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Friend-of-a-Friend in Production Networks: Micro Estimates and Macro Implications*

Hiroyuki Asai† and Makoto Nirei‡

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Abstract

The *friend-of-a-friend* effect is the idea that when two firms share a common trading partner, a new link between them is likely to form. We quantify this effect in production networks, where the shared partner acts as relational capital facilitating new connections. First, we develop a general equilibrium (GE) model that endogenizes firms' link formation and incorporates the friend-of-a-friend mechanism. We show that the GE model simplifies to a *dyad-level logit specification*, enabling us to estimate the friend-of-a-friend effect using a quadruple-based conditional logit that controls for buyer and supplier fixed effects. Analyzing a dynamic panel of Japanese firm-to-firm transactions provides strong evidence of the friend-of-a-friend effect, with a magnitude comparable to other important factors like physical distance and sectoral proximity. Finally, we evaluate the macroeconomic impact through a counterfactual analysis within a calibrated GE model. Results indicate that removing the friend-of-a-friend effect decreases welfare by 0.6% and changes the propagation of firm-level shocks by altering the network structure.

Keywords: Production Networks, Firm Dynamics, Network Formation, Aggregate Productivity

JEL Classification: D21, D24, D57, D85, E22, E23, E61, F21

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1 Introduction

A growing body of research shows that production networks are key to aggregate outcomes. The structure of these networks determines aggregate productivity and controls how individual shocks propagate through the economy (Boehm and Oberfield, 2020; Carvalho et al., 2021). However, we understand relatively little about how these networks form, especially since each firm-to-firm connection is inherently relational rather than simply driven by anonymous market mechanisms.

We focus on a specific relational channel, the *friend-of-a-friend (FoF)* mechanism: when two firms share a common partner, that partner acts as intangible relational capital, promoting link formation between the two firms. This mechanism has been widely discussed in social and organizational networks, including labor markets, scientific and corporate collaboration, and digital platforms (Granovetter, 1973; Powell et al., 1996; Kossinets and Watts, 2006; Leskovec et al., 2008). We can observe a clear, though not conclusive, signature of this mechanism in many types of networks in the form of *clustering*, where two neighbors of a given node are more likely to be neighbors themselves. While directly confirming the FoF mechanism is challenging, we can infer its presence from clustering patterns in observed networks.

However, high clustering does not automatically prove the FoF mechanism. An alternative explanation, known as *homophily*, proposes that clustering can naturally arise from the well-documented tendency to form relationships with proximate counterparts. Specifically, in production networks, a firm's choice to connect with partners close in geography or industry suggests that those partners are also likely to be near each other. This proximity between the partners increases the chances of forming direct links, creating a cluster even without an FoF effect. Therefore, to understand relationship-based link formation and its macroeconomic importance, we must first distinguish the actual FoF effect from the clustering caused by homophily.

This paper directly measures the friend-of-a-friend effect from observed clustering patterns and assesses its macroeconomic significance. We develop a general equilibrium (GE) model with endogenous production network formation, incorporating the FoF and homophily mechanisms, and demonstrate that the link-formation process simplifies to a *dyad-level logit model*. Using panel data from Japanese production networks, we identify a clear FoF effect separate from homophily. Additionally, the calibrated model underscores its macroeconomic importance: the FoF effect encourages link formation and influences how shocks propagate through production networks by changing the network topology.

First, we present several motivating facts that suggest the possibility of the FoF effect and the homophily mechanism, using production network data from Japan. We start by analyzing the local clustering coefficient—the fraction of all possible partner-pairs around firm i that are linked. Compared to the random benchmark, the production network data show a much higher average of this coefficient across firms. While this could indicate the FoF mechanism, the data also support the homophily mechanism. We examine the average pairwise proximity between all potential partners around firm i in terms of sector and geography, and find that the pairwise proximity is actually higher in the data than in the random benchmark. This suggests that partners of firm i are more similar to each other, which could explain the observed clustering. These diagnostics motivate a structural empirical approach that can distinguish the link formation process related to FoF from proximity-driven incentives.

Then, we develop a general equilibrium model that endogenizes the network formation decision of each firm. In a fixed-cost framework following Bernard et al. (2022), each potential link is formed if a random link-formation cost is lower than the expected surplus from that link. This surplus depends on sectoral proximity and physical distance, as implied by firms' production structure. We assume the link formation cost is discounted when a buyer-supplier pair has a common partner in the previous period. A discount parameter governs the intensity of this FoF effect.

Using the model, we then estimate the discount parameter. We first demonstrate that it is estimated as a coefficient of a dyad-level logit model with unobserved heterogeneity of supplier and buyer after controlling for the effects of several sources of homophily. To difference out these two-sided fixed effects in a binary-choice setting, we apply the conditional logit transformation on quadruples proposed by Charbonneau (2017) and Jochmans (2018), and address the combinatorial challenge by randomly sampling informative quadruples following Izumi et al. (2023). The estimation results provide strong evidence of a FoF effect: having a common partner reduces network formation costs by 80%, which is comparable to a 1.75 standard deviation decline in physical distance or a 1.5 standard deviation increase in sectoral proximity for link formation. Interestingly, ignoring homophily significantly inflates the estimated coefficient, which aligns with the idea that the observed clustering results from a mixture of both forces.

Finally, we analyze the macroeconomic implications of FoF by comparing two equilibria in the GE model calibrated to Japan's manufacturing sector in 2015: one with the actual production network structures and another with a counterfactual network that excludes the FoF effect in its formation process. In the counterfactual economy, welfare decreases by 0.6%, mainly because firms cannot produce goods as efficiently as in the baseline due to having fewer suppliers. Decomposition results indicate that the less efficient production propagates to other firms through existing production networks, which amplifies the initial impact on welfare.

We further compare the baseline and the counterfactual economy excluding FoF in terms of shock transmission. We apply the same productivity shock to targeted groups of firms in the two economies, and analyze the resulting gap in welfare responses. Our analysis reveals that shocks to firms that lose their buyers in the CF network affect welfare less in CF than in the baseline, because the downstream propagation of these shocks via their lost buyers is removed. Shocks to firms that lose their suppliers also affect welfare less; however, this mainly reflects that these firms' unit costs increase, making their goods less important as final goods for the household.

Related Literature

This paper connects to various strands of literature. First, it expands on research analyzing the FoF effect in production networks. Atalay et al. (2011), Carvalho and Voigtländer (2015), and Chaney (2014) are the first to introduce an FoF perspective into production and trade networks by including an exogenously specified stochastic process. Recent studies by Panigrahi (2022) and Alfaro-Ureña and Zacchia (2024) construct a sourcing decision process for buyers that incorporates the FoF mechanism and estimate it using production-network data from India and Costa Rica. Building on prior research, we offer a model with a link-formation process where heterogeneity on both sides (supplier and buyer) plays a role, and our GE setting enables us to assess the macroeconomic consequences of FoF. Despite the model's complexity, our estimation approach, rooted in network econometrics, can handle unobserved heterogeneity using fixed effects. Additionally, we highlight the macroeconomic importance of FoF: its quantitative impact on overall welfare and its topological influence on shock propagation.

This paper also adds to the rapidly expanding literature on endogenous production network formation in general equilibrium (e.g., Oberfield, 2018; Eaton et al., 2022; Acemoglu and Azar, 2020; Lim, 2018; Bernard et al., 2022; Dhyne et al., 2021; Kopytov et al., 2022; Elliott et al., 2022; Arkolakis et al., 2023; Miyauchi, 2024). We advance the literature by presenting a portable estimation method that combines our extension of the workhorse fixed-cost-type models (e.g., Lim, 2018; Huneeus, 2020; Bernard et al., 2022) with the dyad-level logit approach. Additionally, our counterfactual analysis underscores the aggregate significance of the FoF effect, emphasizing the need to consider nonmarket factors in firms' networking strategies.

Our estimation approach builds on recent advances in network econometrics (e.g., Graham, 2016; Charbonneau, 2017; Jochmans, 2018; Izumi et al., 2023). We contribute to this literature by providing an underlying GE framework, linking micro-level estimates to the macro-level assessment. Additionally, we leverage the dynamic panel structure of our network data[1] and estimate the effect of existing link structures on link formation, avoiding the simultaneity problem.

Outline

The rest of this paper is organized as follows. Section 2 discusses motivating facts about clustering using the production network in Japan. Section 3 introduces a GE model to analyze the mechanism and key macroeconomic implications quantitatively. Section 4 explains how to identify the key parameters and presents their estimation results. Section 5 examines the aggregate implications using the calibrated model. Section 6 concludes the paper.

2 Motivating Facts

2.1 Data

We use a proprietary firm-level dataset from Tokyo Shoko Research (TSR), a leading credit reporting agency in Japan.[2] TSR gathers comprehensive information on companies through personal interviews or phone surveys, complemented by public sources such as financial statements, corporate registrations, and public relations documents. The data are updated annually, and we have compiled datasets from 2007 to 2022 that cover all sectors in Japan.

[1]This longitudinal dimension of the data is also utilized by Kawakubo and Suzuki (2023), Miyauchi (2024), and Asai and Nirei (2025) to study the dynamic evolution of firm-to-firm networks.

[2]Check Carvalho and Tahbaz-Salehi (2019) and Bacilieri et al. (2023) for comparison with production network data in other countries.

In addition to standard financial information about firms, including the number of employees, sales, four-digit industry classification, and address, the TSR data provide unique details about transaction partners. Each firm reports its suppliers, buyers, and major shareholders up to 24 firms. Despite this reporting limit, the truncation is unlikely to be restrictive in practice for two reasons. First, the proportion of firms reporting exactly 24 suppliers or buyers, which could suggest a boundary due to truncation, is less than 0.1%. Second, we merge self and counterparty reports following Bernard et al. (2019) and Carvalho et al. (2021). Specifically, we combine lists of suppliers (buyers) reported by the firm itself and counterparty lists showing the firm as a buyer (supplier). Therefore, even if the self-reported number of partners hits the cap, as long as the partners report the truncated links, the impact of truncation remains minimal.

For cross-sectional analysis in this section, we use the sample of firms reporting in 2015. Following the structure of the model and its estimation strategy developed in Section 3, the empirical analysis in Section 4 relies on a panel structure of our data and uses the two-year panel (2014-2015). In the appendix, we present summary statistics of the data and confirm that our results are robust to alternative sample years.

2.2 Clustering Coefficient

We first analyze a well-known measure, the *local clustering coefficient*. For simplicity, we treat the network as undirected in this section, while it is considered directed in later estimation sections. The local clustering coefficient, C_i, for an undirected graph, is the proportion of all possible partner-pairs around firm i that are linked to each other, as defined below.

$$C_i = \frac{L_i}{\binom{|\mathcal{P}_i|}{2}} \tag{2.1}$$

where $\mathcal{P}_i$ is the set of partners (suppliers and buyers) of i, and L_i is the number of links between firms in $\mathcal{P}_i$.[3] This can be interpreted as the probability that two distinct partners of i are linked. Figure 1 shows the graphical examples. In the left example, several links between partners of firm i are observed, implying a higher local clustering coefficient of firm i compared to the right example, where there are no links between any pair of the partners of i'.[4]

Figure 2 shows a histogram of the local clustering coefficient for firms in Japan in 2015, with the sample mean marked by a red dashed line. A green dashed line indicates the mean from degree-preserving randomized networks (Maslov and Sneppen, 2002) used as a null-network benchmark, where all links were randomly rewired while preserving the distribution of the number of connection partners.[5] Compared to the null networks, the real data shows

[3] In this section, we exclude firms with $|\mathcal{P}_i| \leq 1$.

[4] A large clustering coefficient, i.e., $E[C_i] \nrightarrow 0$ for $N \to \infty$, is one of the characteristics of so-called *small-world network*. For the details, see Watts and Strogatz (1998).

[5] Note that the mean is one realization of a random variable, as a random sampling procedure is involved to form the null network. In Appendix D.2, we show that the mean value for the random null networks is robust for other random seeds.

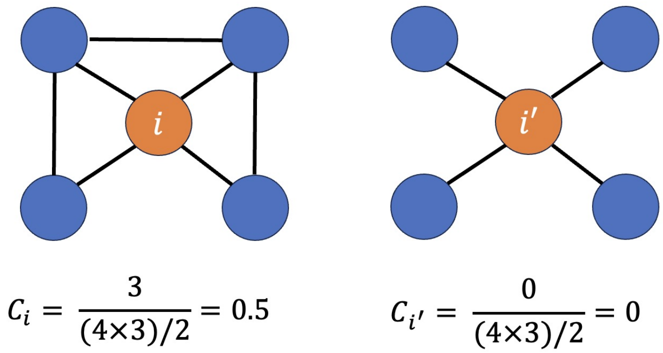

Figure 1: Graphical illustration of clustering coefficient calculation

a higher average. This is consistent with the presence of the friend-of-a-friend (FoF) effect, where a common partner encourages two firms to form a connection.

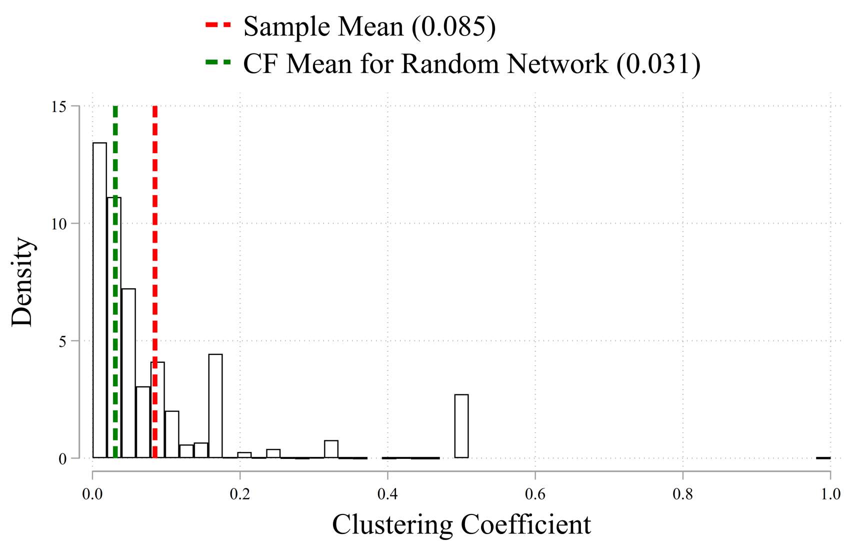

Figure 2: Distribution of clustering coefficients across firms in Japan's production network (2015)

Not only the FoF effect but also homophily can explain the high clustering observed. A well-known fact about production networks is that firms tend to form relationships with other firms in nearby areas and industries close to their own (Bernard et al., 2019; Arkolakis et al., 2023; Miyauchi, 2024). Suppose firm i connects with partners nearby in terms of geography or industry sector. Mechanically, this means that partners of firm i are also likely to be close to each other. Consequently, this proximity between pairs of partners should increase the likelihood that these partners have relationships with each other, even without the FoF effect.

