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Loan Screening under Symmetrically Imperfect Information*

Yun Gao[†] Kenichi Ueda[‡]

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Abstract

We propose a canonical model of loan screening under symmetrically imperfect information. The project quality is unknown to both a lender and a borrower, but it is revealed with noise to the lender with cost. We show that, under meaningful parameter values, there are three types of equilibria: (i) screening and separating equilibrium, (ii) non-screening and pooling equilibrium, and (iii) non-screening and cheap-information-based separating equilibrium. They are all constrained socially optimal. In particular, the screening and separating equilibrium emerges when the average project quality is low or when the interest rate is high.

Keywords: imperfect information, costly screening, lending standard, zombie firms.

JEL codes: G21, G24, G28.

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1 Introduction

We propose a canonical model of loan screening under symmetrically imperfect information, with which we mean, both a lender and a borrower do not know the borrower's type. This assumption is somewhat different from the conventional assumption of imperfect information, that is, asymmetric information, with which borrowers know their types but lenders do not. Our informational setup, we think, better captures real world loans to new projects and also to existed projects under unprecedented changes in business environment, like Covid-19 pandemic.

In our model, borrowers do not have precise information whether they succeed or fail until the production takes place. Moreover, a lender does not know a borrower's type but he has expertise and past data to analyze if the project is promising or not. We assume that a lender can obtain an imperfect signal with a cost on a project's real type, and that the larger the screening cost, the more accurate the signal.

We simplify lenders as banks and borrowers as firms, and construct a one-period model, with five stages as follows: (1) Banks offer loan contracts contingent on signals. (2) Firms apply for loans. (3) Banks screen loan applications with costs by signals and determine the loan terms as pre-specified. (4) A firm chooses to receive or reject the loan. (5) If a firm receives the investment, she will produce goods, and then she chooses whether to repay the loan as promised or go bankrupt.

We show that the economy has three types of Nash equilibria: a screening and separating equilibrium (SS), a non-screening and pooling equilibrium (NP), and a non-screening and cheap-information-based separating equilibrium (NS).[2] We prove that all types of Nash equilibria are constrained social optimal. With typical parameter values, the SS equilibrium is selected.

We analyze debt contracts, but only with two discrete states (success or failure) for the sake of simplicity. In this sense, debt-type contracts and equity-type contracts are barely different

[2]Both lenders and borrowers are usually indifferent between the NS equilibrium and the NP equilibrium, because creditors do not pay to screen in both equilibria. Without loss of generality, we focus on the NP equilibrium, referring as a non-screening equilibrium. Similarly, we refer to the SS equilibrium as a screening equilibrium, hereinafter. However, NS equilibrium becomes distinctively selected if public loan guarantees are introduced. We have discussed this case in the policy implication part in Appendix I Section B.

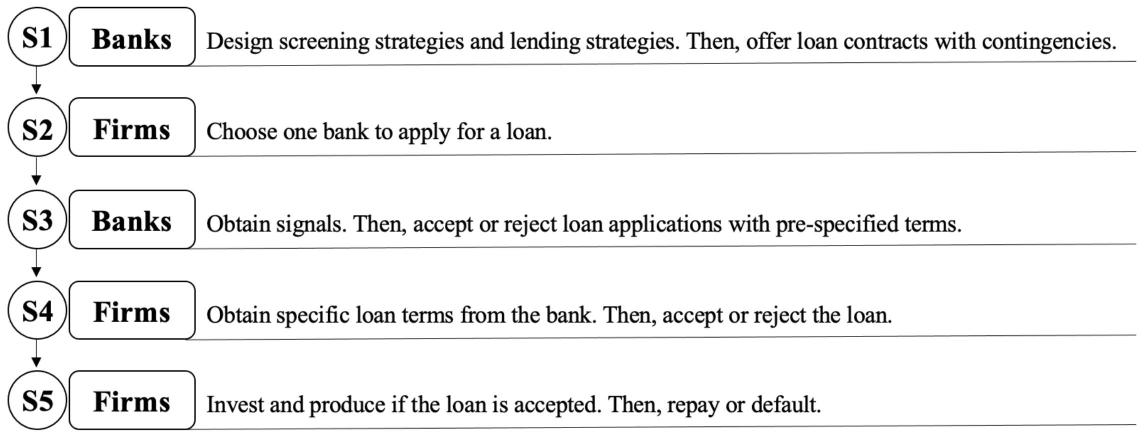


Figure 1. Timeline

from each other. Hence, our model could be extended to analyze equity investment (*e.g.*, venture capital). However, we assume that firm owners control their firms in production. The control right of a firm would be transferred to investors only if a default happens. With this allocation of control rights, our model is more aligned with debt contracts, which are intrinsically different from equity contracts, as in Holmstrom and Tirole (1997), Hart and Moore (1990, 1998), and Hart (2017).[3]

Related Literature

Bisin and Gottardi (2006) theoretically prove that the adverse selection in general equilibrium selects separating equilibrium, not pooling equilibrium. In contrast, our model is not based on the adverse selection, and, therefore, switching to a pooling equilibrium may happen. Indeed, empirically, Dvorkin and Shell (2017) show that under some extreme bad situations, that is, recession, loan demand decreases and loan standard is tightened, and the loan market shuts down. Our model can explain these empirical tendencies.

With additional frictions, the adverse selection in credit markets may select a pooling equilibrium. Dell'Ariccia and Marquez (2006) requires collateral to receive loans. In a boom period, as collateral values increase, banks offer pooling contracts without discriminating bad type borrow-

[3] Ueda (2004) has a detailed discussion on the difference between venture capital and banks.

ers. Figueroa and Leukhina (2015) develop the DSGE model under adverse selection. Given one market interest rate, the loan size is decided as the lending standard which banks could use to achieve self-selection. In a boom, better economic conditions raise overall returns to investment, so loans are made to both good and bad types borrowers, that is, a pooling equilibrium is selected. Although the two papers have different forms of lending standards, the main intuition is similar in that in a boom period, the average project quality increases, the costs associated with tight lending standard exceed the costs associated with financing bad borrowers, and lending to bad firms becomes optimal.

Another closely related paper is by Gorton and Ordonez (2014). In their paper, lenders make loans to borrowers through collateralized loans. The final value of collateral is random. Lenders investigate the final value of collateral but with cost. The information about collateral is independent to project quality, and thus producing information is a waste of resources. They find two equilibria, a separating equilibrium with information sensitive loan contracts, and a pooling equilibrium with information insensitive loan contracts. Our model also assumes imperfect information *ex ante*. However, different from Gorton and Ordonez (2014), we assume banks screen to acquire information about project itself, which can improve social welfare, as shown in Greenwood and Jovanovic (1990) and Townsend and Ueda (2006). In addition, Gorton and Ordonez (2014) assumes the information is always accurate. However, we allow continuous degrees of screening cost and associated signal accuracy.

Lester et al. (2019) also argue that additional information reduces welfare. They incorporate imperfect competition into a standard adverse selection model, and show that a small increase in information (introduction of a free but noisy signal) reduce welfare when the market competition is strong and that additional information has no effect on welfare under perfect competition. In contrast, we assume banks engage meaningful and costly screening efforts.

Regarding costly acquisition of information, Townsend (1979), a seminar paper, introduced the costly state verification, which assumes lenders can verify asset quality *ex post* with costs. Bernanke et al. (1999) applied it to a macroeconomic model on financial accelerator. Wang and Williamson (1998) build a model in which lenders can use costly screening *ex ante* to perfectly observe borrowers' types which borrowers know. The loan contract includes repayment and probability of screening, contingent on borrower types. They concluded that there is only separating equilibrium, because a separating contract can achieve self selection. Good borrowers strictly prefer a separating contract because there is no cross-subsidy to bad borrowers. In contrast, we assume firms do not know their own types, just like banks have no information, and it needs the costly screening that reveals true types though imperfectly. As a result, in our model, there exists a pooling equilibrium in which banks ease lending standard and lend to all firms.

In terms of information processing, Van Nieuwerburgh and Veldkamp (2010) use the reduction in entropy to measure screening technology, which is fairly complicated. Our model assumes a simpler screening technology. The closest paper to our screening technology is Ferrante (2018), who defines banks' costly screening as to select loans that will be good in next period with information accuracy increasing linearly in screening effort. Their cost function of screening effort is quadratic, in order to get a closed-form solution. Also, Freixas and Rochet (2008), in chapter 2, introduce a simplified version of Bayesian learning for moral hazard using binary signals. We follow the latter two literature, and assume binary signals and simple Bayesian laws for banks to update the information of types after the costly screening.

Regarding the policy implications of zombie firms, our results are similar to Caballero et al. (2008). Caballero et al. (2008) focus on the effect of zombie on the economy such as productivity, investment, and employment, and classify firms as zombies based on whether they receive loans lower than market rates. Zombie firms are considered to be produced by an evergreening lending strategy by distressed banks. Banks roll over loans to insolvent borrowers, and unwilling to call in nonperforming loans for the risk of breaking the capital adequacy ratio. Rather, we focus on the eonomic environment, such as a low interest rate and return differences among firms. Moreover, we show how zombie firms are produced by policies, such as public loan guarantees, credit easing, and direct subsidy to business. We show under those policies banks are demotivated to screen loan applicants, in other words, have a non-screening equilibrium and lend to unviable firms, and thus generate zombie firms.

Our model also has implication for venture lending, addressing the theoretical gap regarding the optimal screening costs for venture investors during their pre-investment evaluations of new projects. Regarding pre-investment screening of venture investors, there are not many studies but a few empirical survey results and anecdotal evidences: Kaplan and Strömberg (2004) and Gladstone and Gladstone (2004) show that banks always prepare a detailed pre-investment memorandum for the investment opportunity, on which they spend a great deal of time and effort. Zacharakis and Meyer (2000) show that, despite the time and effort spent on screening, the banks' portfolios fail at a surprisingly high rate. The screening process for venture lending appears to differ from conventional lending and screening literatures in two ways. Firstly, lenders have expertise in screening, though the screening is costly and imperfect. Secondly, borrowers seem unaware of their own types.[4]

2 The Model Setup Under Imperfect Information

2.1 Environment

There are n banks, m firms, and the same m depositors in the market. Each firm has a project that requires 1 unit of investment. Each depositor has 1 unit of funds to save in banks. Overall, there are m units of fund supply in this economy. Banks, as financial intermediaries, borrow from depositors and lend to firms. Banks can invest abroad with fixed return R. Firms are heterogeneous. We assume that firm types are *ex ante* unknown to both the firms themselves and the banks. After receiving loan applications, banks can screen firms but with costly imperfect information on firm types.[5]

[4]These differences might not only apply to venture lendings, but also applies to any business bank loans to small companies (Robb and Robinson, 2014; Berger et al., 2017).

[5]There are few theoretical papers on the pre-investment process of venture lending. Garmaise (2007) assumed investors are more well informed than entrepreneurs and constructed a first price auction game for multiple investors. However, his paper did not model investors' cost of evaluating potential investments.

Firms

Firms are risk neutral. Each firm has no initial wealth, borrows funds to invest in one unit of project, and produce the same final goods. Each firm is either of two types, good or bad, which the firm cannot know *ex ante*. The probability of being a good firm is f, and the probability of being a bad firm is $(1-f)$. Hence, in the market, there are fm good firms and $(1-f)m$ bad firms. This type distribution is public information *ex ante*, although the specific type of each firm is not observed by anyone. A good firm produces G amount of goods from 1 unit of investment, while a bad firm produces B amount. We assume that output G and B are fixed with $G > R > B$, where R is the exogenous international interest rate.

Every firm relies on a loan to finance the project. If a firm does not obtain a loan, the firm exits. A firm's type is perfectly revealed only after the production takes place. The firm chooses to repay the loan in full or default. For the sake of simplicity, we assume that the firm has limited liability and that, if default, all the firm's output would be taken by the bank. Note that, without loss of generality, we consider only debt-type contracts.

Banks

Banks are risk neutral. Banks are identical and issue loan contracts to firms. Each loan contract has a fixed loan size of one.[6]

A bank's strategy in the first stage is to offer a loan contract to a firm with contingencies. In the third stage, it screens the loan applicants, that is, collects information with screening cost C to observe the signal of the firm's type. The signals on firms' types are not perfect, that is, not necessarily the same as firms' real types. We assume that the signal that is obtained by a bank is private information.[7]

For the sake of simplicity, we assume exclusive contracts. Banks are symmetric when posting

[6] We also endogenize the loan size in the earlier working paper and obtain similar results.

[7] The signal is observable but not verifiable by outsiders. Specifically, a bank can use the costly screening to obtain a signal and make an in-house investment evaluation of an entrepreneur.

loan offers. An entrepreneur needs to apply to one bank for a loan. Her signal is revealed only after she applies for a loan.

Regarding the pre-investment screening of venture lending, Gompers et al. (2020) summarize their survey on banks: Banks believe that the quality of the firm management team is more important than any other business-related characteristics when they screen firms.[8] However, the quality of the management team of startups is hard to quantify because it is soft information, as discussed in Liberti and Petersen (2019), which is not stable and highly dependent on the environment. So, banks face those uncertainties, as shown in DeYoung et al. (2008) and Ibrahim (2010). Furthermore, many papers show that entrepreneurs are not particularly successful at evaluating their projects, and might often be overly optimistic. In the extreme, the attitude of the entrepreneur may be an empty signal (see Cooper et al. 1988; Hamilton 2000). Hence, in order to model such uncertainty, we consider the extreme case, and propose a novel assumption, symmetrically imperfect information, that is, neither the creditor nor the borrower know the borrower's type until production takes place.

In our model, we assume that banks obtain signals on firms' types, denoted as $\{\hat{G}, \hat{B}\}$, while the real types of firms are denoted as $\{G, B\}$. The *accuracy* of a signal is assumed to be related to the screening cost C that a bank pays and is considered as an increasing function of screening cost C, that is, $Pr(G|\hat{G}, C)$ or $Pr(B|\hat{B}, C)$. On the other hand, the *correctness* of a signal is expressed as $Pr(\hat{G}|G, C)$ or $Pr(\hat{B}|B, C)$. Note that the accuracy $Pr(G|\hat{G}, C)$ refers to the probability of a signal $\hat{G}$ is accurately assigned to a G firm by a bank under screening cost C, while the correctness $Pr(\hat{G}|G, C)$ refers to the probability of a bank receiving the correct signal $\hat{G}$ on a good firm G under screening cost C.

In the real world, venture capitals and banks spend big efforts to investigate the borrowing firms. For example, MacMillan et al. (1985) and Kollmann and Kuckertz (2010) show the due

[8] Literature on venture capital mostly focus on the post-investment monitoring, including cash-flow rights, voting rights, and exit mechanism. See, for example, Hellmann and Puri (2002); Schmidt (2003); Kaplan and Strömberg (2003); Repullo and Suarez (2004); Bernstein et al. (2016); Gompers (2022); and Sannino (2024).

diligence checklists may well include up to 400 different criteria. Berger et al. (2005) and Ibrahim (2010) show that banks adopt a costly standard screening process before lending to startups. In addition, Zacharakis and Meyer (2000) and Ramsinghani (2014) show that VC-backed firms still fail at a surprisingly high rate, over 20%.

Bank's screening process in this model represents the process of credit scoring in reality. Firstly, based on the screening budget C, a bank makes a list of screening related actions, for example, a site visit and an equipment quality check. Secondly, the bank screens each loan applicant based on the screening check list and obtains a credit score for each applicant. Third, if a bank wants to decide whether to give a loan or not, the bank need to decide a threshold score. Lastly, in this screening procedure, the bank assigns $\hat{G}$ signals to firms whose credit score is above the threshold, and $\hat{B}$ signals otherwise.

A loan contract offered by bank i to firm j includes three parts: a screening strategy, a lending strategy (contingent on screening strategy), and loan repayments (contingent on production realization). We assume that banks commit to loan contract specifications throughout all stages, including contingent plans, when offering loan contracts in stage 1. A bank may reject loan applications. In this case, we assume that banks invest remaining funds in foreign countries at international interest rate R without loss of generality.[9]

Given the lending strategy, the bank decides the screening strategy, that is, the amount of screening cost C. Based on the lending strategy and the screening strategy, the required repayment of a firm becomes contingent on the observed signal.

Theoretically, each bank chooses one from four possible lending strategies: a "lend to $\hat{G}$ only" strategy, a "lend to $\hat{B}$ only" strategy, a "lend to both $\hat{G}$ and $\hat{B}$ with no cross subsidy" strategy, and a "lend to both $\hat{G}$ and $\hat{B}$ with a cross subsidy" strategy. Here, without "cross subsidy," a bank asks for at least R return from each firm, so that a B-type firm cannot repay regardless of signals.

[9] In the earlier working paper, we also analyze a case in which the remaining funds go to the selected firms. There, we drop the assumption of fixed-size loan contracts and endogenize the loan size under the assumption of a constant-returns-to-scale production function. We find our main conclusions still hold.

With "cross subsidy," a bank would require a low return from $\hat{B}$, so that a B-type firm with a $\hat{B}$ signal can repay.

Information Properties

Similar to Martinez-Miera and Repullo (2017) and Ferrante (2018), we use an imperfect and costly screening, assuming the accuracy of screening as an increasing and concave function of the cost with a specific functional form. These papers, including ours, are variants of the costly acquisition of information. Townsend (1979), a seminal paper, introduced the costly state verification, which assumes agents can perfectly verify asset quality *ex post* with costs. Bernanke et al. (1999) applied it to equip a macroeconomic model with a financial accelerator.

Different from most of the literature, we assume the signal from screening is observable to both entrepreneurs and banks, but verifiable only by banks, as the soft information described in Liberti and Petersen (2019). There has been discussions in the contract literature about whether a signal that is observable by outsiders is also verifiable by outsiders (Hart, 1995; Aghion and Tirole, 1997; Baker et al., 2002). The result of the screening can be reduced to a numerical index (e.g., a credit score which can be shared with entrepreneurs). However, assigning numerical values may be judgmental and include a discretionary component (e.g., the detailed investment memorandum confidential to venture capitalist) (Cole et al., 2004; Hertzberg et al., 2010). Hence, it is impossible for entrepreneurs to completely free ride on the screening results, as mentioned in Petersen and Rajan (1994).

Based on these literature, we regard a signal as private information known only to a bank that spends some costs. Together with Bayes' rule, we assume the following specific information properties, for the sake of simplicity but without loss of generality. Table 1 summarizes the key probabilities regarding signals.

1. $Pr(G)$ is equal to f, which is an unconditional probability and known to all agents.

2. The correctness $Pr(\hat{G}|G, C)$ and $Pr(\hat{B}|B, C)$ are denoted as $f_G(C)$ and $f_B(C)$, respectively.

By Bayes' Law, the correctness and the accuracy share the same probabilities.

$$f_G(C) = Pr(G|\hat{G}, C) = \frac{Pr(\hat{G}|G, C)Pr(G)}{Pr(\hat{G})} = Pr(\hat{G}|G, C), \text{and}$$

$$f_B(C) = Pr(B|\hat{B}, C) = \frac{Pr(\hat{B}|B, C)Pr(B)}{Pr(\hat{B})} = Pr(\hat{B}|B, C).$$

3. When banks obtain signals with cost C, the aggregate portion of firms that have $\hat{G}$ signals, $Pr(\hat{G}|C)$, is still equal to f. Note that $Pr(\hat{G}|C) = Pr(\hat{G}|G, C)Pr(G) + Pr(\hat{G}|B, C)Pr(B) = f$. Hence, $Pr(\hat{G}|C) = Pr(G) = f$ and similarly $Pr(\hat{B}|C) = Pr(B) = 1 - f$. The bank knows that f portion of good firms always exists among all loan applicants, by the Law of Large Numbers.

4. The correctness is assumed to naturally have the following properties:

   4.1 The correctness $f_G(\cdot)$ and $f_B(\cdot)$ are increasing, strictly concave, twice continuously differentiable, in terms of C, that is, $\frac{\partial f_G(C)}{\partial C} > 0$, $\frac{\partial^2 f_G(C)}{\partial C^2} < 0$, $\frac{\partial f_B(C)}{\partial C} > 0$, and $\frac{\partial^2 f_B(C)}{\partial C^2} < 0$. When a bank has more screening budget C, then the bank can afford more screening related actions and has a better understanding of the applicant, hence, the probability of signal accuracy increases. However, additional screening related actions are considered to exihibit diminishing returns as C increases.

   4.2 The correctness $f_G(\cdot)$ and $f_B(\cdot)$ are increasing in two parameters: f, the portion of good firms in the market; and $\alpha \in (0, 1)$, the efficiency of screening technology; that is, $\frac{\partial f_G(C)}{\partial f} > 0$, $\frac{\partial f_G(C)}{\partial \alpha} > 0$, $\frac{\partial f_B(C)}{\partial f} > 0$, and $\frac{\partial f_B(C)}{\partial \alpha} > 0$.

5. On the accuracy, the following properties are naturally given:

   5.1 Given the screening cost C, a higher α implies a higher accuracy of a signal $Pr(G|\hat{G}, C)$. If the screening efficiency α increases, for example, by introducing digital transaction records or online meetings, the average cost for one unit of screening decreases. Then,

True type	For G firm	For B firm
Unconditional prob.	$Pr(G) = f$	$Pr(B) = 1 - f$
Accuracy	$Pr(G\|\hat{G}, C) = f_G(C)$	$Pr(B\|\hat{B}, C) = f_B(C)$
Inaccuracy	$Pr(G\|\hat{B}, C) = 1 - f_B(C)$	$Pr(B\|\hat{G}, C) = 1 - f_G(C)$
Signal	For $\hat{G}$ firm	For $\hat{B}$ firm
Unconditional prob.	$Pr(\hat{G}) = f$	$Pr(\hat{B}) = 1 - f$
Correctness	$Pr(\hat{G}\|G, C) = f_G(C)$	$Pr(\hat{B}\|B, C) = f_B(C)$
Incorrectness	$Pr(\hat{G}\|B, C) = 1 - f_B(C)$	$Pr(\hat{B}\|G, C) = 1 - f_G(C)$

Table 1. Key probabilities regarding signals

the bank can conduct more screening under the same budget C, thus, the probability of signal accuracy increases. By Property 2 above, this means $\frac{\partial f_G(C)}{\partial \alpha} > 0$, the same as Property 4.2. Similarly, $\frac{\partial f_B(C)}{\partial \alpha} > 0$.

5.2 Given the screening cost C, a higher f implies a higher accuracy of a signal $Pr(G|\hat{G}, C)$. The lower bound of the accuracy is $Pr(G|\hat{G}, C = 0) = f$. As f increases, this lower bound increases. By Property 2 above, this means $\frac{\partial f_G(C)}{\partial f} > 0$, the same as Property 4.2. Similarly, $\frac{\partial f_B(C)}{\partial f} > 0$.

6. Extreme levels of screening costs are naturally defined in terms of the signal correctness and accuracy by Property 2: $f_G(C = 0) = f$, $f_B(C = 0) = 1 - f$, $\lim_{C \to \infty} f_G(C) = 1$, and $\lim_{C \to \infty} f_B(C) = 1$. When $C = 0$, a bank does not screen firms, and thus $Pr(G|\hat{G}, C = 0) = f$ and $Pr(G|\hat{B}, C = 0) = f$. So, within $\hat{G}$ group and within $\hat{B}$ group, the portion of G are the same, that is, f. On the other extreme, unlimited cost $C \to \infty$ is assumed to give a bank a perfectly accurate signal.

7. For the sake of simplicity, we define a parametric screening function with cost C as follows. Below, e is the Euler number, $f \in (0, 1)$ is the market share of good firms, $\alpha \in (0, 1)$ is the

screening efficiency, and $C \in (0, \infty)$ is the screening cost.

$$Pr(\hat{G}|G, C) = Pr(G|\hat{G}, C) = f_G(C) \equiv 1 - (1 - f)e^{-\alpha C}.$$
$$Pr(\hat{B}|B, C) = Pr(B|\hat{B}, C) = f_B(C) \equiv 1 - fe^{-\alpha C}.$$

This screening function follows existing literature: Ferrante (2018) and Freixas and Rochet (2008). Ferrante (2018) assumes that information accuracy increases linearly in screening effort, and the cost function of screening effort is convex (quardratic). Freixas and Rochet (2008), in Chapter 2, introduce a simplified version of Bayesian learning for moral hazard using binary signals. We assume binary signals and directly assume that information accuracy is a concave function of screening cost, satisfying the five properties above. To obtain a closed-form solution, we assume an exponential function.

