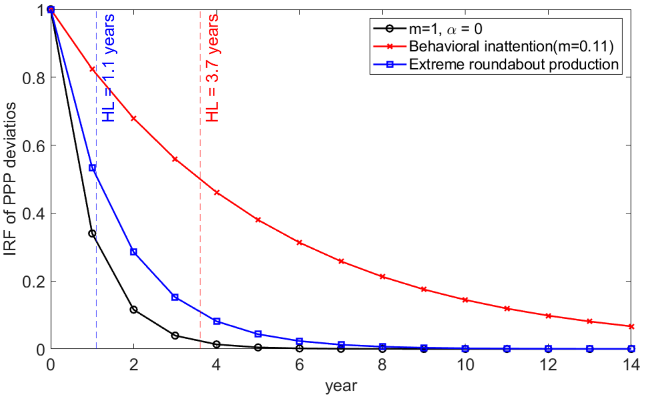
   $$y_{it}(z) = a_{it} n_{it}(z)^{1-\alpha} \Gamma_{it}(z)^{\alpha}$$

   where $\Gamma_{it}(z)$ is the intermediate input demand

   - The marginal cost of production is affected by the general price index
   - Pricing complementarity generates the link btwn $\ln q_{it}$ and $\ln q_t$

Behavioral attention VS. Roundabout production

- Comparing half-lives
- Behavioral inattention based on the estimated m VS. Extreme case of roundabout production ($\alpha = 0.99$)



- Much powerful in generating persistence

Thank you!

Households

- Domestic household solves

$$\max \ \mathbb{E}_0 \sum_{t=0}^{\infty} \delta^t (\ln c_t - \chi n_t)$$

$$\begin{aligned} \text{s.t. } M_t + B_t &= W_t n_t + R_{t-1} B_{t-1} + (M_{t-1} - P_t c_{t-1}) + T_t + \Pi_t \\ M_t &\geq P_t c_t \end{aligned}$$

M_t: nominal money holding, B_t: nominal bond holding, W_t: nominal wage, R_t: nominal interest, P_t: price level, T_t: transfers, Π_t: profits, δ: discount factor

- Foreign household's problem is analogously defined except for the budget const.

$$\begin{aligned} \text{s.t. } M_t^* + \frac{B_t^*}{S_t} &= W_t^* n_t^* + \frac{R_{t-1}}{S_t} B_{t-1}^* + (M_{t-1}^* - P_{t-1}^* c_{t-1}^*) + T_t^* + \Pi_t^* \\ M_t^* &\geq P_t^* c_t^* \end{aligned}$$

S_t: nominal exchange rate

Households (2)

- FOC

$$\frac{W_t}{P_t} = \chi c_t, \; \frac{W_t^*}{P_t^*} = \chi c_t^*$$
$$M_t = P_t c_t, \; M_t^* = P_t^* c_t^*$$

$$\frac{1}{R_t} = \delta \mathbb{E}_t \left[\left(\frac{c_{t+1}}{c_t} \right)^{-1} \frac{P_t}{P_{t+1}} \right] = \beta \mathbb{E}_t \left[\left(\frac{c_{t+1}^*}{c_t^*} \right)^{-1} \frac{S_t}{S_{t+1}} \frac{P_t^*}{P_{t+1}^*} \right]$$
$$q_t \frac{U_{c,t}}{U_{c,t}^*} = q_{t-1} \frac{U_{c,t-1}}{U_{c,t-1}^*} = \cdots = q_0 \frac{U_{c,0}}{U_{c,0}^*} = 1$$

back

CES aggregators

- Home and Foreign (*)
- Consumption of good i

$$c_{it} = \left[\int c_{it}(z)^{(\varepsilon-1)/\varepsilon} dz \right]^{\frac{\varepsilon}{\varepsilon-1}}, \; c^*_{it} = \left[\int c^*_{it}(z)^{(\varepsilon-1)/\varepsilon} dz \right]^{\frac{\varepsilon}{\varepsilon-1}}$$