To demonstrate the prevalence of the homophily effect, for each firm i, we calculate the *average pairwise proximity* among i's partners, considering all unordered pairs $\{p, q\}$. Let the symbol $\binom{\mathcal{X}}{k}$ denote the set of all k-element subsets of the set $\mathcal{X}$. Given a proximity measure $g(p,q)$ defined on unordered pairs $\binom{\mathcal{P}_i}{2}$, the average pairwise proximity $\bar{D}_i^g$ is defined as

$$\bar{D}_i^g = \frac{1}{\binom{|\mathcal{P}_i|}{2}} \sum_{\{p,q\}\in\binom{\mathcal{P}_i}{2}} g(p,q). \tag{2.2}$$

The average pairwise proximity $\bar{D}_i^g$ increases when a firm's partners are closer together. For $g(p,q)$, we examine two common measures of proximity: physical distance and sectoral proximity. Our choice of physical distance is supported by the strong out-of-sample performance of gravity models, which consistently demonstrate that distance reduces bilateral trade in the literature (Head and Mayer, 2014). Sectoral proximity is based on the stability of input–output linkages and their widespread use in measuring technological relatedness and positions along value chains (Antràs et al., 2012). We calculate sectoral proximity as the average of the input coefficient and the output coefficient, following the definition of Bacilieri et al. (2023)[6] using the two-digit input-output table for Japan in 2015.[7]

Figure 3 shows the distributions of both measures, with the sample mean and the null mean overlaid just like before. Both figures provide evidence of homophily's effect on the observed clustering in Figure 2. Partners of a firm tend to be close to each other, both physically and sectorally. This indicates that even without FoF, a pair of partners within a firm is likely to connect, driven by incentives based on sectoral and geographical proximity.

To summarize our reduced-form findings, the production network data show high clustering coefficients, consistent with FoF; however, homophily based on geographical and sectoral proximity, which can also explain the high clustering coefficient, is clearly evident in the data. These facts motivate our structural approach to distinguish FoF from proximity in the following sections.

3 Model

In this section, we develop a GE model with an endogenous production network structure in two steps. First, we characterize an equilibrium where the production network structure is taken as given. Next, we describe how each firm makes decisions regarding network formation. We demonstrate that, under some assumptions, the dyadic link formation equation is endogenously characterized as an equilibrium condition.

[6] Given a typical input-output network matrix X, where rows represent buyers and columns represent suppliers, the input coefficient (input share) A is calculated as $A_{ij} = \frac{X_{ij}}{\sum_j X_{ij}}$, and the output coefficient (sales share) B as $B_{ij} = \frac{X_{ij}}{\sum_i X_{ij}}$. Since we analyze undirected networks in this section, we average the two measures.

[7] See `https://www.soumu.go.jp/english/dgpp_ss/data/io/io15_00001.htm`.

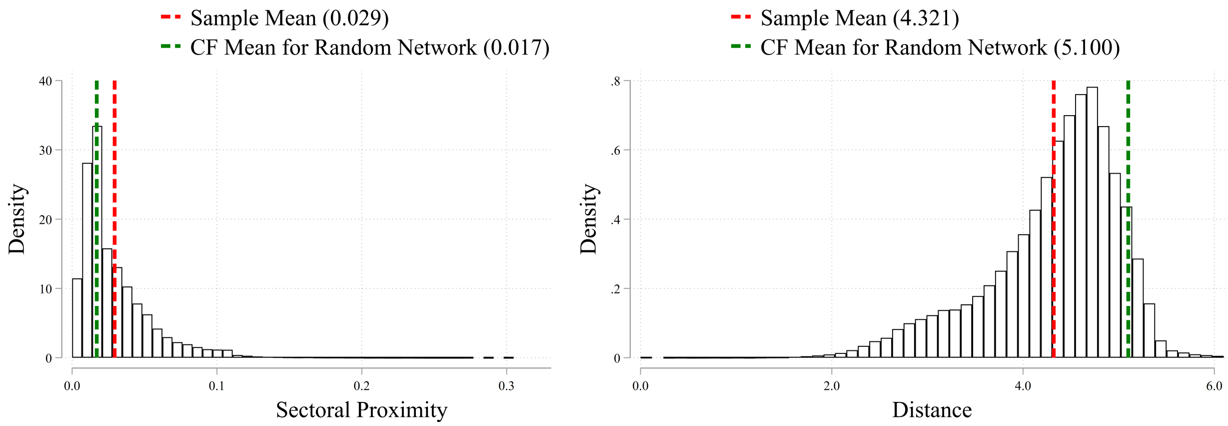

Figure 3: Distributions of average pairwise proximity across firms in Japan's production network (2015)

3.1 Fixed Network Model

3.1.1 Environment

The economy consists of a representative household and a set of firms $\mathcal{N}$ owned by the household. Each firm $i \in \mathcal{N}$ is characterized by fundamental productivity ϕ_i, fundamental demand χ_i, sector k_i, location a_i, set of suppliers $\mathcal{S}_i \subset \mathcal{N}$, and set of buyers $\mathcal{B}_i \subset \mathcal{N}$. By combining labor and intermediate inputs sourced from its own suppliers, each firm produces its own differentiated good.

3.1.2 Households

The representative household supplies a unit of labor inelastically and has CES preferences over final goods, expressed as:

$$U = \left[\sum_{i \in \mathcal{N}} (\chi_i c_i)^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}}. \tag{3.1}$$

Then, each of the demands is

$$c_i = \chi_i^{\sigma-1} \left(\frac{p_i}{P^{Final}} \right)^{-\sigma} U \tag{3.2}$$

$$\text{with } P^{Final} = \left[\sum_{i \in \mathcal{N}} \left(\frac{p_i}{\chi_i} \right)^{1-\sigma} \right]^{\frac{1}{1-\sigma}}. \tag{3.3}$$

3.1.3 Firm production

Each firm produces its output using labor and a bundle of inputs that include intermediate goods sourced from its suppliers. The nested CES production function is given by:

$$y_i = \phi_i \left(\alpha (l_i)^{\frac{\theta-1}{\theta}} + (1-\alpha) (M_i)^{\frac{\theta-1}{\theta}} \right)^{\frac{\theta}{\theta-1}} \tag{3.4}$$

$$\text{with } M_i = \left[\sum_{j \in \mathcal{S}_i} (\lambda_{k_i}^{k_j} x_i^j)^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}} \tag{3.5}$$

where α, θ, σ are the intensity of labor input, the elasticity of substitution between labor and the intermediate goods bundle, and the elasticity of substitution among intermediate goods inputs, respectively. $\lambda_{k_i}^{k_j}$ indicates sectoral proximity between the sector of buyer k_i and that of supplier k_j.[8]

Then, its unit cost is

$$\zeta_i = \frac{1}{\phi_i} \left(\alpha^\theta w^{1-\theta} + (1-\alpha)^\theta (P_i)^{1-\theta} \right)^{\frac{1}{1-\theta}} \tag{3.6}$$

$$\text{with } P_i = \left[\sum_{j \in \mathcal{S}_i} \left(\frac{\tau_{a_i}^{a_j}}{\lambda_{k_i}^{k_j}} p_i^j \right)^{1-\sigma} \right]^{\frac{1}{1-\sigma}} \tag{3.7}$$

where p_i^j is the price charged by supplier j on buyer i, and $\tau_{a_i}^{a_j}$ is an iceberg cost between the locations of buyer a_i and that of supplier a_j.

By defining the demand shifter of firm i as $\Delta_i := \dfrac{M_i}{P_i^{-\sigma}}$, we obtain

$$x_i^j = \left(\lambda_{k_i}^{k_j} \right)^{\sigma-1} \left(\tau_{a_i}^{a_j} p_i^j \right)^{-\sigma} \Delta_i. \tag{3.8}$$

3.1.4 Market Structure

We assume suppliers behave atomistically toward buyers. Under a monopolistic competition setting, based on the CES demand structure and the CRS production structure, a price that maximizes its profit is given by

$$p_i = p_j^i = \mu \zeta_i \tag{3.9}$$

where

$$\mu = \frac{\sigma}{\sigma - 1} \tag{3.10}$$

is a constant markup rate.

[8]When a variable has two firm indices, the subscript denotes the buyer and the superscript denotes the supplier.

From (3.8) and (3.9), the surplus supplier j gains from selling intermediate goods to buyer i is given by:

$$V_i^j = \frac{\mu - 1}{\mu} p_i^j \tau_{a_i}^{a_j} x_i^j \tag{3.11}$$

$$= (\mu - 1)\mu^{-\sigma} \Delta_i \left(\zeta_j\right)^{1-\sigma} \left(\lambda_{k_i}^{k_j}\right)^{\sigma-1} \left(\tau_{a_i}^{a_j}\right)^{1-\sigma} \tag{3.12}$$

Equation (3.12) shows that this surplus depends on two firm-level equilibrium variables (the buyer's demand shifter Δ_i and the supplier's unit cost ζ_j) and two link-level exogenous factors (sectoral proximity $\lambda_{k_i}^{k_j}$ and iceberg cost $\tau_{a_i}^{a_j}$).

3.1.5 Market Clearing

For each good i, we obtain the goods market-clearing condition as follows.

$$y_i = c_i + \sum_{j \in \mathcal{B}_i} \tau_{a_j}^{a_i} x_j^i \tag{3.13}$$

We also obtain the labor market clearing condition (LMC) as follows.

$$L = \sum_{i \in \mathcal{N}} l_i \tag{3.14}$$

3.1.6 Profit

From the markup pricing rule, the profit of firm i can be written as

$$\pi_i = (\mu - 1)\zeta_i y_i. \tag{3.15}$$

Hence, the aggregate profit the household obtains and her income become

$$\Pi = \sum_{i \in \mathcal{N}} \pi_i \tag{3.16}$$

$$I = wL + \Pi. \tag{3.17}$$

3.1.7 Equilibrium

We first define an equilibrium for a given production network.

Definition 1 (Fixed Network Equilibrium)**.** Given the network structure $\mathcal{S}_i$ and $\mathcal{B}_i$ for all $i \in \mathcal{N}$, an exogenous network equilibrium is a pair of allocations and prices such that (1) the household maximizes its utility, (2) firms maximize their profit by producing and setting prices for each of their final and intermediate goods given the networks, and (3) the labor market and the goods market clear. (We take the wage $w = 1$ as the numeraire.)

3.2 Endogenous Network Model

We now describe the link formation process. We consider a two-period economy with $t = 0$ and $t = 1$. The timeline of the economy is summarized in Figure 4. At $t = 0$, the network structure is exogenously given, and the economy is in a fixed-network equilibrium. At the beginning of $t = 1$, two types of shocks occur at the link level. First, each link present at $t = 0$ is exogenously terminated with some probability.[9] Next, for each link that is not present at $t = 0$, a fixed cost to activate the link is drawn. Following the literature (Lim, 2018; Huneeus, 2020; Bernard et al., 2022), we assume the supplier incurs the fixed cost. Then, each supplier compares the realized fixed cost with the expected surplus from activating the link and decides whether to form it; the collection of choices yields the network structure at $t = 1$. Finally, the associated fixed-network equilibrium is realized.

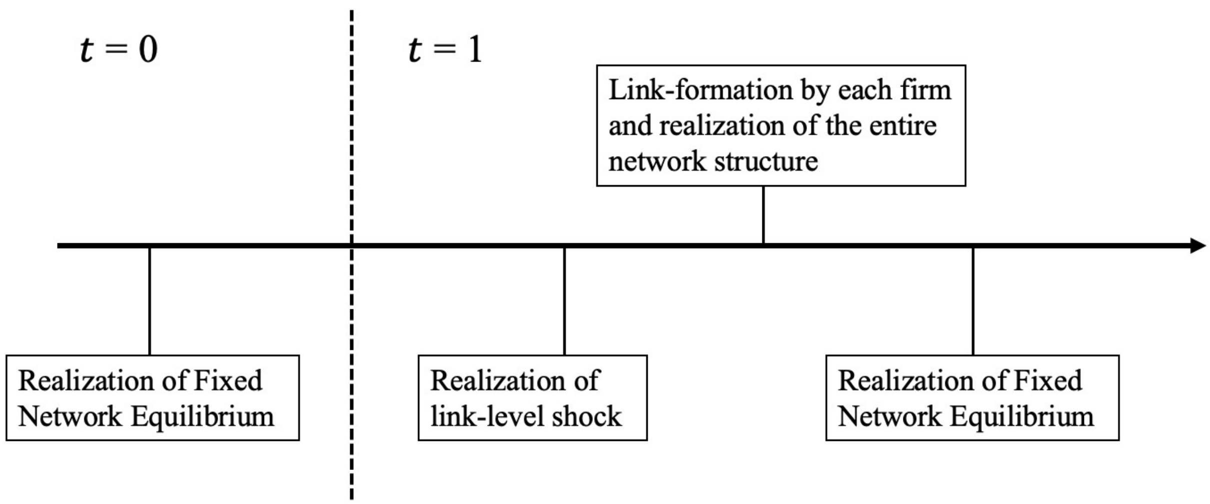

Figure 4: Model Timeline for Link Formation and Equilibrium Realization

3.2.1 Network formation

To formalize suppliers' decisions to form links, we introduce two assumptions. The first assumption is *static expectations*. This assumption governs how a potential supplier j evaluates the expected surplus ($V_i^{j,expected}$) from a new link with buyer i. Recall from (3.12) that this surplus depends on the buyer's demand shifter (Δ_i) and the supplier's unit cost (ζ_j). The static expectations imply that the supplier does not forecast the $t = 1$ values of these equilibrium objects, while both of them could, in reality, change at $t = 1$ due to the rewiring of the production networks.[10] Instead, the supplier evaluates the expected gain using the

[9] This is just to match the data in Section 5.1.