Later, we discuss the effect of improvement of the screening technology. Indeed, Arroyo et al. (2019) and Maurer et al. (2024) empirically show that machine learning and AI can improve the efficiency and accuracy of the screening process. Ahnert et al. (2024) show that digital innovations have made it cheaper for banks to collect and process data.

2.2 Timeline

The model has 5 stages as shown in Figure 1. Ex ante, neither a bank nor a firm know the firm's real type.

- In stage 1, bank i designs screening strategies and lending strategies. Bank i offers its loan contract for firms. The loan terms are contingent on the signal extracted from the loan applicant in the later stage. Bank i commits the contingent contract. The offered loan contracts and banks' commitments are public information.

- In stage 2, firm j selects one bank and applies for a loan. Firm j commits to stay with the bank it selects.

- In stage 3, bank i obtains the private signal by investigating applicant firm j with cost C, and bank i then specifies the loan contract terms contingent on firm j's signal, as described already in the loan offer. This specified term is private information between bank i and firm j. Note that the firm has already committed to a bank and so do not have opportunities to apply for loans to other banks at this stage or later.

- In stage 4, firm j updates the belief of its real type by the specified loan term, and then chooses to accept the loan offer or to reject the offer (and exit the market). Note that in stage 3 and 4, bank i and firm j know firm j's signal, but not yet the real type.

- In stage 5, if obtained a loan, firm j produces goods. Naturally, the real type is revealed to the public. Then, firm j chooses to repay the loan or to default on it.

2.3 Equilibrium Definition

We study a general equilibrium with information friction, essentially following Prescott and Townsend (1984a,b) and Bisin and Gottardi (2006). They use the equilibrium concept known as the lottery equilibirum (similar to the correlated equilibrium), but for the sake of simplicity, we focus on a pure strategy Nash equilibrium. We also focus on symmetric equilibrium for banks, in which banks are identical, adopt the same screening strategy, and offer the same contracts. However, firms can be screened so their strategies and outcomes may vary depending on their types and signals.[10]

Our model is related to the principal-agent problem of the financial contracts. This literature is often based on the assumption of information asymmetry, that is borrowers have private information of their types over banks. There are two classic frictions: adverse selection associated with hidden types and moral hazard associated with hidden efforts.

To model the moral hazard in credit markets, Martinez-Miera and Repullo (2017) assume oligopolistic bank's costly monitoring effort can increase the probability of entrepreneur's success,

[10] This is a kind of a quasi-symmetric equilibrium, in which the number of client firms for each bank can be different and also firms could take different strategies but firms do take the same strategies as explained later. Note that banks can also take different strategies in off-equilibrium paths.

and show that a low international interest rate reduces incentive of monitoring and increases the portion of the non-monitoring loans. They argue that social planner can improve welfare by raising interest rate and, thus, increasing monitoring intensity. However, with competitive banking industry, not an oligopolistic one, we conjecture their model aligns with Prescott and Townsend (1984a,b), who show that for economies with moral hazard, competitive contract markets can generate Pareto optimal equilibrium. We also assume competition among a large number of banks.[11]

Figueroa and Leukhina (2015) develop a macro dynamic stochastic general equilibrium (DSGE) model under adverse selection. They define the loan size as the lending standard, which banks use to achieve self-selection. They show that in a boom period, banks tend to relax constraint on the loan size and have a pooling equilibrium. In a boom, better economic conditions raise overall returns to investment, so larger loans are made, resulting in a worse mix of borrowers. Similarly, Dell'Ariccia and Marquez (2006) model the collateral requirements related to the lending standard. They show that a pooling equilibrium emerges in boom and separating equilibrium in bust. However, Bisin and Gottardi (2006) theoretically prove that the adverse selection in a general equilibrium with a more general contract space selects separating equilibrium, not pooling equilibrium. Wang and Williamson (1998) adopt a costly and perfect screening process and have the same conclusion as Bisin and Gottardi (2006). In contrast, our model is not based on the adverse selection and still has a pooling equilibrium with a very general contract space.

2.4 Notations

For bank i and firm j, bank i's offer to firm j is defined as x_{ij},

$$x_{ij} = \Big(Pr(G|\hat{G}, C), (R_{\hat{G}}, L_{\hat{G}}), (R_{\hat{B}}, L_{\hat{B}}) \Big),$$

[11]There are many papers focusing on strategic interactions among (oligopolistic) investors. For example, Boissay et al. (2016) study the interbank funding market, and Casamatta and Haritchabalet (2007) study the competition and syndication strategy of venture capital firms.

where $Pr(G|\hat{G}, C) \in [0, 1]$ represents the level of signal accuracy or screening intensity. $R_{\hat{G}}$ and $R_{\hat{B}}$ are the required repayments for $\hat{G}$ and $\hat{B}$ firms, respectively. $L_{\hat{G}}$ and $L_{\hat{B}}$ are the loan amounts for $\hat{G}$ and $\hat{B}$ firms, respectively. We adopt a rather strong assumption that the loan amount, which is equal to the investment amount, is either zero or one, $L_{\hat{G}} \in \{0, 1\}$ and $L_{\hat{B}} \in \{0, 1\}$. Also, without loss of generality, $R_{\hat{G}} \in [0, G]$ and $R_{\hat{B}} \in [0, G]$.

In terms of the bank's lending strategy, denote $\hat{\mu} \in [0, 1]$ as the allocation ratio of funds to $\hat{G}$ firms. Depending on lending strategies, the domain of contract x_{ij} is constrained. There are four cases, as below.

Case (1): The lending strategy is "lend to $\hat{G}$ only," that is, $\hat{\mu} = 1$. $\mathbf{S_1}$ is denoted as the constrained strategy set of x_{ij} under the "lend to $\hat{G}$ only" strategy. $Pr(G|\hat{G}, C)$ is an increasing function of C. When $C = 0$, $Pr(G|\hat{G}, C) = f$; when $C > 0$, $Pr(G|\hat{G}, C) > f$ with upper bound $Pr(G|\hat{G}, C) \leq 1$. It is easy to show $R_{\hat{G}} \geq R$. Otherwise, the bank participation constraint is not satisfied. Hence, the constrained strategy can be written as

$$x_{ij} = \Big(Pr(G|\hat{G}, C), (R_{\hat{G}}, L_{\hat{G}}), (R_{\hat{B}}, L_{\hat{B}})\Big) \in [f, 1) \times [R, G] \times \{1\} \times [0, G] \times \{0\} \equiv \mathbf{S_1}.$$

Case (2): The lending strategy is "lend to $\hat{B}$ only," that is, $\hat{\mu} = 0$. $\mathbf{S_2}$ is denoted as the constrained strategy set of x_{ij} under the "lend to $\hat{B}$ only" strategy. It is easy to show $R_{\hat{B}} \geq R$. Otherwise, the bank participation constraint is not satisfied. Hence, the constrained strategy can be written as

$$x_{ij} = \Big(Pr(G|\hat{G}, C), (R_{\hat{G}}, L_{\hat{G}}), (R_{\hat{B}}, L_{\hat{B}})\Big) \in [f, 1) \times [0, G] \times \{0\} \times [R, G] \times \{1\} \equiv \mathbf{S_2}.$$

Case (3): The lending strategy is "lend to both $\hat{G}$ and $\hat{B}$ with no cross subsidy," that is, $\hat{\mu} \in (0, 1)$. If a firm with a bad signal ($\hat{B}$) is indeed a bad firm (B), then the B firm defaults without cross subsidy. Hence, if $R_{\hat{B}} < B$, a bank is sure to lose. In other words, the required repayment must be higher than the output, $R_{\hat{B}} > B$. The constrained strategy can be written as

$$x_{ij} = \Big(Pr(G|\hat{G}, C), (R_{\hat{G}}, L_{\hat{G}}), (R_{\hat{B}}, L_{\hat{B}})\Big) \in [f, 1) \times [R, G] \times \{1\} \times (B, G] \times \{1\} \equiv \mathbf{S_3}.$$

Case (4): The lending strategy is "lend to both $\hat{G}$ and $\hat{B}$ with a cross subsidy," that is, $\hat{\mu} \in (0, 1)$. Particularly, we only consider the case that good firms subsidize bad firms not to default, because bad firms have no room to subsidize good firms. Here, we define a cross subsidy in terms of *ex post* true types of firms (i.e., G and B), instead of the observed types (i.e., $\hat{G}$ and $\hat{B}$). If a bad signal firm ($\hat{B}$) is indeed a bad firm (B), B firm does not default with the cross subsidy. In other words, the required repayment is lower than or equal to the output, $R_{\hat{B}} \leq B$. The constrained strategy can be written as

$$x_{ij} = \Big(Pr(G|\hat{G}, C), (R_{\hat{G}}, L_{\hat{G}}), (R_{\hat{B}}, L_{\hat{B}})\Big) \in [f, 1) \times [R, G] \times \{1\} \times [0, B] \times \{1\} \equiv \mathbf{S_4}.$$

The overall strategy set for x_{ij} is defined naturally as $\mathbf{S} \equiv \mathbf{S_1} \cup \mathbf{S_2} \cup \mathbf{S_3} \cup \mathbf{S_4}$.

There are n banks and m firms in the market. A set of bank i's offers for all firms is denoted as $x_i = \{x_{ij}\}_{j=1}^m \in \mathbf{S}^m$, and other banks' offers as $x_{-i} = \Pi_{h \neq i} x_h$. We assume bank i's x_{ij} is the same for all firms as it contains loan terms contingent on signals. Firm j chooses one bank (or one loan contract offer) from $x_j = \{x_{ij}\}_{i=1}^n \in \mathbf{S}^n$. All loan offers are denoted as $x = \{x_1, ..., x_m\} \in \mathbf{S}^{mn}$. $E\left[\pi^f\left(x_{ij}\right)\right]$ represents firm j's expected profit in stage 2 when firm j faces bank i's offer. $E\left[\pi^b(x_{ij}|x_{-i})\right]$ represents bank i's expected profit from bank i's offer to firm j in stage 1, conditional on any other banks' offers.

3 Optimal Problems And Decisions In Each Stage

We solve the model backward, starting from stage 5.[12]

[12]For the case of perfect information, it is easy to show the existence and uniqueness of a screening separating equilibrium, in which banks only lend to G-type firms, the equilibrium repayment for G-type firms is R, and there is no default. See more detailed discussion in the earlier working paper.

3.1 Stage 5

In stage 5, the active firms are those that have accepted the loan contracts in stage 4. Firms produce, unveil the real types, and then decide whether to repay loans in full or to default and declare bankruptcy. In the case of bankruptcy, we assume that all outputs are seized by the bank. Hence, as long as firms can repay loans, they will repay in full.

Bank's profit is $\pi^b = \hat{\mu}\pi^b_{\hat{G}} + (1-\hat{\mu})\pi^b_{\hat{B}}$. Note that the allocation across true types is not necessarily the same as signal-based allocation $\hat{\mu}$. If firm j repays the loan, $R_{\hat{G}}$ or $R_{\hat{B}}$, the repayment to bank i is contingent on the realization of outputs, G or B. Recall that if firm j declares bankrupt, bank i will take over all firm j's output.

A firm's repayment obligation is contingent on the signal and the loan contract agreed in the previous stages. The active firms in stage 5 can be divided into four cases, where $\hat{\mu}$ is bank's allocation ratio of funds to $\hat{G}$ firms.

(1) $\hat{G}$ firms under a "lend to $\hat{G}$ only" strategy ($\hat{\mu} = 1$) and screening strategy C, with repayment $R_{\hat{G}} \in [0, G]$;

(2) $\hat{B}$ firms under a "lend to $\hat{B}$ only" strategy ($\hat{\mu} = 0$) and screening strategy C, with repayment $R_{\hat{B}} \in [0, G]$;

(3) $\hat{G}$ firms under a "lend to both" strategy ($\hat{\mu} \in (0, 1)$) and screening strategy C, with repayment $R_{\hat{G}} \in [0, G]$;[13]

(4) $\hat{B}$ firms under a "lend to both" strategy ($\hat{\mu} \in (0, 1)$) and screening strategy C, with repayment $R_{\hat{B}} \in [0, G]$.

[13] Here, the "lend to both" strategy include the "lend to both $\hat{G}$ and $\hat{B}$ with no cross subsidy" strategy and the "lend to both $\hat{G}$ and $\hat{B}$ with a cross subsidy" strategy.

3.2 Stage 4

In stage 4, the active firms are those that have received the loan contract offers in stage 3. Each firm is assumed to have applied for one loan contract to one bank in stage 2, but not all applicants are accepted by banks at this stage. The firm with an accepted application may have an updated belief of its own real type, based on the loan terms specified by the bank. The firm with a specified loan contract offer has two options, either accepts the loan contract and produces later in stage 5, or rejects the offer and exits the market in stage 4. Using the updated belief of its type, the threshold of accepting the loan contract and producing goods becomes contingent on the specified loan contract offer, as follows.

(1) $\hat{G}$ firm facing bank's "lend to $\hat{G}$ only" strategy ($\hat{\mu} = 1$) and screening strategy C accepts the loan contract when $Pr(G|\hat{G}, C)(G - R_{\hat{G}}) + Pr(B|\hat{G}, C) \max\{0, B - R_{\hat{G}}\} \geq 0$;

(2) $\hat{B}$ firm facing bank's "lend to $\hat{B}$ only" strategy ($\hat{\mu} = 0$) and screening strategy C accepts the loan contract when $Pr(G|\hat{B}, C)(G - R_{\hat{B}}) + Pr(B|\hat{B}, C) \max\{0, B - R_{\hat{B}}\} \geq 0$;

(3) $\hat{G}$ firm facing bank's "lend to both" strategy ($\hat{\mu} \in (0, 1)$) and screening strategy C accepts the loan contract when $Pr(G|\hat{G}, C)(G - R_{\hat{G}}) + Pr(B|\hat{G}, C) \max\{0, B - R_{\hat{G}}\} \geq 0$;

(4) $\hat{B}$ firm facing bank's "lend to both" strategy ($\hat{\mu} \in (0, 1)$) and screening strategy C accepts the loan contract when $Pr(G|\hat{B}, C)(G - R_{\hat{B}}) + Pr(B|\hat{B}, C) \max\{0, B - R_{\hat{B}}\} \geq 0$.

Note that acceptance conditions in (3) and (4) are the same as in (1) and (2), respectively.

3.3 Stage 3

With cost $C > 0$, bank i screens all the loan applications of firms that have chosen bank i. Through this costly screening, the bank receives a signal for each applicant. Given the signal, the bank offers loan contracts specifying repayments $R_{\hat{G}}$ and $R_{\hat{B}}$ to the selected firms, following contingencies in the offered loan terms in stage 1. Note that the screening strategy (C) and the lending strategy ($\hat{\mu}$) are already announced and committed to in stage 1.

3.4 Stage 2: Firm's Optimal Problem

Firm j selects one bank, and applies for one loan contract, as it maximizes its expected profit by comparing loan offers. Specifically, in stage 2, given all banks' loan contract offers x_j, firm j chooses one offer, x_{ij}, to maximize firm j's expected profit:

$$\max_{x_{ij} \in x_j} E\left[\pi^f(x_{ij})\right], \tag{1}$$

which can be expressed as, for $\forall h \neq i, \forall \tilde{x}_{hj} \in x_j$,

$$E\left[\pi^f(x_{ij})\right] \geq E\left[\pi^f(\tilde{x}_{hj})\right], \tag{2}$$

for a specific x_{ij} to be the maximizer of (1). This becomes the firm's incentive compatibility constraint from the viewpoints of bank i in stage 1.

Depending on the bank's lending strategy that arises in stage 1 in equilibrium, the left-hand-side (LHS) of (2) can be further specified into four cases below.

Stage 2 Case (1): Suppose that "lend to $\hat{G}$ only" ($\hat{\mu} = 1$) is the banks' equilibrium lending strategy, which firms face in stage 2. The signal is revealed to the bank in stage 3. Under this contract, with $\hat{B}$ signal, firm j will not receive loans in stage 4. Moreover, since $R_{\hat{G}} \geq R$ (by the bank participation constraint), B firms *ex post* will default with zero profits in stage 5. Hence, firm's expected profit in stage 2 does not contain the part involving $Pr(B)$. Therefore, firm j's expected profit, the LHS of condition (2), is specified as below: For $\forall h \neq i, \forall \tilde{x}_{hj} \in x_j$,

$$E\left[\pi^f(x_{ij})\right] = Pr(G)Pr(\hat{G}|G, C)(G - R_{\hat{G}}) \geq E\left[\pi^f(\tilde{x}_{hj})\right]. \tag{3}$$

Stage 2 Case (2): Suppose that "lend to $\hat{B}$ only" is the banks' equilibrium lending strategy ($\hat{\mu} = 0$), which firms face in stage 2. By the similar argument as above, condition (2) is specified as below: for $\forall h \neq i, \forall \tilde{x}_{hj} \in x_j$,

$$E\left[\pi^f(x_{ij})\right] = Pr(G)Pr(\hat{B}|G,C)(G - R_{\hat{B}}) \geq E\left[\pi^f(\tilde{x}_{hj})\right]. \tag{4}$$

Stage 2 Case (3): Suppose that "lend to both with no cross subsidy" is the banks' equilibrium lending strategy ($\hat{\mu} \in (0,1)$), which firms face in stage 2. By assumption, $R_{\hat{B}} > B$. A bad firm *ex post* will default with zero profit in stage 5. Hence, the firm expected profit in stage 2 does not contain the part with $Pr(B)$. Those with bad signal $\hat{B}$, but turn out to be good G *ex post*, receive positive profits in stage 5. In stage 2, this probability is $Pr(\hat{B}|G,C)$. The condition (2) is then specified as below: for $\forall h \neq i, \forall \tilde{x}_{hj} \in x_j$,

$$E\left[\pi^f(x_{ij})\right] = Pr(G)[Pr(\hat{G}|G,C)(G - R_{\hat{G}}) + Pr(\hat{B}|G,C)(G - R_{\hat{B}})] \geq E\left[\pi^f(\tilde{x}_{hj})\right]. \tag{5}$$

Stage 2 Case (4): Suppose that "lend to both with a cross subsidy" is the banks' equilibrium lending strategy ($\hat{\mu} \in (0,1)$), which firms face in stage 2. By assumption $R_{\hat{B}} \leq B$, a bad firm *ex post* (B) with a bad signal ($\hat{B}$) will not default and have positive profits *ex post* in stage 5. However, a bad firm *ex post* (B) with a good signal ($\hat{G}$) will default because $R_{\hat{G}} \geq R$. The condition (2) is then specified as below: for $\forall h \neq i, \forall \tilde{x}_{hj} \in x_j$,

$$\begin{aligned} E\left[\pi^f(x_{ij})\right] =& Pr(G)[Pr(\hat{G}|G,C)(G - R_{\hat{G}}) + Pr(\hat{B}|G,C)(G - R_{\hat{B}})] \\ &+ Pr(B)[Pr(\hat{B}|B,C)(B - R_{\hat{B}})] \\ \geq& E\left[\pi^f(\tilde{x}_{hj})\right]. \end{aligned} \tag{6}$$

3.5 Stage 1: Overall Bank's Optimal Problem

Bank i offers a loan contract with contingencies to all firms. The loan term is defined as x_{ij} in the previous section. It includes the screening strategy (C), the lending strategy ($\hat{\mu}$), and the required loan repayments ($R_{\hat{G}}, R_{\hat{B}}$), contingent on the signals obtained in stage 3.

Recall that bank i has four possible lending strategies: a "lend to $\hat{G}$ only" strategy ($\hat{\mu} = 1$), a "lend to $\hat{B}$ only" strategy ($\hat{\mu} = 0$), a "lend to both $\hat{G}$ and $\hat{B}$ with no cross subsidy" strategy ($\hat{\mu} \in (0, 1)$) and a "lend to both $\hat{G}$ and $\hat{B}$ with a cross subsidy" strategy ($\hat{\mu} \in (0, 1)$).

Specifically, in stage 1, bank i designs contracts for all firms $\{x_{ij}\}_{j=1}^{m}$, by maximizing the bank i's expected profit, conditional on the firm's participation constraint (PC), the firm's incentive compatible constraint (IC), and the bank's participation constraint (PC) in equilibrium. Given other banks' strategies x_{-i}, if bank i's offer is not selected, $E\left[\pi^b(x_{ij}|x_{-i})\right] = 0$; otherwise, bank i's offer is selected and $E\left[\pi^b(x_{ij}|x_{-i})\right]$ becomes a function of x_{ij}. Here, the expectation is taken in stage 1 over the probability of the realization of other banks' strategies, x_{-i}.

$$(7) \qquad \max_{\{x_{ij}\}_{j=1}^{m} \in \mathbf{S}^m} \sum_{j=1}^{m} E\left[\pi^b(x_{ij}|x_{-i})\right]$$

$$s.t.$$

$$(8) \qquad E\left[\pi^f\left(x_{ij}\right)\right] \geq 0 \qquad \text{[Firm PC]}$$

$$(9) \qquad E\left[\pi^f\left(x_{ij}\right)\right] \geq E\left[\pi^f(\tilde{x}_{hj})\right], \text{ for } \forall j, h \neq i, \forall \tilde{x}_{hj} \in x_j \qquad \text{[Firm IC]}$$

$$(10) \qquad \sum_{j=1}^{m} E\left[\pi^b(x_{ij}|x_{-i})\right] \geq 0 \qquad \text{[Bank PC]}$$

Note that constraint (9), the incentive compatible (IC) constraint of firms, is derived from stage 2, that is, condition (2).

Note that firms are identical *ex ante*, and without loss of generality, bank i is assumed to offer the same contract to all firms in stage 1. As already discussed, each firm is assumed to choose only one bank, that is, one loan contract. If more than one bank offers the same contract, we assume, for the sake of simplicity, that the market share of such a contract is the same among those banks offering the same contract.[14]

The loan market is perfectly competitive, implying the following Lemma 1.

Lemma 1. *A bank's expected profit from each contract offer to a firm is always zero in equilibrium: for* $\forall i, j, E\left[\pi^b(x_{ij}|x_{-i})\right] = 0$.

The proof is given in Appendix II Section C. Note that this lemma is about a bank's expected profit before even observing signals $\hat{G}$ and $\hat{B}$, and does not exclude the possibility of cross subsidy *ex post* by a bank between G and B firms.

Bank i's maximization problem can be simplified as below, (11) to (14). Since bank i's expected profit from firm j by contract x_{ij} is non-negative, we can focus on bank i's maximization problem of a single contract offer to firm j like (11), as if it is a representative firm. Moreover, by Lemma 1, we can replace bank's participation constraint (10) by bank's zero profit constraint (ZPC) (14).

$$\max_{x_{ij}\in\mathbf{S}} E\left[\pi^b(x_{ij}|x_{-i})\right] \qquad \text{[Bank profit]} \tag{11}$$

$s.t.$

$$E\left[\pi^f\left(x_{ij}\right)\right] \geq 0 \qquad \text{[Firm PC]} \tag{12}$$

$$E\left[\pi^f\left(x_{ij}\right)\right] \geq E\left[\pi^f\left(\tilde{x}_{hj}\right)\right], \text{ for } \forall h \neq i, \forall \tilde{x}_{hj} \in x_j \qquad \text{[Firm IC]} \tag{13}$$

$$E\left[\pi^b(x_{ij}|x_{-i})\right] = 0 \qquad \text{[Bank ZPC]} \tag{14}$$

Note that because banks get zero profit by Lemma 1, a firm would get positive profits and its PC constraint (12) is not binding, under most cases.[15]

[14] This assumption is not so restrictive, as long as monopoly is not selected in this case (see e.g., Ueda (2013).)

[15] If a firm's expected profit $E\left[\pi^f\left(x_{ij}\right)\right] < 0$, the firm will not enter the loan market in stage 1. Hence, there will be no loan contracts in the market (see details in the earlier working paper).

4 Equilibrium

We have the following proposition, regarding the existence of an equilibrium. For one of the cases below, we use a bit of a new equilibrium concept, that is, the *cheap information equilibrium*. It is similar to the cheap talk equilibrium in the sense that the signal is uninformative. However, unlike the cheap talk equilibrium, signal emission is not decided strategically, but signal extraction is.