  - where z denotes the brand z of good i
  - $z \in [0, 1/2]$ is produced in home and $z \in (1/2, 1]$ is produced in foreign

- Aggregate consumption

$$c_t = \left[\int c_{it}^{(\varepsilon-1)/\varepsilon} di \right]^{\frac{\varepsilon}{\varepsilon-1}}, \; c^*_t = \left[\int c_{it}^{*(\varepsilon-1)/\varepsilon} di \right]^{\frac{\varepsilon}{\varepsilon-1}}$$

Resource constraint

- Production function

$$y_{it}(z) = a_{it}n_{it}(z)$$

  - $z \in [0, 1/2]$ are domestic firms

$$y^*_{it}(z) = a^*_{it}n^*_{it}(z)$$

  - $z \in (1/2, 1]$ are foreign firms

- Resource constraint

$$\begin{aligned} c_{it}(z) + (1+\tau)c^*_{it}(z) &= y_{it}(z) \text{ for } z \in [0, 1/2] \\ (1+\tau)c_{it}(z) + c^*_{it}(z) &= y^*_{it}(z) \text{ for } z \in (1/2, 1] \end{aligned}$$

  - Firms supply their goods to home and foreign cities

back

Price indexes for good i

- Under Calvo pricing,

$$\hat{p}_{it} = \lambda(\hat{p}_{it-1} - \pi_t) + (1-\lambda)\hat{p}_{it}^{opt}(m_H, m_F)$$
$$\hat{p}_{it}^{*} = \lambda(\hat{p}_{it-1}^{*} - \pi_t^{*}) + (1-\lambda)\hat{p}_{it}^{opt*}(m_F^{*}, m_H^{*})$$

- $\hat{p}_{it}^{opt}(m_H, m_F)$, $\hat{p}_{it}^{opt*}(m_F^{*}, m_H^{*})$ are the weighted average of reset prices:

$$\hat{p}_{it}^{opt}(m_H, m_F) = \omega\hat{p}_{Hi}(m_H) + (1-\omega)\hat{p}_{Fi}(m_F)$$
$$\hat{p}_{it}^{opt*}(m_F^{*}, m_H^{*}) = \omega\hat{p}_{Fi}(m_F^{*}) + (1-\omega)\hat{p}_{Hi}(m_H^{*})$$

where $1/2 < \omega < 1$ is the degree of home bias as a function of trade costs τ back

Proposition 2

Under the same assumptions in Proposition 1,

$$\rho_q \geq \lambda$$

provided $m \in (0,1]$ *and* $\lambda \in (0,1)$.

- Aggregate the LOP deviations

$$\ln q_{it} = \lambda \ln q_{it-1} + (1-m)(1-\lambda) \ln q_t + \lambda \varepsilon_t^n + \psi \varepsilon_{it}^r$$

to get

$$\ln q_t = \rho_q \ln q_{t-1} + \rho_q \varepsilon_t^n,$$

where

$$\rho_q = \frac{\lambda}{1-(1-m)(1-\lambda)}$$

- back

Proposition 3

Under the same assumptions in Proposition 1,

$$\rho_q \geq \rho_{qi}$$

provided $m \in (0, 1]$, $\lambda \in (0, 1)$, $\tau \in [0, \infty)$, $\varepsilon \in (1, \infty)$, *and* $\sigma_r / \sigma_n \in [0, \infty)$.

- The relationship btwn ρ_q and ρ_{qi} is

$$\rho_q = \left[\frac{1}{1 - (1 - m)(1 - \lambda) \frac{A}{1+A}} \right] \rho_{qi}$$

where

$$A = \psi^2 \frac{1 - \rho_q^2}{\rho_q^2 (1 - \lambda^2)} \left(\frac{\sigma_r}{\sigma_n} \right)^2$$

- back