[10] This assumption is less restrictive than it may appear. First, suppose that the aggregate network structure—except for the link between i and j—remains constant and any GE feedback is shut down. Then, given the CRS production technology, an increase in demand from a new buyer i leaves supplier j's unit cost (ζ_j) unchanged. Similarly, the assumption of atomistic suppliers implies that supplier j takes buyer i's demand shifter (Δ_i) as given, even without static expectations. Therefore, the primary role of static expectations is to fix the global network structure and exclude the GE feedback when the supplier evaluates

realized equilibrium values at $t = 0$ (Δ_i^0 and ζ_j^0). Therefore, the supplier j's expected gain is given by (3.12) evaluated at equilibrium objects $t = 0$:

$$V_i^{j,expected} = (\mu - 1)\mu^{-\sigma}\Delta_i^0 \left(\zeta_j^0\right)^{1-\sigma} \left(\lambda_{k_i}^{k_j}\right)^{\sigma-1} \left(\tau_{a_i}^{a_j}\right)^{1-\sigma} \tag{3.18}$$

An alternative approach could be one of complete information, where firms know the equilibrium outcome of $t = 1$ at the time they decide on links. There are three drawbacks to the complete information setup. First, equilibrium uniqueness is not guaranteed. Second, calculating equilibria is infeasibly costly, unlike under the static-expectations assumption. Third, and most critically, with the complete information setup, we lose the estimable equation shown in the next section: firms' decisions would depend on $t = 1$ equilibrium objects, which themselves depend on the decisions of all other firms, creating an intractable simultaneity problem for estimation.

The second assumption is *friend-of-a-friend discount*: a link-level *i.i.d.* log-logistic cost shock $\tilde{\varepsilon}_i^{j,1}$ is discounted by a factor $\exp(-\beta)$ if the supplier and buyer had a common partner in the previous period.[11] While we are agnostic about the mechanism underlying the FoF discount, a plausible explanation can be provided by reduced information frictions (Chaney, 2014) or a decline in search cost (Carvalho and Voigtländer, 2015).

We define $CommonPartner_i^{j,0}$ as an indicator of whether i and j share a common partner at $t = 0$. It is a directionally agnostic indicator that does not distinguish between common-supplier, common-buyer, or cross-type cases. Then, a potential link between buyer i and supplier j is formed if

$$V_i^{j,expected} \geq \tilde{\varepsilon}_i^{j,1} \exp(-\beta\, CommonPartner_i^{j,0}) \tag{3.19}$$

$$\Leftrightarrow \quad (\mu - 1)\mu^{-\sigma}\Delta_i^0 \left(\zeta_j^0\right)^{1-\sigma} \left(\lambda_{k_i}^{k_j}\right)^{\sigma-1} \left(\tau_{a_i}^{a_j}\right)^{1-\sigma} \geq \tilde{\varepsilon}_i^{j,1} \exp(-\beta\, CommonPartner_i^{j,0}). \tag{3.20}$$

Equation (3.20) has a clear interpretation. Given an idiosyncratic cost shock $\tilde{\varepsilon}_i^{j,1}$, the probability of the link realizing is high if (i) the buyer demands many intermediate goods (large Δ_i),[12] (ii) the supplier produces efficiently (small ζ_j), (iii) the supplier and buyer are close in sector or location (large $\lambda_{k_i}^{k_j}$, small $\tau_{a_i}^{a_j}$), or (iv) they share a common partner ($CommonPartner_i^j = 1$).

3.2.2 Equilibrium

We define an equilibrium concept that includes each firm's networking decision discussed above.

the expected surplus. In reality, however, the network structure fluctuates stochastically due to independent link-level shocks, and any firm action induces endogenous GE adjustments. One important mechanism that is not internalized by suppliers due to the assumption is that acquiring a new buyer raises the supplier's own demand shifter, thereby enhancing the supplier's probability of acquiring new suppliers.

[11] In Appendix E, we analyze other specifications that consider heterogeneity in the intensity of FoF across firm size, number of common partners, or topological location of common partners.

[12] Δ_i becomes large when the firm requires a large intermediate input bundle (large M_i) or when its existing set of suppliers is, on average, expensive (large P_i).

Definition 2 (Endogenous Network Equilibrium at $t = 1$)**.** Given the fixed network equilibrium at $t = 0$ and the realization of link-level shocks, an endogenous network equilibrium at $t = 1$ is the network structure and pair of allocations and prices such that (i) the network structure is determined by each supplier's networking decision in (3.20), and (ii) the associated fixed-network equilibrium holds for that realized network structure.

4 Estimation of the Friend-of-a-Friend Effect

The structural model in the previous section provides us with an estimation equation for the FoF effect. In Section 4.1, we first derive the estimation equation for the FoF effect, leveraging a property of the dyad-level logit model. In Section 4.2, we show the estimation results using the actual dyad-level panel data of Japanese production networks.

4.1 Dyad-level Logit Estimation

Taking logs of both sides of the link-formation inequality (3.20) yields the following dyad-level logit model:

$$\begin{aligned} Y_i^{j,1} =& \mathbb{I}\Big\{\beta \times CommonPartner_i^{j,0} + \gamma_{sector} \times \log \lambda_{k_i}^{k_j} + \gamma_{distance} \times \log \tau_{a_i}^{a_j} \\ & \qquad + \alpha_i^{Buyer,0} + \alpha_j^{Supplier,0} + \varepsilon_i^{j,1} \geq 0\Big\} \end{aligned} \tag{4.1}$$

where $Y_i^{j,1}$ is a dummy variable indicating whether a link is formed between buyer i and supplier j at period 1. $CommonPartner_i^{j,0} = 1$ if there is a common partner of i and j at period 0. $\lambda_{k_i}^{k_j}$ is the input coefficient between buyer sector k_i and supplier sector k_j. $\tau_{a_i}^{a_j}$ is the iceberg cost between buyer location a_i and supplier location a_j. Buyer fixed effect $\alpha_i^{Buyer,0}$ captures how many intermediate goods the buyer demands, and supplier fixed effect $\alpha_j^{Supplier,0}$ captures how efficiently the supplier can produce the goods, both of which are unobservable. $\varepsilon_i^{j,1}$ is a dyad-level error term following a logistic distribution.[13]

Our coefficient of interest is β, which governs the FoF effect. Equation (4.1) shows that by estimating the dyad-level binary response model with firm-level fixed effects and dyad-level controls for sectoral and physical proximity measures, we can directly estimate the FoF effect without calibrating other structural parameters in the GE model.[14]

Estimating models of this type presents two main challenges. The first is an incidental-parameter problem caused by unobserved fixed effects α_i^{Buyer} and $\alpha_j^{Supplier}$, which bias nonlinear estimation as noted by Wooldridge (2010). To address this, we use a method to eliminate the fixed effects through variable transformation proposed by Charbonneau (2017) and Jochmans (2018). For the general form of (4.1) with the assumption that the error term follows a logistic distribution $Y_i^j = \mathbb{1}\{W_i^{j\prime}\theta + \alpha_i + \gamma_j \geq \varepsilon_i^j\}$, consider a quadruple $\sigma = \{i_1, i_2; j_1, j_2\}$ and

[13]While we assume the cost shock is independent of the dyad-level proximity measures, this assumption does not affect estimates of β as long as these characteristics affect the original log-logistic cost multiplicatively. These effects on the cost term are absorbed by γ_{sector} and $\gamma_{distance}$.

[14]Without the static expectations assumption in the model, we cannot consistently estimate β in the reduced-form manner because of simultaneity bias.

define $Z_\sigma = \dfrac{(Y_{i_1}^{j_1} - Y_{i_1}^{j_2}) - (Y_{i_2}^{j_1} - Y_{i_2}^{j_2})}{2}$ and $R_\sigma = (W_{i_1}^{j_1} - W_{i_1}^{j_2}) - (W_{i_2}^{j_1} - W_{i_2}^{j_2})$. Charbonneau (2017) and Jochmans (2018) show that for the transformed variables defined on the quadruple, we obtain

$$Pr\left(Z_\sigma = 1 | R_\sigma, Z_\sigma \in \{-1, 1\}\right) = \frac{\exp(R'_\sigma\theta)}{1 + \exp(R'_\sigma\theta)}. \tag{4.2}$$

Note that the RHS represents the standard conditional-logit probability without any terms related to fixed effects α_i and γ_j; we can easily evaluate its likelihood.[15]

The second issue is its computational burden, as the quadruple space grows exponentially, $O(N^4)$. Following Izumi et al. (2023), we implement a Monte Carlo approximation to the conditional likelihood: we select a uniform random subsample of quadruples that are part of the conditional likelihood ($Z_\sigma \in \{-1, 1\}$) and maximize the log-likelihood in (4.2) on this subsample. Standard errors are calculated using a nonparametric bootstrap. Appendix B.3 shows the details of this sampling procedure.

4.2 Estimation Results

We estimate (4.1) using the variable transformation method and sampling technique mentioned above. We assign period 1 to the year 2015, and based on the same TSR production-network dataset used in Section 2, we utilize data from 2014 and 2015, retaining firms that report their business activity in both years. Following the gravity literature, we assume a power-law relationship between distance and iceberg costs, which means replacing the log of the iceberg cost term with the log of distance does not affect the validity of the estimation of our coefficient of interest, β.[16] The sectoral proximity is calculated based on the input coefficient, according to Acemoglu et al. (2012).[17]

Table 1 shows estimation results for equation (4.1) across several models. Model 1 excludes all homophily terms. Model 2 adds log physical distance, and Model 3 adds log sectoral proximity. Model 4 is our baseline specification, which incorporates all terms implied by the model. Each specification accounts for the fixed effects of buyers and suppliers. Across all models, we observe strong evidence of a friend-of-a-friend effect, indicated by the coefficient on Common Partner. Results from the full model (Model 4) suggest that having a common partner raises the log-odds of link formation by 1.56, making the odds approximately 4.8 times higher. This is comparable to reducing log distance by 2.08 (1.75 standard deviations) or increasing log sectoral proximity by 3.0 (1.5 standard deviations).[18]

[15] As discussed in Izumi et al. (2023), this procedure can be interpreted as an analogy to the standard DID. In DID, we difference out the unobserved fixed effect by utilizing multiple observations in the time dimension. Here, instead, we difference it out by multiple observations in the network dimension. (One firm can be observed several times in a dyad-level data of the snapshot of production networks.)

[16] Given a distance elasticity of the iceberg cost ρ, the coefficient on the log of physical distance corresponds to the coefficient on the log of iceberg cost τ times ρ.

[17] Since the networks we consider in this empirical section are directed, we use the input coefficient, unlike in the motivating facts section, where we treat the network as undirected and take an average of the input and output coefficients. This approach is consistent with the model structure.

[18] The standard deviations of each measure are calculated using all possible pairs of firms in the estimation sample.

Note that once homophily is controlled for, the estimated FoF effect decreases significantly (2.54 $\rightarrow$ 1.92 when adding physical distance, 2.54 $\rightarrow$ 2.28 when adding sectoral proximity, and 2.54 $\rightarrow$ 1.56 when both controls are included). This finding confirms that homophily explains a meaningful portion of the observed clustering, consistent with Section 2.2. Appendix D presents robustness checks demonstrating that our results are stable when varying the sampled years and sectors or when excluding links with ownership ties. In Appendix E, we conduct heterogeneity analysis of the intensity of FoF across firm size, number of common partners, and topological location of common partners.

	Model 1	Model 2	Model 3	Model 4
Common Partner	2.54	1.92	2.28	1.56
	(0.03)	(0.05)	(0.04)	(0.05)
Physical Distance (log)		-0.73		-0.75
		(0.01)		(0.01)
Sectoral Proximity (log)			0.50	0.52
			(0.007)	(0.01)
Supplier FE	Yes	Yes	Yes	Yes
Buyer FE	Yes	Yes	Yes	Yes

Table 1: Estimation results of the dyad-level logit model: coefficients and standard errors.

5 Macroeconomic Evaluation

In this section, we quantitatively evaluate the macroeconomic importance of the FoF effect using the GE model. First, we calibrate the remaining model parameters in Section 5.1. Next, we build a counterfactual network structure that would have realized at $t = 1$ without the FoF effect based on the calibrated model combined with the empirical results on the microeconomic intensity of the FoF effect in Section 5.2. Finally, we analyze the GE behavior under the two networks at $t = 1$: the calibrated baseline networks and the counterfactual networks. By comparing them, we identify several key macroeconomic implications of the FoF effect in Section 5.3.

5.1 Calibration

5.1.1 Aggregate Parameters

Table 2 summarizes the aggregate parameter values, their sources or references, and the data used for setting targets. Regarding the elasticity parameters, we assume a Cobb–Douglas aggregator for labor and the intermediate (IMD) bundle, with an elasticity of substitution of 3 across goods based on Broda and Weinstein (2006). The labor share α in the production function is set to 0.54 to ensure that the ratio of aggregate intermediate inputs sales to the sum of intermediate inputs sales and consumption sales matches the value in Japan's 2015 input-output table.[19] We normalize the aggregate labor supply to $L = 1$.

[19] See `https://www.soumu.go.jp/english/dgpp_ss/data/io/io15_00001.htm`.

Parameter	Description	Value	Target
θ	EoS between labor and IMD bundle	1	Bernard et al. (2022)
σ	EoS between goods	3	Broda and Weinstein (2006)
α	Labor share	0.54	Expenditure share of IMD goods
L	Aggregate Labor Supply	1	Normalization

Table 2: Summary of parameter values, their source/reference, and data for setting targets.