Proposition 1. *There are three types of pure strategy Nash equilibria, under the meaningful parameter values:*[16] *(1) the screening separating equilibrium (SS) with the screening cost $C > 0$; (2) the non-screening pooling equilibrium (NP) with the screening cost $C = 0$ and banks offering the same contract regardless of signals; (3) the non-screening, cheap-information-based separating equilibrium (NS) with the screening cost $C = 0$ and banks offering different contracts contingent on uninformative signals.*

Proof. The complete proof is given in Appendix II Section D. Here is a sketch of the proof. When the benefit of screening is bigger than the cost of screening, the screening separating equilibrium emerges. In other words, because of the noisy screening technology, if the cost is larger than the benefit, the non-screening equilibrium emerges. Furthermore, in the non-screening equilibrium, the optimal screening is zero, implying that signals are randomly assigned to firms. Still, banks could offer contracts based on such uninformative signals, and firms receive the same expected profit in stage 2 regardless of signals.[17] Banks are also indifferent in those contracts because they earn zero profits. Here, both the non-screening, cheap-information based separating equilibria (NS) and the non-screening pooling equilibrium (NP) emerge.

Q. E. D.

[16] Under extreme parameter values, firm's expected profits in stage 2 are negative, in other words, the opportunity cost of borrowing is too high, firms will not borrow to produce and all capital will be invested abroad, and thus the capital flight equilibrium emerges.

[17] Under uninformative signals, within the $\hat{G}$ group and the $\hat{B}$ group, the share of G-type firms are the same, that is, f. Hence, the allocation of contract repayment, $R_{\hat{G}}$ and $R_{\hat{B}}$, does not matter in the equilibrium. Without loss of generality, we focus on the NP equilibrium when NP and NS equilibria are indifferent. Note, however, that an exception emerges with some government policies, which we discuss in the earlier working paper.

5 Characterization Of Equilibrium

5.1 Conditions for Various Equilibra

We can characterize the equilibrium given parameter values. Define Δ as the difference of firm's expected profits in stage 2 under optimal contracts of a "lend to $\hat{G}$ only" strategy, $E\left[\pi^f_{SS}\right]$, and of a "lend to both" strategy, $E\left[\pi^f_{NP}\right]$, as below:

$$E\left[\pi^f_{SS}\right] = f\left[G - R - \frac{1}{\alpha f}\ln[\alpha f(1-f)(G-B)]\right] \tag{15}$$

$$E\left[\pi^f_{NP}\right] = [fG + (1-f)B - R] \tag{16}$$

$$\begin{aligned}\Delta &\equiv E\left[\pi^f_{SS}\right] - E\left[\pi^f_{NP}\right]\\ &= f\left[G - R - \frac{1}{\alpha f}\ln[\alpha f(1-f)(G-B)] - \frac{1}{\alpha f}\right] - [fG + (1-f)B - R]\end{aligned} \tag{17}$$

Depending on the values of parameters, the equilibria can be divided into three cases as below. For each case, the detailed description is given in the Appendix I Section A.

Case (1): When $\Delta \geq 0$, $E\left[\pi^f_{SS}\right] \geq 0$, and $\alpha f(1-f)(G-B) > 1$ are all satisfied, there is a screening separating equilibrium (SS), in which the "lend to $\hat{G}$ only" strategy is the equilibrium strategy.[18] Here, $\alpha f(1-f)(G-B) > 1$ means the equilibrium screening cost (18) is positive.

With the equilibrium contract x^*_{ij}, the equilibrium screening cost C^* and the equilibrium repayment of a $\hat{G}$ firm $R^*_{\hat{G}}$ become:

$$C^* = \frac{1}{\alpha}\ln[\alpha f(1-f)(G-B)] > 0, \tag{18}$$

$$R^*_{\hat{G}} = \frac{R + \frac{C^*}{f} - \frac{B}{\alpha f(G-B)}}{1 - \frac{1}{\alpha f(G-B)}}. \tag{19}$$

Case (2): When $\Delta < 0$, $E\left[\pi^f_{NP}\right] \geq 0$, and $\alpha f(1-f)(G-B) > 1$ are all satisfied, or when

[18] In the earlier working paper version, we provides a comprehensive comparative analysis showing the existence of equilibrium contingent to parameter values.

$E\left[\pi^f_{NP}\right] \geq 0$ and $\alpha f(1-f)(G-B) \leq 1$ are both satisfied, there are non-screening equilibria, that is, non-screening pooling equilibrium (NP) and non-screening cheap-information-based separating equilibrium (NS), depending on the repayment allocation between $R^*_{\hat{G}}$ and $R^*_{\hat{B}}$, as below. Note that $fG + (1-f)B \geq R$ is the firm's participation constraint.[19]

Subcase (2.1): There is a non-screening pooling equilibrium (NP), in which the "lend to both with no cross subsidy" strategy is the equilibrium strategy. With the equilibrium contract x^*_{ij}, the equilibrium screening cost C^*, the equilibrium repayment of a firm regardless of the signal R^*_p , become:

$$C^* = 0, \tag{20}$$

$$R^*_p = R^*_{\hat{G}} = R^*_{\hat{B}} = \frac{R-(1-f)B}{f}. \tag{21}$$

Subcase (2.2): There are a set of non-screening, cheap-information-based separating equilibria (NS), in which the "lend to both with no cross subsidy" strategy is the equilibrium strategy. With the equilibrium contract x^*_{ij}, the equilibrium screening cost C^*, and the equilibrium repayments of a $\hat{G}$ firm and a $\hat{B}$ firm $R^*_{\hat{G}}$ and $R^*_{\hat{B}}$, become:

$$C^* = 0, \tag{22}$$

$$\left(R^*_{\hat{G}}, R^*_{\hat{B}}\right) \in \left\{ R^*_{\hat{G}} \in [R, G], R^*_{\hat{B}} \in (B, G] \middle| fR^*_{\hat{G}} + (1-f)R^*_{\hat{B}} = \frac{R-(1-f)B}{f} \right\}. \tag{23}$$

Here, a pair of $R^*_{\hat{G}}$ and $R^*_{\hat{B}}$ can take many possible values, as long as it satisfies the condition above. The detailed description of each stage is given in the Appendix I Section A.

Subcase (2.3): There are a set of non-screening, cheap-information-based separating equilibria, in which the "lend to both with a cross subsidy" strategy is the equilibrium strategy. With the equilibrium contract x^*_{ij}, the equilibrium screening cost C^*, and the equilibrium repayments of a

[19]When $\Delta = 0$ and also $\alpha f(1-f)(G-B) > 1$, both screening and non-screening equilibria exist. Under this case, there could exist a mixed strategy equilibrium. Since we focus on pure strategy equilibrium, we do not go deep into this case.

$\hat{G}$ firm and a $\hat{B}$ firm $R^*_{\hat{G}}$ and $R^*_{\hat{B}}$, become:

$$C^* = 0, \tag{24}$$

$$\left(R^*_{\hat{G}}, R^*_{\hat{B}}\right) \in \left\{ R^*_{\hat{G}} \in [R, G], R^*_{\hat{B}} \in [0, B] \middle| f R^*_{\hat{G}} + \frac{1-f}{f} R^*_{\hat{B}} = \frac{R - (1-f) f B}{f} \right\}. \tag{25}$$

Case (3): When $\Delta \geq 0$, $E\left[\pi^f_{SS}\right] < 0$, and $\alpha f(1-f)(G-B) > 1$ are all satisfied, it is easy to show $E\left[\pi^f_{NP}\right] < E\left[\pi^f_{SS}\right] < 0$, there will be no domestic investment, thus there is a capital flight equilibrium. All capital will be invested abroad with a fixed return R. Similarly, when $\Delta < 0$, $E\left[\pi^f_{NP}\right] < 0$, and $\alpha f(1-f)(G-B) > 1$ are all satisfied, or when $E\left[\pi^f_{NP}\right] < 0$ and $\alpha f(1-f)(G-B) \leq 1$ are both satisfied, there is a capital flight equilibrium.

5.2 GDP and National Income (NI)

We can calculate the GDP and the NI for each of screening and non-screening equilibria. Gross domestic product (GDP) consists of the outputs of firms. National income (NI) consists of the income of banks, firms, and depositors. Hence, when there are international investment flows, GDP may drop while NI may not.[20]

The expected NI and GDP in "SS" and "NP" equilibria can be expressed as below.[21]

$$E(GDP_{SS}) = \left[fG - \frac{1}{\alpha} - \frac{1}{\alpha} ln(\alpha f(1-f)(G-B)) \right] m \tag{26}$$

$$E(NI_{SS}) = \left[fG - \frac{1}{\alpha} + (1-f)R - \frac{1}{\alpha} ln(\alpha f(1-f)(G-B)) \right] m \tag{27}$$

$$E(GDP_{NP}) = [fG + (1-f)B]m \tag{28}$$

$$E(NI_{NP}) = [fG + (1-f)B]m \tag{29}$$

[20] Recall that we assume there are n banks and m firms, and there are total m units of funding in the market, which are owned by depositors. Investors can save money in banks, or invest abroad with a fixed return R. Banks absorb the savings and lend them to firms or invest abroad with a fixed return R. As shown already, in any equilibrium, the depositor's or investor's return of a unit funding is R.

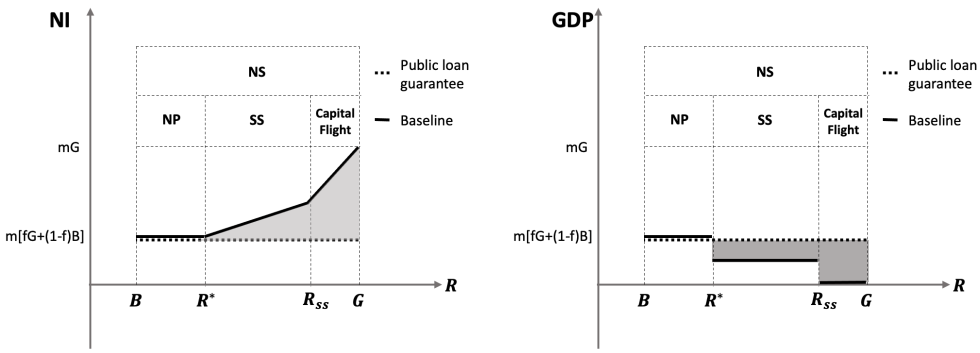
[21] The expected NI and GDP in "NS" equilibrium are same as "NP" equilibrium, thus, omitted.

In the earlier working paper, we show how the equilibrium changes as well as GDP and NI change for different values of five parameters: f, R, B, G, or α, *ceteris paribus*. Figure 2 shows the case that the screening equilibrium exists for some $R \in (B, G)$, in which the solid lines show the levels of NI and GDP of three types of equilibria, depending on the level of R relative to returns G and B.[22] The full description can be found in the Appendix I Section B.

Here is the summary of findings: When the efficiency of screening is low, the economy ends up with a non-screening equilibrium because the screening cost is simply higher than the loss associated with the higher default without screening. When the average project quality is high, likely so in booms, the economy ends up in the non-screening equilibrium because the probability of default is sufficiently low, even with a random selection of borrowers. In addition, a non-screening equilibrium is selected when the international interest rate is low, or when a good firm's output is high, or when a bad firm's output is high (*e.g.*, the cost of default is low, or the recovery upon default is high).

The national income (NI) depends on equilibrium. In the screening equilibrium, the national income becomes larger, (i) when the efficiency of screening increases, (ii) when the average project quality increases, (iii) when the international interest rate increases, (iv) when a good firm's output increases, or (v) when a bad firm's output increases. Similarly, the gross domestic product (GDP) becomes larger under mostly the same conditions. However, the international interest rate does not affect the GDP, because the income from foreign investments is not included in the GDP.

[22] Specifically, the parameters in this case need to satisfy $\alpha f(1-f)(G-B) > 1$, $E\left[\pi^f_{NP}\right] \geq 0$ for $R \in [B, R^*]$, and $E\left[\pi^f_{SS}\right] \geq 0$ for $R \in (R^*, R_{SS}]$. The detailed calculation can be found in the earlier working paper.



(a) Changes in NI

(b) Changes in GDP

Figure 2. Changes in NI and GDP under public loan guarantees

In the non-screening equilibrium, the national income is equal to the GDP. Both become larger under mostly the same conditions as in the screening equilibrium. However, the screening efficiency and the international interest rate do not affect the national income or the GDP in the non-screening equilibrium because no screening and no divergent to foreign investments occur in the non-screening equilibrium.

Obviously, the advancement of bank screening technology would improve the social welfare in the screening equilibrium. The improvement of screening technology reduces the screening cost, motivates banks to use screening, and improves social welfare.

6 Social Planner's Problem

Here, we consider a constrained social planner's problem under imperfect information, as described in the previous section. In other words, neither the social planner nor firms know firms' types until production. The social planner needs to use screening cost C per firm to observe the signal of firm's type. The social planner is benevolent, thus, has zero profit.

Consider the two lending strategies: a screening strategy and a non-screening strategy. The social planner chooses z_j for all firm: $\forall j \in \{1, 2, ..., m\}$. $z_j = (Pr(G|\hat{G}, C_j), L_{\hat{G},j}, L_{\hat{B},j}) \in [f, 1) \times \{0, 1\} \times \{0, 1\}$. Without externality, it is natural to have the following proposition.

Proposition 2. *Both the screening and the non-screening Nash equilibria achieve the constrained social optima.*

The proof is given in Appendix II Section E. The social planner achieves the same allocation as in the decentralized equilibrium, supporting the welfare theorem.

Note that in the case of a social planner's problem under perfect information, firms' types are public information. The social planner will lend to G-type firms only. There are fm amount of G-type firms in the market, so fm amount of funds will fund domestic firms. $(1-f)m$ amount of funds will go to foreign markets with return R per unit of funds. The national income is thus $NI = [fG + (1-f)R]m$.

7 Policy Implications

Apparently, any policy intervention is distortive, because the laissez-faire equilibrium achieves the constrained social optimum. For example, the government subsidies lower banks' screening incentive and cause welfare loss.

We discuss three policies. The first policy is credit easing, which means that the central bank lowers the funding cost of banks, if banks lend domestically. The second policy is a business subsidy, which means that the government gives away a fixed amount of money to firms. The third policy is a public loan guarantee, which means that the government covers the loss of banks if firms default on bank loans.

In principle, these policies bring welfare loss by lowering the funding cost of creditors, and incentivizing creditors to loosen or even stop screening and invest in unviable borrowers. The shaded areas in Figure 2 show the welfare loss of a public loan guarantee. The detailed description of the welfare loss is given in the Appendix I Section B. The results aligns with the empirical findings regarding SMEs' supporting programs introduced or reinforced amid the Covid-19 pandemic, for Japan, as in Hoshi et al. (2022), for Germany, as in Dörr et al. (2021), for Spain, as in Martin et al. (2024), and for Europe wide, as in Bighelli et al. (2022).

Accordingly, our model has implications on the banking sector. Martin et al. (2024) show that credit guarantees granted in Spain during the pandemic have caused welfare loss. Aligning with Martin et al. (2024), our model can show that the introduction of the public credit guarantee will bring welfare loss, since the investors no longer have the incentive to screen. Caballero et al. (2008) consider zombie firms being produced by bank's evergreening lending strategy and show the effects of zombie firms on the economy. Our model can also show how zombie firms are produced by policies, such as public loan guarantees, credit easing, and direct subsidy to business. We show under those policies banks are demotivated to screen loan applicants, in other words, have a non-screening equilibrium and lend to unviable firms, thus, generating zombie firms.

Another closely related paper is by Gorton and Ordonez (2014). In their paper, banks make collateral loans to borrowers. The final value of collateral is random. Banks investigate the final value of collateral but with costs. The information about collateral is independent to project quality and, thus, costly acquiring the information is a waste of resources. They find two equilibria, a separating equilibrium and a pooling equilibrium. Our model also assumes imperfect information *ex ante*. However, different from Gorton and Ordonez (2014), we assume banks screen to acquire information about the project itself, which can improve social welfare, as in the case with Greenwood and Jovanovic (1990) and Townsend and Ueda (2006). In addition, Gorton and Ordonez (2014) assumes the information is always accurate. However, we allow continuous degrees of screening cost and associated signal accuracy.

8 Conclusion

We have proposed a new simple theory that captures a key feature of bank lending, especially towards new projects or existing ones under unprecedented economic shocks, for which theoretical studies have rarely conducted so far. Specifically, we studied banks' optimal screening efforts on lending under symmetrically imperfect information. Unlike the conventional setting of adverse selection with asymmetric information, we focus on a case that neither firms nor banks are aware of firms' types *ex ante*. We believe that this setup is realistic: Many startups or small firms seem optimistic but uncertain about whether they will succeed when they borrow loans. We then allow banks to choose their screening efforts, which we formulate as costly and imperfect signal extraction. The more a bank spends in the screening process, the more accurate a signal of the firm's type a bank can obtain.

Depending on the parameter values, we show that three types of equilibria emerge: the screening separating equilibrium (SS), the non-screening pooling equilibrium (NP), and the non-screening and cheap-information based separating equilibrium (NS). However, the outcomes of the NP and NS equilibria are the same in terms of the ex-ante expected profits for banks and firms. Moreover, we prove that all these equilibira are all socially optimal. Obviously, any policy interventions on venture lending or SME financing are detrimental to the economy.

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Appendix I

A Characterization of Equilibrium: Detailed Description

Case (1): When $\Delta \geq 0$, $\alpha f(1-f)(G-B) > 1$, and $G - R - \frac{1}{\alpha f}\ln[\alpha f(1-f)(G-B)] - \frac{1}{\alpha f} \geq 0$, there is a screening separating equilibrium (SS), in which the "lend to $\hat{G}$ only" strategy is the equilibrium strategy. Here, $\alpha f(1-f)(G-B) > 1$ means the positive screening cost, and $G - R - \frac{1}{\alpha f}\ln[\alpha f(1-f)(G-B)] - \frac{1}{\alpha f} \geq 0$ is the firm's participation constraint. Note that we discuss the case of zero screening cost, that is, $\alpha f(1-f)(G-B) = 1$, in case 2 below.

In the equilibrium contract x^*_{ij}, the equilibrium screening cost C^* and the equilibrium repayment of a $\hat{G}$ firm $R^*_{\hat{G}}$ are as below:

$$C^* = \frac{1}{\alpha}\ln[\alpha f(1-f)(G-B)] > 0,$$
$$R^*_{\hat{G}} = \frac{R + \frac{C^*}{f} - \frac{B}{\alpha f(G-B)}}{1 - \frac{1}{\alpha f(G-B)}}.$$

In stage 1, bank i uses the "lend to $\hat{G}$ only" strategy and offers the contract x^*_{ij}, and bank i's expected profit from firm j before screening is

$$E\left[\pi^b(x^*_{ij})\right] = Pr(\hat{G})[Pr(G|\hat{G},C^*)R^*_{\hat{G}} + Pr(B|\hat{G},C^*)B - R] - C^* = 0.$$

In stage 2, firm j's expected profit before screening is

$$E\left[\pi^f\left(x^*_{ij}\right)\right] = Pr(G)Pr(\hat{G}|G,C^*)(G - R^*_{\hat{G}}) = f\left[G - R - \frac{1}{\alpha f}\ln[\alpha f(1-f)(G-B)] - \frac{1}{\alpha f}\right].$$

In stage 3, bank i's expected profit from firm j after screening is contingent on the signal of firm j,

$$
\begin{aligned}
&E\left[\pi^b(x_{ij}^*)|\hat{G}\right] = Pr(G|\hat{G}, C^*)R_{\hat{G}}^* + Pr(B|\hat{G}, C^*)B - R - C^*/Pr(\hat{G}) = 0. \\
&E\left[\pi^b(x_{ij}^*)|\hat{B}\right] = 0.
\end{aligned}
$$

In stage 4, firm j's expected profit after screening is contingent on signals,

$$
\begin{aligned}
&E\left[\pi^f\left(x_{ij}^*\right)|\hat{G}\right] = Pr(G|\hat{G}, C^*)(G - R_{\hat{G}}^*) = G - R - \frac{1}{\alpha f}\ln[\alpha f(1-f)(G-B)] - \frac{1}{\alpha f}. \\
&E\left[\pi^f\left(x_{ij}^*\right)|\hat{B}\right] = 0.
\end{aligned}
$$

In stage 5, firm j's profit after production is contingent on signals and real types,

$$
\begin{aligned}
&\pi^f\left(x_{ij}^*|\hat{G}, G\right) = G - R_{\hat{G}}^* = G - \frac{R + \frac{C^*}{f} - \frac{B}{\alpha f(G-B)}}{1 - \frac{1}{\alpha f(G-B)}}. \\
&\pi^f\left(x_{ij}^*|\hat{G}, B\right) = 0.
\end{aligned}
$$

In stage 5, bank i's profit from firm j after production is contingent on signals and real types,

$$
\begin{aligned}
&\pi^b\left(x_{ij}^*|\hat{G}, G\right) = R_{\hat{G}}^* - R - \frac{C^*}{f} = \frac{R + \frac{C^*}{f} - \frac{B}{\alpha f(G-B)}}{1 - \frac{1}{\alpha f(G-B)}} - R - \frac{1}{\alpha f^2}\ln[\alpha f(1-f)(G-B)]. \\
&\pi^b\left(x_{ij}^*|\hat{G}, B\right) = B - R - \frac{C^*}{f} = B - R - \frac{1}{\alpha f^2}\ln[\alpha f(1-f)(G-B)]. \\
&\pi^b\left(x_{ij}^*|\hat{B}, B\right) = 0. \\
&\pi^b\left(x_{ij}^*|\hat{B}, G\right) = 0.
\end{aligned}
$$

Case (2): When $\Delta < 0$, $\alpha f(1-f)(G-B) > 1$, and $fG + (1-f)B \geq R$,
or when $\alpha f(1-f)(G-B) \leq 1$ and $fG + (1-f)B \geq R$, there are non-screening equilibria, including non-screening pooling equilibrium (NP) and non-screening cheap-information-based separating equilibrium (NS), depending on the repayment allocation between $R_{\hat{G}}^*$ and $R_{\hat{B}}^*$, as below. Here,

$fG + (1-f)B \geq R$ is the firm's participation constraint.[23]

Subcase (2.1): There is a non-screening pooling equilibrium (NP), in which the "lend to both with no cross subsidy" strategy is the equilibrium strategy. In the equilibrium contract x_{ij}^*, C^* is the equilibrium screening cost, and R_p^* is the equilibrium repayment of a firm, regardless of the signal, as below:

$$
\begin{aligned}
&C^* = 0, \\
&R_p^* = R_{\hat{G}}^* = R_{\hat{B}}^* = \frac{R - (1-f)B}{f}.
\end{aligned}
$$

In stage 1, bank i uses the "lend to both with no cross subsidy" strategy and offers the contract x_{ij}^*, and bank i's expected profit from firm j before getting any signals is

$$
\begin{aligned}
E\left[\pi^b(x_{ij}^*)\right] =& Pr(\hat{G})[Pr(G|\hat{G}, C^*)R_p^* + Pr(B|\hat{G}, C^*)B] + Pr(\hat{B})[Pr(G|\hat{B}, C^*)R_p^* + Pr(B|\hat{B}, C^*)B] \\
&- R - C^* = 0.
\end{aligned}
$$

In stage 2, firm j's expected profit before getting any signals is

$$
E\left[\pi^f(x_{ij}^*)\right] = Pr(G)\left[Pr(\hat{G}|G, C^*)(G - R_p^*) + Pr(\hat{B}|G, C^*)(G - R_p^*)\right] = fG + (1-f)B - R.
$$

In stage 3, bank i's expected profit from firm j after getting any signals is not contingent on signals,

$$
\begin{aligned}
E\left[\pi^b(x_{ij}^*)|\hat{G}\right] &= Pr(G|\hat{G}, C^*)R_p^* + Pr(B|\hat{G}, C^*)B - R - C^* = 0. \\
E\left[\pi^b(x_{ij}^*)|\hat{B}\right] &= Pr(G|\hat{B}, C^*)R_p^* + Pr(B|\hat{B}, C^*)B - R - C^* = 0.
\end{aligned}
$$

[23] When $\Delta = 0$ and also $\alpha f(1-f)(G-B) > 1$, both screening and non-screening equilibria exist. Under this case, there could exist a mixed strategy equilibrium. Since we focus on pure strategy equilibrium, we do not go deep into this case.