5.1.2 Micro Parameters

To calibrate firm-level fundamental productivity, we use the TSR data from the manufacturing sector in 2015 ($|\mathcal{N}| = 8142$).[20] Production network structure $\mathcal{S}_i$ and $\mathcal{B}_i$ are set to the actual network structure. Fundamental productivity ϕ_i is calibrated to the number of employees, and we assume fundamental demand $\chi_i = 1$. Figure 5 shows a binned scatter of model-implied vs observed log employment (both normalized to sum to one), which exhibits a fairly good fit with $R^2 > 0.99$. In the appendix, we also confirm a good fit of non-targeted moments. In line with the empirical analysis in Section 4, the industrial weight on inter-

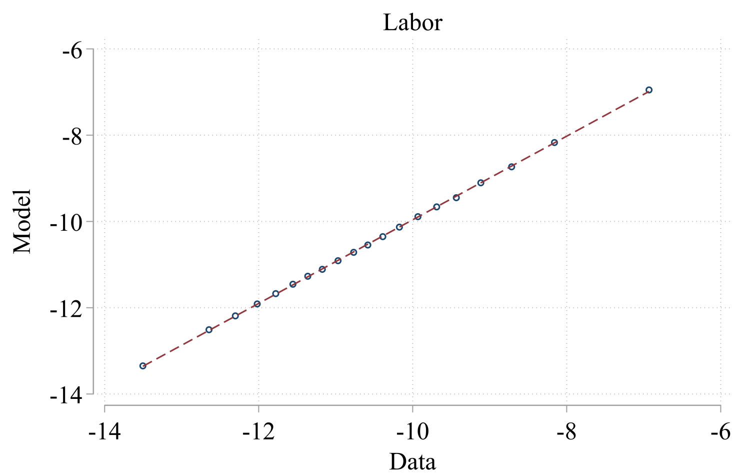

Figure 5: Number of Employees (Log): Model vs. Data

mediate goods $\lambda_k^{k'}$ is determined based on the input-output table in Japan at 2015.[21] Also, we adopt a power-law iceberg costs $\tau_a^{a'}$ with the distance elasticity $\rho = 0.23$, following Kano et al. (2013).

5.2 Construction of Counterfactual Network

Next, we show how to build a counterfactual network that would have existed without the FoF effect at $t = 1$ in the model. In Appendix A.1, by combining Bayes' rule with the

[20]We drop firms that do not report the sales, the number of employees, or the profit, and firms that do not have any suppliers in the sample.

[21]See `https://www.soumu.go.jp/english/dgpp_ss/data/io/io15_00001.htm`.

rare-event interpretation[22] of the logit (where the odds ratio approximates the relative risk), we demonstrate that the conditional probability that a FoF-backed dyad formed at $t = 1$ in the baseline (with FoF) also emerges at $t = 1$ in the counterfactual (without FoF) is approximately $\frac{1}{\exp(\beta)}$. Based on this result, we construct the counterfactual network as follows.[23]

1. Among all the observed links $\{link_{ij}\}$, extract the links that are newly created in that year and have a common partner from the previous year, denoted as $\{link_{ij}^{New,\ CP}\}$.

2. Among the $\{link_{ij}^{New,\ CP}\}$, randomly keep a $\frac{1}{\exp(\beta)}$ fraction of them and create the set of the counterfactual links, $\{link_{ij}^{New,\ CP,\ CF}\}$, which are newly created and share a common partner from last year.

3. Combine the remaining links not selected in step 1 ($\{link_{ij}\} \setminus \{link_{ij}^{New,\ CP}\}$) and the $\{link_{ij}^{New,\ CP,\ CF}\}$, and generate a counterfactual set of links $\{link_{ij}^{CF}\}$.[24]

5.3 Aggregate Implications of Friend-of-a-friend Effect

Finally, we compare two economies: the baseline economy with the network structure being $\{link_{ij}\}$, and a counterfactual economy with $\{link_{ij}^{\mathrm{CF}}\}$. Note that the procedure above requires a random sampling process, so the counterfactual network structure is not uniquely determined. Therefore, we conduct the sampling one hundred times and generate one hundred counterfactual network structures, then calculate the associated one hundred equilibrium outcomes. We report the sample mean of these outcomes hereafter. In Appendix D.2, we present the standard deviation, minimum value, and maximum value of each outcome, which shows quite small variation among samples and supports the robustness of our implications.

5.3.1 Macroeconomic Relevance of the Friend-of-a-Friend Effect

First, we examine the direct impact of the FoF effect on macroeconomic variables. Table 3 shows percentage changes in key variables for the counterfactual (CF) economy without FoF, compared to the baseline. The clustering coefficient is the average of the undirected local clustering coefficient introduced in Section 2. The results highlight the macroeconomic importance of the FoF effect: without FoF, the number of links decreases by 1.1%, the global clustering coefficient declines by 13.7%, and welfare drops by 0.58%. Given that the average

[22]This assumption finds strong justification in the production network data. The sample comprises 8,142 firms, implying 66,284,022 potential directed links. Given 28,106 links that existed in 2014 ($t = 0$), the number of potential links that could be formed in 2015 ($t = 1$) is 66,255,916. As only 1,732 new links were actually created among these potential pairs, the 'rare-event' interpretation for link creation is highly applicable.

[23]Regarding the value of β, we use the baseline estimated value of 1.56 in Model 4 of Table 1, which is close to the estimated value of 1.61 for only manufacturing sectors.

[24]Consistent with the model's two-period timing ($t = 0, 1$), we shut down FoF only on the 2014–2015 formation margin, leaving legacy links intact. This corresponds to a one-year intervention; in a long-run steady state without FoF in the real economy, macro impacts would be larger.

annual growth rate of real GDP in Japan from 2000 to 2024 is 0.70%,[25] the aggregate effect of this mechanism is substantial.

	Number of Links	Clustering Coefficient	U
CF	-1.1	-13.7	-0.58

Table 3: Aggregate Impact of Removing the Friend-of-a-Friend (FoF) Effect: % changes to the baseline

To investigate the mechanism behind the welfare change, we develop Proposition 1, which decomposes welfare changes due to a change in the network structure under the assumption of $\theta = 1$ (Cobb-Douglas). To handle the discrete nature of link deletions, we exploit characteristics of the logarithmic mean without relying on differential calculus, following Sato (1976) and Vartia (1976). The proof is in Appendix A.2.

Proposition 1 (Decomposition of change in U): Suppose $\theta = 1$. When the network structure changes and supplier sets slightly decrease from $\{\mathcal{S}_i\}_i$ to $\{\mathcal{S}'_i\}_i$, using household expenditure share $\tilde{w}_i$ and input expenditure share $\tilde{A}_i^j = \frac{p_j x_i^j}{\zeta_i y_i}$, the welfare change can be approximately decomposed based on the change in *Network Productivity* Φ_i as follows.

$$\frac{\Delta U}{U} = \frac{1}{\sigma - 1} \sum_{i \in \mathcal{N}} \tilde{w}_i \frac{\Delta \Phi_i}{\Phi_i} \tag{5.1}$$

$$\frac{\Delta \Phi_i}{\Phi_i} = \underbrace{- \sum_{j \in \mathcal{S}_i \setminus \mathcal{S}'_i} \tilde{A}_i^j}_{\text{Firm LoV Effect}} + \underbrace{\sum_{j \in \mathcal{S}'_i \cap \mathcal{S}_i} \tilde{A}_i^j \frac{\Delta \Phi_j}{\Phi_j}}_{\text{Spillover Effect}} \tag{5.2}$$

The proposition is intuitive. The first equation (5.1) implies that the welfare change is driven by changes in firm productivity, weighted by the household's expenditure share. The second equation (5.2) illustrates how changes in firm productivity are determined. The first term captures the direct effect of supplier losses on productivity. In a love-of-variety structure, the loss of a supplier reduces productivity according to its importance. The second term reflects the indirect network effect of the productivity decline. If a firm's productivity decreases, the decline propagates to its buyers through production networks, leading to further spread. In this way, changes in productivity are characterized by the fixed point in (5.2).

Table 4 shows the decomposition. It emphasizes the significance of both channels: the direct effect from supplier losses and the indirect spillover through production networks. This suggests that not only do firms benefit directly from the FoF effect by gaining more partners, but also their (indirect) partners gain from the FoF effect.

[25] See `https://data.imf.org/en/Datasets`.

	U	Firm LoV	Spillover
CF	-0.58	-0.27	-0.32

Table 4: Decomposition of the Aggregate Welfare Loss from Removing FoF: % change

5.3.2 Friend-of-a-Friend Effect in Shock Propagation

Next, we study how the FoF effect changes the propagation of idiosyncratic productivity shocks. Changes in the production network affect not only the level of aggregate welfare discussed in Section 5.3.1, but also the mapping from firm-level shocks to aggregate outcomes, because they modify each firm's topological position in the network and thus its influence on others.

To quantify this channel, we compare the welfare response to the same productivity shock under two network structures: the baseline network and the counterfactual (CF) network without FoF-induced links. We first partition firms into four (possibly overlapping) groups $\mathcal{G} \subset \mathcal{N}$ according to how their topological position changes when we remove the FoF effect: (i) buyer-loss firms (those that lose buyers in CF), (ii) supplier-loss firms (those that lose suppliers), (iii) FoF-intermediary firms (common partners for links that disappear in CF), and (iv) others, i.e., firms outside groups (i)–(iii).

For each group $\mathcal{G} \subset \mathcal{N}$, we conduct the following experiment. Under each network structure, CF and the baseline, we increase the fundamental productivity of all firms in $\mathcal{G}$ by 10 percent, holding the productivity of all other firms fixed, and compute the associated welfare response. Let

$$\Delta^{\mathcal{G}} U^{Network} = U^{Network}_{\phi_{shock}, \mathcal{G}} - U^{Network}, \qquad Network \in \{base, CF\},$$

denote the welfare change under a *Network* induced by this shock. We summarize the difference in welfare responses by the *Welfare Response Gap* (WRG) for group $\mathcal{G}$, defined as

$$\mathrm{WRG}(\mathcal{G}) := \frac{\Delta^{\mathcal{G}} U^{\mathrm{CF}}(\mathcal{G})/U^{\mathrm{CF}}}{\Delta^{\mathcal{G}} U^{\mathrm{base}}(\mathcal{G})/U^{\mathrm{base}}} - 1.$$

A positive (negative) value of $\mathrm{WRG}(\mathcal{G})$ indicates that shocks to firms in group $\mathcal{G}$ have stronger (weaker) aggregate consequences in the CF network, meaning that these firms become more (less) influential in the transmission of productivity shocks once the FoF effect is removed.

Table 5 shows the WRG values for each shocked group as a percentage. First, the results for groups (i) and (ii) indicate that firms experiencing buyer-loss and supplier-loss transmit positive productivity shocks less strongly under CF compared to the baseline. By losing partners, these firms become less influential in CF, meaning shocks to them have less impact on overall welfare. In contrast, the result for group (iv) suggests that firms that are neither link-loss firms nor FoF intermediaries become relatively more important in CF, as the significance shifts away from link-loss firms. The result for group (iii) indicates that FoF intermediaries face two opposing forces with minimal overall change.[26] On the one hand,

[26] Table D.7 shows that the standard deviation of the moment across random samples of links used to build counterfactual networks is 0.1. This aligns with the reported Welfare Response Gap (average across the samples) in Table 5, indicating that the WRG for FoF intermediaries is not significantly different from 0.

they lose their significance because their partners, link-loss firms, also become less important. Through production networks, the change in the importance of these partners affects the FoF intermediaries. On the other hand, they benefit from the reallocation of importance away from the link-loss firms. These opposing effects cancel out, resulting in a negligible net difference.

Shocked Group	(i) Buyer-loss	(ii) Supplier-Loss	(iii) FoF Intermediaries	(iv) Others
Welfare Response Gap	-3.2	-1.4	-0.1	1.2

Table 5: Gap in the welfare responses (%) to the same productivity shock between CF and baseline, by shocked group

To further characterize the Welfare Response Gap (WRG) metrics, we first develop an aggregation framework that links micro-level productivity shocks to the aggregate welfare response $\frac{dU}{U}$ under $\theta = 1$. This framework allows us to decompose the welfare impact into direct and indirect channels. We then use this decomposition to analyze the WRG for each shocked group. Unless otherwise noted, we use bold lowercase to denote column vectors and bold uppercase to denote matrices, where rows represent buyers and columns represent suppliers.

Using the household expenditure share vector $\tilde{\boldsymbol{w}}$ and the input expenditure share matrix $\tilde{\boldsymbol{A}}$, the welfare response to small shocks in each firm's fundamental productivity is determined as:

$$\frac{dU}{U} = \sum_{i \in \mathcal{N}} \lambda_i \frac{d\phi_i}{\phi_i}, \tag{5.3}$$

where the *influence vector* $\boldsymbol{\lambda}' := \tilde{\boldsymbol{w}}' \left(I - \tilde{\boldsymbol{A}}\right)^{-1}$ can be decomposed into the *direct influence vector* $(\boldsymbol{\lambda}^{Direct})' := \tilde{\boldsymbol{w}}'$ and the *indirect influence vector* $(\boldsymbol{\lambda}^{Indirect})' := \tilde{\boldsymbol{w}}' \sum_{n=1}^{\infty} \tilde{\boldsymbol{A}}^n$ as:

$$\boldsymbol{\lambda} = \boldsymbol{\lambda}^{Direct} + \boldsymbol{\lambda}^{Indirect}. \tag{5.4}$$

The derivation is provided in Appendix A.3.

Equation (5.3) illustrates how micro-level productivity shocks combine into the welfare response. Unlike the traditional revenue-based Domar weight used under perfect competition (Hulten, 1978; Acemoglu et al., 2012), the presence of markups in our model requires using alternative weights, consistent with Baqaee and Farhi (2020).[27]

The interpretation of the influence vector is straightforward. First, when a positive shock to firm i's fundamental productivity decreases its unit cost and price, it directly lowers the

[27]Equation (5.3) is a special case of Theorem 1 in Baqaee and Farhi (2020). In their framework, the change in the aggregate output (welfare) is measured by the sum of *cost-based Domar weighted* productivity shocks and the *allocative efficiency* term. The cost-based Domar weights match our influence vector. The *allocative efficiency* term reflects changes in the markup and allocation of primary factors. In our model, these terms are zero because (i) the markup remains unchanged, (ii) labor is the sole primary factor, and (iii) the elasticity of substitution between labor and the intermediate input bundle is one.)

household price index based on the household expenditure share of good i ($\tilde{w}_i$), as shown in the direct influence vector in (5.4). At the same time, the lower price reduces input costs for firm j, which buys from firm i, according to i's share in j's input costs ($\tilde{A}^i_j$). This cost reduction then spreads through the supply chain network $\tilde{\boldsymbol{A}}$ and is eventually reflected in the household price index, weighted by expenditure shares. The indirect effect is captured by the indirect influence vector in (5.4), with the n-th power term representing the n-step pass-through along production networks from the original shocks.

The aggregation framework utilizing the influence vector functions as a diagnostic tool to understand the mechanisms behind the Welfare Response Gap (WRG), as we state in the following proposition.

Proposition 2 (Decomposition of WRG): Suppose $\theta = 1$. The Welfare Response Gap (WRG) for group $\mathcal{G}$ is decomposed using the influence vectors from the baseline economy and those from the counterfactual economy as follows.

$$\mathrm{WRG}(\mathcal{G}) = \left(\underbrace{\sum_{i \in \mathcal{G}} \left[\lambda_i^{Direct,CF} - \lambda_i^{Direct,\ base} \right]}_{\text{Direct Channel}} + \underbrace{\sum_{i \in \mathcal{G}} \left[\lambda_i^{Indirect,CF} - \lambda_i^{Indirect,\ base} \right]}_{\text{Indirect Channel}} \right) \Big/ \sum_{i \in \mathcal{G}} \lambda_i^{base} \tag{5.5}$$

The proof is given in Appendix A.3.

Equation (5.5) breaks down WRG($\mathcal{G}$) into two components—the direct channel (the sum of changes in direct weights for the shocked group $\mathcal{G}$) and the indirect channel (the sum of changes in indirect weights for the shocked group $\mathcal{G}$), with a normalization term in the denominator. This breakdown clarifies whether, given WRG($\mathcal{G}$), the change is due to firms in the shocked group $\mathcal{G}$ altering their direct exposure to the household (direct channel) or because they change their importance within the supply chain networks (indirect channel).

	(i) Buyer-loss		(ii) Supplier-Loss		(iii) FoF		(iv) Others	
	Direct	Indirect	Direct	Indirect	Direct	Indirect	Direct	Indirect
WRG	-0.26	-2.85	-0.88	-0.47	-0.02	-0.11	0.47	0.83

Table 6: Decomposition of Welfare Response Gap (WRG), by shocked group

Table 6 shows a breakdown of the Welfare Response Gap (WRG) for each shocked group into the direct and indirect channels implied by Proposition 2. For buyer-loss firms, the indirect channel predominantly drives the gap: although their household-expenditure share remains relatively unchanged, once the FoF-generated buyers disappear in CF, productivity shocks to these firms no longer spill over along downstream paths, significantly reducing propagation. Conversely, firms experiencing supplier losses show substantial effects in both channels: losing suppliers increases their unit costs, leading them to be replaced both by households (reducing the direct weight) and by their buyers (diminishing pass-through at the intensive margin). The 'others' category absorbs both of the influence transferred from link-loss firms.[28] FoF intermediaries show a small net negative gap, consistent with offsetting forces—reduced influence through neighbors' diminished influence,[29] but partial reweighting elsewhere.

6 Conclusion

This paper quantifies the friend-of-a-friend (FoF) effect in production networks and assesses its macroeconomic relevance. Using firm-to-firm transaction data from Japan, we show that production networks display high clustering together with strong geographic and sectoral homophily. These patterns are consistent with FoF but cannot, by themselves, separate FoF from proximity-driven link formation.

We then develop a general equilibrium model with endogenous network formation in which suppliers face fixed costs of forming links and enjoy a discount in these costs when they and a potential buyer share a common partner. Under a static-expectations assumption, the model yields a dyad-level logit link formation equation with buyer and supplier fixed effects and dyad-level controls for sectoral and physical proximity. Applying a quadruple-based conditional logit estimator, we find strong evidence of FoF: we estimate that sharing a common partner reduces effective link-formation costs by about 80%, a magnitude comparable to a 1.75 standard deviation decrease in physical distance or a 1.5 standard deviation increase in sectoral proximity. Controlling for homophily is crucial; failing to do so substantially overestimates the FoF coefficient.

Embedding the estimated FoF parameter into a calibrated GE model for Japan's 2015 manufacturing sector, we show that shutting down the FoF mechanism on the formation margin decreases aggregate welfare by 0.6%, mainly because firms cannot produce goods as efficiently due to having fewer suppliers. Decomposition results indicate that this decreased efficiency spreads across the network, amplifying the negative impact on welfare. We also show that removing FoF changes the mapping from firm-level productivity shocks to aggregate outcomes by reallocating influence across firms in the supply chain.

Beyond the specific FoF mechanism we study, our approach provides a portable way to link micro-level estimates of network formation to macro-level outcomes. Applying this framework to other relational channels, alternative institutional settings, and richer dynamic

[28] It can be rigorously demonstrated that the sum of both types of influence vectors across all firms remains constant. Specifically, a decrease in the sum of an influence vector for one group of firms corresponds to an increase in the sum for the other group. Clearly, the sum of the direct influence vector across firms equals 1 because it represents the total household expenditure share. Regarding the sum of the indirect influence vector across firms, from $(\boldsymbol{\lambda}^{Indirect})' = \tilde{\boldsymbol{w}}' \sum_{n=1}^{\infty} \tilde{\boldsymbol{A}}^n$, we find that $(\boldsymbol{\lambda}^{Indirect})'\mathbf{1} = \tilde{\boldsymbol{w}}' \sum_{n=1}^{\infty} \tilde{\boldsymbol{A}}^n \mathbf{1} = \tilde{\boldsymbol{w}}' \sum_{n=1}^{\infty} (1-\alpha)^n \mathbf{1} = \frac{1-\alpha}{\alpha} \tilde{\boldsymbol{w}}' \mathbf{1} = \frac{1-\alpha}{\alpha}$, where the second equality reflects that the row sum of $\tilde{A}$ is $1-\alpha$. Under the Cobb-Douglas structure, a $(1-\alpha)$ share of each firm's total costs is allocated to intermediate input.

[29] This spillover mechanism can be clearly shown using the influence vector. Since $\boldsymbol{\lambda}$ satisfies $\boldsymbol{\lambda}' = \tilde{\boldsymbol{w}}' \left(I - \tilde{\boldsymbol{A}}\right)^{-1}$, we immediately obtain $\boldsymbol{\lambda} = \tilde{\boldsymbol{w}} + \tilde{\boldsymbol{A}}' \boldsymbol{\lambda}$. This implies that a firm's influence is affected by the weighted sum of the influence of its buyers.

network environments is a promising direction for future research on the origins and consequences of production networks.

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A Derivation

A.1 Quantitative Relevance of β for Sampling

We relate the estimated friend-of-a-friend coefficient β obtained in Section 4.2 to the construction of counterfactual networks without this effect in Section 5.2.

Let Y_i^j denote the original realization of links, and $Y_i^{j,CF}$ the counterfactual realization of the links. Suppose friend-of-a-friend does not influence link formation in the counterfactual economy, with the same realization of the idiosyncratic cost shock ε_i^j. Clearly, this does not affect link realization without common partners. Hence, we focus on the link realization with a common partner.

Note that when $\beta > 0$, this must decrease the value of the latent variables in the logit model (4.1) given the identical realization of the cost shock term (ε_i^j) in both the baseline and counterfactual economies. Therefore, links that are not originally observed ($Y_i^j = 0$) should also not be realized in the counterfactual ($Y_i^{j,CF} = 0$).

For links initially observed with common partners ($Y_i^{j,CP} = 1$), we need to examine the conditional probability of $Y_i^{j,CP,CF}$ being 1 given $Y_i^{j,CP} = 1$, because the fact that $Y_i^{j,CP} = 1$ alters the conditional distribution of ε_i^j given $Y_i^{j,CP} = 1$ from the original (unconditional) logistic distribution. For clarity, let X_i^j represent all the dependent variables except for the common partner dummy and γ denote its coefficients. Note that, by Bayes' theorem, we derive

$$\Pr\left(Y_i^{j,CP,CF} = 1 | Y_i^{j,CP} = 1, X_i^j\right) = \frac{\Pr\left(Y_i^{j,CP,CF} = 1 \wedge Y_i^{j,CP} = 1 | X_i^j\right)}{\Pr\left(Y_i^{j,CP} = 1 | X_i^j\right)} \tag{A.1}$$

$$= \frac{\Pr\left(\gamma X_i^j \geq \varepsilon_i^j \wedge \beta + \gamma X_i^j \geq \varepsilon_i^j\right)}{\Pr\left(\beta + \gamma X_i^j \geq \varepsilon_i^j\right)}. \tag{A.2}$$

From $\beta \geq 0$, we have $\gamma X_i^j \geq \varepsilon_i^j \Rightarrow \beta + \gamma X_i^j \geq \varepsilon_i^j$. Therefore, we have

$$\Pr\left(Y_i^{j,CP,CF} = 1 | Y_i^{j,CP} = 1, X_i^j\right) = \frac{\Pr\left(\gamma X_i^j \geq \varepsilon_i^j\right)}{\Pr\left(\beta + \gamma X_i^j \geq \varepsilon_i^j\right)} \tag{A.3}$$

$$= \frac{\exp(\gamma X_i^j)}{1 + \exp(\gamma X_i^j)} \Big/ \frac{\exp(\beta + \gamma X_i^j)}{1 + \exp(\beta + \gamma X_i^j)}. \tag{A.4}$$

Given the fact that the number of new links is much smaller than all the possible links,[30] the probabilities $\frac{\exp(\gamma X_i^j)}{1 + \exp(\gamma X_i^j)}$ and $\frac{\exp(\beta + \gamma X_i^j)}{1 + \exp(\beta + \gamma X_i^j)}$ are very small, so $\exp(\gamma X_i^j)$ and $\exp(\beta + \gamma X_i^j)$ are much smaller than one. Hence $1 + \exp(\gamma X_i^j) \simeq 1$ and $1 + \exp(\beta + \gamma X_i^j) \simeq 1$, which

[30] For the justification of this, see footnote[22].

implies

$$\Pr\left(Y_i^{j,CP,CF} = 1 | Y_i^{j,CP} = 1, X_i^j\right) = \frac{\exp(\gamma X_i^j)}{\exp(\beta + \gamma X_i^j)} \tag{A.5}$$

$$= \frac{1}{\exp(\beta)}. \tag{A.6}$$

Essentially, this is the same as the rare-disease approximation (e.g., Cornfield (1951)) in epidemiology: when the outcome is rare, the odds ratio (A.5) closely approximates the probability ratio (A.4).

A.2 Proof for Proposition 1

In this section, we show how to derive Proposition 1. For a generic equilibrium object, X corresponds to the network structure $\{\mathcal{S}_i\}_i$, and X' corresponds to $\{\mathcal{S}_i'\}_i$. We denote the difference $X' - X$ as ΔX.

We define *network productivity* as follows.

$$\Phi_i = \zeta_i^{1-\sigma} \tag{A.7}$$

Then, from (3.6) and (3.7), we have

$$\Phi_i = \left(\frac{1}{\phi_i}\right)^{1-\sigma} \mu^{1-\sigma} \left[\sum_{j \in \mathcal{S}_i} \Phi_j\right]^{1-\alpha} \tag{A.8}$$

Taking logs of both sides, we have

$$\log(\Phi_i) = (1-\sigma)\log\left(\frac{1}{\phi_i}\right) + \log(\mu^{1-\sigma}) + (1-\alpha)\log\left[\sum_{j \in \mathcal{S}_i} \Phi_j\right]. \tag{A.9}$$

We also have

$$\log(\Phi_i') = (1-\sigma)\log\left(\frac{1}{\phi_i}\right) + \log(\mu^{1-\sigma}) + (1-\alpha)\log\left[\sum_{j \in \mathcal{S}_i'} \Phi_j'\right]. \tag{A.10}$$

Taking the difference, we have

$$\log(\Phi_i') - \log(\Phi_i) = (1-\alpha)\left(\log\left[\sum_{j \in \mathcal{S}_i'} \Phi_j'\right] - \log\left[\sum_{j \in \mathcal{S}_i} \Phi_j\right]\right) \tag{A.11}$$

$$= (1-\alpha)\left(\sum_{j \in \mathcal{S}_i'} \Phi_j' - \sum_{j \in \mathcal{S}_i} \Phi_j\right) \frac{\log\left[\sum_{j \in \mathcal{S}_i'} \Phi_j'\right] - \log\left[\sum_{j \in \mathcal{S}_i} \Phi_j\right]}{\left(\sum_{j \in \mathcal{S}_i'} \Phi_j' - \sum_{j \in \mathcal{S}_i} \Phi_j\right)} \tag{A.12}$$

Since $\mathcal{S}_i' \subseteq \mathcal{S}_i$, we have

$$\sum_{j\in\mathcal{S}_i'} \Phi_j' - \sum_{j\in\mathcal{S}_i} \Phi_j = \sum_{j\in\mathcal{S}_i'\cap\mathcal{S}_i} \left(\Phi_j' - \Phi_j\right) - \sum_{j\in\mathcal{S}_i\setminus\mathcal{S}_i'} \Phi_j. \tag{A.13}$$

Therefore, we have

$$\log\left(\frac{\Phi_i'}{\Phi_i}\right) = (1-\alpha)\left(\sum_{j\in\mathcal{S}_i'\cap\mathcal{S}_i} \left(\Phi_j' - \Phi_j\right) - \sum_{j\in\mathcal{S}_i\setminus\mathcal{S}_i'} \Phi_j\right) \frac{\log\left[\sum_{j\in\mathcal{S}_i'} \Phi_j'\right] - \log\left[\sum_{j\in\mathcal{S}_i} \Phi_j\right]}{\left(\sum_{j\in\mathcal{S}_i'} \Phi_j' - \sum_{j\in\mathcal{S}_i} \Phi_j\right)} \tag{A.14}$$

$$\begin{aligned}\log\left(1+\frac{\Phi_i'-\Phi_i}{\Phi_i}\right) = (1-\alpha)&\left(\sum_{j\in\mathcal{S}_i'\cap\mathcal{S}_i} \frac{\Phi_j}{\sum_{j\in\mathcal{S}_i}\Phi_j}\frac{\Phi_j'-\Phi_j}{\Phi_j} - \sum_{j\in\mathcal{S}_i\setminus\mathcal{S}_i'} \frac{\Phi_j}{\sum_{j\in\mathcal{S}_i}\Phi_j}\right) \\ &\times \left(\sum_{j\in\mathcal{S}_i}\Phi_j\right)\frac{\log\left[\sum_{j\in\mathcal{S}_i'} \Phi_j'\right] - \log\left[\sum_{j\in\mathcal{S}_i} \Phi_j\right]}{\left(\sum_{j\in\mathcal{S}_i'} \Phi_j' - \sum_{j\in\mathcal{S}_i} \Phi_j\right)}\end{aligned} \tag{A.15}$$