In stage 4, firm j's expected profit after getting any signals is not contingent on signals,

$$E\left[\pi^f(x_{ij}^*)|\hat{G}\right] = Pr(G|\hat{G}, C^*)(G - R_p^*) = fG + (1-f)B - R.$$
$$E\left[\pi^f(x_{ij}^*)|\hat{B}\right] = Pr(G|\hat{B}, C^*)(G - R_p^*) = fG + (1-f)B - R.$$

In stage 5, firm j's profit after production is contingent on real types (not on signals),

$$\pi^f\left(x_{ij}^*|\hat{G}, G\right) = \pi^f\left(x_{ij}^*|\hat{B}, G\right) = G - R_p^* = G - \frac{R-(1-f)B}{f}.$$
$$\pi^f\left(x_{ij}^*|\hat{G}, B\right) = \pi^f\left(x_{ij}^*|\hat{B}, B\right) = 0.$$

In stage 5, bank i's profit from firm j after production is contingent on real types (not on signals),

$$\pi^b\left(x_{ij}^*|\hat{G}, G\right) = \pi^b\left(x_{ij}^*|\hat{B}, G\right) = R_p^* - R - C^* = \frac{R-(1-f)B}{f} - R > 0.$$
$$\pi^b\left(x_{ij}^*|\hat{G}, B\right) = \pi^b\left(x_{ij}^*|\hat{B}, B\right) = B - R < 0.$$

Subcase (2.2): There are a set of non-screening, cheap-information-based separating equilibria (NS), in which the "lend to both with no cross subsidy" strategy is the equilibrium strategy. In the equilibrium contract x_{ij}^*, C^* is the equilibrium screening cost, and $R_{\hat{G}}^*$ and $R_{\hat{B}}^*$ are the equilibrium repayments of a $\hat{G}$ firm and a $\hat{B}$ firm, as below:

$$C^* = 0,$$
$$\left(R_{\hat{G}}^*, R_{\hat{B}}^*\right) \in \left\{R_{\hat{G}}^* \in [R, G], R_{\hat{B}}^* \in (B, G] \,\middle|\, fR_{\hat{G}}^* + (1-f)R_{\hat{B}}^* = \frac{R-(1-f)B}{f}\right\}.$$

Here, a pair of $R_{\hat{G}}^*$ and $R_{\hat{B}}^*$ can take many possible values, as long as it satisfies the condition above.

In stage 1, bank i uses the "lend to both with no cross subsidy" strategy and offers the contract

x^*_{ij}, and bank i's expected profit from firm j before separating firms with cheap information is

$$E\left[\pi^b(x^*_{ij})\right] = Pr(\hat{G})[Pr(G|\hat{G},C^*)R^*_{\hat{G}} + Pr(B|\hat{G},C^*)B] + Pr(\hat{B})[Pr(G|\hat{B},C^*)R^*_{\hat{B}} + Pr(B|\hat{B},C^*)B]$$
$$- R - C^* = 0.$$

In stage 2, firm j's expected profit before separating firms with cheap information is

$$E\left[\pi^f(x^*_{ij})\right] = Pr(G)\left[Pr(\hat{G}|G,C^*)(G - R^*_{\hat{G}}) + Pr(\hat{B}|G,C^*)(G - R^*_{\hat{B}})\right] = fG + (1-f)B - R.$$

In stage 3, bank i's expected profit from firm j after separating firms with cheap information is contingent on signals,

$$E\left[\pi^b(x^*_{ij})|\hat{G}\right] = Pr(G|\hat{G},C^*)R^*_{\hat{G}} + Pr(B|\hat{G},C^*)B - R - C^* = fR^*_{\hat{G}} + (1-f)B - R.$$
$$E\left[\pi^b(x^*_{ij})|\hat{B}\right] = Pr(G|\hat{B},C^*)R^*_{\hat{B}} + Pr(B|\hat{B},C^*)B - R - C^* = fR^*_{\hat{B}} + (1-f)B - R.$$

In stage 4, firm j's expected profit after separating firms with cheap information is contingent on signals,

$$E\left[\pi^f(x^*_{ij})|\hat{G}\right] = Pr(G|\hat{G},C^*)(G - R^*_{\hat{G}}) = f(G - R^*_{\hat{G}})$$
$$E\left[\pi^f(x^*_{ij})|\hat{B}\right] = Pr(G|\hat{B},C^*)(G - R^*_{\hat{B}}) = f(G - R^*_{\hat{B}})$$

In stage 5, firm j's profit after production is contingent on signals and real types,

$$\pi^f\left(x^*_{ij}|\hat{G},G\right)=G-R^*_{\hat{G}}.$$
$$\pi^f\left(x^*_{ij}|\hat{B},G\right)=G-R^*_{\hat{B}}.$$
$$\pi^f\left(x^*_{ij}|\hat{G},B\right)=0.$$
$$\pi^f\left(x^*_{ij}|\hat{B},B\right)=0.$$

In stage 5, bank i's profit from firm j after production is contingent on signals and real types,

$$\pi^b\left(x^*_{ij}|\hat{G},G\right)=R^*_{\hat{G}}-R-C^*=R^*_{\hat{G}}-R.$$
$$\pi^b\left(x^*_{ij}|\hat{B},G\right)=R^*_{\hat{B}}-R-C^*=R^*_{\hat{B}}-R.$$
$$\pi^b\left(x^*_{ij}|\hat{G},B\right)=B-R.$$
$$\pi^b\left(x^*_{ij}|\hat{B},B\right)=B-R.$$

Subcase (2.3): There are a set of non-screening, cheap-information-based separating equilibria, in which the "lend to both with a cross subsidy" strategy is the equilibrium strategy. In the equilibrium contract x^*_{ij}, C^* is the equilibrium screening cost, and $R^*_{\hat{G}}$ and $R^*_{\hat{B}}$ are the equilibrium repayments of a $\hat{G}$ firm and a $\hat{B}$ firm, as below:

$$C^*=0,$$
$$\left(R^*_{\hat{G}},R^*_{\hat{B}}\right)\in\left\{R^*_{\hat{G}}\in[R,G],R^*_{\hat{B}}\in[0,B]\,\middle|\,fR^*_{\hat{G}}+\frac{1-f}{f}R^*_{\hat{B}}=\frac{R-(1-f)fB}{f}\right\}.$$

In stage 1, bank i uses the "lend to both with a cross subsidy" strategy and offers the contract x^*_{ij}, and bank i's expected profit from firm j before separating firms with cheap information is

$$E\left[\pi^b(x^*_{ij})\right]=Pr(\hat{G})\left[Pr(G|\hat{G},C^*)R^*_{\hat{G}}+Pr(B|\hat{G},C^*)B\right]+Pr(\hat{B})R^*_{\hat{B}}-R-C^*=0.$$

In stage 2, firm j's expected profit before separating firms with cheap information is

$$
\begin{aligned}
E\left[\pi^f(x^*_{ij})\right] =& Pr(G)\left[Pr(\hat{G}|G,C^*)(G-R^*_{\hat{G}})+Pr(\hat{B}|G,C^*)(G-R^*_{\hat{B}})\right] \\
&+Pr(B)\left[Pr(\hat{B}|B,C^*)(B-R^*_{\hat{B}})\right] \\
=& f[G-fR^*_{\hat{G}}-(1-f)R^*_{\hat{B}}]+(1-f)(1-f)(B-R^*_{\hat{B}}) \\
=& fG+(1-f)^2B-f^2R^*_{\hat{G}}-(1-f)R^*_{\hat{B}} \\
=& fG+(1-f)B-R.
\end{aligned}
$$

In stage 3, bank i's expected profit from firm j after separating firms with cheap information is contingent on signals,

$$
\begin{aligned}
E\left[\pi^b(x^*_{ij})|\hat{G}\right] &= Pr(G|\hat{G},C^*)R^*_{\hat{G}}+Pr(B|\hat{G},C^*)B-R-C^* = fR^*_{\hat{G}}+(1-f)B-R. \\
E\left[\pi^b(x^*_{ij})|\hat{B}\right] &= R^*_{\hat{B}}-R-C^* = R^*_{\hat{B}}-R.
\end{aligned}
$$

In stage 4, firm j's expected profit after separating firms with cheap information is contingent on signals,

$$
\begin{aligned}
E\left[\pi^f(x^*_{ij})|\hat{G}\right] &= Pr(G|\hat{G},C^*)(G-R^*_{\hat{G}}) = f(G-R^*_{\hat{G}}) \\
E\left[\pi^f(x^*_{ij})|\hat{B}\right] &= Pr(G|\hat{B},C^*)(G-R^*_{\hat{B}})+Pr(B|\hat{B},C^*)(B-R^*_{\hat{B}}) = fG+(1-f)B-R^*_{\hat{B}}
\end{aligned}
$$

In stage 5, firm j's profit after production is contingent on signals and real types,

$$
\begin{aligned}
\pi^f\left(x^*_{ij}|\hat{G},G\right) &= G-R^*_{\hat{G}}. \\
\pi^f\left(x^*_{ij}|\hat{B},G\right) &= G-R^*_{\hat{B}}. \\
\pi^f\left(x^*_{ij}|\hat{G},B\right) &= 0. \\
\pi^f\left(x^*_{ij}|\hat{B},B\right) &= B-R^*_{\hat{B}}.
\end{aligned}
$$

In stage 5, bank i's profit from firm j after production is contingent on signals and real types,

$$\pi^b\left(x_{ij}^*|\hat{G},G\right) = R_{\hat{G}}^* - R - C^* = R_{\hat{G}}^* - R.$$
$$\pi^b\left(x_{ij}^*|\hat{B},G\right) = R_{\hat{B}}^* - R - C^* = R_{\hat{B}}^* - R.$$
$$\pi^b\left(x_{ij}^*|\hat{G},B\right) = B - R.$$
$$\pi^b\left(x_{ij}^*|\hat{B},B\right) = R_{\hat{B}}^* - R.$$

B Policy Implications: Welfare Loss

In this section, we discuss three policies. The first policy is credit easing, which means that the central bank lowers the funding cost of banks if banks lend domestically. The second policy is a business subsidy, which means that the government gives away a fixed amount of money to firms. The third policy is a public loan guarantee, which means that the government covers the loss of banks if banks' lending to firms are defaulted.

B.1 Credit Easing

Credit easing was adopted by some central banks, such as the Bank of Japan since 2009 and the Bank of England since 2016.[24] The central bank offers banks a lower funding cost than the international interest rate R, if banks lend to domestic borrowers. Let τ denote as the level of the subsidy and let $\tilde{R}$ denote as domestic interest rate. Bank's zero profit condition means arbitrage free with international interest rate, that is, $\tilde{R} = (1-\tau)R$. Thus, the banks' funding cost is $(1-\tau)R$ per unit of loan, and the central bank provides a subsidy of τR per unit of loan. The cost of credit easing is financed by the lump sum tax (inflation tax) on consumers. With the introduction of credit easing policy, we have the following proposition, regarding the existence of an equilibrium.

[24]For Japan, the Comprehensive Monetary Easing (CME) was introduced in late 2009, in which the Bank of Japan expanded its policy toolkit to include outright purchases of corporate bonds and commercial papers. See Fasano-Filho et al. (2012). For UK, the Corporate Bond Purchase Scheme (CBPS) conducted by the Bank of England was introduced in 2016. See D'Amico and Kaminska (2019).

The complete proof can be found in the earlier working paper.

Proposition 3. *Under the credit easing policy, there are four types of pure strategy Nash Equilibria, depending on the parameter values: (1) the screening separating equilibrium (SS) with the screening cost $C > 0$; (2) the non-screening pooling equilibrium (NP) with the screening cost $C = 0$ and banks offering the same contract regardless of signals; (3) the non-screening, cheap-information-based separating equilibrium (NS) with the screening cost $C = 0$ and banks offering different contracts contingent on uninformative signals; (4) the capital flight equilibrium with all capital invested abroad.*[25]

$$\Delta_\tau \equiv f\left[G-(1-\tau)R-\frac{1}{\alpha f}\ln[\alpha f(1-f)(G-B)]-\frac{1}{\alpha f}\right]-[fG+(1-f)B-(1-\tau)R].$$

Define Δ_τ as the difference in the firm's expected profits in stage 2 between equilibrium contracts of the SS and NP under credit easing policy, as above. The equilibria are described as below.

(i) When $\alpha f(1-f)(G-B) > 1$, $\Delta_\tau \geq 0$, and $G-(1-\tau)R-\frac{1}{\alpha f}\ln[\alpha f(1-f)(G-B)]-\frac{1}{\alpha f} \geq 0$, there is a screening separating equilibrium (SS), in which the "lend to $\hat{G}$ only" strategy is the equilibrium strategy. Following, C^*_τ is the equilibrium screening cost, and $R^*_{\hat{G}\tau}$ is the equilibrium repayment of a $\hat{G}$ firm.

$$C^*_\tau = \frac{1}{\alpha}\ln[\alpha f(1-f)(G-B)].$$
$$R^*_{\hat{G}\tau} = \frac{(1-\tau)R+\frac{C^*}{f}-\frac{B}{\alpha f(G-B)}}{1-\frac{1}{\alpha f(G-B)}}.$$

(ii) When $\alpha f(1-f)(G-B) > 1$, $\Delta_\tau < 0$,[26] and $fG+(1-f)B \geq (1-\tau)R$, or when $\alpha f(1-f)(G-B) \leq$

[25]Since the non-screening, cheap-information-based separating equilibrium (NS) is indifferent from non-screening pooling equilibrium (NP), in terms of firm's and bank's expected profits. Here, the discussion of NS equilibrium is omitted.

[26]When $\alpha f(1-f)(G-B) > 1$, $\Delta_\tau = 0$, both screening and non-screening equilibria exist. Under this case, there could exist a mixed strategy equilibrium. Since we focus on pure strategy equilibrium, we do not go deep into this case.

1 and $fG + (1-f)B \geq (1-\tau)R$, there is a non-screening pooling equilibrium (NP), in which the "lend to both with no cross subsidy" strategy is is the equilibrium strategy.[27] Following, C_τ^* is the equilibrium screening cost, and $R_{p\tau}^*$ is the equilibrium repayment of a firm, regardless of the signal.

$$C_\tau^* = 0.$$
$$R_{p\tau}^* = \frac{(1-\tau)R - (1-f)B}{f}.$$

(iii) Otherwise, there will be no domestic investment, thus there is a capital flight equilibrium. All capital will be invested abroad with a fixed return R.

We omit the full description of firm's and bank's expected profits but compare firm's expected profit in stage 2 in the credit easing case to the baseline (as in Proposition 1). The complete proof is given in the working paper.

In the screening separating equilibrium, the equilibrium screening cost is the same as the baseline. $\hat{G}$ firm's required repayment is smaller than the baseline, but firm's expected profit in stage 2, denoted as $E\left[\pi_{SS\tau}^f(x_{ij}^*)\right]$, is bigger than the baseline by $f\tau R$.

$$E\left[\pi_{SS\tau}^f(x_{ij}^*)\right] = f\left[G - (1-\tau)R - \frac{1}{\alpha f}\ln[\alpha f(1-f)(G-B)] - \frac{1}{\alpha f}\right]$$

In the non-screening pooling equilibrium, firm's required repayment is smaller than the baseline, but firm's expected profit in stage 2, denoted as $E\left[\pi_{NP\tau}^f(x_{ij}^*)\right]$, is bigger than the baseline by τR.

$$E\left[\pi_{NP\tau}^f(x_{ij}^*)\right] = fG + (1-f)B - (1-\tau)R.$$

Next, we calculate the NI and the GDP of the screening equilibrium and the non-screening equilibrium, as below. In the screening equilibrium, NI becomes lower than the baseline level, while GDP remains the same as the baseline level. In the non-screening equilibrium, NI and GDP

[27] Here, we skip the discussion of the non-screening, cheap-information-based separating equilibria, in which everyone is indifferent to the non-screening pooling equilibrium, following the same logic as in the baseline.

remain the same as the baseline levels.

$$E(NI_{SS\tau}) = \left[fG - \frac{1}{\alpha} + (1-f)(1-\tau)R - \frac{1}{\alpha}ln(\alpha f(1-f)(G-B))\right]m \tag{30}$$

$$E(GDP_{SS\tau}) = \left[fG - \frac{1}{\alpha} - \frac{1}{\alpha}ln(\alpha f(1-f)(G-B))\right]m \tag{31}$$

$$E(NI_{NP\tau}) = [fG + (1-f)B]m \tag{32}$$

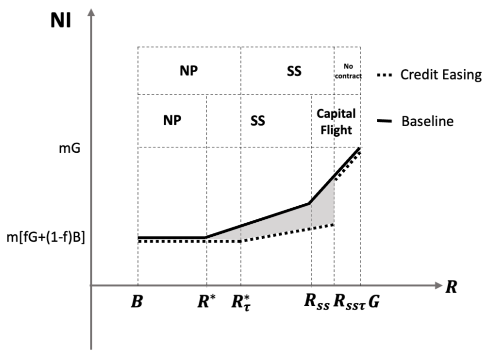
$$E(GDP_{NP\tau}) = [fG + (1-f)B]m \tag{33}$$

Let us look at the effects of changes in R. Credit easing affects the switching thresholds of equilibria. Let R^*_τ denote the interest rate that satisfies $\Delta_\tau(R^*_\tau) = 0$ under credit easing. Compared to the baseline, $(1-\tau)R^*_\tau = R^*$. In addition, let $R_{SS\tau}$ denote the interest rate that satisfies $E\left[\pi^f_{SS\tau}(R_{SS\tau})\right] = 0$, that is, $(1-\tau)R_{SS\tau} = R_{SS}$. Also let $R_{NP\tau}$ denote the interest rate that satisfies $E\left[\pi^f_{NP\tau}(R_{NP\tau})\right] = 0$, that is, $(1-\tau)R_{NP\tau} = R_{NP}$.

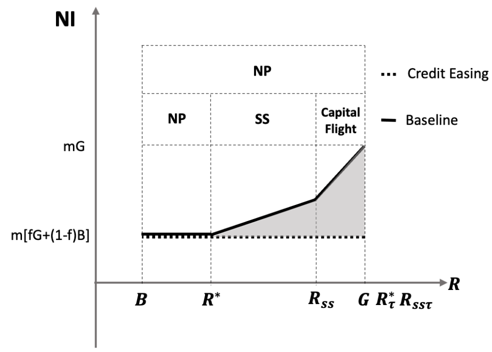
In Figure 3, the upper row shows the case that screening equilibrium exists with $R \in (B, G)$ in the baseline, while the lower row shows the case that $\forall R \in (B, G)$, screening equilibrium does not exist in the baseline.[28] For columns, we discuss two examples of parameter settings. The left panels (a) and (c) show the case that τ is small enough that satisfies $R_{SS\tau} < G$ or $R_{NP\tau} < G$, while the right panels (b) and (d) show the case that τ is big enough that satisfies $R_{SS\tau} \geq G$ or $R_{NP\tau} \geq G$.[29] Figure 4 shows the case of GDP instead of NI.

[28] Regarding the parameterizations for the existence of screening equilibrium, see details in the working paper.

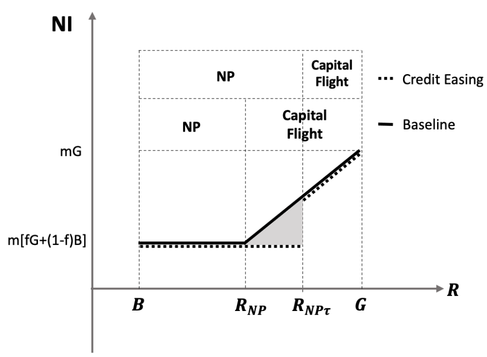
[29] Here, we omit the case $R^*_\tau < G < R_{SS\tau}$, in which $\forall R \in [B, R^*_\tau]$, there is a non-screening equilibrium while $\forall R \in (R^*_\tau, G]$, there is a screening equilibrium. The key message is the same as the two examples in Figure 3.



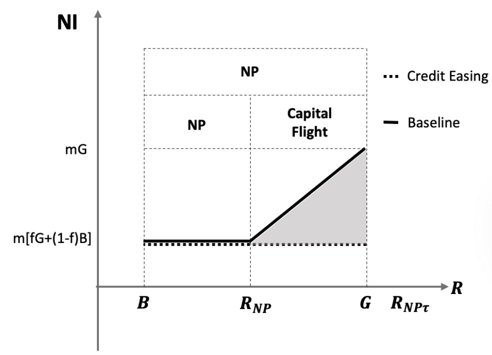
(a) $R_{SS\tau} < G$



(b) $R^*_\tau \geq G$

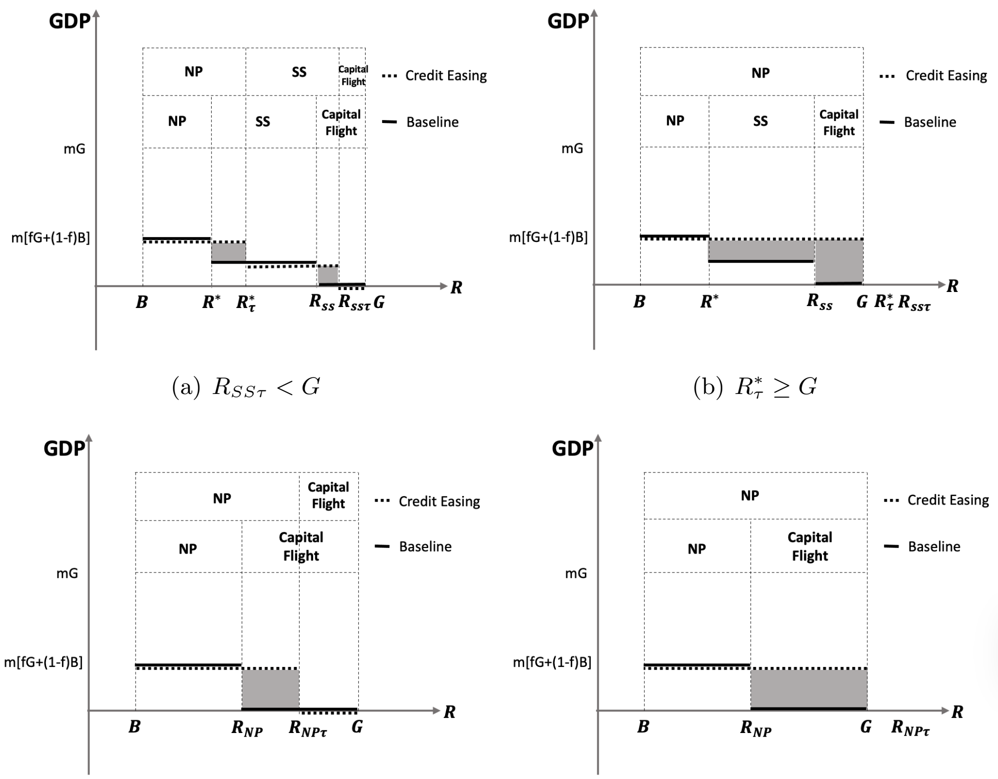


(c) $R_{NP\tau} < G$



(d) $R_{NP\tau} \geq G$

Figure 3. Changes in NI under credit easing



(a) $R_{SS\tau} < G$

(b) $R^*_\tau \geq G$

(c) $R_{NP\tau} < G$

(d) $R_{NP\tau} \geq G$

Figure 4. Changes in GDP under credit easing

The shaded areas are the changes of NI and GDP. With credit easing, NI decreases in screening equilibrium and remains the same in non-screening equilibrium, while GDP remains the same for both screening and non-screening equilibria. However, all thresholds move rightward. With such a change, as seen from the figures, NI decreases for some values of R, while GDP increases for some values of R.

Intuitively, credit easing encourages banks to allocate excess fund to the domestic market rather than investing abroad. Hence, firms are more likely to get funds, but together with a lower funding cost, and some of them are nonviable firms under the international interest rate. As a result, the average asset quality deteriorates and NI decreases compared to the baseline. However, given that more firms get loans to produce, GDP increases under some parameter values compared to the baseline. Still, NI, not GDP, captures social welfare, which are shown to decline with credit easing, compared to the screening equilibrium under the baseline.