With respect to the second term, we utilize the characteristics of the logarithmic mean following Sato (1976) and Vartia (1976). Since $\log(x)$ is concave, for scalars $a \geq b > 0$, we have

$$\frac{1}{a} \leq \frac{\log a - \log b}{a-b} \leq \frac{1}{b} \tag{A.16}$$

$$\Leftrightarrow a \geq \frac{a-b}{\log a - \log b} \geq b. \tag{A.17}$$

Therefore, for generic scalar variables $a', b' > 0$, we have

$$\min\{a', b'\} \leq \frac{a'-b'}{\log a' - \log b'} \leq \max\{a', b'\}. \tag{A.18}$$

Here, by substituting $\sum_{j\in\mathcal{S}_i'} \Phi_j'$ into a' and $\sum_{j\in\mathcal{S}_i} \Phi_j$ into b', we have

$$\min\left\{\sum_{j\in\mathcal{S}_i'} \Phi_j', \sum_{j\in\mathcal{S}_i} \Phi_j\right\} \leq \frac{\sum_{j\in\mathcal{S}_i'} \Phi_j' - \sum_{j\in\mathcal{S}_i} \Phi_j}{\log\left[\sum_{j\in\mathcal{S}_i'} \Phi_j'\right] - \log\left[\sum_{j\in\mathcal{S}_i} \Phi_j\right]} \leq \max\left\{\sum_{j\in\mathcal{S}_i'} \Phi_j', \sum_{j\in\mathcal{S}_i} \Phi_j\right\}. \tag{A.19}$$

Letting $\vartheta_i := \left(\sum_{j\in\mathcal{S}_i}\Phi_j\right)\frac{\log\left[\sum_{j\in\mathcal{S}_i'} \Phi_j'\right] - \log\left[\sum_{j\in\mathcal{S}_i} \Phi_j\right]}{\left(\sum_{j\in\mathcal{S}_i'} \Phi_j' - \sum_{j\in\mathcal{S}_i} \Phi_j\right)}$, we rewrite (A.15) as

$$\log\left(1+\frac{\Delta\Phi_i}{\Phi_i}\right) = \vartheta_i\left(\sum_{j\in\mathcal{S}_i'\cap\mathcal{S}_i} \tilde{A}_i^j\frac{\Delta\Phi_j}{\Phi_j} - \sum_{j\in\mathcal{S}_i\setminus\mathcal{S}_i'} \tilde{A}_i^j\right) \tag{A.20}$$

where $\tilde{A}_i^j := (1-\alpha)\dfrac{\Phi_j}{\sum_{j'\in\mathcal{S}_i}\Phi_{j'}}$ is input expenditure share of supplier j's good in buyer i's production.

Note that from (A.19), we have

$$\frac{\sum_{j\in\mathcal{S}_i}\Phi_j}{\min\left\{\sum_{j\in\mathcal{S}_i'}\Phi_j',\sum_{j\in\mathcal{S}_i}\Phi_j\right\}} \leq \vartheta_i \leq \frac{\sum_{j\in\mathcal{S}_i}\Phi_j}{\max\left\{\sum_{j\in\mathcal{S}_i'}\Phi_j',\sum_{j\in\mathcal{S}_i}\Phi_j\right\}}. \tag{A.21}$$

By assuming $\sum_{j\in\mathcal{S}_i'}\Phi_j' \simeq \sum_{j\in\mathcal{S}_i}\Phi_j$ (i.e., the sum of suppliers' network productivity of each firm do not change a lot), we have $\vartheta_i \simeq 1$ and finally obtain an approximated expression of the changes in the network productivity.[31]

$$\frac{\Delta\Phi_i}{\Phi_i} = \sum_{j\in\mathcal{S}_i'\cap\mathcal{S}_i}\tilde{A}_i^j\frac{\Delta\Phi_j}{\Phi_j} - \sum_{j\in\mathcal{S}_i\setminus\mathcal{S}_i'}\tilde{A}_i^j. \tag{A.22}$$

We also have

$$P_H = \mu\left[\sum_{i\in\mathcal{N}}\Phi_i\right]^{\frac{1}{1-\sigma}}. \tag{A.23}$$

Taking logs of both sides, we have

$$\log(P_H) = \log(\mu) + \frac{1}{1-\sigma}\log\left[\sum_{i\in\mathcal{N}}\Phi_i\right] \tag{A.24}$$

Then, we have

$$\begin{aligned}
\frac{\Delta P_H}{P_H} &= \frac{1}{1-\sigma}\frac{\sum_{i\in\mathcal{N}}\Delta\Phi_i}{\sum_{i\in\mathcal{N}}\Phi_i} && \text{(A.25)}\\
&= \frac{1}{1-\sigma}\sum_{i\in\mathcal{N}}\frac{\Phi_i}{\sum_{i\in\mathcal{N}}\Phi_i}\frac{\Delta\Phi_i}{\Phi_i} && \text{(A.26)}\\
-\frac{\Delta P_H}{P_H} &= \frac{1}{\sigma-1}\sum_{i\in\mathcal{N}}\tilde{w}_i\frac{\Delta\Phi_i}{\Phi_i} && \text{(A.27)}
\end{aligned}$$

where $\tilde{w}_i := \dfrac{\Phi_i}{\sum_{i\in\mathcal{N}}\Phi_i}$ is the household expenditure share.

[31] When $\sum_{j\in\mathcal{S}_i'}\Phi_j'$ differ from $\sum_{j\in\mathcal{S}_i}\Phi_j$ significantly, the second order term due to the covariance between ϑ_i and $\left(\sum_{j\in\mathcal{S}_i'\cap\mathcal{S}_i}\tilde{A}_i^j\frac{\Delta\Phi_j}{\Phi_j} - \sum_{j\in\mathcal{S}_i\setminus\mathcal{S}_i'}\tilde{A}_i^j\right)$ in (A.20) becomes non-negligible. Similar discussion about the second term due to covariance of small changes can be found Baqaee and Farhi (2019).

Under the assumption that $\theta = 1$,

$$I = wL + \Pi \tag{A.28}$$

$$= wL + \sum_{i \in \mathcal{N}} (\mu - 1)\zeta_i y_i \tag{A.29}$$

$$= wL + \sum_{i \in \mathcal{N}} (\mu - 1)\frac{1}{\alpha} w l_i \tag{A.30}$$

$$= \left(1 + \frac{\mu - 1}{\alpha}\right) wL \tag{A.31}$$

holds. Therefore, given that the wage is the numeraire ($w = 1$) and the labor supply is inelastic in the model, we obtain

$$\frac{\Delta I}{I} = 0. \tag{A.32}$$

Then, we have

$$U = \frac{I}{P_H} \tag{A.33}$$

$$\frac{\Delta U}{U} = \frac{\Delta I}{I} - \frac{\Delta P_H}{P_H} \tag{A.34}$$

$$= -\frac{\Delta P_H}{P_H}. \tag{A.35}$$

Therefore, we finally obtain

$$\frac{\Delta U}{U} = -\frac{\Delta P^H}{P^H} = \frac{1}{\sigma - 1} \sum_{i \in \mathcal{N}} \tilde{w}_i \frac{\Delta \Phi_i}{\Phi_i} \tag{A.36}$$

$$\frac{\Delta \Phi_i}{\Phi_i} = \underbrace{- \sum_{j \in \mathcal{S}_i \setminus \mathcal{S}'_i} \tilde{A}^j_i}_{\text{Firm LoV Effect}} + \underbrace{\sum_{j \in \mathcal{S}'_i \cap \mathcal{S}_i} \tilde{A}^j_i \frac{\Delta \Phi_j}{\Phi_j}}_{\text{Spillover Effect}}. \tag{A.37}$$

A.3 Proof for Proposition 2

In this section, we show how to derive Proposition 2.

We define network productivity as follows.

$$\Phi_i = \zeta_i^{1-\sigma} \tag{A.38}$$

Then, from (3.6) and (3.7), we have

$$\Phi_i = \left(\frac{1}{\phi_i}\right)^{1-\sigma} \mu^{1-\sigma} \left[\sum_{j \in \mathcal{S}_i} \Phi_j\right]^{1-\alpha} \tag{A.39}$$

Taking logs of both sides, we have

$$\log(\Phi_i) = (1-\sigma)\log\left(\frac{1}{\phi_i}\right) + \log(\mu^{1-\sigma}) + (1-\alpha)\log\left[\sum_{j\in\mathcal{S}_i}\Phi_j\right]. \tag{A.40}$$

Taking first order differentiation, we obtain

$$\frac{d\Phi_i}{\Phi_i} = (\sigma-1)\frac{d\phi_i}{\phi_i} + \sum_{j\in\mathcal{S}_i}\tilde{A}_i^j\frac{d\Phi_j}{\Phi_j} \tag{A.41}$$

where $\tilde{A}_i^j := (1-\alpha)\frac{\Phi_j}{\sum_{j\in\mathcal{S}_i}\Phi_i}$ is input expenditure share of supplier j's good in buyer i's production.

In a matrix form, we obtain

$$\frac{d\boldsymbol{\Phi}}{\boldsymbol{\Phi}} = (\sigma-1)\frac{d\boldsymbol{\phi}}{\boldsymbol{\phi}} + \tilde{\boldsymbol{A}}\frac{d\boldsymbol{\Phi}}{\boldsymbol{\Phi}} \tag{A.42}$$

$$\frac{d\boldsymbol{\Phi}}{\boldsymbol{\Phi}} = (\sigma-1)\left(I-\tilde{\boldsymbol{A}}\right)^{-1}\frac{d\boldsymbol{\phi}}{\boldsymbol{\phi}} \tag{A.43}$$

We also have

$$P_H = \mu\left[\sum_{i\in\mathcal{N}}\Phi_i\right]^{\frac{1}{1-\sigma}}. \tag{A.44}$$

Taking logs of both sides, we have

$$\log(P_H) = \log(\mu) + \frac{1}{1-\sigma}\log\left[\sum_{i\in\mathcal{N}}\Phi_i\right] \tag{A.45}$$

Then, we have

$$\frac{dP_H}{P_H} = -\frac{1}{\sigma-1}\sum_{i\in\mathcal{N}}\tilde{w}_i\frac{d\Phi_i}{\Phi_i} \tag{A.46}$$

where $\tilde{w}_i := \frac{\Phi_i}{\sum_{i\in\mathcal{N}}\Phi_i}$ is the household expenditure share.

Substituting the equation (A.43), in a matrix form, we obtain

$$\frac{dP_H}{P_H} = -\tilde{\boldsymbol{w}}'\left(I-\tilde{\boldsymbol{A}}\right)^{-1}\frac{d\boldsymbol{\phi}}{\boldsymbol{\phi}} \tag{A.47}$$

Under the assumption that $\theta = 1$,

$$I = wL + \Pi \tag{A.48}$$

$$= \left(1 + \frac{\mu-1}{\alpha}\right)wL \tag{A.49}$$

holds. Therefore, given that the wage is the numeraire ($w = 1$) and the labor supply is inelastic in the model, we obtain

$$\frac{dI}{I} = 0. \tag{A.50}$$

Then, we have

$$U = \frac{I}{P_H} \tag{A.51}$$

$$\frac{dU}{U} = -\frac{dP_H}{P_H}. \tag{A.52}$$

Finally, we obtain

$$\frac{dU}{U} = \tilde{\boldsymbol{w}}' \left(I - \tilde{\boldsymbol{A}}\right)^{-1} \frac{d\boldsymbol{\phi}}{\boldsymbol{\phi}} \tag{A.53}$$

By letting $\boldsymbol{\lambda}' = \tilde{\boldsymbol{w}}' \left(I - \tilde{\boldsymbol{A}}\right)^{-1}$, we have

$$\frac{dU}{U} = \sum_{i \in \mathcal{N}} \lambda_i \frac{d\phi_i}{\phi_i}. \tag{A.54}$$

Furthermore, we can have the Neumann series of $\boldsymbol{\lambda}$ under the condition that the spectral radius of $\tilde{\boldsymbol{A}}$ is less than 1, which holds from the labor input share $\alpha > 0$ in our model. (i.e., the pass-through of the productivity change in suppliers is smaller than 1 due to the existence of the other inputs, labor.)

$$\boldsymbol{\lambda}' = \tilde{\boldsymbol{w}}' \left(I - \tilde{\boldsymbol{A}}\right)^{-1} \tag{A.55}$$

$$= \tilde{\boldsymbol{w}}' \left(I + \sum_{n=1}^{\infty} \tilde{\boldsymbol{A}}^n\right) \tag{A.56}$$

$$= \underbrace{\tilde{\boldsymbol{w}}'}_{=\boldsymbol{\lambda}^{Direct'}} + \underbrace{\tilde{\boldsymbol{w}}' \sum_{n=1}^{\infty} \tilde{\boldsymbol{A}}^n}_{=\boldsymbol{\lambda}^{Indirect'}} \tag{A.57}$$

Therefore, we obtain

$$\frac{dU}{U} = \sum_{i \in \mathcal{N}} \left(\lambda_i^{Direct} + \lambda_i^{Indirect}\right) \frac{d\phi_i}{\phi_i}, \tag{A.58}$$

and the equation (5.5) can be directly derived from this.

B Algorithm

B.1 Backward and Forward Fixed Points

Given a production network $\{S_i, B_i\}_{i\in\mathcal{N}}$, the price vector $\{\zeta_i\}_{i\in\mathcal{N}}$ and the allocation vector y_i satisfy the following systems.

$$\zeta_i = \frac{1}{\phi_i}\left(\alpha^\theta w^{1-\theta} + (1-\alpha)^\theta \left[\sum_{j\in\mathcal{S}_i}\left(\frac{\tau_{a_i}^{a_j}}{\lambda_{k_i}^{k_j}}\mu\zeta_j\right)^{1-\sigma}\right]^{\frac{1-\theta}{1-\sigma}}\right)^{\frac{1}{1-\theta}} \tag{B.1}$$

$$y_i = \chi_i^{\sigma-1}\left(\frac{p_i}{P^{Final}}\right)^{-\sigma} U + \sum_{j\in\mathcal{B}_i}\tau_{a_j}^{a_i}\left(\lambda_{k_j}^{k_i}\right)^{\sigma-1}\left(\frac{\tau_{a_j}^{a_i}p_j^i}{P_j}\right)^{-\sigma}(1-\alpha)\left(\frac{P_j}{\zeta_j}\right)^{-\theta} y_j \tag{B.2}$$

Following the terminology of Bernard et al. (2022), we refer to (B.1) as the backward fixed point (BFP), since it aggregates costs over upstream suppliers and determines the unit cost of the downstream buyer, and to (B.2) as the forward fixed point (FFP), since it aggregates demand downstream and determines the production of the upstream suppliers. Following Lim (2018), Huneeus (2020), and Carvalho et al. (2021), we treat these mappings as contraction mappings in the price and allocation vector, so simple fixed-point iteration on $\{\zeta_i\}_i$ and $\{y_i\}_i$ converges to the unique solution.

B.2 Solution Algorithm for Equilibrium Given Networks

Given the characterization in Section B.1, the fixed-network equilibrium for a production network $\{S_i, B_i\}_{i\in\mathcal{N}}$ is obtained by solving the backward and forward fixed points once at an arbitrary scale and then rescaling quantities to satisfy the labor-market clearing condition (3.14). Because technologies are constant returns to scale and preferences are homothetic, once unit costs and price indices are determined, the forward system (B.2) is linear in aggregate demand U: all real quantities are homogeneous of degree one in U, while prices are invariant to U.

1. Normalize the wage to $w = 1$ and solve the backward fixed point (B.1) to obtain unit costs $\{\zeta_i\}_i$, firm prices $p_i = \mu\zeta_i$, and the associated price indices (P_j and P^{Final}).

2. Fix an arbitrary normalization of aggregate demand, e.g. $U = 1$, and solve the forward fixed point (B.2) for outputs $\{y_i(1)\}_i$. Using firms' cost-minimization conditions, compute the implied firm-level labor demands $\{l_i(1)\}_i$ and aggregate labor demand $L^d(1) = \sum_{i\in\mathcal{N}} l_i(1)$.

3. By homogeneity, for any U we have $y_i(U) = U\, y_i(1)$ and $L^d(U) = U\, L^d(1)$. Given aggregate labor supply L, set $U^* = L/L^d(1)$ and obtain the equilibrium allocation by rescaling all quantity variables, e.g. $y_i^* = U^* y_i(1)$ and $l_i^* = U^* l_i(1)$, with an analogous scaling for intermediate inputs and consumption. Prices and unit costs remain those computed in step 1.

B.3 Sampling Procedure

To avoid the computational burden associated with the very large sample size, we follow the sampling procedure proposed by Izumi et al. (2023). Figure B.1 illustrates quadruples with $Z_\sigma = 1$ and $Z_\sigma = -1$, respectively.

0. Construct a dyadic dataset that consists of buyer–supplier pairs (i, j) at $t = 1$ such that the link between i and j does not exist at $t = 0$.

1. From the N newly formed trading pairs at $t = 1$, draw N quadruples $\sigma = \{i_1, i_2; j_1, j_2\}$.

2. Keep the quadruples satisfying

   - $Z_\sigma = 1$ (a link between buyer i_1 and supplier j_1 is realized, whereas a link between buyer i_1 and supplier j_2 is not; similarly, a link between buyer i_2 and supplier j_2 is realized, whereas a link between buyer i_2 and supplier j_1 is not),
   - neither a link between buyer i_1 and supplier j_2 nor a link between buyer i_2 and supplier j_1 existed at $t = 0$.

3. Repeat steps 1–2 until N quadruples are collected.

4. Duplicate these N quadruples (with $Z_\sigma = 1$) and swap j_1 and j_2 within each duplicate to construct N quadruples with $Z_\sigma = -1$.

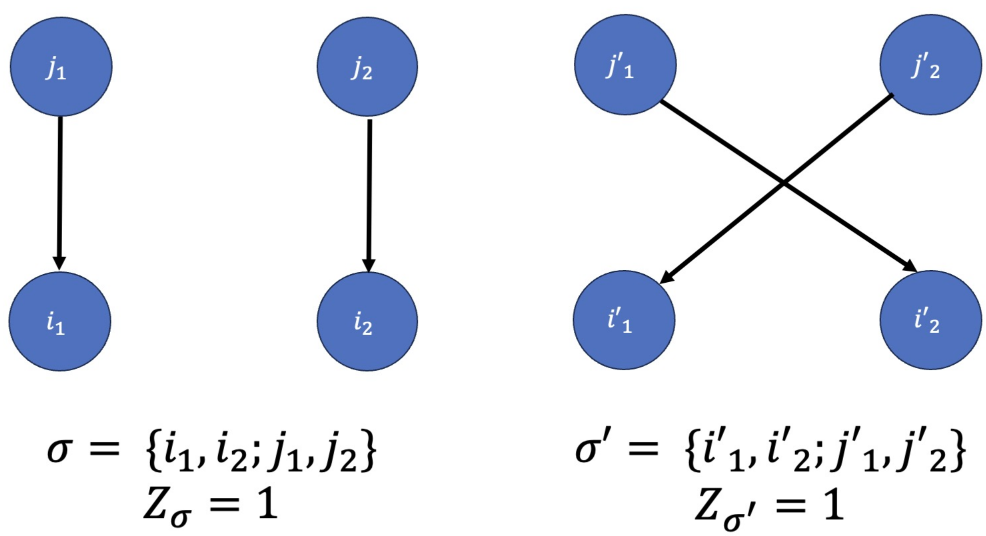

Figure B.1: Visualization of Quadruples for Conditional Logit Estimation

C Data

C.1 Summary Statistics

	Number of Suppliers			Number of Buyers		
	Existing	New	Dropped	Existing	New	Dropped
Mean	6.37	0.57	0.47	6.70	0.61	0.50
SD	40.92	3.56	3.48	41.57	3.51	2.52
Mean (log)	1.14	0.24	0.20	1.19	0.27	0.23
SD (log)	0.95	0.49	0.44	0.96	0.50	0.47

Table C.1: Firm-Level Summary Statistics of Transaction Links (2015)

	Number of Suppliers			Number of Buyers		
	Existing	New	Dropped	Existing	New	Dropped
2008	5.70	0.79	0.56	6.40	0.91	0.63
2009	5.77	0.78	0.57	6.45	0.84	0.62
2010	5.83	0.73	0.53	6.47	0.79	0.58
2011	6.10	0.67	0.51	6.50	0.72	0.55
2012	6.14	0.64	0.56	6.53	0.68	0.60
2013	5.98	0.64	0.52	6.57	0.69	0.55
2014	6.29	0.60	0.48	6.68	0.64	0.51
2015	6.37	0.57	0.47	6.70	0.61	0.50
2016	6.41	0.56	0.49	6.72	0.59	0.49
2017	6.50	0.55	0.46	6.79	0.58	0.48
2018	6.58	0.54	0.45	6.82	0.56	0.47
2019	6.67	0.44	0.42	6.87	0.46	0.44
2020	6.72	0.44	0.45	6.90	0.46	0.46
2021	7.19	0.59	0.59	7.54	0.59	0.60

Table C.2: Yearly Averages for Firm-Level Link Dynamics

	Number of Suppliers			Number of Buyers		
	Existing	New	Dropped	Existing	New	Dropped
Full Sample	6.37	0.57	0.47	6.70	0.61	0.50
Manufacturing	8.51	0.72	0.66	8.97	0.70	0.63

Table C.3: Comparison of Firm-Level Link Dynamics: Full Sample vs. Manufacturing Sector (2015)

D Robustness Checks

D.1 Robustness to Sample Selection

In this section, we conduct a series of robustness checks. First, we demonstrate the robustness of the empirical findings in Section 4.2. Table D.1 reports the estimation results excluding firms in the financial sector. Table D.2 presents the results when links associated with capital (equity) relationships are excluded. Table D.3 shows the results for manufacturing firms only. Figure D.1 displays the estimation results for the baseline model (Model 4) across alternative sampling years from 2008 to 2020, with the 95% confidence interval indicated by the shaded region. The robustness of the estimation results supports the presence of a pervasive friend-of-a-friend mechanism in the link-formation process of production networks.

Second, Table D.4 demonstrates the robustness of the macroeconomic relevance of FoF analyzed in Section 5. We conduct the same exercise as in that section using alternative sample periods and report the same moments as in Table 3. The stability of the reported moments across years underscores the macroeconomic importance of FoF.

	Model 1	Model 2	Model 3	Model 4
Common Partner	2.54	1.87	2.29	1.52
	(0.03)	(0.04)	(0.04)	(0.05)
Physical Distance (log)		-0.78		-0.79
		(0.01)		(0.01)
Sectoral Proximity (log)			0.48	0.50
			(0.007)	(0.01)
Supplier FE	Yes	Yes	Yes	Yes
Buyer FE	Yes	Yes	Yes	Yes

Table D.1: Estimation results for dyad-level logit model excluding financial sectors, 2015

	Model 1	Model 2	Model 3	Model 4
Common Partner	2.50	1.84	2.25	1.48
	(0.03)	(0.05)	(0.04)	(0.05)
Physical Distance (log)		-0.76		-0.77
		(0.01)		(0.01)
Sectoral Proximity (log)			0.49	0.52
			(0.006)	(0.02)
Supplier FE	Yes	Yes	Yes	Yes
Buyer FE	Yes	Yes	Yes	Yes

Table D.2: Estimation results for dyad-level logit model excluding the links associated with capital relationships, 2015

	Model 1	Model 2	Model 3	Model 4
Common Partner	1.90	1.76	1.73	1.60
	(0.43)	(0.58)	(0.46)	(0.62)
Physical Distance (log)		-0.64		-0.68
		(0.16)		(0.22)
Sectoral Proximity (log)			0.51	0.56
			(0.14)	(0.19)
Supplier FE	Yes	Yes	Yes	Yes
Buyer FE	Yes	Yes	Yes	Yes

Table D.3: Estimation results for dyad-level logit model only for manufacturing sectors, 2015

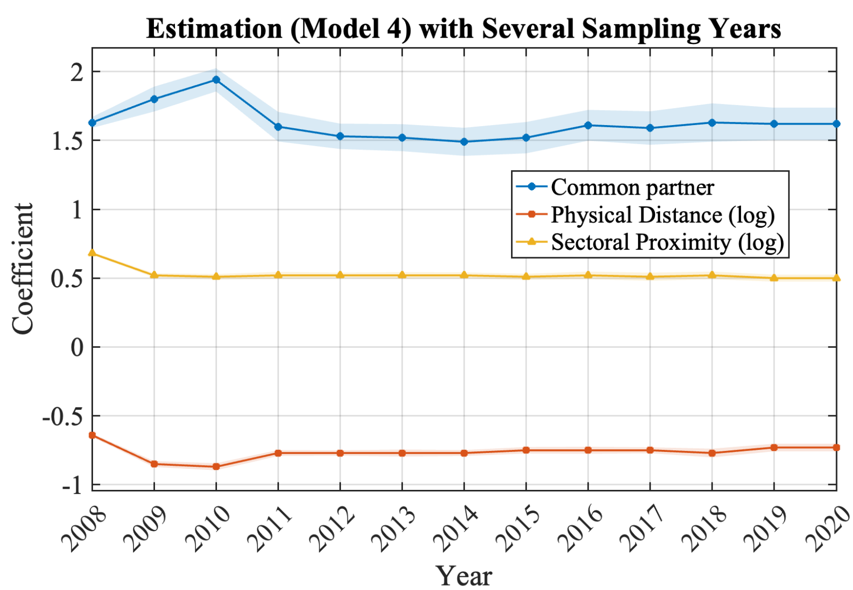

Figure D.1: Stability of Estimated Coefficients (Model 4) Across Different Sampl Years

Year	Number of Links	Clustering Coefficient	U
2009	-2.0	-17.7	-2.37
2010	-1.7	-15.0	-0.96
2011	-1.8	-15.5	-1.37
2012	-1.8	-13.9	-2.22
2013	-1.7	-15.5	-1.29
2014	-1.1	-8.8	-0.79
2015	-1.1	-13.7	-0.58
2016	-1.1	-10.5	-0.60
2017	-1.3	-11.1	-0.77
2018	-1.0	-9.8	-0.39
2019	-0.9	-10.1	-0.50
2020	-0.8	-8.9	-0.46

Table D.4: Stability of Macroeconomic Relevance of FoF: Counterfactual (no FoF) vs. baseline (% changes) by Year

D.2 Effect of the Random Sampling on the Reported Moments

In this section, we assess the variation caused by sampling in the reported moments from Sections 2.2 and 5. Since both the random network in Section 2.2 and the counterfactual network without the friend-of-a-friend effect in Section 5 depend on random sampling, the resulting moments can differ, which may impact reproducibility. We address this issue by providing summary statistics across multiple random draws, demonstrating that the moments are highly consistent.

Table D.5 reports summary statistics for the measures in Section 2 across 10 random samples (degree-preserving randomization; Maslov and Sneppen (2002)). Tables D.6 report summary statistics for changes in aggregate variables in the counterfactual economy without the friend-of-a-friend effect (corresponding to Table 3 and Table 4), based on 100 random samples following Section 5.2. All reported moments have small coefficients of variation, and the implications in the main results hold at both their minimum and maximum values. Table D.7 and Table D.8 report summary statistics for shock propagation analysis (corresponding to Table 5 and Table 6). As discussed in the main body of the paper, the standard deviation of the moments related to group (iii), FoF intermediaries, is comparable to its mean. This implies these moments are not significantly different from 0.