B.2 Business Subsidy

During the COVID-19 pandemic, many countries introduced subsidies to business, especially SMEs, (e.g., Blanchard et al. (2020) for Europe and the US case and Hoshi et al. (2022) for Japan's case). We model this policy as government subsidy S to all firms. The subsidy can be considered as a temporary increase of firm's output from G and B to $(G+S)$ and $(B+S)$. For the sake of simplicity, we assume that the size of the increase is exogenous.[30] With the introduction of this business subsidy policy, we have the following proposition, regarding the existence of an equilibrium. The complete proof is given in the working paper.

Proposition 4. *With business subsidy, there are four types of pure strategy Nash Equilibria, depending on the parameter values: (1) the screening separating equilibrium (SS) with the screening cost $C>0$; (2) the non-screening pooling equilibrium (NP) with the screening cost $C=0$ and banks offering the same contract regardless of signals; (3) the non-screening, cheap-information-based separating equilibrium (NS) with the screening cost $C=0$ and banks offering different contracts contingent on uninformative signals; (4) the capital flight equilibrium with all capital invested abroad.*[31]

$$\Delta_s \equiv f\left[G+S-R-\frac{1}{\alpha f}\ln[\alpha f(1-f)(G-B)]-\frac{1}{\alpha f}\right]-[fG+(1-f)B+S-R].$$

Define Δ_τ as the difference in the firm's expected profits in stage 2 between equilibrium contracts

[30] There is another type of subsidy that the government subsidizes a part of required repayment, that is $\phi R_{\hat{G}}, \phi R_{\hat{B}}$. Such subsidy can be considered as a temporary increase of firm's output from G and B to $G+\phi R_{\hat{G}}$ and $B+\phi R_{\hat{G}}$. The calculation of equilibrium is a bit complicated in this case.

[31] Since the non-screening, cheap-information-based separating equilibrium (NS) is indifferent from non-screening pooling equilibrium (NP), in terms of firm's and bank's expected profits. Here, the discussion of NS equilibrium is omitted.

of the SS and NP under business subsidy, as above. The equilibria are described as below.

(i) When $\alpha f(1-f)(G-B) > 1$, $\Delta_s \geq 0$, and $G+S-R-\frac{1}{\alpha f}\ln[\alpha f(1-f)(G-B)]-\frac{1}{\alpha f} \geq 0$, there exists a screening separating equilibrium, in which the "lend to $\hat{G}$ only" strategy is the equilibrium strategy. Following, C_s^* is the equilibrium screening cost, and $R_{\hat{G}s}^*$ is the equilibrium repayment of a $\hat{G}$ firm.

$$C_s^* = \frac{1}{\alpha}\ln[\alpha f(1-f)(G-B)].$$
$$R_{\hat{G}s}^* = \frac{R+\frac{C^*}{f}-\frac{B+S}{\alpha f(G-B)}}{1-\frac{1}{\alpha f(G-B)}}.$$

(ii) When $\alpha f(1-f)(G-B) > 1$, $\Delta_s < 0$, and $fG+(1-f)B+S-R \geq 0$, or when $\alpha f(1-f)(G-B) \leq 1$ and $fG+(1-f)B+S-R \geq 0$, there exists a non-screening pooling equilibrium (NP), in which the "lend to both with no cross subsidy" strategy is the equilibrium strategy. Following, C_s^* is the equilibrium screening cost, and R_{ps}^* is the equilibrium repayment of a firm, regardless of the signal.

$$C_s^* = 0.$$
$$R_{ps}^* = \frac{R-(1-f)(B+S)}{f}.$$

(iii) Otherwise, there will be no domestic investment, thus there is a capital flight equilibrium. All capital will be invested abroad with a fixed return R.

We omit the full description of firm's and bank's expected profits but compare firm's expected profit in stage 2 in the business subsidy case to the baseline (as in Proposition 1). The complete proof is given in the working paper.

In the screening equilibrium, the equilibrium screening cost is the same as the baseline. $\hat{G}$ firm's required repayment is smaller than the baseline, but firm's expected profit in stage 2, denoted as

$E\left[\pi^f_{SSs}(x^*_{ij})\right]$, is bigger than the baseline, due to the subsidy.

$$E\left[\pi^f_{SSs}(x^*_{ij})\right] = f\left[G + S - R - \frac{1}{\alpha f}\ln[\alpha f(1-f)(G-B)] - \frac{1}{\alpha f}\right].$$

In the non-screening equilibrium, firm's required repayment is smaller than the baseline, but firm's expected profit in stage 2, denoted as $E\left[\pi^f_{NPs}(x^*_{ij})\right]$, is bigger than the baseline, due to the subsidy.

$$E\left[\pi^f_{NPs}(x^*_{ij})\right] = fG + (1-f)B + S - R.$$

We can calculate the NI and the GDP of the screening equilibrium and the non-screening equilibrium as below. In both screening and non-screening equilibria, both NI and GDP are the same as in the baseline.

$$E(NI_{SSs}) = \left[fG - \frac{1}{\alpha} + (1-f)R - \frac{1}{\alpha}ln(\alpha f(1-f)(G-B))\right]m \tag{34}$$

$$E(GDP_{SSs}) = \left[fG - \frac{1}{\alpha} - \frac{1}{\alpha}ln(\alpha f(1-f)(G-B))\right]m \tag{35}$$

$$E(NI_{NPs}) = [fG + (1-f)B]m \tag{36}$$

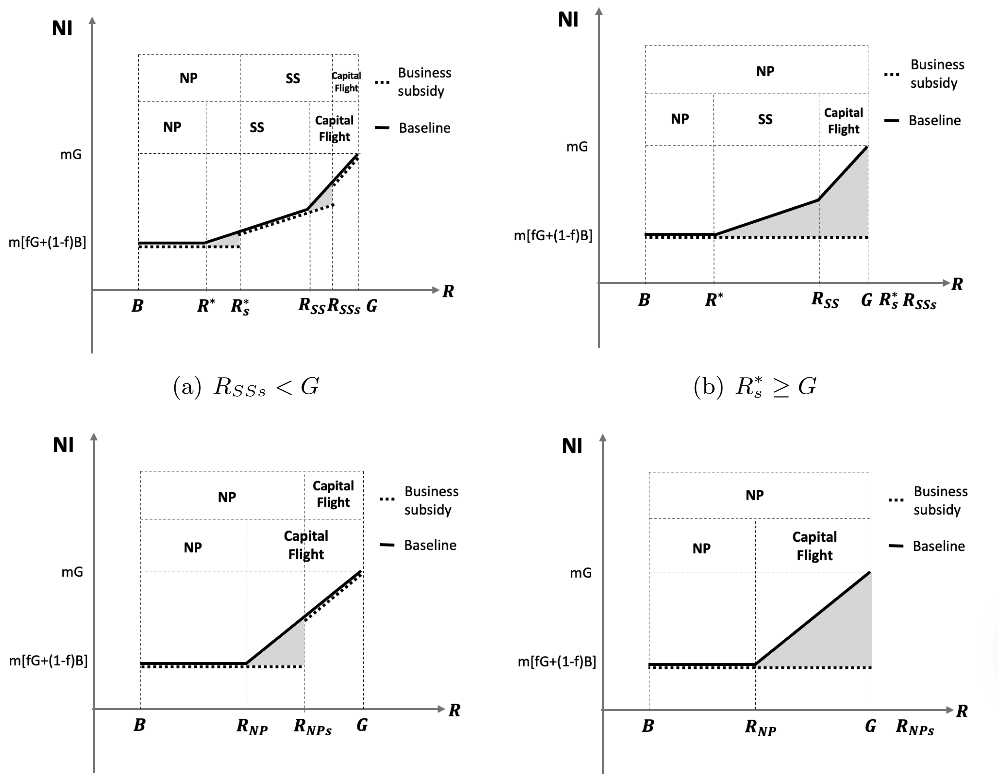
$$E(GDP_{NPs}) = [fG + (1-f)B]m \tag{37}$$

(38)

Let us look at the effects of changes in R. Business subsidies affect the switching thresholds of screening and non-screening equilibria. Let R^*_s define as $\Delta_s(R^*_s) = 0$. Compared to the baseline, $\Delta_s > \Delta$, $R^*_s > R^*$. Let R_{SSs} define as $E\left[\pi^f_{SSs}(R_{SSs})\right] = 0$. We can show that $R_{SSs} > R_{SS}$. Let R_{NPs} define as $E\left[\pi^f_{NPs}(R_{NPs})\right] = 0$. We can show that $R_{NPs} > R_{NP}$.

In figure 5, the upper row shows the case that the screening equilibrium exists with $R \in (B, G)$ in the baseline. The lower row shows the case that screening equilibrium does not exist for any

$R \in (B, G)$ in the baseline. The left column panels (a) and (c) show the case that the subsidy S is small enough that satisfies $R_{SSs} < G$ or $R_{NPs} < G$, while the right column panels (b) and (d) show the case that the subsidy S is big enough that satisfies $R_{SSs} \geq G$ or $R_{NPs} \geq G$.[32] Figure 6 shows the case of GDP instead of NI.



(a) $R_{SSs} < G$

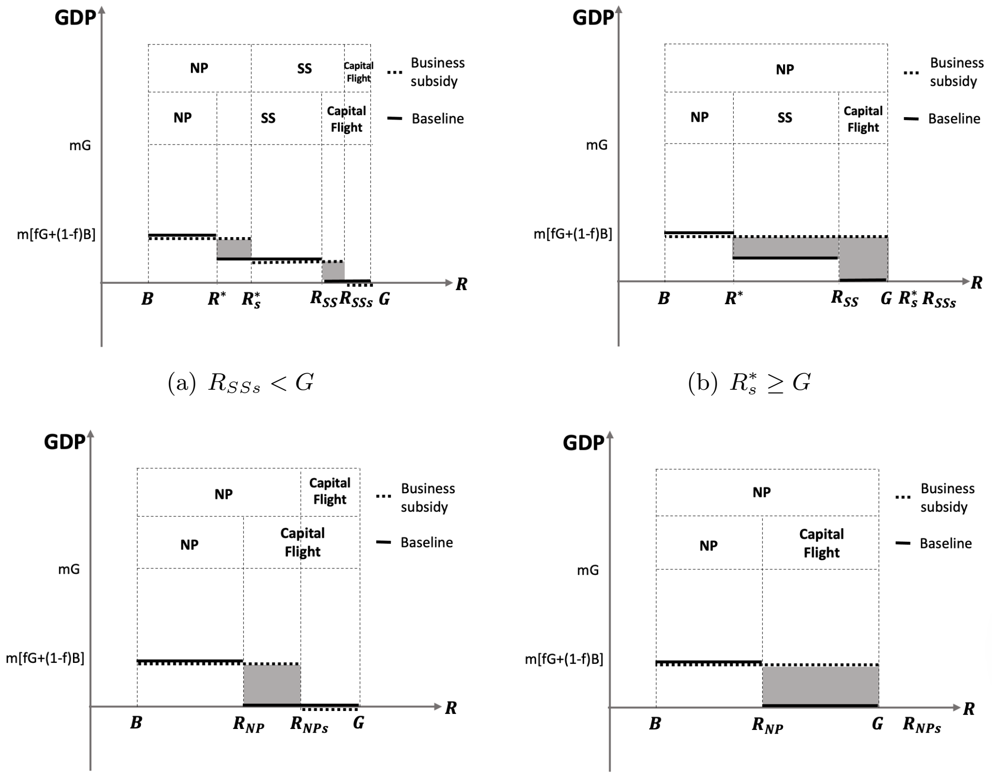
(b) $R_s^* \geq G$

(c) $R_{NPs} < G$

(d) $R_{NPs} \geq G$

Figure 5. Changes in NI under business subsidy

[32]Here, we omit the case $R_s^* < G < R_{SSs}$, in which $\forall R \in [B, R_s^*]$, there is a non-screening equilibrium; $\forall R \in (R_s^*, G]$, there is a screening equilibrium. The key outcomes are the same as the two examples in Figure 5.



(a) $R_{SSs} < G$

(b) $R_s^* \geq G$

(c) $R_{NPs} < G$

(d) $R_{NPs} \geq G$

Figure 6. Changes in GDP under business subsidy

The shaded areas show the changes in expected NI and expected GDP. With direct business subsidy, NI and GDP keep the same levels in screening and non-screening equilibria as in the baseline. However, all thresholds move rightward. As shown in Figure 4, NI decreases for some values of R, while GDP increases for some values of R.

Intuitively, the business subsidy increases firm's output temporarily, which encourages banks to offer excess loans and allocate excess fund to the domestic market. Direct business subsidy reallocates resources between depositors and firms with distortion, and income loss (NI) occurs as banks allocate capital excessively in the domestic market,which includes an increase in overall output (GDP) but reduce overall welfare.

B.3 Public Loan Guarantees

Public loan guarantees have been used by some countries. In particular, those used in Japan are quite generous, often 100 percent in the past decade or so, and strengthened during the COVID-19 pandemic, see Hoshi et al. (2022). We model a simple version of public loan guarantees, as follows. The government provides a complete loan guarantee plan *ex ante* with the required loan rate contingent on the signal, $R_{\hat{G}}$ or $R_{\hat{B}}$. The government expenditure on loan guarantees comes from the lump sum tax, paid by consumers. For a given loan contract, if a borrower repays, the borrower will pay the required loan rate in the contract; if a borrower defaults, the lender will acquire the borrower's output, and the government will pay the rest of the repayment to achieve the full repayment at $R_{\hat{G}}$ or $R_{\hat{B}}$.

Hence, with the introduction of public loan guarantees, we have the following proposition regarding the existence of an equilibrium.

Proposition 5. *Under public loan guarantees, there always exists a non-screening, cheap-information-based separating Nash equilibrium (NS), in which the "lend to both with a cross subsidy" strategy is is the equilibrium strategy.*[33]

The equilibria are described as below: C^* is the equilibrium screening cost, and $R^*_{\hat{G}}$ and $R^*_{\hat{B}}$ are the equilibrium repayments of a $\hat{G}$ firm and a $\hat{B}$ firm.

$$C^* = 0.$$

$$R^*_{\hat{B}} = 0.$$

$$R^*_{\hat{G}} = \frac{R}{f}.$$

We omit the full description of firm's and bank's expected profits but show firm's expected

[33] Under public loan guarantees, non-screening pooling strategy is dominated by non-screening, cheap-information-based separating strategy. Also, screening separating equilibrium vanishes.

profit in stage 2, $E\left[\pi^{f}_{NS}(x^{*}_{ij})\right]$ as below.

$$E\left[\pi^{f}_{NS}(x^{*}_{ij})\right] = \begin{cases} f(G-R)+(1-f)^2B, & \text{if} \quad fG \geq R, \\ (1-f)[fG+(1-f)B], & \text{if} \quad fG < R. \end{cases} \tag{39}$$

Note that if $R^{*}_{\hat{G}} \leq G$, that is, $fG \geq R$, the defaulted firms are B firms with $\hat{G}$ signals. If $fG < R$, the defaulted firms are all G and B firms with $\hat{G}$ signals. In addition, a depositor's expected return are denoted as $E\left(\pi^{d}_{NS}\right)$, as below.

$$E\left(\pi^{d}_{NS}\right) = \begin{cases} fR+f(1-f)B, & \text{if} \quad fG \geq R, \\ f[fG+(1-f)B], & \text{if} \quad fG < R. \end{cases} \tag{40}$$

Here is a sketch of the proof. When public loan guarantees are provided, banks have no loss when lending to bad firms, so banks have no incentive to screen firms nor invest abroad. Here, non-screening euiqlibrium emerges. Furthermore, under public loan guarantees and non-screening strategy, banks can choose $R^{*}_{\hat{G}}$ and $R^{*}_{\hat{B}}$ freely, as long as they satisfy $fR^{*}_{\hat{G}} + (1-f)R^{*}_{\hat{B}} = R$, irrelevant to firms' default rate.[34] Banks then choose them to maximize firm's expected profits in the competitive loan market. Here, firms prefer the "lend to both with a cross subsidy" strategy to the "lend to both without cross subsidy" strategy, because the former strategy has a lower default rate and provides firms with a higher expected profit.[35] This brings a non-screening, cheap-information-based separating Nash equilibrium under public loan guarantees. Note that this optimal loan contract offers a bad firm (with $\hat{B}$ signals) a lower loan rate. The complete proof is given in the working paper.

[34] This is derived from bank's zero profit constraint, because the government will guarantee that banks will receive the contract repayments.

[35] By definition, under the "lend to both without cross subsidy" strategy, B type firms default with 100 percent, while under the "lend to both with a cross subsidy" strategy, B type firms with $\hat{B}$ signals do not default.

NI and GDP are expressed as below,

$$E(NI_{NS}) = [fG + (1-f)B]m \tag{41}$$

$$E(GDP_{NS}) = [fG + (1-f)B]m \tag{42}$$

Let us look at the effects of changes in R. In Figure 7, the left panel implies the case that screening equilibrium exists with some $R \in (B, G)$ in the baseline and the right panel shows the case that screening equilibrium does not exist for any $R \in (B, G)$ in the baseline. On the other hand, with public loan guarantees, there always exists non-screening, cheap-information-based separating equilibrium, for $\forall R \in (B, G)$. Figure 8 shows the case of GDP instead of NI for the same setup of parameter values as in Figure 7.

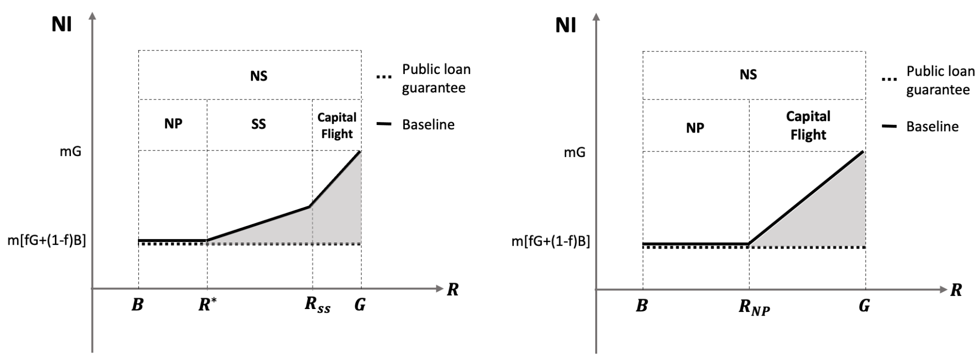


Figure 7. Changes in NI under public loan guarantees

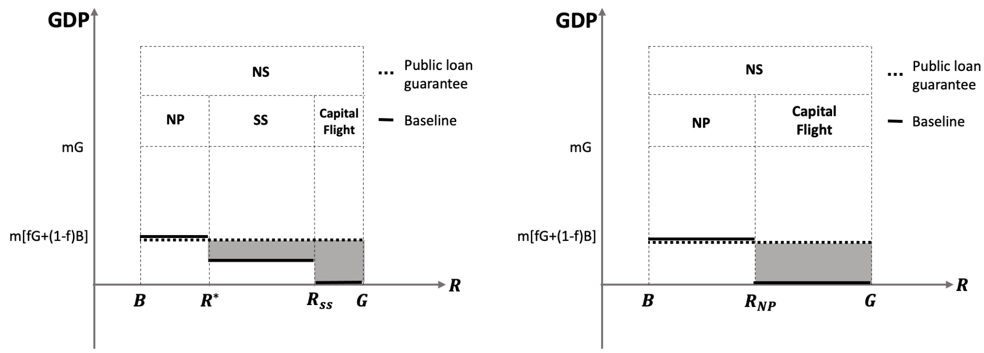


Figure 8. Changes in GDP under public loan guarantees

The shaded areas show the changes in expected NI and expected GDP. Under public loan guarantees, there always exists non-screening, cheap-information-based separating equilibrium (NS). The levels of expected NI and GDP in the NS equilibrium in the public loan guarantee case are the same as in the NP equilibrium in the baseline. The expected NI in NS equilibrium is lower than in the SS equilibrium, while the expected GDP in NS equilibrium is higher than in the SS equilibrium. This is because in the SS equilibrium, banks invest f portion of capital in the domestic market and the rest in the foreign market with return R, while in the NS equilibrium, banks lose the incentive of investing abroad, and invest all capital to domestic borrowers, in other words, more bad firms are financed in the market. Since banks facilitate as many firms as possible to produce, GDP increases under some values of R, for which the screening equilibrium and no contract cases emerge in the baseline.

Here, banks do not make efforts to screen firms but provide loans with low rate to firms with bad signals ($R^*_{\hat{B}} = 0$) so that those firms survive perfectly. This mechanism seems to explain the emergence of zombie firms under public loan guarantees. Intuitively, public loan guarantees generate a moral hazard problem in the sense that, for a sizable range of parameter values, banks might prefer screening *ex ante* in the baseline but stop screening completely after introducing public loan guarantees.

Interestingly, 100 percent public loan guarantees generate a bigger welfare loss than the credit easing in our model. Recall that the credit easing does not eliminate screening equilibrium when the subsidy level τ is small. However, when τ is big, the credit easing is as bad as public loan guarantees, in terms of welfare loss.

Appendix II

C Proof of Lemma 1

First, we prove a bank's expected profit from a firm cannot be positive in equilibrium by contradiction. Suppose firm j chooses bank i, bank i obtains a positive profit from firm j in equilibrium, denoted as $E\left[\pi^b(x_{ij}|x_{-i})\right] = 2\epsilon > 0$, and firm j's expected profit as $E\left[\pi^f\left(x_{ij}\right)\right]$. Then, there exists a profitable deviation by another bank h which offers almost the same contract as bank j, but gives ϵ to firm j. Then, bank h's profit becomes $E[\pi^b(x_{hj}|x_{-i})] = \epsilon$ and firm j's profit becomes $E[\pi^f(x_{hj})] = E\left[\pi^f\left(x_{ij}\right)\right] + \epsilon$ by choosing bank h. Then, bank i loses the client firm j, and $E\left[\pi^b(x_{ij}|x_{-i})\right] = 0$. This contradicts the assumption.

Second, we prove a bank's expected profit from a firm cannot be negative in equilibrium by contradiction. Suppose bank i obtains a negative profit from firm j in equilibrium, $E\left[\pi^b(x_{ij}|x_{-i})\right] < 0$. By bank participation constraint (10), there must be some firm k which gives positive profit to bank i, $E\left[\pi^b(x_{ik}|x_{-i})\right] > 0$. However, bank profit cannot be positive from firm k as shown above, which is a contradiction.

Therefore, it must be the case that $E\left[\pi^b(x_{ik}|x_{-i})\right] = 0$.

D Proof of Proposition 1

Here is the outline of the proof: Depending on lending strategies, a bank's expected profit $E\left[\pi^b(x_{ij}|x_{-i})\right]$ and a firm's expected profit $E\left[\pi^f\left(x_{ij}\right)\right]$ can be simplified. Specific expressions of the constrained profit maximization problem for bank i, (11) - (14), can be described in four cases, consistent with four types of lending strategies, similar to stage 2. Under each case, we specify the function forms of bank's and firm's expected profits, simplify firm's IC constraint, and solve bank's optimal problem. Then, we discuss the existence of the equilibrium lending strategy, depending on specific parameter values.

D.1 Stage 1 Case (1)

Suppose the "lend to $\hat{G}$ only" ($\hat{\mu} = 1$) is the equilibrium lending strategy in stage 1, before observing signals. Bank i offers the "lend to $\hat{G}$ only" strategy, then receives loan applicantions, and screens the applicant firms. If bank i obtains $\hat{G}$ signal for applicant firm j in stage 3, then bank i would provide the loan to firm j. With $\hat{B}$ signal, bank i would not provide the loan to firm j. The bank's optimal problem, with respect to a firm in stage 1, is as below.

(43)

$$\max_{x_{ij} \in \mathbf{S_1}} E\left[\pi^b(x_{ij}|x_{-i})\right] = Pr(\hat{G})\left[Pr(G|\hat{G},C)R_{\hat{G}} + Pr(B|\hat{G},C)B - R - \frac{C}{f}\right] \qquad \text{[Bank profit]}$$

$s.t.$

(44)

$$E\left[\pi^f\left(x_{ij}\right)\right] = Pr(G)Pr(\hat{G}|G,C)(G - R_{\hat{G}}) \geq 0 \qquad \text{[Firm PC]}$$

(45)

$$E\left[\pi^f\left(x_{ij}\right)\right] = Pr(G)Pr(\hat{G}|G,C)(G - R_{\hat{G}}) \geq E\left[\pi^f(\tilde{x}_{hj})\right], \text{ for } \forall h \neq i, \forall \tilde{x}_{hj} \in x_j \qquad \text{[Firm IC]}$$

(46)

$$E\left[\pi^b(x_{ij}|x_{-i})\right] = Pr(\hat{G})\left[Pr(G|\hat{G},C)R_{\hat{G}} + Pr(B|\hat{G},C)B - R - \frac{C}{f}\right] = 0 \qquad \text{[Bank ZPC]}$$

Here, the probability of getting signal $\hat{G}$, $Pr(\hat{G}) = Pr(\hat{G}|C) = f$ is entered in equation (43) and (46) with signal accuracy $Pr(G|\hat{G},C)$, because the bank's expected profit maximizing problem is formulated in stage 1, before observing any signals. On the other hand, for firm's expected profit in (44) and (45), the probability of true type $Pr(G)$ enters with signal correctness $Pr(\hat{G}|G,C)$.

Regarding the screening cost, we assume that the overall screening cost mC is evenly charged to all borrowers, which are $\hat{G}$ firms in this case. All applicant firms are screened, and assigned signals $\hat{G}$ and $\hat{B}$, where the ratios of $\hat{G}$ and $\hat{B}$ firms in the market $Pr(\hat{G}) = f$ and $Pr(\hat{B}) = 1 - f$, which are consistent with the distribution of the true types. Only $\hat{G}$ firms are offered contracts and charged with the screening cost. So, in each contract to a $\hat{G}$ firm, screening cost $C/Pr(\hat{G}) = C/f$ is subtracted.

Subcases of Case (1) IC constraint We further simplify the firm's incentive compatible (IC) constraint (45). The right-hand-side (RHS) represents any possible deviations that firm j takes an offer from bank h other than bank i. Instead of solving the RHS part, that is, $E\left[\pi^f(\tilde{x}_{hj})\right]$, we use an equivalent condition that describes the highest possible expected profit of firms by taking bank h's offer, as (47) below. Obviously bank h is deviating from the equilibrium strategy to lure firm j. The deviant bank h's offer can be chosen from the strategy set $\mathbf{S}$ but it should deliver strictly positive profits for individually rational bank h. We define such a restricted strategy set for the deviant bank h as $\mathbf{X}_j = \{\tilde{x}_{hj} \in \mathbf{S} | E\left[\pi^b(\tilde{x}_{hj}|x_{-h})\right] > 0\}$.

Conditional on the lending strategies of the deviant bank h, the sufficient condition for the firm's IC constraint can be divided into four maximization problems. Here, the restricted strategy set for bank $h \neq i$, $\mathbf{X}_j$ is divided into $\mathbf{X}_{1j}$, $\mathbf{X}_{2j}$, $\mathbf{X}_{3j}$, and $\mathbf{X}_{4j}$, defined as $\mathbf{X}_{1j} = \{\tilde{x}_{hj} \in \mathbf{S_1} | E\left[\pi^b(\tilde{x}_{hj}|x_{-h})\right] > 0\}$, $\mathbf{X}_{2j} = \{\tilde{x}_{hj} \in \mathbf{S_2} | E\left[\pi^b(\tilde{x}_{hj}|x_{-h})\right] > 0\}$, $\mathbf{X}_{3j} = \{\tilde{x}_{hj} \in \mathbf{S_3} | E\left[\pi^b(\tilde{x}_{hj}|x_{-h})\right] > 0\}$, and $\mathbf{X}_{4j} = \{\tilde{x}_{hj} \in \mathbf{S_4} | E\left[\pi^b(\tilde{x}_{hj}|x_{-h})\right] > 0\}$. Recall that $\mathbf{S_1}, \mathbf{S_2}, \mathbf{S_3}$, and $\mathbf{S_4}$ represent the set of bank's loan contract offers under lending strategies "lend to $\hat{G}$ only," "lend to $\hat{B}$ only," "lend to both $\hat{G}$ and $\hat{B}$ with no cross subsidy," and "lend to both $\hat{G}$ and $\hat{B}$ with a cross subsidy." Using these, the right hand side of the IC constraint (45) can be expressed as below.

$$\max\Bigg\{\max_{\tilde{x}_{hj}\in\mathbf{X}_{1j}} Pr(G)Pr(\hat{G}|G,\tilde{C})(G-\tilde{R}_{\hat{G}}),$$

(47)

$$\max_{\tilde{x}_{hj}\in\mathbf{X}_{2j}} Pr(G)Pr(\hat{B}|G,\tilde{C})(G-\tilde{R}_{\hat{B}}),$$

$$\max_{\tilde{x}_{hj}\in\mathbf{X}_{3j}} Pr(G)[Pr(\hat{G}|G,\tilde{C})(G-\tilde{R}_{\hat{G}}) + Pr(\hat{B}|G,\tilde{C})(G-\tilde{R}_{\hat{B}})],$$

$$\max_{\tilde{x}_{hj}\in\mathbf{X}_{4j}} Pr(G)[Pr(\hat{G}|G,\tilde{C})(G-\tilde{R}_{\hat{G}}) + Pr(\hat{B}|G,\tilde{C})(G-\tilde{R}_{\hat{B}})] + Pr(B)[Pr(\hat{B}|B,\tilde{C})(B-\tilde{R}_{\hat{B}})].\Bigg\}$$

To further simplify (47), we calculate the four maximization problems of the deviant bank h under the restricted strategy sets $\mathbf{X}_{1j}$, $\mathbf{X}_{2j}$, $\mathbf{X}_{3j}$, and $\mathbf{X}_{4j}$, and denote as RHS-X1, RHS-X2, RHS-X3, and RHS-X4.

RHS-X1: If the deviant bank h chooses the "lend to $\hat{G}$ only" strategy, firm j's highest possible expected profit by choosing the deviant bank h's offer can be assessed as (48) below.

$$\max_{\tilde{x}_{hj}\in\mathbf{X}_{1j}} E\left[\pi^f(\tilde{x}_{hj})\right] = Pr(G)Pr(\hat{G}|G,\tilde{C})(G-\tilde{R}_{\hat{G}}) \tag{48}$$

Because $\tilde{x}_{hj} \in \mathbf{X}_{1j}$, the loan contract of the deviant bank h should give deviant bank h a strictly positive expected profit.

$$E\left[\pi^b(\tilde{x}_{hj}|x_{-h})\right] = Pr(\hat{G})\left[Pr(G|\hat{G},\tilde{C})\tilde{R}_{\hat{G}} + Pr(B|\hat{G},\tilde{C})B - R - \frac{\tilde{C}}{f}\right] = \epsilon > 0. \tag{49}$$

Here, using $Pr(\hat{G}|G,C) = 1-(1-f)e^{-\alpha C}$, and $Pr(G) = Pr(\hat{G}) = f$, (48) and (49) become

$$E\left[\pi^f(\tilde{x}_{hj})\right] = f\left[1-(1-f)e^{-\alpha\tilde{C}}\right](G-\tilde{R}_{\hat{G}}). \tag{50}$$

$$E\left[\pi^b(\tilde{x}_{hj}|x_{-h})\right] = f\left[\left[1-(1-f)e^{-\alpha\tilde{C}}\right]\tilde{R}_{\hat{G}} + (1-f)e^{-\alpha\tilde{C}}B - R - \frac{\tilde{C}}{f}\right] = \epsilon. \tag{51}$$

By substituting (51) to (50) to eliminate $\tilde{R}_{\hat{G}}$, we obtain

$$E\left[\pi^f(\tilde{x}_{hj})\right] = f\left[G-(1-f)e^{-\alpha\tilde{C}}(G-B) - R - \frac{\tilde{C}}{f} - \frac{\epsilon}{f}\right]. \tag{52}$$

Here, firm j's expected profit becomes a function of bank's screening cost $\tilde{C}$. Then, we can find the loan contract, specifically $\tilde{C}$, by the deviant bank h that offers firm j the highest expected profit. To see it, we take the first derivative of the firm's profit (52) with respect to $\tilde{C}$,

$$\frac{\partial E\left[\pi^f(\tilde{x}_{hj})\right]}{\partial\tilde{C}} = f\left[\alpha(1-f)(G-B)e^{-\alpha\tilde{C}} - \frac{1}{f}\right]. \tag{53}$$

It is easy to calculate that the second order condition is always negative. Because $e^{-\alpha\tilde{C}}|_{C=0} = 1$ and decreases with C, there are two cases regarding the firms' profit.

First, when $\alpha f(1-f)(G-B) > 1$, we have $\frac{\partial E\left[\pi^f(\tilde{x}_{hj})\right]}{\partial \tilde{C}}|_{\tilde{C}=0} > 0$ and $\frac{\partial E\left[\pi^f(\tilde{x}_{hj})\right]}{\partial \tilde{C}}|_{\tilde{C}\to\infty} < 0$, that is, $E\left[\pi^f(\tilde{x}_{hj})\right]$ is an inverted-U shape function of $\tilde{C}$, hitting zero at some point. In this case, there is an internal solution $\tilde{C}^* > 0$, at which $\frac{\partial E\left[\pi^f(\tilde{x}_{hj})\right]}{\partial \tilde{C}} = 0$, that is,

$$e^{-\alpha\tilde{C}^*} = \frac{1}{\alpha f(1-f)(G-B)} \tag{54}$$

Second, when $\alpha f(1-f)(G-B) \leq 1$, it is always the case that $\frac{\partial E\left[\pi^f(\tilde{x}_{hj})\right]}{\partial \tilde{C}}|_{\tilde{C}=0} \leq 0$. In this case, $E\left[\pi^f(\tilde{x}_{hj})\right]$ is a monotonically decreasing function of $\tilde{C}$, so a corner solution $\tilde{C}^* = 0$ should be chosen, given the screening cost is non-negative.

The optimal screening cost is, thus, characterized as below.

$$\tilde{C}_1^* = \begin{cases} \frac{1}{\alpha}\ln[\alpha f(1-f)(G-B)], & \text{if} \quad \alpha f(1-f)(G-B) > 1, \\ 0, & \text{otherwise.} \end{cases} \tag{55}$$

Zero screening cost implies that the bank makes no screening efforts. Let $\tilde{x}_{hj}^{1*} \in \mathbf{X}_{1j}$ denote the bank h's loan contract that uses screening cost $\tilde{C}_1^*$. By substituting (54) and (55) into (52), firm j's highest expected profit from the deviant bank h's offer is as below.

$$E\left[\pi^f(\tilde{x}_{hj}^{1*})\right] = \begin{cases} f\left[G - R - \frac{1}{\alpha f}\ln[\alpha f(1-f)(G-B)] - \frac{1}{\alpha f} - \frac{\epsilon}{f}\right], & \text{if} \quad \alpha f(1-f)(G-B) > 1, \\ f\left[fG + (1-f)B - R - \frac{\epsilon}{f}\right], & \text{otherwise.} \end{cases}$$

Since $E\left[\pi^f(\tilde{x}_{hj}^{1*})\right]$ is a decreasing function of ϵ in either case, firm j's expected profit will be higher if the deviant bank h's expected profit ϵ is smaller. So, the highest expected profit for firm

j would be realized if the deviant bank h takes ϵ to the limit of zero.

(56)

$$\sup E\left[\pi^f(\tilde{x}_{hj}^{1*})\right] =$$
$$\begin{cases} \lim_{\epsilon\to 0} E\left[\pi^f(\tilde{x}_{hj}^{1*})\right] = f\left[G - R - \frac{1}{\alpha f}\ln[\alpha f(1-f)(G-B)] - \frac{1}{\alpha f}\right], & \text{if} \quad \alpha f(1-f)(G-B) > 1, \\ \lim_{\epsilon\to 0} E\left[\pi^f(\tilde{x}_{hj}^{1*})\right] = f[fG + (1-f)B - R], & \text{otherwise.} \end{cases}$$

RHS-X2: If the deviant bank h chooses the "lend to $\hat{B}$ only" strategy, firm j's highest possible expected profit by choosing the deviant bank h's offer can be assessed as (57) below.

$$\max_{\tilde{x}_{hj}\in\mathbf{X}_{2j}} E\left[\pi^f(\tilde{x}_{hj})\right] = Pr(G)Pr(\hat{B}|G,\tilde{C})(G - \tilde{R}_{\hat{B}}) \tag{57}$$

Because $\tilde{x}_{hj} \in \mathbf{X}_{2j}$, the loan contract of the deviant bank h should give deviant bank h a strictly positive expected profit.

$$E\left[\pi^b(\tilde{x}_{hj}|x_{-h})\right] = Pr(\hat{B})\left[Pr(G|\hat{B},\tilde{C})\tilde{R}_{\hat{B}} + Pr(B|\hat{B},\tilde{C})B - R - \frac{\tilde{C}}{1-f}\right] = \epsilon > 0. \tag{58}$$

Here, using $Pr(\hat{B}|G,\tilde{C}) = (1-f)e^{-\alpha\tilde{C}}$, (57) becomes

$$E\left[\pi^f(\tilde{x}_{hj})\right] = f(1-f)e^{-\alpha\tilde{C}}(G - \tilde{R}_{\hat{B}}). \tag{59}$$

Using $Pr(G|\hat{B},\tilde{C}) = fe^{-\alpha\tilde{C}}$, (58) becomes

$$E\left[\pi^b(\tilde{x}_{hj}|x_{-h})\right] = (1-f)\left[fe^{-\alpha\tilde{C}}\tilde{R}_{\hat{B}} + \left(1 - fe^{-\alpha\tilde{C}}\right)B - R - \frac{\tilde{C}}{1-f}\right] = \epsilon. \tag{60}$$

By substituting (60) to (59) to eliminate $\tilde{R}_{\hat{B}}$, we obtain

$$E\left[\pi^f(\tilde{x}_{hj})\right] = f(1-f)e^{-\alpha\tilde{C}}G + (1-f)\left[\left(1 - fe^{-\alpha\tilde{C}}\right)B - R - \frac{\tilde{C}}{1-f} - \frac{\epsilon}{1-f}\right]. \tag{61}$$

Then, we can find the loan contract, the screening cost $\tilde{C}$, by the deviant bank h that offers firm j the highest expected profit. The first derivative is calculated as in (62), which is always negative. Hence, $E\left[\pi^f(\tilde{x}_{hj})\right]$ is a decreasing function of $\tilde{C}$, so the corner solution $\tilde{C}_2^* = 0$ should be chosen.

$$\frac{\partial E\left[\pi^f(\tilde{x}_{hj})\right]}{\partial \tilde{C}} = -\alpha f(1-f)(G-B)e^{-\alpha \tilde{C}} - 1 < 0 \tag{62}$$

$$\tilde{C}_2^* = 0. \tag{63}$$

Let $\tilde{x}_{hj}^{2*} \in \mathbf{X}_{2j}$ denote the bank h's loan contract which uses screening cost $\tilde{C}_2^* = 0$. By substituting (63) into (61), firm j's highest expected profit from the deviant bank h's offer is as below.

$$E\left[\pi^f\left(\tilde{x}_{hj}^{2*}\right)\right] = (1-f)\left[fG + (1-f)B - R - \frac{\epsilon}{1-f}\right].$$

Since $E\left[\pi^f(\tilde{x}_{hj}^{2*})\right]$ is a decreasing function of ϵ, firm j's expected profit will be higher if the deviant bank h's expected profit ϵ is smaller. So, the highest expected profit for firm j would be realized if the deviant bank h takes ϵ to the limit of zero.

$$\sup E\left[\pi^f\left(\tilde{x}_{hj}^{2*}\right)\right] = \lim_{\epsilon \to 0} E\left[\pi^f(\tilde{x}_{hj}^{2*})\right] = (1-f)[fG + (1-f)B - R].$$

RHS-X3: If the deviant bank h chooses the "lend to both with no cross subsidy" strategy, firm j's highest possible expected profit by choosing the deviant bank h's offer can be assessed by (64) below.

$$\max_{\tilde{x}_{hj} \in \mathbf{X}_{3j}} E\left[\pi^f(\tilde{x}_{hj})\right] = Pr(G)\left[Pr(\hat{G}|G,\tilde{C})(G - \tilde{R}_{\hat{G}}) + Pr(\hat{B}|G,\tilde{C})(G - \tilde{R}_{\hat{B}})\right] \tag{64}$$

Because $\tilde{x}_{hj} \in \mathbf{X}_{3j}$, the loan contract of the deviant bank h should give deviant bank h a strictly positive expected profit.

$$E\left[\pi^b(\tilde{x}_{hj}|x_{-h})\right] = Pr(\hat{G})\left[Pr(G|\hat{G},\tilde{C})\tilde{R}_{\hat{G}} + Pr(B|\hat{G},\tilde{C})B\right] + \tag{65}$$
$$Pr(\hat{B})\left[Pr(G|\hat{B},\tilde{C})\tilde{R}_{\hat{B}} + Pr(B|\hat{B},\tilde{C})B\right] - R - \tilde{C} = \epsilon > 0.$$

Here, using $Pr(\hat{G}|G,C) = Pr(G|\hat{G},C) = 1-(1-f)e^{-\alpha C}$ and $Pr(\hat{B}|B,C) = Pr(B|\hat{B},C) = 1 - fe^{-\alpha C}$, (64) and (65) become as below.

$$E\left[\pi^f(\tilde{x}_{hj})\right] = f\left[\left(1-(1-f)e^{-\alpha\tilde{C}}\right)(G-\tilde{R}_{\hat{G}}) + (1-f)e^{-\alpha\tilde{C}}(G-\tilde{R}_{\hat{B}})\right]. \tag{66}$$

$$E\left[\pi^b(\tilde{x}_{hj}|x_{-h})\right] = f\left[\left(1-(1-f)e^{-\alpha\tilde{C}}\right)\tilde{R}_{\hat{G}} + (1-f)e^{-\alpha\tilde{C}}B\right] + \tag{67}$$
$$(1-f)\left[fe^{-\alpha\tilde{C}}\tilde{R}_{\hat{B}} + \left(1-fe^{-\alpha\tilde{C}}\right)B\right] - R - \tilde{C} = \epsilon.$$

By substituting (67) to (66) to eliminate $\tilde{R}_{\hat{G}}$ and $\tilde{R}_{\hat{B}}$, we obtain

$$\begin{aligned} E\left[\pi^f(\tilde{x}_{hj})\right] &= f\left[G - \left(1-(1-f)e^{-\alpha\tilde{C}}\right)\tilde{R}_{\hat{G}} - (1-f)e^{-\alpha\tilde{C}}\tilde{R}_{\hat{B}}\right] \\ &= fG + f(1-f)e^{-\alpha\tilde{C}}B + (1-f)\left(1-fe^{-\alpha\tilde{C}}\right)B - R - \tilde{C} - \epsilon \\ &= fG + (1-f)B - R - \tilde{C} - \epsilon. \end{aligned} \tag{68}$$

Then, we can find the loan contract, specifically the screening cost $\tilde{C}$, by the deviant bank h that offers firm j the highest expected profit. The first derivative is calculated as in (69), which is always negative. $E\left[\pi^f(\tilde{x}_{hj})\right]$ is a decreasing function of $\tilde{C}$, so the corner solution $\tilde{C}_3^* = 0$ should be chosen.

$$\frac{\partial E\left[\pi^f(\tilde{x}_{hj})\right]}{\partial\tilde{C}} = -1 < 0. \tag{69}$$

$$\tilde{C}_3^* = 0. \tag{70}$$

Let $\tilde{x}_{hj}^{3*} \in \mathbf{X}_{3j}$ denote the bank h's loan contract which uses screening cost $\tilde{C}_3^* = 0$. By substituting

(70) into (68), firm j's highest expected profit from the deviant bank h's offer is as below.

$$E\left[\pi^f\left(\tilde{x}_{hj}^{3*}\right)\right] = fG + (1-f)B - R - \epsilon.$$

Since $E\left[\pi^f(\tilde{x}_{hj}^{3*})\right]$ is a decreasing function of ϵ, firm j's expected profit will be higher if the deviant bank h's expected profit ϵ is smaller. So, the highest expected profit for firm j would be realized if the deviant bank h takes ϵ to the limit of zero.

$$\sup E\left[\pi^f\left(\tilde{x}_{hj}^{3*}\right)\right] = \lim_{\epsilon\to 0} E\left[\pi^f(\tilde{x}_{hj}^{3*})\right] = fG + (1-f)B - R.$$

.

RHS-X4: If the deviant bank h chooses the "lend to both with a cross subsidy" strategy, firm j's highest possible expected profit by choosing the deviant bank h's offer can be assessed by (71) below.

$$\max_{\tilde{x}_{hj}\in\mathbf{X}_{4j}} E\left[\pi^f(\tilde{x}_{hj})\right] = Pr(G)\left[Pr(\hat{G}|G,\tilde{C})(G-\tilde{R}_{\hat{G}}) + Pr(\hat{B}|G,\tilde{C})(G-\tilde{R}_{\hat{B}})\right] + Pr(B)\left[Pr(\hat{B}|B,\tilde{C})(B-\tilde{R}_{\hat{B}})\right] \tag{71}$$

Note that unlike (64) in case (IC-3), cross-subsidy implies $R_B < B$, so that a firm can repay in full R_B even with the bad realization of the firm's type. Because $\tilde{x}_{hj}\in\mathbf{X}_{4j}$, the loan contract of the deviant bank h should give deviant bank h a strictly positive expected profit.

$$E\left[\pi^b(\tilde{x}_{hj}|x_{-h})\right] = Pr(\hat{G})[Pr(G|\hat{G},\tilde{C})\tilde{R}_{\hat{G}} + Pr(B|\hat{G},\tilde{C})B] + \Pr(\hat{B})\tilde{R}_{\hat{B}} - R - \tilde{C} = \epsilon > 0. \tag{72}$$

Here, using $Pr(\hat{G}|G,C) = Pr(G|\hat{G},C) = 1-(1-f)e^{-\alpha C}$ and $Pr(\hat{B}|B,C) = Pr(B|\hat{B},C) =$

$1 - fe^{-\alpha C}$, (71) and (72) become as below.

$$E\left[\pi^f(\tilde{x}_{hj})\right] = f\left[\left(1-(1-f)e^{-\alpha C}\right)(G - \tilde{R}_{\hat{G}}) + (1-f)e^{-\alpha C}(G - \tilde{R}_{\hat{B}})\right]$$

$$+ (1-f)\left[\left(1 - fe^{-\alpha C}\right)(B - \tilde{R}_{\hat{B}})\right]. \tag{73}$$

$$E\left[\pi^b(\tilde{x}_{hj}|x_{-h})\right] = f\left[\left(1-(1-f)e^{-\alpha C}\right)\tilde{R}_{\hat{G}} + (1-f)e^{-\alpha C}B\right] + (1-f)\tilde{R}_{\hat{B}} - R - \tilde{C} = \epsilon. \tag{74}$$

By substituting (74) to (73), to eliminate $\tilde{R}_{\hat{G}}$ and $\tilde{R}_{\hat{B}}$, we obtain

$$\begin{aligned} E\left[\pi^f(\tilde{x}_{hj})\right] =& fG + (1-f)\left(1 - fe^{-\alpha C}\right)B - f\left[\left(1-(1-f)e^{-\alpha C}\right)\tilde{R}_{\hat{G}} + (1-f)e^{-\alpha C}\tilde{R}_{\hat{B}}\right] \\ & - (1-f)\left(1 - fe^{-\alpha C}\right)\tilde{R}_{\hat{B}} \\ =& fG + (1-f)\left(1 - fe^{-\alpha C}\right)B - f\left(1-(1-f)e^{-\alpha C}\right)\tilde{R}_{\hat{G}} - (1-f)\tilde{R}_{\hat{B}} \\ =& fG + (1-f)B - R - \tilde{C} - \epsilon. \end{aligned} \tag{75}$$

Then, we can find the loan contract, specifically the screening cost $\tilde{C}$, by the deviant bank h that offers firm j the highest expected profit. The first derivative is calculated as in (76), which is always negative. $E\left[\pi^f(\tilde{x}_{hj})\right]$ is a decreasing function of $\tilde{C}$, so the corner solution $\tilde{C}_4^* = 0$ should be chosen.

$$\frac{\partial E\left[\pi^f(\tilde{x}_{hj})\right]}{\partial \tilde{C}} = -1 < 0. \tag{76}$$

$$\tilde{C}_4^* = 0. \tag{77}$$

Let $\tilde{x}_{hj}^{4*} \in \mathbf{X}_{4j}$ denote the bank h's loan contract which uses screening cost $\tilde{C}_4^* = 0$. By substituting (77) into (75), firm j's highest expected profit from the deviant bank h's offer is as below.

$$E\left[\pi^f\left(\tilde{x}_{hj}^{4*}\right)\right] = fG + (1-f)B - R - \epsilon.$$

Since $E\left[\pi^f(\tilde{x}_{hj}^{4*})\right]$ is a decreasing function of ϵ, firm j's expected profit will be higher if the deviant bank h's expected profit ϵ is smaller. So, the highest expected profit for firm j would be realized if the deviant bank h takes ϵ to the limit of zero.

$$\sup E\left[\pi^f\left(\tilde{x}_{hj}^{4*}\right)\right] = \lim_{\epsilon\to 0} E\left[\pi^f(\tilde{x}_{hj}^{4*})\right] = fG + (1-f)B - R.$$

Next, by summarizing RHS-X1, RHS-X2, RHS-X3, and RHS-X4, the right hand side of the incentive compatible constraint (47) can be expressed as below.

$$\begin{aligned}
\max\Bigg\{&\left\{ f\left[G - R - \frac{1}{\alpha f}\ln[\alpha f(1-f)(G-B)] - \frac{1}{\alpha f}\right]\Bigg|_{\alpha f(1-f)(G-B)>1},\right.\\
&\left. f[fG+(1-f)B-R]\Bigg|_{\alpha f(1-f)(G-B)\leq 1}\right\},\\
&(1-f)[fG+(1-f)B-R],\\
&fG+(1-f)B-R.\Bigg\}
\end{aligned} \tag{78}$$

Here, the first two lines represent RHS-X1, the third line represents RHS-X2. The fourth line represents RHS-X3 and RHS-X4. The third line is smaller than the fourth line, since the third line is equal to the fourth line multiplied by $1-f<1$. Therefore, we will not consider RHS-X2. Then, we can simplify (78) as below.

$$\begin{aligned}
\max\Bigg\{&\left\{ f\left[G - R - \frac{1}{\alpha f}\ln[\alpha f(1-f)(G-B)] - \frac{1}{\alpha f}\right]\Bigg|_{\alpha f(1-f)(G-B)>1},\right.\\
&\left. f[fG+(1-f)B-R]\Bigg|_{\alpha f(1-f)(G-B)\leq 1}\right\},\\
&fG+(1-f)B-R.\Bigg\}
\end{aligned} \tag{79}$$

When $\alpha f(1-f)(G-B)>1$, define Δ as the first line RHS-X1 minus the third line RHS-X3 (or RHS-X4) in (79), as below.

$$\Delta \equiv f\left[G - R - \frac{1}{\alpha f}\ln[\alpha f(1-f)(G-B)] - \frac{1}{\alpha f}\right] - [fG + (1-f)B - R]. \tag{80}$$

Here, if $\Delta > 0$, then the "lend to $\hat{G}$ only" strategy should be chosen by a deviant bank over the "lend to both" strategy. Firm j's IC constraint (79), originally (45), in which "lend to $\hat{G}$ only" is the equilibrium strategy, can be further classified into four subcases as below.