	N	Mean	SD	Min	Max
Global Clustering Coefficient	10	0.031	0.0003	0.031	0.032
Average Physical Distance	10	5.10	0.002	5.10	5.10
Average Sectoral Proximity	10	0.017	0.0001	0.017	0.017

Table D.5: Robustness of Random Network Benchmarks to Sampling Variation in Section 2.2

	N	Mean	SD	Min	Max
Link Density	100	-1.1	0.007	-1.2	-1.1
Clustering	100	-13.7	0.92	-15.9	-11.0
U	100	-0.58	0.047	-0.68	-0.46
Firm LoV	100	-0.27	0.029	-0.32	-0.19
Spillover	100	-0.32	0.028	-0.37	-0.25

Table D.6: Robustness of Aggregate Implications in Table 3 and Table 4 to Sampling Variation

	N	Mean	SD	Min	Max
(i) Buyer-loss	100	-3.177	0.336	-3.877	-2.227
(ii) Supplier-loss	100	-1.356	0.170	-1.684	-0.910
(iii) FoF Intermediaries	100	-0.126	0.106	-0.390	0.168
(iv) Others	100	1.227	0.114	0.945	1.455

Table D.7: Robustness of WRG in Table 5 to Sampling Variation

		N	Mean	SD	Min	Max
(i) Buyer-loss	Direct	100	-0.257	0.115	-0.495	-0.009
	Indirect	100	-2.852	0.281	-3.486	-2.071
(ii) Supplier-loss	Direct	100	-0.880	0.112	-1.124	-0.617
	Indirect	100	-0.466	0.096	-0.666	-0.245
(iii) FoF Intermediaries	Direct	100	-0.021	0.065	-0.214	0.157
	Indirect	100	-0.115	0.093	-0.322	0.103
(iv) Others	Direct	100	0.471	0.054	0.347	0.584
	Indirect	100	0.831	0.073	0.655	0.977

Table D.8: Robustness of WRG Direct/Indirect Channels in Table 6 to Sampling Variation

D.3 Non-targeted Moment in the Calibrated Model

As a validation check, we investigate the binned scatter plots of the log of sales and profit of the calibrated model in Section 5 and the data. While they are non-targeted moments, we observe good fits between the model moments and the data.

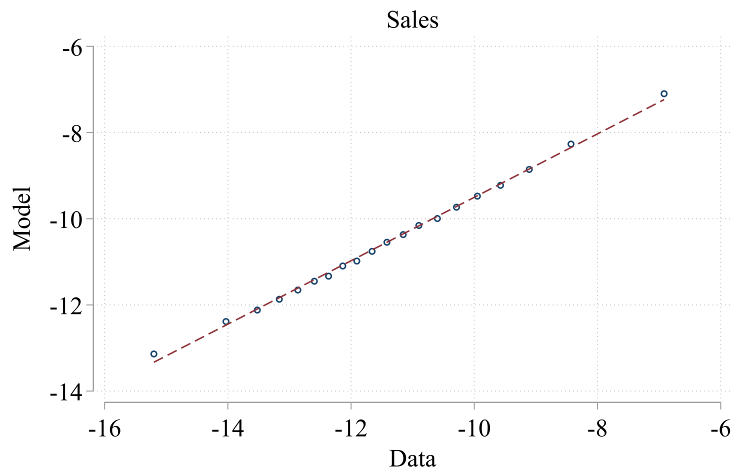

Figure D.2: Sales (Log): Model vs. Data

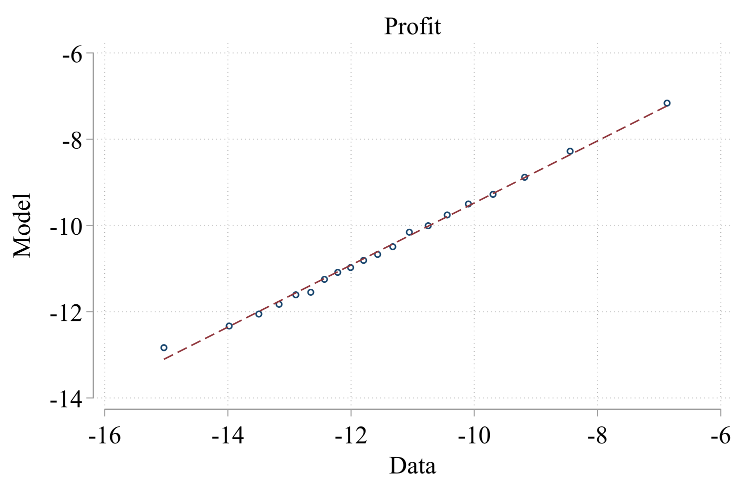

Figure D.3: Profit (Log): Model vs. Data

E Heterogeneity Analysis

This section examines how the FoF effect in our baseline dyad-level logit estimation (4.1) varies across economic and topological dimensions. We measure all heterogeneity shifters at $t = 0$ and include the same set of controls (log distance, sectoral proximity, and two-way fixed effects) to preserve the model's timing logic and ensure comparability with the main estimates.

E.1 Firm Size

A large body of research indicates that the FoF mechanism interacts with firm size (Ravasz and Barabási, 2003; Kozitsin et al., 2023; Manova et al., 2025), which motivates including a size-contingent FoF term instead of assuming homogeneity. We proxy the size of potential buyer i by the number of suppliers ($\#supplier_i$), and the size of potential supplier j by the number of buyers ($\#buyer_j$), summarizing both with their mean-centered log sum as $k_i^j = \log(\#supplier_i + 1) + \log(\#buyer_j + 1) - \overline{(\log(\#supplier_i + 1) + \log(\#buyer_i + 1))}$.[32] We then estimate an extended version of the baseline dyad-level logit specification (4.1):

$$Y_i^{j,1} = \mathbb{I}\left\{\beta \times CommonPartner_i^{j,0} + \beta_{cross} \times CommonPartner_i^{j,0} \times k_i^{j,0} \right. \\ \left. + Controls + \varepsilon_i^{j,1} \geq 0\right\}, \tag{E.1}$$

where the $Controls$ term includes all the controls (log distance, sectoral proximity, and the two-way fixed effects) used in the baseline.[33] The coefficient of interest is β_{cross}, and if it is positive (negative), larger (smaller) firms benefit more from FoF.

Table E.1 reports the estimation results using several samples for a robustness check, following Appendix D. We find a strong negative size gradient in the FoF effect. The coefficient on Common Partner × Firm Size is approximately -0.4, which is comparable to the coefficient on Common Partner, given that the standard deviation of k_i^j is around 1.97. This pattern remains when excluding firms in financial sectors or links with capital relationships, providing robust evidence that FoF is more helpful to small firms; for large pairs, alternative search channels or internal capabilities likely substitute for the FoF effect.

E.2 Number of Common Partners

Applied network studies often argue that while additional common partners increase the likelihood of link formation, their marginal contribution quickly declines (Snijders et al., 2006; Arora and Osadchiy, 2025). To quantify the speed of this saturation within our identification strategy, we use the geometrically weighted edgewise shared partner (GWESP) kernel, following the literature on exponential random graph models (ERGM) (Wasserman and Pattison, 1996; Snijders et al., 2006; Hunter, 2007).

[32] Adding 1 prevents taking the log of zero.

[33] We do not include the first-order terms of the size effects $k_i^j = \log(\#supplier_i) + \log(\#buyer_j)$ since the buyer fixed effect α_i^{Buyer} absorbs $\log(\#supplier_i)$, and the supplier fixed effect $\alpha_j^{Supplier}$ absorbs $\log(\#buyer_j)$.

	Baseline	Excl. Financial Sector	Excl. Capital Links
	(1)	(2)	(3)
Common Partner	2.430	2.409	2.411
	(0.096)	(0.109)	(0.104)
Common Partner Firm Size	-0.416	-0.409	-0.418
	(0.032)	(0.031)	(0.035)
Physical Distance (log)	-0.739	-0.746	-0.740
	(0.016)	(0.014)	(0.016)
Sectoral Proximity (log)	0.476	0.476	0.473
	(0.016)	(0.017)	(0.016)
Supplier FE	Yes	Yes	Yes
Buyer FE	Yes	Yes	Yes

Table E.1: Estimation results for dyad-level logit model with size heterogeneity across baseline and subsamples

Let n_i^j be the logarithm of the number of common partners between buyer i and supplier j. Then, using a parameter ϕ, the GWESP-type kernel is defined as

$$g_\phi(n_i^j) = \frac{1 - e^{-\phi n_i^j}}{1 - e^{-\phi}}. \tag{E.2}$$

Although in our estimation n_i^j is a continuous log measure, it is useful to recall the behavior of this kernel when applied directly to an integer count. If we let n denote an integer and evaluate $g_\phi(n)$ at that integer, we obtain

$$\frac{1 - e^{-\phi n}}{1 - e^{-\phi}} = \sum_{s=1}^{n} e^{-\phi(s-1)}, \tag{E.3}$$

which makes clear that ϕ governs how quickly the marginal effect of additional (common-partner) units declines: a larger ϕ implies a faster geometric decay in the incremental contribution of extra common partners. The same interpretation carries over when we apply $g_\phi(\cdot)$ to the log number of common partners.

Using this, we estimate equation (E.4) while accounting for the heterogeneity in firm size. Note that since a large number of common partners between a supplier and a buyer directly suggests that both have many partners, ignoring the heterogeneity due to firm size when estimating the effect of the number of common partners results in a significant downward bias. To avoid this, we consider both effects simultaneously.[34]

$$Y_i^{j,1} = \mathbb{I}\left\{\beta \times g_\phi(n_i^{j,0}) + \beta_{cross} \times g_\phi(n_i^{j,0}) \times k_i^{j,0} + Controls + \varepsilon_i^{j,1} \geq 0\right\} \tag{E.4}$$

Since we cannot directly estimate ϕ in a standard logit, we perform a grid search over ϕ that minimizes AIC following Diessner et al. (2023). [35]

[34]The same reasoning indicates an upward bias in the estimate in (E.1). This suggests that the true heterogeneity due to firm size is more negative, which does not alter the conclusion above.

[35]For each ϕ, we estimate equation (E.4) fixing ϕ and compute the AIC of the model. Then, we compare the various AICs corresponding to each ϕ and select $\hat{\phi}$ that minimizes AIC.

Table E.2 reports the estimation results. Our GWESP estimates indicate early convergence in the marginal contribution of shared partners. The GWESP decay $\phi = 0.9$ implies that each +1 log-unit increase in the number of common partners results in only $e^{-0.9} \simeq 0.41$ of the previous marginal gain. Therefore, once initial trust or information barriers are overcome, additional overlaps contribute relatively little.

	Baseline	Excl. Financial Sector	Excl. Capital Links
	(1)	(2)	(3)
Hyperparameter:			
ϕ	0.899	0.907	0.870
	(0.086)	(0.087)	(0.089)
Coefficients:			
$g_\phi(n)$	2.286	2.273	2.015
	(0.091)	(0.097)	(0.095)
$g_\phi(n)\times$ Firm Size	-0.402	-0.397	-0.374
	(0.025)	(0.025)	(0.028)
Physical Distance (log)	-0.741	-0.748	-0.742
	(0.016)	(0.015)	(0.015)
Sectoral Proximity (log)	0.472	0.472	0.478
	(0.016)	(0.017)	(0.015)
Supplier FE	Yes	Yes	Yes
Buyer FE	Yes	Yes	Yes

Table E.2: Estimation results for dyad-level logit model with a geometrically weighted edgewise shared partner (GWESP) kernel across baseline and subsamples

E.3 Position of Common Partners (Directed Motifs)

Prior evidence shows that the topological location of common partners (directed motifs) provides information about link-formation patterns. (Romero and Kleinberg, 2010; Yin et al., 2020; Di Vece et al., 2024) To evaluate the heterogeneity caused by different topological positions of common partners in our framework, we first categorize common partners for buyer i and supplier j into four groups: common supplier (CS) to both i and j, common buyer (CB) of both, intermediary-forward (IF), which is a buyer of supplier j and a supplier of buyer i, and intermediary-reverse (IR), which is a supplier of supplier j and a buyer of buyer i. For each $position \in \{CS, CB, IF, IR\}$, we define the share within the (classified)[36] common partner set as:

$$S^j_{i,position} = \frac{\#CP^j_{i,position}}{\#CP^j_{i,CS} + \#CP^j_{i,CB} + \#CP^j_{i,IF} + \#CP^j_{i,IR}} \tag{E.5}$$

[36]Since a common partner can be both a common supplier and a common buyer at the same time, the total number of classified common partners may exceed the total number of common partners.

so that the shares sum to one, thereby isolating compositional differences and netting out the effect of the number of (classified) common partners. Then, we estimate

$$Y_i^{j,1} = \mathbb{I}\left\{\beta_{CS} S_{i,CS}^{j,0} + \beta_{CB} S_{i,CB}^{j,0} + \beta_{IF} S_{i,IF}^{j,0} + \beta_{IR} S_{i,IR}^{j,0} + Controls + \varepsilon_i^{j,1} \geq 0\right\}. \tag{E.6}$$

Table E.3 presents the estimation results. Breaking down by directed motifs shows that the strongest FoF effect occurs when the common partner buys from the potential supplier j and sells to the potential buyer i (intermediary-forward (IF)): the IF coefficient is 1.69, higher than CC (1.43), IR (1.42), and CS (0.46). A natural interpretation is that IF captures technological relatedness beyond our sectoral proximity measure: when the common partner k is IF, the potential supplier j already produces an intermediate input good that the potential buyer i indirectly uses through k. This result indicates a limitation of our measure of technological proximity.

	Baseline	Excl. Financial Sector	Excl. Capital Links
	(1)	(2)	(3)
Share of CS	0.463	0.542	0.412
	(0.236)	(0.202)	(0.220)
Share of CB	1.429	1.448	1.228
	(0.093)	(0.093)	(0.093)
Share of IF	1.690	1.727	1.393
	(0.089)	(0.091)	(0.087)
Share of IR	1.420	1.381	1.095
	(0.089)	(0.085)	(0.089)
Physical Distance (log)	-0.782	-0.800	-0.784
	(0.011)	(0.012)	(0.012)
Sectoral Proximity (log)	0.525	0.508	0.527
	(0.011)	(0.012)	(0.012)
Supplier FE	Yes	Yes	Yes
Buyer FE	Yes	Yes	Yes

Table E.3: Estimation results for dyad-level logit model with directional heterogeneity across baseline and subsamples