Subcase (1-a): IC constraint: When $\alpha f(1-f)(G-B) > 1$, and $\Delta > 0$, the "lend to $\hat{G}$ only" contract provides firm j the highest expected profit compared to contracts under other lending strategies. Here, while bank i chooses the "lend to $\hat{G}$ only" strategy, the deviant bank h may also choose the "lend to $\hat{G}$ only" strategy, and the firm j's IC constraint (45) becomes

$$Pr(G)Pr(\hat{G}|G,C)(G - R_{\hat{G}}) \geq f\left[G - R - \frac{1}{\alpha f}\ln[\alpha f(1-f)(G-B)] - \frac{1}{\alpha f}\right]. \tag{81a}$$

which is

$$f f_G(C)(G - R_{\hat{G}}) \geq f\left[G - R - \frac{1}{\alpha f}\ln[\alpha f(1-f)(G-B)] - \frac{1}{\alpha f}\right]. \tag{81b}$$

Subcase (1-b): IC constraint: When $\alpha f(1-f)(G-B) > 1$, and $\Delta < 0$, the "lend to both" contract provides firm j the highest expected profit compared to contracts under other lending strategies. "Lend to both" strategies include case (IC-3) "lend to both with no cross subsidy" and case (IC-4) "lend to both with a cross subsidy." Here, while the bank i chooses the "lend to $\hat{G}$ only" strategy, the deviant bank h may choose the "lend to both" strategy, and the firm j's IC constraint (45) becomes

$$Pr(G)Pr(\hat{G}|G,C)(G - R_{\hat{G}}) \geq fG + (1-f)B - R. \tag{82}$$

Subcase (1-c): IC constraint: When $\alpha f(1-f)(G-B) > 1$, and $\Delta = 0$, the "lend to $\hat{G}$ only" contract and the "lend to both" contract can provide the same highest firm's expected profit. Under this case, there could appear a mixed strategy equilibrium.[36] We do not go deep into this case, and we assume all banks choose the same pure strategy. Without loss of generality, case (1-c) is combined with case (1-a). Hereinafter, we do not consider this case separately.

Subcase (1-d): IC constraint: When $\alpha(1-f)(G-B) \leq 1$, the "lend to both" contract provides firm j the highest expected profit compared to contracts under other lending strategies, because in (79) the second line is f multiplying the third line, and f is smaller than 1. Here, while the bank i chooses the "lend to $\hat{G}$ only" strategy, the deviant bank h may choose the "lend to both" strategy, and the firm j's IC constraint (45) becomes

$$Pr(G)Pr(\hat{G}|G,C)(G - R_{\hat{G}}) \geq fG + (1-f)B - R. \tag{83}$$

Next, we incorporate subcases of IC constraint, (1-a), (1-b), and (1-d) into the bank's optimal problem.

Subcases of Case (1) Bank's Problem

Subcase (1-a) Bank's Problem In subcase (1-a), $\alpha f(1-f)(G-B) > 1$ and $\Delta \geq 0$, the deviant bank h adopts the "lend to $\hat{G}$ only" strategy. Firm's IC constraint becomes (81b). Bank

[36] This mixed strategy equilibrium can be achieved by asymmetric pure strategies or by lottery by each bank.

i's optimal problem is expressed as below.

$$\max_{x_{ij} \in \mathbf{S_1}} f \left[f_G(C) R_{\hat{G}} + (1 - f_G(C))B - R - \frac{C}{f} \right] \qquad \text{[Bank profit]}$$

$s.t.$

$$f_G(C)(G - R_{\hat{G}}) \geq 0 \qquad \text{[Firm IR]}$$

$$f_G(C)(G - R_{\hat{G}}) \geq G - R - \frac{1}{\alpha f} \ln[\alpha f(1-f)(G-B)] - \frac{1}{\alpha f} \qquad \text{[Firm IC]}$$

$$f \left[f_G(C) R_{\hat{G}} + (1 - f_G(C))B - R - \frac{C}{f} \right] = 0 \qquad \text{[Bank ZPC]}$$

Note that if the RHS of the firm's IC constraint is less than zero, only the firm's IR constraint binds, and vice versa. Here, we analyze the case with the firm's IC constraint, that is, the RHS of the firm's IC constraint is positive without firm's IR constraint.

By banks' zero-profit constraint,

$$R_{\hat{G}} = \frac{R + C/f - (1 - f_G(C))B}{f_G(C)} \equiv k(C).$$

We substitute $R_{\hat{G}}$ to the firm's IC constraint as below.

$$f_G(C)(G - k(C)) \geq G - R - \frac{1}{\alpha f} ln(\alpha f(1-f)(G-B)) - \frac{1}{\alpha f}.$$

Then, we form the Lagrangian as below.

$$\begin{aligned} \mathcal{L}(C) =& f \left[f_G(C)k(C) + (1 - f_G(C))B - R - \frac{C}{f} \right] + \lambda \Big\{ f_G(C)(G - k(C)) - \Big[G - R - \\ & \frac{1}{\alpha f} ln(\alpha f(1-f)(G-B)) - \frac{1}{\alpha f} \Big] \Big\} \end{aligned} \tag{84}$$

A bank maximizes this Lagrangian by choosing $C \geq 0$. The first order condition is written as below.

(85)

$$0 = \frac{\partial \mathcal{L}}{\partial C} = f\left[f'_G(C)k(C) + f_G(C)k'(C) - f'_G(C)B - \frac{1}{f}\right] + \lambda[f[f'_G(C)(G - k(C)) - f_G(C)k'(C)]].$$

In the Lagrangian (84), the firm's IC constraint should be binding, and $\lambda > 0$. Recall that in stage 2, firm j's maximization problem (1) is derived from the first order condition with respect to C to maximize firm j's expected profit by (deviant) bank as (53), that is,

$$\frac{\partial E\left[\pi^f(x_{ij})\right]}{\partial C} = f[f'_G(C)(G - k(C)) - f_G(C)k'(C)] = 0. \tag{86}$$

By (86), we have $f'_G(C)G = f'_G(C)k(C) + f_G(C)k'(C)$. By substituting (86) to (85), the first order condition becomes

$$\begin{aligned} 0 = \frac{\partial \mathcal{L}}{\partial C} &= f\left[f'_G(C)k(C) + f_G(C)k'(C) - f'_G(C)B - \frac{1}{f}\right]. \\ 0 &= f\left[f'_G(C)(G - B) - \frac{1}{f}\right]. \end{aligned} \tag{87}$$

Here, $f'_G(C) = \alpha(1-f)e^{-\alpha C} > 0$, $f'_G(C)|_{C=0} = \alpha(1-f)$ and the second derivative $f''_G(C) < 0$ is negative. We can calculate $\frac{\partial \mathcal{L}}{\partial C}|_{C=0} = \alpha(1-f)(G-B) - \frac{1}{f} > 0$, $\frac{\partial^2 \mathcal{L}}{\partial C^2} < 0$. Hence, the Lagrangian (84) is an increasing and concave function of C, and there is an internal solution of C^*.

In summary, when $\alpha f(1-f)(G-B) > 1$, $\Delta \geq 0$, and $G - R - \frac{1}{\alpha f}\ln[\alpha f(1-f)(G-B)] - \frac{1}{\alpha f} \geq 0$,[37] the "lend to $\hat{G}$ only" strategy by banks is an equilibrium strategy. The equilibrium loan contract uses the "lend to $\hat{G}$ only" lending strategy, and the screening strategy and the required repayment

[37] This condition is from firm's PC constraint. $E\left[\pi^f\left(x_{ij}\right)\right]$ represents firm's expected profit before screening in stage 2, which is non-negative by firm's participation constraint. Otherwise, firms will not enter the market in the first place. $E\left[\pi^f\left(x^*_{ij}\right)\right] = Pr(G)Pr(\hat{G}|G, C)(G - R_{\hat{G}}) = f\left[G - R - \frac{1}{\alpha f}\ln[\alpha f(1-f)(G-B)] - \frac{1}{\alpha f}\right] \geq 0$.

is characterized in the loan contract x^*_{ij} , where

$$C^* = \frac{1}{\alpha}\ln[\alpha f(1-f)(G-B)].$$
$$R^*_{\hat{G}} = \frac{R + \frac{C^*}{f} - \frac{B}{\alpha f(G-B)}}{1 - \frac{1}{\alpha f(G-B)}}.$$

Both are derived from (87). $R^*_{\hat{B}}$ can be arbitrary because banks do not lend to $\hat{B}$ firms.

In summary, for subcase (1-a), we have the following result.

Lemma 2. *When $\alpha f(1-f)(G-B) > 1$, $\Delta \geq 0$, and $G - R - \frac{1}{\alpha f}\ln[\alpha f(1-f)(G-B)] - \frac{1}{\alpha f} \geq 0$, the "lend to $\hat{G}$ only" strategy by banks is an equilibrium strategy.*

Subcases (1-b) and (1-d) Bank's Problem In subcase (1-b), $\alpha f(1-f)(G-B) > 1$, and $\Delta < 0$, bank i offers the "lend to $\hat{G}$ only" strategy in equilibrium. The deviant bank h offers the "lend to both" strategy to give firms the highest expected profit in a deviation among all possible deviation offers. In this case, we have the IC constraint (82).

However, under the "lend to $\hat{G}$ only" strategy, firms at most receive the following expected profit, derived as the same way as for the first line of (79), which is

$$f\left[G - R - \frac{1}{\alpha f}\ln[\alpha f(1-f)(G-B)] - \frac{1}{\alpha f}\right].$$

Under the "lend to both" strategy, firms receive the third line of (79). By our current assumption here $\Delta < 0$,

$$f\left[G - R - \frac{1}{\alpha f}\ln[\alpha f(1-f)(G-B)] - \frac{1}{\alpha f}\right] < fG + (1-f)B - R.$$

Therefore, in this parametric values case, bank i does not take the "lend to $\hat{G}$ only" strategy in an equilibrium. In other words, this case cannot be taken in an equilibrium.

In subcase (1-d), $\alpha f(1-f)(G-B) \leq 1$, bank i offers the "lend to $\hat{G}$ only" strategy in equilibrium.

The deviant bank h offers the "lend to both" strategy to give firms the highest expected profit in a deviation among all possible deviation offers. In this case, we have the IC constraint (83).

However, under the "lend to $\hat{G}$ only" strategy, firms at most receive the following expected profit, derived as the same way as for the second line of (79), which is

$$f[fG + (1-f)B - R].$$

Under the "lend to both" strategy, firms receive the third line of (79). We have

$$f[fG + (1-f)B - R] < fG + (1-f)B - R.$$

Therefore, in this parametric values case, bank i does not take the "lend to $\hat{G}$ only" strategy in an equilibrium. In other words, this case cannot be taken in an equilibrium.

In summary, for subcases (1-b) and (1-d) , we have the following result.

Lemma 3. *When $\alpha f(1-f)(G-B) > 1$ and $\Delta < 0$, or when $\alpha f(1-f)(G-B) \leq 1$, the "lend to $\hat{G}$ only" strategy cannot be an equilibrium strategy.*

D.2 Stage 1 Case (2)

Suppose the "lend to $\hat{B}$ only" ($\hat{\mu} = 0$) is the equilibrium lending strategy in stage 1 before observing signals. Bank i offers the "lend to $\hat{B}$ only" strategy, then receive loan applicants, and screens the applicant firms. If bank i obtains $\hat{B}$ signal for firm j, then bank i would provide the loan to firm j. With the $\hat{G}$ signal, bank i would not provide the loan to firm j.

The bank optimal problem per firm in stage 1 is defined as below.

(88)

$$\max_{x_{ij}\in\mathbf{S_2}} E\left[\pi^b(x_{ij}|x_{-i})\right] = Pr(\hat{B})\left[Pr(G|\hat{B},C)R_{\hat{B}} + Pr(B|\hat{B},C)B - R - \frac{C}{1-f}\right] \quad \text{[Bank profit]}$$

(89)

$s.t.$

(90)

$$E\left[\pi^f\left(x_{ij}\right)\right] = Pr(G)Pr(\hat{B}|G,C)(G-R_{\hat{B}}) \geq 0 \quad \text{[Firm PC]}$$

(91)

$$E\left[\pi^f\left(x_{ij}\right)\right] = Pr(G)Pr(\hat{B}|G,C)(G-R_{\hat{B}}) \geq E\left[\pi^f(\tilde{x}_{hj})\right], \text{ for } \forall h \neq i, \forall \tilde{x}_{hj} \in x_j \quad \text{[Firm IC]}$$

(92)

$$E\left[\pi^b(x_{ij}|x_{-i})\right] = Pr(\hat{B})\left[Pr(G|\hat{B},C)R_{\hat{B}} + Pr(B|\hat{B},C)B - R - \frac{C}{1-f}\right] = 0 \quad \text{[Bank ZPC]}$$

Now, we prove that this case cannot be taken in an equilibrium.

Under the "lend to $\hat{B}$ only" strategy, firms can receive at most the following expected profit, derived as the same way as for the third line of (78), which is

$$f[fG + (1-f)B - R].$$

Suppose the deviant bank h offers the "lend to both" strategy, then firms can receive at most the fourth line of (78). We have

$$f[fG + (1-f)B - R] < fG + (1-f)B - R.$$

Therefore, in this parametric values case, bank i does not take the "lend to $\hat{B}$ only" strategy in an equilibrium. In other words, this case cannot be taken in an equilibrium. We have the following lemma.

In summary, for stage 1 case (2), we have the following result.

Lemma 4. *The "lend to $\hat{B}$ only" strategy cannot be an equilibrium strategy, under any parameter values.*

D.3 Stage 1 Case (3)

Suppose the "lend to both with no cross subsidy" ($\hat{\mu} \in (0,1)$, $R_{\hat{B}} > B$), is the equilibrium lending strategy in stage 1 before observing signals. Bank i offers the lending strategy "lend to both with no cross subsidy", then receives loan applicants, and screens the applicant firms. If bank i obtains $\hat{G}$ signal for firm j, then bank i would provide the loan rate $R_{\hat{G}}$ to firm j. With $\hat{B}$ signal, bank i would provide the loan rate $R_{\hat{B}}$ to firm j.

The bank optimal problem per firm is defined as below.

$$\max_{x_{ij} \in \mathbf{S_3}} E\left[\pi^b(x_{ij}|x_{-i})\right] = Pr(\hat{G})\left[Pr(G|\hat{G},C)R_{\hat{G}} + Pr(B|\hat{G},C)B\right] + Pr(\hat{B})\left[Pr(G|\hat{B},C)R_{\hat{B}} + \right.$$

(93)

$$\left. Pr(B|\hat{B},C)B\right] - R - C \qquad \text{[Bank profit]}$$

s.t.

(94)

$$E\left[\pi^f(x_{ij})\right] = Pr(G)\left[Pr(\hat{G}|G,C)(G - R_{\hat{G}}) + Pr(\hat{B}|G,C)(G - R_{\hat{B}})\right] \geq 0 \qquad \text{[Firm PC]}$$

$$E\left[\pi^f(x_{ij})\right] = Pr(G)\left[Pr(\hat{G}|G,C)(G - R_{\hat{G}}) + Pr(\hat{B}|G,C)(G - R_{\hat{B}})\right] \geq E\left[\pi^f(\tilde{x}_{hj})\right],$$

(95)

$$\text{for } \forall h \neq i, \forall \tilde{x}_{hj} \in x_j \qquad \text{[Firm IC]}$$

$$E\left[\pi^b(x_{ij}|x_{-i})\right] = Pr(\hat{G})\left[Pr(G|\hat{G},C)R_{\hat{G}} + Pr(B|\hat{G},C)B\right] + Pr(\hat{B})\left[Pr(G|\hat{B},C)R_{\hat{B}} + \right.$$

(96)

$$\left. Pr(B|\hat{B},C)B\right] - R - C = 0 \qquad \text{[Bank ZPC]}$$

Subcases of Case (3) IC constraint By the same argument to derive (79), firm j's IC constraint (95) can be expressed as below:

$$
\begin{aligned}
&Pr(G)\left[Pr(\hat{G}|G,C)(G-R_{\hat{G}})+Pr(\hat{B}|G,C)(G-R_{\hat{B}})\right]\\
&\geq \max\left\{\left\{f\left[G-R-\frac{1}{\alpha f}\ln[\alpha f(1-f)(G-B)]-\frac{1}{\alpha f}\right]\Bigg|_{\alpha f(1-f)(G-B)>1},\right.\right.\\
&\left. f[fG+(1-f)B-R]\Bigg|_{\alpha f(1-f)(G-B)\leq 1}\right\},\\
&\left. fG+(1-f)B-R.\right\}. \qquad (97)
\end{aligned}
$$

Then, firm j's IC constraint (97) can be simplified into four cases as below.

Subcase (3-a): IC constraint: When $\alpha f(1-f)(G-B)>1$, and $\Delta>0$, the "lend to $\hat{G}$ only" contract provides firm j the highest expected profit, compared to contracts under other lending strategies. Hence, while bank i chooses the "lend to both with no cross subsidy" strategy, the deviant bank h may choose the "lend to $\hat{G}$ only" strategy, and firm j's IC constraint (97) becomes

(98)
$$
Pr(G)\left[Pr(\hat{G}|G,C)(G-R_{\hat{G}})+Pr(\hat{B}|G,C)(G-R_{\hat{B}})\right]\geq f\left[G-R-\frac{1}{\alpha}ln[\alpha(1-f)(G-B)]-\frac{1}{\alpha}\right].
$$

Subcase (3-b): IC constraint: When $\alpha f(1-f)(G-B)>1$, and $\Delta<0$, the "lend to both" contract provides firm j the highest expected profit, compared to contracts under other lending strategies. Firm j's IC constraint (97) becomes

$$
Pr(G)\left[Pr(\hat{G}|G,C)(G-R_{\hat{G}})+Pr(\hat{B}|G,C)(G-R_{\hat{B}})\right]\geq fG+(1-f)B-R. \qquad (99)
$$

Subcase (3-c): IC constraint: When $\alpha f(1-f)(G-B)>1$, and $\Delta=0$, the "lend to $\hat{G}$ only" contract and the "lend to both" contract can provide the same highest firm's expected profit.

Under this case, there could exist a mixed strategy equilibrium. Since we focus on pure strategy equilibrium, we do not go deep into this case, and we assume all banks choose the same pure strategy. Without loss of generality, case (3-c) is combined with case (3-b). Hereinafter, we do not construct this case separately.

Subcase (3-d): IC constraint: When $\alpha(1-f)(G-B) \leq 1$, the "lend to both" contract provides firm j with the highest expected profit compared to contracts under other lending strategies. Firm j's IC constraint (97) becomes

$$Pr(G)\left[Pr(\hat{G}|G,C)(G-R_{\hat{G}})+Pr(\hat{B}|G,C)(G-R_{\hat{B}})\right] \geq fG+(1-f)B-R. \tag{100}$$

This is the same constraint as in case (3-b), (99). Next, we incorporate subcases of IC constraint, (3-a), (3-b), and (3-d) into bank's optimal problem.

Subcases of Case (3) Bank's Problem

Subcase (3-a) Bank's Problem In case (3-a), $\alpha f(1-f)(G-B) > 1$ and $\Delta > 0$, bank i offers the "lend to both with no cross subsidy" strategy in equilibrium. The deviant bank h offers the "lend to $\hat{G}$ only" strategy to give firms the highest expected profit in a deviation among all possible deviation offers. In this case, we have the IC constraint (98).

However, under the "lend to both with no cross subsidy" strategy, firms at most receive the following expected profit, derived in the same way as for the third line of (79), which is

$$fG+(1-f)B-R.$$

Under the "lend to $\hat{G}$ only" strategy, firms receive the first line of (79). By our current assumption here $\Delta > 0$,

$$f\left[G - R - \frac{1}{\alpha f}\ln[\alpha f(1-f)(G-B)] - \frac{1}{\alpha f}\right] > fG + (1-f)B - R.$$

Therefore, in this parametric values case, bank i does not take the "lend to both with no cross subsidy" strategy in an equilibrium. In other words, this case cannot be taken in an equilibrium.

Subcases (3-b) and (3-d) Bank's Problem In subcases (3-b) and (3-d), firm's IC constraint is (109). Here, the bank i's optimal problem is simplified as below:

$$\max_{x_{ij}\in \mathbf{S_3}} f[f_G(C)R_{\hat{G}} + (1-f_G(C))B] + (1-f)[f_B(C)B + (1-f_B(C))R_{\hat{B}}] - R - C \quad \text{[Bank profit]}$$

$s.t.$

$$f\left[f_G(C)\left(G - R_{\hat{G}}\right) + (1-f_G(C))\left(G - R_{\hat{B}}\right)\right] \geq 0 \quad \text{[Firm PC]}$$

$$f\left[f_G(C)\left(G - R_{\hat{G}}\right) + (1-f_G(C))\left(G - R_{\hat{B}}\right)\right] \geq fG + (1-f)B - R \quad \text{[Firm IC]}$$

$$f[f_G(C)R_{\hat{G}} + (1-f_G(C))B] + (1-f)[f_B(C)B + (1-f_B(C))R_{\hat{B}}] - R - C = 0 \quad \text{[Bank ZPC]}$$

Recall that $Pr(\hat{G}|G,C) = Pr(G|\hat{G},C) = f_G(C) = 1-(1-f)e^{-\alpha C}$, $Pr(\hat{B}|B,C) = Pr(B|\hat{B},C) = f_B(C) = 1-fe^{-\alpha C}$. Then, we have $f_B(C) = 1-f\frac{1-f_G(C)}{1-f}$. By banks' zero-profit constraint, $R_{\hat{G}}, R_{\hat{B}}$ are functions of C, as following:

$$R_{\hat{G}} \equiv g(C) = \frac{R + C - (1-f)B}{f f_G(C)} - \frac{1-f_G(C)}{f_G(C)}b(C).$$
$$R_{\hat{B}} \equiv b(C) = \frac{R + C - (1-f)B}{f(1-f_G(C))} - \frac{f_G(C)}{1-f_G(C)}g(C).$$

We substitute $R_{\hat{G}}, R_{\hat{B}}$ to the firm's IC constraint as below:

$$fG + (1-f)B - R - C \geq fG + (1-f)B - R,$$

which can be further simplified as

$$C \leq 0.$$

There are two subcases. First, the RHS of firm's IC constraint is less than zero. We can show that firm's IR constraint is violated, as in case (1-a). No firms enter in stage 1, and, hence, there is no equilibrium in this case. Second, the RHS of firm's IC constraint is non-negative, which indicates that $fG + (1-f)B \geq R$. Given that the screening cost is non-negative, we have the optimal screening cost $C^* = 0$.

In the equilibrium, $C^* = 0$. Bank ZPC means that

$$fR^*_{\hat{G}} + (1-f)R^*_{\hat{B}} = \frac{R - (1-f)B}{f} \in [R, G]. \tag{101}$$

Note that a firm chooses one bank in stage 2 and cannot switch to other banks (loan contracts) in later stages, implying that a firm only values the expected profit *ex ante*, that is, the expected repayment in stage 2. Here, $R^*_{\hat{G}}$ and $R^*_{\hat{B}}$ may be different, but it does not affect firm's strategy in stage 2 as long as satisfying (101), because firms do not know whether they will get $\hat{G}$ or $\hat{B}$ signals. The firm's expected profit in stage 2 as below.

$$\begin{aligned} E\left[\pi^f(x^*_{ij})\right] &= Pr(G)\left[Pr(\hat{G}|G,C)(G - R^*_{\hat{G}}) + Pr(\hat{B}|G,C)(G - R^*_{\hat{B}})\right] \\ &= f\left[f(G - R^*_{\hat{G}}) + (1-f)(G - R^*_{\hat{B}})\right] \\ &= f\left[G - \left(fR^*_{\hat{G}} + (1-f)R^*_{\hat{B}}\right)\right] \end{aligned} \tag{102}$$

Here, the remaining question is whether banks want to offer different $R^*_{\hat{G}}$ and $R^*_{\hat{B}}$. Remember that under $C^* = 0$, $R^*_{\hat{G}}$ and $R^*_{\hat{B}}$ do not convey any information, and firms are indifferent between $R^*_{\hat{G}}$ and $R^*_{\hat{B}}$ as long as (101) is satisfied. This means banks are also indifferent between offering symmetric strategy and offering asymmetric strategies.

In summary, when $\alpha f(1-f)(G-B) > 1$, $\Delta \leq 0$, and $fG + (1-f)B \geq R$; or when $\alpha f(1-f)(G-B) \leq 1$ and $fG + (1-f)B \geq R$,[38] the "lend to both with no cross subsidy" strategy by banks is the equilibrium strategy of a set of non-screening, cheap-information-based equilibria. The equilibrium contract uses the "lend to both with no cross subsidy" strategy, and the screening strategy and the required repayment is characterized in the loan contract x_{ij}^* , where

$$C^* = 0.$$

$$fR_{\hat{G}}^* + (1-f)R_{\hat{B}}^* = \frac{R-(1-f)B}{f}.$$

In particular, there is a non-screening pooling equilibrium contract, in which

$$R_{\hat{G}}^* = R_{\hat{B}}^* \equiv R_p^* = \frac{R-(1-f)B}{f} > R.$$

Because $R_p^* \geq R \geq B$, the assumption of no cross subsidy holds.

In summary, for stage 1 case (3), we have the following result.

Lemma 5. *When $\alpha(1-f)(G-B) > 1$, $\Delta < 0$, and $fG+(1-f)B \geq R$, or when $\alpha(1-f)(G-B) \leq 1$ and $fG+(1-f)B \geq R$, the "lend to both with no cross subsidy" strategy by banks is an equilibrium strategy.*

D.4 Stage 1 Case (4)

Suppose the "lend to both with a cross subsidy" ($\hat{\mu} \in (0,1)$, $R_{\hat{B}} \leq B$), is the equilibrium lending strategy in stage 1 before observing signals. Bank i provides the "lend to both with a cross subsidy" strategy, then receives loan applicants and screens the applicant firms. If bank i obtains $\hat{G}$ signal for firm j, then bank i would provide the loan rate $R_{\hat{G}}$ to firm j. With $\hat{B}$ signal, bank i would provide the loan rate $R_{\hat{B}}$ to firm j.

[38]This condition is from firm's IR constraint. $E\left[\pi^f\left(x_{ij}\right)\right]$ represents firm's expected profit before screening in stage 2, which is non-negative by firm's participation constraint. Otherwise, firms will not enter the market in the first place. By (101) and (102), $E\left[\pi^f\left(x_{ij}^*\right)\right] = fG + (1-f)B - R \geq 0$.

The bank optimal problem per firm is defined as below.

(103)

$$\max_{x_{ij}\in\mathbf{S_4}} E\left[\pi^b(x_{ij}|x_{-i})\right] = Pr(\hat{G})\left[Pr(G|\hat{G},C)R_{\hat{G}} + Pr(B|\hat{G},C)B\right] + Pr(\hat{B})R_{\hat{B}} - R - C \quad \text{[Bank profit]}$$

$s.t.$

$$E\left[\pi^f(x_{ij})\right] = Pr(G)\left[Pr(\hat{G}|G,C)(G-R_{\hat{G}}) + Pr(\hat{B}|G,C)(G-R_{\hat{B}})\right]$$

(104)

$$+\,Pr(B)\left[Pr(\hat{B}|B,C)(B-R_{\hat{B}})\right] \geq 0 \qquad \text{[Firm PC]}$$

$$E\left[\pi^f(x_{ij})\right] = Pr(G)\left[Pr(\hat{G}|G,C)(G-R_{\hat{G}}) + Pr(\hat{B}|G,C)(G-R_{\hat{B}})\right]$$

(105)

$$+\,Pr(B)\left[Pr(\hat{B}|B,C)(B-R_{\hat{B}})]\right] \geq E\left[\pi^f(\tilde{x}_{hj})\right],\ \text{for } \forall h\neq i, \forall \tilde{x}_{hj}\in x_j \quad \text{[Firm IC]}$$

(106)

$$E\left[\pi^b(x_{ij}|x_{-i})\right] = Pr(\hat{G})\left[Pr(G|\hat{G},C)R_{\hat{G}} + Pr(B|\hat{G},C)B\right] + \Pr(\hat{B})R_{\hat{B}} - R - C = 0 \quad \text{[Bank ZPC]}$$

Subcases of Case (4) IC constraint By the same argument used to derive (79), firm's IC constraint (105) can be expressed as below:

$$\begin{aligned}
&Pr(G)[Pr(\hat{G}|G,C)(G-R_{\hat{G}}) + Pr(\hat{B}|G,C)(G-R_{\hat{B}})] + Pr(B)[Pr(\hat{B}|B,C)(B-R_{\hat{B}})] \\
&\geq \max\left\{\left\{f\left[G-R-\frac{1}{\alpha f}\ln[\alpha f(1-f)(G-B)] - \frac{1}{\alpha f}\right]\right|_{\alpha f(1-f)(G-B)>1}, \right. \\
&\left. f[fG+(1-f)B-R]\right|_{\alpha f(1-f)(G-B)\leq 1}\Bigg\}, \\
&\left. fG+(1-f)B-R.\right\}. \qquad (107)
\end{aligned}$$

Then, firm j's IC constraint (107) can be simplified into four cases as below.

Subcase (4-a): IC constraint: When $\alpha f(1-f)(G-B) > 1$ and $\Delta > 0$, the "lend to $\hat{G}$ only" contract provides firm j the highest expected profit, compared to contracts under other lending strategies. Hence, while bank i chooses the "lend to both with a cross subsidy" strategy, the deviant bank h may choose the "lend to $\hat{G}$ only" strategy, and the firm j's IC constraint (107) becomes

$$\begin{aligned}&Pr(G)\left[Pr(\hat{G}|G,C)(G-R_{\hat{G}})+Pr(\hat{B}|G,C)(G-R_{\hat{B}})\right]\\&+Pr(B)\left[Pr(\hat{B}|B,C)(B-R_{\hat{B}})\right]\geq f\left[G-R-\frac{1}{\alpha}ln[\alpha(1-f)(G-B)]-\frac{1}{\alpha}\right].\end{aligned}\tag{108}$$

Subcase (4-b): IC constraint: When $\alpha f(1-f)(G-B) > 1$ and $\Delta < 0$, the "lend to both" contract provides firm j with the highest expected profit compared to contracts under other lending strategies, and firm j's IC constraint (107) becomes

$$\begin{aligned}&Pr(G)\left[Pr(\hat{G}|G,C)(G-R_{\hat{G}})+Pr(\hat{B}|G,C)(G-R_{\hat{B}})\right]\\&+Pr(B)\left[Pr(\hat{B}|B,C)(B-R_{\hat{B}})\right]\geq fG+(1-f)B-R.\end{aligned}\tag{109}$$

Subcase (4-c): IC constraint: When $\alpha f(1-f)(G-B) > 1$ and $\Delta = 0$, the "lend to $\hat{G}$ only" contract and the "lend to both" contract can provide the same highest firm's expected profit. Under this case, there could be a mixed strategy equilibrium appear. Since we focus on pure strategy equilibrium, we do not go deep into this case, and we assume all banks choose the same pure strategy. Without loss of generality, case (4-c) is combined with case (4-b). Hereinafter, we do not construct this case separately.

Subcase (4-d): IC constraint: When $\alpha(1-f)(G-B) \leq 1$, the "lend to both" contract provides firm j with the highest expected profit compared to contracts under other lending strategies, and the firm j's IC constraint (107) becomes

$$\begin{aligned} & Pr(G)\left[Pr(\hat{G}|G,C)(G-R_{\hat{G}})+Pr(\hat{B}|G,C)(G-R_{\hat{B}})\right] \\ & + Pr(B)\left[Pr(\hat{B}|B,C)(B-R_{\hat{B}})\right] \geq fG+(1-f)B-R. \end{aligned} \tag{110}$$

This is the same constraint as in case (4-b) (109).

Next, we incorporate subcases of IC constraint, (4-a), (4-b), and (4-d) into the bank's optimal problem.

Subcases of Case (4) Bank's Problem

Subcase (4-a) Bank's Problem In subcase (4-a), $\alpha f(1-f)(G-B) > 1$ and $\Delta > 0$, bank i offers the "lend to both with a cross subsidy" in equilibrium. The deviant bank h offers the "lend to $\hat{G}$ only" strategy to give firms the highest expected profit in a deviation among all possible deviation offers. In this case, we have the IC constraint (108).

However, under the "lend to both with a cross subsidy" strategy, firms at most receive the following expected profit, derived in the same way as for the third line of (79), which is

$$fG+(1-f)B-R.$$

Under the "lend to $\hat{G}$ only" strategy, firms receive the first line of (79). By our current assumption here $\Delta > 0$,

$$f\left[G-R-\frac{1}{\alpha f}\ln[\alpha f(1-f)(G-B)]-\frac{1}{\alpha f}\right] > fG+(1-f)B-R.$$

Therefore, in this parametric values case, bank i does not take the "lend to both with a cross subsidy" strategy in an equilibrium. In other words, this case cannot be taken in an equilibrium.

Subcases (4-b) and (4-d) Bank's Problem In subcases (4-b) and (4-d), firm's IC constraint is (109). Here, the bank i's optimal problem is simplified as below:

$$\max_{x_{ij}\in\mathbf{S_4}} f[f_G(C)R_{\hat{G}} + (1-f_G(C))B] + (1-f)R_{\hat{B}} - R - C \quad \text{[Bank profit]}$$

$s.t.$

$$f[G - f_G(C)R_{\hat{G}} - (1-f_G(C))R_{\hat{B}}] + (1-f)f_B(C)(B - R_{\hat{B}}) \geq 0 \quad \text{[Firm PC]}$$

$$f[G - f_G(C)R_{\hat{G}} - (1-f_G(C))R_{\hat{B}}] + (1-f)f_B(C)(B - R_{\hat{B}}) \geq fG + (1-f)B - R \quad \text{[Firm IC]}$$

$$f[f_G(C)R_{\hat{G}} + (1-f_G(C))B] + (1-f)R_{\hat{B}} - R - C = 0 \quad \text{[Bank ZPC]}$$

By banks' zero-profit constraint, $R_{\hat{G}}, R_{\hat{B}}$ are functions of C as following:

$$R_{\hat{G}} \equiv h(C) = \frac{R + C - f(1-f_G(C))B}{ff_G(C)} - \frac{1-f}{ff_G(C)}l(C),$$
$$R_{\hat{B}} \equiv l(C) = \frac{R + C - f(1-f_G(C))B}{1-f} - \frac{ff_G(C)}{1-f}h(C).$$

Recall that $Pr(\hat{G}|G,C) = Pr(G|\hat{G},C) = f_G(C) = 1-(1-f)e^{-\alpha C}$, $Pr(\hat{B}|B,C) = Pr(B|\hat{B},C) = f_B(C) = 1 - fe^{-\alpha C}$. Then, we have $f_B(C) = 1 - f\frac{1-f_G(C)}{1-f}$. We substitute $R_{\hat{G}}, R_{\hat{B}}, f_B(C)$ to the IC constraint and it becomes

$$fG + (1-f)B - R - C \geq fG + (1-f)B - R,$$

which can be further simplified as

$$C \leq 0.$$

There are two subcases. First, the RHS of firm's IC constraint is less than zero. We can show that firm's IR constraint is violated, as in case (1-a). No firms enter in stage 1, hence, there is no equilibrium in this case. Second, the RHS of firm's IC constraint is non-negative, which indicates that $fG + (1-f)B \geq R$. Given that the screening cost is non-negative, we have the optimal screening cost $C^* = 0$.

In the equilibrium, $C^* = 0$. Bank ZPC means that

$$f^2 R^*_{\hat{G}} + (1-f)R^*_{\hat{B}} = R - (1-f)fB \tag{111}$$

Note that a firm chooses one bank in stage 2, and cannot switch to other banks (loan contracts) in later stages, implying that a firm only values the expected profit *ex ante*, that is, the expected repayment in stage 2. Here, $R^*_{\hat{G}}$ and $R^*_{\hat{B}}$ may be different, but it does not affect firm's strategy in stage 2 as long as satisfying (111), because firms do not know whether they will get $\hat{G}$ or $\hat{B}$ signals. The firm's equilibrium expected profit in stage 2 is as below.

$$\begin{aligned}
E\left[\pi^f(x^*_{ij})\right] =& Pr(G)\left[Pr(\hat{G}|G,C)(G - R^*_{\hat{G}}) + Pr(\hat{B}|G,C)(G - R^*_{\hat{B}})\right] \\
&+ Pr(B)\left[Pr(\hat{B}|B,C)(B - R^*_{\hat{B}})\right] \\
=& f\left[f(G - R^*_{\hat{G}}) + (1-f)(G - R^*_{\hat{B}})\right] + (1-f)^2(B - R^*_{\hat{B}}) \\
=& fG + (1-f)^2 B - [f^2 R^*_{\hat{G}} + (1-f)R^*_{\hat{B}}] \qquad (112)
\end{aligned}$$

Here, the remaining question is whether banks want to offer different $R^*_{\hat{G}}$ and $R^*_{\hat{B}}$. Remember that under $C^* = 0$, $R^*_{\hat{G}}$ and $R^*_{\hat{B}}$ do not convey any information, and firms are indifferent between $R^*_{\hat{G}}$ and $R^*_{\hat{B}}$, as long as (111) is satisfied. This means banks are also indifferent between offering pooling strategy and offering cheap information strategies.

In summary, when $\alpha f(1-f)(G-B) > 1$, $\Delta \leq 0$, and $fG + (1-f)B \geq R$, or when $\alpha f(1-f)(G-B) \leq 1$ and $fG + (1-f)B \geq R$,[39] the "lend to both with a cross subsidy" strategy by banks is the equilibrium strategy of a set of cheap information non-screening equilibria. The equilibrium contract uses the "lend to both with a cross subsidy" strategy, and the screening strategy and the

[39] This condition is from firm's PC constraint. $E\left[\pi^f\left(x_{ij}\right)\right]$ represents firm's expected profit before screening in stage 2, which is non-negative by firm's participation constraint. Otherwise, firms will not enter the market in the first place. By (111) and (112), $E\left[\pi^f\left(x^*_{ij}\right)\right] = fG + (1-f)B - R \geq 0$.

required repayment is characterized in the loan contract x_{ij}^* , where

$$C^* = 0.$$
$$fR_{\hat{G}}^* + \frac{1-f}{f}R_{\hat{B}}^* = \frac{R-(1-f)fB}{f}.$$

Let $R_{\hat{G}}^* = R_{\hat{B}}^* \equiv R_p^*$, by (111), we have $R_{\hat{G}}^* = R_{\hat{B}}^* \equiv R_p^* = \frac{R-(1-f)fB}{1-(1-f)f} > \frac{B-(1-f)fB}{1-(1-f)f} = B$. Since $R_p^* \geq B$, the assumption of cross subsidy does not hold. Under "lend to both with a cross subsidy" strategy, there is no pooling Nash equilibrium.

In summary, for stage 1 case (4), we have the following result.

Lemma 6. *When $\alpha(1-f)(G-B) > 1$, $\Delta < 0$, and $fG+(1-f)B \geq R$, or when $\alpha(1-f)(G-B) \leq 1$ and $fG+(1-f)B \geq R$, the "lend to both with a cross subsidy" strategy by banks is an equilibrium strategy.*

E Proof of Proposition 2

For the case of screening strategy, $z_j = (Pr_j(G|\hat{G}, C_j), L_{\hat{G},j}, L_{\hat{B},j}) \in [f, 1) \times \{1\} \times \{0\}$. We consider the "lend to $\hat{G}$ only" strategy. Denote economy-wide net output as W_s.

$$\max_{\{z_j\}_{j=1}^m} \quad W_s \equiv \sum_{j=1}^{m} Pr(\hat{G})[Pr(G|\hat{G}, C_j)G + Pr(B|\hat{G}, C_j)B - R] - C_j$$
$$s.t. \quad Pr(\hat{G})[Pr(\hat{G})[Pr(G|\hat{G}, C_j)G + Pr(B|\hat{G}, C_j)B - R] - C_j \geq 0, \text{ for } \forall j \in \{1, 2, ..., m\}$$

The Lagrangian is as below.

$$\mathcal{L} = \sum_{j=1}^{m} \Big\{Pr(\hat{G})[Pr(G|\hat{G}, C_j)G + Pr(B|\hat{G}, C_j)B - R] - C_j\Big\} - \sum_{j=1}^{m} \lambda_j \Big\{Pr(\hat{G})[Pr(\hat{G})[Pr(G|\hat{G}, C_j)G + Pr(B|\hat{G}, C_j)B - R] - C_j\Big\}$$

By $Pr(G|\hat{G}, C_j) = f_G(C_j)$, the Lagrangian can be rewritten as below.

$$\mathcal{L} = \sum_{j=1}^{m} \{f[f_G(C_j)G + (1 - f_G(C_j))B - R] - C_j\} - \sum_{j=1}^{m} \lambda_j \{f[f_G(C_j)G + (1 - f_G(C_j))B - R] - C_j\}$$

The first order conditions are for $\forall j \in \{1, 2, ..., m\}$,

$$\frac{\partial \mathcal{L}}{\partial C_j} = (1 + \lambda_j)(f f'_G(C_j)(G - B) - 1) = 0.$$

Firms are identical, thus, $C_1 = C_2 = ... = C_m \equiv C$. The first order conditions imply $f f'_G(C^*)(G - B) - 1 = 0$. Recall that in the decentralized baseline, the first order condition in the screening equilibrium is $f'_G(C^*)(G - B) - 1/f = 0$. Therefore, the optimal screening cost of the screening strategy in the social planner's problem is the same as the optimal screening cost in the screening equilibrium in the decentralised economy. The optimal overall output is

$$W_s^* = \Big[fG - \frac{1}{\alpha} - \frac{1}{\alpha} ln(\alpha f(1 - f)(G - B))\Big]m.$$

For the case of non-screening strategy, $z_j = (Pr_j(G|\hat{G}, C), L_{\hat{G},j}, L_{\hat{B},j}) \in [f, 1) \times \{1\} \times \{1\}$. Denote economy-wide net output as W_p.

$$\begin{aligned}
\max_{\{z_j\}_{j=1}^m} \quad & W_p \equiv \sum_{j=1}^{m} Pr(\hat{G})[Pr(G|\hat{G}, C_j)G + Pr(B|\hat{G}, C_j)B] + Pr(\hat{B})[Pr(G|\hat{B}, C_j)G + Pr(B|\hat{B}, C_j)B] \\
& - R - C_j \\
s.t. \quad & Pr(\hat{G})[Pr(G|\hat{G}, C_j)G + Pr(B|\hat{G}, C_j)B] + Pr(\hat{B})[Pr(G|\hat{B}, C_j)G + Pr(B|\hat{B}, C_j)B] \\
& - R - C_j \geq 0, \text{ for } \forall j \in \{1, 2, ..., m\}
\end{aligned}$$

The Lagrangian is as below.

$$\mathcal{L} =$$
$$\sum_{j=1}^{m} \Big\{ Pr(\hat{G})[Pr(G|\hat{G},C_j)G + Pr(B|\hat{G},C_j)B] + Pr(\hat{B})[Pr(G|\hat{B},C_j)G + Pr(B|\hat{B},C_j)B] - R - C_j \Big\} -$$
$$\sum_{j=1}^{m} \lambda_j \Big\{ Pr(\hat{G})[Pr(G|\hat{G},C_j)G + Pr(B|\hat{G},C_j)B] + Pr(\hat{B})[Pr(G|\hat{B},C_j)G + Pr(B|\hat{B},C_j)B] - R - C_j \Big\}$$

where $Pr(\hat{B})Pr(G|\hat{B},C_j) = Pr(\hat{B})\frac{Pr(\hat{B}|G,C_j)Pr(G)}{Pr(\hat{B})} = Pr(\hat{B}|G,C_j)Pr(G) = Pr(\hat{G})Pr(B|\hat{G},C_j)$. By Simplifying the Lagrangian, we have

$$\mathcal{L} = \sum_{j=1}^{m}[fG + (1-f)B - R - C_j] - \sum_{j=1}^{m} \lambda_j[fG + (1-f)B - R - C_j]$$

The first order conditions are for $\forall j \in \{1, 2, ..., m\}$,

$$\frac{\partial \mathcal{L}}{\partial C_j} = -(1 + \lambda_j) < 0.$$

Firms are identical. $C_1 = C_2 = ... = C_m \equiv C$. The first order conditions imply $C^* = 0$, So the optimal screening cost of the non-screening strategy in the social planner's problem is the same as the optimal screening cost in the non-screening equilibrium in the decentralised economy. The optimal overall output is

$$W_p^* = [fG + (1-f)B - R]m.$$

The condition of switching between the screening equilibrium and the non-screening equilibrium depends on Δ_w, is consistent with Δ in the decentralized baseline, as shown below. When $\Delta_w > 0$, the optimal solution of the social planner's problem is the same as the screening equilibrium. When $\Delta_w \leq 0$, the optimal solution of the social planner's problem is the same as the non-screening equilibrium.

$$
\begin{aligned}
\Delta_w =& W_s - W_p \\
=& \left[fG - \frac{1}{\alpha} - \frac{1}{\alpha} ln(\alpha f(1-f)(G-B)) \right] m \\
& - [fG + (1-f)B - R]m \\
=& m\Delta
\end{aligned}
$$